

Agendas in Multi-Issue Bargaining: When to Sweat the Small Stuff*

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This Draft: January 2006

Abstract

In practice, negotiators deal with numerous issues by ordering them in an agenda, yet in theory separating components of a decision can preclude Pareto-improving tradeoffs. Why then do negotiators address issues separately, rather than all at once? Moreover, what determines the order issues get addressed, and what effect does it have on the final agreement? I characterize an extension of Rubinstein bargaining to the multiple-issue setting and show it has a simple and unique equilibrium agenda. Under this equilibrium issue-by-issue bargaining can ameliorate ex-ante bargaining risk, leading to Pareto improvements. Issue bargaining can also allow bargainers to make *implicit*, if not *explicit*, tradeoffs between issues, and can dramatically change a negotiation's final agreement, with large distributional consequences. My results suggest that issue-by-issue bargaining may be socially optimal and preferred by one or both parties if players are sufficiently asymmetric, or if bargaining frictions are large.

*I am extremely grateful to Dan Benjamin, Drew Fudenberg, Gabor Futo, Matthew Gentzkow, Jerry Green, Emir Kamenica, David Laibson, Mihai Manea, Markus Möbius, Al Roth and workshop participants at Dartmouth, Harvard, Kellogg, and Yale for numerous insightful comments on this and previous drafts.

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Here we have the oldest and most naive canon of justice, *fair play*. Justice, at this level, is good will operating among men of roughly equal power, their readiness to come to terms with one another, to strike a compromise— or, in the case of others, to *force* them to accept such a compromise.

Friedrich Nietzsche; *Towards a Genealogy of Morals*, 1887

1 Introduction

People, firms and nations must regularly commit to common action in situations with conflicting interests, often by means of a bargaining procedure. Indeed as Nietzsche (derogatorily) points out, distributive notions of justice and fairness are often by-product of some underlying bargaining game, under conditions of roughly equal power.

Our intuition on the outcome of such games draws heavily from the Rubinstein model of alternating-offers bargaining. However in practice, most negotiations occur over several separately-addressed issues, an aspect of bargaining the Rubinstein model does not capture. For example political parties bargain over the structure of multiple programs such as taxes, welfare outlays, foreign policy, and domestic spending, but rarely all in the same bill. Examples of multi-issue bargaining are ubiquitous; nations negotiate trade terms in multiple markets; unions and firms bargain over wages, hours, and benefits; couples must decide both which movie and which restaurant to patronize.

In this paper I examine an extension of the Rubinstein model which allows bargainers to address multiple issues. To do this, the procedure must allow bargainers a much wider array of strategies. Possible actions include tendering an offer on any issue, revising past offers, accepting any standing offer, making counter-offers on received proposals, and tabling issues to begin debating another. Underlying this process is a latent risk that bargaining may breakdown, suffered by both parties whenever a disagreement arises over how an issue is to be settled. I show this extension has a unique solution under subgame-perfection and a monotonicity condition, allowing full characterization of equilibrium bargaining.

Since my framework allows bargainers to raise any unresolved issue at any time, the order in which issues arise is a natural vehicle for understanding agenda formation. I show the equilibrium agenda can be produced by a straightforward algorithm which uses only *ordinal* information on how much each bargainer values each issue. This low-informational requirement leads

to the possibility of strong agenda inefficiency; it is possible that both bargainers would prefer that issues be addressed in an order different from the way they are in the unique equilibrium. In addition, the degree to which agenda inefficiency can exist is a function of how correlated player's preferences are.

Using this unique agenda we also ask a more fundamental question about issue-by issue bargaining; why separate issues at all when separating issues may preclude Pareto-improving cross-issue tradeoffs? Here, I provide two possible explanations. First, the distributional effects of separating issues may systematically shift the outcome of bargaining in favor of one party, causing him to prefer the separation of issues. More broadly, if bargaining frictions create significant advantages to moving first, separating issues typically increases *both* agents ex-ante utility. This occurs because as bargaining frictions get large, separate issue bargaining not only ameliorates the ex-ante risk of moving first or second, but also enables bargainers to *implicitly* make inter-issue tradeoffs that were made *explicitly* under collective bargaining. These observations lead to several predictions as to when separation of issues will occur. Issue-by-issue bargaining will be optimal if players are sufficiently asymmetric or if bargaining frictions are large, a comparative static I suggest organizes many real-world observations.

The paper is organized as follows. In Section 2 I illustrate the questions surrounding agenda bargaining, and briefly discuss previous approaches the literature has taken towards answering them. In Section 3 I present my model and characterize its solution. Section 4 presents the algorithm that determines the equilibrium, then characterizes and bounds its inefficiencies. Section 5 discusses predictions which arise from these results and Section 6 concludes.

2 Agenda Approaches

Figure 1 illustrates a type of efficiency loss that can result from separate issue bargaining. Two agents have preferences over two issues with linear frontiers but differing marginal rates of substitution. Assuming both agents total utility is simply the sum of their utilities on issues 1 and 2, the combined bargaining problem is represented by the polygon on the right. Recall that the Nash bargaining solution maximizes the product of agent's utilities; for linear frontiers this is satisfied at the midpoint (Nash 1950). If the agents were to Nash bargain over the two issues separately, they would reach the point labeled $Solution(1) + Solution(2)$. This point however, is strictly

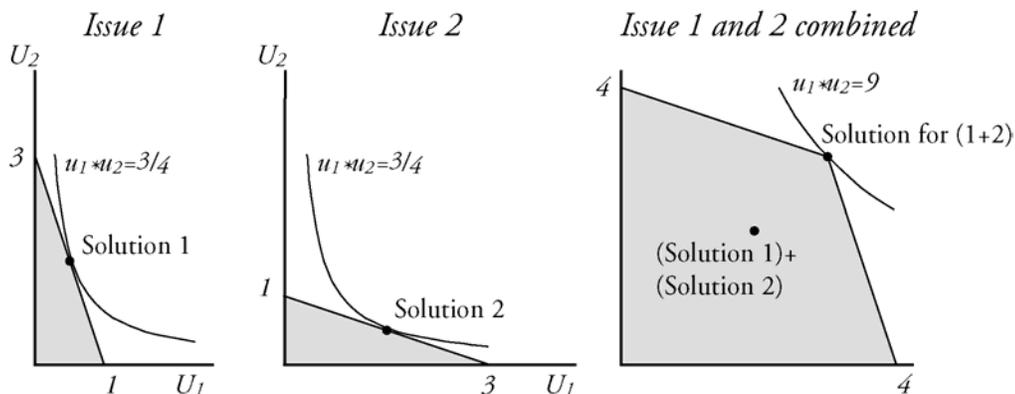


Figure 1: Inefficiency in separate vs. single-issue bargaining.

dominated by the point $Solution(1 + 2)$, which would result if the players Nash bargained over the combined problem. Intuitively, under separate issue bargaining both agents would be better off if they could forfeit all utility on the issue they care about less in exchange for a full concession on the issue they care about more. Since Nash bargaining over the combined problem would result in an efficient total allocation, all such Pareto improving tradeoffs would be exhausted if the agents had bargained over the collective problem.

This example illustrates two barriers to understanding the strategic importance of agendas. First, in this setting it is hard to understand why agents would choose to bargain over issues separately at all. Second, note that the order in which issues are addressed is irrelevant to either player. Anecdotally though, issue order can have large effects on the outcome of a negotiation, and addressing issues separately is often seen as desirable. In these respects, Nash bargaining is inadequate as a multi-issue model. A richer axiomatic framework can produce more reasonable total allocations, see for example O'Neill, Samet, Weiner & Winter (2001). To understand how issues are ordered and what effect that order has, however, requires a non-cooperative approach. In my framework, which issue to broach next is a strategic choice, and what I call the agenda is simply the order in which agents choose to raise those issues.

2.1 Past Attempts

This non-cooperative approach is part of a rapidly growing literature on multi-issue bargaining. Fershtman (1990) studies the case of 2 issues which are both pie-splitting problems. Here a unique subgame-perfect equilibrium is easy to compute, and the effects of going first on either the large or small pie are analyzed.

Another early approach was to simply extend the Rubinstein model by adding a preliminary period in which agents bargain over an agenda. This is the approach taken by Busch and Horstmann (1997); their agents first bargain over an ordering then bargain sequentially over the issue as set forth by that ordering. The addition of a preliminary stage may seem artificial though, and makes the ordering of issues less endogenous than one would like. A model in which agents are free to discuss any issue at any time may be a more natural way of endogenizing the order of discussion; that is the approach taken here. My model also raises the possibility that bargainers can actually find themselves ordering issues inefficiently; i.e. there can exist a different agenda both bargainers would prefer. In my model this happens when bargainers' preferences are highly correlated, and is not possible under a preliminary agenda-setting model. Several recent papers have also taken this approach, including Inderst (2000) and Weinberger (2000). However, these papers are plagued by multiple equilibria even in very simple split-the-pie settings.

Alternatively, a closely related and well developed literature considers the problem of allocations in committee settings. This literature focuses primarily on three things; the impact of coalition formation on final agreements; the role of agendas in achieving coalitional stability, and the non-cooperative foundations of core-allocation implementation. Generally speaking, imposing stationarity on the set of admissible strategies allows papers such as Perry and Reny (1994), and Winter (1997) to provide a non-cooperative basis for achieving core allocations, the latter in multi-issue settings.

The paper with the most similar approach to the one taken here is In and Serrano (2001), which studies a procedure similar to ours but in which the original Rubinstein formulation is followed more literally. In and Serrano provide a partial characterization of the multiple stationary equilibria which emerge, again under the special case of agents splitting zero-sum pies. They also analyze a model in which agents can choose to bargain over issues separately or collectively. Unfortunately, they obtain the negative result that only collective (not issue-by-issue) bargaining is a subgame-perfect.

2.2 Uniqueness and the Agenda

In contrast to these papers, the model I present allows a completely endogenous agenda to emerge, yet obtains a unique subgame-perfect equilibrium when a natural monotonicity restriction is imposed. This uniqueness allows me to study in what order issues will be broached, what effect issue-by-issue bargaining will have on agreements, and when efficiency gains or losses will occur. My model escapes the negative result of In and Serrano by allowing bargainers much more conversational flexibility; I allow offers to be tabled, revised, accepted or rejected at any time. Hopefully this also increases the model's descriptive realism, mirroring the back and forth interchange of actual negotiations. Perhaps most importantly, in my model I can unambiguously characterize when separate-issue bargaining will be preferred by one or both players.

3 Multi-Issue Bargaining Problems

The bargaining literature is one of the oldest in game theory, and my setup follows the canonical model. Two parties have the opportunity to reach agreement on an outcome from a set X of possible agreements. They have agreed to bargain over which $x \in X$ will obtain, and have agreed to abide by the results of a bargaining procedure. Both parties' preferences over X are common knowledge, and both perceive that if they fail to reach agreement the outcome will be some fixed event $D \in X$. For example X could be all possible trading terms for all markets in which 2 countries trade, and D could be the event that no trade takes place. We assume that X is a compact, connected subset of Euclidean space, and that it divides naturally into separate issues along subsets of its dimensions. The set L of separate issues in X is endogenously determined by player's preferences; in this model separate issues are disjoint subspaces of X between which both player's preferences are additively separable.¹

¹This separation comes with a loss of generality, but dramatically simplifies the analysis and has natural appeal. Intuitively, if two components of a decision problem are neither compliments nor substitutes for either player, it seems natural that bargainers would consider them separate issues. See the Appendix on separate issues for a more complete discussion.

3.1 Issues

Formally, let $X \subset \mathbf{R}^n$ and let P be a partition of the dimensions of X into $m \leq n$ subsets. For simplicity I specify X and P such that both players' utilities are additively separable between the subspaces of X indexed by P . Furthermore, let P be the finest partition for which this additive co-separability is true and let X be imbedded in \mathbf{R}^n in such a way that all such additive co-separabilities are captured by P . For example if $X \subset \mathbf{R}^3$ then $P = \{\{1, 3\}, \{2\}\}$ if and only if for both players $i \in \{1, 2\}$:

There exist functions f_i and g_i such that player i 's preferences can be represented as

$$U_i(\langle x_1, x_2, x_3 \rangle) = f_i(\langle x_1, x_3 \rangle) + g_i(\langle x_2 \rangle)$$

and where for at least some player j , U_j is not further separable; i.e. $\nexists f'$ and g' such that

$$f_j(\langle x_1, x_3 \rangle) = f'(\langle x_1 \rangle) + g'(\langle x_3 \rangle).$$

Notation 1 For convenience we denote the set of these additively separable subspaces of X as the set $L = \{I_1, I_2, I_3, \dots, I_m\}$ the set of **separate issues** in X , and $D_1, D_2, D_3, \dots, D_m$ the corresponding components of D , the disagreement outcome.

3.2 The Basic Issue-by-Issue Model

I model multi-issue bargaining as a game in which two players alternate turns making offers.² There is no predetermined bound on the number of rounds of negotiation. Intuitively, players go back and forth making offers until 2 consecutive offers are feasible. All asymmetries between players are exogenous. That is, the first mover is determined prior to play (perhaps by coin flip), and any differences between players are determined solely by their preferences. The bargaining procedure I study is simple:

²Alternating-offer models, though common in the literature, may seem unrealistic. However, alternating offers are often the only stable outcome of a more flexible game, where at any time players are free to make an offer or remain silent. This will be the case if the channel through which bargainers communicate sufficiently discourages both simultaneous and rapid-fire individual offers. See Maskin & Tirole (1988) for an example in a price-setting model, and the appendix of this paper for an extension which relaxes this procedure.

1. The first mover chooses an issue $I \in L$ and proposes that the bargainers choose agreement $a \in I$. Outcome a becomes the **standing offer** on I .
2. The second mover can then choose any issue $J \in L$, and propose agreement $a' \in J$.
 - If $I = J$, and $a = a'$, then the players have **reached an agreement**, and can now only make offers on the set $L - I$.³
 - If $I = J$, but $a \neq a'$, then player 2 has made a counter-proposal and the players have **disagreed** on I . Then with probability $(1 - \beta)$ bargaining **breaks down**⁴ and issue I can never be broached again; bargaining continues over issues $L - I$. With probability β the players continue bargaining over the set L , and a' becomes the standing proposal.
 - If $p \neq q$, then the second mover has **tabled** the first mover's offer on I ; outcome a is the standing proposal on I and a' becomes the standing proposal on J .

The game continues with players alternating offers until all issues in L are either resolved or have ended in disagreement. Whenever it is player i 's turn she can make a proposal on any issue J which has not broken down or been settled.

3. Whenever a proposal is made on issue J and not by the player who made the standing offer, either:
 - (a) the proposals are feasible and issue J is now settled, or
 - (b) a risk $(1 - \beta)$ of breakdown on J occurs. If breakdown does not occur then the new offer replaces the standing offer.
4. If player i makes an offer on an issue J for which i also made the standing offer, then i has **revised** her offer, incurring a risk $(1 - \beta)$ of breakdown. If breakdown does not occur then i 's new offer is the standing offer.

³Abusing notation slightly, I write $L - I$ instead of $L - \{I\}$.

⁴In my model, the incentive for bargainers to reach agreement is that every time they disagree on an issue I they bear a risk of $(1 - \beta)$ that I will revert to its disagreement outcome. This assumption is discussed in the section on payoffs and in the Appendix, section 7.3.

5. An issue is settled whenever two offers in a row by different agents propose the same agreement.

3.2.1 Payoffs

Completing the model, both players are assumed to have continuous preferences U_i over the set of agreements X . Whenever an issue is settled both players receive their payoffs on that issue, regardless of the future path of play. Furthermore, when bargaining breaks down both agents receive their disagreement payoffs on that issue, but can continue bargaining on any remaining issue. Time-discounting is assumed to be negligible over the course of negotiations; all bargaining frictions derive from the risk of breakdown induced by disagreement. This full separability of issues models bargaining situations in which the separate implementation of issues is feasible, and in which bargainers dispassionately treat past disagreements as sunk.⁵ This payoff structure and the additive separability of U_i over the issues in L is equivalent to the first of our axioms:

Axiom 1 *For each player i there exists continuous functions*

$$u_i : \cup I_j \longrightarrow R \text{ and } U_i : X \longrightarrow R$$

such that the preferences \succsim_i of player i are represented on every issue $I \in L$ by u_i , and such that player i 's total utility $U(x)$ is just the sum $\sum u_i$ of her utilities over the components of x , i.e. the projections of x into the subspaces indexed by L .

Note that when the set L is simply $\{X\}$, this model is equivalent to Rubinstein's (1982) bargaining game of alternating offers. This model differs from others recent attempts, however, in how it extends to multiple issues . Figure 2 shows the extensive form of traditional Rubinstein bargaining.

In this extensive form, bargainers take turns making offers and eliciting a responses from the other. Thus each stage game actually comprises a move by both players, with consecutive stages differing in which player gets to make the first move. Hence most extensions are not truly alternating-move games. In particular the first move of the first player (by player 1 in Figure 2) is simply an offer, not an acceptance or rejection followed by an offer. I propose a model in which this asymmetry is removed but which is equivalent to the above form when the number of issues is one. Figure 3 is the extensive form of the first two stages:

⁵Ex-ante we do not assume agents cannot condition on past disagreements, just that the past is sunk with respect to their utilities. For a more complete discussion of this

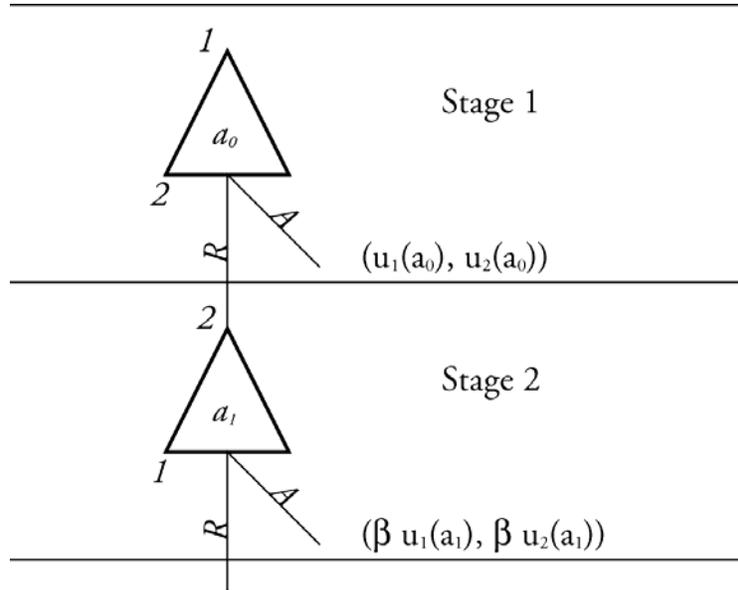


Figure 2: The first 2 periods of Rubinstein bargaining.

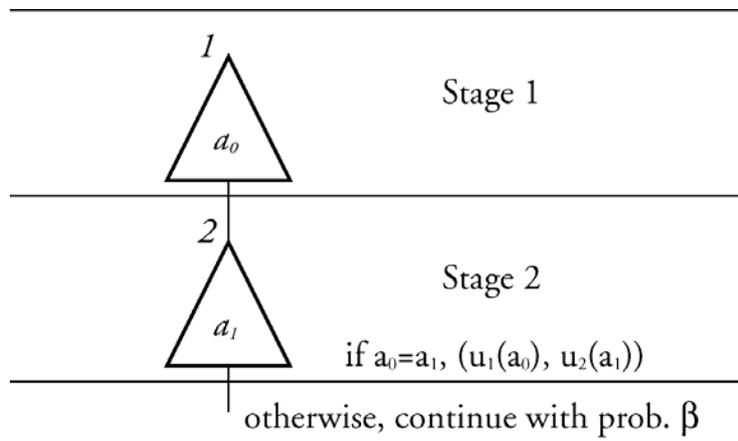


Figure 3: The first 2 periods of my modified game.

In this game form players really do alternate making offers. The role played above by the acceptance/rejection move is instead merged into the transition rule from one stage to the next. Here, identical offers signal agreement while mismatched offers lead to disagreement and the possibility of breakdown. It is easy to see that this simplification has no substantive effect on the structure of one issue bargaining models above. When extended to multiple issues however, this removal of an acceptance and rejection stage tremendously simplifies the set of equilibria, as we will demonstrate in the next section. In specific, this modification allows bargainers to table proposals and consider other issues first, without requiring (though not excluding) the agent's immediate rejection of a received proposal.

Also, this model differs from existing models in which β is treated as a rate of time discounting. Here we reinterpret β as the ability of the first mover to force the second mover to either accept her proposal or face the hazards of disagreement, not an expression of the impatience of agents.⁶ This reinterpretation has an obvious advantage. Given that most negotiations take place over the span of hours or days and not years, any β significantly less than 1 is very difficult to calibrate with reasonable levels of time discounting. In practice though, there are often significant asymmetries in bargaining outcomes, requiring implausibly high bargaining frictions (low β 's) to exist over short time spans. The reinterpretation of β as a probability of breakdown makes plausible values significantly lower than 1, providing a more compelling explanation for the existence of sizable advantages from being the first mover.⁷ Again, this modification has no structural effect on the Rubinstein model in the single-issue case but dramatically changes the equilibrium outcome in the multi-issue case.

3.3 Equilibrium Bargaining

Now I characterize the equilibrium path of play in this model. In this paper I employ **Monotonicity**⁸ to restrict attention to simple equilibria, and **Subgame-Perfection** to eliminate non-credible threats. This is much

structure and the implications of relaxing it, see the Appendix on separate issues.

⁶Indeed, the traditional Rubinstein model has been reinterpreted in just this way in Binmore, Rubinstein and Wolinsky (1986).

⁷This intuition follows from the unique equilibrium payoff to each bargainer and its dependence on β , which we will characterize as a comparative static in the next section.

⁸This refinement imposes a very natural monotonicity condition on the rejection of offers. Intuitively, holding all other issues constant if player 1 makes a worse offer on issue I (demands greater concessions from 2,) it should only make player 2 *more* likely to reject 1's offer. See the proof of Theorem 3 in the Appendix for a full definition.

weaker than Stationarity, the refinement commonly used in the bargaining literature. From here on I refer to subgame-perfect equilibria as *SPE* (or simply equilibria) and monotone subgame-perfect equilibria as *MSPE*. It turns out that under a few simplifying assumptions on the structure of the set of issues L , restricting attention to MSPE leaves us with a unique equilibrium path and outcome. This stands in contrast to existing models of multi-issue bargaining which either have a multiplicity of stationary equilibria supporting an infinite number of non-stationary equilibria, or the added complication of incomplete information. The structure we place on every issue $I \in L$ mirrors Rubinstein's original formulation. The first assumption eliminates redundancies:

Axiom 2 *For every issue I and two distinct agreements $x, y \in I$ it is not the case that both*

$$u_1(x) = u_1(y) \text{ and } u_2(x) = u_2(y).$$

Were this ever violated we could simply identify the two agreements and bargain over the subsequent problem.

Axiom 3 *For every issue I_j the corresponding disagreement outcome D_j has utility 0 for both agents, and happens when the other agent gets her best agreement.*

In other words,

$$u_1(D_j) = u_2(D_j) = 0 = u_1(B_j^2) = u_2(B_j^1), \text{ where}$$

Notation 2 B_j^i *is the best agreement for i in issue I_j .*

This axiom comes without loss of generality, since assuming additive separability allows us to replace the function

$$U_i(x) = u_i(x_1) + u_i(x_2) + \dots + u_i(x_m)$$

with the function

$$U'_i(x) = [u_i(x_1) + z_1] + [u_i(x_2) + z_2] + \dots + [u_i(x_m) + z_m]$$

where z_j is any Real number.

The next axioms refer to characteristics of the Pareto frontier or the set of **efficient agreements**, where

Definition 1 An agreement $x \in I$ is **efficient** if and only if $\neg \exists y \in I$ with $y \succ_i x$ for $i = 1, 2$.

Axiom 4 For every issue I the Pareto Frontier of I is strictly monotone. In other words if an agreement x is efficient there is no other agreement y such that $y \succ_i x$ for both players.

This is satisfied in every bargaining situation where parties can make small monetary transfers to one another.

Axiom 5 For every issue I there is a unique pair of efficient agreements $a, b \in I$ such that $u_1(b) = \beta u_1(a)$ and $u_2(a) = \beta u_2(b)$.

A sufficient condition for this assumption to be satisfied is if for every I the utility possibility set $\{ \langle u_1(x), u_2(x) \rangle : x \in I \}$ is convex. This holds trivially if the bargaining set also includes lotteries over agreements. A weaker condition that also produces axiom 5 is that the Pareto frontier of every issue I can be described by some concave function f .

In his original 1982 paper Rubinstein shows that under similar axioms his bargaining model admits a unique subgame-perfect equilibrium and that the pair $\langle a, b \rangle$ from axiom 5 completely determines the equilibrium outcome. That is:

Theorem 1 (Rubinstein 1982) Under axioms 1–5, the Rubinstein model of bargaining on a single issue I has a unique SPE characterized by the pair $\langle a, b \rangle$. In that equilibrium if player 1 (2) goes first she proposes a (b) and player 2 (1) immediately accepts.

Notation 3 For every issue I we denote the unique pair of agreements $a, b \in I$ as the **Rubinstein solution** to issue I , giving us the solution value for player i

$$r_I^i = \begin{cases} a & \text{if } i = 1, \\ b & \text{if } i = 2. \end{cases}$$

An additional theorem which will be extremely useful in examining the efficiency of agreements under the multi-issue case was shown by Binmore in 1987.

Theorem 2 (Binmore 1987) If we take the single issue bargaining problem I and let β approach 1, the game of alternating offers implements the Nash bargaining solution. That is as $\beta \rightarrow 1$, r_I^i converges to the $\arg \max_{x \in I} \{u_1(x) \cdot u_2(x)\}$.

Now we turn our attention to the full multi-issue game of alternating offers.

3.4 The Multi-Issue Equilibrium

We inductively define two functions that will determine the equilibrium path of offers and counter-offers in the multi-issue game with fully separable agreements. These jointly defined functions specify two things:

1. the issue on which an offer is made by a player when faced with the ability to do so,
2. and the value function of facing such a set.

Notation 4 Denote the *value function* for player i when faced with a subset $S \subset L$ as $V_i(S)$, and

Notation 5 Denote the *offer function* for player i when faced with a subset $S \subset L$ as $O_i(S)$.

Definition 2 We define $O_i(S)$ and $V_i(S)$ recursively by induction on the $|S| \in \mathbf{N}$, letting

$$\begin{aligned} O_i(S) &= \{I\} \text{ when } |S| = 1, S = \{I\}, \\ V_1(S) &= u_1(a) \text{ when } S = \{I\} \text{ and } r_I^1 = a, \\ \text{and } V_2(S) &= u_2(b) \text{ when } S = \{I\} \text{ and } r_I^2 = b. \end{aligned}$$

In words, each player makes the first offer on the last unacceptable or untouched issue if given the chance, and propose what they would have under the standard Rubinstein model. Continuing,

$$\begin{aligned} O_i(S) &= \arg \max_{x \in S} (V_i(\{x\}) + \beta V_i(S - \{x\})), \text{ when } |S| = 2, \\ V_i(S) &= V_i(O_i(S)) + \beta V_i(S - O_i(S)), \text{ when } |S| = 2, \\ O_i(S) &= \\ \arg \max_{x \in S} &(V_i(\{x\}) + \beta V_i(O_{-i}(S - \{x\})) + V_i(S - \{x\} - O_{-i}(S - \{x\}))) \\ &\text{when } |S| \geq 3, \\ V_i(S) &= V_i(O_i(S)) + \beta V_i(O_{-i}(S - O_i(S))) + V_i(S - O_i(S) - O_{-i}(S - O_i(S))) \\ &\text{when } |S| \geq 3. \end{aligned}$$

So player i chooses which of two issues left to make an offer on by maximizing the sum of a first mover payoff on one and second mover payoff (β times first mover payoff) on the other. When choosing between n choices the agent

maximizes the payoff of choosing x and leaving her opponent to choose over $S - \{x\}$, allowing him to choose over $S - \{x\} - O_{-i}(S - \{x\})$, and so on.⁹

To simplify our proofs we adopt the following notation:

Notation 6 Let the **state of** I be represented by $s : L \rightarrow \cup I_j$, where

$$s(I_j) = \begin{cases} D_j & \text{if no offer has been made on } I_j, \\ x_j^i \in I_j & \text{if } i \text{ made the last proposal } x \text{ on } I_j, \text{ which is still open,} \\ \bar{x}_j \in I_j & \text{if } I_j \text{ has been settled with } x, \\ \bar{D}_j & \text{if } I_j \text{ has broken down.} \end{cases}$$

Notation 7 Let S_h represent the **state of** L after history h , where

$$S_h = \langle (s(I_1), s(I_2), \dots, s(I_m)) \rangle$$

This leads us to our first proposition the characterization of a SPE in the multi-issue game of alternating offers.

Proposition 1 (Equilibrium for Multi-Issue Bargaining) *A bargaining game of alternating offers $\langle L, (\succsim_i) \rangle$ which satisfies assumptions 1 – 5 admits the strategies described below as a SPE.*

The strategies involve two stages; a proposal / counter-proposal stage and an acceptance stage.

3.4.1 Proposal Stage

Definition 3 *When it is Player i 's turn, she first looks at the set of **unacceptable issues**,*

$$\mathcal{T}_i = \{I \in L : \begin{cases} I \text{ is open, and} \\ u_i(s(I)) < \beta u_i(r_I^i) & \text{if } -i \text{ made the last offer on } I, \\ u_i(s(I)) < u_i(r_I^i) & \text{otherwise;} \end{cases}$$

where

Notation 8 *An issue I is **open** if neither agreement or breakdown on I has occurred.*

⁹In defining $O_i(S)$ we ignore the case in which an argmax is multi-valued. This will generically be the case, and when it is not the analysis does not change in a substantive way.

This is the set of issues on which either no one has bid or the standing bid is off-equilibrium path, i.e. less than the Rubinstein solution to that issue.

If the set \mathcal{T}_i is non-empty, i proposes $r_i^{O_i(\mathcal{T}_i)}$ on issue $O_i(\mathcal{T}_i)$. When $\mathcal{T}_i = \emptyset$ player i leaves the proposal stage and enters the:

3.4.2 Acceptance Stage

Definition 4 *Player i looks at the set of **pending issues**,*

$$\mathcal{U}_i = \{I_j \in L : I_j \text{ is open and } -i \text{ made the standing offer}\}.$$

If the set \mathcal{U} is non-empty she agrees to the standing proposal on $O_i(\mathcal{U})$ by proposing $s(O_i(\mathcal{U}))$. If the set \mathcal{U} is empty, then player i has no legal moves left and is done bargaining.

To prove this proposition when we have more than one issue we first show that the bargaining game of alternating offers satisfies the **one-stage deviation principle**. That is,

Lemma 1 *Any strategy profiles s_1 and s_2 form a subgame-perfect equilibrium if and only if there does not exist a history up to time t such that either player can profit by changing her action at t alone.*

Proof of Lemma 1 *We will show that players payoffs are continuous in the limit as time goes to infinity.*

Let $|L| = m$.

Note that after an offer has been made on an issue, any subsequent offer either settles the issue or risks breakdown, and players cannot make offers on issues which have either broken down or have been settled. Hence the probability that at time t fewer than m breakdowns or settlements have occurred is a Poisson process.

Therefore $\forall \varepsilon > 0$ there exists a time t such that the probability that any issues remain open at t is $< \varepsilon$. But only open issues affect player's payoffs. Consequently the bargaining process satisfies **continuity at infinity**,¹⁰ which implies the one-stage deviation principle.

This finishes lemma 1. \square

We now use the one-stage deviation principle to prove our proposition.

¹⁰For a proof that continuity at infinity implies the one-stage deviation principle see Fudenberg and Tirole (1991).

Proof of Proposition 1 Let s_1 and s_2 be the strategies of players 1 and 2 as described in the proposition.

We look at a history h and state S_h . Note that s_1 and s_2 treat identically any history which results in the same state, so we can suppress the history and refer to S . Recall that:

$$\mathcal{T}_i = \begin{cases} I \text{ is open,} \\ I \in L : \begin{array}{ll} u_i(s(I)) < \beta u_i(r_I^i) & \text{if } -i \text{ made the last offer on } I, \\ u_i(s(I)) < u_i(r_I^i) & \text{otherwise;} \end{array} \end{cases}$$

Without loss of generality assume it is player 1's move after history h . We examine two possible sets of history/state pairs; pairs in which $\mathcal{T}_1 \neq \emptyset$ and in which $\mathcal{T}_1 = \emptyset$.

Case 1 (Proposal Stage) Assume $\mathcal{T}_1 \neq \emptyset$.

If player 1 deviates from s_1 once and then never again, she either:

1. makes an offer of b different than $r_1^{O_1(\mathcal{T}_1)}$,
2. bids on an issue $J \neq O_1(\mathcal{T}_1)$,
3. or both.

If she does the first she has either bid too aggressively or not enough. If she has bid too aggressively, then:

$$u_2(b) < \beta u_2(r_{O_1(\mathcal{T}_1)}^2).$$

But then b will not be accepted, issue $O_1(\mathcal{T}_1)$ remains in \mathcal{T}_{-1} , and player 1 has simply lost a move.

Claim: If players return to playing s_1 and s_2 , this is equivalent to player 1 having made an offer of r_J^1 on issue J where

$$J = \arg \min_{I \in \mathcal{T}_1} u_2(r_I^2) - u_2(r_I^1).$$

We postpone proving this claim since it is an immediate corollary of our analysis in section 6.

Since $O_1(\mathcal{T}_1)$ was the argmax of all possible choices in \mathcal{T}_1 , this leaves 1 weakly worse off. Hence 1 did not profit from bidding too aggressively.

If 1 has not bid aggressively enough, she could have gotten a bid strictly better for himself accepted.

If 1 bids on an issue $J \neq O_1(T_1)$, by the construction of the function $O_1(\cdot)$ she can only have made herself worse off (weakly) since $O_1(\cdot)$ is precisely the arg max of player 1's utility under s_1 and s_2 . If she does the third and has bid r_J^1 then the same argument as in the second applies. If she has bid $b \neq r_J^1$ then the same argument as in the first applies.

This finishes case 1.

Case 2 (Acceptance Stage) Assume $T_1 = \emptyset$.

Recall

$$\mathcal{U}_i = \{I_j \in L : I_j \text{ is open and } -i \text{ made the standing offer}\}.$$

If $\mathcal{U}_1 = \emptyset$, then we are done.

Otherwise, by deviating player 1 either:

1. makes an offer of b different than $s(O_1(\mathcal{U}_1))$,
2. proposes on an issue $J \neq O_1(\mathcal{U}_1)$,
3. or both.

If she does the first or the third she risks breakdown, and receives at most

$$\beta u_1(r_J^1) \leq s(O_1(\mathcal{U})).$$

If she does the second she either doesn't change the outcome (J is an issue she would have accepted eventually anyway), or risks breakdown unnecessarily revising a proposal, which doesn't improve her payoff. If she does both and has bid r_J^1 then the same argument as in the second applies. If she has bid $b \neq r_J^1$ then the same argument as in the second case applies.

This finishes case 2.

So we have proven Proposition 1. \square

We now turn to the more difficult task of proving that the subgame-perfect strategies described above are essentially unique. This is our second proposition:

Proposition 2 (Uniqueness of Equilibrium) *If monotonic strategies s'_1 and s'_2 are subgame perfect, then all issues must be settled exactly as in s_1 and s_2 , our equilibrium strategies from Proposition 1.*

I postpone proving this theorem until the mathematical appendix in order to characterize the order in which issues get addressed.

4 The Agenda

4.1 Characterizing the Equilibrium Agenda

Again, consider the set of issues $L = \{I_1, I_2, I_3, \dots, I_m\}$. Recall that for each issue I_j , there are two agreements $a_j, b_j \in I_j$, where

$$u_1(b_j) = \beta u_1(a_j) \text{ and } u_2(a_j) = \beta u_2(b_j)$$

From this we see that each player has a corresponding utility surplus associated with being the first mover on any given issue. In other words, player 1 earns $u_1(a_j) - u_1(b_j)$ more utility from proposing rather than accepting the equilibrium agreement on issue I_j , and vice versa for player 2. We use this surplus to define, \prec_i the **issue order relation** for player i over the set of issue L .

Definition 5 Let \prec_i be the order relation which ranks the issues of L from greatest to least utility surplus for player i .¹¹

For expositional simplicity, we adopt the following notation:

Notation 9 Let $w^i(S) : \mathcal{P}(L) \longrightarrow L$ be the **worst issue** for player i from the set of issues $S \subset L$, i.e.

$$w^i(S) = I_j, \text{ where } S = \{I_j \prec_i I_x \prec_i \dots \prec_i I_y\}$$

We now define inductively a sequence a_j^i which we will call the **agenda ordering**. This issue a_j^i will be the j^{th} issue raised if player i moves first. Hence this sequence fully describes the agenda which arises in equilibrium bargaining.¹²

Definition 6 The agenda ordering sequence $a_1^i, a_2^i, a_3^i, \dots, a_m^i$ is defined recursively as follows:

$$a_m^i = \begin{cases} w^i(L) & \text{if } m \text{ is even,} \\ w^{-i}(L) & \text{if } m \text{ is odd} \end{cases}, \text{ and}$$

$$\forall j < m, a_j^i = \begin{cases} w^i(S) : S = L - \{a_{j+1}^i, \dots, a_m^i\} & \text{if } j \text{ is even,} \\ w^{-i}(S) : S = L - \{a_{j+1}^i, \dots, a_m^i\} & \text{if } j \text{ is odd.} \end{cases}$$

¹¹For simplicity we abuse notation and look at a players (cardinal) preferences over issue surpluses, rather than agreements. I ignore the case of equality of surplus; this does not substantially changes any subsequent results.

¹²Special thanks to Gabor Futo, who suggested the form of this sequence.

In words, the following algorithm determines the order in which issues will be considered:

Algorithm 1 (Endogenous Agenda) *If there are an even (odd) number of issues, the first (second) mover's worst issue will be considered last. This process repeats using the player's ranking over remaining issues, with the second mover now the first mover and vice versa.*

It is worthwhile to note several interesting properties of this order function, an example of which is illustrated in Figure 4.

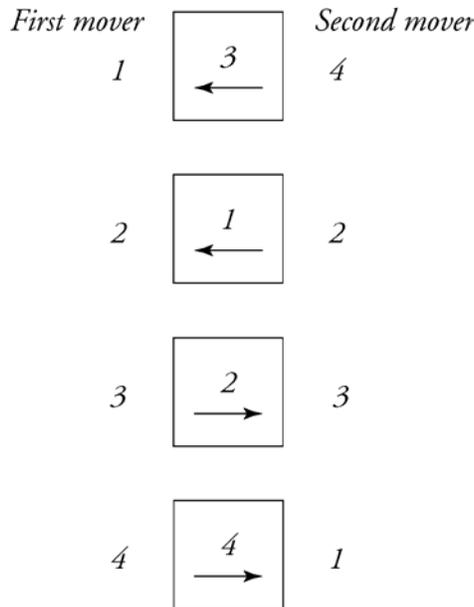


Figure 4: An example of the agenda algorithm. The left and right columns are how players rank the issues, and the center numbered arrows indicate the order of offers.

Remark 1 *Note that the agenda which arises endogenously depends only on the ways players rank issues, and not on the relative importance of those issues.*

The example in Figure 4 demonstrates the basic comparative static that determines when a player makes an offer on an issue. Note that the first

mover makes her first offer on her second most important issue and only latter makes an offer on her most important issue. This is because the first mover's most important issue is of little importance to the second mover and addressing it is not urgent.

Proposition 3 (Endogenous Agenda) *The inductive function defined above defines the unique subgame-perfect agenda of the multi-issue bargaining game (generically). That is if the first mover is i , she makes the first offer on issue a_1^i , then player $-i$ makes the first offer on issue a_2^i, \dots*

Remark 2 *By Zermelo's argument we know that there exists a unique subgame-perfect agenda (generically), and that all subgame-perfect agendas hold the same value for the first mover.*

Remark 3 *It is trivial to note that for $|L| = m \in \{1, 2\}$ the algorithm produces the equilibrium agenda, and easy enough to prove by inspection for $m = 3$.*

Proof of Proposition 3 *We proceed by strong induction on the number of issues in L .*

First, note that if we prove player 1 cannot increase her payoff by deviating, then we are done since after 1 has moved 2 faces an identical problem, only on a game with one fewer issues.

Let us assume that $\forall n < k, |L| = n \Rightarrow O_i$ determines a subgame-perfect equilibrium agenda, which attains for player 1 her optimal payoff. We will now show it holds true for $n = k$. First we prove a Lemma which will simplify the proof of our proposition.

Lemma 2 (First-Mover Advantage) *For any bargaining game S with a number of issues $n < k$, both players weakly prefer moving first to moving second.*

Proof of Lemma 2 *First, note that by our inductive hypothesis that the agenda algorithm determines the unique subgame-perfect distribution of issues.*

We will show that moving first, a player can always achieve the same distribution which would have occurred had they moved second. Since she could have achieved what she would have gotten had she moved second, she must at least weakly prefer what she gets when she moves first.

For expositional simplicity we prove this first for when the number of issues n is even, then we show it for n odd. Recall that:

$$a_1^i, a_2^i, \dots, a_n^i$$

is that agenda for the game when player i moves first, and

$$a_1^{-i}, a_2^{-i}, \dots, a_n^{-i}$$

when i moves second. Since n is even, when i moves first she proposes on issues

$$a_1^i, a_3^i, \dots, a_{n-1}^i$$

and on issues

$$a_2^{-i}, a_4^{-i}, \dots, a_n^{-i}$$

when she moves second.

We now show she can have achieved what she would have received when moving second, when she moves first. How? Player i could deviate from the strategy laid out by $a_1^i, a_2^i, \dots, a_n^i$ by first proposing on player $-i$'s worst issue $w^{-i}(L)$ instead of a_1^i . Player i is then the second mover in the game $S - \{w^{-i}(S)\}$. But note that:

$$w^{-i}(L) = a_n^{-i}.$$

Applying the agenda algorithm to the remaining game $L - \{a_n^{-i}\}$ gives us precisely the sequence $a_1^{-i}, a_2^{-i}, \dots, a_{n-1}^{-i}$ as an agenda which maximizes the payoff to player $-i$. The resulting agenda is

$$a_n^{-i}, a_1^{-i}, a_2^{-i}, \dots, a_{n-1}^{-i}$$

giving player i the first move on issues

$$a_n^{-i}, a_2^{-i}, a_4^{-i}, \dots, a_{n-2}^{-i}.$$

But this is precisely what she gets when she goes second. This demonstrates our first-mover advantage lemma in even games.

Now note that when n is odd, an analogous argument holds. When player i moves first, she proposes on issues $a_1^i, a_3^i, \dots, a_n^i$ when she moves first, and issues $a_2^{-i}, a_4^{-i}, \dots, a_{n-1}^{-i}$ when she moves second. If she would have preferred $a_2^{-i}, a_4^{-i}, \dots, a_{n-1}^{-i}$ to $a_1^i, a_3^i, \dots, a_n^i$ however, she could have achieved something *strictly* better moving first, by first proposing on her worst issue $w^i(S)$. Applying the agenda algorithm, if player i moves first on her worst issue, she receives:

$$a_2^{-i}, a_4^{-i}, \dots, a_{n-1}^{-i} \cup w^i(L).$$

That is, she would have received everything she got by moving second, plus the additional benefit of her worst issue. Since she could have received something strictly better than what she would have received by moving second, our first-mover advantage lemma holds for odd games as well.

Therefore we are done with Lemma 2. \square

Now for any game $|L| = k$, let the sequence $a_1^i, a_2^i, \dots, a_k^i$ be the agenda determined by the algorithm. Note that once the first mover has chosen an issue to make the first offer on, by our inductive hypothesis the order determined by the algorithm on the remaining game maximizes her payoff. But if the first mover proposed first on a_1^i , the subsequent agenda would simply be a_2^i, \dots, a_k^i . Therefore to prove that the sequence $a_1^i, a_2^i, \dots, a_k^i$ is a subgame-perfect agenda it is enough to show that the first mover cannot profitably deviate in the first round. Towards this we examine 2 cases:

1. i deviates by proposing first on something she proposes on under $a_1^i, a_2^i, \dots, a_k^i$, but not a_1^i ,
2. i deviates by proposing first on something she does not propose on under $a_1^i, a_2^i, \dots, a_k^i$.

Case 1 Assume that player i chooses to move first on issue a_j^i , something she would have proposed on but not a_1^i .

By assumption j is odd and $j \neq 1$. By our inductive assumption the game proceeds on the remaining game $S - \{a_j^i\}$ by the agenda algorithm. Let that sequence be $b_1^{-i}, b_2^{-i}, \dots, b_{k-1}^{-i}$. Note that:

$$a_k^i = w^i(S).$$

now if $j < k$, then a_k^i is still in the reduced game, and hence

$$b_{k-1}^{-i} = w^i(S) = a_k^i.$$

Repeating this argument, we get that:

$$\forall l > j, b_{l-1}^{-i} = a_l^i.$$

Then when $l = j$, b_{l-1}^{-i} would have been a_j^i , but that issue has already been broached. Therefore

$$b_{l-1}^{-i} = w^{-i}(\{a_1^i, a_2^i, \dots, a_{j-1}^i\}),$$

and the sequence continues, (possibly) differently then $a_1^i, a_2^i, \dots, a_l^i$. The final agenda is then:

$$a_j^i, b_1^{-i}, b_2^{-i}, \dots, b_{k-1}^{-i} = a_j^i, b_1^{-i}, b_2^{-i}, \dots, b_{j-1}^{-i}, a_{j+1}^i, a_{j+2}^i, \dots, a_k^i.$$

Note that this differs from $a_1^i, a_2^i, \dots, a_k^i$ only up to issue b_{j-1}^{-i} . Now note that under $a_1^i, a_2^i, \dots, a_k^i$ player i proposes on issues

$$a_1^i, a_3^i, \dots, a_{j-2}^i, a_j^i, a_{j+2}^i, \dots, a_{k-1}^i,$$

and under $a_j^i, b_1^{-i}, b_2^{-i}, \dots, b_{j-1}^{-i}, a_{j+1}^i, a_{j+2}^i, \dots, a_k^i$, proposes on

$$a_j^i, b_2^{-i}, b_4^{-i}, \dots, b_{j-1}^{-i}, a_{j+2}^i, a_{j+4}^i, \dots, a_{k-1}^i,$$

Now note that

$$a_1^i, a_3^i, \dots, a_{j-2}^i$$

is also what player i proposes on when she moves first on the game $\{a_1^i, a_2^i, \dots, a_{j-1}^i\}$. But $b_2^{-i}, b_4^{-i}, \dots, b_{j-1}^{-i}$ is what player i gets when she moves second on the game $\{a_1^i, a_2^i, \dots, a_{j-1}^i\}$.

By our first-mover advantage lemma, this cannot leave player i better off. Hence the initial deviation of i to a_j^i leaves him weakly worse off.

This finishes case 1.

Case 2 Assume that player i chooses to move first on issue a_j^i , something she does not originally propose on.

By assumption j is even. By our inductive assumption the game proceeds on the remaining game $L - \{a_j^i\}$ by the agenda algorithm. Let that sequence be $b_1^{-i}, b_2^{-i}, \dots, b_{k-1}^{-i}$. Similar to Case 1, we have that:

$$\forall l > j, b_{l-1}^{-i} = a_l^i.$$

Note b_{j-1}^{-i} (which $-i$ proposes on) would have been

$$w^i(\{a_1^i, a_2^i, \dots, a_j^i\}) = a_j^i.$$

But i has already proposed on a_j^i , so

$$b_{j-1}^{-i} = w^i(\{a_1^i, a_2^i, \dots, a_{j-1}^i\}).$$

Hence, looking only at the distribution of issues $a_1^i, a_2^i, \dots, a_j^i$ (since all other issues are distributed identically with or without the initial deviation,) we see that under the deviation of moving first by proposing first on a_j^i , player i gets a_j^i , plus those issues she gets by moving second in the odd game $\{a_1^i, a_2^i, \dots, a_{j-1}^i\}$, while if she had not initially deviated she would have gotten those issues she receives when moving first on $\{a_1^i, a_2^i, \dots, a_j^i\}$.

Second, note that since j is even the agenda algorithm assigns the same issues to player i regardless if she goes first in the game $\{a_1^i, a_2^i, \dots, a_j^i\}$ or the game $\{a_1^i, a_2^i, \dots, a_{j-1}^i\}$. This is because in an even game the first mover's worst issue always gets addressed last and goes to the second mover; deleting it therefore does not affect the payoff of the first mover.

But now note that moving first in the game $\{a_1^i, a_2^i, \dots, a_{j-1}^i\}$, player i could have achieved what she would have achieved by moving second *plus* the issue a_{j-1}^i , just by moving first on a_{j-1}^i . By our inductive hypothesis this could not have helped player i . Summarizing, we have that:

What i would have received had she not initially deviated equals
what she gets by moving first in the game $\{a_1^i, a_2^i, \dots, a_j^i\} =$
what she gets by moving first in the game $\{a_1^i, a_2^i, \dots, a_{j-1}^i\}$,

which is better (by our inductive hypothesis) for i than what she gets by moving second in $\{a_1^i, a_2^i, \dots, a_{j-1}^i\}$ *plus* the issue a_{j-1}^i .

But recall that by initially deviating by first proposing on a_j^i , player i received what she gets by moving second in $\{a_1^i, a_2^i, \dots, a_{j-1}^i\}$ *plus* the issue a_j^i . But by our agenda algorithm, $a_j^i = w^i(\{a_1^i, a_2^i, \dots, a_j^i\})$, and is in particular worse for i than a_{j-1}^i . Therefore she prefers not to have initially deviated by proposing on a_j^i .

This finishes case 2.

This completes our proof of Proposition 3. \square

Now that we have characterized the subgame-perfect agenda, note an immediate corollary:

Corollary 1 *Any deviation by player i from the strategy given by our algorithm which results in a different distribution of issues results in a strictly dominated list of issues for i .*

That is, there is a one-to-one function f from the set of issues i proposes on to the set she proposes on after the deviation such that i ranks x weakly

higher than $f(x)$. This follows immediately from the fact that the agenda was determined only by the way players *ranked* various issues, not by *relative magnitudes* of utility surplus earned on those issues.

4.2 Efficiency and Inefficiency in the Agenda

Descriptively, this procedure mirrors the way agendas form in many actual bargaining situations. The order in which issues are discussed is determined by when agents choose to raise them in the course of free-flowing dialog. One interesting feature of this procedure is that it raises the possibility of an *inefficient agenda*. That is, there can exist a different distribution of first-proposal rights that both agents would find preferable to the one which occurs in equilibrium. This is easiest to see in a situation where preferences are perfectly correlated. For example, assume that both players rank all issues identically. The agenda algorithm gives the first player her odd ranked issues and the second player her even ranked issues. Depending on the actual utility surpluses however, the players may want to trade issues $\{1, 5\}$ for issues $\{2, 4\}$. Despite this possibility, we can prove a partial-efficiency result which guarantees that the final distribution of first proposals on issues cannot be drastically inefficient.

Proposition 4 (No 1-to-1 Trades) *Let S be the set of issues i proposes on in equilibrium, and let T be the set of issues $-i$ proposes on in equilibrium. Then there does not exist an $s \in S$ and $t \in T$ such that i and $-i$ would prefer to trade respective first proposer advantages on s and t .*

Proof of Proposition 4 We must show that $\nexists s, t$ such that i ranks $t \succsim s$ and $-i$ ranks $s \succ t$.

Let $s = a_j^i$ and $t = a_k^i$ where $a_1^i, a_2^i, \dots, a_m^i$ is the equilibrium agenda. Note that

$$a_j^i = w^{-i}(\{a_j^i, a_{j-1}^i, \dots, a_1^i\}),$$

and

$$a_k^i = w^i(\{a_k^i, a_{k-1}^i, \dots, a_1^i\}).$$

This means that $-i$ will not trade issue a_k^i for a_j^i if $k < j$. But likewise, i will not trade issue a_j^i for a_k^i if $j < k$.

Since $j \neq k$, one player must be unwilling to trade.

This proves Proposition 4. \square

Additionally, inefficiencies in the agenda arise mostly when the ranking of issues between parties are highly correlated. Note that high ranking correlation occurs when parties largely agree on which issues are worth large amounts and when the Pareto frontiers of each issue have relatively equal slopes. Losses due to an inefficient agenda are also a function of β since as the risk of breakdown vanishes, so does the importance of the agenda. Consequently, as the risk of breakdown vanishes the losses due to inefficient agendas go to 0. Offsetting agenda-based losses though, β 's strictly less than 1 tend to decrease another form of inefficiency found in separate-issue bargaining.

4.3 Efficiency and Inefficiency in the Separation of Issues

In Figure 5 we use the fact that as $\beta \rightarrow 1$ the bargaining solution approaches the Nash solution (Theorem 2) to demonstrate another basic inefficiency of multi-issue bargaining. That is, as $\beta \rightarrow 1$ separate-issue bargainers lose the ability to exploit differing marginal rates of substitution between issues.

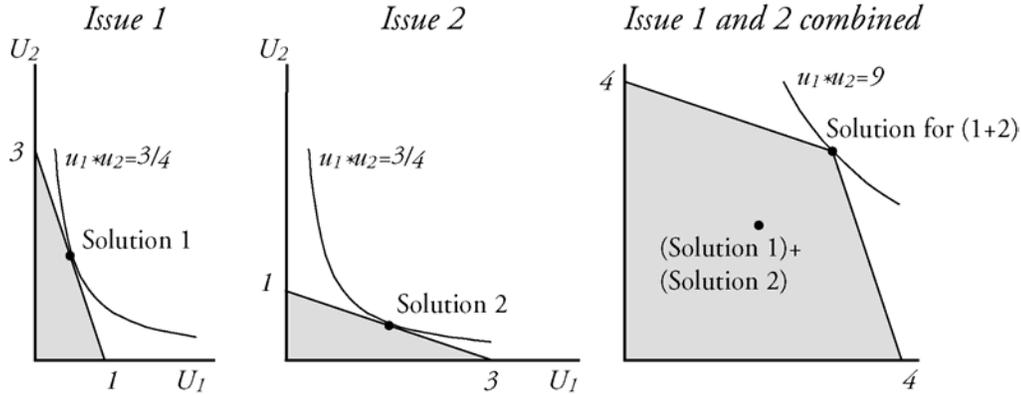


Figure 5: Inefficiency in separate vs. single-issue bargaining.

When issues 1 and 2 are bargained on separately the agents attain a utility of

$$\left(\frac{1}{2}, \frac{1}{2}\right) + \left(1\frac{1}{2}, \frac{1}{2}\right) = (2, 2).$$

However if the agents bargain over the two issues collectively they attain utility $(3, 3)$. This leads to the following observation:

Remark 4 As $\beta \rightarrow 1$ the separate issue bargaining solution is (generically) inefficient in the combined problem.

That is, if the bargaining solutions of each issue have differing marginal rates of substitution then in the combined problem the issue-by-issue solution will not lie on the Pareto frontier.

As the risk of breakdown increases however, the relationship between efficiency and separation becomes more complex. Continuing with the same example, Figure 6 demonstrates what happens as the risk of breakdown becomes non-negligible.

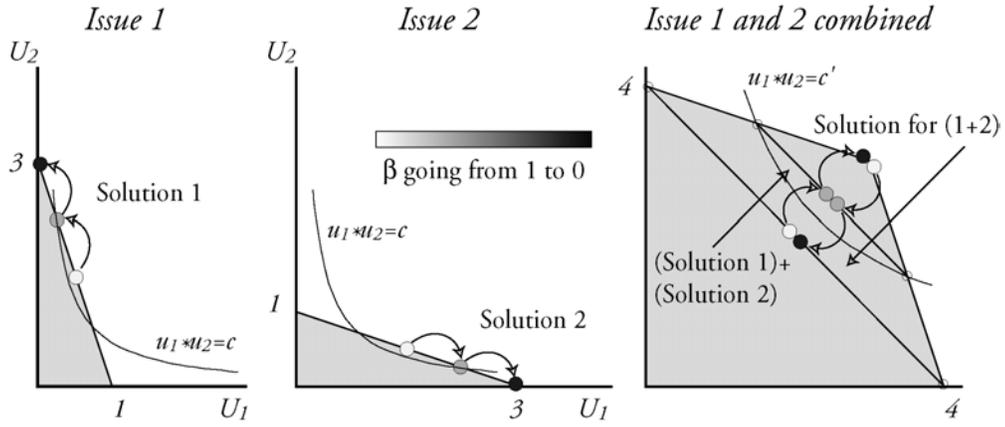


Figure 6: As breakdown becomes likely, issue-by-issue bargaining eliminates ex-ante risk.

Note that if player 1 moves first she proposes on issue 2 and vice versa for player 2; hence the same distribution occurs regardless of who moves first. Solving for the solution of each issue yields:

$$\left(\frac{\beta}{1+\beta}, \frac{3}{1+\beta}\right)$$

for issues 1,

$$\left(\frac{3}{1+\beta}, \frac{\beta}{1+\beta}\right)$$

for issue 2, and

$$\left(\frac{12\beta}{3+\beta}, \frac{12}{3+\beta}\right), \left(\frac{12}{3+\beta}, \frac{12\beta}{3+\beta}\right)$$

on the combined problem if player 1 or 2 moves first, respectively.

As $\beta \rightarrow 0$ the settlement point of the first issue goes to $(0, 3)$ and the second issue goes to $(3, 0)$. So as $\beta \rightarrow 0$ separate-issue bargainers earn

$$(0, 3) + (3, 0) = (3, 3).$$

In contrast collective bargainers move from $(3, 3)$ when $\beta = 1$ to an even gamble over $(0, 4)$ and $(4, 0)$ as $\beta \rightarrow 0$. Ex-ante this earns bargainers an expected payoff of $(2, 2)$. So as the risk of breakdown increases, bargaining separately both ameliorates the ex-ante risk induced by first-mover advantage and *implicitly* induces agents to make efficient tradeoffs between issues. This suggests:

Proposition 5 (Benefits of Issue Bargaining) *As $\beta \rightarrow 0$ both players weakly prefer bargaining over issues to bargaining over the combined problem.*

Proof of Proposition 5 *We show directly that the endogenous agenda leads both players to weakly prefer issue bargaining.*

Let the set of issues be L , and recall that B_j^i is the best agreement for i in issue $I_j \in L$.

Note that as $\beta \rightarrow 0$ the solution of the combined problem is:

$$(B_1^1 + B_2^1 + \dots + B_m^1, 0), (0, B_1^2 + B_2^2 + \dots + B_m^2)$$

if player 1 or 2 moves first, respectively. This is because

$$B_1^i + B_2^i + \dots + B_m^i = B^i(I_1 + I_2 + \dots + I_m).$$

Now bargaining issue by issue, the worst player i can obtain is:

$$B_1^i + B_3^i + B_5^i + \dots$$

if i moves first and

$$B_2^i + B_4^i + B_6^i + \dots$$

if i moves second. This is because the agent can always adopt the strategy of proposing on her highest ranked remaining issue. But ex-ante this is the same payoff as bargaining on the combined problem.

So at worst player i is indifferent between issue and combined bargaining. This proves Proposition 5. \square

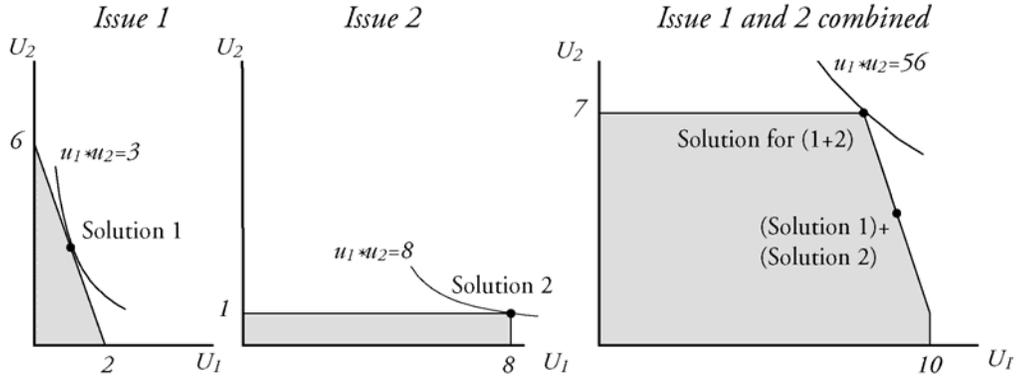


Figure 7: Players disagree as to whether bargaining over issues is preferable.

Note that the natural converse is not true. That is, it is not always the case that as $\beta \rightarrow 1$ both players prefer to bargain collectively. Figure 7 provides a counter-example:

For $\beta = 1$, issue 1 settles at $(1, 3)$, issue 2 at $(8, 1)$, and the combined issues at $(8, 7)$. But then player 1 gets 8 if bargaining collectively, and 9 if bargaining proceeds issue by issue. This implies that for a range of β player 1 will strictly prefer bargaining over issues 1 and 2 separately. This suggests that the distributional effects of issue-by-issue bargaining can significantly shift the distribution from one bargainer to the other.

5 Discussion, Efficiency and Separation

Proposition 5 provides a positive rationale for issue-by-issue bargaining; when bargaining frictions are high agents will find discussing issues separately Pareto-dominates combining them. This dominance can be viewed as the result of two distinct effects.

First, in the presence of large bargaining frictions the first mover (on any issue) enjoys an extreme advantage; since rejecting her offer carries with it high costs, in the limit the first mover can demand her best outcome. This imposes significant ex-ante risk on both players. Separating issues ameliorates this risk; agents endogenously partition issues into first mover sets, essentially splitting first mover rents. The greater the number of issues is, the greater are the gains to be had through this division.

Second, as bargaining frictions increase, the strategic process of issue

selection *implicitly* makes Pareto optimal tradeoffs which might have gone unimplemented in issue bargaining. That is, as bargaining frictions grow the gains to be had by moving first on any particular issue increase. But these gains are precisely a function of the bargainer's marginal rates of substitution between issues. As the relative gains on any issue shift towards a player, the probability she will be the first mover on that issue increases. Hence, the strategic exchange of offers begins to make those Pareto-improving tradeoffs that bargaining over the combined problem would have achieved.

Further, note that in Figure 7, player 1 would prefer issue-by-issue to collective bargaining even if she knew *ex-ante* she would go first under either procedure. This indicates that separate-issue bargaining may be robust to richer models in which the linking of issues is endogenous. In specific, if it were sufficient that both bargainers had to agree to issue-bargaining *ex-ante*, situations in which separation represent a Pareto improvement would suffice to produce issue separation. The example of Figure 7 however, would survive even a more stringent procedure in which the first mover could decide whether issues should be separated or grouped.

6 Conclusion

The informal literature on bargaining (for example see Raiffa's *The Art and Science of Negotiation*) instructs negotiators to bundle issues whenever possible¹³ in order to exploit inter-issue tradeoffs. By contrast, many real-world bargaining situations proceed sequentially over several distinct, unbundled issues. In this paper I have tried to understand the costs and benefits of both systems and to characterize the conditions under which each system is optimal. Instrumental to this is the fact that under reasonable restrictions my model has a unique equilibrium prediction.

My results suggest that when bargaining frictions are low, linking issues is generically more efficient than bargaining issue-by-issue. Congruent with intuition, this is due to bargainers being unable to commit to mutually-beneficial tradeoffs. However when bargaining frictions are large, these tradeoffs which are made *explicitly* under collective bargaining begin to be made *implicitly* by the order in which issue endogenously arise. In these situations bargaining issue-by-issue can benefit *both* parties by ameliorating the *ex-ante* risk that stem from a first-mover advantage. These results suggest that issue-by-issue bargaining can be desirable, a counterpoint to the informal literature on bargaining. Normatively, when negotiating un-

¹³Suggestively, the informal literature calls the multi-issue case *integrative* bargaining.

der large threat of breakdown bargainers should try to split the problem into numerous smaller issues, a prediction which accords well with casual observation.

The degree to which player's preferences are correlated can also effect the relative merits of bargaining systems, but in a less straightforward manner. When negotiators' ordinal rankings of issues largely agree, the possibility of being stuck in an inefficient agenda can be high. Countervailing this is the fact that when players rank issues similarly, the possibility of drastically differing marginal rates of substitution diminishes, and with it the gains to be had from inter-issue tradeoffs.

7 Appendix: Discussion of Modelling Assumptions

7.1 Issue Separation and Selection

Intuitively, additive separability seems *sufficient* grounds for players to regard choices as intellectually distinct. That is, if it becomes common knowledge that two parts of an overall agreement are neither complements nor substitutes for either party, it seems reasonable they would commonly regard them as separate issues. In the absence of correspondingly strong *necessary* grounds we adopt the modeling choice of additively separable issues, which brings with it three strong advantages. First, the model's tractability and hence its empirical usefulness are greatly benefited by this choice. Second, the set of issues becomes endogenous and determined by the more primitive notion of players' preferences. Third, additive separability serves to focus attention solely on those aspects of multi-issue strategy driven by the relative importance of issues, not by complementarities in preferences.

Additionally in many interesting bargaining situations additive separability is a compelling assumption, modeling decisions aside. Firms negotiating joint ventures in disjoint markets will have just such preferences if they are profit maximizing. Utilitarian political leaders bargaining over international policy will also, provided the policies being considered primarily effect disjoint sets of the population.

7.2 Example of Issue Complementarities

I present an example which is suggestive of what may happen if we relax the assumption of issue-separability. Intuitively, if one bargainer finds two issues to be complements and the other does not, the first would want those issue discussed simultaneously while her opponent would not. While this is hard

to incorporate in a non-cooperative framework, using the Nash bargaining solution as a guide permits the following example.

Suppose 2 agents are negotiating the split of two pies, but for agent 1 the pies are complements. We can model this by assuming there are 2 issues I and J , where:

$$I = \{(x_I, y_I) : x_I + y_I = 1\}, J = \{(x_J, y_J) : x_J + y_J = 1\},$$

$$u_1(x_I, x_J) = x_I + x_J, \text{ and } u_2(y_I, y_J) = y_I \cdot y_J.$$

The Nash solutions for either issue separately are simply

$$(x, y) = (0.5, 0.5),$$

yielding utilities 1 and $\frac{1}{4}$ for players 1 and 2, respectively.

The Nash solution for the combined problem solves

$$\max(x_I + x_J) \cdot (y_I \cdot y_J), \text{ such that}$$

$$x_I + y_I = 1 \text{ and } x_J + y_J = 1.$$

Solving, we obtain the split

$$(x, y) = \left(\frac{1}{3}, \frac{2}{3}\right)$$

on both issues, yielding utilities $\frac{2}{3}$ and $\frac{4}{9}$ for players 1 and 2, respectively.

Since $\frac{4}{9} > \frac{1}{4}$, the player with complementarities (player 2) prefers that issues be discussed simultaneously, while player 1 prefers issue-by-issue bargaining. Intuitively, the second player can extract a higher payoff when her inter-issue complementarities are reflected in the total utility possibility set.

7.3 Forms of Breakdown Risk

Another important modelling issue is the risk of breakdown the agents perceive. The exact form this risk takes critically informs how the bargaining proceeds. Here we assume that should parties disagree on any issue, resolution on that issue is threatened but the overall negotiations are not. This models a situation in which bargaining takes place over issues that can be implemented separately. For example peace talks between nations may address the division of several disjoint sets of disputed territory. If an agreement could not be reached on any particular territory, it seems feasible to implement agreements in other areas over which consensus had been reached. When firms negotiate strategic alliances over disjoint markets or over disjoint territories partial agreements are often reached while more contentious

proposals remain unimplemented. When two people are deciding where to eat dinner and what movie to watch, it may be possible for them to eat separately but still meet up to see a movie.

In contrast, many situations provide issues over which separate implementation of agreements is not possible; an example being the labor problem mentioned earlier. Generally, a firm and a worker must agree on all major aspects of a job before they can legally contract on the terms of employment. Two firms considering a joint venture may have to agree on every aspect before the project becomes feasible. A car dealer and customer must decide on a model, options package, and financing before trade can take place. In all of these examples failure to reach agreement on any issue produces the disagreement outcome over all the issues.

7.4 Differing Risks

Finally, the assumption of one β across players and issues is easily relaxed without substantively changing our equilibrium analysis. Most naturally, some issue may be more volatile than others, with disagreements leading much more readily to the collapse of talks. These issues would provide larger first-mover advantages and hence tend to be broached sooner than less controversial issues. This may seem counterintuitive; in many real-world negotiations more volatile issues are often addressed later. This could be modeled as a time-varying β ; issues broached later may be less likely to break down due to a built up of good-faith between the bargainers. Often though, issues are seen as volatile when the set of agreements both parties would prefer to the status quo is very small; a feature which would encourage later discussion since it would reduce first-mover surpluses.

8 Appendix: Proof of Equilibrium Uniqueness

Theorem 3 *If monotonic strategies s'_1 and s'_2 are subgame-perfect, all issues must be settled exactly as in s_1 and s_2 , our equilibrium strategies from Proposition 1, where*

Definition 7 *Strategies s'_1 and s'_2 are **Monotonic** if and only if:*

For every state X , if \exists a history h which leads to state X and after which player j rejects player $-j$'s offer $x_I \in I$ with probability $p > 0$, then after every state X' such that:

1. X and X' differ only on issue I ,

2. player $-j$ also made the standing offer $x'_I \in I$ in X' , and
 3. player j likes x_I strictly more than x'_I ($u_j(x_I) > u_j(x'_I)$),
- player j must reject $-j$'s offer on I .

Remark 5 Note that this is significantly weaker than the stationarity found in the existing literature. Monotonicity requires only that holding all other issues constant, a worse offer from $-i$ to i on an issue makes i more likely to reject that offer.

Proof of Theorem 3 We prove equilibrium uniqueness by iterated conditional dominance.

The proof of our main result proceeds as follows. We first establish that in equilibrium the set of offers that can be made and the set that can be rejected are very small. We then show that constrained to actions within these sets, the order issues can be broached is generically unique and gives rise to the same agreements.

Proceeding then, we first prove:

Lemma 3 (Restriction of Strategies) *If Monotonic strategies s_1^j and s_2^j are subgame-perfect, then on any issue K , player j cannot:*

1. offer an $x \in K$ such that $u_j(x) < u_j(r_K^j)$, or
 2. accept an $x \in K$ such that $u_j(x) < \beta u_j(r_K^j)$,
- if $\forall L \neq K$ and L still open, $u_i(x_L^i) < u_i(r_L^i)$.

In words, no player can offer strictly less than her Rubinstein solution (what she offers in the single-issue case), or accept anything which is worse for her than her opponent's Rubinstein solution if all other standing offers do not ask for strictly more than their Rubinstein values.

Proof of Lemma 3 For expositional simplicity I prove the Lemma in the 2 issue case; the proof extends easily by identical arguments to the n issue case.

Assume players are bargaining over 2 issues, I and J , and recall B_I^i denotes the best outcome for agent i on issue I . Without loss of generality we focus on issue I ; let $f(x)$ be the function which maps player 1's utility to the highest utility player 2 can earn on I when 1 earns x ; i.e.

$$f(x) = \max\{y : (x, y) \in I\}.$$

Now assume the state of the game is $\langle x_I^1, x_J^1 \rangle$, i.e. no issue has broken down, and for both issues the standing offer x was made by player 1. Further assume that

$$\forall S, u_2(x_S^1) > \beta u_2(B_S^2).$$

In words, assume that 1 has offered 2 more than β times 2's best possible outcome on every issue. Note that this is a possible subgame of the bargaining game for any number of issues; player 1 can make offer x_I^1 on I , 2 can propose on J , and then player 1 can reject and propose x_J^2 . Since every standing offer was made by player 1, it must be player 2's turn in this subgame.

Now in any SPE, player 2 cannot reject on I . Why? Rejecting on I cannot help on I , since the probability of breakdown already negates any possible gains on I from rejecting.

Further, rejecting on I cannot help on J . Why? Any change to the state of issue J would involve a risk of breakdown on that issue; hence 2 has nothing to gain on any issue other than I . Switching the roles of the two issues, we see that:

Conclusion 1 *In such a state player 2 cannot reject on I , and must accept both issues.*

Furthermore, if the state is $\langle x_I^2, x_J^1 \rangle$, it is 1's turn,

$$\begin{aligned} u_1(x_I^2) &< \beta f^{-1}(\beta u_2(B_I^2)), \text{ and} \\ u_2(x_J^1) &> \beta u_2(B_J^2), \end{aligned}$$

then in any SPE player 1 cannot accept x_I^2 . Why? Player 1 could reject x_I^2 and propose an agreement on I arbitrarily close to $(f^{-1}(\beta u_2(B_I^2)), \beta u_2(B_I^2))$, which by our first conclusion player 2 must accept. This earns player 1 a payoff of $\beta f^{-1}(\beta u_2(B_I^2))$ on issue I and leaves all other payoffs the same.

Conclusion 2 *Hence in such a state player 1 can never accept such a x_I^2 .*

But note then that in state $\langle x_I^1, x_J^1 \rangle$ where

$$\begin{aligned} u_2(x_I^1) &> \beta f(\beta f^{-1}(\beta u_2(B_I^2))), \text{ and} \\ \forall S \neq I, u_2(x_S^1) &> \beta u_2(B_S^2), \end{aligned}$$

player 2 cannot reject on I . Why?

By the same argument which established our first conclusion, if player 2 rejects x_J^1 ,

1. 1 will not accept any offer which gives him less than $\beta f^{-1}(\beta u_2(B_I^2))$, and
2. if she rejects will not propose anything which earns him less than $f^{-1}(\beta u_2(B_I^2))$.

In either case 2 will not earn more than accepting x_J^1 .

Further, player 2 cannot reject on J . Why? If player 2 rejects and counter-proposes on J either:

1. player 1 accepts player 2's proposal, or
2. player 1 rejects player 2's proposal and counter-proposes.

If the first happens, 2 has been hurt on J and by the original Rubinstein argument must immediately accept x_J^1 . If the second happens it cannot have helped 2 on J . Could it have helped on issue I ? Only if 1's counter-proposal x is weakly worse for 2 than $\beta u_2(B_J^2)$. But

$$\beta u_2(B_J^2) \geq u_2(x) \Rightarrow u_2(x_J^1) > u_2(x).$$

But then by Monotonicity player 2 must immediately reject i 's counter-proposal x . Hence player 2 cannot act on I while the state is $\langle x_I^1, y_J^1 \rangle$, whenever

$$u_2(y_J^1) \leq \beta u_2(B_J^2).$$

But in both of these situations player 2 is strictly worse off.

Conclusion 3 *Hence in such a state player 2 cannot reject x_I^1 , and must accept both issues.*

Now assume that we have shown that whenever player 1 has made both pending offers, player 2 must accept both offers immediately if they are strictly better for 2 than (m_I, m_J) , with

$$u_2(m_I) \geq u_2(r_I^1) \text{ and } u_2(m_J) \geq u_2(r_J^1).$$

We now show 2 claims:

Claim 1 *If the state is $\langle x_I^1, x_J^1 \rangle$, where:*

1. $u_2(x_I^1) > \beta f(\beta f^{-1}(m_I))$, and
2. $u_2(x_J^1) > m_J$,

then player 2 must accept both issues, and

Claim 2 *if the state is $\langle x_I^2, x_J^1 \rangle$, it is 1's turn,*

$$u_1(x_I^2) < \beta f^{-1}(m_I), \text{ and} \\ u_2(x_J^1) > m_J,$$

then in any SPE player 1 must reject x_I^2 .

To establish these claims first note that if the state is $\langle x_I^2, x_J^1 \rangle$, it is 1's turn,

$$u_1(x_I^2) < \beta f^{-1}(m_I), \text{ and } u_2(x_J^1) > m_J,$$

then in any SPE player 1 cannot accept x_I^2 . Why? Just as before player 1 could reject x_I^2 and propose an agreement on I arbitrarily close to $(f^{-1}(m_I), m_I)$, which by our first conclusion 2 must accept. This earns player 1 a payoff of $\beta f^{-1}(m_I)$ on issue I and leaves all other payoffs the same.

But then player 1 cannot accept on issue I .

This proves Claim 2. \square

Now towards our first claim; can player 1 reject on issue J ? No, since if player 2 rejects and counter-proposes on J either:

1. player 1 accepts player 2's proposal, or
2. player 1 rejects player 2's proposal and counter-proposes.

If the first happens, 2 has been hurt on J , and by the original Rubinstein argument must immediately accept x_J^1 . If the second happens it cannot have helped 2 on J .

Can it have helped on issue I ? Only if 1's counter-proposal x is weakly worse for 2 than m_J . But

$$m_J \geq u_2(x) \Rightarrow u_2(x_J^1) > u_2(x).$$

But then by Monotonicity player 2 must then reject x 's counter-proposal x . Hence player 2 cannot act on I while the state is $\langle x_I^1, y_J^1 \rangle$, whenever

$$u_2(y_J^1) \leq m_j.$$

But in both of these situations player 2 is strictly worse off.

This proves Claim 1. \square

Iterating the preceding arguments and noting that our selection of issue I came without loss of generality, we arrive at the following conclusions:

Conclusion 4 *Player 1 cannot accept $x_K^2 \in K \in \{I, J\}$ if*

$$\begin{aligned} u_1(x_K^2) &< \beta f^{-1}(\beta f(\beta f^{-1}(\beta f(\beta f^{-1}(\dots \beta f(\beta f^{-1}(\beta u_2(B_K^2))))\dots))))), \\ &\text{and } \forall L \neq K \text{ and } L \text{ open,} \\ u_2(x_{-K}^1) &> \beta f(\beta f^{-1}(\beta f(\beta f^{-1}(\dots \beta f(\beta f^{-1}(\beta u_2(B_{-K}^2))))\dots))). \end{aligned}$$

Similarly,

Conclusion 5 *Player 2 must immediately accept both x_K^1 and x_{-K}^1 if*

$$\begin{aligned} u_2(x_K^1) &> \beta f(\beta f^{-1}(\beta f(\beta f^{-1}(\dots \beta f(\beta f^{-1}(\beta u_2(B_K^2))))\dots))), \\ &\text{and } \forall L \neq K \text{ and } L \text{ open,} \\ u_2(x_{-K}^1) &> \beta f^{-1}(\beta f(\beta f^{-1}(\beta f(\beta f^{-1}(\dots \beta f(\beta f^{-1}(\beta u_2(B_{-K}^2))))\dots)))). \end{aligned}$$

And as an immediate consequence,

Conclusion 6 *Player 1 cannot propose $x_K^1 \in K$ where*

$$\begin{aligned} u_1(x_K^1) &< f^{-1}(\beta f(\beta f^{-1}(\beta f(\beta f^{-1}(\dots \beta f(\beta f^{-1}(\beta u_2(B_K^2))))\dots))))), \\ &\text{if } \forall L \neq K \text{ and } L \text{ open,} \\ u_2(x_{-K}^1) &> \beta f^{-1}(\beta f(\beta f^{-1}(\beta f(\beta f^{-1}(\dots \beta f(\beta f^{-1}(\beta u_2(B_{-K}^2))))\dots)))). \end{aligned}$$

But note that these conditions converge to Rubinstein solutions, i.e.

$$\begin{aligned} \beta f^{-1}(\beta f(\beta f^{-1}(\beta f(\beta f^{-1}(\dots \beta f(\beta f^{-1}(\beta u_2(B_K^2))))\dots)))) &\longrightarrow u_1(r_K^2), \\ &\text{and} \\ \beta f(\beta f^{-1}(\beta f(\beta f^{-1}(\dots \beta f(\beta f^{-1}(\beta u_2(B_{-K}^2))))\dots))) &\longrightarrow u_2(r_{-K}^1). \end{aligned}$$

Finally, note that we can repeat these arguments with players 1 and 2 switched, and recall the choice of I came without loss of generality.

Then these last three conclusions imply Lemma 3. \square

Now we use Lemma 3 to limit the types of outcomes that can occur in equilibrium.

Lemma 4 (No Extreme Advantage) *If Monotonic strategies s'_1 and s'_2 are subgame-perfect, then players can not settle on $\langle \bar{x}_I, \bar{x}_J \rangle$,¹⁴ where $u_i(x_I) > u_i(r_I^i)$ and $u_i(x_J) > u_i(r_J^i)$.*

¹⁴Recall that an overbar denotes a final agreement.

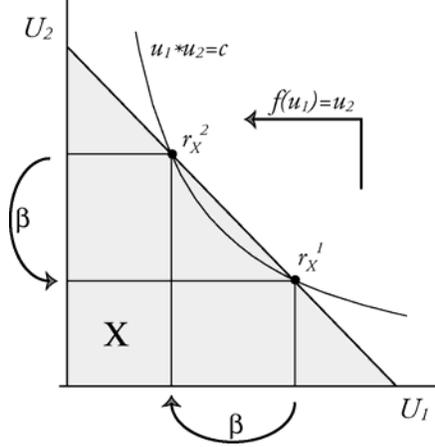


Figure 8: Rubinstein solutions and the iterative process.

Proof of Lemma 4 *We proceed by contradiction.*

Without loss of generality, we assume it is player 1 who has received better than her Rubinstein values on both issues.

By the original Rubinstein argument player 1 must have gone last. Without loss of generality let us assume issue I settled last, at time t .

This implies that player 2 moved at time $t - 1$, and must have moved on issue J , since if not J must have already settled and player 2 could not have offered 1 better than her Rubinstein payoff on I (by subgame-perfection.) Continuing, player 1 moved at time $t - 2$, necessarily on issue J (else she could not have closed issue I at time t .) Now without loss of generality, subgame-perfection lets us focus on equilibrium paths of the form:

$$\langle \dots, x_I^2, x_J^1, \bar{x}_J^2, \bar{x}_I^1 \rangle.$$

But note that

$$u_2(x_I^2) < u_2(r_I^2),$$

yet 2 accepts and offer x_J^1 such that

$$u_2(x_J^1) < \beta u_2(r_J^2).$$

This violates Lemma 3, a contradiction.

This proves Lemma 4. \square

Remark 6 *Note that it also cannot be the case that $u_i(x_I) > u_i(r_I^1)$ and $u_i(x_J) = u_i(r_J^1)$, or vice-versa. Player 2 could have changed her time $t - 1$ move to revising her offer on issue I to arbitrarily close to r_I^2 , which by Lemma 3 would then have been accepted. This would have earned 2 a strictly higher payoff.*

Finally, we prove that all payoffs that can be realized under some MSPE must be bounded by the two Rubinstein payoffs on each issue. In other words,

Lemma 5 (No Extreme Payoffs) *If Monotonic strategies s'_1 and s'_2 are subgame-perfect, then players can not settle on $\langle \bar{x}_I, \bar{x}_J \rangle$, where $u_i(x_I) > u_i(r_I^1)$ and $u_j(x_J) > u_j(r_J^1)$.*

Proof of Lemma 5 *As with Lemma 4, we can impose a tremendous amount of structure on the end of the equilibrium path without loss of generality.*

First note that if $i = j$, then we are done by Lemma 4. Toward a contradiction, assume that:

$$u_1(\bar{x}_I) > u_1(r_I^1) \text{ and } u_2(\bar{x}_J) > u_2(r_J^2).$$

Without loss of generality, let player 1 have moved last. Again, subgame-perfection implies issue I must have settled last. Backward-constructing the equilibrium path, we arrive at:

$$\langle \dots, x_I^2, x_J^1, \bar{x}_J^2, \bar{x}_I^1 \rangle.$$

But note that player 1 could have accepted player 2's offer on issue I immediately. This would strictly increase 1's payoff, since she would then get a minimum of

$$u_1(r_J^2) = \beta u_1(r_I^1)$$

on issue J . This contradicts subgame-perfection.

This proves Lemma 5. \square

But from Lemmas 3 – 5 it is immediate that only very specific equilibrium paths are possible. Whenever a player proposes she must propose her Rubinstein solution for that issue, and this can not be rejected by her opponent. The way Section 3.4 characterizes my solution is precisely the backwards-inductive process both players face when they are restricted to

these types of offers and know their opponent is as well. Hence the solution characterized in this paper is generically unique among monotone subgame-perfect equilibria of the multi-issue bargaining game. Note that mixed strategies were not ruled out ex-ante, but are ruled out by the MSPE refinement.

This proves Theorem 3. \square

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