PART II

International Trade and Welfare
Theory and Measurement of Trade Dependence and Interdependence

In Chapter 2, we discussed the analysis of time-series data of a single country's imports or exports with the object in mind of assessing quantitatively the separate influences of supply and demand on foreign trade. The present chapter, in contrast, employs a cross-section approach to the analysis of import or export data for many countries at a single point in time. Our discussion here will be cast in a general-equilibrium setting, in which it is of little consequence to identify the separate influences of demand and supply.

The cross-sectional nature of the data that compose the analysis will necessitate somewhat different interpretations of the results as compared with those in Chapter 2. In that chapter, the questions that were posed were specific to the country being analyzed. For example, we implicitly asked what would happen to Country A's imports if it suffered a 10 percent inflation. The questions to be discussed in the present chapter are, in contrast, comparative inquiries. That is, we will discuss why Country A imports more than Country B at a specific point in time, and why the flow of goods from A to B exceeds the flow from C to D. Nonetheless, it will be of some importance to know whether the cross-section conclusions have implications for specific countries. For instance, if we know why A imports more than B, we might be able to say that if B becomes more like A, then B's imports will diminish. We will thus be discussing issues such as this later in the chapter.

While our previous treatment of partial-equilibrium demand functions was most relevant for international monetary relations, the cross-section general-equilibrium functions of the present chapter will be discussed in the context of the pure theory of international trade. Those familiar with the pure theory have no doubt already recognized the theoretical issues at stake in trying to answer the question of why A imports (exports) more than B. That is, are the composition and quantity of a country's imports (exports) explained by technological differences as suggested by the Ricardian comparative advantage theory, by factor endowments as suggested by the Heckscher–Ohlin
theory, by differences in tastes, or by a technological gap? The recent proliferation of theories purporting to explain the central forces behind the international flow of goods and services will necessarily require empirical efforts if economists are to settle on a generally acceptable theory.\(^1\) Although the empirical work to be described in this chapter is still in the embryonic stage, we can expect additional analysis of this kind, which is likely to be of considerable use in supporting or rejecting these various competing hypotheses.

The second part of this chapter dealing with multicountry trade flows poses a question that has been largely ignored by the trade theorists who have generally labored in a two-country world. In such a world the determinants of trade flows are the same as the determinants of imports. Trade theorists have consequently offered few suggestions as to why pairs of countries become trading partners. Investigators of trade flows therefore have had to construct and test their own theories.

Three characteristics of the data distinguish this chapter from the preceding ones: (1) the use of cross-section data already mentioned; (2) the exclusion of price variables; and (3) the inclusion of static variables such as distance and trade preferences. The exclusion of price variables stems directly from the general-equilibrium nature of this analysis. In such a setting prices are endogenous and merely adjust to equate supply and demand.\(^2\) We will not be satisfied with a tautological statement such as that A exports relatively little because her prices are high. Rather we will wish to know why A’s prices are high. It should be strongly emphasized that the exclusion of price variables in no way implies that prices are not effective in allocating resources. On the contrary, prices are assumed to adjust quickly, and supply and demand are

\(^{1}\) Huber [9] presents a useful summary and interesting tests of these theories. He identifies seven competing theories purporting to explain comparative advantage in manufactured goods: (1) factor proportions; (2) human skills; (5) economies of scale and trade barriers; (4) stage of production; (5) technological gap; (6) product cycle; and (7) preference similarity. The relevant references can be found in the notes to Table 1 of Huber’s paper.

\(^{2}\) This point can be clarified by considering demand and supply schedules as follows

\[
q^d = f(p, D_1, D_2, \ldots, D_n)
\]

\[
q^g = g(p, S_1, S_2, \ldots, S_m)
\]

which indicate that the quantity demanded depends on the price and a set of demand factors \(D_i\), and that supply behaves similarly. We can solve these equations for the market-clearing quantity

\[
q = q^d = q^g = \bar{q}(D_1, D_2, \ldots, D_n, S_1, S_2, \ldots, S_m)
\]

That is, the observed quantity depends on the demand and supply factors but not on the price variable.

These readers familiar with simultaneous equations methods (see, for example, Johnston [11]) will recognize this as reduced-form estimation, which is free of the simultaneity problems associated with the more traditional demand analysis such as that discussed in Chapter 2.

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**THEORY AND MEASUREMENT OF GENERAL-EQUILIBRIUM TRADE SECTORS**

In this section we will be concerned with the determinants of the level of a country’s imports and exports in a general-equilibrium setting. We will rely on the opportunity-cost theory of international trade to suggest explanatory variables and their probable effects on the dependent variable, imports (exports). The points to be discussed will be best illuminated in the context of a three-good world. There will be one home good that cannot be traded in the foreign market due, say, to transport costs or perhaps to the nature of the good, services being an example. In addition, there will be two foreign goods. A typical country will have resources suited to the production of only one of the foreign goods and will accordingly export that good and import the other. Such a situation is depicted in Figures 6.1 and 6.2.

Figure 6.1 depicts the consumption possibilities of the country in the absence of trade. The production possibilities curve \(AB\) indicates the maxi-
appropriate international-trading ratio. Consumption possibilities are therefore extended toward importables along such lines. A possible consumption point is indicated by $E$ requiring production at $P$, imports of $ED$, and exports of $PD$.

Consider now a second country identical in all respects to the first country except that it has a smaller resource endowment and therefore a production possibilities curve somewhat closer to the origin. Such a country is sure to have a smaller foreign sector, that is, a smaller value of exports or imports. This suggests that we write

$$V_i = V_i' = F_i = f(Y_i)$$

indicating that, in equilibrium, the value of exports of Country $i$, $V_i$, and the value of imports, $V_i'$, equal the value of the foreign sector $F_i$, which is a function of the GNP of Country $i$, $Y_i$.

There are of course many reasons why Equation (6.1) will not hold. On the supply side, all countries are not endowed with roughly the same distribution of resources. Those countries with resource endowments especially suited to the production of exports will have a correspondingly larger foreign sector. Such a situation is depicted in Figures 6.3 and 6.4. The country with relatively more resources suited to the production of exportables is indicated in Figure 6.3. Exports are $DP$ and imports $ED$, which are clearly larger than the corresponding values in Figure 6.4, $D'P'$ and $E'D'$. Yet both countries...
may have the same level of GNP, depending of course on the prices chosen to measure the value of the output bundles.3

Similarly, a country with a great diversity of resources allowing the production of all goods will have a reduced foreign sector. This situation is depicted in Figure 6.5 with a production possibilities surface ABC. A consumption point such as E may require no foreign trading at all if it falls on the production surface ABC.

These supply-side considerations lead to the following amendment to Equation (6.1). Resource endowments will have an increasing effect on the size of the foreign sector according to the following classification: balanced endowment, skewed toward the production of domestic goods, skewed toward the production of international goods.4 A similar statement can be made for the demand side. That is, the demand characteristics can be ranked in the order of increasing stimulus to the size of the foreign sector: heavy demand for home goods, heavy demand for own international goods, heavy demand for importables.

A third influence that will disturb Equation (6.1) is the cost of international trade. Clearly a country that must incur unusually heavy costs to engage in international trade will have a correspondingly reduced foreign sector. Transport costs and tariffs are natural candidates to be included. More subtle trade obstacles such as language and cultural differences and political hostilities should also be included. The economic costs may be thought of as rotating downward the trading lines TT' in Figure 6.2, thereby making importables more expensive relative to exportables and reducing the ability of the country to consume the foreign goods. The psychic costs such as cultural barriers may be thought of as shifting the utility surface in favor of home goods and own international goods.

Finally we must admit that the data we have available will not have been generated under general-equilibrium conditions. Countries that were experiencing inflation would experience greater imports and reduced exports. Prices may lag behind the demand shifts that naturally occur in a growing world economy. A related factor completely ignored in our theoretical discussion is the presence of autonomous capital flows. For instance, a short-term capital inflow would substitute for the price adjustment needed to cure a balance-of-trade deficit. Furthermore, we ought to allow for capital flows that are long-term in nature. This suggests that those countries that import capital either due to autonomous transactions or due to accommodating adjustments of reserves in a disequilibrium period will have greater imports and reduced exports. The converse is true for capital exporters.

With the foregoing factors taken into account, our theoretical model may be summarized as:

\[
B = \text{a variable which reflects disequilibrium and capital flows. The letter } B \text{ stands for balance.}
\]

\[
E = \text{resource endowment.}
\]

\[
F = \text{general-equilibrium value of the foreign sector.}
\]

3 This brings up another source of difficulty. Home goods are relatively expensive in Figure 6.3. How are we to calculate GNP figures for countries with different price structures? See Balassa [1] and Linnemann [16, p. 69] for comments on this problem.

4 It will be noted that we have abstracted from the existence of economies of scale. Such economies can be thought of as affecting the shape of the production possibilities surface. However, in the absence of market imperfections they will not necessarily alter the level of trade in one direction or the other.
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\[ i = \text{subscript indicating Country } i \]
\[ R = \text{general resistance to trade (transport costs, tariffs, etc.)} \]
\[ U = \text{utility or demand structure.} \]
\[ V^X, V^M = \text{value of exports and imports} \]
\[ Y = \text{gross national product.} \]

\[ F_i = f(Y_i, E_i, U_i, R_i) \quad (6.2) \]
\[ V^X_i = g(F_i, B_i) \quad \text{and} \quad V^M_i = h(F_i, B_i) \quad (6.3) \]

Equation (6.2) indicates that the size of the foreign sector in a general-equilibrium context will be determined by GNP, resource endowment, utility structure, and resistance factors. The actual value of imports \( V^M \) and exports \( V^X \) will depend on this general-equilibrium foreign sector and the nature of the current disequilibrium \( B \). This same description, it may be noted, is appropriate for disaggregated commodity groups.

Such a model is much too general for statistical application. Three variables (resource endowment, utility, and resistance) represent collections of influences and must be more precisely and more narrowly defined within the constraint of data availability. One possible approach is to use variables that directly measure these influences. For instance, in the case of resource endowment we might be able to use such variables as capital stock, expenditures on research and development, geographic area, average temperature, and average rainfall. To date, this has not been the procedure followed. Rather, it has been argued that all countries have roughly the same resource endowments and demand structures except the countries differ in population and income. Accordingly, population and income are used as proxies for resource endowment and utility structure. In addition, the problem of disequilibrium has been ignored, and \( V^X_i \) and \( V^M_i \) have been set equal to \( F_i \).

An empirical example of what we have been discussing can be found in work by Chenery [3, p. 634] (although his work is not particularly aimed at the subject matter of this chapter). Taking a cross section of the value of imports \( V^X_i \), gross national product \( Y_i \), and population size \( N_i \) for 62 countries in 1952-54, Chenery reached the following result

\[ \log V^M = 20.4 + 0.987 \log Y_i - 0.281 \log N_i \quad (0.69) \]
\[ \log (Y_i / N_i) = 0.045 \]

The reported \( R^2 \) adjusted for degrees of freedom was 0.81, a respectable figure. He also presented similar results for disaggregated commodity groups.

The population variable \( N \) may be associated with both the utility structure \( U \) and the resource endowment \( E \). On the demand side, countries with greater populations will have heavy demands for home goods since foreign goods are dispensed with in order to feed, clothe, and shelter the inhabitants. On the supply side, countries with very small populations will have produc-

\[ \text{tion possibilities skewed toward exportables (Figure 6.3), will therefore specialize, and will have sizable exports. Countries with somewhat larger populations will be able to diversify production (Figure 6.5) and will have somewhat fewer exports. Finally, countries with very large populations will have production possibilities skewed toward domestic goods and will have correspondingly reduced foreign sectors. We see, therefore, that both demand and supply considerations suggest that countries with large populations will have correspondingly small foreign sectors.} \]

Now it may be objected that this explanation of the coefficient on \( N \) is contrived, at best. However, if we consider that our objective is to tie such empirical results to the traditional opportunity-cost theory of international trade, it seems plausible to argue that the negative influence of population is not inconsistent with that theory. In contrast to our view, it may be of interest to point out that some of the empirical studies bearing on this and related issues have all too often demonstrated a willingness to discard the existing theory. Linnebaum [16, p. 15], for example, has suggested that the negative influence of \( N \) may be explained by economies of scale in production and significant barriers (tariff, transport, etc.) to international trade.⁶

It is perhaps worth emphasizing at this point that Chenery’s model using income and population as the only explanatory variables is based on the proposition that individual countries follow generally the same path of development and consequently have similar resources/tastes at any point along that path. This raises the question of the extent to which an individual country can use the foregoing statistical results as an indication of its own future

⁶ That is, he suggested that the negative influence of \( N \) may be explained by noting that in the relationship

\[ F \quad \gamma \quad \frac{Y^\beta}{N^\gamma} \]

we should expect \( \beta \) to be zero and \( \gamma \) negative. His argument is that in a world in which economies of scale and significant barriers to trade exist, industries would prefer to locate only in the largest markets to avoid the greatest transport costs. The smaller markets would not be able to support their own industries because of the economies of scale. A large local market is thus looked on as a source of comparative advantage. With a constant per capita income \( Y/N \), an increase in \( N \) will increase the market size, thus allowing more industries to locate in the market in question and thereby reducing the dependence on trade. Accordingly, the constant \( \gamma \) will be negative. Increases in per capita income with constant population, on the other hand, will increase the market size, thereby resulting in additional domestic industries but at the same time creating a demand for other (new?) products that cannot be economically produced in the country. The net effect, and consequently \( \beta \), may be zero. Thus, the influence of \( N \) on \( F \) would be simply \( \gamma \), which is negative.

Inasmuch as both our interpretation and Linnebaum’s are consistent with the observations, we are unable to establish any presumption as to which is to be preferred. It may be that the depressing effect of population on the foreign sector reflects a shift of demand and productive ability away from international goods as indicated by the opportunity cost theory. On the other hand, it is also possible that increases in population allow for greater diversification of home production and reduce dependence on international trade. This is a matter accordingly that remains open theoretically.
growth over time. Or to put this in another way, are the cross-section results consistent with what we would expect from time-series?  

One can think of situations in which time-series and cross-section data would provide significantly different implications. For instance, the arrows in Figure 6.6 relating to \( V_P^T \) and \( Y_t \) can be interpreted as hypothetical indi-

![Cross-Section Regression Line](image)

**FIGURE 6.6**

Hypothetical Growth Paths of Imports in Relation to GNP

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4 Chenery [3, p. 633] argues on this matter that his statistical results can be interpreted as short-run growth paths:

Historically, the growth of a country takes place in an environment in which trading possibilities and technology are constantly changing. The growth functions derived from cross section analysis, on the other hand, represent the adaptation of countries at different levels of income to conditions of technology and trade existing at one time. Ideally, they may be thought of as indicating the path that a typical country would follow if its income increased so rapidly that conditions of trade and technology were relatively constant.

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5 It is noteworthy that a question similar to this one is being asked on a theoretical level in the literature dealing with the effects of growth on international trade. This literature is much less restrictive, however, than the empirical work being cited. Thus, for example, in Johnson's model [10], allowance is made for intercountry differences in factor endowments, factor intensities in production, and propensities to consume the different goods. With reference to our discussion in the preceding footnote, population growth is shown by Johnson to be "ultra-anti-trade-biased" in the manufacturing country and to range from being "ultra-pro-trade-biased" to "anti-trade-biased" in the agricultural country.

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The recent growth history of several countries. The cross-section regression line shown would not capture the growth path of any of the individual countries. Some empirical support for the divergence of time-series as compared with cross-section results has been provided by Steuer and Vovoras [22], who subjected Chenery's results to a rather severe test. On the other hand, the annual cross-section results calculated by Puliafi and Pulliamen [20] for 1948–60 in his model of world-trade flows were relatively stable. A similarly stable pattern was found by Glezser [6] for 1954, 1958, and 1961 for imports, but there were differences in the parameter estimates for exports.

In the most extensive examination of this issue in a related context, Chenery and Taylor [4] explored the stability of cross-section estimates over time, compared cross-section with time-series estimates for individual countries, and attempted to project the time-series on the basis of the cross-section results. They concluded that the cross-section estimates captured the impact of the universal factors affecting all countries. The variation of characteristics among countries may nevertheless still cause substantial discrepancy between the cross-section and time-series implications, although we might expect that additional explanatory variables would narrow this discrepancy. A fully specified cross-section study might be able to capture the medium-run growth paths of individual economies. But such a study would probably not capture short-run cyclical patterns, nor would it capture the long-run patterns when technology/tastes may change considerably.

On the whole, therefore, this question of the applicability of cross-section results to time-series considerations must be approached with caution from the standpoint of individual countries.

A second empirical example that may help to elucidate our discussion can be taken from the work by Glezser [6], which has just been cited. For 61 countries for the year 1961, his results were:

\[
\log V_P^T = -0.3 + 0.87 \log Y_t - 0.14 \log N_t + 0.24 \log D_i + 0.05 \log P_t
\]

\[
\log V_P^T = -0.8 + 1.03 \log Y_t - 0.26 \log N_t + 0.20 \log D_i + 0.05 \log P_t
\]

where \( D_i \) measures the closedness of other markets and \( P_t \) reflects membership in a preference group. Both these variables fall into the class of resistance variables, which we have associated above with the symbol \( R_i \). The actual definition used for the vicinity variable is

\[
D_i = \sum_{j \neq i} \frac{V_{ij}}{D_{ij}}
\]

6 Kuznets [15, p. 2] has also expressed skepticism about using cross-section findings for inferring growth trends.

7 Glezser actually used \( \log Y_t/Y_t \) as a dependent variable. The coefficients reported in the text are adjusted for consistency with the Chenery study.
where $D_{ij}$ is the distance from Country $i$ to Country $j$. Thus, $D_{ij}$ will be large when Country $i$ is close to large markets and, correspondingly, the foreign sector will be large.\footnote{For example, Glejser's results indicate that Belgium, which is relatively closest to large markets, would enjoy twice as much trade, ceteris paribus, as, say Australia, which suffers the most from distance. Pahlaifer's results [20] for 1949–60 referred to above also call attention to the importance of distance; these results show a slight reduction in the effects of distance during this period.}

Attention should be drawn to the fact that the coefficients for exports and imports in Equations (6.5) and (6.6) are not the same. This suggests that the equation of $V^x_i$ and $V^m_i$ to $I_i$ in (6.2) and (6.3) is not appropriate. Examination of the coefficients in Equations (6.5) and (6.6) indicates that increases in GNP and reductions in population will add more to exports than to imports, and consequently improve the trade balance.\footnote{This also suggests, as Glejser [6, p. 52] has pointed out, that the so-called (Sombart) law of declining foreign trade does not hold for the export side. For further critical discussion and empirical testing of this law, see Kuznets [15, esp. pp. 2–26].}

The result just noted should be interpreted with considerable care. Our earlier discussion has indicated that a difference between $V^x_i$ and $V^m_i$ can occur either because of an autonomous capital flow or because of a short-run disequilibrium in the balance of payments. The autonomous capital flow offers the easiest way out. The richer countries may experience an autonomous and willing capital outflow in favor of the poorer countries either in the form of development aid or as private capital transactions. However, the same result has been found [2] for trade in manufactured products among the industrialized countries. This rules out the development aid argument, but leaves open the possibility of private capital flows.\footnote{We are indebted to Th. Peeters for this observation.}

If, instead, we turn to the short-run imbalance explanation, we are led to the conclusion that the wealthier countries enjoy payments surpluses while their less fortunate neighbors suffer payments deficits. The general-equilibrium opportunity-cost theory offers no insights into the nature of such disequilibria. Rather, we must rely on a partial equilibrium explanation to the effect that short-run deficits result from excessive price inflation and/or rapid growth, the former phenomenon especially associated with the poorer nations.

Our discussion to this point has concentrated on aggregate imports and exports. Nonetheless, almost all that has been mentioned applies equally well to individual commodity groups. An analysis of this kind applied to disaggregated commodity classes would be intended to disclose the nature of comparative advantage. That is, it would be designed to disclose the determinants of the commodity composition of imports (exports). As might be expected, income and population turn out not to be particularly powerful explanatory variables for the trade in many commodity classes, particularly those requiring specific resources that are not found in most countries, oil being an example.\footnote{Thus, Chenery's [3, p. 634] result for petroleum-product imports was $\log P^m_i = 0.88 + 1.0 \log Y_i - 0.44 \log N_i$ with an adjusted $R^2$ of only 0.57.}

Consequently, greater effort will be required to generate explanatory variables that accurately reflect the influence of resource endowment on these disaggregated import (export) data.

### THE THEORY AND MEASUREMENT OF TRADE FLOWS

Having completed our discussion of the determinants of import and export levels, we now turn to an analysis of trade flows. These flows will be denoted by $V_{ij}$, the value of the flow of goods from Country $i$ to Country $j$. We can think of these flows as forming an $n \times n$ matrix of values, where $n$ is the number of countries:

$$
\begin{bmatrix}
V^x_i & V^m_i & \cdots & V^x_j & \cdots & V^m_n \\
V^x_{i1} & 0 & \cdots & V^x_{ij} & \cdots & V^x_{in} \\
V^x_{i2} & V^m_{i1} & \cdots & V^m_{ij} & \cdots & V^m_{in} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
V^x_{in} & V^m_{i1} & \cdots & 0 & \cdots & V^m_{in}
\end{bmatrix}
$$

The row and column totals, $V^x_i$ and $V^m_i$, indicate the total value of exports and imports of the particular country in question. We would like to explain the trade flows, that is, the elements of the matrix.

There are two possible approaches that can be used for this purpose. We can assume that the values on the margins of the above matrix, $V^x_i$ and $V^m_i$, are known, and use them as explanatory variables. Alternatively, we can use the more fundamental explanatory variables of the previous section, such as income and population, to explain jointly the levels of imports and exports, and the values of the trade flows. Most of the studies to be discussed have attempted the latter, more difficult task. We shall do likewise and instead of using the actual values of exports and imports as explanatory variables, we will use $F_i$ to denote that the size of the foreign sector is being explained by
the regression as well as the value of the trade flows. This point will be brought out more fully below.

Three models have been used to describe the trade flows. The earliest appealed to physical laws of gravitation and electrical forces to arrive at the conclusion that the flow of goods from Country $i$ to Country $j$ equals the product of the potential trade or trade capacity measured by $F_i$, the value of the foreign sector at the two points ($F_i \times F_j$), divided by the resistance or distance (perhaps squared). While the economic meaning of such “gravity” models is not altogether clear, we will present below a specification similar to that implied by these models.

A second approach, employed by Linne mann [16], specifies a Walrasian general-equilibrium model, with each country having its own supply and a set of demands for the goods of all other countries. Solving for the flows, we would obtain the usual general-equilibrium result that each flow depends on everything else. But we would have some reason to believe that the particular flow $V_{ij}$ would be most influenced by export-supply factors in Country $i$ and import-demand factors in Country $j$. We might therefore write

$$V_{ij} = h(F_i, F_j) = h[f(Y_i, E_i, U_i, R_i), f(Y_j, E_j, U_j, R_j)]$$

(6.8)

We could also take the simple additional step of including a resistance variable $R_{ij}$ to indicate the level of trade resistance specific to the flow from $i$ to $j$.

The third description of trade flows is based on a probability model, which we have adapted from the work of Savage and Deutsch [21]. It is characterized by demanders being assigned to suppliers in a random fashion. There are three reasons why we prefer this (or this kind of) model. In the first place, the general-equilibrium model does not suggest the form of the function $h$ in Equation (6.8). We will argue presently that this form can be represented by the simple product $F_i \times F_j$. Secondly, as noted below, statistical problems of heteroscedasticity and autocorrelation become clear when the probability model is used. Finally, the probability model will serve to tie these studies together with other work, especially that of Savage and Deutsch just mentioned.

Turning now to the details of the probability model, we will view world trade as being generated by thousands of small independent transactions.

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**THEORY AND MEASUREMENT OF TRADE FLOWS**

Each transaction will be of a certain size $\beta$ and will involve one exporting country and one importing country. Since Country $i$ enjoys a share of world trade $f_i = F_i/T$, where $T$ is total world trade, we will quite naturally assume that the probability that a particular transaction involves Country $i$ as the exporter is $f_i$. In similar fashion, the probability that Country $j$ is the home of the importer is $f_j$. For the moment we will assume that the selections of exporter and importer are independently performed and we will write the probability that the flow travels from $i$ to $j$ as

$$p_{ij} = f_i f_j$$

(6.9)

Let us assume further that all transactions are of size $\beta$, and that $N$ of these transactions occur. We can then write total world trade as

$$T = N \beta$$

(6.10)

and calculate the expected flow from Country $i$ to Country $j$: 15

$$V_{ij} = N \beta f_i f_j$$

(6.11)

This equation expresses the fact that Country $i$ is prepared to export $F_i$ worth of goods, which, because of the randomness in the assignment of suppliers to demanders, is distributed evenly among the importers proportional to the size of each demand $F_j$.

15 The text describes a multinomial probability distribution. Feller [5, p. 141] gives the expected value as

$$E(V_{ij}) = N \beta f_i f_j$$

with the corresponding regression equation given by

$$V_{ij} = N \beta f_i f_j + \epsilon_{ij}$$

$$E(\epsilon_{ij}) = 0$$

The corresponding variance is (Feller [5, p. 214])

$$\text{Var}(\epsilon_{ij}) = \text{Var}(V_{ij}) = N \beta f_i f_j (1 - f_i f_j) = \beta F_i F_j \left[ 1 - \frac{(F_i/T)(F_j/T)}{T} \right]$$

We observe that for $f_i f_j < 1/2$ the variance is an increasing function of $f_i f_j$. This situation is described as heteroscedasticity and will necessitate some alteration of ordinary least squares. (See Footnote 18.)

A second source of statistical difficulty stems from the correlation of the error terms. Ordinary least squares requires that the error terms be independently generated. In our model, however, flows with positive error terms (deviations from the expected value) will tend to be offset by negative errors elsewhere. The actual covariance is (Feller [5, p. 224, problem 20])

$$\text{Cov}(V_{ij}, V_{jk}) = N \beta^2 f_i f_j f_k$$

15 It should be pointed out that the probability model just presented is but one among many candidates. Its most glaring weakness is that it makes no allowance for the fact that $p_{ij}$ and $V_{ij}$ are zero, that is, that there is no trade flow internal to any country. We could conceivably generalize the model to include internal flows. Alternatively we could amend...
discussion particularly at the resistance factors, \( g(R_{ij}) \). First, however, we would like to illustrate the empirical link between the trade-flow regressions and the foreign-sector regressions of the previous section. Equation (6.13) includes two unmeasured foreign-sector terms. This suggests that we should be able to read the foreign-sector expressions directly from the trade-flow equation. For example, Linnemann [16, p. 82] provides the following trade flow result:

\[
\log V_{ij} = \alpha_0 + 0.99 \log Y_i - 0.2 \log N_i + 0.85 \log Y_j - 0.15 \log N_j + \log g(R_{ij})
\]  

(6.14)

where \( g(R_{ij}) \) refers to resistance variables to be identified subsequently. Taking the \( F_i \) and \( F_j \) terms from this equation, we obtain

\[
\log V_{ij}' = \log F_i = \alpha_0' + 0.85 \log Y_i - 0.15 \log N_i
\]  

(6.15)

\[
\log V_{ij}'' = \log F_j = \alpha_0'' + 0.99 \log Y_j - 0.2 \log N_j
\]  

(6.16)

These equations are almost identical to Gleiser's results reported in Equations (6.5) and (6.6), a fact that tends to support our probability model and its consequent equation (6.13). Furthermore, this illustrates the fact that a regression of the form (6.14) explains the total value of the foreign sector as well as the levels of flows to and from individual countries.

Let us now focus our attention on the \( g(R_{ij}) \) term that appears in our theoretical model represented by Equation (6.13) and in a regression such as the one indicated in Equation (6.14). We have already mentioned a wide

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18 With the exception of Pihlson's [18] work, all the results to date have been fitted by ordinary least squares with a log-linear function. Footnotes 15 and 17 suggest that this is not the optimum fitting procedure. The model we have presented in Equation (6.13) can be written

\[
V_{ij} = \frac{F_i F_j g(R_{ij})}{T} + u_{ij}
\]

where \( u_{ij} \) is an error term with

\[
E(u_{ij}) = 0
\]

\[
\text{Var} (u_{ij}) = N\beta f f_i g(R_{ij})[1 - f f_j g(R_{ij})]
\]

\[
\text{Cov} (u_{ij}, u_{ij}') = -N\beta f f_i f_j g(R_{ij})g(R_{ij}')
\]

The covariance terms can perhaps be neglected as they are surely smaller than the variance terms by a factor of one hundred (since no country has more than a tenth of world trade, \( f f_j < 0.01 \)). Maximum likelihood estimation, assuming the \( u_{ij} \) are distributed normally, requires the minimization of

\[
\sum \left[ V_{ij} - \frac{F_i F_j g(R_{ij})}{T} \right]^2
\]

This compares with the log-linear ordinary least squares minimization of

\[
\sum [\log V_{ij} - \log F_i - \log F_j - \log g(R_{ij})]^2
\]
variety of influences subsumed in \( g(R_{ij}) \), which will tend to impede or enhance trade between individual countries. While the existing empirical work does not do full justice to the rich theoretical possibilities of this term, the results that have been obtained are nevertheless interesting and provocative.

The simplest resistance factor is distance itself. Tinbergen [25, p. 273], for example, fitted a regression of the form (6.14) and obtained

\[
g(R_{ij}) = D_{ij}^{-0.89} \tag{6.17}
\]

where \( D_{ij} \) is the distance from Country \( i \) to Country \( j \). Distance is seen to have a significant depressing effect on trade with an elasticity less than unity.

Recognizing the simplistic nature of this specification, Tinbergen [25, p. 266] altered the resistance variables to

\[
g(R_{ij}) = D_{ij}^{-2}C_i^p D_{ij}^{2r} \tag{6.18}
\]

where \( C_i \) is a dummy variable for neighboring or adjacent countries, and \( P_c \) and \( P_b \) are dummy variables indicating Commonwealth and Benelux preference. His actual estimates had the proper signs and reasonably small standard errors but the \( r^2 \) climbed only slightly.

The use of the dummy variables to reflect preference-group membership has interesting implications for the analysis of such groups. The estimated coefficient on the dummy variable can be used to calculate the extent to which intermember flows were augmented. For instance, Tinbergen found that Benelux association increased the flows between members by 10 percent. It is not clear, however, from such information whether the increase in intermember flows was at the expense of the member-to-nonmember flows or whether it reflected a general increase in trade of the member nations. The answer to such a question can be gleaned from the work reported in the previous section on the foreign sector. That is, Glejsen’s [6] results indicated that Belgium enjoyed a 60 percent increase in her foreign sector due to membership in preference groups (including colonial preference). Such quantitative information may be useful therefore insofar as it yields conclusions as to the trade-diverting/trade-creating effects of customs unions.39

39 Another approach to the analysis of customs unions, suggested by Linnemann [16, p. 179], is to observe that two countries will have a total trade with the rest of the world given by

\[
F_{i+} = \alpha Y_i^0 N_i^0 + \alpha Y_j^0 N_j^0 - 2 \beta \gamma_i^0 N_i^0 \gamma_j^0 N_j^0 D_{ij}^{-2}
\]

which indicates the sum of the total trade of the two countries minus their internal trade. If, however, these countries were fully integrated, we would have (assuming the resulting GNP is simply the sum of the individual ones)

\[
F_{i+} = \alpha (Y_i + Y_j) \theta (N_i + N_j)^{\theta^1}
\]

Although full integration is unlikely, a comparison of these two values might prove useful as a guide to the static effects of a customs union on trade with the rest of the world.

\[
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\]

The distance term discussed above has one troubling feature. As the distance shrinks to zero, the term \( D_{ij}^{-5} \) explodes, indicating very heavy trading for countries that are close to each other. This point has led Pöyhönen [18] to his unique scaling of the distance term

\[
g(R_{ij}) = (1 + \gamma D_{ij})^{-1} \quad \tag{6.19}
\]

This describes resistance to trade as being a constant resistance plus a part that increments with distance. Since the resistance term reflects costs, we can think of this formulation as allowing for both fixed and variable costs. If the value of \( \gamma \) is large, variable costs will dominate. If it is small, variable costs will not have a great influence. This specification converges to the earlier one \( g(R_{ij}) = D_{ij}^{-2} \) as \( \gamma \) grows.30 Though this specification is clearly preferred to the earlier one on the grounds of generality, it has a computational drawback because it cannot be linearized by applying logarithms, and ordinary least squares cannot be used for the estimation.31 With this computational burden in mind, Pöyhönen found it necessary to revert to the first specification.

In a later article, Pöyhönen [19] presented some theoretical arguments that led to the specifications used by Pulliainen [20]. These authors argued, in effect, that individual countries are not natural units in determining trade flows. The proper unit is the market area, comprising many countries. While trade between these market areas is adequately described by the flow equation we have already discussed, a second allocation of the trade is necessary for the within-market-area distribution. Such a description is based on the notion that information and distribution come in bundles that refer to a whole market, hereby inducing a business decision that refers to the whole market. For example, a U.S. businessman typically decides to export to the South American market. He rarely would make the decision to export to Brazil alone, since the heavy fixed costs associated with gathering information, making the decision, and establishing a distribution network need be incurred only once for either decision.

These considerations suggest the following equation

\[
V_{ij} = \frac{E_i}{F_i} \left( \frac{F_j}{T} \right) \left( \frac{F_i}{F_j} \right) g(R_{ij}, R_{ii}, R_{jj}, R_{ij}) \tag{6.20}
\]

where \( F_i \) is the foreign sector of the market area to which Country \( i \) belongs and \( R_{ij} \) is a general resistance between market areas \( i \) and \( j \). This equation is

30 The value of \( \gamma \) that Pöyhönen reported, 0.00157/nautical mile, is a relatively small number reflecting the importance of fixed costs. For instance, variable costs do not equal fixed costs up to a distance of 700 nautical miles. We should also observe that the adjacent-country variable used by Tinbergen (28) and reported in Equation (6.18) can be interpreted as reflecting the influence of fixed costs.

31 Pöyhönen minimized the sum of squared residuals in logarithms:

\[
\sum_{ij} \left[ \log Y_{ij} - \log \hat{\gamma} - \log \hat{\gamma} - \log \hat{\gamma} - \log \hat{\gamma} - \hat{\beta} \log Y_i + \hat{\beta} \log (1 + \hat{\gamma} D_{ij}) \right]^2
\]
interpreted as follows. The center terms \( F_s(F_s/T) \) are familiar and describe the flow of trade from Market area \( I \) to Market area \( J \). That is, Area \( I \) is prepared to export \( F_s \) worth of goods, which is distributed evenly among the market areas proportional to their demands \( F_s \). A share of this flow \( F_s/F_s \) originates in the \( i \)th country. Similarly, the flow from Country \( i \) to Market area \( J \) is distributed evenly among the members of the market area proportional to their demands \( F_s \). The variable \( R_{ij} \) can be thought of as affecting the flow of goods from \( I \) to \( J \), while other resistance terms alter the distribution of that flow among the members of the market area.

We can now cancel the identical terms in Equation (6.20) to arrive at precisely the same equation as before with the exception of the new resistance variables. The market-area consideration, therefore, affects only the trade resistance factors.

Pulliainen’s [20, p. 82] resistance variables were actually

\[
g(\text{r}_{ij}) = D^*_i Y_i Y_j (1 + |C_i^* - C_j^*|)^{v}
\]  

(6.21)

where \( Y_i \) and \( Y_j \) are the aggregate gross-domestic products of Market areas \( I \) and \( J \), and \( C_i^* \) and \( C_j^* \) are the long-range mean temperatures of Countries \( i \) and \( j \). In light of the above discussion that the market-area consideration has an effect only on the resistance factors, it is surprising to find \( Y_i \) and \( Y_j \) as explanatory variables. Pulliainen offers the appealing explanation that increases in trade between the two markets associated with increases in \( Y_i \) and \( Y_j \) are accompanied by a reduction in trade resistance as distribution, information, and sales networks are established. To use our symbols, we would write

\[
R_{ij} = r_{ij} Y_i Y_j
\]  

(6.22)

where \( r_{ij} \) reflects resistances associated with other variables.

The other contribution of Pulliainen is the use of the temperature variable \((1 - |C_i^* - C_j^*|)\) to reflect differences in resource endowments associated with different mean temperatures. As the temperatures become more separated, the countries in question become less similar in resource base, export more dissimilar products, and may trade more. On the other hand, tastes may be molded by the domestic production, and the consequent dissimilarity of tastes may induce countries with dissimilar resources to trade less.

It should be recognized that the temperature-disparity variable does not reflect resistance to trade in the previous sense of a monetary or psychic penalty. Rather, it reflects the simple fact that pairs of traders will not do business unless the supplier and demander have the same good in mind. What this suggests is that we must find some way of dealing with differences in the commodity composition of trade. One way to handle this is in terms of the aggregate relation as above with variables that indicate dissimilarities in resource base. Alternatively, at least in principle, one may perform the analysis on disaggregated commodity classes. A third approach, suggested by Linne mann [16], is to use variables in the aggregate relation that indicate the interaction of \( i \)'s export structure with \( j \)'s import structure as

\[
C_{ij} = \sum_k \frac{V_{ik}^* V_{jk}^*}{V_{ik}^2 V_{jk}^2}
\]

(6.23)

where \( V_{ik}^*/V_{ik}^2 \) is the proportion in value terms of exports of Commodity \( k \) by Country \( i \), and \( V_{jk}^*/V_{jk}^2 \) is the proportion of imports of the same good by Country \( j \).

A similar interaction term \( C_{ij}^* \) results naturally from the aggregation over commodities

\[
V_{ij} = \sum_k \frac{F_{ik} F_{jk}^* g(R_{ij})}{T_k}
\]

(6.24)

where \( F_{ik}^* \) is the export supply of the \( k \)th commodity by the \( i \)th country, \( F_{ik} \) is the import demand of the \( j \)th country, \( R_{ij} \) is the trade resistance specific to the \( k \)th commodity, and \( T_k \) is the world value of \( k \)-commodity trade. This equation can be altered as follows

\[
V_{ij} = \frac{F_i F_j^* \sum_k \frac{F_{ik}^* F_{jk}^* g(R_{ij})}{T_k}}{T_i F_j^* \sum_k \frac{F_{ik}^* F_{jk}^* g(R_{ij})}{T_k}} \left( \frac{T_k}{T} \right)^{-1}
\]

(6.25)

with

\[
C_{ij}^* = \sum_k \frac{F_{ik}^* F_{jk}^* g(R_{ij})}{T_k} \left( \frac{T_k}{T} \right)^{-1}
\]

We may first note with regard to the foregoing equations that the appropriate interaction term is not the simple inner product given in Equation (6.22), but rather a weighted inner product \( C_{ij}^* \). The factor \( T/T_k \) weights the commodity in inverse proportion to its importance in world trade. The larger the commodity class, the smaller its weight. The other factor \( g(R_{ij})/g(R_{ij}) \) works in the opposite direction. There is a presumption that resistance to trade is least [large \( g(R_{ij}) \)] for those commodities that form the bulk of world trade. Accordingly the larger commodity class should be given a somewhat larger weight. These two counteracting considerations may possibly make the unweighted inner product \( C_{ij} \) very close to the properly weighted term.

Secondly, there is some question as to whether the use of the commodity composition of imports and exports as explanatory variables is legitimate. As we noted earlier, the explanation of trade flows, given the levels of imports and exports, is presumably a much easier task than the explanation of the
trade flows with only the more fundamental variables. Had we been concerned with the easier task, we would have used a relation such as

$$V_{ij} = \frac{V^x_i V^y_j g(R_{ij})}{T}$$

(6.26)

where $V^x_i$ and $V^y_j$ are the actual levels of exports and imports, rather than Equation (6.13) in our earlier discussion. While the use of the interaction term does not weaken the hypothesis to the extent that the use of $V^x_i$ and $V^y_j$ would, it nonetheless is open to question on the same grounds. More fundamental explanatory variables reflecting the interaction of resource endowments or tastes would seem to be more appropriate choices, for example, the temperature disparity variable discussed above.

The work we have been describing to this point, using such fundamental explanatory variables as income and population, should be thought of as testing both the theory of the total foreign sector and the theory of trade flows, since the two theories must be valid to produce a good fit. Suppose, however, that we wanted to test the flow theory alone; we could use a regression such as (6.26), in which the values of exports and imports are explanatory variables. An alternative is the use of dummy variables $c_i$ and $c_j$

$$V_{ij} = c_i c_j g(R_{ij})$$

(6.27)

This says that $i$ and $j$ have foreign sectors of size $c_i$ and $c_j$, the determinants of which are not of concern. Thus, Equation (6.27) would provide a good fit quite independent of the validity of what is being posited in the theoretical description of the total foreign sector.

It is instructive to compare Equation (6.27) with a hybrid specification that Pöyhönenn [18] and Pulliaimen [20] have employed in their work

$$V_{ij} = c_i c_j Y^x_i Y^y_j g(R_{ij})$$

(6.28)

Equation (6.28) is the same as (6.27) except for the inclusion of the income terms, $Y_i$ and $Y_j$. However, since (6.28) contains the dummy variables, $c_i$ and $c_j$, this specification is a test only of the theory of trade flows.38

We may note, in conclusion, two other approaches that have been used to analyze trade flows. While these approaches may be of some interest in describing the structure of the trade matrix in question, they are cruder than what we have been discussing in the sense that no attempt generally is made

38 It is worth noting that Pöyhönenn calculated the coefficients on the income terms to be quite close to 0.5, in comparison with coefficients in the 0.8 to 1.0 range obtained by all other investigators, including Pöyhönenn’s own research associate, Pulliaimen. The only satisfactory explanation of this phenomenon is the existence of a computational error. That is, the minimization of a value such as Pöyhönenn’s (see Footnote 21) necessarily involves an iterative technique. It is common for iterative techniques either to converge to a local minimum or to converge so slowly that the program feels convergence is complete long before the true values are reached. The fact that Pöyhönenn’s initial values are 0.5, therefore, makes 0.52 and 0.504 as final values very suspect.

to explain why, economically speaking, the distribution of trade flows is more skewed than would be assumed on the basis of a simple probability model. No resort is made, in other words, to variables such as distance, trade preferences, etc., which constitute the economic forces behind the observed flows.

The first approach is represented by the work of Savage and Deutsch [21], who developed a simple probability model to describe the generation of international flows. While our model described earlier in Equation (6.11) has been adapted from the Savage-Deutsch model, there are nevertheless two important differences to be noted. Their model takes into account the fact that there is no flow internal to any country; that is, they restrict $p_{ij}$ and $V_{ii}$ to zero. Secondly, they allow the size of a transaction (consignment, in their terminology) to vary with mean $\beta$ and variance $\sigma^2$. Using this model, Savage and Deutsch estimated the expected flows from knowledge of the marginal totals, $V^x_i$, $V^y_j$, and explored for significant differences between the actual and expected flows, which of course discovered.39 The logical subsequent step of regressing these deviations on resistance variables was performed with some success by Linnemann [16, p. 183]. Closely related to the Savage and Deutsch approach is that of Uribe, Theil, and de Leeuw [26], which sought to predict trade flows from knowledge of the future import and export totals, $V^x_i$, $V^y_j$, with an adjustment for the fact that present flows do not conform to the hypothetical simple pattern such as that indicated by our Equation (6.11).40

39 More specifically the authors assumed

$$p_{ij} = \begin{cases} SP_i Q_j & \text{for } i \neq j \\ 0 & \text{for } i = j \end{cases}$$

with the normalizing constant

$$S = (1 - \sum_k p_{ik} Q_k)^{-1}$$

The numbers $P_i$ and $Q_j$ will be approximately the probabilities of having $i$ as the origin and $j$ as the terminal point. The authors then calculated the expected values of $V^x_i$ and $V^y_j$ as

$$E(V^x_i) = TSP_i (1 - Q_i)$$

$$E(V^y_j) = TSP_j (1 - P_j)$$

where $T$ is total world trade. These equations were then solved for estimates of $P_i$ and $Q_j$, which were used to calculate the expected flows as

$$E(V_{ij}) = TSP_i Q_j$$

40 A simple prediction of $V_{ij}$ suggested by Uribe, Theil and de Leeuw, is

$$V_{ij} = \frac{\sum_i PV^x_i}{T}$$

This prediction can be adjusted to allow for the divergence of the actual flow from the expected flow in a base period as follows

$$V'_{ij} = \frac{\sum_i PV^x_i}{T} \times \frac{V_{ij}}{\sum_i PV^x_i / T_0}$$

That is, if $V_{ij}$ in the base period $V_{ij}$ exceeds its expected value $(\sum_i PV^x_i / T_0)$, we will adjust our estimate of $V_{ij}$ accordingly.
CONCLUSION

The review in this chapter has suggested a number of possibly fruitful areas for research into the factors determining the level and flow pattern of a country's foreign trade. The significance of such research must be understood in the context of seeking a broader understanding of the empirical basis of the pure theory of international trade. This is something a number of the studies cited have failed to make clear.

The starting point for investigation is to be found in the economic models depicted in Equations (6.2) and (6.8) and in the probability model of trade flows in Equation (6.11). The need for further theoretical refinement and empirical specification of these models should be evident from our discussion. It would appear especially worthwhile to attempt to give greater empirical content to the resource endowment, utility structure, and resistance variables represented in the models.

In particular, the relation between the cross-section and time-series analysis of the determinants of the size of a country's foreign sector should be investigated in order to see whether some typical growth path for this sector can be identified. This observation points toward a pooling of cross-section and time-series data in a single study. It is also important to be certain that the model used in analyzing a country's foreign sector is consistent with the model of international trade flows that has been used. In both types of investigation, the significance of such resistance factors as distance, market familiarity, and preference arrangements has been made clear. These are certainly factors that deserve more careful study.

REFERENCES


