Brand Performance Volatility Arising from Marketing Spending Behavior under Competition

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March 9, 2011

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ABSTRACT

While volatile marketing spending, as opposed to even-level spending, may improve a brand's financial performance, it can also increase the volatility of performance, which is not a desirable outcome. This paper analyzes how revenue and cash-flow volatility are influenced by own and competitive marketing spending volatility, by the level of marketing spending, by the responsiveness of own marketing spending, and by competitive reactivity. From market response theory, we derive propositions about the influence of these variables on revenue and cash-flow volatility. Based on a broad sample of 99 pharmaceutical brands in four clinical categories and four European countries, the authors test for the empirical relevance of the propositions and assess the magnitude of the different sources of marketing-induced performance volatility.

The authors find broad support for the predicted volatility effects. The results suggest that volatile marketing spending may incur negative financial side effects such as greater financing costs or higher opportunity costs of cash holdings. Thus common volatility-increasing marketing practices such as advertising pulsing may be effective at the top-line, but could turn out to be ineffective after all costs are taken into account.

Keywords: Revenue/Cash-Flow Volatility, Marketing Volatility, Econometric Models, Marketing Metrics

Introduction

In order to increase impact, marketing executives often deploy their resources in spending bursts, i.e. regimes characterized by on-again, off-again marketing actions, including advertising campaigns, sales promotions and new-product launches. Inasfar as such *pulsing* causes revenues and cash flows to become more volatile, it may have an unintended negative consequence on their firms' financial performance. Indeed, senior executives dislike cash-flow volatility since it increases investors' perceived uncertainty about future cash flows, incurs additional financing costs for the firm and, accordingly, hurts firm value (e.g., McInnis 2010). Top executives appear to be actively involved in earnings management in order to diminish cash-flow volatility, and are even willing to sacrifice real economic gains (Chapman and Steenburgh 2011).

Such revenue or cash-flow volatility has traditionally not been of major concern to marketers. However, as marketing moves away from a sole focus on customer-demand impact to a focus on firm-value impact (e.g., Rust et al. 2004; Srinivasan and Hanssens 2009), it adopts an investor perspective, which includes a strong concern about the stability of revenues and cash flows. Marketing spending may indeed be a key source of revenue and cash-flow volatility, which is controllable by the company. For example, Rao and Bharadwaj (2008) show analytically that marketing actions can affect the probability distribution of sales as well as the cash requirements, which in turn influences the distribution of cash flows.

Marketing actions that are routinely carried out by brand managers potentially generate performance volatility. Specifically, how brand managers allocate marketing spending over time may affect cash-flow volatility because it impacts the volatility of brand revenues as well as that of costs. Following the pulsing argument, marketing managers may increase marketing spending volatility to improve brand sales, but that may also increase revenue and cash-flow volatility. As long as they are not aware of the potential negative financial effects of such policy, they have no incentive to reduce spending volatility and the resulting cash-flow volatility. Thus, there may be a potential conflict between sales-impact maximization (a typical marketing objective) and stable cash-flow generation (a typical finance objective). This study examines this potential conflict both theoretically and empirically.

From financial portfolio theory, we know that volatility can be reduced through diversification. For example, a beverage manufacturer could ensure that a major campaign for its ice-tea brand does not coincide with another campaign for its bottled-water brand. This suggests that marketing may employ volatile expenditure practices at the brand level and diversify away any induced volatility at the portfolio level by simply strategically coordinating marketing activities across brands. As a result, we would expect expenditures to be predominantly negatively correlated. The analysis of inter-brand correlation patterns in our empirical data set , however, shows that it is predominantly zero. Hence, there seems to be no coordination across brands or divisions, at least in these data. In addition, we will demonstrate that even if a firm followed a perfect expenditure hedging strategy that maximizes brand pulsing effects while keeping total expenditures constant does not eliminate revenue and cash-flow variance. In fact, the variance at the portfolio level increases with brand-specific expenditure volatility. It is therefore important to study the *drivers* and *consequences* of this *volatility* at the *brand level*.

Our second contribution is to develop general insights into marketing-induced volatility effects at the brand level, based on market response theory. We consider the *level* and the *volatility* of marketing spending – as well as the customer *responsiveness* to marketing spending as important drivers of revenue and cash-flow volatility. In addition, we derive results on the impact of competitor behavior on volatility. Generally, this analysis suggests that the direction and the strength of volatility effects depend on the type and strength of competition, leading to some surprising implications.

Since volatility effects cannot be universally predicted under competitive conditions, we use a database of 99 pharmaceutical brands from four European countries to test for the direction and the magnitude of the impact of the various volatility drivers. This provides our third contribution, which is of an empirical nature.

The remainder of the article is organized as follows. Following a brief literature review, we develop propositions about the effects of marketing spending on revenue and cash-flow volatility at the portfolio and the brand level. Next we describe our research methodology to measure the effects in an empirical study. We present empirical results and discuss the theoretical and managerial implications of our findings. The article concludes with a synthesis of the findings, limitations and suggestions for future research.

Prior Research on Marketing-Related Volatility Effects

Many marketers believe in the effectiveness of a pulsing strategy, which implies an uneven distribution of expenditures over time. A survey among media-planners in the U.S. reports that almost 70% use pulsing to increase the effectiveness of their spending (Leckenby and Kim 1994). Indeed, theoretical and empirical studies suggest that pulsing may lead to higher revenues and cash flows under certain conditions, e.g., the existence of a differential stimulus effect (Simon 1982). Hence, both marketing theory and practice suggest that marketing managers use pulsing tactics in order to increase revenues and cash flows; but these studies did not consider potential financial side effects that arise from the increased volatility. As Rao and Bharadwaj

(2008) show in an analytical model, shareholders' wealth can be negatively affected by volatile marketing expenditures.

Only a few empirical studies have addressed the relationship between marketing and revenue/cash-flow volatility to date. Raju (1992) examines the drivers of category sales variability and finds that the magnitude of discounts is positively associated with sales volatility whereas frequency is negatively correlated. By adopting an EGARCH approach, Vakratsas (2008) shows that marketing-mix variables including price, advertising, and distribution affect market-share volatility via the error term. Gruca and Rego (2005) analyze the impact of satisfaction on cash-flow volatility at the firm level, and conclude that satisfied customers are an important asset because they lower the volatility of cash flows. This argument has also been supported by Anderson, Fornell, and Mazvancheryl (2004) and Fornell et al. (2006).

While these prior studies provide valuable insights, they do not address the volatility impact on *brand* revenues and cash flows originating from marketing spending *level*, spending *variation* and sales *responsiveness*. Such a focus is highly actionable for marketing managers, and still offers a close link to the company's financial performance.

Marketing-Induced Volatility Effects

In this section, we analyze various effects marketing-induced volatility has on sales and cash flows. We first address the question whether marketing-induced volatility at the brand level can be diversified away at the portfolio level. We then turn our attention to the drivers and sources of revenue and cash-flow volatility at the individual brand level.

Market Response Theory

Our conceptual development is rooted in market response theory. We start from the premise that aggregated brand sales follow a concave relationship with marketing expenditures. A concave response function is theoretically attractive because it implies diminishing returns which are a prerequisite for marketing budget optimization. It is by far the most frequent type of response function encountered in empirical research (Hanssens, Parsons, and Schultz 2001). Since a concave log-log response model also turns out to best represent our data, our theory development is fully consistent with the subsequent empirical analysis. Finally, the results may be generalized to other types of response, such as an S-shaped or a differential stimulus response. Assuming rational, profit-maximizing behavior, budgets only vary within the concave zone of these functions, which is the only assumption we make.

By varying conditions such as responsiveness to marketing, we derive propositions on our focal volatility variables. Specifically, we consider two measures of volatility: the *variance* and the *range* (i.e. the difference between maximum and minimum values) of marketing expenditures, revenues, and cash flows. Variance is a common measure of variability; we will focus on this variable to derive our propositions. Range is another useful metric of volatility (Alizadeh, Brandt, and Diebold 2002), especially in our brand-level context.

Portfolio-level Volatility and Diversification

Firms usually manage a portfolio of products, not just a single product. Following financial portfolio theory (Markowitz 1952), management could try to diversify its marketing-induced volatility away by strategically coordinating marketing activities across brands and/or regional markets. For example, a beverage manufacturer could ensure that a major campaign for its ice-tea brand does not coincide with another campaign for its bottled-water brand in the same market. If both brands alternate their marketing spending, the two-brand expenditures are perfectly negatively correlated and total spending remains constant, i.e. portfolio expenditure variance is zero. Consistent with portfolio theory, this would be a perfect hedge of marketing expenditures. Without doubt, such a perfect hedging strategy significantly reduces cash-flow volatility of the portfolio as it eliminates any volatility in marketing-related cash outflow.

Managing the volatility of expenditures, revenues, and cash flows of a product portfolio, however, is quite different from managing the volatility of a financial portfolio of risky securities. In *financial portfolio* selection, the objective is to maximize portfolio return, which is the weighted average of individual security returns, while reducing portfolio variance to a minimum. The investment proportion is the decision variable, and it is relatively easy to change the proportion of securities in the portfolio when necessary. Variances and correlations among securities are exogenous.

In *product portfolio* management, the objective is to maximize the sum of product profits or cash flows, respectively, while the portfolio's cash-flow variance should be as low as possible. Quite in contrast, variances and correlations among brand expenditures are no longer exogenous but are the result of the (coordinated or not) management of the marketing budget. Reducing or increasing the share of a brand's cash flow in portfolio cash flows to a desired level is not possible, at least in the short run. Indeed, it might not be desirable to change the share of a brand’s cash flows just to reduce cash flow volatility. Note that a firm’s product portfolio is an outcome of important strategic decisions, coupled with major marketing/R&D/manufacturing efforts. Instead, firms can change the way of spending on marketing activities to reduce cash flow volatility. Hence, objective and decision variables are quite different leading to different outcomes of diversification in "real" product portfolios compared to financial portfolios.

Specifically, investors can hedge the variance in stock returns through reallocation of funds, while marketing managers can hedge the variance in cash flows through coordination of marketing spending. One example is a perfect expenditure hedging strategy that allows for maximal pulsing with zero variance in total expenditures through spending alternation. However, we demonstrate that even such a perfect hedging strategy in expenditures does not result in a zero variance in total revenues and cash flows.

Perfect expenditure hedging strategy. To show this, let us first formalize the perfect expenditure hedging strategy. Let MKT_i denote expenditures on product *i* in a portfolio of size *N*. Total portfolio expenditures MKT_P is the sum of all product expenditures. A perfect hedging strategy would be to pulse in an alternating way across all products. The budget in the first period is spent on the first product, then in the second period on the second product, and so forth. Essentially, this strategy realizes maximal pulsing effects, since product spending bumps between zero and the total spending level, and a minimum of total expenditure volatility. While portfolio expenditure variance is zero, variances of product expenditures are equal. We can derive the following lemma about the correlation of expenditures among products.

LEMMA 1. *A perfect expenditure hedging strategy of successive alternating spending bursts across products of a portfolio implies that expenditures are correlated with 1/(1-N).*

PROOF. Note that the variance of portfolio expenditures is given by

$$\begin{aligned} Var\left(MKT_P\right) &= Var\left(\sum_{i=1}^{N} MKT_i\right) = \sum_{i=1}^{N} Var\left(MKT_i\right) + \sum_{i=1,i\neq j}^{N} \sum_{j=1}^{N} Cov\left(MKT_i, MKT_j\right) \\ &= \sum_{i=1}^{N} Var\left(MKT_i\right) + \sum_{i=1,i\neq j}^{N} \sum_{j=1}^{N} \rho_{ij} \left[Var\left(MKT_i\right) Var\left(MKT_j\right)\right]^{\frac{1}{2}}, \end{aligned} \tag{1}$$

where ρ_{ij} measures the correlation coefficient. There are *N* variance terms and *N*(*N*-1) covariance terms. Because $Var(MKT_i)=Var(MKT_j)$, $\forall i,j$, we can write for Equation (1)

$$Var\left(\sum_{i=1}^{N} MKT_i\right) = N \cdot Var\left(MKT_i\right) + N\left(N-1\right)\rho_{ij} Var\left(MKT_i\right). \tag{2}$$

Since portfolio variance is zero, i.e. $Var\left(\sum_{i=1}^{N} MKT_i\right) = 0$, we can solve for $\rho_{ij} = 1/(1-N)$. □

For the case of two products, as an example, correlation is –1, consistent with our hedging example above. It decreases in absolute values for larger product portfolios.

What are the implications of this hedging strategy for the volatility of portfolio cash flows? The firm avoids any volatility on the expenditure side through a coordinated expenditure plan. But, it turns out that zero variance in expenditures does not automatically transfer into zero variance in portfolio cash flows. The cash-flow variance of the portfolio rather increases with the variance of individual product expenditures. The following theoretical analysis reveals this fundamental result.

Definitions. Let P_i denote price of product *i*, C_i marginal cost, and $Q_i(MKT_i, \mathbf{MKT}_j)$ unit sales that depends on own marketing expenditures MKT_i and expenditures by other products *j* of the portfolio summarized in the row vector $\mathbf{MKT}_j$, with $j = 1, 2, \ldots, N$ and $j \neq i$. Revenues, RV_i, and cash flows, CF_i, are given by the following expressions:

$$RV_i\left(MKT_i\right) = P_i \cdot Q_i\left(MKT_i, \mathbf{MKT}_j\right) \tag{3}$$

$$CF_i\left(MKT_i, \mathbf{MKT}_j\right) = \left(P_i - C_i\right) Q_i\left(MKT_i, \mathbf{MKT}_j\right) - MKT_i . \tag{4}$$

$Q_i(MKT_i, \mathbf{MKT}_j)$ is a nonlinear, twice differentiable function with $Q'_{ii} = \partial Q_i / \partial MKT_i$ and $\mathbf{Q}'_{ij} = \partial \mathbf{Q}_i / \partial \mathbf{MKT}_j$. Q'_{ii} measures the own demand effect with respect to *i*'s expenditures which is positive. Q'_{ij} captures the cross-effect of product *j*'s expenditures on demand for *i*. The cross-effect may be negative (product substitute) or positive (product complement). Let MKT_i be a random variable with mean (average spending level), μ_i, and variance, $Var(MKT_i)$. Since $Q_i(MKT_i, \mathbf{MKT}_j)$ is nonlinear we need to approximate its variance (Greene 2006). For this purpose, we use the linear Taylor series approximation and choose the vector **μ**, without loss of generality, as expansion point:

$$Q_i\left(MKT_i, \mathbf{MKT}_j\right) \cong Q_i\left(\mu_i, \boldsymbol{\mu}_j\right) + Q'_{ii}\left(\mu_i\right) MKT_i - Q'_{ii}\left(\mu_i\right)\mu_i + \mathbf{Q}'_{ij}\left(\boldsymbol{\mu}_j\right)\mathbf{MKT}_j - \mathbf{Q}'_{ij}\left(\boldsymbol{\mu}_j\right)\boldsymbol{\mu}_j . \quad (5)$$

Inserting (5) into (3) and (4) and summing across all N products produces expressions for portfolio revenues and cash flows:

$$\begin{aligned} RV_P\left(\mathbf{MKT}\right) \cong & \sum_i P_i Q_i\left(\boldsymbol{\mu}\right) + \left[P_i Q'_{ii}\left(\mu_i\right) + \sum_{j \neq i} P_j Q'_{ji}\left(\mu_i\right)\right] MKT_i \\ & - \sum_i \left[P_i Q'_{ii}\left(\mu_i\right) + \sum_{j \neq i} P_j Q'_{ji}\left(\mu_i\right)\right]\mu_i \end{aligned} \quad (6)$$

$$\begin{aligned} CF_P\left(\mathbf{MKT}\right) \cong & \sum_i \left(P_i - C_i\right) Q_i\left(\boldsymbol{\mu}\right) + \left[\left(P_i - C_i\right) Q'_{ii}\left(\mu_i\right) + \sum_{j \neq i}\left(P_j - C_j\right) Q'_{ji}\left(\mu_i\right)\right] MKT_i \\ & - \sum_i \left[\left(P_i - C_i\right) Q'_{ii}\left(\mu_i\right) + \sum_{j \neq i}\left(P_j - C_j\right) Q'_{ji}\left(\mu_i\right)\right]\mu_i - \sum_i MKT_i \end{aligned} . \quad (7)$$

Let $CF'_i = \left(P_i - C_i\right) Q'_{ii}\left(\mu_i\right) + \sum_{j \neq i}\left(P_j - C_j\right) Q'_{ji}\left(\mu_i\right)$ denote the marginal net cash-flow effect with respect to product i's marketing expenditures. It measures the profit-margin weighted sum of own and cross-demand effects which is always positive if products are complements. For substitutes it may also be negative depending on the margin structure of the portfolio; but it is clearly not in the interest of the firm to have cannibalization reduce total cash flows.

From Equation (7), we can derive the variance of portfolio cash flows as follows:

$$\begin{aligned} Var\left[CF_P\left(\mathbf{MKT}\right)\right] \cong & \sum_i \left(CF'_i\right)^2 Var\left(MKT_i\right) + \sum_{i \neq j}\sum_j \rho_{ij} CF'_i CF'_j \left[Var\left(MKT_i\right) Var\left(MKT_j\right)\right]^{\frac{1}{2}} \\ & + Var\left(MKT_P\right). \end{aligned} \quad (8)$$

Discussion of portfolio volatility effects. Expression (8) enables us to analyze the effects of a perfect expenditure hedging strategy as described above on the volatility of total cash flows. Recall that our perfect expenditure hedging strategy implies $Var(MKT_P) = 0$ and $Var(MKT_i)=Var(MKT_j)$. Therefore

$$Var\left[CF_P\left(\mathbf{MKT}\right)\right] \cong \left[\sum_i \left(CF_i'\right)^2 + \sum_{i \neq j} \sum_j \rho_{ij} CF_i' CF_j'\right] Var\left(MKT_i\right). \tag{9}$$

PROPOSITION 1. *Under a perfect expenditure hedging strategy of successive alternating spending bursts across products, the variance of portfolio cash flows increases with the variance of individual product expenditures, unless* $\sum_i \left(CF_i'\right)^2 + \sum_{i \neq j} \sum_j \rho_{ij} CF_i' CF_j' = 0$.

PROOF. Since $Var(CF_P) \geq 0$ and $Var(MKT_i) > 0$, $\sum_i \left(CF_i'\right)^2 + \sum_{i \neq j} \sum_j \rho_{ij} CF_i' CF_j' \geq 0$ and therefore $\partial Var\left(CF_P\right) / \partial Var\left(MKT_i\right) \geq 0$. □

Proposition 1 implies that our perfect expenditure hedging strategy does not necessarily wipes out cash flow volatility but rather increases it with larger product-specific expenditure variance. The condition under which portfolio cash-flow volatility is zero is hardly met in reality and depends on marginal demand effects. These marginal effects are by large exogenous and thus cannot be easily managed by the firm. The following corollary shows that the chance for zero variance of total cash flows totally vanishes in large product portfolios.

COROLLARY 1. *For large product portfolios, the variance of portfolio cash flows increases with the variance of individual product expenditures according to*

$$\partial Var\left(CF_P\right) / \partial Var\left(MKT_i\right) \cong \sum_i \left(CF_i'\right)^2 .$$

PROOF. From Lemma 1, we know that $\rho_{ij} = -1/\left(N-1\right)$, $\forall i,j$. We can therefore write

$$Var\left[CF_P\left(\mathbf{MKT}\right)\right] \cong \left[\sum_i \left(CF_i'\right)^2 - \frac{\sum_{i \neq j} \sum_j \rho_{ij} CF_i' CF_j'}{N-1}\right] Var\left(MKT_i\right).$$

For $N \to \infty$ the second term in the bracket vanishes leading to the marginal effect in the corollary, which is always positive. □

Our theoretical portfolio analysis provides important insights into the effects of volatile expenditure patterns. While volatility in marketing expenditures can be completely diversified away by adopting a simple coordination plan, cash-flow volatility at the portfolio level cannot be easily diversified away. Instead, it increases with the marketing-induced expenditure volatility at the individual product level. It is therefore important to understand the drivers and sources of revenue and cash-flow volatility at the product or brand level, respectively, which is the focus of the remainder of our paper.

Brand-level Volatility: Impact of Own Marketing

We start the discussion of volatility effects with the impact of own marketing spending behavior on the volatility of brand revenues, followed by its effects on the volatility of brand cash flows. Our general argument is that the volatility, the average level and the sales responsiveness of marketing expenditures together affect the volatility of revenues and cash flows. By sales responsiveness we mean the lift in sales that can be related to the increase in marketing expenditures. It is measured by the slope parameter of the response function.

In the theoretical analysis, we assume that both own marketing and competitive marketing expenditures influence brand sales. The impact of competitive marketing on brand sales is measured by its cross-effect. Because of potential competitive interactions, there is a connection between own marketing expenditures and competitive expenditures that needs to be reflected in the volatility analysis. We capture this interaction by the observed correlation between own and competitive expenditures.

Definitions and Assumptions. Let $Q(MKT, CMKT)$ measure unit brand sales that depends on own marketing expenditures, *MKT*, and the cumulative marketing expenditures by competitors *CMKT*. *Q* is a nonlinear, twice differentiable function with $Q'(MKT) > 0$ and $Q''(MKT) < 0$. Q' measures the marginal own demand effect with respect to *MKT*. Assuming profit maximization together with response functions (e.g., S-shaped) that suggest pulsing as an optimal strategy implies that firms operate in the concave part of the response function. Hence, our assumptions about $Q''(MKT)$ still hold. Q_c' captures the cross-effect of competitive expenditures, *CMKT*, on demand. This effect may be substitutive ($Q_c' < 0$) or market expanding ($Q_c' > 0$). Let $\varepsilon = Q' \cdot MKT/Q$ denote the elasticity of sales with respect to own marketing expenditures and $\varepsilon_c = Q_c' \cdot CMKT/Q$ be the cross-elasticity with respect to competitive expenditures.

Using the linear Taylor series approximation and mean expenditures levels μ and μ_c for own and competitive expenditures, respectively, as expansion points gives:

$$Q(MKT, CMKT) \cong Q(\mu, \mu_c) + Q'(\mu)MKT - Q'(\mu)\mu + Q_c'(\mu_c)CMKT - Q_c'(\mu_c)\mu_c. \quad (10)$$

From Equation (10) together with (3) and (4), we obtain the variance of revenues

$$\begin{aligned} Var\left[RV(MKT, CMKT)\right] \cong\ & P^2\left[Q'(\mu)\right]^2 Var(MKT) + P^2\left[Q_c'(\mu_c)\right]^2 Var(CMKT) \\ & + 2P^2 \rho Q'(\mu) Q_c'(\mu_c)\left[Var(MKT)Var(CMKT)\right]^{\frac{1}{2}}, \end{aligned} \quad (11)$$

and the variance of cash flows

$$\begin{aligned} Var\left[CF(MKT, CMKT)\right] \cong\ & \left[(P-C)Q'(\mu)-1\right]^2 Var(MKT) + (P-C)^2\left[Q_c'(\mu_c)\right]^2 Var(CMKT) \\ & + 2(P-C)^2 \rho Q'(\mu) Q_c'(\mu_c)\left[Var(MKT)Var(CMKT)\right]^{\frac{1}{2}}, \end{aligned} \quad (12)$$

where ρ measures the correlation between own and competitive marketing expenditures.

From Dorfman and Steiner (1954), we know that the profit-maximizing marketing budget must satisfy the first-order condition $MKT^* = \varepsilon^*(P-C)Q^*$, where the star indicates that variables are in their optimum. This relation also holds in a competitive Nash equilibrium (Fischer et al. 2011), where ε^* and Q^* reflect equilibrium values and depend on equilibrium competitive expenditures as defined in (10). We will use μ^*, the optimal equilibrium mean expenditure level, as a useful reference point in the subsequent analysis. Let us also introduce $\tilde{\mu}$, the near-optimal expenditure level that is derived from current parameter values according to

$$\tilde{\mu} = \varepsilon\left(P - C\right)Q\,. \tag{13}$$

Fischer et al. (2011) show that using this relation as a periodic rule to determine the optimal budget under Nash competition quickly converges to the true optimum. In addition, we use the coefficient of variation as a normalized measure of the volatility of own and competitive marketing expenditures. They are defined as $CV = SD(MKT)/\mu$ and $CV_c = SD(CMKT)/\mu_c$, where CV denotes the coefficient of variation and SD is the standard deviation.

Finally, we assume that unit profit contribution and mean expenditure levels for own and competitive marketing are always strictly positive, i.e. $(P\text{-}C)$, μ, $\mu_c > 0$, and therefore $Var(CMKT) > 0$ and $Var(MKT) > 0$. We also assume $Q'(MKT) \neq 0$ and $Q_c'\left(CMKT\right) \neq 0$.

Effects on Revenue Volatility. We derive the following propositions on revenue volatility.

PROPOSITION 2A. *Ceteris paribus, a higher variance of own expenditures increases the variance of revenues if* $\varepsilon CV > -\rho\,\varepsilon_c\,CV_c$.

PROPOSITION 2B. *Ceteris paribus, a higher mean level of own expenditures decreases the variance of revenues if* $\varepsilon CV > -\rho\,\varepsilon_c\,CV_c$.

PROPOSITION 2C. *Ceteris paribus, a higher marketing responsiveness increases the variance of revenues if* $\varepsilon CV > -\rho\,\varepsilon_c\,CV_c$.

PROOFS. See Web Appendix. □

Apparently, the postulated effects of revenue volatility depend on the condition that $\varepsilon CV > -\rho\,\varepsilon_c\,CV_c$. At first glance, this result may appear surprising as revenues always increase with own expenditures, albeit at a decreasing rate. Hence, we would expect that higher own expenditure variance also always translates into higher revenue variance, consistent with the monotonic shape of the response function. In fact, the intuition is not wrong if we consider a situation without competition. Then, $\rho = 0$, and we have the condition $\varepsilon CV > 0$, which is always satisfied since ε and $CV > 0$. Under competition, however, revenues are also affected by competitive actions. A higher expenditure volatility, as an example, does not necessarily increase the volatility of revenues but may in fact decrease it. Whether this situation arises depends on the type and intensity of competitive interaction.

We note that there is always a positive effect on revenue volatility if competitive behavior is accommodating ($\rho < 0$) and cross-effects are substitutive ($\varepsilon_c < 0$), or if competitive behavior is counteractive ($\rho > 0$) and cross-effects are market-expanding ($\varepsilon_c > 0$). The reality in many competitive markets, however, is that cross-effects are substitutive ($\varepsilon_c < 0$) and competitive interaction is counteractive ($\rho > 0$). A (counterintuitive) negative effect on revenue volatility does occur in that situation if $-\rho\,\varepsilon_c\,CV_c > \varepsilon\,CV$. This inequality implies that the demand-effective volatility of competitive expenditures, as represented by $\varepsilon_c\,CV_c$, must be higher than the demand-effective volatility of own expenditures, $\varepsilon\,CV$. It increases with the strength of the cross-effect and the normalized variance of competitive expenditures. In addition, own and competitive expenditures must be positively correlated. It is because of this competitive interaction that a higher variance in own expenditures entails a competitive reaction that may overcompensate the volatility induced by own expenditure volatility.

Effects on Cash-Flow Volatility. The results on revenue volatility cannot be automatically transferred to cash-flow volatility since an increase (decrease) in revenues is also associated with an increase (decrease) in costs.

PROPOSITION 3A. *Ceteris paribus, a higher variance of own expenditures increases the variance of cash flows if* $\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2 > -\rho\frac{\varepsilon_c\,CV_c}{\varepsilon\,CV}$.

PROOF. See Web Appendix. □

Cash-flow volatility always increases if $\rho = 0$, i.e. if there is no competitive interaction. Consistent with the effect on revenue volatility, however, a positive effect on cash-flow volatility is not universally guaranteed under regular competitive conditions ($\rho > 0$ and $\varepsilon_c < 0$).

COROLLARY 2. *Under regular competitive conditions ($\rho > 0$ and $\varepsilon_c < 0$), there is always an expenditure level close enough to the optimal Dorfman-Steiner level when higher variance of own expenditures leads to a lower variance of cash flows.*

PROOF. We need to show that $\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2 < k,$ where $k = -\rho \frac{\varepsilon_c \, CV_c}{\varepsilon \, CV}$. From $\varepsilon_c < 0$ and ρ, ε, CV, $CV_c > 0$, it follows $k > 0$. Assume expenditures are set at the optimal level, i.e. $\mu = \mu^*$. Since $\mu^* = \varepsilon^* \left(P - C\right) Q^*$, it follows with (13) that $\tilde{\mu} = \mu^* = \varepsilon^* \left(P - C\right) Q^*$. Let $\kappa \in \mathbb{R}$ be an arbitrary constant that measures how close actual expenditures are to the optimal level: $\mu = \left|\mu^* - \kappa\right|$. For $\kappa \to 0$, $\mu \to \mu^*$ and $\tilde{\mu} \to \mu^*$. As a consequence, $\left|\tilde{\mu} - \mu\right| \to 0$ and therefore $\left.\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2\right|_{\kappa \to 0} = 0$.

Hence, there is a κ small enough to satisfy $\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2 < k$. □

How can we explain that range around the optimal level where higher expenditure variance reduces cash-flow variance? From the flat maximum principle (e.g., Tull et al. 1986), we know that the cash-flow curve is flat around the optimum. A large variation of marketing expenditures is associated with only a small variation in cash flows. While cash-flow variance always increases if competitors do not react, counteractive behavior and substitutive effects can overcompensate changes in cash flows if they are small, as is the case around the maximum. As a result, cash-flow variance decreases.

PROPOSITION 3B. *Ceteris paribus, the variance of cash flows follows a U-shape with higher mean levels of marketing expenditures if* $\varepsilon CV > -\rho \varepsilon_c CV_c$.

PROOF. See Web Appendix. □

This proposition implies that the first derivative of Equation (12) has a root, which defines the minimum of cash-flow variance. In contrast to the variance of revenues, the relationship between the variance of cash flows and the mean expenditure level is no longer monotonic. The following corollary characterizes this relationship more precisely.

COROLLARY 3. *Under regular competitive conditions (*$\rho > 0$ *and* $\varepsilon_c < 0$*), the variance of cash flows starts to increase at a level lower than the optimal Dorfman-Steiner level.*

PROOF. See Web Appendix. □

Interestingly, under regular competitive conditions, the optimal mean level of marketing expenditures is associated with lower variance in revenues but higher variance of cash flows compared to a lower expenditure level. Considering the financial costs of higher cash-flow volatility, the optimal mean expenditure level may therefore be below the Dorfman-Steiner level.

PROPOSITION 3C. *Ceteris paribus, a higher marketing responsiveness increases the variance of cash flows if* $\mu < \tilde{\mu}\left(\varepsilon CV + \rho \varepsilon_c CV_c\right) / \varepsilon CV$ *and* $\varepsilon CV > -\rho \varepsilon_c CV_c$. *For* $\mu > \tilde{\mu}\left(\varepsilon CV + \rho \varepsilon_c CV_c\right) / \varepsilon CV$ *, the variance of cash flows decreases with a higher marketing responsiveness.*

PROOF. See Web Appendix. □

Proposition 3C states that the effect of an increased marketing responsiveness on cash-flow volatility depends on the level of marketing expenditures. In fact, this interaction effect with the level of marketing expenditures is non-monotonic. While cash-flow volatility generally increases with higher marketing responsiveness, this relation turns into the opposite at a point close to the optimal expenditure level. One explanation for this effect is due to the fact that every additional dollar spent beyond the optimal level incurs a loss. The loss, however, is less the greater the responsiveness of demand. Or technically, the cash-flow function is less steep. Therefore, (negative) cash-flows vary to a lesser extent with expenditures beyond the profit-maximizing level if sales responsiveness is larger.

Brand-level Volatility: Impact of Competitive Marketing and Interaction

We now turn our focus to two effects that arise from competitive interaction. Specifically, we consider the impact of the volatility of competitive expenditures and the correlation between own and competitive expenditures on revenue and cash-flow volatility.

Competitive-expenditure volatility. The effects of competitive-expenditure variance are the same on revenue and cash-flow variance. The conditions for the direction of the effects, however, are different depending on the type of cross-effect. Specifically, we specify the following conditions under which propositions 4A and 4B hold:

$$\text{If } \varepsilon_c < 0 \text{ and } \varepsilon_c CV_c < -\rho\,\varepsilon\,CV, \text{ then} \tag{14a}$$

$$\text{If } \varepsilon_c > 0 \text{ and } \varepsilon_c CV_c > -\rho\,\varepsilon\,CV, \text{ then} \tag{14b}$$

PROPOSITION 4A. *Ceteris paribus, a higher variance of competitive expenditures increases the variance of revenues.*

PROPOSITION 4B. *Ceteris paribus, a higher variance of competitive expenditures increases the variance of cash flows.*

PROOFS. See Web Appendix. □

The effects of competitive-expenditure volatility are symmetric to the effect of own-expenditure volatility on revenue volatility (see proposition 2A again). Whether the variance of revenues and cash flows increases with higher competitive-expenditure variance depends on the strengths of demand-effective volatilities and the type and intensity of competitive interaction. If there is no interaction, i.e. $\rho = 0$, we have the apparent result that volatility in our focal variables always increases. It does not depend on the direction of the cross-effect because variance itself has no directional meaning. The picture changes when we consider a situation with competitive interaction. Under regular competitive conditions ($\rho > 0$ and $\varepsilon_c < 0$), both sides of inequality (14a) are positive. In absolute terms, the demand-effective volatility of competitive expenditures must be greater than the correlation-weighted demand-effective volatility of own expenditures. Since cross-effects are often considerably smaller than own effects, the condition may not be met and the effect reverses. I.e., a larger variance in competitive expenditures decreases the variance of own revenues and cash flows, which is counterintuitive but a direct implication of propositions 4A and 4B.

Competitive interaction. The propositions on the effects of the correlation between own and competitive marketing expenditures on revenue and cash-flow volatility are identical.

PROPOSITION 5A. *Ceteris paribus, a higher correlation between own and competitive marketing expenditures increases the variance of revenues if* $\varepsilon_c > 0$. *The variance of revenues decreases if* $\varepsilon_c < 0$.

PROPOSITION 5B. *Ceteris paribus, a higher correlation between own and competitive marketing expenditures increases the variance of cash flows if* $\varepsilon_c > 0$. *The variance of cash flows decreases if* $\varepsilon_c < 0$.

PROOFS. See Web Appendix. □

Our last propositions state that an increase (decrease) in counteractive (accommodating) competitive behavior ($d\rho > 0$) increases the variance in revenues and cash flows if cross-effects are market-expanding. It decreases volatilities of the focal variables if cross-effects are substitutive. These results follow directly from the properties of the response function. Substitutive competitive expenditures, for example, reduce own brand sales and therefore compensate an increase in sales due to larger own expenditures. If competitive expenditures follow own expenditures more closely, i.e. ρ is higher, the compensation effect is greater and variance in brand sales declines.

Summary and Predicted Volatility Effects in the Empirical Analysis

Our theoretical volatility analysis reveals a number of important, sometimes counterintuitive insights. First, we note that the variance, the level, and the sales responsiveness of own expenditures do impact the volatility of brand revenues and cash flows. Second, the effects are different for revenue and cash-flow volatility. Since marketing expenditures both positively and negatively affect cash flows the relationship with the variance of cash flows is often non-monotonic. Third, while the direction of the volatility effects is usually well-defined for a situation without competitive interaction, it is not at all clear under the conditions of competition, producing sometimes counterintuitive results. Generally, under regular competitive conditions with substitutive cross-effects and counteractive firm behavior, the direction of volatility effects depends on the structure and intensity of competition which can only be decided on empirical grounds.

In our subsequent empirical analysis, we use the results from market response estimation to predict the direction of volatility effects based on our propositions. The analysis of several drug categories reveals that these markets are, on average, consistent with the conception of a competitive market, i.e. cross-effects are substitutive and firm behavior is counteractive.

For the revenue volatility effects of own marketing expenditures, we conclude that the statements of proposition 2A-2C hold. We also predict a volatility-increasing effect of the variance and the sales responsiveness of own expenditures with respect to cash flows. Recall that the direction of the effects reverses for budgets within a close range around the optimal level (proposition 3A) and higher than that level (proposition 3C). For the average brand in our dataset, the critical range around the optimal spending level is ± 10%. Since brands in our dataset, on average, underspend by more than 10%, we predict that both conditions for the positive volatility effects of proposition 3A and 3C are met. We also expect to find the postulated U-shaped relationship between the level of own marketing expenditures and cash-flow volatility (proposition 3B). Because of data limitations, however, we cannot empirically test the inverted U-shaped two-way interaction effect of the expenditure level with the responsiveness on cash-flow volatility (see again proposition 3C).

For competitive-expenditure volatility, we expect to find a counterintuitive negative impact on both revenue and cash-flow variance (proposition 4A and 4B). Since cross-effects are, on average, substitutive and reaction behavior is counteractive, we predict a negative effect of the correlation of own and competitive marketing expenditures on revenue and cash-flow variance (proposition 5A and 5B). We note that in reality, reaction occurs with a certain time lag that may lead to divergent correlation structures. With quarterly data as ours, however, this effect vanishes and we should observe a positive correlation if expenditures are synchronized.

Data

Data on prescription drugs from two therapeutic areas (cardio-vascular and gastro-intestinal) that cover four product categories are available. Two categories, calcium channel blockers and ACE inhibitors, comprise drugs for the treatment of cardio-vascular diseases. Drugs in the two other categories, H2 antagonists and proton pump inhibitors, are used in gastro-intestinal therapies. These four categories are among the largest prescription-drug categories. They differ in their therapeutic principles to treat diseases like hypertension or acid related gastro-intestinal disorders. Data, collected by IMS Health, are available on a quarterly basis for a time period of 10 years (1987-1996) covering the growth and maturity phases of the analyzed categories. They include unit sales (normalized over different application forms of the drug and transformed into daily dosages by a brand-specific dosage factor), revenues, and aggregate marketing expenditures on detailing, journal advertising and other communications media. Detailing has the lion's share in expenditures with more than 90%. Monetary values are in 1996 US$ and have been deflated by country-specific consumption price indexes. The data cover four European countries: France, Germany, Italy, and the UK, and comprise sixteen product markets (4 categories × 4 countries). We analyze data on 99 brands, which were marketed by 26 pharmaceutical firms.

Table 1 shows the descriptive statistics of the variables used in the estimation equations. Revenues average about $9.2 million per quarter, cash flows are about $5.0 million, and average marketing spending amounts to about $1.0 million. There is also considerable variation in the data across brands and time, as indicated by the standard deviations and the volatility measures in Table 1. Volatility is particularly high with respect to marketing spending. Moving variance is about $151.1 million (or $0.4 million in terms of standard deviation) and moving range is about $0.8 million, virtually as high as the mean spending. We report on the operationalization of these variables subsequently. Plots of marketing spending over time (not shown) reveal substantial volatility for many brands in our sample.

=== Insert Table 1 about here ===

Methodology

Our empirical study focuses on the volatility of brand revenues and brand cash flows (propositions 2A to 5A). It proceeds in two steps. The first step estimates a *market response model* that relates brand unit sales to relevant variables, among them own and competitive marketing expenditures. This market response model provides us with brand-specific estimates of marketing responsiveness. In a second step, these responsiveness estimates are used as predictor variables in a model that explains differences in volatility of revenues and cash flows (*volatility models*). In addition, we use the results of the market response model to remove the effects of exogenous factors such as seasonality and trend from the brand sales time series. Such factors are outside the control of management and are therefore not relevant for the study of marketing spending impact on volatility. Brand expenditures are not subject to trend or seasonality as specification tests revealed.

Market Response Model

Specification. Following recent research on pharmaceuticals (e.g., Fischer and Albers 2010), we specify a log-log sales response model for each of the two therapeutic areas (cardio-vascular drugs and gastro-intestinal drugs). Let sales of drug $i \in I_k$, with I_k as country-specific index set, in country $k \in K$, with K=4, and in period $t \in T_i$, with T_i as brand-specific index set, be defined as follows:

$$\begin{aligned} \ln Q_{ikt} &= \alpha_{0ik} + \alpha_{1ik} \ln MKT_{ikt} + \alpha_{2ik} \ln MKT_{ikt-1} + \alpha_{3k} \ln CMKT_{ikt} + \alpha_{4k} \ln CMKT_{ikt-1} \\ &\quad + \alpha_{5k} \ln GDP_{kt} + \alpha_{6k} ET_{ikt} + \sum\nolimits_{l=1}^{K} \sum\nolimits_{h=1}^{H-1} \beta_{lh} SD_{ht} \times CTY_{lk} + u_{ikt}, \\ &\quad \text{with } u_{ikt} = \varphi_1 u_{ikt-1} + \varphi_2 u_{ikt-2} + \eta_{ikt}, \text{ and } \eta_{ikt} \text{ i.i.d. } N\left(0, \sigma_\eta^2\right), \end{aligned} \tag{15}$$

where *GDP* measures the gross domestic product, *ET* denotes the elapsed time since launch of the brand, *SD* is a quarterly seasonal dummy variable, *CTY* is a country dummy variable, and all other terms are defined as earlier. The disturbance term u shows an autoregressive structure of second order, where φ is an autocorrelation coefficient, and η is a white-noise error term with zero mean and variance σ_η^2. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are parameter vectors to be estimated.

We tested several alternative response models such as a linear model and a semi-log model. We also estimated an S-shaped model that allows for saturation and extended our log-log model by a differential stimulus variable that captures any extra demand lift due to pulsing (Simon 1982). Based on the Schwartz Information Criterion and Davidson-McKinnon comparative test (Greene 2006), we find that specification (15) best represents our data.

Our brand sales model includes variables that are relevant to the international markets over the ten-year sample period. Specifically, it incorporates own and competitive marketing expenditures, including lagged effects. To account for substitution effects across categories in a therapeutic area we treat brands from other categories as competitors. The coefficients associated with previous quarter's own and competitive marketing expenditures capture lagged effects. Sales dynamics are also represented by seasonal dummies, a trend variable (elapsed time since launch of a brand) to control for life-cycle effects, a country's gross domestic product (GDP) as a proxy of the overall economic condition of a country, and the autoregressive error structure. Compared to other dynamic specifications, time-series specification tests indicated that this structure best represents the expenditure dynamics. The total (long-term) effect on sales in period t is simply the sum of the two effects, i.e., α_{1ik} and α_{2ik}. We use this measure as marketing responsiveness measure, *RESP*, in the subsequent volatility equation (16). We account for brand heterogeneity in demand, e.g., quality, brand equity, order of entry, by estimating brand-specific fixed effects. *Distribution* and *price are not relevant* variables in our context. In the European countries covered by our data, pharmacies are required to list every approved drug, resulting in 100% distribution for the drugs in our sample. Prices were highly regulated during the observation period and therefore not used as a tactical marketing instrument.

Estimation. We estimate the brand sales model (17) with generalized least squares. Marketing expenditures might be endogenous introducing a correlation with the error term that leads to biased or inconsistent estimates. We are not concerned about a potential *cross-sectional* correlation since we control for that by including brand-specific intercepts. To investigate a potential error correlation over time we apply the Hausman-test to the model in first differences and instrument expenditure variables with their three and four-period lagged values. They are uncorrelated with the error term by construction in a first difference model and therefore serve as valid instruments (Greene 2006). We do not find evidence for endogeneity in this dataset. As a consequence, we do not expect potential simultaneity issues in our volatility models.

Volatility Models

Structural Equations. Let *V*(*REV*) denote the volatility of revenues measured in terms of variance or range, respectively, *V*(*MKT*) represent the volatility of own marketing expenditures, *A*(*MKT*) be the average level of own marketing expenditures, *V*(*CMKT*) denote the volatility of competitive marketing expenditures, *CORR* represent the correlation between own and competitive marketing expenditures, *RESP* denote total marketing responsiveness (including lagged and current effects), **X** denote a vector including the remaining variables of the brand sales model as specified in Equation (15) (i.e., brand-fixed effects to control for order of entry, quality, etc., trend, seasonality, and *GDP* as surrogate for general demand), $\boldsymbol{\gamma}$ be a parameter vector to be estimated, and ν be an error term with variance ξ. Omitting brand, country, and time subscripts for the moment, we specify the revenue volatility model as follows:

$$V\left(REV\right)=\gamma_0 V\left(MKT\right)^{\gamma_1} A\left(MKT\right)^{\gamma_2} V\left(CMKT\right)^{\gamma_3} Exp\left(\gamma_4 CORR+\gamma_5 RESP+\mathbf{X}\boldsymbol{\gamma}+\nu\right), \quad \text{with } \nu \sim N\left(0,\xi\right). \tag{16}$$

We assume the relationship between revenue volatility and its drivers to be multiplicative. Thus the variables interact with each other, consistent with the results from the theoretical discussion. The correlation between own and competitive marketing expenditures and the estimated marketing responsiveness parameter appear as part of an exponential function because they may become negative. The parameters $\gamma_{1\text{-}3}$ can be directly interpreted as elasticities and facilitate the comparison of volatility drivers. We subsequently describe how we transform the dataset to remove the *X*-variables, which are not the focus in this study.

Since cash flows are constructed from revenues and costs, revenue volatility enters the cash-flow volatility equation:

$$V(CF) = \delta_0 V(REV)^{\delta_1} V(MKT)^{\delta_2} A(MKT)^{\delta_3} \operatorname{Exp}\left[\delta_4 A(MKT) + \upsilon\right], \quad \text{with } \upsilon \sim N(0, \psi), \tag{17}$$

where $V(CF)$ denotes the volatility of cash flows, $\boldsymbol{\delta}$ is a parameter vector to be estimated, and υ represents an error term with variance ψ. The effects of competitive-marketing-expenditure volatility, competitive reaction, marketing responsiveness, and *X*-variables on cash-flow volatility are mediated through revenue volatility. In addition, revenue volatility mediates the impact of own expenditures. Since own expenditures also enter the cash-flow equation as cost we expect an additional direct effect on cash-flow volatility. Finally, note that specification (17) allows for a U-shaped influence of the level of marketing expenditures on cash-flow volatility, consistent with our proposition 3B. This situation occurs if $\delta_3<0$ and $\delta_4>0$. We further allow the error terms to be correlated across the two equations (16) and (17).

Data Transformation. By using the estimates of the brand sales model, we remove the effects of exogenous market factors such as seasonality, trend, and overall economic condition (measured by the GDP), and derive an adjusted unit-sales time-series for each brand. We multiply the unit sales with the brand's unit price and arrive at adjusted brand revenues. We then multiply the adjusted revenues by a cash contribution margin of 85% that is typical for original prescription drugs. From these gross cash flows we subtract the marketing expenditures and arrive at the final variable of adjusted brand cash flows.

The volatility of the adjusted revenues and cash flows is measured by the variance or range of these quantities over a time period of 8 quarters. Consequently, we use the first two available years of sales for each brand as an initialization period. We compute the volatility measure of the subsequent period by dropping the first period and including the information of the following period. We continue until the end of the brand-specific time series and thus obtain a time series of moving volatility measures of adjusted revenues and cash flows (moving-window analysis). This procedure is also applied to compute moving volatilities for own and competitive marketing expenditures and the moving average of own marketing expenditures. We denote moving volatilities with *MV* and moving averages with *MA*.

The application of moving-window analysis is well established in the accounting literature (e.g., Kothari 2001) and is justified for two reasons. First, it increases sample size and therefore improves the power of statistical tests. Note that observations are inevitably lost due to the calculation of the volatility measures. Second, it accounts for possible dynamic effects. Capital markets research has shown that it often takes some time until economic effects have fully materialized in earnings volatility.

Estimation Equations. The use of moving windows is helpful to increase the power of statistical tests due to the increase in degrees of freedom, but it is also likely to generate serially correlated errors in the time series of adjusted revenues and cash flows. We therefore transform expressions (16) and (17) into a series of relative differences. By taking the total differentials of the log-transformed equations (16) and (17), we obtain (see the Web Appendix for details):

$$\frac{\Delta MV(AREV)_{ikt}}{MV(AREV)_{ikt-1}} = \gamma_1 \frac{\Delta MV(MKT)_{ikt}}{MV(MKT)_{ikt-1}} + \gamma_2 \frac{\Delta MA(MKT)_{ikt}}{MA(MKT)_{ikt-1}} + \gamma_3 \frac{\Delta MV(CMKT)_{ikt}}{MV(CMKT)_{ikt-1}} + \gamma_4 \Delta MA(CORR)_{ikt} + \Delta v_{ikt}, \quad (18)$$

$$\frac{\Delta MV\left(ACF\right)_{ikt}}{MV\left(ACF\right)_{ikt-1}} = \delta_1 \frac{\Delta MV\left(AREV\right)_{ikt}}{MV\left(AREV\right)_{ikt-1}} + \delta_2 \frac{\Delta MV\left(MKT\right)_{ikt}}{MV\left(MKT\right)_{ikt-1}} + \delta_3 \frac{\Delta MA\left(MKT\right)_{ikt}}{MA\left(MKT\right)_{ikt-1}} + \delta_4 \Delta MA\left(MKT\right)_{ikt} + \Delta \upsilon_{ikt}, \quad (19)$$

where,

$MV(AREV)_{ikt}$	=	Moving volatility of adjusted revenues of brand *i* in country *k* and period *t*
$MV(MKT)_{ikt}$	=	Moving volatility of marketing expenditures of brand *i* in country *k* and period *t*
$MA(MKT)_{ikt}$	=	Moving average of marketing expenditures of brand *i* in country *k* and period *t*
$MV(CMKT)_{ikt}$	=	Moving volatility of marketing expenditures of brand *i*'s competitors in country *k* and period *t*
$MA(CORR)_{ikt}$	=	Moving average correlation between own and competitive marketing expenditures of brand *i* in country *k* and period *t*
$MV(ACF)_{ikt}$	=	Moving volatility of adjusted cash flows of brand *i* in country *k* and period *t*
Δ	=	First-difference operator.

Equations (18) and (19) represent the original equations (16) and (17) in terms of relative differences. Unlike absolute differences, this representation not only reduces serial correlation, but also controls for brand-size effects. For example, bigger brands are expected to have larger absolute changes in revenues, cash flows and marketing spending.

Equations (18) and (19) establish an equation system with possibly correlated errors across equations. Revenue volatility is the only endogenous variable occurring at the right hand side of Equation (19). Thus, the system is recursive and Generalized Least Squares (GLS), which allows for cross-equation error correlation, provides efficient estimates (Zellner 1962). Since first-differencing may not completely remove serial correlation we also allow for equation-specific autocorrelation coefficients in the variance-covariance matrix.

The first-differencing procedure eliminates the time-invariant marketing responsiveness variable that is part of the revenue volatility model (16). To measure its influence, we linearize (16) first via log-transformation and then build a cross-sectional regression model by obtaining averages of all time-varying variables. The resulting equation can be estimated with Ordinary

Least Squares (OLS). However, the marketing-responsiveness parameters of the first stage are measured with sampling error that vanishes in the limit. As a consequence, OLS estimates from the second stage regression will be consistent but their standard errors may be biased (Murphy and Topel 1985). Following Nijs, Srinivasan, and Pauwels (2007), we obtain corrected standard errors by a bootstrapping procedure with 10,000 replications. First-differencing also eliminates brand-specific factors such as quality that may explain different volatility levels among brands. Note that, together with the procedure to adjust revenues, we have therefore completely removed the impact of the *X*-variables of Equation (16) in our final estimation equations.

Results

Brand Sales Model

The log-log brand sales model describes sales evolution in the markets very well. The average total marketing elasticity, weighted by relative standard errors to account for estimation uncertainty, is .192, which is well in line with recently reported results (e.g., Fischer and Albers 2010). The impact of competitive marketing activities is negative, with a mean value of -.01. In general, there is substantial variation in the marketing responsiveness estimates, which we use as a predictor in our volatility models. Recall that we use the total effect which is the sum of current and lagged marketing responsiveness.

Volatility Models

Table 2 shows the estimation results for the revenue and cash-flow volatility models by using either (adjusted) variance or range as dependent variable. Our focal predictor variables explain a substantial part of variance in observed (i.e. unadjusted) revenue and cash-flow volatility in estimation and holdout samples underlining the relevance of marketing activities for performance volatility. To form holdout samples we excluded the last four quarters (20% of total cases) in the first-difference models and the last 20 brands (20% of cases) in the cross-sectional model.

=== Insert Table 2 about here ===

In the following discussion we focus on variance as volatility measure and on the results from first-difference models. Since the effect of marketing responsiveness, which does not vary within but across brands, can only be estimated by a cross-sectional model we also report on the results of the cross-sectional regression model. In this model, we also include time-invariant control variables, order of entry, quality, average price, and average time in market. These controls, however, do not add explanatory power to the model ($F_{4, 89}$=.158, p>.10). We note that due to the missing time variation and the substantially lower number of observations in this model, the effects for the time-varying variables should be interpreted with caution.

We discuss first estimates from the revenue volatility model and then turn to the cash-flow volatility model. The volatility of marketing expenditures measured by their variance increases the volatility of revenues and supports our first prediction, with an estimated elasticity of .273 (p<.05).

The first-difference model also supports our second prediction on the influence of the level of marketing expenditures on revenue volatility; but the coefficient is not significant at p<.05. We obtain a significant negative effect from the cross-sectional regression (-1.99, p<.05). Note that this variable has been divided by average brand unit sales in order to control for brand-size effects. The effect comes out stronger in a pure cross-sectional regression.

Marketing responsiveness drives revenue volatility (8.11, p<.05), supporting our third prediction. The associated elasticity of .811 (=8.11×.10) is substantial. As predicted, we find evidence for a negative effect of the volatility of competitive marketing expenditures on revenue volatility. The effect, however, is only marginally significant in the cross-sectional regression (-.222, $p<.10$). The correlation of own and competitive marketing expenditures shows a significant negative effect on revenue volatility (-.262, $p<.05$).

As expected, revenue volatility is an important driver of cash-flow volatility, with an elasticity of 1.36 ($p<.05$). Its lower boundary value is the squared profit margin, which would be achieved if cash flows consisted only of revenues multiplied by the profit margin. The direct effect of the volatility of marketing expenditures is positive and significant, with a value of .535 ($p<.05$). This coefficient represents the volatility effect due to the cost component of marketing expenditures. In order to fully evaluate the predicted effect of expenditure volatility on cash-flow volatility, we need to consider the total effect.

Table 3 displays the total effects in terms of elasticity, which facilitates the interpretation and comparison of the magnitude of effects. The total effect of expenditure volatility on cash-flow volatility amounts to .906 ($=1.36\times.273+.535$; $p<.05$). Hence, we find strong support for our prediction. Interestingly, this elasticity is more than three times higher than for revenue volatility. We also find strong support for the expected U-shaped influence of the level of marketing expenditures on cash flows (-2.38, $p<.05$ and .003, $p<.05$; see table 3). The direction of the influence of marketing responsiveness on cash-flow volatility is also consistent with our prediction. Its elasticity is high with a value of 1.10 ($p<.05$). The volatility effect of the volatility of competitive marketing expenditures is not significant, which may be due to the fact that the estimated cross-effects are rather small and not uniform in sign across all categories. We find, however, support for the expected cash-flow volatility effect of the correlation of own and competitive marketing expenditures though the associated elasticity is modest (-.123; $p<.05$).

=== Insert Table 3 about here ===

Both tables 2 and 3 show also the results for models when we take range instead of variance as volatility measure. Overall, the results are consistent with the results using variance as volatility measure.

Robustness of Findings

We performed several analyses to verify the robustness of these results. First, we varied the window of the volatility measures. Instead of 8 quarters we computed volatility measures based on 4 quarters and 12 quarters. The results were similar but model fit deteriorated, underlining that the 8-quarter window is the best choice for our dataset. Second, we created volatility variables that do not overlap over time periods. For example, the first observation of an 8-quarter-based variance variable includes the first 8 quarters, the second observation is based on the subsequent 8 quarters, and so forth. This procedure reduces the sample size to only 292 observations. The results did not change materially, though the standard errors increased. Third, we verified whether the results are influenced by collinearity. The condition indices of the models were well below the critical value of 30 (Greene 2006).

Discussion

Managerial Implications

Our study provides insights that invite marketing decision makers to think differently about the consequences of their actions. First, our results suggest that higher marketing spending volatility leads to a higher volatility of revenues as well as cash flows. Thus managers who decide on the timing of media plans, promotion plans, product launches etc. can influence the volatility of both the top-line and bottom-line performance of their brands. Since marketing costs grow linearly while revenues grow at a decreasing rate, their impact on cash-flow volatility is larger than on revenue volatility. Second, stronger market response parameters also translate into higher volatility of revenues and cash flows. Thus, on the one hand, larger response parameters are good news for the marketing manager because his/her expenditures produce higher returns. On the other hand, higher responsiveness has a dark side since it makes revenues and cash flows more volatile, even if spending volatility itself does not change. Third, we find that a higher mean level of marketing expenditures *reduces* revenue volatility, holding spending volatility constant. Higher spending also decreases the cash-flow volatility for typical non-monotonic cash-flow distributions up to a certain level. That level is *lower* than the optimal spending level.

Some marketing tactics, such as advertising campaigns, are used frequently and involve a volatile deployment of the marketing budget. Sometimes these tactics improve a brand's top-line results, sometimes they do not, but in either case, we expect them to have an effect on the volatility of both revenues and cash flows, as our theoretical and empirical analysis suggest. Since volatility incurs additional financial costs, even revenue-effective volatile marketing tactics may turn out to be harmful to the bottom line. This creates a managerial tradeoff. If the effect of marketing volatility on the level of revenues/cash flows is small or non-existent, there is no need to increase marketing volatility, and in fact it should be avoided. If the effect on sales is high, managers need to find the right balance between that positive impact and its negative financial side effect.

Similarly, different brands have different levels of marketing spending and our results show that those with higher spending *levels* enjoy protection against performance volatility, especially cash-flow volatility, so long as their expenditures are economically reasonable, i.e. they are not too far beyond the Dorfman-Steiner optimal levels. On the other hand, higher spending *quality* (as assessed by responsiveness) comes at a cost of increased performance volatility in connection with volatile spending behavior, and in that sense good marketing and stable business performance are difficult to reconcile.

Research Implications

Our findings contribute to the advancement of knowledge in marketing. Cash-flow volatility has been overlooked in marketing for a long time (Srivastava, Shervani, and Fahey 1998). While recent research (e.g. Gruca and Rego 2005; Rao and Bharadwaj 2008) has focused on marketing's potential to *reduce* volatility and its associated financial side effects at the firm level, our study is the first to describe its potential to *increase* volatility. We do so by relying on extant market response theory, which allows us to make the formal connection between marketing spending, marketing responsiveness and revenue and cash-flow volatility.

The empirical application of our theoretical framework to a large dataset from the pharmaceutical industry creates interesting findings. We learn that the volatility of marketing expenditures has a sizeable effect on revenue volatility, and its effect on cash-flow volatility is even higher. Marketing responsiveness has a strong effect on revenue volatility, which translates into a substantial impact on cash-flow volatility. Finally, our analysis shows that higher marketing-spending levels help reduce the volatility of revenues and cash flows. This conclusion is similar to the firm-level finding of McAlister, Srinivasan, and Kim (2007) that marketing spending reduces the systematic risk of the firm. Thus while generous marketing spending may be favorable for the firm in the long run, at the same time, allocating these budgets across brands in a pulsing pattern may be less favorable.

Revenue and cash-flow volatility may also arise from other marketing-mix activities such as promotions and new-product introductions. The extant promotion literature shows that promotions lead to higher revenues, at least in the short run (e.g., Blattberg, Briesch, and Fox 1995). Even though the long-term consequences of promotion activities may not be beneficial to the firm's bottom line, this marketing tactic is heavily used by CPG companies and retailers because of its immediate impact on revenues (e.g. Srinivasan et al. 2004). In addition, the financial benefits of new-product introductions have been acknowledged (e.g., Pauwels et al. 2004). For example, Sorescu and Spanjol (2008) report that the average CPG company in their sample introduced 237 new products during the period 1985-2003. Typically, these new-product introductions are not evenly distributed across the year, possibly causing performance volatility.

Limitations and Future Research

Our research is subject to limitations that may stimulate future research. First, we have quantified the magnitude of volatility drivers in eight prescription-drug markets. It would be interesting to extend this analysis to other industries. Second, we do not claim to have analyzed all marketing-related drivers of volatility. In particular, new-product introductions and sales promotions may also contribute to volatility, and should be explored in future research. Third, we discussed the cost-benefit trade-off of volatility-driving marketing practices such as pulsing, but we did not derive the optimal level of volatility. Future research should develop a normative model that includes the benefits and the costs of volatility-increasing marketing activities.

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TABLE 1

Descriptive Statistics (Period = Quarter)

Level variables	*Mean*	*Std. dev.*	*Volatility variables*	*Mean*	*Std. dev.*
Unit sales in daily dosages (,000)	17,817	20,392	Moving variance of adjusted revenues in th. US$	7,524,430	40,764,000
Revenues in US$ (,000)	9,342	10,400	Moving variance of adjusted cash flows in th. US$	3,310,330	17,186,000
Cash flows in US$(,000)	5,022	6,385	Moving variance of marketing expenditures in th. US$	151,134	347,868
Marketing expenditures in US$ (,000)	1,053	872	Moving variance of competitive marketing expenditures in th. US$	2,300,460	2,525,510
Competitive marketing expenditures in US$ (,000)	5,008	3,390	Moving range of adjusted revenues in th. US$	3,754	6,749
Moving average of marketing expenditures in US$ (,000)	960	732	Moving range of adjusted cash flows in th. US$	2,692	4,383
Moving average correlation between own and competitive marketing expenditures	0.35	0.40	Moving range of marketing expenditures in th. US$	854	756
			Moving range of competitive marketing expenditures in th. US$	3,675	2,322

Notes: All variables before log-transformation that is used in estimation. All values in 1996 dollars deflated by country-specific consumption price index.

TABLE 2

Estimation Results for the Volatility Models

	Expected sign	*Revenue volatility*				*Cash-flow volatility*	
		First-difference model		Cross-sectional model		First-difference model	
		Dependent variable					
		Variance	Range	Variance	Range	Variance	Range
Constant		-		-11.120 (9.70)	-6.537 (4.883)	-	
Volatility of revenues	+	-		-		1.359 (.029)***	1.155 (.018)***
Volatility of marketing expenditures	+	.273 (.024)***	.101 (.024)***	1.926 (.699)***	2.214 (.819)***	.535 (.032)***	.227 (.023)***
Level of marketing expenditures	–	-.245 (.237)	.139 (.076)	-1.988[1] (.665)***	-1.008[1] (.327)***	-2.042 (.369)***	-.307 (.079)***
Exp(Level of marketing expenditures)	+	-		-		.003 (.001)***	.001 (.000)***
Volatility of competitive marketing expenditures	–	-.006 (.023)	.018 (.020)	-.222 (.143)*	-.449 (.285)*	-	
Correlation between own and competitive marketing expenditures	–	-.262 (.095)***	-.066 (.026)**	-2.337 (3.19)	-1.019 (1.481)	-	
Marketing responsiveness	+	-		8.110 (4.75)**	3.974 (2.078)**	-	
Variance explained in estimation/holdout samples[2]		.245/.202	.339/.215	.201/.600	.241/.647	.724/.585	.771/.721
Total no. of observations		2,104		99		2,104	

Notes: Standard errors in parentheses. One-sided t-test applies to unidirectional expectations, two-sided t-tests otherwise. *** p < .01; ** p < .05; * p < .10

[1] Level of marketing expenditures was divided by the mean level of unit sales for a brand to account for brand size effects.

[2] Variance in log-transformed focal volatility variable explained by predictor variables. Estimation sample includes 80%, holdout sample 20% of cases.

TABLE 3

Total Effects in Terms of Elasticity (When Applicable)

	Expected sign	*Revenue Volatility*		*Cash-flow volatility*	
Dependent variable variance					
Variance of marketing expenditures	+	.273	(.024)***	.906	(.070)***
Level of marketing expenditures[1)]	–	-.245	(.237)	-2.375	(.490)***
Exp(Level of marketing expenditures)[1)]	+	-		.003	(.001)***
Marketing responsiveness[2)]	+	.811	(.474)**	1.102	(.645)**
Variance of competitive marketing expenditures	–	-.006	(.023)	-.009	(.031)
Correlation between own and competitive marketing expenditures[2)]	–	-.090	(.033)***	-.123	(.045)***
Dependent variable range					
Range of marketing expenditures	+	.101	(.024)***	.344	(.051)***
Level of marketing expenditures[1)]	–	.139	(.076)	-.147	(.118)
Exp(Level of marketing expenditures)[1)]	+			.001	(.000)***
Marketing responsiveness[2)]	+	.397	(.208)**	.459	(.240)**
Range of competitive marketing expenditures	–	.018	(.020)	.021	(.023)
Correlation between own and competitive marketing expenditures[2)]	–	-.023	(.009)***	-.026	(.010)***

Notes: (Approximated) standard errors in parentheses. Results are based on first difference models except for marketing responsiveness which are based on cross-sectional models. *** $p < .01$; ** $p < .05$

[1)] Results reflect parameters of a non-monotonic function, not elasticities.

[2)] Elasticities are not constant and are evaluated at sample means for responsiveness and expenditure correlations, respectively.

Technical Web Appendix

Accompanying

Brand Performance Volatility Arising from Marketing Spending Behavior under Competition

Proofs

PROOF OF PROPOSITION 2A. We need to show that $\frac{\partial Var\left[RV\left(MKT,CMKT\right)\right]}{\partial Var\left(MKT\right)}>0$. Taking the first derivative of Equation (11) w.r.t. *Var*(*MKT*) and setting > 0 gives,

$$P^2\left[Q'\left(\mu\right)\right]^2+P^2\rho Q'\left(\mu\right)Q_c'\left(\mu_c\right)\left[\frac{Var\left(CMKT\right)}{Var\left(MKT\right)}\right]^{\frac{1}{2}}>0\,, \tag{20}$$

which we divide by $P^2Q'(\mu)$ and rearrange to

$$Q'\left(\mu\right)\sqrt{Var\left(MKT\right)}>-\rho Q_c'\left(\mu_c\right)\sqrt{Var\left(CMKT\right)}\,. \tag{21}$$

Recall our expressions for elasticities, $\varepsilon = Q' \cdot \mu/Q$ and $\varepsilon_c = Q_c' \cdot \mu_c/Q$, and coefficients of variation, $CV = SD(MKT)/\mu$ and $CV_c = SD(CMKT)/\mu_c$. Dividing (21) by Q and expanding the l.hs. with μ and the r.h.s. with μ_c produces

$$\varepsilon\, CV > -\rho\, \varepsilon_c\, CV_c\ ,$$

which is equivalent to the condition in proposition 2A. □

PROOF OF PROPOSITION 2B. We need to show that $\frac{\partial Var\left[RV\left(MKT,CMKT\right)\right]}{\partial \mu}<0$.

Taking the first derivative of Equation (11) w.r.t. μ and setting < 0 gives,

$$2P^2Q''\left(\mu\right)Q'\left(\mu\right)Var\left(MKT\right)+2P^2\rho Q''\left(\mu\right)Q_c'\left(\mu_c\right)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}}<0\,. \tag{22}$$

Note that $Q''(\mu) < 0$. Dividing (22) by $2P^2Q''(\mu)Var(MKT)$ and rearranging the result gives

$$Q'\left(\mu\right)\sqrt{Var\left(MKT\right)}>-\rho Q'\left(\mu_c\right)\sqrt{Var\left(CMKT\right)}\,, \tag{23}$$

which is equivalent to expression (21). As done before, we can transform this expression to

$$\varepsilon\, CV > -\rho\, \varepsilon_c\, CV_c\ ,$$

which is equivalent to the condition in proposition 2B. □

PROOF OF PROPOSITION 2C. We need to show that $\frac{\partial Var\left[RV\left(MKT,CMKT\right)\right]}{\partial Q'\left(\mu\right)}>0$.

Taking the first derivative of Equation (11) w.r.t. $Q'(\mu)$ and setting > 0 gives,

$$2P^2Q'(\mu)Var(MKT)+2P^2\rho Q_c'(\mu_c)\left[Var(MKT)Var(CMKT)\right]^{\frac{1}{2}}>0\,, \tag{24}$$

which we divide by $P^2Q'(\mu)[Var(MKT)]^{1/2}$ and rearrange to

$$Q'(\mu)\sqrt{Var(MKT)}>-\rho Q_c'(\mu_c)\sqrt{Var(CMKT)}\,, \tag{25}$$

which is equivalent to expression (21). As done before, we can transform this expression to

$$\varepsilon\, CV>-\rho\,\varepsilon_c\, CV_c\ ,$$

which is equivalent to the condition in proposition 2C. □

PROOF OF PROPOSITION 3A. We need to show that $\dfrac{\partial Var\left[CF(MKT,CMKT)\right]}{\partial Var(MKT)}>0$. Taking the first derivative of Equation (12) w.r.t. $Var(MKT)$ and setting > 0 gives,

$$\left[(P-C)Q'(\mu)-1\right]^2+(P-C)^2\rho Q'(\mu)Q_c'(\mu_c)\left[\frac{Var(CMKT)}{Var(MKT)}\right]^{\frac{1}{2}}>0\,. \tag{26}$$

Selectively, we expand terms of (26) with Q, μ, and μ_c, respectively, to obtain

$$\frac{\left[\varepsilon(P-C)Q-\mu\right]^2}{\mu^2}>-(P-C)^2\rho\varepsilon\varepsilon_c\frac{Q^2}{\mu\mu_c}\left[\frac{Var(CMKT)}{Var(MKT)}\right]^{\frac{1}{2}}. \tag{27}$$

Dividing this expression by $\dfrac{\varepsilon^2(P-C)^2Q^2}{\mu^2}$ gives,

$$\frac{\left[\varepsilon(P-C)Q-\mu\right]^2}{\varepsilon^2(P-C)^2Q^2}>-\rho\frac{\varepsilon_c}{\varepsilon}\frac{\mu}{\mu_c}\left[\frac{Var(CMKT)}{Var(MKT)}\right]^{\frac{1}{2}}. \tag{28}$$

Substituting for the near-optimal expenditure level $\tilde{\mu}$ according to Equation (13) and the coefficients of variation, CV and CV_c, we can write for (28)

$$\left(\frac{\tilde{\mu}-\mu}{\tilde{\mu}}\right)^2>-\rho\frac{\varepsilon_c\, CV_c}{\varepsilon\, CV}\ ,$$

which is equivalent to the condition in proposition 3A. □

PROOF OF PROPOSITION 3B. This proposition implies that the first derivative of (12) has a root. Hence,

$$\frac{\partial Var\left[CF\left(MKT,CMKT\right)\right]}{\partial\mu}=0$$

$$=2\left(P-C\right)Q''\left(\mu\right)\left[\left(P-C\right)Q'\left(\mu\right)-1\right]Var\left(MKT\right) \quad (29)$$

$$+2\left(P-C\right)^2\rho Q''\left(\mu\right)Q_c'\left(\mu_c\right)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}}.$$

Dividing (29) by $Q''<0$, expanding Q' and Q_c' with Q, μ, and μ_c, respectively, and substituting terms for elasticities, coefficients of variation and $\tilde{\mu}$ from (13), we solve for the root

$$\mu_0=\tilde{\mu}\left(\frac{\varepsilon CV+\rho\varepsilon_c CV_c}{\varepsilon CV}\right). \quad (30)$$

Because $\varepsilon CV>-\rho\varepsilon_c CV_c$, the root is defined for positive mean expenditure levels. For a U-shaped relation, we must show that $\partial Var\left[CF\left(MKT,CMKT\right)\right]/\partial\mu\big|_{\mu<\mu_0}<0$ and $\partial Var\left[CF\left(MKT,CMKT\right)\right]/\partial\mu\big|_{\mu>\mu_0}>0$. Rewriting Equation (29) by substituting terms for elasticities, coefficients of variation and $\tilde{\mu}$, these inequalities imply $\tilde{\mu}\left(\frac{\varepsilon CV+\rho\varepsilon_c CV_c}{\varepsilon CV}\right)-\mu>0$ and $\tilde{\mu}\left(\frac{\varepsilon CV+\rho\varepsilon_c CV_c}{\varepsilon CV}\right)-\mu<0$, respectively. Let $\mu=z\mu_0$, with $z>0$. Note that $\mu<\mu_0$ for $z<$ 1 and $\mu>\mu_0$ for

$z>1$. Substituting μ_0 for (30), we easily verify that $\partial Var\left[CF\left(MKT\right)\right]/\partial\mu\big|_{z<1}<0$ and $\partial Var\left[CF\left(MKT\right)\right]/\partial\mu\big|_{z>1}>0$. □

PROOF OF COROLLARY 3. Set $\mu=\mu^*$, which implies with (13) that $\tilde{\mu}=\mu^*$. Because $Q_c'<0$ and $\rho>0$, $\left(\varepsilon CV+\rho\varepsilon_c CV_c\right)/\varepsilon CV<1$. Then, result (30) implies that $\mu_0<\mu^*$. □

PROOF OF PROPOSITION 3C. The derivative of (12) w.r.t. $Q'(\mu)$ is given by

$$\frac{\partial Var\left[CF\left(MKT,CMKT\right)\right]}{\partial Q'\left(\mu\right)}=2\left(P-C\right)\left[\left(P-C\right)Q'\left(\mu\right)-1\right]Var\left(MKT\right)$$
$$+2\left(P-C\right)^{2}\rho Q_c'\left(\mu_c\right)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}}. \tag{31}$$

Note that Equation (29), the derivative of (12) w.r.t μ, equals Equation (31) scaled by $Q''\left(\mu\right)$. Hence, both derivatives have the same root, as given by Equation (30). However, because $Q''<0$, the inequality conditions for $\partial Var\left[CF\left(MKT,CMKT\right)\right]\big/\partial Q'\left(\mu\right)$ at $\mu<\mu_0$ and $\mu>\mu_0$, respectively, are reversed. Following the chain of proof for proposition 3B, we can proof that these inequalities hold. Specifically,

$\partial Var\left[CF\left(MKT,CMKT\right)\right]\big/\partial Q'\left(\mu\right)\Big|_{\mu<\mu_0}>0\Rightarrow\mu<\tilde{\mu}\left(\frac{\varepsilon CV+\rho\,\varepsilon_c\,CV_c}{\varepsilon CV}\right)$, with $\varepsilon CV>-\rho\varepsilon_c CV_c$

and

$\partial Var\left[CF\left(MKT,CMKT\right)\right]\big/\partial Q'\left(\mu\right)\Big|_{\mu>\mu_0}<0\Rightarrow\mu>\tilde{\mu}\left(\frac{\varepsilon CV+\rho\,\varepsilon_c\,CV_c}{\varepsilon CV}\right)$,

which is equivalent to the conditions in proposition 3C. □

PROOF OF PROPOSITION 4A. We need to show that $\frac{\partial Var\left[RV\left(MKT,CMKT\right)\right]}{\partial Var\left(CMKT\right)}>0$. Taking the first derivative of Equation (11) w.r.t. *Var*(*CMKT*) and setting > 0 gives,

$$P^2\left[Q_c'\left(\mu_c\right)\right]^2+P^2\rho Q'\left(\mu\right)Q_c'\left(\mu_c\right)\left[\frac{Var\left(MKT\right)}{Var\left(CMKT\right)}\right]^{\frac{1}{2}}>0,$$

which we divide by P^2 to obtain

$$\left[Q_c'\left(\mu_c\right)\right]^2+\rho Q'\left(\mu\right)Q_c'\left(\mu_c\right)\left[\frac{Var\left(MKT\right)}{Var\left(CMKT\right)}\right]^{\frac{1}{2}}>0. \tag{32}$$

Consider first a substitutive cross-effect, i.e. $Q_c'\left(\mu_c\right)<0$ and therefore $\varepsilon_c<0$. Dividing (32) by $Q_c'\left(\mu_c\right)$ and rearranging the result gives,

$$Q_c'\left(\mu_c\right)\sqrt{Var\left(CMKT\right)}<-\rho Q'\left(\mu\right)\sqrt{Var\left(MKT\right)}. \tag{33}$$

Dividing (21) by Q and expanding the l.hs. with μ_c and the r.h.s. μ, we can rewrite inequality (33) as

$$\varepsilon_c CV_c < -\rho\varepsilon CV \quad ,$$

which proofs condition (14a) for proposition 4A.

Consider now a market-expanding cross-effect, i.e. $Q_c'(\mu_c) > 0$ and therefore $\varepsilon_c > 0$. Divide (32) by $Q_c'(\mu_c)$ and rearrange the result to

$$Q_c'(\mu_c)\sqrt{Var(CMKT)} > -\rho Q'(\mu)\sqrt{Var(MKT)}\ . \tag{34}$$

Again, we can rewrite this expression in terms of elasticities and coefficients of variation

$$\varepsilon_c CV_c > -\rho\varepsilon CV \quad ,$$

which proofs condition (14b) for proposition 4A. □

PROOF OF PROPOSITION 4B. We need to show that $\frac{\partial Var\left[CF(MKT, CMKT)\right]}{\partial Var(CMKT)} > 0$. Taking the first derivative of Equation (12) w.r.t. $Var(CMKT)$ and setting > 0 gives,

$$(P-C)^2\left[Q_c'(\mu_c)\right]^2 + (P-C)^2\rho Q'(\mu)Q_c'(\mu_c)\left[\frac{Var(MKT)}{Var(CMKT)}\right]^{\frac{1}{2}} > 0,$$

which we divide by $(P\text{-}C)^2$ to obtain

$$\left[Q_c'(\mu_c)\right]^2 + \rho Q'(\mu)Q_c'(\mu_c)\left[\frac{Var(MKT)}{Var(CMKT)}\right]^{\frac{1}{2}} > 0\ . \tag{35}$$

Note that this inequality is identical to inequality (32). With inequalities (33) and (34), we already showed that the conditions (14a) and (14b) satisfy inequality (35). □

PROOF OF PROPOSITION 5A. The first derivative of Equation (11) w.r.t. ρ is given by

$$\frac{\partial Var\left[RV(MKT, CMKT)\right]}{\partial\rho} = 2P^2Q'(\mu)Q_c'(\mu_c)\left[Var(MKT)Var(CMKT)\right]^{\frac{1}{2}} \tag{36}$$

Again, we can rewrite this expression in terms of elasticities and coefficients of variation

$$\frac{\partial Var\left[RV(MKT, CMKT)\right]}{\partial\rho} = 2(PQ)^2\varepsilon CV\varepsilon_c CV_c\ . \tag{37}$$

Since P, Q, ε, CV, and CV_c are always strictly positive, it is easy to show that

$$\frac{\partial Var\left[RV\left(MKT,CMKT\right)\right]}{\partial\rho}>0 \quad \text{iff} \quad \varepsilon_c>0$$

and

$$\frac{\partial Var\left[RV\left(MKT,CMKT\right)\right]}{\partial\rho}<0 \quad \text{iff} \quad \varepsilon_c<0. \ \square$$

PROOF OF PROPOSITION 5B. The first derivative of Equation (12) w.r.t. ρ is given by

$$\frac{\partial Var\left[CF\left(MKT,CMKT\right)\right]}{\partial\rho}=2\left(P-C\right)^2 Q'\left(\mu\right)Q_c'\left(\mu_c\right)\left[Var\left(MKT\right)Var\left(CMKT\right)\right]^{\frac{1}{2}},$$

which can be rewritten as

$$\frac{\partial Var\left[CF\left(MKT,CMKT\right)\right]}{\partial\rho}=2\left(P-Q\right)^2 \varepsilon CV\varepsilon_c CV_c\,. \tag{38}$$

Note that (P-C)² is always strictly positive. Following the chain of proof for proposition 5A, it is easy to show that proposition 5B also holds. □

Derivation of Estimation Equations

In this section, we derive the estimation equations (18) and (19) from equations (16) and (17). We start with a log transformation of equations (16) and (17),

$$\begin{aligned}\ln V\left(REV\right)=\ln\gamma_0+\gamma_1\ln V\left(MKT\right)+\gamma_2\ln A\left(MKT\right)+\gamma_3\ln V\left(CMKT\right)+\gamma_4 CORR+\\ \gamma_5 RESP+\mathbf{X\gamma}+\nu,\end{aligned} \tag{39}$$

$$\ln V\left(CF\right)=\ln\delta_0+\delta_1\ln V\left(REV\right)+\delta_2\ln V\left(MKT\right)+\delta_3\ln A\left(MKT\right)+\delta_4 A\left(MKT\right)+\upsilon. \tag{40}$$

Then we take the total differentials to obtain,

$$\begin{aligned}\frac{1}{V\left(REV\right)}dV\left(REV\right)=\frac{\gamma_1}{V\left(MKT\right)}dV\left(MKT\right)+\frac{\gamma_2}{A\left(MKT\right)}dA\left(MKT\right)\\ +\frac{\gamma_3}{V\left(CMKT\right)}dV\left(CMKT\right)+\gamma_4 dCORR+\gamma_5 dRESP+\mathbf{dX\gamma}+d\nu,\end{aligned} \tag{41}$$

$$\begin{aligned}\frac{1}{V\left(CF\right)}dV\left(CF\right)=\frac{\delta_1}{V\left(REV\right)}dV\left(REV\right)+\frac{\delta_2}{V\left(MKT\right)}dV\left(MKT\right)+\frac{\delta_3}{A\left(MKT\right)}dA\left(MKT\right)\\ +\delta_4 dA\left(MKT\right)+d\upsilon,\end{aligned} \tag{42}$$

which define the structural form of our estimation equations (18) and (19).