1 Theories of International Trade

This chapter offers a review of the standard Heckscher-Ohlin model of international trade that forms a basis for later empirical analysis of the sources of comparative advantage. The two fundamental hypotheses of this model are (1) there are factors of production that are immobile between countries, and (2) these factors are used in different combinations to produce different goods. A country will then possess a comparative advantage in good $X$ if the country is relatively well endowed with factors that are used intensively in the production of $X$. Although a long list of restrictive assumptions is necessary to make this proposition a precisely defined mathematical truth, it nonetheless is hard to imagine a model in which the possession of immobile factors of production is not a source of comparative advantage. Moreover, with a broad definition of factors it becomes evident that the proposition that endowments are the source of comparative advantage is tautological. For example, the technological differences in the Ricardian model can be thought to arise from differing endowments of knowledge capital.

The way in which endowments confer comparative advantage is not tautological, however. In the simple traditional model with assumptions including constant returns to scale and equal numbers of factors and goods, trade is a linear function of the endowments. If the list of assumptions is altered, the relation between trade and endowments becomes nonlinear in ways that depend on the assumption that is altered. A linear relation between net exports and factor endowments serves as the working hypothesis for the empirical work subsequently presented. This choice is dictated by reasons of convenience and conceptual clarity, since, among the many convenient functional forms that could be used in the data analysis, only the linear form can be derived formally from a fully described general equilibrium model. An important purpose of this chapter is to alert the reader that there is a myriad of assumptions that could be made—and to discuss the kind of nonlinearities that might consequently be expected in the data set. Though overall the impression that is likely to be created is that the linear model is very fragile, it will be shown that linearity is preserved if there are nontraded goods, if there are more goods than factors, if consumption shares are income dependent, and even if there are certain kinds of transportation costs and tariff charges.

The traditional assumptions underlying the $2 \times 2$ textbook trade model are listed in section 1.1; the resulting theorems are derived in section 1.2; and the effects of departures from the assumptions are discussed
in section 1.3. This chapter is meant to be a fairly complete statement of international trade theory as it relates to the determinants of the composition of trade and provided it seems useful in studying empirical data. Hypotheticals, such as "autarky prices," that have no observable counterparts are excluded from discussion, as are models that do not produce adequately clear descriptions of the relation between the composition of trade and its observable determinants.\footnote{The discussion in this chapter is detailed when a specific relation between trade and factors endowments can be derived, but the discussion is brief when the theory can provide only an impression. The subsequent empirical work rests primarily on the simplest model and its extension to high dimensions. It is therefore sufficient to read sections 1.1 and 1.2 and the part of section 1.3 dealing with dimensionality.}

1.2 The Basic Trade Theorems

The core of general equilibrium trade theory consists of four theorems that describe the responsiveness of outputs and factor prices to changes in output prices and factor supplies and a fifth that identifies which commodities are exported and which are imported. The factor price equalization theorem deals with the responsiveness of factor prices to factor supplies, holding fixed output prices. The Stolper-Samuelson theorem describes the relation between factor prices and output prices, holding fixed factor supplies. The Rybczynski theorem links outputs to factor supplies, given output prices. And the Samuelson reciprocity relations describe the correspondence between the Stolper-Samuelson effects and the Rybczynski effects. The fifth theorem, the Heckscher-Ohlin theorem, identifies the structure of trade as a function of either (a) the difference between autarky (pretrade) prices and posttrade prices or (b) factor supplies. Since autarky prices are generally unobservable, this book will make use of only the quantity version of the Heckscher-Ohlin theorem.\footnote{The factor price equalization theorem: All countries have identical factor prices.}

**Proof** Let the goods be labeled $X_1$ and $X_2$ and let the factors be labor ($L$) and capital ($K$). A unit-value isozquant for commodity $X_1$ is depicted in figure 1.1. This is the set of combinations of capital and labor minimally

![Figure 1.1](image)

Unit-value isozquant and isocost lines.
required to produce a unit value of output. Because commodity prices are assumed equalized by trade and because technologies are assumed identical, all countries have the same unit-value isoquant. The three straight lines labeled $C_1$, $C_2$, and $C_3$ in figure 1.1 are three hypothetical unit isoquant lines defined as combinations of capital and labor that cost a unit value. A unit isoquant line is defined algebraically as $1 = w_k K + w_L L$, where $w_k$ and $w_L$ are the rental rates for capital and labor services, respectively. Each isoquant line has slope equal to $-w_k/w_L$, and from the fact that the three isoquant lines in figure 1.1 are parallel we may infer that the ratio of $w_k$ to $w_L$ is the same in each case. Of these three unit isoquant lines, only line $C_2$, which is tangent to the unit-value isoquant, is consistent with competitive equilibrium. If line $C_1$ were the unit-cost line, producers could hire a unit value of inputs and produce more than a unit value of output. In an attempt to exploit this profit opportunity by hiring inputs, producers would drive up the input costs, thereby shifting the unit-cost line downward, a process that continues until profit opportunities are exhausted, that is, until the unit-value isoquant and the unit-cost line are tangent. Likewise, if the isoquant line were $C_3$, there would be no way for producers even to break even, and either factor costs would have to fall or the commodity would not be produced.

The unit-value isoquants for both commodities are depicted in the "Lerner diagram," figure 1.2 (Lerner, 1952). Only one unit-cost line is consistent with the production of both goods in equilibrium, since only one line is tangent to both unit-value isoquants. This line determines unique values for the factor rental rates $w_k$ and $w_L$. Hence factor prices are equalized. Though this argument has referred to unit-value isoquants, the assumption of constant returns makes it apply to any level of output.

This argument is deficient in one respect. There is no assurance that the unit-value isoquants admit only a single tangent isoquant line. An example to the contrary is illustrated in figure 1.3, in which there is a high wage equilibrium with $X_1$ produced with a capital intensive technique and a low wage equilibrium with $X_1$ produced with a labor intensive technique. The commodities $X_1$ and $X_2$ are said in this case to exhibit a factor intensity reversal, since one of the commodities can be produced efficiently with capital/labor ratios either more or less than the capital/labor ratio of the other, depending on the relative factor returns.

When there is more than one possible equilibrium set of factor returns, the choice among them depends on the factor endowment of the country. The allocation of a given endowment of capital and labor between the industries is illustrated in figure 1.4, where the vector $V_1$ indicates the factors allocated to industry 1 and the vector $V_2$ indicates the factors allocated to industry 2. These two vectors must lie along the expansion paths of the two industries and they must sum to the endowment vector $(K, L)$. If the endowment vector $(K, L)$ lies outside the cone between the
two expansion vectors, it is impossible to find two vectors along the expansion vectors that sum to the endowment vector, and consequently the factor prices associated with that cone are inconsistent with equilibrium. Return again to figure 1.3; this implies that the choice between the two factor price ratios depends on the endowment ratio. If the endowment is in a cone $B$ with relative capital abundance, high wages are selected. If the endowment is in cone $D$ with relative labor abundance, low wages are selected. If labor is even more abundant, and the endowment vector lies in region $E$, then the country specializes in $X_1$, and wages are still lower. Relative factor prices may then be found by placing an isocost line tangent to the $X_1$ isoquant at the endowment point.

The regions $B$ and $D$ in figure 1.3 are called cones of diversification. Countries with endowments in the same cone of diversification produce both goods using the same techniques and also have the same factor prices. Countries with endowments in different cones use different techniques and have different factor prices. Countries with endowments in none of the cones specialize in the production of one of the goods.

In order to ensure factor price equalization it is therefore necessary to invoke the following.

**Factor Endowment Similarity Condition** Given the equilibrium commodity prices and the consequent family of cones of diversification, all countries either have factor endowments in the same cone or identical factor endowment ratios.

The cones of diversification will be wider, the more dissimilar are the productive techniques, and this condition can be stated as above that endowments must be more similar than (equilibrium) input intensities.

It may be noted in passing that this demonstration has made use of assumptions 1–5 with the exception of the part of 2a describing the immobility of factors between countries. Because factor prices are equalized, the economic incentive for migration is eliminated, and international mobility of factors is immaterial, provided such mobility does not affect the assumption about factor endowment similarity. Assumption 6 is used only in the limited sense that the production side of the economy maximizes the value of output.

From the internationally determined prices of goods, we have been able to derive the factor prices and the factor input intensities (defined precisely below). These intensities are depicted in figure 1.2, where $(a_{K1}^*, a_{L1}^*)$ and $(a_{K2}^*, a_{L2}^*)$ are the amounts of capital and labor required to produce a unit value of $X_1$ and $X_2$, respectively. The input requirements $a_{ij}^*$ per unit value of output can be transformed into input requirements $a_{ij}$ per unit of output by multiplying by product prices: $a_{ij} = a_{ij}^* p_i$, where $p_i$ is the product price. The input requirements $a_{ij}$ are called **factor input intensities**, and can be collected into a factor intensity matrix symbolized by $A$.

The following definitions refer to relative factor intensities and relative factor abundance.

**Factor Intensity Definition** Commodity $X_1$ is said to be the relatively capital intensive commodity if

$$a_{K1}/a_{L1} > a_{K2}/a_{L2}.$$  

**Factor Abundance Definition** A country endowed with capital $K$ and labor $L$ is said to be relatively abundant in capital if its share of the world's capital stock exceeds its share of the world's labor force:

$$K/K_w > L/L_w,$$

or equivalently if the country is more capital abundant than the world as a whole:

$$K/L > K_w/L_w,$$

where the $w$ subscript refers to world totals.  

Having established factor price equalization, we may now demonstrate the Heckscher-Ohlin theorem.
THE HECKSCHER-OHLIN THEOREM A country with balanced trade will export the commodity that uses intensively its relatively abundant factor and will import the commodity that uses intensively its relatively scarce factor.

Proof Denoting the output levels by \( X_1 \) and \( X_2 \), we may set factor supply equal to factor demand to obtain

\[
K = a_{k_1}X_1 + a_{k_2}X_2,
\]

\[
L = a_{l_1}X_1 + a_{l_2}X_2,
\]

which is a system that can be solved for outputs as a function of the endowments. If \( V \) represents the vector of endowments \((K, L)\) and \( X \) the vector of outputs \((X_1, X_2)\), then these equations may be written in matrix form as \( V = AX \), or inverted as

\[
X = A^{-1}V,
\] (1.1)

where the inverse exists provided the relative input intensities are unequal: \( a_{k_1}/a_{l_1} \neq a_{k_2}/a_{l_2} \). Because of the linearity of these equations (and the consequent unresponsiveness of total world outputs to factor migration), we can also write total world outputs \( X_w \) as a function of total world endowments \( V_w \):

\[
X_w = A^{-1}V_w.
\]

Since the relative prices of goods are given in world markets and hence are the same for all countries, assumption 6 implies that each country consumes commodities in the same proportions, that is,

\[
C = sX_w,
\]

where \( s \) is the country’s consumption share of world output and \( C \) is its consumption vector.

Trade balance requires that the value of production equal the value of consumption, that is, \( p'X = p'C = sp'X_w \), where \( p \) is the price vector \((p_1, p_2)\). Thus if trade is balanced, the consumption share is the ratio of own GNP to world GNP:

\[
s = p'X/p'X_w.
\] (1.2)

The vector of net exports is the difference between production and consumption:

\[
T = X - C = A^{-1}V - sA^{-1}V_w = A^{-1}(V - sV_w),
\] (1.3)

which is \( A^{-1} \) times the vector of excess factor supplies:

\[
V - sV_w = \begin{bmatrix} K-K_w \cr L-L_w \end{bmatrix} = \begin{bmatrix} K_w & (K/K_w) - s \\\cr L_w & (L/L_w) - s \end{bmatrix}.
\]

It will now be demonstrated that if the country in question is relatively capital abundant, \( K/K_w > L/L_w \), then this excess factor supply vector has signs \((+, -)\). This follows from the fact that the consumption share is a weighted average of the capital share and the labor share

\[
s = \frac{p'X}{p'X_w} = \frac{p'A^{-1}V}{p'A^{-1}V_w} = \frac{w'V}{w'V_w} = \frac{w_kK_w(K/K_w) + w_LL_w(L/L_w)}{w_kK_w + w_LL_w},
\]

where \( w \) is the factor reward vector: \( w = (A')^{-1}p \). Thus \( s \) must fall between \( K/K_w \) and \( L/L_w \), and consequently \( K/K_w > s \) is equivalent to \( K/K_w > L/L_w \).

To determine the sign of the net export vector of a capital abundant country, we need to determine the effect of premultiplying a vector with signs \((+, -)\) by the inverse of the matrix \( A \):

\[
A^{-1} = \begin{bmatrix} a_{k_1} & a_{k_2} \\
ar_{l_1} & a_{l_2} \end{bmatrix}^{-1} = \begin{bmatrix} a_{l_1} & -a_{k_2} \\
-a_{l_2} & a_{k_1} \end{bmatrix} / |A|,
\]

where the determinant is

\[
|A| = (a_{k_1}a_{l_2} - a_{l_1}a_{k_2}) = a_{l_1}a_{l_2} \left( \frac{a_{k_1}}{a_{l_1}} - \frac{a_{k_2}}{a_{l_2}} \right).
\]

If \( X_1 \) is the capital intensive industry, then \( |A| > 0 \), and \( A^{-1} \) has the sign pattern

\[
A^{-1} = \begin{bmatrix} + & - \\
- & + \end{bmatrix}.
\]
If the country is abundant in capital, the vector of excess factor supplies has sign pattern (+, −), and trade therefore has sign pattern
\[ T = \begin{bmatrix} + & - \\ - & + \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix} , \]
meaning the capital abundant country exports the capital intensive commodity \( X_1 \) and imports the labor intensive commodity \( X_2 \). All other cases of the Heckscher-Ohlin theorem follow similarly.3

The primary intent of this book is to study the linkages between trade and factor endowments, but for completeness I will briefly state the other basic trade theorems.

**THE RYBCZYNSKI THEOREM** At constant commodity prices, an increase in the supply of a factor will lead to an increase in the output of the commodity that uses that factor intensively and a reduction in the output of the other commodity.

This follows directly from equation (1.1) and the sign patterns for \( A^{-1} \). A somewhat more precise version of the theorem can be stated in terms of the percentage changes in outputs induced by percentage changes in inputs. From equation (1.1) we have
\[ \dot{X}_1 = (dX_1)/X_1 = (a_{L2}dK - a_{K2}dL)/(a_{L2}K - a_{K2}L). \]

Thus
\[ (a_{L2}K - a_{K2}L)\dot{X}_1 = a_{L2}K\dot{K} - a_{K2}L\dot{L}, \]
\[ a_{L2}K(\dot{X}_1 - \dot{K}) = a_{K2}L(\dot{X}_1 - \dot{L}), \]
or
\[ (\dot{X}_1 - \dot{K}) = (\dot{X}_1 - \dot{L})(a_{K2}/a_{L2})(K/L). \]

Thus if commodity one is the capital intensive commodity, \( a_{K1}/a_{L1} > K/L > a_{K2}/a_{L2} \), we must have \( |\dot{X}_1 - \dot{K}| < |\dot{X}_1 - \dot{L}| \), that is, \( X_1 \) must grow at a rate more similar to \( K \) than to \( L \), and we must have \( \text{sign}(\dot{X}_1 - \dot{K}) = \text{sign}(\dot{X}_1 - \dot{L}) \). This implies that either \( \dot{L} < \dot{K} < X_1 \) or \( \dot{K} < \dot{L} < X_1 \), which establishes.

**JONES'S MAGNIFICATION RESULT** At constant commodity prices, \( a_{K1}/a_{L1} > a_{K2}/a_{L2} \) and \( K > L \) imply \( \dot{X}_1 > \dot{K} > \dot{L} > \dot{X}_2 \).

The next result links factor prices to commodity prices. This linkage is implied by the zero-profit conditions
\[ A'w = p, \]
(1.4)
where \( p \) is the vector of prices \( (p_1, p_2) \) and \( w \) the vector of factor rewards. This condition asserts that the price of a commodity is equal to the cost of producing it. Differentiating this expression produces \( A'(dw) = dp, \) since \( (dA')w = 0 \) by cost minimization.4 Writing this as
\[ dw = (A')^{-1} dp, \]
(1.5)
and again referring to the sign pattern of the inverse of a \( 2 \times 2 \) positive matrix, we obtain.

**THE STOLPER SAMUELSON THEOREM** An increase in the price of the import good leads to an increase in the return to the scarce factor and a reduction in the return to the abundant factor.

Finally, by comparing (1.5) with (1.1) we obtain a result due to Samuelson (1953/1954):

**THE RECIPROCITY RELATIONS**
\[ \partial X_i/\partial V_j = \partial w_i/\partial p_j, \quad i = K, L, \quad j = 1, 2. \]
In words, the partial derivative of the supply of commodity \( j \) with respect to the total availability of factor \( i \) is equal to the partial derivative of the wage of factor \( i \) with respect to the price of commodity \( j \). These reciprocity relations are used in Chapter 7 to obtain indirect estimates of the effects of tariffs on factor returns from direct observation of the responsiveness of output to factor supplies.

### 1.3 The Effects of Departures from Assumptions

It takes neither great observational skills nor keen inquisitiveness to make one question the six assumptions and to make one wonder whether the results hold up if these assumptions are relaxed. A particularly troubling observation is the great international disparity in wage rates. For example, the agricultural wage rates reported in Table 1.1 vary from a low of $0.046 per hour in India to a high of $2.04 per hour in Denmark. Part of these differences might be explained by skill differences, but agricultural wages seem unlikely to include a reward for skills that is
Table 1.1
Agricultural wage rates ($/hour): 1972 or closest year data available

<table>
<thead>
<tr>
<th>Country</th>
<th>Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARG</td>
<td>.31</td>
</tr>
<tr>
<td>AUST</td>
<td>.97</td>
</tr>
<tr>
<td>BLUX</td>
<td>1.30</td>
</tr>
<tr>
<td>CAN</td>
<td>1.75</td>
</tr>
<tr>
<td>CHLE</td>
<td>.18</td>
</tr>
<tr>
<td>CSTR</td>
<td>.22</td>
</tr>
<tr>
<td>CYP</td>
<td>.61</td>
</tr>
<tr>
<td>DEN</td>
<td>2.04</td>
</tr>
<tr>
<td>ELSAL</td>
<td>.63</td>
</tr>
<tr>
<td>FIN</td>
<td>.96</td>
</tr>
<tr>
<td>FRA</td>
<td>.48</td>
</tr>
<tr>
<td>GER</td>
<td>1.35</td>
</tr>
<tr>
<td>GANA</td>
<td>.18</td>
</tr>
<tr>
<td>INDA</td>
<td>.046</td>
</tr>
<tr>
<td>IRE</td>
<td>1.04</td>
</tr>
<tr>
<td>ISRL</td>
<td>.71</td>
</tr>
<tr>
<td>JAP</td>
<td>.69</td>
</tr>
<tr>
<td>KOR</td>
<td>.25</td>
</tr>
<tr>
<td>MALA</td>
<td>.17</td>
</tr>
<tr>
<td>MAUR</td>
<td>.17</td>
</tr>
<tr>
<td>MEX</td>
<td>.22</td>
</tr>
<tr>
<td>NUZE</td>
<td>.86</td>
</tr>
<tr>
<td>NOR</td>
<td>1.79</td>
</tr>
<tr>
<td>PHIL</td>
<td>.062</td>
</tr>
<tr>
<td>PORT</td>
<td>.38</td>
</tr>
<tr>
<td>SRIL</td>
<td>.078</td>
</tr>
<tr>
<td>SPAIN</td>
<td>.38</td>
</tr>
<tr>
<td>SWE</td>
<td>2.80</td>
</tr>
<tr>
<td>TURK</td>
<td>.34</td>
</tr>
<tr>
<td>UK</td>
<td>1.34</td>
</tr>
<tr>
<td>US</td>
<td>1.76</td>
</tr>
<tr>
<td>YUG</td>
<td>.49</td>
</tr>
</tbody>
</table>


sufficiently variable to account for the data in table 1.1. This observation encourages a search for assumptions that do not necessarily imply factor price equalization.

1.3.1 Dimensionality

There surely are more than 2 commodities, and more than 2 factors as well. In an intertemporal model one might suppose that there are only 2 factors, labor and land, but these ought to be disaggregated along several quality dimensions. These primary factors can be combined to produce capital goods. But at a specific time, past savings decisions can be taken as given, and capital can be treated as a fixed input. At any instant there will be many types of capital that are relatively well suited to the production of particular commodities and that cannot be transformed into other kinds of capital. For example, human capital and machinery are not fungible except in the long run, and within a year, say, can be treated as separate factors of production. Identifying the particular categories of capital is a very delicate conceptual and empirical issue that interfaces with the mobility assumption. Automobile-stamping plants might be treated as a category of capital that is relatively suited to production of automobiles but not especially useful in the medium run to the production of textiles. For that matter, automobile workers have a certain amount of human capital invested in the auto industry because of special skills and mobility costs. An explanation of trade in terms of exceedingly fine measures of capital would be too close to a tautology to be very satisfying, but simple distinctions between human and physical capital are appealing. In any case, it is clear that there are more than 2 factors of production, and it is essential that the model be enlarged.

The Even Model, $n = m > 2$ The factor price equalization theorem holds in the even case regardless of the number of factors. Imagine the 3-dimensional picture analogous to figure 1.2. The three unit-value isoquants instead of being lines are surfaces—think of three bowls—and the isocost lines are planes instead of straight lines. Only one of these planes rests squarely on all three bowls; hence there is only one set of factor prices consistent with the production of all three goods. As in the 2-dimensional case, factor intensity reversals can occur, but, regardless, for any set of commodity prices there will be a set of cones of diversification, and all countries in the same cone will have the same factor rewards.

The Heckscher-Ohlin theorem in one sense is dramatically altered, but in another sense is not altered at all by high dimensionality with $m = n$. The basic equations of the model, (1.1)–(1.3), which determine the output vector, the consumption share, and the trade vector, need not be altered. If factor prices are equalized, the factor intensity matrix $A$ will be the same in each country, and equations (1.1)–(1.3) are an immediate consequence. However, the interpretation of these equations in the $2 \times 2$ case depends on properties of the inverse of a $2 \times 2$ positive matrix. Namely, the inverse of the factor intensity matrix $A$ has one of two distinct sign patterns, depending on the ordering of the capital intensities of the 2 goods. Things become much more complex in higher dimensions. Suppose the country is abundant in the first factor and scarce in the other two in the sense that the excess factor supply vector $V - sV_u$ has sign pattern $(+, -, -)$. You can then determine the sign of the first element of the trade vector $T = A^{-1}(V - sV_u)$ from knowledge of only the sign of $A^{-1}$ in two cases:

\[\begin{bmatrix} + & - & - \end{bmatrix} \begin{bmatrix} + \\ - \\ - \end{bmatrix} = \begin{bmatrix} + \\ - \\ - \end{bmatrix},\]

\[\begin{bmatrix} - & + & + \end{bmatrix} \begin{bmatrix} + \\ + \\ - \end{bmatrix} = \begin{bmatrix} + \\ - \\ - \end{bmatrix}.\]
What are the circumstances in which the first row of $A^{-1}$ has one of these two sign patterns? Consider the inverse of $A$ with factors capital $(K)$, labor $(L)$, and land $(M)$:

$$
A = \begin{bmatrix}
    a_{K1} & a_{K2} & a_{K3} \\
    a_{L1} & a_{L2} & a_{L3} \\
    a_{M1} & a_{M2} & a_{M3}
\end{bmatrix}.
$$

To find the first row of $A^{-1}$ we need to compute the cofactors

$$
C_{11} = \det \begin{bmatrix}
    a_{L2} & a_{L3} \\
    a_{M2} & a_{M3}
\end{bmatrix} = a_{L2}a_{L3}(a_{M3} - a_{M2}),
$$

$$
C_{12} = -\det \begin{bmatrix}
    a_{K2} & a_{K3} \\
    a_{M2} & a_{M3}
\end{bmatrix} = a_{M2}a_{M3}(a_{K3} - a_{K2}),
$$

$$
C_{13} = \det \begin{bmatrix}
    a_{K2} & a_{K3} \\
    a_{L2} & a_{L3}
\end{bmatrix} = a_{L2}a_{L3}(a_{K3} - a_{K2}),
$$

and the determinant of $A$:

$$
|A| = a_{K1}C_{11} + a_{L1}C_{12} + a_{M1}C_{13}.
$$

Then the relevant elements of $B = A^{-1}$ are

$$
B_{11} = C_{11}/|A|,
$$

$$
B_{12} = C_{12}/|A|,
$$

$$
B_{13} = C_{13}/|A|.
$$

The condition that we are seeking is that $B_{11}$ has sign opposite of $B_{12}$ and $B_{13}$. By inspection of the cofactors this places rather bizarre and complicated restrictions on the factor intensities in industries 2 and 3. If industry 2 is less land intensive than 3 (relative to labor), $C_{11} > 0$, then industry 2 must be more capital intensive (relative to land) than 3, but 2 must also be less capital intensive (relative to labor) than 3.

In higher dimensions it thus becomes impossible to state the Heckscher-Ohlin theorem in a useful way analogous to its statement in the 2-dimensional case. But who really cares? All this is saying is that there exists a simple algorithm for determining the sign of the trade vector $T$ in the 2-dimensional case, but the algorithm becomes more complex in higher dimensions. The degree of complexity remains trivial for a computer given dimensions well above 3, even though 3 strains the human mind. What we might do to assist limited human beings is to rephrase the Heckscher-Ohlin theorem in a correct and informative way. To do this, note that the vector of factors required to produce the vector of outputs $X$ is the vector $AX$. Thus

**DEFINITION** The factors embodied in net exports form the vector $AT$, where $A$ is the matrix of factor intensities and $T$ is the vector of net exports.

The vector $AT$ can have positive or negative elements, meaning that factor services (embodied in goods) can be either exported or imported. Then we can rewrite equation (1.3) as Vanek (1968) does,

$$
AT = V - sV_w,
$$

meaning that the factors embodied in net exports equal the excess factor supplies. This set of equations serves as the foundation for the subsequent data analysis and are referred to as the **Heckscher-Ohlin-Vanek (HOV) equations**. In order to express these equations in the words of the Heckscher-Ohlin theorem, we need a definition of factor abundance applicable to the multidimensional case:

**FACTOR ABUNDANCE DEFINITION** A country is said to be abundant in factor $i$ if its share of the world’s supply of the factor exceeds its consumption share ($V_i/V_w > s$).

Note that if trade is balanced, $B = 0$, then equation (1.6) implies that

$$
s = p'X/p'X_w = p'A^{-1}V/p'A^{-1}V_w = w'V/w'V_w = \sum w_iV_i(V_i/V_w)/\sum w_iV_i.
$$

In words, $s$ is a weighted average of the abundance ratios $V_i/V_w$ with weights equal to world earnings $w_iV_i$. Thus the factor abundance definition compares the abundance ratio of a selected factor with an average of all abundance ratios. With this definition, we can interpret equation (1.6) as the

**HECKSCHER-OLIN-VANEK THEOREM** A country will export the services of abundant factors and import the services of scarce factors.

This way of reexpressing the Heckscher-Ohlin theorem properly emphasizes the point that it is factor services that are being exchanged through trade. Commodities serve only as a bundle within which factor services are wrapped.
There is no simple algorithm in the $m = n > 2$ case for linking the signs of the trade vector to the signs of the excess factor supply vector. For the same reasons, the linkages of commodities to factors implied by the Stolper-Samuelson and the Rybczynski theorems become incomprehensibly complex. All three theorems depend on the sign pattern of the inverse of the factor intensity matrix, which in 2 dimensions is a function of rather straightforward qualitative information, namely, the sign of the determinant. In higher dimensions, the required qualitative information becomes so complex that it is just as convenient to condition these theorems on complete quantitative knowledge of the factor intensity matrix $A$. Namely, the derivatives of outputs with respect to endowments are collected in the matrix $A^{-1}$, and the derivatives of factor prices with respect to commodity prices are collected in the matrix $A''^{-1}$. Measuring $A^{-1}$ is an important empirical task that can be done, as in Chipman (1977/1978), by regressing internal goods and factor prices on traded goods prices or, as in this work, by regressing outputs on factor supplies or trade on excess factor supplies.

**More Goods Than Factors, $m < n$** If there are more goods than factors, there are new ways for the factor endowment similarity condition to be violated. Barring that possibility, the model remains basically the same as the $m = n > 2$ model. Consider, for example, the 3-good 2-factor case depicted in figure 1.5 with unit-value isoquants for goods $X_1$, $X_2$, $X_3$. This figure is similar to figure 1.3. As in that case, if the endowments of all countries are in the same cone of diversification, then all countries will have the same factor prices. Here, however, at least 1 of the 3 goods will then not be produced; for example, if the endowments are all in cone $B$, $X_3$ is not produced. The lack of supply of $X_3$ would then put upward pressure on the price of $X_3$. A rise in the price of $X_3$ shifts the unit-value isoquant in toward the origin until figure 1.6 occurs, at which point all three goods can be produced at nonnegative profit. What this means is that similarity in factor endowments will imply that commodity prices will adjust so that an equilibrium such as depicted in figure 1.6 occurs, with all countries having the same factor rewards and all using the same factor intensities $A$.

The matrix $A$ is not square, however, and cannot be inverted. The set of equations $AX = V$ leaves $X$ indeterminate. ($X$ has $n$ elements but there are only $m$ equations, $m < n$.) Outputs, and consequently trade, have a degree of indeterminativeness equal to $n - m$. As an illustration consider the 1-factor 2-good (Ricardian) model. All countries have straight line production possibilities curves with the same slopes. A country could attain its optimal consumption point with no trade by matching production and consumption. Alternatively, the production point can be anywhere on the production possibilities line, and the optimal consumption point can still be achieved by a suitable choice of trade. Although trade is indeterminate, the net flow of factor services is not. That is, equation (1.6) still holds. What is happening is that any given net export of factor services $V - sV_*$ can be achieved in many different bundles of net exports $T$.

A way to resolve the indeterminacy of trade and production is to hypothesize small international transportation costs that deter trade but are otherwise negligible. In the 1-factor Ricardian model, no trade would then
occur. For the more general model suppose that it costs $c_j$ to exchange commodity $j$ internationally. If these costs are small enough that factor price equalization is imperceptibly altered, we can suppose that the economy minimizes transportation costs $\sum c_j |T_j|$ subject to the constraint $AT = V - sV_w$. Whatever the solution to this linear programming problem may be, it will have the property that it is a linear function of the excess factor supply vector $V - sV_w$.

**More Factors Than Goods** Results necessarily change more substantially if there are more factors than goods. Then it is impossible to derive the (internal) factor prices from the (external) commodity prices. This can be understood easily with reference to a model with 3 factors and 2 goods. A figure analogous to figure 1.2 is 3 dimensional, with each factor measured on a different axis. The unit-value isoquants are surfaces (bowls), and the isocost surface is a plane. In the $3 \times 3$ case, we have three bowls upon which to rest the plane. If there are only 2 goods, then there are only two bowls and the plane cannot be supported by these bowls alone. Thus there are many sets of factor prices that could support the production of both goods. Corresponding to each of these factor price planes there is a minimum cost combination of inputs. The factor exhaustion equation can be written as

$$
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22} \\
a_{31} & a_{32}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix},
$$

where the elements $a_{ij}$ depend on the particular factor price plane. For most values of $a_{ij}$ this system cannot be solved for $(X_1, X_2)$. It is necessary to select factor prices such that the given endowment vector goes through the line connecting the two tangency points $(a_{11}, a_{21}, a_{31})$ and $(a_{12}, a_{22}, a_{32})$. Thus the factor matrix $A$ will change in a complicated way as the endowment vector changes. The resulting relation between trade and endowments is highly complex.

**1.3.2 Factor Endowment Similarity**

If countries are sufficiently similar in their factor endowments, the factor price equalization theorem and a version of the Heckscher-Ohlin theorem hold provided the number of factors is less than or equal to the number of commodities. But if countries are sufficiently dissimilar, they may specialize production in different subsets of commodities, or they may use different input combinations to produce the same commodity. If the first occurs, countries will be said to be **completely specialized**. If the second occurs, and if the country is not completely specialized, the technology is said to admit economically relevant **factor intensity reversals**. In either case, factor prices will not be equalized. If countries are completely specialized, the content of the Heckscher-Ohlin theorem is preserved, though the precise linear form is not. If factor intensity reversals occur, even the content of this theorem can be reversed.

Consider, for example, the model with 3 goods and 2 factors depicted in figure 1.5. Countries with endowment vectors located in cone $B$ will specialize in the production of the relatively capital intensive commodities. Countries with less capital compared to labor, such that their endowment vectors are located in cone $D$, will have lower wage rates and will specialize in production of the two relatively labor intensive commodities. The outputs per man and the corresponding wage rates as a function of capital per man implied by figure 1.5 are depicted in figures 1.7 and 1.8.
The section on international trade discusses the role of trade in economic growth. Both these mobility assumptions are used in explaining the balance of payments. The higher the price of goods and services, the lower the demand for foreign products. This is because of the increased demand for home-produced goods, leading to a decrease in the demand for foreign goods. The result is an increase in the value of the domestic currency, leading to an increase in the price of goods and services. This, in turn, leads to an increase in the demand for foreign goods, leading to a decrease in the value of the foreign currency. As a result, the trade balance improves, leading to an increase in the wealth of the country.
quotient of equilibrium in the factor markets is equalised. The factor prices are equalised across different factors such as labor and capital. The market is in equilibrium when the real return on capital is equal to the return on labor.

\[ \frac{P_L}{P_K} = \frac{W}{r} \]

where \( P_L \) is the price of labor, \( P_K \) is the price of capital, \( W \) is the wage rate, and \( r \) is the interest rate.

The equation can be solved for \( X \), a function of \( Y \) and \( Z \):

\[ X(Y, Z) = \frac{W}{r} \cdot \frac{P_L}{P_K} \]

where \( Y \) is the output level and \( Z \) is the input level.

In the model for the number of mobile factors, the number of mobile factors is equal to the number of goods, \( M \), and the mobile factor share, \( X \), is equal to the number of mobile factors in the economy. The mobile factors include labor and capital.

The number of mobile factors is a function of the number of goods, \( Y \), and the mobile factor share, \( X \), is a function of the number of goods, \( Y \), and the mobile factor share, \( X \).

\[ X(Y, M) = \frac{M}{Y} \]

where \( M \) is the number of mobile factors and \( Y \) is the output level.

In the model for the number of goods, the number of goods is equal to the number of mobile factors, \( M \), and the number of goods, \( Y \), is equal to the number of mobile factors, \( M \), and the number of goods, \( Y \).

\[ Y(M, M) = \frac{M}{M} \]

where \( M \) is the number of mobile factors and \( M \) is the number of goods.

In the model for the number of goods, the number of goods, \( Y \), is a function of the number of mobile factors, \( M \), and the number of goods, \( Y \), is a function of the number of mobile factors, \( M \), and the number of goods, \( Y \).

\[ Y(M, M) = \frac{M}{M} \]

where \( M \) is the number of mobile factors and \( M \) is the number of goods.
The condition back into which we have this...

\[ f(x) = f(A) + \sum \lambda_i c_i(x) \]

Thus, we can solve for \( x \). This condition is equivalent to solving the set of equations:

\[ \begin{align*}
\frac{\partial f(x)}{\partial x_j} &= \sum \lambda_i \frac{\partial c_i(x)}{\partial x_j} \\
\lambda_i &\geq 0, \quad \lambda_i \in \mathbb{R}^n, \quad \sum \lambda_i c_i(x) = 0
\end{align*} \]

To check how to deal with trade imbalances at different levels, we can consider the effects of price differences induced by trade imbalances.

**Transshipment Costs, Terminals, and Other Trade Impediments**

**Results**

In the context of the problem of production in this model with more goods, the effect of trade imbalances is crucial. The trade imbalances can be found by imposing the trade balance condition:

\[\begin{align*}
\sum_{i=1}^{n} A_{i} \cdot s &= \sum \lambda_i c_i(x) \\
\sum_{i=1}^{n} A_{i} \cdot s &= \sum \lambda_i c_i(x)
\end{align*}\]

where *A* is the matrix of trade imbalances of the prolonged good. Also, the *c* matrix of trade imbalances of the prolonged good.
\[
\left(\frac{(d/dx)^2}{x} \right) = \lambda
\]

From the trade balance condition, we solve for the money income level: $I_T = (C - X)$, where $I_T$ is assumed to be the trade balance condition, $I_T = (C - X)$, where $I_T$ is assumed to be equal to the world price. The trade balance condition is determined by

\[
L_A = \sum_{i=1}^{n} \frac{d^i}{dA^i} = C
\]

where $L_A$ are the internal price $A$.

We assume that the countries choose the commodity vector to maximize with a function of $A$. The final result is:

\[
A_{M1-\Theta1-d} = A_1 - V = X
\]

which is a diagonal matrix with $d$ on the diagonal. Therefore, the relation between outcomes and inputs becomes

\[
fo_{d \Theta_1 - M} = \{m/d\theta\} = \{d\} = V
\]

where $M$ is a diagonal matrix with $d$ on the diagonal and $d$ is a diagonal matrix.

\[
(\theta X) + (d) = (\theta) (X + d) = \theta + (\theta X + d)
\]

In matrix form this can be written

\[
(\theta X) + (d) = (\theta) (X + d) = \theta + (\theta X + d)
\]
For the union label.

The impact of the union label on the industrial relations of textile workers ("The Struggle of the Union"")

Chapter I

Theories of International Trade

Table 1.1

1995

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Note: The data for 1995 is estimated based on previous years.

The impact of the union label on the industrial relations of textile workers ("The Struggle of the Union")

Chapter I

Theories of International Trade

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1995

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Note: The data for 1995 is estimated based on previous years.
Chapter I

Theories of Intersectoral Trade

...
There is a fundamental amount of trade in new goods.

Theorem of International Trade

If there are no scale economies and the technology is the same, then the production of goods in different countries is not affected by trade flows.

Theorem of Comparative Advantage

Countries have a relative advantage in the production of goods if the opportunity cost of producing another good is lower.

The theorem states that countries should specialize in the production of goods in which they have a comparative advantage and trade with each other to gain from increased efficiency and lower prices.

Public Resources

Climate change is an important determinant of the location of economic activity. The availability of natural resources, such as forests and fishing grounds, can influence the distribution of economic activity.

The existence of industries, such as agriculture and fishing, is influenced by the availability of natural resources and the climate. Climate change can have significant impacts on the productivity of these industries, as well as on the distribution of economic activity.
Economies of Scale - The model described in section 1.2 points that the increased trade in goods can have very substantial welfare benefits. The fact that trade is open to new exporters, the positive externalities in the production process, and the fact that positive externalities can be captured by the tradable good, mean that the model can be used to analyze the implications of trade in a more general context.

The model described in section 1.2 points that the increased trade in goods can have very substantial welfare benefits. The fact that trade is open to new exporters, the positive externalities in the production process, and the fact that positive externalities can be captured by the tradable good, mean that the model can be used to analyze the implications of trade in a more general context.
These hypotheses lend to describing rather than analyzing. They also

Vernon (1969) and Khandelwal, Mehra, and
Hunt (1967) and Poole (1961), Vernon, (1966), and
Vernon, (1967). (1961) stepped over time, often at a very disaggregated level. In contrast, the cross

An economic model to accommodate assumptions technological

Theoretical differences in common patterns as well, which are

Technological Differences in common patterns during the

not import of the nonagricultural product of the nonagricultural product. This

Interindustry theory: The intermediary product is labor intensive, then

intermediate inputs in the intermediary product, but not in the intermediary produced are

production function subject to increasing returns. Each country

Hirschman's model of trade in differentiated products, with demand

the Heckscher-Ohlin model based on factor endowment theory,

Factor endowment theory of production, derived from their

Hirschman's model of trade in differentiated products, with demand

so that it is the standard product which is the

The Heckscher-Ohlin model is much more satisfying since it accounts

on the other hand, is determined by the production of the other

The difference in product and technology is that within each factory, the

The advantage that I want to develop in this book concerning economies of scale, but the precise relation

the good subject to constant returns to scale. But the precise relation
Theorem of International Trade

The theorem of international trade states that if a country has a comparative advantage in producing a good, it should specialize in producing that good and trade for other goods. This theorem is based on the principle of comparative advantage, which suggests that countries should produce and export goods in which they have a comparative advantage, and import goods in which they have a comparative disadvantage.

The theorem is illustrated by the Heckscher-Ohlin model, which assumes that countries have different factor endowments and that goods are produced using different combinations of labor, capital, and other resources. In this model, countries specialize in producing goods for which they have a comparative advantage, and trade to obtain goods that they cannot produce efficiently.

The theorem of international trade has significant implications for trade policy and economic development. It suggests that countries should focus on developing industries in which they have a comparative advantage, rather than trying to produce every good domestically. This can lead to increased efficiency and lower costs, which can be passed on to consumers in the form of lower prices.

The theorem also highlights the importance of factor endowments in determining the pattern of international trade. Countries with abundant labor and scarce resources tend to specialize in labor-intensive goods, while those with abundant resources and scarce labor tend to specialize in resource-intensive goods. This can lead to a more diversified global economy, with a greater variety of goods and services available to consumers worldwide.
Capital is listed as one of the inputs into a production process in an economic text. The text discusses the importance of capital in production and how it relates to the concept of capital efficiency. The text also mentions the importance of understanding the role of capital in economic models and how it affects production outcomes.

The traditional production theory that the capital is one of the inputs into a production process is based on the assumption that capital is a necessary input to produce goods and services. The text suggests that understanding the role of capital in economic models is crucial for analyzing production outcomes and making informed decisions.

The text also mentions the concept of capital efficiency, which refers to the ability of a production process to produce goods and services with the least amount of capital. The text suggests that understanding capital efficiency is important for improving production outcomes and making more efficient use of capital resources.

The text also highlights the importance of understanding the relationship between capital and labor in production processes. The text suggests that a better understanding of this relationship can lead to more efficient production outcomes and improved economic performance.

The text also mentions the importance of understanding the role of capital in economic models, such as the Neoclassical model. The text suggests that understanding these models is crucial for making informed decisions and analyzing economic outcomes.

The text also highlights the importance of understanding the role of capital in economic models, such as the Neoclassical model. The text suggests that understanding these models is crucial for making informed decisions and analyzing economic outcomes.
\( T \) of the consumption function can be used to solve the usual way for \( X \) as a function of \( A, d \) and \( \beta \).

The capital stock \( K \) is defined as the value of intermediate goods in

\[ T = X^{\frac{\beta}{1-\beta}}. \]

\( \beta \) is the full employment condition is

\( \beta \) the product price of \( \alpha \) in a vector of \( \alpha \) is

\( \beta \) in a function of \( \alpha \).
that with capital as an input along with other factors, a deeper treatment of capital than is embraced by the traditional model for be avoided. Any study of dynamic, changes would study reduce problems of defining and modeling capital in a dynamic world can there exist essential elements in the static trade theories, and the very difficult issue of neutrality by the change that internal capital neutrality. As discussed by Kemp (1973) and Ether (1979), this model can be applied to single systems, and using the cost minimization strictly variable. These results also apply to single-phased models, as

Chapter 1

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