Persistence Modeling for Assessing Marketing Strategy Performance

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INTRODUCTION

The question of long-run market response lies at the heart of any marketing strategy that tries to create a sustainable competitive advantage for the firm or brand. A key challenge, however, is that only short-run results of marketing actions are readily observable. Persistence modeling addresses the problem of long-run market-response quantification by combining into one measure of "net long-run impact" the chain reaction of consumer response, firm feedback and competitor response that emerges following the initial marketing action.

In this paper, we (i) summarize recent marketing-strategic insights that have been accumulated through various persistence modeling applications, (ii) provide an introduction to some of the most frequently used persistence modeling techniques, and (iii) identify some other strategic research questions where persistence modeling may prove to be particularly valuable.

CONCEPTS

Suppose you are a marketing executive contemplating the launch of a costly marketing campaign whose objective is to lift the sagging sales performance of a brand. Naturally, the anticipated sales increase as a result of this campaign should be a primary criterion to decide whether or not to make the engagement. Indeed, the campaign should be viewed as an investment of the company's time, money and reputation. This investment is expected to yield a net positive return that is higher than that of alternative uses of these scarce resources.

However, the potential impact of marketing campaigns on sales and, ultimately, profits, extends well beyond the spending period. Both the marketing literature and managerial experience teach us, for example, that advertising effects are subject to a wear-in or build-up phase, followed by a wear-out phase (Hanssens, Parsons and Schultz 2001). These and other over-time effects of marketing apply not only to *consumer response*. Indeed, a successful campaign may result in *feedback effects* on internal decision making, for example when the observed sales lift attributed to advertising results in subsequent increases in advertising budget allocations. Similarly, competitors may imitate or *retaliate* against a campaign that is perceived as a threat to their business performance.

Whether or not our hypothetical marketing campaign is ultimately successful will depend on the *combined* forces of consumer response, performance feedback and competitive reactions, and how these forces shape the financial return to the initial campaign. Therefore, an accurate assessment of marketing effectiveness should pay particular attention to so-called *long-run* sales response, i.e. movements in business performance that continue in the future but can still be attributed to short-term marketing activity, as well as the corresponding long-run spending and/or pricing implications, i.e. subsequent marketing-mix adjustments that persist over time.

The question of long-run sales response lies at the heart of marketing strategy, which tries to create a *sustainable* competitive advantage for the firm or the brand. However, academics are understandably surprised at reported empirical results that 85% of all promotions are losing money to the promoters, and that only half of the advertising expenditures generate economic benefits to the advertisers (Abraham and Lodish 1990). Practitioners are concerned to observe virtually entire industries go through prolonged money-losing periods, such as the U.S. airlines in the early 1990s and again in the early 2000s, and increasingly feel the pinch of demonstrating the long-run revenue generation of their marketing budgets (Slywotzky and Shapiro 1993). Thus there is an urgent need to better assess the long-run impact of marketing strategies.

A key challenge is that only short-term results of marketing actions are readily observable, yet at the same time, most agree that short-term profit maximization is not the best paradigm for allocating resources. American businesses in general and the marketing discipline in particular, have repeatedly been criticized for their short-run orientation (Wind and Robertson 1983). Long-term profit maximization is considerably more difficult to operationalize, however, because there is little consensus of what constitutes the long run, and because market conditions continuously change, making it difficult to relate future outcomes to current actions (Dekimpe and Hanssens 1995a).

Do marketing investments themselves help shape the future by contributing to changing market conditions or by affecting the competitors' long-run position? Certain well-publicized marketing events have been said to change market conditions forever. For example, in the early nineties Compaq launched an aggressively-priced high-quality line of products, which is widely believed to have opened up the home market for personal computers. Zantac's sustained marketing campaign raised its market share to 50% in the anti-ulcer medication market, while Tagamet's share gradually eroded to 23% over the same 6-year time span (Slywotzky and Shapiro 1993). Much of this evidence is anecdotal, though, and, until recently, there was no broad body of knowledge allowing us to *precisely* measure the degree to which marketing efforts affect the long-term evolution of the market place.

Indeed, standard managerial tools were of little help in increasing our understanding of observable long-term marketing effects, or in offering guidelines for long-term resource allocation in evolving or changing markets. Marketing's focus has been on "short-run forecasting and optimization procedures, while assuming an essentially stable environment" (Wind and Robertson 1983, p. 13). However, recent empirical research suggests that 60 percent of market performance variables, and 78 percent of sales variables, are not stable, but rather evolve over time (Dekimpe and Hanssens 1995b, p. G114). If marketing as a management discipline is to develop strategic relevance at the highest level of decision making, it should provide answers to questions about the drivers of evolving, and therefore long-run, business performance.

Persistence models address the problem of quantification of short- and long-run market response in evolving environments (Dekimpe and Hanssens 1995a). We will describe these techniques in some detail in the next section. At a conceptual level, marketing actions have persistent effects on sales if (1) the sales environment is evolving (as opposed to stable or stationary), and (2) this sales evolution is related to the marketing actions. For example, Dekimpe and Hanssens (1995a) found that a home-improvement chain's price-oriented print advertising had a high short-run impact with limited sales persistence (mainly short-run benefits), while TV spending had a low short-run impact with substantial sales persistence (mainly long-run benefits). The application illustrated that marketing can indeed have persistent performance (in casu, sales) effects which can be quantified empirically.

Persistence models have their methodological roots in the econometrics and time-series literature, and have been used in a number of social-science disciplines, including macro-economics and finance. For example, economists have used persistence models to determine which major world events have altered long-term trends in economic activity, and finance researchers have used them to investigate the long-term effects of monetary policy on stock-market returns. The growing use of persistence models in marketing is due mainly to the field's long-standing interest in determining the short-run and long-run effects of various marketing activities on market performance. Examples include the sales impact of advertising campaigns (Dekimpe and Hanssens 1995a), price promotions (Dekimpe, Hanssens and Silva-Risso 1999), distribution changes (Bronnenberg, Mahajan and Vanhonacker 2000), channel additions (Deleersnyder et al. 2002) and new-product introductions (Pauwels, Silva-Risso, Srinivasan and Hanssens 2004; Pauwels and Srinivasan 2004).

Marketing persistence models are not restricted to one level of data aggregation such as firm performance or individual consumer choice. They have been estimated at various levels of aggregation, ranging from market shares (Franses, Srinivasan and Boswijk 2001), to brand sales (Dekimpe and Hanssens 1999) to category demand (e.g. Nijs, Dekimpe, Steenkamp and Hanssens 2001) to macro-economic indicators (Jung and Seldon 1995). Persistence modeling has also been applied to ever-smaller levels of aggregation, such as the individual-store level (Horváth, Leeflang and Wittink 2001) or particular consumer segments (Lim, Currim and Andrews 2003). Even the emerging discipline of one-on-one marketing uses persistence models, for example to measure the effectiveness of various customer acquisition channels (Villanueva, Yoo and Hanssens 2003) or to compare marketing's impact on customer acquisition vs. customer retention (Yoo, Hanssens and Powers 2003). So long as a sufficient number of equally-spaced performance and marketing data are available, and regardless of aggregation level, persistence models can be used to distinguish between marketing's short- and long-run impact, and to combine consumer response, firm feedback and competitor response in one measure of "net" long-term impact.

TECHNICAL OVERVIEW

Persistence modeling is a multi-step process, as depicted in Figure 1. In a first step, unit-root tests are used to determine whether or not the different variables are stable or evolving. In case several of the variables are found to have a unit root, one subsequently tests for cointegration. Depending on the outcome of these two preliminary steps, one estimates a Vector-AutoRegressive (VAR) model in the levels, in the differences, or in error-correction format. Finally, the parameter estimates from this VAR model are used to derive Impulse Response Functions (IRFs), from which various summary statistics on the short- and-long-run dynamics of the system can be derived. We now briefly elaborate on each of these steps.

Figure 1 about here

Unit-root testing: are performance and marketing variables stable or evolving?

The distinction between stability and evolution is formalized through the *unit-root* concept. Following Dekimpe and Hanssens (1995a), we first consider the simple case where the over-time behavior of the variable of interest (e.g. a brand's sales S_t) is described by a first-order autoregressive process:

$$(1 - \phi L)\, S_t = c + u_t\,, \qquad (1a)$$

where ϕ is an autoregressive parameter, L the lag operator (i.e. $L^k S_t = S_{t-k}$), u_t a residual series of zero-mean, constant-variance (σ^2_u) and uncorrelated random shocks, and c a constant. Note that Equation (1a) may also be written in the more familiar form

$$S_t = c + \phi\, S_{t-1} + u_t\,, \qquad (1b)$$

which corresponds to a simple regression model of S_t on its own past, with u_t the usual i.i.d. residuals. Applying successive backward substitutions allows us to write equation (1) as

$$S_t = [c / (1 - \phi)] + u_t + \phi\, u_{t-1} + \phi^2\, u_{t-2} + \dots, \qquad (2)$$

in which the present value of S_t is explained as a weighted sum of random shocks. Depending on the value of ϕ, two scenarios can be distinguished.[1] When $|\phi|<1$, the impact of past shocks diminishes and eventually becomes negligible. Hence, each shock has only a temporary impact. In that case, the series has a fixed mean $c/(1-\phi)$ and a finite variance $\sigma^2_u/(1-\phi^2)$. Such a series is called stable. When $|\phi|=1$, however, (2) becomes:

[1] Strictly speaking, one could also consider the situation where $|\phi|>1$, in which case past shocks become more and more important, causing the series to explode to plus or minus infinity. Situations where the past becomes ever more important are, however, unrealistic in marketing.

$S_t = (c + c + ...) + u_t + u_{t-1} + ...,$ (3)

implying that each random shock has a permanent effect on the subsequent values of *S*. In that case, no fixed mean is observed, and the variance increases with time. Sales do not revert to a historical level, but instead wander freely in one direction or another, i.e. they *evolve*. Distinguishing between both situations involves checking whether the parameter ϕ in Equation (1) is smaller than or equal to one.[2]

Numerous tests have been developed to distinguish stable from evolving patterns. One popular test, due to Dickey and Fuller (1979), is based on the following test equation:

$$(1 - L)\, S_t = \Delta S_t = a_0 + b\, S_{t-1} + a_1 \Delta S_{t-1} + ... + a_m \Delta S_{t-m} + u_t \,. \tag{4}$$

The *t*-statistic of *b* is compared with critical values and the unit-root null hypothesis is rejected if the obtained value is larger in absolute value than the critical value. The *m* ΔS_{t-j} terms reflect temporary sales fluctuations, and are added to make u_t white noise. Because of these additional terms, one often refers to this test as the "augmented" Dickey-Fuller (ADF) test. The ADF test was used, for example, in Dekimpe and Hanssens (1999). They analyzed a monthly sample of five years of market performance (number of prescriptions), market support (national advertising and number of sales calls to doctors) and pricing (price differential relative to the main challenger) data for a major brand in a prescription drug market. Based on the Schwartz (SBC) criterion (cf. infra), a value of *m* varying between 0 (price differential & sales-calls series) and 2 (prescription series) was selected. The *t*-statistic of the *b*-parameter in Equation (4) was smaller in absolute value than the 5%- critical value for each of the variables, implying the presence of a unit root in each of them.

Key decisions to be made when implementing ADF-like unit-root tests are (i) the treatment (inclusion/omission) of various deterministic components, (ii) the determination of the number of augmented (ΔS_{t-j}) terms, and (iii) whether or not allowance is made for structural breaks in the data. First, Equation (4) tests whether or

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[2] The previous discussion used the first-order autoregressive model to introduce the concepts of stability, evolution and unit roots. The findings can easily be generalized to the more complex autoregressive moving-average process $\Phi(L)S_t = c + \Theta(L)u_t$. Indeed, the stable/evolving character of a series is completely determined by whether or not some of the roots of the autoregressive polynomial $\Phi(L) = (1 - \phi_1 L - ... - \phi_p L^p)$ are equal to one.

not temporary shocks may cause a permanent deviation from the series' fixed mean level. When dealing with temporally disaggregated (less than annual) data, marketing researchers may want to add deterministic seasonal dummy variables to the test equation to allow this mean level to vary across different periods of the year. Their inclusion does not affect the critical value of the ADF test. This is not the case, however, when a deterministic trend is added to the test equation, in which case one tests whether shocks can initiate a permanent deviation from that predetermined trend line. Assessing whether or not a deterministic trend should be added is intricate because the unit-root test is conditional on its presence, while standard tests for the presence of a deterministic trend are, in turn, conditional on the presence of a unit root. An often-used test sequence to resolve this issue is described in Enders (1995, pp. 256-257), and a marketing application may be found in Nijs et al. (2001).

A second critical issue in the implementation of ADF tests is the determination of the number of augmented terms. Two popular order-determination procedures are the application of fit indices such as the AIC or SBC criterion (see e.g. Nijs et al. 2001; Srinivasan, Pauwels, Hanssens and Dekimpe 2003), or the top-down approach advocated by Perron (1994). The latter approach, used in a marketing setting by Deleersnyder et al. (2002), starts with a maximal value of m, and successively reduces this value until a model is found where the last lag is significant, while the next-higher lag is not.

Finally, a decision has to be made whether or not to allow for a structural break in the data-generating process. Indeed, the shocks considered in Equations (1-4) are expected to be regularly occurring, small shocks that will not alter the underlying data-generating process. This assumption may no longer be tenable for shocks associated with, e.g., a new-product introduction (see e.g. Pauwels and Srinivasan 2003; Dekimpe et al. 1997) or an Internet channel addition (Deleersnyder et al. 2002). Such shocks tend to be large, infrequent, and may alter the (long-run) properties of the time series. A failure to account for these special events has been shown to bias unit-root tests towards finding evolution. In that case, one would erroneously conclude that all (regular) shocks have a long-run impact, while (i) these shocks cause only a temporary deviation from a fixed mean (deterministic trend), and (ii) only the special events caused a permanent shift in the level (intercept and/or slope) of an otherwise level (trend) stationary series. Appropriate adjustments to Equation

(4) to account for such special event(s) have been proposed by Perron (1994) and Zivot and Andrews (1992), among others.

Other developments that are relevant to applied marketing researchers deal with the design of unit-root tests that incorporate the logical consistency requirements of market shares (Franses, Srinivasan and Boswijk 2001), and the use of outlier-robust unit-root (and cointegration, cf. infra) tests as described in Franses, Kloek and Lucas (1999).

Cointegration tests: does a long-run equilibrium exist between evolving series?

Evolving variables are said to be cointegrated when a linear combination exists between them that results in stable residuals. Even though each of the individual variables can move far away from its previously held positions, this long-run equilibrium prevents them from wandering apart.[3] Such long-run equilibria can emerge because of a variety of reasons. Among them, certain budgeting rules (e.g. percentage-of-sales allocation rules) imply that sales successes eventually translate into higher marketing spending. Similarly, competitive decision rules can result in firms' marketing spending levels never to deviate too far from each other. Finally, customers' limited budgets may cause different price levels to be associated with different long-run demand levels, which would imply a cointegration relationship between sales and prices.

Consider, without loss of generality, a three-variable example where a brand's sales (S), marketing support (M) and its competitors' marketing support (CM) are all evolving (i.e. they all have a unit root). The existence of a perfect equilibrium relationship between these three variables would imply (see Powers et al. 1991 for a more in-depth discussion):

$$S_t = \beta_0 + \beta_1 M_t + \beta_2 CM_t \qquad (5)$$

In practice, however, we are unlikely to observe a perfect equilibrium in every single period. A more realistic requirement is that its deviations are mean-reverting (stable) around zero, i.e. $e_{S,t}$ in Eq. (6) should no longer be evolving, even though each of the other variables in the equation is:

[3] One could argue that two mean-stationary series are also in long-run equilibrium, as each series deviates only temporarily from its mean level, and hence, from the other. However, this situation is conceptually different from a cointegrating equilibrium, in which a series can wander away from its previously-held positions, but not from the other.

$S_t = \beta_0 + \beta_1 M_t + \beta_2 CM_t + e_{S,t}$ (6)

A simple testing procedure for cointegration, proposed by Engle and Granger (1987), is to estimate (6) using OLS, and test the residuals $e_{S,t}$ for a unit root using standard unit-root tests (without intercept in the test equation, and using updated critical values as listed in Engle and Yoo 1987). A marketing application of the Engle-and-Granger (EG) approach to cointegration testing can be found in Baghestani (1991), among others. Lately, Johansen's Full Information Maximum Likelihood (FIML) approach has become increasingly popular to test for cointegration. The latter test was applied in Dekimpe and Hanssens (1999, p. 406) in their analysis of a prescription drugs market (see before). It was found that even though each of the individual series (prescriptions, advertising, sales calls and price differential) was evolving, the four variables were tied together in a long-run equilibrium that prevented them from wandering too far apart from each other.

As with the unit-root tests, cointegration tests have also been extended to allow for structural breaks; see e.g. Gregory and Hansen (1996) for a technical discussion, or Kornelis (2002) for marketing applications.

VAR models: how to capture the dynamics in a system of variables?

The third step in persistence modeling is to specify a vector-autoregressive model to link the (short-run) movements of the different variables under consideration. Depending on the outcomes of the preceding unit-root and cointegration tests, these VAR models are specified in the levels (no unit roots), in the differences (unit roots without cointegration), or in error-correction format (cointegration).[4]

For expository purposes, we first consider a model in levels, and focus on a simple-three equation model linking own sales performance (S), own marketing spending (M) and competitive marketing spending (CM). The corresponding VAR model (in which, for ease of notation, all deterministic components are omitted) becomes:

[4] In case only a subset of the variables has a unit root or is cointegrated, mixed models are specified.

$$\begin{bmatrix} S_t \\ M_t \\ CM_t \end{bmatrix} = \begin{bmatrix} \pi_{11}^1 & \pi_{12}^1 & \pi_{13}^1 \\ \pi_{21}^1 & \pi_{22}^1 & \pi_{23}^1 \\ \pi_{31}^1 & \pi_{32}^1 & \pi_{33}^1 \end{bmatrix} \begin{bmatrix} S_{t-1} \\ M_{t-1} \\ CM_{t-1} \end{bmatrix} + \ldots + \begin{bmatrix} \pi_{11}^J & \pi_{12}^J & \pi_{13}^J \\ \pi_{21}^J & \pi_{22}^J & \pi_{23}^J \\ \pi_{31}^J & \pi_{32}^J & \pi_{33}^J \end{bmatrix} \begin{bmatrix} S_{t-J} \\ M_{t-J} \\ CM_{t-J} \end{bmatrix} + \begin{bmatrix} u_{S,t} \\ u_{M,t} \\ u_{CM,t} \end{bmatrix}, \tag{7}$$

where J is the order of the model, and where $\vec{u} = [u_{S,t}\ u_{M,t}\ u_{CM,t}]' \sim N(0, \Sigma)$. This specification is very flexible, and reflects the forces or channels of influence discussed earlier: delayed response (π_{12}^j, j=1, … J), purchase reinforcement (π_{11}^j), performance feedback (π_{21}^j), inertia in decision making (π_{22}^j) and competitive reactions (π_{32}^j). Only instantaneous effects are not included directly, but these are reflected in the variance-covariance matrix of the residuals (Σ). Estimation of these models is straightforward: (i) all explanatory variables are predetermined, so there is no concern over the identification issues that are often encountered when specifying structural multiple-equation models, and (ii) all equations in the system have the same explanatory variables so that OLS estimation can be applied without loss of efficiency.

However, this flexibility comes at a certain cost. First, the number of parameters may become exuberant. For J = 8, for example, the VAR model in equation (7) will estimate 9 x 8 =72 autoregressive parameters. If, however, one considers a system with 5 endogenous variables, this number increases to 25 x 8=200. Several authors (see e.g. Pesaran, Pierse and Lee 1993; Dekimpe and Hanssens 1995a) have therefore restricted all parameters with $|t|<1$ to zero.[5] While this may alleviate the problem of estimating and interpreting so many parameters, it is unlikely to fully eliminate it. As a consequence, VAR modelers typically do not interpret the individual parameters themselves, but rather focus on the impulse-response functions (IRFs) derived from these parameters. As discussed in more detail in Section 2.4, IRFs trace, over time, the incremental performance and spending implications of an initial one-period change in one of the support variables. In so doing, they provide a concise summary of the information contained in this multitude of parameters, a summary that lends itself well to a graphical and easy-to-interpret representation (cf. infra).

[5] Note that this may necessitate the use of SUR, rather than OLS, estimation, as the equations may now have a different set of explanatory variables.

Second, no direct estimate is provided of the instantaneous effects. The residual correlation matrix can be used to establish the presence of such an effect, but not its direction. Various procedures have been used in the marketing literature to deal with this issue, such as an a priori imposition of a certain causal ordering on the variables (i.e. imposing that an instantaneous effect can occur in one, but not the other, direction) as in Dekimpe and Hanssens (1995a), a sensitivity analysis of various causal orderings (see e.g. Dekimpe, Hanssens and Silva-Risso 1999), or accounting for expected instantaneous effects in the other variables when deriving the impulse-response functions (cf. Section 2.4), as implemented in Nijs et al. (2001).

If some of the variables have a unit root, the VAR model in Eq. (7) is specified in the differences; e.g. S_t, S_{t-1}, … are replaced by ΔS_t, ΔS_{t-1},… If the variables are cointegrated as well, this model in differences is augmented with the lagged residuals of the respective long-run equilibrium relationships (cf. Eq. 6), resulting in the following specification:

$$\begin{bmatrix} \Delta S_t \\ \Delta M_t \\ \Delta CM_t \end{bmatrix} = \begin{bmatrix} \alpha_S & 0 & 0 \\ 0 & \alpha_M & 0 \\ 0 & 0 & \alpha_{CM} \end{bmatrix} \begin{bmatrix} e_{S,t-1} \\ e_{M,t-1} \\ e_{CM,t-1} \end{bmatrix} + \sum_{j=1}^{J} \begin{bmatrix} \pi_{11}^j & \pi_{12}^j & \pi_{13}^j \\ \pi_{21}^j & \pi_{22}^j & \pi_{23}^j \\ \pi_{31}^j & \pi_{32}^j & \pi_{33}^j \end{bmatrix} \begin{bmatrix} \Delta S_{t-j} \\ \Delta M_{t-j} \\ \Delta CM_{t-j} \end{bmatrix} + \begin{bmatrix} u_{S,t} \\ u_{M,t} \\ u_{CM,t} \end{bmatrix}. \quad (8)$$

The addition of the error-correction terms $[\alpha_S e_{S,t-1} \ \alpha_M e_{M,t-1} \ \alpha_{CM} e_{CM,t-1}]'$ implies that in every period there is a partial adjustment towards restoring the underlying, temporarily disturbed, long-run equilibrium. Said differently, the system partially corrects for the previously observed deviations $[e_{S,t-1} \ e_{M,t-1} \ e_{CM,t-1}]'$, and the respective α-coefficients reflect the speed of adjustment of the corresponding dependent variable towards the equilibrium. A good review on the implementation issues involved can be found in Franses (2001). In the earlier prescription-drugs example, Dekimpe and Hanssens (1999) had identified that all 4 series in their sample were evolving (cf. Section 2.1), and that a long-run equilibrium relationship existed between them (cf. Section 2.2). They therefore estimated a four-equation VAR model that was specified in the differences, whereby each equation was augmented with a lagged error-correction term (i.e. the lagged residuals from the equilibrium relationship); see their Table 2 for an overview of the resulting parameter estimates.

Impulse-response function derivation

An impulse-response function (IRF) traces the incremental effect of a one-unit (or one-standard deviation) shock in one of the variables on the future values of the other endogenous variables. The first steps of this process are depicted in Appendix B (where we consider, for expository purposes, a VAR model of order 1). IRFs can also been seen as the difference between two forecasts: a first extrapolation based on an information set that does not take the marketing shock into account, and another prediction based on an extended information set that takes this action into account. As such, IRFs trace the incremental effect of the marketing action reflected in the shock. Note that marketing actions (e.g. a price promotion) are operationalized as deviations from a benchmark, which is derived as the expected value of the marketing mix-variable (e.g. the price) as predicted through the dynamic structure of the VAR model. See Pauwels, Hanssens and Siddarth 2002 for an extensive discussion on this issue.

A graphical illustration of some IRFs, taken from Nijs et al. (2001), is given in Figure 2. The top panel shows the IRF tracing the incremental performance impact of a price-promotion shock in the stationary Dutch detergent market. Because of the chain reaction of events reflected in this IRF, we see various fluctuations over time; for example, a typical stockpiling effect, feedback rules, and competitive reactions. Eventually, however, any incremental effect disappears. This does not imply that no more detergents are sold, but rather that no *additional* sales can be attributed to the initial promotion. In contrast, in the evolving dairy-creamer market shown in the bottom panel of Figure 2, we see that this incremental effect stabilizes at a non-zero, or persistent, level. In that case, we have identified a long-run effect, as the initial promotion keeps on generating extra sales. Behavioral explanations for this phenomenon could be that newly attracted customers make regular repeat purchases, that the existing customer base has increased its usage rate, etc…

Insert Figure 2 about here

While impulse-response functions are useful summary devices, the multitude of numbers (periods) involved still makes them somewhat awkward to compare across brands, markets, or marketing-mix instruments. To reduce this set of numbers to a more manageable size, one often (see e.g. Nijs et al. 2001; Srinivasan et al. 2003; Pauwels and Srinivasan 2004) derives various summary statistics from them, such as:

(i) the ***immediate*** performance impact of the marketing-mix shock;

(ii) the ***long-run*** or permanent (persistent) impact, which is the value to which the IRF converges,

(iii) the cumulative effect before this convergence level is obtained. This cumulative effect is often called the total ***short-run*** effect. For stationary series, this reflects the area under the curve. In case of a persistent effect, one can compute the combined (cumulative effect) over the time span it takes before the persistent effect is obtained. The time interval before convergence is obtained is often referred to as the dust-settling period (Dekimpe and Hanssens 1999; Nijs et al. 2001).[6]

In the impulse-response derivation of Appendix B, no instantaneous effects are captured, i.e. a shock in one of the variables does not result in a non-zero shock value in the other variables. Moreover, since all variables in the VAR model are predetermined, instantaneous effects are not captured through any of the π_{ij} parameters. In order to capture such instantaneous effects, the approach by Evans and Wells (1983) has become popular in recent marketing applications (see e.g. Nijs et al. 2001; Srinivasan et al. 2003). The information in the residual variance-covariance matrix of the VAR model is used to derive a vector of *expected* instantaneous shock values following an initiating shock in one of the variables.[7] This expected shock vector, rather than the [0 1 0]' vector used in Appendix B, is subsequently traced through the system in order to derive its incremental impact on the future values of the various endogenous variables. This procedure was adopted in Dekimpe and Hanssens' (1999) analysis of a prescription drug market (see also Sections 2.1-2.3). Impulse-response functions were used to quantify the immediate, short- and long-run performance, spending and profit implications of changes in, respectively, advertising support, the number of sales calls, and the price differential with a major competitor. Focusing on their long-term conclusions, increases in calling support failed to produce persistent sales gains, but were costly in the long run. Narrowing the price gap with its competitors improved the brand's long-run profitability, even though this strategy

[6] In panel B, the dust-settling period is defined in terms of the last period that has an impact significantly different from the nonzero asymptotic value (see Nijs et al. 2001 for details).

[7] Assuming multivariate normality of the residuals of the VAR model, it is easy to show that the expected shock values in the other variables after a one-unit shock to the *i*-th variable are given by $[\sigma_{ij}/\sigma_{ii}]$, with the σ elements derived from the estimated residual variance-covariance matrix of the VAR model .

contributed to the long-run sales erosion of the brand. Finally, the observed reductions in advertising support had a negative impact on long-run sales levels as well.

Last, but not least, we briefly discuss the use of forecast error variance decompositions (FEVD). For each post-shock time period, the IRF shows the total impact of the shock on each endogenous variable in the system. The FEVD measures the relative contribution of each shock component on that total shock impact. For example, if an advertising shock lifts sales four weeks into the future, the FEVD would assess how much of that sales lift is due to consumer response, to competitive reaction, to advertising decision rules, etc… FEVD has been used in recent marketing studies by Hanssens (1998) and by Pauwels et al. (2004).

NEW DIRECTIONS

As Appendix A indicates, empirical work in marketing persistence models is developing rapidly. We conjecture that the main reasons for this diffusion are as follows:

- Persistence modeling makes a clear, quantifiable *distinction between short-run and long-run* marketing effectiveness. Persistence modeling provides a much needed and workable definition of *long run*, based on the difference between temporary and permanent movements in the data.
- Persistence modeling uses a *system's approach* to market response, e.g. it combines the forces of customer response, competitive reaction and firm decision rules. It allows for the decomposition of the total observed long-run effect of a marketing action as a *chain rule* formed by these three forces. Thus it relates well to the complexities of real-world marketing strategy.
- As databases expand both longitudinally and cross-sectionally, *new application areas* of persistence modeling have emerged. For example, the cross-sectional variation in persistence estimates derived across numerous categories and brands, has lead to various empirical generalizations on long-run marketing effectiveness and their antecedents (see Appendix A for a review).

Earlier applications dealt predominantly with a quantification of the long-run effectiveness of a variety of marketing-mix instruments, such as advertising, promotions, and distribution changes. Recently, however, we have witnessed the application of persistence modeling to a new set of relevant strategic questions, four of which we briefly examine below.

Is competitive retaliation necessary or discretionary?

How do competitors react to each other's price-promotion and advertising attacks? What are the reasons for the observed reaction behavior? Steenkamp et al. (2003) answer these questions by performing a large-scale empirical study on the short-run and long-run reactions to promotion and advertising attacks in over 400 consumer product categories, over a four-year time span.

The main finding of the study is that competitive reaction is predominantly passive. When it is present, it is usually retaliatory in the same instrument, i.e., promotion attacks are countered with promotions, and advertising attacks are countered with advertising. There are very few long-run consequences of any type of reaction behavior. The authors are able to draw these inferences because their models examine the 'chain reaction' of consumer and competitor response following the initial advertising or promotion campaign.

The study also reports on a number of moderating effects, such as power asymmetry, promotional intensity and perishability of the product category, that support the presence of a certain amount of rationality in competitive reaction behavior. Finally, by linking reaction behavior to both cross and own marketing effectiveness, they demonstrate that passive behavior is often a sound strategy. On the other hand, firms that opt to retaliate often use ineffective instruments, resulting in "spoiled arms." Accommodating behavior is observed in only a minority of cases, and often results in a missed sales opportunity when promotional support is reduced.

The authors' overall conclusion is that the ultimate impact of most promotion and advertising campaigns depends primarily on the nature of consumer response, not the vigilance of competitors. In order words, the strong link in the chain reaction is the consumer. This is an important finding for marketing strategy, especially as it counters a prevailing belief in the management strategy literature that the ultimate effectiveness of an action depends largely on the defenders' response.

Marketing and firm valuation

While marketing scientists are understandably focused on consumer and competitor response to marketing actions, it is equally important to study how these actions influence investor behavior. In particular, do investors place a premium value on firms that advertise heavily? Do they value new-product activity and/or promotional campaigns?

The finance discipline has long established that stock prices follow random walks, i.e. new information that is profit relevant is incorporated immediately and fully in valuation. As a result, stock prices are always evolving, and persistence models may be used to uncover how marketing actions influence that evolution, above and beyond their sales- and profit impact.

This principle has been used in two contexts to date. First, Pauwels, Silva-Risso, Srinivasan and Hanssens (2004) contrasted investor reactions to auto companies' new-product introductions vs. price promotions over a five-year period. They found that new-product introductions have a gradually increasing influence on stock price, all else equal. On the other hand, price promotions generally detract firm value, even though they may successfully stimulate demand. Thus, investors view new-product activity as long-term value generating, and promotions as long-term value destroying. The authors estimate the net market value addition/subtraction of a typical innovation/promotion shock to be in the tens to hundreds of millions of dollars.

Second, Joshi and Hanssens (2003) examine the influence of advertising campaigns on the valuation of firms in the personal computer industry over a ten-year long period. They found that advertising has a small, but positive long-term effect on stock prices, again after controlling for advertising's direct impact on sales and profits. Thus, investors view advertising as a signal of firm strength and are willing to pay a premium for it. The market value addition of an advertising shock in that industry is estimated at several tens of millions of dollars.

Marketing and customer equity

With the advent of customer and prospect databases and the proliferation of direct marketing, marketers are increasingly viewing their customers as strategic assets, and strive to maximize their firm's customer equity (defined as the sum of the lifetime values of all customers). Because of its connection to cash flows, customer equity is a tangible metric of firm performance, yet at the same time it embodies the marketing concept. Consequently, there is considerable interest in the relationship between marketing spending and customer equity.

Persistence modeling is well suited to address this research question when applied to tracking data of customer and prospect movements and transactions, say on a weekly basis. For example, suppose a new customer is acquired via advertising in a given week. This acquisition can start a chain reaction of subsequent customer and prospect movements as follows:

- the customer generates a stream of revenues from purchases;
- the customer generates word-of-mouth which leads to the subsequent acquisition of new customers;
- the success of advertising feeds forward into future advertising spending;
- etc…

All these events add to the customer equity of the firm, and in this way a long-term customer equity effect of the original advertising emerges. This long-term impact can be measured by persistence modeling, as was done recently for an on-line service company in Villanueva, Yoo and Hanssens (2003). This study found substantial differences in the lifetime values of customers acquired through different channels.

Diagnosing marketing turnarounds

Early work in persistence modeling revealed that a substantial fraction of market performance measures is stationary over time, especially market-share measures (Dekimpe and Hanssens 1995b). However, stationary performance over extended periods of time is not necessarily compatible with firms' objectives for sustained profitable growth.

This apparent contradiction between managerial goals and observed performance is resolved when applying persistence modeling to *moving windows*, i.e. subsamples of time that capture only the last few years of a firm or brand's history. This was done for a frequently purchased product category by Pauwels and Hanssens (2003). They found that, even though the market is stationary over the entire seven-year period under study, each brand goes through successive performance regimes of growth, stability and decline. Furthermore, the authors use persistence modeling to distinguish between turnarounds caused by time itself, by single vs. sustained marketing actions, and by competitive activity.

Inertia in marketing decision making

Even though profit-maximizing rules for price setting exist, prices in practice often exhibit inertia or stickiness, i.e. a tendency to depend on past prices. Persistence modeling may be used to assess the prevalence of price inertia as well as its economic consequences. A pioneering study by Srinivasan, Pauwels & Nijs (2003) demonstrated the extent to which retail prices are driven by past pricing history, brand demand, brand acquisition cost, category management, store brand performance and store traffic. The results show that retail prices are mainly driven by past retail prices (50%), followed by product acquisition costs (25%) and demand feedback (12.5%). While this dependence on past prices benefits the long-run sales performance of both manufacturers and retailers, it hurts retailers' financial performance in the long run. In contrast, demand-based pricing benefits the response levels of all performance variables.

CONCLUSION

Marketing strategy aims at developing a sustainable competitive advantage to the firm or the brand. Therefore, an important aspect of marketing strategy research should be concerned with the long-run impact of marketing actions on business performance.

Persistence modeling provides one such approach, based on the important principle that marketing success depends on the combined influence of customers, competition and the behavior of the firm itself. By carefully measuring the chain reactions that unfold over time as the result of a marketing action, persistence modeling quantifies both the magnitude and the duration of marketing's impact on business performance. As longitudinal marketing databases continue to improve in scope and in quality, we expect that these techniques will find increased use among academic scholars as well as advanced practitioners of marketing strategy.

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FIGURE 1: OVERVIEW OF PERSISTENCE MODELING PROCEDURE

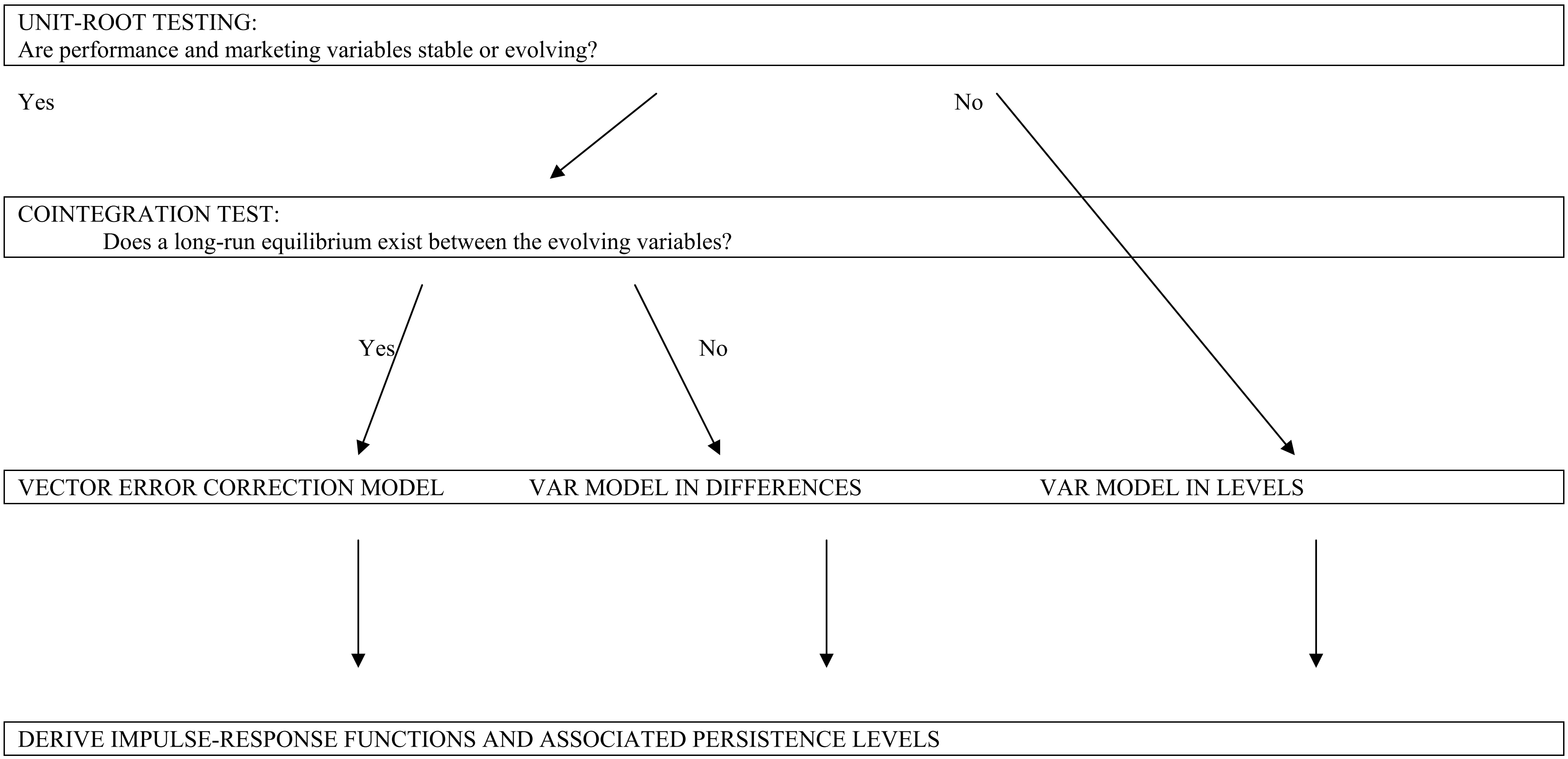
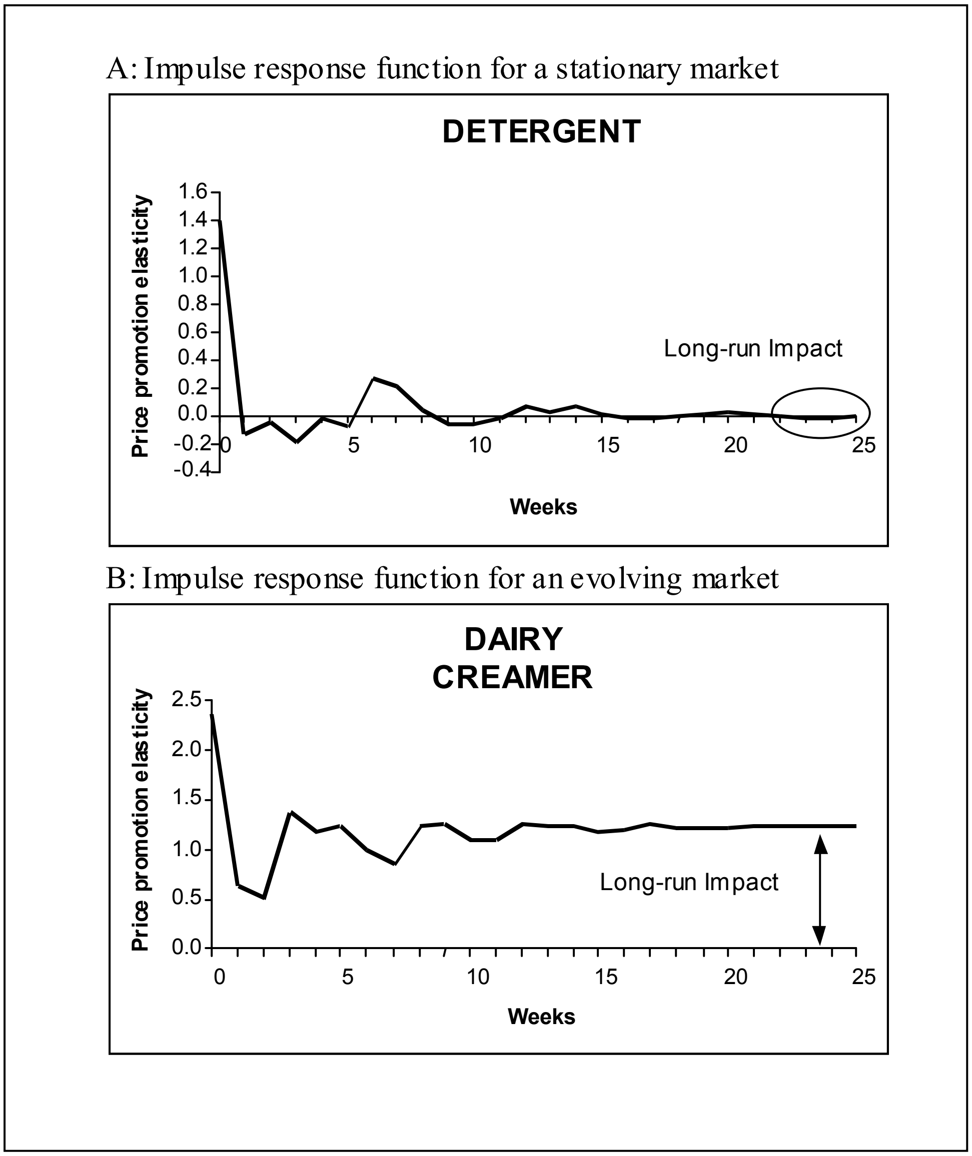


FIGURE 2: IMPULSE RESPONSE FUNCTIONS



APPENDIX A

STRATEGIC INSIGHTS FROM PERSISTENCE MODELING

Study	***Contribution***
Baghestani (1991)	Advertising has a long run impact on sales if both variables are (a) evolving and (b) in long-run equilibrium (cointegrated).
Bronnenberg et al. (2000)	Distribution coverage drives long-run market shares, especially the coverage evolution early in the life cycle.
Cavaliere and Tassinari (2001)	Advertising is not a long-run driver of aggregate whisky consumption in Italy.
Chowdhury (1994)	No long run equilibrium (cointegration) relationship is found between UK aggregate advertising spending and a variety of macro-economic variables.
Dekimpe & Hanssens (1995a)	Persistence measures quantify marketing's long-run effectiveness. Image-oriented and price-oriented advertising messages have a differential short- and long-run effect.
Dekimpe & Hanssens (1995b)	Sales series are mostly evolving, while a majority of market-share series is stationary.
Dekimpe & Hanssens (1999)	Different strategic scenarios (business as usual, escalation, hysteresis and evolving business practice) have different long-run profitability implications.
Dekimpe et al. (1999)	Little evidence of long-run promotional effects is found in FPCG markets.
Dekimpe et al. (1997)	New product introductions may cause structural breaks in otherwise stationary loyalty patterns
Franses (1994)	Gompertz growth models with non-constant market potential can be written in error-correction format.
Franses et al. (1999)	Outlier-robust unit-root and cointegration tests are called for in promotion-intensive scanner environments.
Franses et al. (2001)	Unit root and cointegration tests which account for the logical consistency of market shares.
Hanssens (1998)	Factory orders and sales are in a long-run equilibrium, but shocks to either have different long-run consequences
Hanssens & Ouyang (2001)	Derivation of advertising allocation rules (in terms of triggering versus maintenance spending) under hysteresis conditions
Johnson et al. (1992)	The long-run consumption of alcoholic beverages is not price sensitive.
Joshi and Hanssens (2003)	Advertising has a long-run positive effect on firm valuation.
Jung & Seldon (1995)	Aggregate US advertising spending is in long-run equilibrium with aggregate personal consumption expenditures.
McCullough & Waldon (1998)	Network and national spot advertising are substitutes.
Nijs, Dekimpe, Steenkamp and Hanssens (2001)	Limited long-run category expansion effects of price promotions. The impact differs in terms of the marketing intensity, competitive structure, and competitive conduct in the industry.
Pauwels and Srinivasan (2003)	Permanent performance effects are observed from store brand entry, but these effects differ between manufacturers and retailers, and between premium-price and second-tier national brands.
Pauwels and Hanssens (2003)	Brands in mature markets go through different performance regimes, which are influenced by their marketing policies
Pauwels et al. (2002)	The decomposition of the promotional sales spike in category-incidence, brand-switching and purchase-quantity effects differs depending on the time frame considered (short versus long run).

Pauwels, Srinivasan, Silva-Risso and Hanssens (2004)	Investor markets reward product innovation but punish promotional initiatives by automobile manufacturers.
Srinivasan & Bass (2000)	Stable market shares are consistent with evolving sales if brand and category sales are cointegrated
Srinivasan, Popkowski Leszczyc and Bass (2000)	Temporary, gradual and structural price changes have a different impact on market shares.
Srinivasan, Pauwels, Hanssens and Dekimpe (2003)	Price promotions have a differential performance impact for retailers versus manufacturers.
Srinivasan, Pauwels and Nijs (2003)	Retail prices exhibit a high, but varying, degree of inertia.
Steenkamp, Nijs, Hanssens and Dekimpe (2003)	Competitive reactions to promotion and advertising attacks are often passive. This rarely involves a missed sales opportunity. If reaction occurs, if often involves spoiled arms.
Villanueva, Yoo and Hanssens (2003)	Customers acquired through different channels have different lifetime values.
Zanias (1994)	Feedback effects occur between sales and advertising. The importance of cointegration analysis is demonstrated with respect to Granger causality testing and multi-step forecasting.

APPENDIX B

IMPULSE-RESPONSE FUNCTIONS: MATHEMATICAL DERIVATION

$$\begin{bmatrix} S_t \\ M_t \\ CM_t \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \begin{bmatrix} S_{t\text{-}1} \\ M_{t\text{-}1} \\ CM_{t\text{-}1} \end{bmatrix} + \begin{bmatrix} u_{S,t} \\ u_{M,t} \\ u_{CM,t} \end{bmatrix}, \quad \text{(B1)}$$

one sets $[u_S, u_M, u_{CM}]$ = $[0,0,0]$ prior to t
$[0,1,0]$ at time t
$[0,0,0]$ after t

and computes (simulates) the future values for the various endogenous variables, i.e.

$$\begin{bmatrix} S_t \\ M_t \\ CM_t \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} S_{t+1} \\ M_{t+1} \\ CM_{t+1} \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \pi_{12} \\ \pi_{22} \\ \pi_{32} \end{bmatrix},$$

$$\begin{bmatrix} S_{t+2} \\ M_{t+2} \\ CM_{t+2} \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{bmatrix} \begin{bmatrix} \pi_{12} \\ \pi_{22} \\ \pi_{32} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \pi_{11}\pi_{12} + \pi_{12}\pi_{22} + \pi_{13}\pi_{32} \\ \pi_{21}\pi_{12} + \pi_{22}\pi_{22} + \pi_{23}\pi_{32} \\ \pi_{31}\pi_{12} + \pi_{32}\pi_{22} + \pi_{33}\pi_{32} \end{bmatrix},$$

Etc…