Notes

Chapter 1

1. In preparing this chapter I have been fortunate to have copies of many of the chapters to the *Handbook of International Economics*, published by North-Holland. I have drawn especially on the excellent general review by Jones and Neary (1982), Ethier's (1982c) discussion of dimensionality, Helpman's (1982) chapter on increasing returns, and Deardorff's (1982b) review of the empirical literature.

2. For theorems using autarky prices, consult Deardorff (1980), (1982a) and Ethier (1982c).

3. Trade need not be exactly balanced to have the Heckscher-Ohlin result. If B is the trade balance, then $B = p'X - p'C = p'X - sp'X_w$, and the consumption share becomes

$s = (p'X - B)/p'X_w$.

This consumption share can still be between the endowment shares, provided the trade imbalance is not too great, in which case the excess factor supply vector has the desired sign pattern. This condition, $L/L_w < s < K/K_w$, implies

$1 - (K/K_w)/(Y/Y_w) < B/Y < 1 - (L/L_w)/(Y/Y_w)$,

where Y and Y_w are own and world GNP. If the trade balance goes outside of this interval, then both goods are exported ($B > 0$) or both are imported ($B < 0$).

4. If F is the production function for X_1, then the unit isoquant is $1 = F(a_{K1}, a_{L1})$. Minimizing cost subject to this constraint implies $w_K = \lambda F_K$ and $w_L = \lambda F_L$, where λ is the Lagrange multiplier. But along the unit isoquant $0 = F_K da_{K1} + F_2 da_{L1} = \lambda^{-1}(w_K da_{K1} + w_L da_{L1})$.

5. Ethier (1982c) provides an excellent and thorough review of the problems of dimensionality, though often referring to autarky prices, which for empirical reasons I shall not use.

6. Defining a factor intensity reversal is a difficult matter in high dimensions. If the factor intensity matrix A is a continuous function of the vector of factor prices w, then in 2 dimensions there are no factor intensity reversals, provided A is nonsingular for all w. In higher dimensions, this is not enough. See Ethier (1982c) for references and discussion.

7. Ethier (1974) has shown that if A is strictly positive, each column of A^{-1} must have at least one negative and one positive element. A proof is simple: If δ is the vector with a 1 in the first element and 0's elsewhere, then the first column of A^{-1} is $z = A^{-1}\delta$, which can be rewritten $\delta = Az$. Since A is strictly positive, the only way for this equation to be satisfied is for z to have both positive and negative elements. By the same logic each row of A^{-1} must have at least one negative and one positive element.

8. The reader may verify that the derivative of GNP $= \sum_j p_j X_j$ with respect to the number of workers is the wage rate. GNP per worker is found by summing the three output functions. In the regions of complete specialization these are just the individual output functions. In the cones B and D, two functions need to be added, producing the straight line segments in figure 1.7. If $pf(k)$ stands for GNP per worker and pfL for GNP, then the derivative of GNP with respect to L is $pf - pkf'$, which implies that the wage rate can be found by extending a straight line tangent to $pf(k)$ until the line intersects the vertical axis. In the two cones B and D, f is linear in k, and the wage rate is correspondingly constant. (It should also be noted that Y/L is convex.)

9. With constant returns to scale we may write $F(L, M, K) = LF(1, M/L, K/L)$, and the marginal product of capital depends on only the ratios M/L, K/L. If M/L is fixed, constant marginal product of capital implies that K/L fixed.

10. Notice that a parameter of the function h_1 is w_K/p_1. This might lead you to question the Stolper-Samuelson result, since a change in p_1 seems to necessitate a change in F_1^*. It is left as

an exercise to verify that the Stolper-Samuelson result holds in the small country case (w_K fixed).

11. This discussion takes factor prices to be equalized, but factor price equalization is not assured, even for infinitesimally different countries, if there are nontraded goods. For examples of sufficient conditions for factor price equalization see Ethier (1972) and Woodland (1982, pp. 228–232).

12. Another way to get to the same result is to imagine firms that produce both X_1 and X_2, but that use all of the X_2 as intermediate inputs in the production of X_1. The final output of such a firm is $X_1 = F(K_1^*, L_1^*, G(K_1 - K_1^*, L_1 - L_1^*, X_{12})) - X_{12}$, where K_1 and L_1 are the total resources used by the firm. If the allocation of capital and labor K_1^*, L_1^* and intermediate product X_{12} are done optimally, output can be written as a function of K_1 and L_1: $X_1 = F^*(K_1, L_1)$. This completely internalizes the intermediate input, and since F^* can be shown to exhibit constant returns to scale, the model is unaltered by the existence of intermediate inputs.

13. Cost minimization implies $(dB)p(1+p) + (dA_L)w = 0$. Thus differentiating (1.13) produces

(i) $[I - B(1+\rho)]d(p) = [Bp, A_L][d(1+\rho), d(w)]'$.

Define final output as X minus the time augmented intermediate inputs $F = [I - B(1+\rho)]X$. Then using (1.14), we obtain

(ii) $[I - B(1+\rho)]^{-1}F = [Bp, A_L]'^{-1}V$

Equation (i) is analogous to (1.5), from which we derive the Stolper-Samuelson result. Reciprocity follows from a comparison of (i) and (ii).

14. Ethier's (1979b) paper is basically a comment on Steedman and Metcalfe (1973, 1977), who conclude (1973, p. 50) that "the existence of heterogenous capital goods does lead to a breakdown of the logic of the HOS theory and hence to that of its major conclusions." Metcalfe and Steedman (1981) object to Ethier's (1979b) contrary conclusion, but Ethier's reply is unbending. The logic of the HOS theory is affected, since factor intensities depend on commodity prices, but the conclusions are unaffected. In particular, the cross-section data analysis reported in this book need not be overly concerned with the definition of capital.

Chapter 2

1. See Hilton (1981) for discussion of demand models that give some justification for using net exports as the dependent variables and interpreting the estimated coefficients as factor cost differences. Deardorff (1982b) also offers a model that explains autarky price differences in terms of factor intensities and a measure of relative factor abundance.

Chapter 3

1. The clustering results based on the regression estimates are discussed in a separate paper available on request.

Chapter 4

1. See the discussion in the chapter on Chile by T. Jeanneret in Balassa (1971).

Chapter 5

1. Estimates formed by omitting an observation can be easily computed. The least-squares estimate based on the whole data set, $\hat{\beta}$, and the estimate with observation i omitted, $\hat{\beta}_i$, are related by the formula $\hat{\beta} - \hat{\beta}_i = (X'X)^{-1}x_i e_i/(1 - h_i)$, where X is the matrix of explanatory variables with row i equal to x_i', e_i is the ith residual, and h_i is the ith diagonal element of $X(X'X)^{-1}X'$. The t-value attaching to a dummy variable that selects the ith observation is $t_i = e_i/s_i(1 - h_i)^{1/2}$, where s_i is the estimate of the residual variances if the ith observation is omitted. The usual estimate of the residual variance s^2 is related to s_i^2 by the formula $(n - k - 1)s_i^2 = (n - k)s^2 - e_i^2/(1 - h_i)$, where $n - k$ is the degrees of freedom of s^2.

The effect of deletions on t-values can also be a useful statistic. For example, the omission of the extreme observation from the data depicted in figure 5.1 has a dramatic effect on the t-value, but has an unclear effect on the estimate. The computation of the t-value is somewhat more troublesome, and the material that follows is based on the hope that t-sensitivity is revealed by estimate sensitivity.

2. Any three-step procedure, in which least-squares residuals are used to estimate a model of systematic heteroscedasticity and then the model is estimated with weighted regression, cannot adequately deal with a scatter such as figure 5.1 because the least-squares residuals would not suggest heteroscedasticity, and weighted least squares would be the same as ordinary least squares. The real error in this procedure is that inaccuracy in selecting the weights does not affect the standard errors in the weighted regression formula, and the standard errors are consequently underestimated. A full maximum likelihood solution with the inverse of the estimated information matrix as the covariance matrix can be expected to yield much larger standard errors of the coefficients in the case of a data set such as the one depicted in figure 5.1.

3. The seminal paper is by James and Stein (1961), who produce an estimator with smaller risk than least squares. Ridge regression proposed by Hoerl and Kennard (1970) is also based on quadratic constraints such as (3). Judge et al. (1980) have an extensive discussion.

4. This curve has been called the "ridge trace" by Hoerl and Kennard (1970) and the "curve decolletage" by Dickey (1975).

5. Wages and salaries per full-time equivalent worker, U.S. National Income and Products Accounts.

Chapter 6

1. The consumption shares are also recoverable if the model is enlarged to allow trade imbalances. If B indicates the trade surplus, then the consumption share is $(Y - B)/Y_w$, which replaces Y/Y_w in (6.1) and (6.2). Consequently, to (6.1) and (6.4) we must add the term $B(\beta_{iL}L_w + \beta_{iK}K_w)/Y_w = BQ_{iw}/Y_w$, which suggests regressing trade on endowments and the trade imbalance, with the coefficient on the trade imbalance interpreted as the output share Q_{iw}/Y_w. This is not further explored because the trade imbalance ought to be related to rates of return to capital at home and abroad, and consequently be itself a function of factor supplies. This causes econometric difficulties if all resources are not measured.

2. A Bayes estimator, corrected for measurement errors, can be written as $(X'X - D + H^*)^{-1}(X'Y + H^*b^*)$, where D is a diagonal matrix with error variances on the diagonal, Y and X are arrays of data, H^* is the prior precision matrix, and b^* is the prior mean vector. If $b^* = 0$, a sensitivity analysis of this formula with respect to changes in H^* can be interpreted as a sensitivity analysis with respect to $H^* - D$, and this Bayesian sensitivity analysis can be thought to include the possibility of some measurement error. In this book, however, b^* is not the zero vector.

Chapter 7

1. These shares were computed from data on domestic consumption (final demand less net exports) from the 1967 *Input-Output Structure of the U.S. Economy*, U.S. Department of Commerce, volume 1. A correspondence between the BEA categories and the SITC categories was formed from the correspondence between BEA and SIC of the U.S. Department of Commerce and from the correspondence between SIC and SITC in *U.S. Foreign Trade Statistics, Classifications and Cross Classifications*, 1968, U.S. Department of Commerce, Bureau of the Census.

Appendix B

1. A complete description of the full data set is available from Professor Bowen.

2. This corresponds to "gross domestic capital formation" as defined in the U.N. system of national accounts (SNA). See United Nations Statistical Office, *A System of National Accounts*, Studies in Methods, series F, no. 2, rev. 3 (New York: United Nations, 1968).