

Debt, Equity, and Taxes

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Debt, Equity, and Taxes

Abstract

In this study, we extend Miller's (1977) capital structure analysis by adding potentially high personal taxes on dividends and share repurchases, and by focusing on mature firms with at least some pre-existing equity. We demonstrate that personal taxes on equity distributions push new equity financing to an inferior corner, but they do not push internal equity (i.e., reinvested free cash flows) to a corner. Therefore an interior capital structure solution remains, in which firms are indifferent between using debt and internal equity financing, while preferring to avoid issuing additional external equity. Interestingly, many attributes of Miller's model survive high personal taxes on equity distributions, including the aggregate debt-equity ratio, the identity of the marginal investment clientele, and investors' portfolio allocations between debt and equity securities. Nevertheless, many unique insights also arise.

Debt, Equity, and Taxes

In 1977, Miller laid out a basic paradigm of debt and taxes that continues to frame much of the capital structure debate today. Miller treats the capital structure decision as a simple choice between issuing debt versus external equity to finance investment. If the firm issues debt, the firm pays out tax-deductible interest to bondholders, avoiding the corporate tax but subjecting investors to heavy personal taxes. If the firm issues equity, the firm uses non-deductible dividends or share repurchases to distribute profits, incurring the corporate tax but only subjecting investors to personal dividend or capital gains taxes. Miller assumes away much of these personal taxes on dividends and capital gains, arguing firms can defer taxable equity distributions indefinitely and/or there is a very low tax rate on distributions. As a result, the marginal investment clientele faces much higher personal taxes on debt than on equity, which offsets the corporate tax advantage of debt. An interior capital structure equilibrium results, in which firms and the marginal investor are indifferent between debt and equity financing.

In practice, taxable distributions to shareholders are common. Fama and French (2001) document that the percent of dividend-paying firms has declined substantially over time, from 52.8 percent in 1973 to 20.8 percent in 1999. Nevertheless, aggregate dividends (gross share repurchases) as a percent of aggregate earnings actually increased from 34.0 percent (3.4 percent) in 1973-1977 to 39.3 percent (35.5 percent) in 1993-1998.¹ In addition, the amount of taxable dividends reported on individual income tax returns increased from \$38.8 billion in 1980 to \$132.5 billion in 1999, and the amount of

¹ The increase in aggregate dividends from 1973-1977 to 1993-1998 period is not monotonic. Specifically, the aggregate dividend percentage reached a peak of 56.9 percent in 1988-1992 before dropping back to 39.3 percent in 1993-1998.

taxable net capital gains from all sources (including share repurchases) increased from \$32.7 billion in 1980 to \$552.6 billion in 1999 (*IRS Statistics of Income*). By largely assuming away dividend and capital gains taxes on distributions, therefore, Miller appears to understate the personal tax on equity (Green and Hollifield, 2001).

In this study, we extend Miller's analysis by adding potentially high personal distribution taxes. In addition, we focus on mature firms with at least some pre-existing equity. To alter pre-existing capital structure, mature firms can not only issue new equity and pay off debt, but they also can issue debt and distribute cash to shareholders via dividends or share repurchases, which trigger immediate tax costs. Miller implicitly abstracts from this second capital structure mechanism by essentially modeling firms as start-ups choosing capital structures from scratch, with no immediate tax consequences.

In the special case of zero personal taxes on equity distributions, we find shareholders are indifferent regarding the firm's use of debt versus external equity (i.e., new equity issues), consistent with Miller's model. Also as contemplated by Miller, increasing equity distribution taxes pushes external equity financing to an inferior corner. In contrast to Miller's analysis, however, high personal taxes on equity distributions do not result in an all-debt financing solution, at least for mature firms. An internal capital structure solution remains intact because shareholders remain indifferent regarding the firm's use of debt versus internal equity. That is, shareholders remain indifferent regarding the firm's choice to either (a) distribute free cash flows and increase debt, or (b) retain free cash flows and decrease debt.² Given this indifference between debt and

² This property of our model is consistent with the evidence in Fama and French (2002) that variation in investment and earnings, which determines free cash flows, is mostly absorbed by debt.

internal equity, many attributes of Miller's equilibrium survive high personal taxes on equity distributions. For example Miller's aggregate debt-equity ratio, the identity of the marginal investment clientele, and investors' portfolio allocations between debt and equity securities all remain intact.

Our interior capital structure equilibrium is robust to distribution taxes because investors treat the distribution tax on internal equity as a sunk cost. In our model, firms must eventually distribute earnings, and in equilibrium, the present value tax cost is the same whenever distributions occur. Given the sunk-cost nature of the distribution tax, our equilibrium is feasible whether firms use high-tax dividends, low-tax share repurchases, or any combination of the two methods to distribute equity.

Despite the similarities between our model and Miller's model, adding equity distribution taxes yields a variety of unique insights. First, Miller's analysis implies firms should be indifferent between debt and external equity financing, whereas in practice, debt largely dominates external equity financing. Consistent with observed practice, our model implies mature firms should prefer both debt and internal equity financing relative to external equity financing whenever the personal tax rate on equity distributions is positive.³ Second, our model implies there is a potentially large tax wedge between the costs of using internal and external equity capital, which should affect capital budgeting decisions and weighted average cost of capital (WACC) calculations. Current textbook discussions often ignore this tax wedge. Third, our model implies that

³ Of course, information asymmetry between managers and investors may contribute to the observed preference for internal versus external equity financing (see Myers and Majluf, 1984, Myers, 1984), but our model implies taxes could play an important, previously ignored role in observed financing pecking orders.

in equilibrium, the timing of corporate distributions is irrelevant. Therefore firms can distribute equity when desired without concern of imposing incremental taxes on shareholders, essentially because the distribution tax is a sunk cost.⁴

Another unique implication of our model is that exchanging debt for internal equity by issuing debt and distributing earnings increases firm value. However, the exchange does not enhance shareholder wealth because it triggers an immediate distribution tax that shareholders recognize on their own accounts, which perfectly offsets the increase in firm value. Hence maximizing firm value is not equivalent to maximizing shareholder wealth. This implies that market evidence of a large debt-tax shield (see, e.g., Kemsley and Nissim, 2001) does not necessarily imply low-debt firms leave shareholder money on the table.

The paper proceeds as follows. In Section 1 we specify our assumptions and definitions. In Section 2 we derive equilibrium distribution and capital structure policy in an economy with a single investment clientele. In Section 3 we extend our model to a general equilibrium setting with multiple clienteles and tax rates. In Section 4 we describe the properties and implications of the model. We conclude in Section 5.

1. Assumptions and Definitions

1.1 Miller's Basic Assumptions

Our objective is to isolate the effects of adding equity distribution taxes to Miller's model, while focusing on mature firms. Therefore we begin by adopting five of

⁴ This result is very different from the result in DeAngelo and Masulis (1980a, Section 5), in which taxable equity distributions disappear entirely, largely because we focus on mature firms with pre-existing capital structures (see Section 1.3 for details).

Miller's basic assumptions. First, we assume firms have unlimited access to the debt market to either buy or sell bonds. Second, we assume all agents face constant, linear tax rates. However, we allow for cross-agent variation in tax rates (e.g., tax-exempts vs. individuals). Third we assume zero financial distress costs from debt and assume there is no information asymmetry. We set these potentially important nontax factors aside for future research. Fourth, we assume short sales (borrowing) by investors are limited. Without loss of generality, we assume the limit is zero, but any finite limit is sufficient for our model. Fifth, we assume agents are risk-neutral, modeling cash flows as certain. However, this risk-neutrality assumption may not be as restrictive as it appears. When analyzing the effect of taxes on financial market equilibrium, Williams (2001) specifies reasonable conditions under which these cash flows can be regarded as the certainty equivalents of the actual risky flows.⁵

1.2 The Tax Environment

We now describe the tax environment. The corporate tax rate (t) is constant for all firms. Each investor faces tax rates on ordinary income (t) and capital gains (g). g is the accrual-equivalent capital-gains tax rate, and $t > g$.⁶ Dividends and interest are subject to tax at rate t . In addition there is a combined tax rate on corporate cash

⁵ In particular, Williams (2001) demonstrates that under relatively mild assumptions, which are met by the assumptions of our model plus the inclusion of a set of futures contracts that spans the risk factors in firm cash flows, the equivalent martingale measure for valuing risky cash flows is independent of the tax system and constant across differently taxed cash flows.

⁶ Capital gains taxes are only levied upon sale of assets, so deferring capital gains decreases the present value of the tax. The accrual-equivalent capital gains tax rate accounts for this deferral benefit.

distributions to shareholders (d), which reflects expectations regarding the mix of dividends and share repurchases.

Carefully defining d is not trivial because a distribution has two tax effects. First, distributions trigger a direct tax cost for shareholders – the ordinary income tax on dividends and the capital gains tax on share repurchases. Green and Hollifield (2001) demonstrate the overall tax cost of repurchases is material. In our model, however, the distribution tax burden of a repurchase is limited to the acceleration of gains that tendering shareholders would otherwise recognize at a later point in time (e.g., through liquidity-motivated trades). Thus, the tax cost of a repurchase is simply the lost time value from capital gains tax deferral.

Second, distributions often alter stock prices, which can result in a capital gain or loss. This change in stock prices is related to the familiar ex-dividend effect (see, e.g., Elton and Gruber, 1970), but is not restricted to dividends. We allow for the possibility that share repurchases also affect stock prices. In our model, this ex-distribution stock price effect is determined endogenously. Prior to deriving the equilibrium, therefore, we must define d as a function of the change in stock price. To do so, we first define g as the decline in the market value of a firm upon distribution of \$1 from the firm. We then define d in terms of g as follows:

$$d = d(g) \equiv a(t - g) + (1 - g)g; \quad 0 \leq a \leq 1. \quad (1)$$

To understand (1), consider the extreme cases in which $a = 1$ and $a = 0$. If a firm uses dividends exclusively, $a = 1$, and (1) simplifies to $d = t - gg$. This reflects the direct tax on the dividend offset by the capital gains tax benefit from a reduction in share prices. At the other extreme if a firm uses share repurchases exclusively, and if we make the

extreme assumption that there is no direct tax on share repurchases, $\mathbf{a} = 0$, and (1) simplifies to $d = (1-\mathbf{g})g$. Note that in this case, there is no distribution tax if the decline in the market value of the firm equals the magnitude of the repurchase because \mathbf{g} would equal unity and d would equal 0. This is as it should be, because share repurchases reduce the number of shares outstanding, so if market value falls by the amount of the repurchase, then market value and the number of shares outstanding would fall proportionately, stock price would remain constant, and there would be no distribution tax.

Miller (1977) essentially assumes \mathbf{a} and d are small, whereas we generalize Miller's model by allowing for positive, potentially high values for \mathbf{a} and d . We believe this is an important generalization because \mathbf{a} and d are likely to be positive even if a firm solely uses tax-favored share repurchases to distribute equity. The use of dividends increases d further.

Although the use of dividends in lieu of share repurchases increases \mathbf{a} and d , we abstract from the determinants of the choice between the two distribution methods and allow for any mix. In practice we observe both types of distributions, and prior research has not fully resolved why firms choose one method or the other. Although share repurchases are tax-advantaged relative to dividends, Chowdhry and Nanda (1994) demonstrate that under certain conditions the price impact of share repurchases could discourage their use. Hausch and Seward (1993) demonstrate that under certain conditions the signaling attributes of dividends are more effective than the signaling attributes of share repurchases. Jagannathan et al. (2000) and Guay and Harford (2000) provide evidence that firms with stable, permanent earnings tend to use dividends,

whereas firms with transitory shocks to earnings tend to use share repurchases. Hence many factors could affect the choice between dividends and share repurchases, so in this study, we simply allow for both. Rather than focus on the choice of distribution method, we focus on the timing of corporate distributions.

1.3 Corporate Activities

We now consider the investment and financial policy activities of the firm. We assume firm n has operating earnings before interest and taxes of $X_n(i)$ and operating assets of $O_n(i)$ for future period i . The firm optimizes value by investing in all available positive NPV operating projects based on a discount rate equal to the after-tax borrowing rate of r . Therefore, operating assets (O_n) should vary over time according to variation in the set of available positive NPV projects. The pretax rate of return on bonds is r_b , and as previously assumed, the corporate income tax rate is t . Therefore $r = (1-t)r_b$.⁷ Debt is available to the firm in infinite supply, so the marginal cost of debt is a constant unaffected by financial policy choice. That is, the investment and financial policy decisions of the firm are separable.

The only restrictions we impose on the sequences of operating income and assets are:

⁷ Alternatively, we could model the required after-tax return to investors as an exogenous constant and derive the pretax interest rate endogenously. Beyond notational differences, this alternative would not affect our model because we are not interested in deriving comparative statics of how equilibrium changes if the tax regime changes (i.e., we treat tax rates as exogenous constants).

$$\begin{aligned}
O_n(i) &> 0 \quad \forall i, \\
\frac{X_n(i)}{O_n(i)} &> r_b \quad \forall i, \quad \text{and} \\
\lim_{i \rightarrow \infty} \frac{O_n(i+1)}{O_n(i)} &< 1 + r.
\end{aligned} \tag{A1}$$

The first condition requires the firm to maintain positive operating assets at all times.

The second condition is that the firm must generate an average return on investment in excess of the discount rate, which follows from our earlier assumption that the firm only chooses positive NPV operating projects. The third condition ensures the firm does not grow faster than the discount rate. Without this condition, the firm would have infinite value (given the second condition).

In addition to operating assets, the firm also has a sequence of net financial assets over time, $F_n(i)$, which generates an after-tax return of r . A negative value for $F_n(i)$ represents the debt level of the firm. The firm also chooses a sequence of distributions, $D_n(i)$, to pay to its shareholders. The time path of financial assets is subject to the following financing constraint:

$$F_n(i) = (1+r)F_n(i-1) + X_n(i)(1-t) - [O_n(i) - O_n(i-1)] - D_n(i). \tag{2}$$

This constraint reflects basic accounting identities. In particular, financial assets in period i equal $(1+r)$ times financial assets in period $i-1$, plus after-tax operating earnings, less net new investment in operating assets, and less distributions.

Using (2) we can define two central terms we use to describe our model – free cash flow and internal equity. We define free cash flow as the increase in net financial assets prior to making a distribution, if any. Thus free cash flow is $rF_n(i-1) + X_n(i)(1-t) - [O_n(i) - O_n(i-1)]$. We define internal equity as the portion of free cash flow the firm does not distribute to shareholders. Hence to “use internal equity” means to retain free cash

flow rather than to distribute it to shareholders. Along with the issuance of debt and new external equity, internal equity is one of the firm's three sources of financing.

We impose two restrictions on distribution policy. Any distribution policy that satisfies both conditions is defined as a feasible distribution policy. First, distributions cannot be negative, or $D_n(i) \geq 0 \forall i$. Second, we place the following transversality constraint on the amount of debt and financial assets the firm may accumulate:

$$\infty > \lim_{i \rightarrow \infty} \frac{F_n(i)}{O_n(i)} > -\infty. \quad (\text{A2})$$

The practical effect of this restriction is to prevent the firm from paying more distributions than the operations of the firm can ultimately support, and from evading payouts forever through accumulation inside the firm. Relative to the model in Miller (1977), which implicitly assumes firms can largely avoid distribution taxes forever through retention, (A2) is a key innovation.⁸ The assumption is based on the recognition that firms can merely delay these taxes, although they may do so for a very long time. (A2) is consistent with the trapped-equity models in Auerbach (1979), Bradford (1981), and King (1977), except in contrast to these prior models we allow for the use of share repurchases as well as dividends to distribute cash.

Finally, we assume the firm is "mature." In particular, we assume the firm has positive pre-existing equity, consisting of either paid-in capital or retained earnings, at the

⁸ The Accumulated Earnings Tax (IRC Sections 531-537) helps support this assumption. Tax regulators impose the punitive Accumulated Earnings Tax on a corporation's undistributed earnings to the extent the retained earnings are not required to meet the reasonable business needs of the company. Even before tax regulators become involved, however, shareholders often put pressure on firms with large amounts of accumulated earnings to distribute the profits (see, e.g., "A Cash-Rich Microsoft Faces Shareholder Call for Dividend, *Wall Street Journal*, January 2, 2002).

time it chooses its capital structure and distribution policies. This assumption is important because, as we later demonstrate, issuing additional external equity is sub-optimal, and without pre-existing equity, distribution is a moot issue. Intuitively, a mature firm is one that is beyond the start-up stage where external equity often is the principal source of capital because of initially low and highly uncertain cash flows and because of low marginal tax rates, which reduce or eliminate the benefits of debt. Thus, the firm might have had strong motives to issue equity in the past, even though such motives no longer exist and external equity is sub-optimal.

In addition, even if a firm is initially debt-financed, surviving firms typically accumulate internal equity because it is infeasible to structure contingent debt contracts with interest and repayment provisions that exactly offset the firm's income in all states.⁹ Therefore, even if firms attempt to use debt to zero out their taxable income, the firms will either have unexpectedly high earnings and accumulate retained earnings after paying interest, or have unexpectedly low earnings and become bankrupt and disappear. For these reasons, we believe it is reasonable to assume mature firms have positive existing equity, despite their strict incentive to avoid issuing new external equity.

2. Equilibrium Capital Structure and Distribution Policy

We treat the choice between dividends and share repurchases as exogenous to the model, so we are particularly interested in the choice regarding how much cash to distribute each period. As demonstrated later, external equity is strictly inferior to debt

⁹ While this is possible in principle (see, e.g., DeAngelo and Masulis, 1980a), it would violate the substance over form doctrine, so the Internal Revenue Service would reclassify the debt as equity and disallow the deduction for interest.

for any positive tax rate on distributions. Given a significant tax rate on equity distributions, therefore the only financing margin that could potentially lead to an interior capital structure solution is the margin between debt and internal equity. The firm chooses its mix of debt and internal equity through distribution timing policy. Paying an incremental dollar of free cash flow out to shareholders at a point in time requires the firm to sell (i.e., issue) an incremental dollar of debt. Similarly, retaining an incremental dollar of free cash flow requires the firm to buy (e.g., pay off) an incremental dollar of debt. Given this link between distributions and capital structure, identifying an interior solution for distribution timing policy is equivalent to identifying an interior solution between debt and internal equity. Therefore we now turn to distribution timing policy, beginning in a no-tax setting.

2.1 Distribution Timing Policy without Taxes

Given assumptions (A1) and (A2) in a no-tax setting, the present values of all feasible distribution policies are the same. More formally, we provide the following lemma.

Lemma 1

Every distribution policy that satisfies (A2) has the same present value. That is, there exists a V_n such that

$$\sum_{i=1}^{\infty} \frac{D_n(i)}{(1+r)^i} = V_n \quad \forall \text{ feasible } D_n(\cdot).$$

Proof: See Appendix A.

Lemma 1 states that any change in distribution policy must be present-value preserving to be feasible. Intuitively it simply reflects the dividend displacement property of Miller and Modigliani (1961) – a dollar distribution displaces a dollar of

market value, so assuming perfect markets with no taxes, the timing of distributions is irrelevant.

2.2 Distribution Timing Policy with Taxes

To add taxes, we begin by assuming there is only one investment clientele (denoted by m), taxed at rate t_m on ordinary income, at rate g_m on capital gains, and rate d_m on distributions they receive, where d_m is related to t_m and g_m by (1). Given this single clientele, we can prove the following proposition.

Proposition 1

All feasible distribution policies are equally attractive if

$$(1-d_m)(1-t)(1-g_m) = (1-d_m)(1-t_m).$$

$$(1-t)(1-g_m) = (1-t_m). \quad (3)$$

Moreover, if (3) holds, shareholders are indifferent regarding distribution timing policy, leading to an interior capital structure solution in which they are indifferent between debt and internal equity. In contrast, if the left-hand side (LHS) of (3) is greater than the right-hand side (RHS) (high-tax investors), then the optimum distribution policy is to delay payouts as long as possible. If the RHS of (3) exceeds the LHS (low-tax investors), then the optimum distribution policy is to pay out cash as early as possible. Finally, (3) implies the “ex-distribution” market value drop due to the payment of a dollar distribution is:

$$g = 1 - d_m = \frac{1 - (1-a)g_m - at_m}{1 - g_m}. \quad (4)$$

Proof: See Appendix A.

Intuitively, the LHS of (3) represents the total corporate and personal taxes on returns to internal equity, and the RHS of the equation represents the total personal tax on returns to debt. Hence the equation reflects the fundamental equilibrium condition in Miller (1977) that total corporate and personal taxes on equity must equal the tax on debt.

Note that the distribution tax appears on both sides of (3), so it drops out of the equation. Miller (1977) largely drops the distribution tax by assuming distributions from equity are either taxed at a very low rate or need never occur, i.e., firms often can defer payouts indefinitely. This is not required for equilibrium in our model because distribution taxes represent sunk costs investors will incur whether the firm uses internal equity or debt to finance investment.¹⁰

To illustrate, note that if the firm uses internal equity, all earnings on the reinvested cash flows will generate distribution taxes in the future. On the other hand, if the firm pays out free cash flow as a distribution and issues debt to finance investment, the payout will trigger an immediate distribution tax. Either way investors will incur the distribution tax at some point in time, and either way the present value of the tax is the same, so it drops out of the equation. This is true whether the firm uses dividends or repurchases to distribute equity.

Equation (4) corresponds to the familiar expression for ex-dividend price falloffs necessary for the dividend displacement property of Miller and Modigliani (1961) to hold

¹⁰ Our equilibrium also is different from the equilibrium in DeAngelo and Masulis (1980a). DeAngelo and Masulis (1980a, Section 5) obtain indifference between debt and capital gains, but to do so, they assume state-contingent debt (which is not feasible in practice), and they conclude there will be zero dividends. In contrast, our equilibrium between debt and internal equity does not require state-contingent debt, and it allows for positive dividends.

on an after-tax basis, as first derived by Elton and Gruber (1970). However, (4) is more generic and applies to any form of distribution with any tax burden. Intuitively, investors account for future distribution taxes when they value the firm's equity, so distributing equity only decreases firm value by the after-personal-tax value of the distribution.

To gain further insights into (4), consider two special cases. First, if the firm uses dividends to distribute cash, then as noted earlier, $a = 1$ and $d_m = t_m - g g_m$. Substituting this characterization of d_m into (4) yields:

$$g = \frac{1-t_m}{1-g_m} = 1-t; \quad d_m = \frac{t_m - g_m}{1-g_m} = t. \quad (5)$$

The first expression for g in (5) is the Elton and Gruber (1970) ex-dividend expression.

The second expression for g in (5), which is derived from (3), demonstrates that in equilibrium the ex-dividend effect is driven by the corporate tax rate. Moreover, in equilibrium, the tax burden of a dividend equals the corporate tax rate.

Second, if the firm uses share repurchases to distribute cash, and if we make the extreme assumption that the direct tax burden of a repurchase is zero, then $a = 0$, (4) simplifies to $g = 1$, and $d_m = 0$. Unsurprisingly, therefore, if the firm can distribute cash without cost, market value falls dollar for dollar in distributions. Following a mixed strategy of dividends and share repurchases leads to intermediate values for g and for d_m , weighted according to the relative use of share repurchases and dividends.

3. General Equilibrium with Multiple Clienteles

The analysis in Proposition 1 characterizes a knife-edge equilibrium in which firms are only indifferent between debt and internal equity if investor tax rates exactly

balance the corporate tax rate. We now consider a more general setting with multiple clienteles, as in Miller (1977), facing different tax rates on interest, capital gains, and distributions. We shall demonstrate that the distribution-capital structure indifference equilibrium is more robust in this context. Specifically, a general equilibrium is obtained in which the marginal clientele between debt and equity satisfies (3), as long as the aggregate capital structure provides a sufficient supply of equity (debt) for all investors who face a lower (higher) tax rate on equity than on debt.

To help characterize our equilibrium, we let W^* equal the aggregate wealth held by low-tax investors who face a lower or equal tax rate on debt versus equity, i.e., those investors for whom the RHS of (3) is at least as great as the LHS. We then must add two new assumptions. First, we assume that at each point in time W^* exceeds the aggregate pre-distribution debt level of firms in the economy. Second, we assume annual distributions are not “too large” in the sense that the fraction of market value distributed each year is less than r_b (this assumption can be relaxed if $\mathbf{a} < 1$). Given these assumptions, we now describe the general equilibrium.

Proposition 2

The general equilibrium has the following characteristics.

a) Any investor for whom the RHS of (3) exceeds the LHS (low-tax investors) will invest exclusively in debt. Any investor for whom the LHS of (3) exceeds the RHS (high-tax investors) will invest exclusively in equity. Any investor for whom the LHS of (3) equals the RHS of (3) will be indifferent between debt and internal equity. Thus,

$$(1 - \mathbf{t})(1 - g) < (1 - t) \Rightarrow \text{invest in debt.}$$

$$(1 - \mathbf{t})(1 - g) > (1 - t) \Rightarrow \text{invest in equity.}$$

$$(1 - \mathbf{t})(1 - g) = (1 - t) \Rightarrow \text{indifferent (marginal)}.$$

- b) If $d_m > 0$, external equity is strictly inferior to debt and internal equity.
- c) Any feasible distribution policy is optimal for any individual firm.
- d) The aggregate debt level is fixed at W^* , while no individual firm will have a unique optimal capital structure.
- e) The market value of equity in each firm is $E_n = gV_n$, where g is given by (4) and V_n is a constant, defined in Lemma 1.

Proof: See Appendix A.

To see how the equilibrium is obtained, consider what would happen if firms in aggregate have debt levels less than W^* , conditional on a set of intended distribution policies. In this case, low-tax investors for whom $(1 - t) > (1 - \mathbf{t})(1 - g)$ would demand more debt and would be the marginal investor. If so, then Proposition 1 implies firms would have an incentive to accelerate distributions from internal equity and issue replacement debt. Firms will continue to accelerate distributions and issue debt until the aggregate debt level reaches W^* , at which time the marginal clientele shifts to where (3) holds and firms no longer have an incentive to change distribution timing policy. Similarly, if the aggregate debt level, conditional on a set of intended distribution policies, exceeds W^* , the supply of debt would exceed demand. In this case, firms would have an incentive to delay distributions from internal equity and pay off debt until the marginal clientele shifts to (3). In equilibrium, therefore, the supplies of debt and equity will adjust to ensure the marginal clientele has tax rates consistent with (3). This implies the relative tax costs of debt and internal equity are equal, so distribution timing policy and the choice between debt and internal equity are irrelevant.

4. Properties of the Equilibrium

4.1 Role for the Distribution Tax

Miller (1977) argues that a low tax rate on equity is required to obtain equilibrium. In contrast, Proposition 2 indicates that many properties of our equilibrium are completely independent from the level of tax on distributions (d_m or \mathbf{a}). In particular, the identity of the marginal clientele, the aggregate debt-equity mix, and the choice of portfolios by investors are all independent of the magnitude of the tax on distributions. In addition, any feasible distribution timing policy is optimal for any individual firm, independent from d_m , which implies indifference between using debt and internal equity. This is true because given the marginal clientele necessary for equilibrium, the present value of the distribution tax is the same regardless of when the firm pays the distribution. Thus, the tax is unavoidable and treated as a sunk cost.

Indeed there are only two features of the equilibrium that are a function of d_m . First, the price of equity decreases in d_m because $g = 1 - d_m$. Therefore using share repurchases to minimize distribution taxes should increase firm value. Second, if d_m (\mathbf{a}) is positive, then external equity is inferior to debt and internal equity. Furthermore the inferiority of external equity increases in d_m . Therefore the personal tax on equity drives external equity to an inferior corner. Miller (1977) contemplates that a sufficiently high personal tax rate on equity could lead to such a corner solution. As Miller fails to consider, however, pushing external equity to a corner does not necessarily imply a corner solution for capital structure; we still obtain an interior solution between debt and internal equity.

The choice between debt and internal equity is independent from d_m because as a sunk cost its effective rate is zero. Given an effective tax rate of zero on distributions from equity, the personal tax disadvantage of debt remains intact and equilibrium is obtained. In contrast d_m is not a sunk cost when focusing on the margin between debt and external equity, so d_m must actually be zero to obtain indifference between these two sources of capital, absent nontax considerations.

The preference for debt and internal equity when $d_m > 0$ is, we believe, at least somewhat descriptive of the economy, in which firms use large amounts of debt and internal equity and use relatively little external equity. Of course firms often use external equity to finance investment in the start-up phase, which is why we model the activities of mature firms. In the start-up phase, internal equity often is unavailable. In addition, the financial distress costs of debt could create a preference for equity in certain situations (see DeAngelo and Masulis, 1980b), or information asymmetry could prevent a firm with positive NPV investment opportunities from obtaining debt financing, compelling it to issue equity despite the large tax penalty. As previously noted, therefore, we assume the pre-existence of equity for the firm.¹¹ Proposition 2 then states if $d_m > 0$, there is a strict tax disadvantage to issuing new external equity.

4.2 Buying Bonds

A common concern raised regarding firms that purchase large amounts of bonds and other marketable securities is that interest earned within the firm is subject to both

¹¹ Note that our model implies a firm cannot simply repurchase shares to eliminate the tax disadvantage of its pre-existing external equity. Specifically, if a firm issues debt and repurchases external equity, the repurchase triggers the equity distribution tax, which represents the tax wedge between debt (or internal equity) and external equity capital.

corporate and personal distribution taxes (i.e., double taxation), whereas interest earned directly by investors is only subject to a single layer of tax. In our model this strategy only destroys value if a firm issues external equity to buy the bonds. If the firm simply invests free cash flows in bonds, the distribution tax drops out of the analysis as a sunk cost. Therefore firms can invest internal equity in bonds without an incremental tax penalty, subject of course to the transversality condition.

4.3 The Cost of Capital

We can express key features of our model in terms of the costs of using debt, internal equity, and external equity. Note that in all three cases the alternative investment opportunity within our model is a taxable bond, yielding an after-tax return equal to $r_b(1-t_m)$. Of course, stocks and bonds provide identical after-tax returns for the marginal clientele.

The cost of debt financing equals the after-tax return on a taxable bond, $r_b(1-t_m)$, adjusted for the corporate tax benefit of debt (numerator) and the personal tax investors pay on bond interest (denominator), or:

$$\text{Cost of Debt} = \frac{r_b(1-t_m)(1-t)}{(1-t_m)} = r_b(1-t). \quad (6)$$

In this case the actual investment (a taxable bond) is equivalent to the alternative investment (a taxable bond), so the two $(1-t_m)$ terms cancel out and only the corporate tax benefit of debt remains.

The cost of internal equity financing equals the after-tax return on a taxable bond, $r_b(1-t_m)$, adjusted for the immediate distribution tax investors would have to pay to withdraw the internal equity for the alternative taxable bond investment (numerator). We also must adjust the equation for the distribution and capital gains taxes investors will

eventually incur if the firm reinvests the internal equity (denominator). Thus, the cost of internal equity is:

$$\text{Cost of Internal Equity} = \frac{(1-d_m)r_b(1-t_m)}{(1-d_m)(1-g_m)} = \frac{r_b(1-t_m)}{(1-g_m)}. \quad (7)$$

The cost of external equity is equivalent to the cost of internal equity except no distribution tax must be incurred to withdraw the equity for an alternative investment.

Therefore the $(1-d_m)$ term drops out of the numerator of (7) yielding:

$$\text{Cost of External Equity} = \frac{r_b(1-t_m)}{(1-d_m)(1-g_m)} = \frac{r_b(1-t_m)}{1-(1-a)g_m - at_m}. \quad (8)$$

Comparing (6) and (7) indicates the cost of debt is equal to the cost of internal equity as long as $(1-t_m)/(1-g_m) = (1-t)$, which is the equilibrium condition expressed in (3). Comparing (6) and (7) to (8) indicates the cost of external equity is equal to the cost of debt and internal equity if $\alpha = d_m = 0$. This essentially is the special case assumed in Miller (1977), although under certain conditions, Miller's model leads to an interior equilibrium if d_m is slightly positive. In our model, external equity becomes more costly than the other two forms of financing for any positive value of d_m (\mathbf{a}).

In terms of the familiar after-tax weighted average cost of capital formula (WACC), if we assume equilibrium holds so $(1-t_m)/(1-g_m) = (1-t)$, then (6), (7), and (8) imply the personal tax on equity distributions creates a wedge between the cost of debt and internal equity financing on the one hand, and the cost of external equity financing on the other. Therefore the model implies internal equity should be grouped with debt in WACC calculations, instead of following the common practice of grouping internal equity with external equity (see, e.g., Brealey and Myers, 2000, p. 543). Of course, this

result abstracts from the potential nontax costs of debt, which future research could incorporate in the model.

4.4 Firm Value Versus Shareholder Wealth

An interesting implication of our model is that substituting debt for internal equity, by issuing debt and distributing cash to shareholders (which occurs any time debt issues and distributions occur in the same period) increases firm value but does not affect shareholder wealth. To illustrate, consider a firm substituting debt for internal equity by issuing a dollar of debt and distributing a dollar of cash to shareholders. When distributing the dollar of cash, firm value decreases by the after-personal-tax value of the distribution, or g . On the other hand, issuing a dollar of debt increases firm value by a full dollar. Hence the combined effect of the two transactions is to increase firm value by $1-g$. However, shareholders pay for the additional firm value by incurring a net personal tax equal to $1-g$ according to (4), which perfectly offsets the increase in firm value resulting from the debt-tax shield acquired in the transactions.

A related implication is that distributing a dollar of free cash flow increases a firm's market value premium relative to book value (the valuation premium). This occurs because market value only decreases by g , whereas book value decreases by a full dollar. One way to interpret this result is to note that the recorded book value of the firm ignores an important liability investors capitalize into firm value; namely, future equity distribution taxes. When the firm distributes cash, it reduces the unrecorded tax liability as shareholders pay the tax on their own accounts. If firms were to record this liability for future distribution taxes, then the valuation premium would be invariant to distribution policy and capital structure.

5. Conclusion

In this paper, we extend the capital structure model in Miller (1977) by focusing on mature firms with pre-existing equity that can only be decreased through taxable dividend and share repurchase distributions to shareholders. In contrast to Miller's analysis, we demonstrate it is not necessary to have low personal taxes on equity to obtain an interior capital structure equilibrium. An interior solution is possible in the presence of substantial taxes on dividends and share repurchases, even in the absence of financial distress costs from debt. In our equilibrium, these taxes push external equity financing alone to an inferior corner. Firms remain indifferent between debt and internal equity financing, regardless of the magnitude of the tax rate on distributions. Indeed, the tax cost of distributing cash to shareholders is irrelevant for most aspects of the equilibrium because it drops out as a sunk cost, although it does affect stock prices.

The model implies it is insufficient to view capital structure in terms of debt versus equity because the tax wedge between internal and external equity is potentially large. Capital budgeting procedures and weighted average cost of capital formulas should account for all three forms of financing. The model also implies the timing of distributions does not affect the present value tax burden, so firms are free to distribute equity whenever nontax factors suggest it is optimal to do so. A final implication of our model is that exchanging debt for internal equity increases firm value but does not increase shareholder wealth because shareholders essentially pay for the additional firm value through personal tax payments. Therefore market evidence of positive tax benefits

from debt (see, e.g., Kemsley and Nissim, 2001) does not necessarily imply leverage enhances shareholder wealth.

Our objective in this study has been to turn a single dial in Miller's model, which is to add material equity distribution taxes while focusing on mature firms. Like Miller (1977), therefore, we have abstract from potentially important factors, including information asymmetry, financial distress costs from debt, and potential shifts in clienteles when firms alter their distribution policies. In addition, we treat the choice between dividends and share repurchases as an exogenous decision. Future research could build these additional factors into the model, leading to a more complete understanding of debt, equity, and taxes.

Appendix A. Proofs

Proof of Lemma 1

Consider two feasible distribution policies, $D^1_n(\cdot)$ and $D^2_n(\cdot)$, that both satisfy (A2). Given (1), we can derive the corresponding net financial assets under both policies, $F^1_n(\cdot)$ and $F^2_n(\cdot)$. Define $D^*_n(\cdot)$ and $F^*_n(\cdot)$ as the differences between the distribution and net financial asset sequences, respectively. The present value of $D^*_n(\cdot)$ is the difference between the present values of the two distribution streams. Also, given (1), $F^*_n(\cdot)$ is related to $D^*_n(\cdot)$ as follows:

$$\begin{aligned} F^*_n(i) &= (1+r)F^*_n(i-1) - D^*_n(i) \\ &= -\sum_{j=1}^i (1+r)^{i-j} D^*_n(j) \\ &= -(1+r)^i \sum_{j=1}^i \frac{D^*_n(j)}{(1+r)^j}. \end{aligned}$$

The limit of the sum is the present value of $D^*_n(\cdot)$. If this sum is not zero, then $F^*_n(\cdot)$ increases at rate r . However, according to the transversality condition (A1), $O_n(\cdot)$ increases at a rate lower than r . Consequentially, from (A2), $F^1_n(\cdot)$ and $F^2_n(\cdot)$ both increase at rates lower than r , implying that $F^*_n(\cdot)$ does as well. Thus the present value of $D^*_n(\cdot)$ must be zero, implying that the present values of $D^1_n(\cdot)$ and $D^2_n(\cdot)$ are identical.

Q.E.D.

Intuitively, this proof suggests that if distribution policy changes in a way that does not preserve the present value of the distribution stream, the change in distributions will either increase or decrease the net financial assets of the firm. Given the compounding of interest, this difference in net financial assets will build up over time and

become infinitely large in the limit. This would be inconsistent with the transversality condition, which requires net financial assets to grow at a slower long-run rate.

Proof of Proposition 1

To consider changes in distribution policy, we focus on a single-period acceleration of a dollar of present value distributions from period $i+1$ to period i . Every feasible distribution policy differs from every other feasible distribution policy by a linear combination of such one-period shifts in distributions (a direct consequence of Lemma 1); thus, if one-period distribution shifts are value neutral, all distribution policies are equally attractive. The specific change in policy we consider is:

Policy Δ

$$D_n^*(i) = D_n(i) + 1,$$

$$D_n^*(i+1) = D_n(i+1) - (1+r), \text{ and}$$

$$D_n^*(j) = D_n(j) \quad \forall j \notin \{i, i+1\}.$$

Descriptively, the distribution policy change (Policy Δ) has the firm increase its distribution by a dollar at time i , borrow that dollar, and cut its distribution (from what it would otherwise have been) by $\$(1+r)$ dollars at time $i+1$. The policy change is feasible (assuming that $D_n(i+1) \geq 1+r$) because it does not alter the present value of the distribution sequence (by Lemma 1), and all feasible distribution policies are equally attractive if Policy Δ has no effect on shareholder utility.

If the firm implements Policy Δ by accelerating the distribution, shareholder m will receive an extra dollar at time i , taxed at rate $d_m(\mathbf{g})$, and then invest the after-tax distribution at an after-tax rate of return r_m^* for one period. From (1),

$$d_m(\mathbf{g}) = \mathbf{a}(t_m - g_m) + (1 - \mathbf{g})g_m.$$

By accelerating the distribution by one period, shareholder m relinquishes the distribution that would have been available in period $i+1$, equal to $1+(1-t)r_b$.

Paying an extra distribution at time i also reduces the market value of the firm's stock by γ , through the ex-distribution effect. The distribution forgone in period $i+1$ then causes the market value to return to its initial trajectory (because all subsequent distributions are unaffected by Policy Δ). This results in a capital gain accrual of γ in period $i+1$. Note that the second period capital gain accrual is only a $\gamma / [1+(1-t)r_b]$ fraction of the foregone distribution in that period. Thus, the distribution tax rate in period $i+1$ is

$$d_m \left(\frac{\mathbf{g}}{1+(1-t)r_b} \right) = \mathbf{a}(t_m - g_m) + \frac{1+(1-t)r_b - \mathbf{g}}{1+(1-t)r_b} g_m.$$

All of these wealth effects from the acceleration of the distribution for shareholder m can be summarized as follows:

$$\Delta W_{m,i} = 1 - d_m(\mathbf{g}) - \frac{[1+r_b(1-t)] \left[1 - d_m \left(\frac{\mathbf{g}}{1+r_b(1-t)} \right) \right]}{1+r_m^*}.$$

$$\Delta W_{m,i} = [1 - \mathbf{a}(t_m - g_m) - (1-\mathbf{g})g_m] - \frac{[1+r_b(1-t)][1 - \mathbf{a}(t_m - g_m)] - [1+r_b(1-t) - \mathbf{g}]g_m}{1+r_m^*} \quad (\text{B1})$$

where $\Delta W_{m,i}$ is the change in wealth for shareholder m , measured at period i . The first term on the RHS of equation (B1) represents the benefit shareholder m receives from the accelerated distribution in period i , net of all tax effects. The second term on the RHS of the equation represents the present value of the cost of relinquishing the distribution in period $i+1$. If the benefit from the accelerated distribution equals the cost, then ΔW is

zero, and Policy Δ is a neutral shift. Using equation (B1) and setting $\Delta W = 0$, we derive the following equilibrium condition:

$$1 - \mathbf{a}(t_m - g_m) - (1 - \mathbf{g})g_m = \frac{[1 + r_b(1 - t)][1 - \mathbf{a}(t_m - g_m)] - [1 + r_b(1 - t) - \mathbf{g}]g_m}{1 + r_m^*}. \quad (\text{B2})$$

We now consider a new investor in the firm an instant after the distribution payout at time i . The price such an investor is willing to pay for a given percentage of the firm is affected by Policy Δ through its changes in future distributions. In particular, the reduction in purchase price resulting from the distribution at time i (which strips equity from the firm) must exactly compensate the investor for the lower distribution at time $i+1$ (after also accounting for the anticipated market value change of \mathbf{g} at that time). Thus,

$$\mathbf{g} = \frac{[1 + r_b(1 - t)][1 - \mathbf{a}(t_m - g_m)] - [1 + r_b(1 - t) - \mathbf{g}]g_m}{1 + r_m^*}. \quad (\text{B3})$$

Rearranging this yields:

$$\begin{aligned} \mathbf{g}(1 + r_m^*) &= \{[1 - \mathbf{a}(t_m - g_m) - g_m][1 + r_b(1 - t)] + g_m \mathbf{g}\} \\ \mathbf{g} &= \frac{[1 - \mathbf{a}(t_m - g_m) - g_m][1 + r_b(1 - t)]}{1 + r_m^* - g_m}. \end{aligned} \quad (\text{B4})$$

Note that the RHS of (B2) is identical to the RHS of (B3). Thus, their left-hand sides are also equal:

$$\begin{aligned} 1 - \mathbf{a}(t_m - g_m) - g_m(1 - \mathbf{g}) &= \mathbf{g}. \\ \mathbf{g} &= \frac{1 - (1 - \mathbf{a})g_m - \mathbf{a}t_m}{1 - g_m}. \end{aligned} \quad (\text{B5})$$

Substituting (B5) into (1):

$$d_m = \mathbf{a}(t_m - g_m) + \left(1 - \frac{1 - (1 - \mathbf{a})g_m - \mathbf{a}t_m}{1 - g_m}\right)g_m = \frac{\mathbf{a}(t_m - g_m)}{1 - g_m}.$$

Equation (4) follows immediately from the last two equations.

Setting the right-hand sides of (B4) and (B5) equal to each other, and noting that a positive demand for bonds implies that $r_m^* = (1-t_m)r_b$, allows us to express the neutral ex-distribution price effect in terms of corporate and individual tax rates as follows:

$$\frac{1 - (1-a)g_m - at_m}{1 - g_m} = \frac{[1 - a(t_m - g_m) - g_m][1 + r_b(1-t)]}{1 + r_m^* - g_m}.$$

$$1 + r_b(1-t_m) - g_m = (1 - g_m)[1 + r_b(1-t)].$$

$$r_b(1-t_m) = (1 - g_m)r_b(1-t).$$

$$(1-t)(1 - g_m) = (1-t_m). \quad (3)$$

If equation (3) holds, all distribution policies are equally optimal.

Accelerating the distribution will increase shareholder wealth if the drop in market value on the ex-distribution day, or γ , is less than $(1-d_m-g_m)/(1-g_m)$. Given equation (B4), it can be demonstrated that this implies the RHS of (3) exceeds the LHS (the derivation is directly analogous to the above derivation of (3)), meaning that the after-tax return on debt is greater than the after-tax return on equity. In this case, Policy Δ increases shareholder wealth because it accelerates the distribution and allows shareholders to invest the funds in debt. Indeed, the firm will accelerate distributions as much as possible. The converse follows from a similar argument. Therefore the only interior solution occurs when equation (3) holds. Q.E.D.

Proof of Proposition 2

Given the conjectured portfolios, the market would clear if firms collectively issue a quantity of debt equal to W^* at each point in time. By assumption, that level of debt could be achieved solely by issuing debt and paying distributions, without resorting to external equity. Thus, the conjectured behavior is feasible.

To demonstrate that the conjectured behavior is optimal, we first assume that (3) holds for the marginal clientele and determine market values (prices) of firms and optimal corporate financial policy. Later, given these prices and corporate decisions, we will confirm that investors choose the conjectured portfolios and that (3) does indeed hold for the marginal clientele.

First, given that (3) holds for the marginal clientele, Proposition 1 implies that the ex-distribution effect, g , obeys (4), and since $a > 0$ (by assumption), $g < 1$. Further, Lemma 1 implies that for any feasible distribution policy, the present value of all future distributions equals V_n . Clearly, one such feasible policy is to distribute V_n immediately and then make no further distributions. If the firm follows such a policy, then the market value would decline by gV_n . After the super-distribution, the value of the firm would be 0 (since there would be no future distributions). Thus, the market value of the firm prior to making any distributions must be gV_n . Proposition 1 ensures that any distribution policy is equivalent to any other, so regardless of what distribution policy is chosen, the market value of the firm, E_n , is gV_n .

Next, we consider the decision of whether to issue new equity. Since issuing \$1 of equity increases V_n by \$1, it increases the market value of firm by $g < 1$. Thus, shareholders immediately lose $1 - g$ in the transaction. Because issuing equity triggers an immediate loss of value without any countervailing benefit, it is strictly suboptimal in this model, and new equity is strictly inferior to debt and internal equity.

When focusing on the optimality of distribution policy, recall that Proposition 1 has already demonstrated that firm distribution policy is irrelevant for the conjectured marginal clientele. Thus any set of distribution policies by firms that maintains the

aggregate debt level at W^* is weakly optimal for companies since no firm has an incentive to change financial policy. By assumption, such an equilibrium can exist because absent any distributions, aggregate debt would be less than W^* , and increases in distributions would lead to more debt.

The final step in the proof of equilibrium is the demonstration that investors optimally choose the conjectured portfolios. Consider an investor with tax rates of t and g on ordinary income and capital gains, respectively. By (1), this investor faces a distribution tax rate of $d = \mathbf{a}(t - g) + (1 - \mathbf{g})g$. If the investor chooses debt, the rate of return is $(1 - t)r_b$. If the investor chooses equity, the rate of return is

$$r_e(i) = \frac{[E_n(i+1) - E_n(i)] + (1-d)D_n(i+1) - g[E_n(i+1) - E_n(i) + \mathbf{g}D_n(i+1)]}{E_n(i)}. \quad (\text{B6})$$

Note that $E_n(i)$ is the market value of the firm immediately after the distribution at time i . The return has three parts, the increase in market value, the after-tax distribution (including capital gains/losses on the “ex-distribution” price drop), and the capital gains tax on price appreciation (pre-distribution). As shown above, $E_n(i)$ equals $\mathbf{g}V_n(i)$. Based on Lemma 1, the following must hold.

$$V_n(i+1) = [1 + (1-t)r_b]V_n(i) - D_n(i+1). \quad (\text{B7})$$

Substituting (B7) into (B6):

$$r_e(i) = \frac{(1-g)\mathbf{g}\{[1 + (1-t)r_b]V_n(i) - D_n(i+1) - V_n(i)\} + [1 - \mathbf{a}t - (1-\mathbf{a})g]D_n(i+1)}{E_n(i)}.$$

$$r_e(i) = \frac{(1-g)(1-t)r_b E_n(i) + [1 - \mathbf{a}t - (1-\mathbf{a})g - (1-g)\mathbf{g}]D_n(i+1)}{E_n(i)}. \quad (\text{B8})$$

Substituting (4) into (B8):

$$r_e(i) = (1-g)(1-t)r_b + \left[1 - \mathbf{a}t - (1-\mathbf{a})g - (1-g) \frac{1 - \mathbf{a}t_m - (1-\mathbf{a})g_m}{1-g_m} \right] \frac{D_n(i+1)}{E_n(i)}. \quad (\text{B9})$$

Consider three groups of investors. First, for those for whom (3) holds, the term in brackets is zero (because in that case, $t = t_m$ and $g = g_m$), so (B9) reduces to $(1-g)(1-t)r_b$, which by (3) equals $(1-t)r_b$, the return on debt. Thus, as conjectured, this clientele is indifferent between debt and equity and is the marginal clientele.

Now consider the group of investors for whom the LHS of (3) exceeds the RHS. The term in brackets in (B9) is negative for this clientele (because $(1-t)/(1-g) < 1-t = (1-t_m)/(1-g_m)$). By assumption $D_n(i+1) / E_n(i) < r_b$, so

$$r_e(i) > (1-g)(1-t)r_b + \left[1 - \mathbf{a}t - (1-\mathbf{a})g - (1-g) \frac{1 - \mathbf{a}t_m - (1-\mathbf{a})g_m}{1-g_m} \right] r_b.$$

$$r_e(i) > \left[(1-g) \frac{1-t_m}{1-g_m} + 1 - \mathbf{a}t - (1-\mathbf{a})g - (1-g) \frac{1 - \mathbf{a}t_m - (1-\mathbf{a})g_m}{1-g_m} \right] r_b.$$

$$r_e(i) > \left[\frac{1-g}{1-g_m} (1-\mathbf{a})(g_m - t_m) + 1 - \mathbf{a}t - (1-\mathbf{a})g \right] r_b.$$

$$r_e(i) > \left[(1-\mathbf{a}) \frac{1-g}{1-g_m} (1-t_m) + \mathbf{a}(1-t) \right] r_b.$$

$$r_e(i) > [(1-\mathbf{a})(1-g)(1-t) + \mathbf{a}(1-t)] r_b.$$

$$r_e(i) > [(1-\mathbf{a})(1-t) + \mathbf{a}(1-t)] r_b.$$

$$r_e(i) > (1-t)r_b.$$

This demonstrates that, for this clientele, the return on equity exceeds the return on debt, so the conjectured portfolio choice is optimal. A similar argument can be used to show that for investors such that the RHS of (3) exceeds the LHS, the return on debt is higher than the return on equity. Thus, all of the conjectured portfolios are optimal. Q.E.D.

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