Information Aggregation, Security Design, and Currency Swaps

Bhagwan Chowdhry

University of California, Los Angeles

Mark Grinblatt

University of California, Los Angeles and National Bureau of Economic Research

David Levine

University of California, Los Angeles

A security design model shows that multinational firms needing to finance their operations should issue different securities to investors in different countries in order to aggregate their disparate information about domestic and foreign cash flows. However, if the firm becomes bankrupt, investors may face uncertain costs of reorganizing assets in a foreign country and thus may value foreign assets at their average value. This penalizes superior firms with low reorganization costs. Such firms minimize the adverse selection penalty by designing securities that allocate all the cash flow in bankruptcy to investors for which the adverse selection costs are the smallest given the exchange rate. We show that this sharing rule can be implemented with currency swaps because these instruments allow the priorities of claims in bankruptcy to switch depending on the exchange rate.

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I. Introduction

One of the long-held tenets of financial economics, dating back to the Modigliani and Miller capital structure propositions, is that in a perfect capital market, the precise packaging and marketing of securities are irrelevant. However, the practice of finance since the 1980s is largely noted for the proliferation of new contractual arrangements that package security payoffs in different ways. As Ross (1989) suggested in his presidential address to the American Finance Association, we still do not understand why firms go through the trouble of creating such seemingly redundant derivative assets and liabilities. Furthermore, firms often market securities to different clienteles and raise financing from several different sources even when a single financial intermediary may be capable of satisfying their financing requirements. In this paper, we provide a rationale for the design and marketing of such securities by firms.

We model a situation in which different agents possess signals about different components of a firm's future aggregate cash flows. Here, domestic investors receive private signals about a multinational firm's domestic cash flows and foreign investors receive private signals about its foreign cash flows. Firms with high cash flows prefer full revelation of investor information because less informed investors value uncertain cash flows at their mean values. We show that whenever a firm issues two distinct securities to domestic and foreign investors, equilibrium prices of the two distinct securities reveal all investor signals about domestic and foreign future cash flows. This can explain why firms issue multiple securities to multiple investors.

If bankruptcy is possible, the securities issued can be fixed-income securities. Our paper focuses on the optimal design of fixed-income securities when future cash flows and exchange rates are uncertain. The pricing of fixed-income securities depends on the probability of bankruptcy and the payoffs to fixed-income claimants conditional on bankruptcy. These, in turn, depend on the joint distribution of future cash flows and the future exchange rate. As a consequence, information revelation about future cash flows affects the pricing of fixed-income securities.

Equilibrium prices communicate investor information because investors submit rational reservation price functions that depend on their private information as well as on the prices of securities that they observe. An "auctioneer" aggregates these reservation price functions to determine equilibrium prices. Intuitively, investors are effectively submitting conditional bids that are not firm unless all investors know that no investor has regrets after observing the prices of securities. This is analogous to the situation in which a domestic and a foreign bank are

more willing to jointly lend money to a multinational firm at a small credit spread than each would be acting alone.

Full revelation of investor information in our model does not, however, preclude informational asymmetry about the future value of assets in bankruptcy, which could differ for domestic and foreign investors. The differences in valuations arise because in bankruptcy, a firm's assets may need to be reorganized to realize their full value. It seems intuitive and natural that the costs of reorganizing the firm's domestic assets would be higher for foreign investors and vice versa. Our model assumes that there are potential costs of reorganization if domestic assets are reorganized by foreign investors and, conversely, if foreign assets are reorganized by domestic investors. Equilibrium prices reveal all investor information about the firm's domestic and foreign cash flows but not the potential costs of reorganization. As a consequence of uncertain potential costs of organization, the equilibrium obtained is one in which investors pool firms with high and low reorganization costs. This pooling penalizes the firm that has low reorganization costs. We refer to this as an adverse selection penalty. Such a firm then has an incentive to create security instruments that minimize this penalty. Our main results focus on how this firm, which is assumed to know that it has low reorganization costs, can design securities that minimize this adverse selection penalty. Firms with high reorganization costs will mimic this security design.

One way of dealing with this adverse selection problem is to create extreme securities that "spin off" domestic and foreign assets: in bankruptcy, to provide no foreign assets to domestic investors and vice versa. In this case, all investors know that domestic assets will be reorganized by domestic investors and vice versa, and hence there will be no adverse selection penalty. However, securities like this are infeasible when, because of contractual, regulatory, or operational frictions, contracts must be written on aggregate cash flows from all assets, domestic and foreign; this is perhaps the very reason the firm exists as a multinational corporation.

When contracts are written on aggregate cash flows from all assets, firms with low reorganization costs cannot avoid the adverse selection penalty because domestic investors value cash flows from domestic assets at their full value but impute an average cost associated with reorganizing foreign assets in bankruptcy; conversely, foreign investors value cash flows from foreign assets at their full value but impute an average cost associated with reorganizing domestic assets in bankruptcy. Optimal security design requires that securities, in bankruptcy, minimize expected payoffs from foreign assets to domestic investors and from domestic assets to foreign investors. This optimization problem faces constraints in that contracts on aggregate cash flows, unlike "spin-off"

contracts, cannot minimize both sets of reorganization costs independently.

Our solution to this constrained optimization problem recognizes that one investor type—either domestic or foreign—imposes greater adverse selection costs given a realized exchange rate at the time of bankruptcy. The optimal contracts on aggregate cash flows therefore are those that minimize adverse selection costs conditional on the future exchange rate. By minimizing adverse selection costs for each realization of the future exchange rate, such contracts also minimize the ex ante cost of adverse selection, which the firm implicitly pays at the time of issue in the pricing of its securities.

We show that properly designed currency swaps, which are essentially contracts whose payoffs are contingent on the future exchange rate, ensure that in bankruptcy, only investors of one type, domestic or foreign, are owed money (i.e., have positive contractual payoffs) when investors of the other type owe money (i.e., have negative contractual payoffs) and that the investors that are owed money are precisely the ones that impose the smaller adverse selection costs given the future realization of exchange rates. It is this feature that allows swaps to minimize adverse selection and dominate seemingly identical debt contracts, even in the absence of hedging needs. Both pari passu domestic and foreign debt and any senior-subordinated debt structures are always suboptimal. Unlike a pair of currency swaps, these fixed-income designs do not allow the order of priority in bankruptcy to depend on the realized exchange rate. As a consequence, financing packages that include properly designed currency swaps may appear to be cheaper than those based strictly on straight debt.

Our paper makes three contributions: (1) We show that firms issue distinct securities to multiple clienteles in order to aggregate disparate investor information. (2) Our results on security design are motivated by issue cost minimization designed to minimize adverse selection costs caused by unresolved information asymmetries. (3) We show that instruments such as currency swaps, by allowing switching of priorities in different states in bankruptcy reorganization, can implement a security design that minimizes adverse selection costs. We thus provide an explanation for the use of derivative instruments by many corporations that is based purely on issue cost minimization. Hedging and risk-sharing motives play no role in our framework because our approach assumes risk neutrality. Consistent with our argument, a Harvard Business School case on Walt Disney Company's yen financing (Allen 1987) points out that the use of currency swaps is often motivated by efficient financing considerations.

The paper is outlined as follows. Section II presents the model. Sec-

tion III derives the results. Section IV discusses related literature. Section V concludes the paper.

II. The Model

A. The General Setup

We study the problem of a risk-neutral firm with superior information that finances its operations by issuing optimally designed securities to price-taking, risk-neutral domestic and foreign investors. Specifically, consider a multinational firm operating and issuing securities in two countries: domestic, denoted *X*, and foreign, denoted *Y*. The firm's projects, which require financing of *I*, generate cash flows

$$(x+z_x)+s(y+z_y)+\iota\theta,$$

where $x + z_x$ is domestic cash flows associated with country X, $y + z_y$ is foreign cash flows associated with country Y, s is the exchange rate (which is a contractible variable), θ is a noncontractible, nonnegative cash flow, and ι is an indicator variable that takes the value one if the firm is able to pay off its promises to security holders and zero in the event of bankruptcy.

We shall see that because θ is not contractible, it generates the possibility of bankruptcy. In bankruptcy, if the firm is reorganized by domestic investors, the foreign cash flow described above is reduced by reorganization cost z_y (in foreign currency units). If the firm is reorganized by foreign investors, the domestic cash flow is reduced by reorganization cost z_x ; the original management can reorganize without costs. Thus the cash flow component, y, can be thought of as the domestic investors' reservation value of foreign assets (in foreign currency units) when the firm is bankrupt. Similarly, x can be thought of as the foreign investors' reservation value of domestic assets when the firm is bankrupt.

The firm's objective is to issue claims to the cash flows to maximize the value of preexisting ownership claims (loosely referred to as "equity") while obtaining financing of at least *I*. The sequence of events is as follows:

1. Nonnegative domestic cash flow, $x + z_x$, foreign cash flow, $y + z_y$, and the reorganization costs, z_x and z_y , are determined and revealed to the firm. Domestic investors learn $x + z_x$. Foreign investors learn $y + z_y$.

- 2. The firm designs and markets fixed-income securities to investors in countries *X* and *Y* with promised payoffs that are contingent only on the realized exchange rate s. Equilibrium prices are determined.
- 3. The exchange rate s and cash flow shock θ are determined; all agents learn cash flows, shocks, and (if the firm is bankrupt) reorganization costs.
- 4. The firm either pays its contractual obligations fully or goes bankrupt. If bankrupt, it reorganizes by negotiating a settlement with fixed-income claimants.

B. An Overview of the Main Results

In equilibrium, the firm issues two securities, X and Y, the prices of which reveal $x+z_x$ and $y+z_y$. Domestic investors, despite having inferred $y+z_y$, cannot distinguish a high- z_y , low-y firm from a low- z_y , high-y firm (similarly for foreign investors). This gives rise to an adverse selection problem because investors remain less informed than the firm that observes both cash flows and reorganization costs. A firm with low reorganization costs, that is, $z_x < E(z_x|x+z_x)$ and $z_y < E(z_y|y+z_y)$, receives a less than fair price for its securities. It is penalized because domestic investors use the average value of y and foreign investors use the average value of x to bid for the firm's securities.

The main result is that, in state s, it is optimal for the firm to allocate all the cash flow to either security X or security Y depending on the exchange rate s. The optimal security design allocates the cash flow in each state to the investor type that values it most (i.e., demands the lowest adverse selection premium). In this way, the firm with low reorganization costs minimizes the adverse selection cost it faces, state by state. This feature of the optimal design can be implemented by issuing currency swaps to the investors (see fig. 1). Figure 1 also demonstrates that it would be suboptimal to issue the same security to both domestic and foreign investors since one of the two investor types has higher adverse selection for any realization of the exchange rate. For simplicity and without loss of generality, we denote X as the security that is issued to domestic investors and Y as the security that is issued to foreign investors.

We now describe the model and the results in detail.

¹ Following Townsend (1979) and Diamond (1984), we assume that the nonverifiability of payoffs makes the issuance of fixed-income claims desirable. However, straight debt need not be the optimal fixed-income security to issue here because there is a publicly observable and verifiable state variable, the exchange rate *s*, on which contractual payoffs can be made contingent.

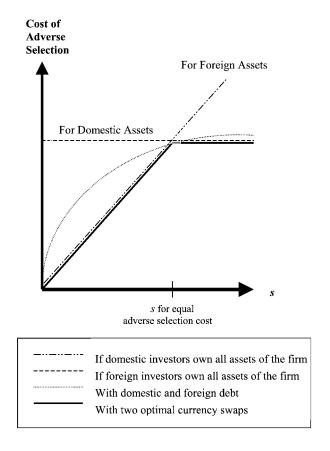


Fig. 1.—Cost of adverse selection as a function of the exchange rate s

C. Cash Flows to Investors

The net cash flow available for distribution depends not only on whether or not the firm is bankrupt but, if it is bankrupt, on who reorganizes the firm's assets. Because of this feature of the model, different investors value the same asset differently, even though (in equilibrium) they are symmetrically informed. The difference in valuation generates an information asymmetry between the firm and its investors that cannot be resolved completely but can be mitigated by security design.

In nonbankrupt states, the firm is operated by the original managers, and there is a bonus cash flow θ from the absence of the need to operate

under bankruptcy constraints.² The total cash flow in the absence of bankruptcy is denoted by

$$A(s, \theta, x + z_x, y + z_y) \equiv (x + z_x) + s(y + z_y) + \theta.$$

When the firm falls into bankruptcy, the aggregate contractual payments on its fixed-income securities, denoted F(s), exceed the available cash flow, that is,

$$F(s) > A(s, \theta, x + z_x, y + z_y).$$

The inability to contract on the shock θ is what leads to the possibility of bankruptcy.

We make the following distributional assumptions: (1) The joint distribution of $[x, z_x, y, z_y, s, \theta]$ is common knowledge. (2) The distribution $[s, \theta]$ is independent of $[x, z_x, y, z_y]$. (3) The conditional expectation functions

$$\bar{x}(x+z_x) \equiv E(x|x+z_x),$$

$$\bar{y}(y+z_y) \equiv E(y|y+z_y),$$

are monotonically increasing in their arguments.³ (4) For any given $x + z_x$ and $y + z_y$, there exists a firm with both the smallest $z_x \ge 0$ and the smallest $z_y \ge 0$. Note that since the distribution of $[s, \theta]$ and the securities issued by the firm are common knowledge, so is the probability of bankruptcy if (as is indeed the case) the equilibrium pricing of securities leads all investors to learn $x + z_x$ and $y + z_y$.

The inefficiency of bankruptcy has two aspects to it. One is a general inefficiency, represented by the loss of the nonnegative θ . Another inefficiency is represented by the potential reorganization costs of z_x and z_x .

Clearly, in bankruptcy, the firm cannot meet its aggregate contractually promised payment. Hence, there must be some rule that divides up what the firm can pay in bankruptcy to its cash flow claimants. In principle, the rule for determining the realized payoffs in bankrupt states need not be tied to the promised payoffs. As a practical matter, however, what a cash flow claimant gets in bankruptcy is related to what he is promised. In this vein, we assume that fractional ownership of assets by fixed-income claimants in bankruptcy is proportional to the promises made to these claimants. Let $F_X(s)$ and $F_Y(s)$ denote the prom-

² Consistent with a long literature on bankruptcy, we assume that it is less efficient to operate the firm in bankruptcy. Titman (1984), e.g., discusses how difficult it is for firms in financial distress to efficiently employ their existing assets. One reason is that the optimal liquidation policy for different financial claimants is not the same as the efficient liquidation policy for the firm's assets.

³ This condition obtains when x and $x + z_{\infty}$ as well as y and $y + z_{\infty}$ are affiliated (i.e., loosely speaking, positively correlated; see Milgrom and Weber [1982, theorem 5]).

ised payments to the holders of securities X and Y, respectively. Hence the fraction of the firm owned by domestic investors in bankruptcy is

$$f_X(s) \equiv \begin{cases} 1 & \text{if } F_X(s) \ge 0, \ F_Y(s) < 0 \\ \\ \frac{F_X(s)}{F_X(s) + F_Y(s)} & \text{if } F_X(s) \ge 0, \ F_Y(s) \ge 0 \\ \\ 0 & \text{if } F_X(s) < 0, \ F_Y(s) \ge 0. \end{cases}$$

For symmetry, the fraction of the firm owned by foreign investors in bankruptcy is $f_X(s) = 1 - f_X(s)$. When bankruptcy occurs in our model, s, x, z_x, y , and z_y are revealed, and negotiations go on between the managers and the senior claimants over the payment owed to the claimants. Because domestic investors incur costs in reorganizing foreign assets, they have a reservation value of $f_X(s)[(x+z_x)+sy]$ for their claim, which is less than the full value $f_X(s)[(x+z_x)+s(y+z_y)]$ obtained when assets are reorganized by the existing management. Similarly, foreign investors have a reservation value of $f_Y(s)[x+s(y+z_y)]$, which is less than the full value $f_Y(s)[(x+z_x)+s(y+z_y)]$.

We assume for notational simplicity and without loss of generality that all of the bargaining power in bankruptcy negotiations belongs to the equity-maximizing managers. Hence, equity-maximizing managers can reorganize the firm by paying the reservation values

$$C_X \equiv f_X(s)[(x+z_x)+sy]$$

to domestic investors and

$$C_{y} \equiv [1 - f_{x}(s)][x + s(y + z_{y})]$$

to foreign investors and keeping the rent

$$f_X(s) s z_y + [1 - f_X(s)] z_x$$

for equity holders.

D. The Equilibrium

The mechanism for market clearing in this model is one in which investors announce reservation price schedules that are nontrivial func-

⁴The deadweight losses associated with the costly reorganization of assets by fixed-income investors suggest a rationale for why absolute priority is often violated in practice. Reorganizations and debt restructurings occur in bankruptcy, which allow equity holders to maintain a measure of control of the firm. In the United States, e.g., Chapter 11 bankruptcy involves a reorganization of the firm as an operating entity.

tions of both their private information and market prices.⁵ These reservation price schedules can be viewed as a number of rational and competitive conditional bids. Price is determined by an "auctioneer" to clear the announced reservation price schedules.

It may be useful to picture a real-world analogy. Consider an investment bank using its sales force to shop among partially informed investors for the best deal for a corporate client. It offers a security to domestic investors, informs them that it will also be offering another security to foreign investors, and asks domestic investors what they will price the security for. Domestic investors return a price quote. This price quote is not firm until domestic investors see foreign investors' price for their security, and it is common knowledge that both sets of investors are satisfied with their quotes knowing each other's quotes. The bank then approaches foreign investors and enters into an analogous negotiation. This process may iterate back and forth as the investment bank shuttles pricing information back and forth between the two investor types until an equilibrium pair of prices is reached in which all investors know that no investors have regrets. This description of the negotiations between different parties is similar to Allen's (1987) description of negotiations between Walt Disney, Goldman Sachs, a French utility, and a Japanese financial intermediary about the terms of a currency swap as part of Disney's financing package.

III. Results

Our key results are derived in several steps. In equilibrium, the firm issues two securities, the prices of which reveal $x + z_x$ and $y + z_y$. To illustrate the intuition behind full revelation, subsections A and B analyze the simpler case in which bankruptcy occurs with probability one. To further simplify matters, we assume in this section that $F_X(s)$ and $F_Y(s)$ are nonnegative. Domestic investors remain less informed than the firm insofar as they can infer only $y + z_y$ whereas the firm observes both $y + z_y$ and z_y (similarly for foreign investors). This gives rise to an adverse

⁵ The requirement that reservation price schedules be nontrivial functions of private information resolves a paradox in the rational expectations literature, specifically, how the market can know private information when investors have equilibrium strategies that do not depend on private information. Beja (1976) first noted that some models that use the standard rational expectations equilibrium as their equilibrium definition exhibit this fundamental paradox (see also Radner 1979). Admati (1989) provides an intuitive discussion of this and related issues. An alternative resolution of the paradox is to introduce noise traders, as in Hellwig (1980) and Kyle (1989), among others. This has the advantage of also explaining the incentive to acquire information, along with resolving Beja's paradox. We employ a variation of the noise trader assumption in our model when we assume uncertain reorganization costs. Thus, except for degenerate parameter values, the existence of uncertain reorganization costs obviates the need for the *requirement* that demand be sensitive to private information; demand will generally have this property in our model.

selection problem for a firm with low reorganization costs. The profit of such a firm from security issuance is shown to be increasing in the expected portion of the domestic cash flow going to the domestic investor, $E[f_X(s)]$, and decreasing in the expected portion of the foreign cash flow going to the domestic investor, $E[f_X(s)s]$. The reverse is true for the portions to the foreign investor. In subsection C, we relax all the simplifying assumptions. The main result here is that, in state s, it is optimal for the firm to allocate all the cash flow to either domestic or foreign investors depending on whether $x^* - \bar{x}$ is above or below $s(y^* - \bar{y})$, where x^* and y^* refer to values for the pool-leading firm with the lowest reorganization costs z_x and z_y . We show that this sharing rule can be implemented with currency swaps.

A. Information Aggregation

To understand information aggregation and revelation more clearly, let us focus on the case in which $\theta = 0$, and to further simplify matters assume that $F_X(s)$ and $F_Y(s)$ are nonnegative. Suppose that because of sufficiently tight bounds on s, $F_X(s) + F_Y(s) > (x + z_x) + s(y + z_y)$ with certainty. In this case, the payoffs to the investors are simply what they get in bankruptcy, which occurs with probability one.

Let V_X denote the value of security X to domestic investors under full revelation, that is, if investors knew both $x + z_x$ and $y + z_y$. Then

$$V_X = E[f_X(s)[(x + z_x) + sy]|x + z_x, y + z_y]$$

= $\alpha_X(x + z_x) + \beta_X \bar{y}(y + z_y),$

where $\alpha_X \equiv E[f_X(s)]$ and $\beta_X \equiv E[f_X(s)s]$. Similarly, let V_Y denote the value of security Y to foreign investors under full revelation. Then

$$V_Y = E[f_Y(s)[x + s(y + z_y)]|x + z_x, y + z_y]$$

= $\alpha_Y \bar{x}(x + z_x) + \beta_Y(y + z_y),$

where $\alpha_Y \equiv E[f_Y(s)]$ and $\beta_Y \equiv E[f_Y(s)s]$. Note that $\alpha_X + \alpha_Y = 1$ and $\beta_X + \beta_Y = E(s)$.

Let $g_X(P_y|x+z_x)$ denote the reservation price schedule for security X by domestic investors who privately observe $x+z_x$ but also condition on the price for the other security P_y . Similarly, let $g_Y(P_x|y+z_y)$ denote the reservation price schedule for security Y by foreign investors. For these reservation price schedules to be rational, they must be consistent with fully revealing prices. In other words, if P_X and P_Y were replaced by values of these securities under full revelation, rational reservation price schedules must give the value of these respective securities. The ratio-

nality conditions can then be written as

$$V_X = g_X(V_Y|x + z_x),$$

$$V_Y = g_Y(V_X|y + z_y).$$

The only solution for the reservation price schedules satisfying these rationality conditions is 6

$$g_X(P_Y|x+z_x) = \alpha_X(x+z_x) + \beta_X \bar{y} \left[\frac{P_Y - \alpha_Y \bar{x}(x+z_x)}{\beta_Y} \right],$$

$$g_Y(P_X|y+z_y) = \alpha_Y \bar{x} \left[\frac{P_X - \beta_X \bar{y}(y+z_y)}{\alpha_Y} \right] + \beta_Y(y+z_y).$$

Remark 1. When the firm issues security X to domestic investors and security Y to foreign investors, the equilibrium conditions

$$P_X = g_X(P_Y|x + z_x),$$

$$P_Y = g_Y(P_X|y + z_y)$$

generate fully revealing prices with

$$P_X = \alpha_X(x + z_x) + \beta_X \bar{y}(y + z_y),$$

$$P_Y = \alpha_Y \bar{x}(x + z_x) + \beta_Y(y + z_y).$$

Proof. See the Appendix.

Because the two securities are different in bankruptcy, prices reveal private information. Securities can differ here for two reasons. First, if $\alpha_x/\beta_x \neq \alpha_y/\beta_y$, the securities have different relative sensitivities to the domestic and foreign cash flow components. Second, reorganization costs generate an asymmetry in the payoffs of these securities to domestic and foreign investors.

If reorganization costs were zero and if $\alpha_X/\beta_X = \alpha_Y/\beta_Y$, the two securities would be identical and the reservation price schedules would be insensitive to investors' private information. To demonstrate this, note that in this case, the reservation price schedules characterized

⁶ The proof of this is trivial. Assume that the g functions are those specified plus two separate arbitrary functions, $h_X(\cdot | x + z_x)$ and $h_Y(\cdot | y + z_y)$, respectively. The solution of the two rationality conditions would then imply that each of the h functions is identically zero.

above degenerate to

$$g_X(P_Y|x) = \alpha_X x + \frac{\beta_X}{\beta_Y}(P_Y - \alpha_Y x) = \frac{\beta_X}{\beta_Y}P_Y$$

$$g_{y}(P_{X}|y) = \frac{\alpha_{Y}}{\alpha_{X}}(P_{X} - \beta_{X}y) + \beta_{y}y = \frac{\alpha_{Y}}{\alpha_{X}}P_{X}.$$

Notice here that the reservation price schedules of both domestic and foreign investors are insensitive to their private information. Although a nonmarket mechanism, such as a direct revelation mechanism in which investors credibly announce their private signals to each other, could uncover the hidden information, our view is that a market mechanism is limited to revelation of reservation price schedules, and since private information is not reflected in such schedules, it cannot be reflected in market equilibrium prices.

This example reflects the key role of security differentiation in revealing private information. It also illustrates why firms raise financing from several different sources even when a single financial intermediary may be capable of satisfying the financing requirements of any given firm.

B. Uncertainty, Adverse Selection, and the Implications for Security Design

In this subsection, we show that optimal security design requires that securities, in bankruptcy, minimize expected payoffs from foreign assets to domestic investors and from domestic assets to foreign investors. This mitigates adverse selection arising from residual information asymmetry between the firm and its investors. We continue to consider the case in which the firm is bankrupt with probability one.

Notice that even when all investor information is revealed, the information advantage of the issuing firm relative to its investors is not eliminated. For example, foreign investors observing a high price for a security X issued to domestic investors cannot distinguish between a high value of x and a high z_x . Thus foreign investors must pool firms with high x's and low z_x 's together with firms that have low x's and high z_x 's in formulating the function $\bar{x}(x+z_x)$. As we show below, the firms with high reorganization costs benefit by being in the pool and get favorable pricing on the securities they issue to foreign investors. The firms with low reorganization costs receive unfair pricing on securities issued to foreign investors. They will do anything they can to either break out of the pool or alter the composition of the intrinsic security values in the pool so that their security pricing is not so disadvantageous. This, as we shall show, is accomplished via security design. The low-x, high-z, firms

will remain in the pool by mimicking the high-x, low- z_x firms' security design. In this pooling equilibrium, all firms issue the same securities. (A similar argument applies when domestic investors observe the price of security Y issued to foreign investors.) This suggests that security design is dictated by the preferences of firm types with $x > \bar{x}(x+z_x)$ and $y > \bar{y}(y+z_y)$, that is, firms with low reorganization costs, which, in effect, finance their projects at unfavorable rates.

To show that the design of the securities issued affects the financing revenue for firms with low reorganization costs, observe that the difference between revenue and cost from security issuance for these firms is⁷

$$P_{X} - C_{X} = [\alpha_{X}(x + z_{x}) + \beta_{X}\bar{y}(y + z_{y})] - [\alpha_{X}(x + z_{x}) + \beta_{X}y],$$

$$P_{Y} - C_{Y} = [\alpha_{Y}\bar{x}(x + z_{x}) + \beta_{Y}(y + z_{y})] - [\alpha_{Y}x + \beta_{Y}(y + z_{y})].$$

Substituting the adding up constraints,

$$\alpha_X + \alpha_Y = 1,$$

$$\beta_X + \beta_Y = E(s),$$

into the equations above implies

$$P - C = (P_X + P_Y) - (C_X + C_Y)$$

$$= -\{[x - \bar{x}(x + z_x)] + E(s)[y - \bar{y}(y + z_y)]\}$$

$$+ \alpha_X[x - \bar{x}(x + z_x)] + \beta_Y[y - \bar{y}(y + z_y)].$$

Notice that the firm's aggregate profit in this fully revealing equilibrium equals P-C and is increasing in α_X and β_Y for firms for which $x>\bar{x}(x+z_x)$ and $y>\bar{y}(y+z_y)$. A high value of α_X and a low value of β_X reduce the adverse selection faced by pool-leading firms. If the contracts were not constrained to be written on aggregate cash flows, the firm could eliminate adverse selection by choosing $\alpha_X=1$ and $\beta_X=0$, which essentially spins off domestic assets to domestic investors and, by the adding-up constraint, foreign assets to foreign investors. However, since $\alpha_X=E[f_X(s)]$ and $\beta_X=E[f_X(s)s]$, α_X and β_X are related to one another. This relation makes the security design problem, that is, the choice of $f_X(s)$, more complex than simply maximizing α_X and minimizing β_X . In the next subsection, we shall analyze the optimal security design, accounting for this constraint.

⁷ Note that in this example, we are ignoring the financing constraint, revenue of at least *I*, and focusing on what maximizes the profit to preexisting claimants from auctioning off the firm. This is necessary because the example assumes that the firm is always bankrupt, which constrains the amount of revenue raised.

C. Optimal Security Design

In the case we have analyzed so far, we assumed that the firm was always bankrupt. Here, we relax this assumption and analyze the optimal security design. Note that in contrast to the previous examples, we also allow $F_X(s)$ and $F_Y(s)$ to be negative. To simplify notation, let F denote F(s), which equals $F_X(s) + F_Y(s)$; let A denote $A(s, \theta, x + z_x, y + z_y)$; let \bar{x} denote $\bar{x}(x+z_x)$; and let \bar{y} denote $\bar{y}(y+z_y)$. Note that if $F_x(s) < 0$ or $F_{\nu}(s) < 0$ in bankruptcy (implying one is positive), there will be a transfer payment between investors holding security X and those holding security Y. Thus, in addition to out of bankruptcy payments, bankrupt states in which transfers occur will add an additional term to the expressions for the prices of the two securities given in the previous example. However, such a transfer payment does not affect the aggregate proceeds, P, of the firm's securities issuance. To simplify notation further, let NB be the set of nonbankrupt states $(A \ge F)$, B be the set of states in which the firm is bankrupt (A < F), T be a subset of B consisting of transfer payment states in which the firm owes money to investors holding security X and is owed money by investors holding security Y or vice versa $(F_x(s) < 0 \text{ or } F_y(s) < 0 \text{ but not both}), E^k \text{ be the expectation conditional}$ on being in set k, and π^k be the probability of being in set k.

As in the previous examples, the pricing of two distinct securities reveals the private information signals $x + z_x$ and $y + z_y$.

Remark 2. When the firm issues security X to domestic investors and security Y to foreign investors, the equilibrium conditions

$$P_X = g_X(P_Y|x + z_x),$$

$$P_Y = g_Y(P_X|y + z_y)$$

generate fully revealing prices with

$$P_{X} = K_{X} + \alpha_{X}(x + z_{x}) + \beta_{X}\bar{y}(y + z_{y}),$$

$$P_{Y} = K_{Y} + \alpha_{Y}\bar{x}(x + z_{x}) + \beta_{Y}(y + z_{y}),$$

where

$$\begin{split} &\alpha_{j} \equiv \pi^{B} E^{B}[f_{j}(s)], \\ &\beta_{j} \equiv \pi^{B} E^{B}[f_{j}(s)s], \\ &K_{X} \equiv \pi^{NB} E^{NB}[F_{X}(s)] + \pi^{T} E^{T}[\min[0, F_{X}(s)] + \max[0, -F_{Y}(s)]], \\ &K_{Y} \equiv \pi^{NB} E^{NB}[F_{Y}(s)] + \pi^{T} E^{T}[\max[0, -F_{X}(s)] + \min[0, F_{Y}(s)]]. \end{split}$$

Proof. See the Appendix.

Note that, in bankruptcy, the firm is buying its assets back at the

reservation prices of domestic and foreign investors, leaving a surplus for equity holders. Thus the equity holders of the firm have cash not only in nonbankrupt states but also in bankrupt states because absolute priority is violated in our reorganizational-type bankruptcy. Specifically, equity value, *W*, is cash raised,⁸

$$P = \pi^{NB} E^{NB}(F) + \pi^{B} E^{B}[f_{X}(s)[(x + z_{x}) + s\bar{y}]] + [1 - f_{X}(s)][\bar{x} + s(y + z_{y})]],$$

less cash invested, I, plus the value of the cash flows from the assets financed by investing I accounting for the deadweight loss of bankruptcy (given the firm's information),

$$J = E(A) - \pi^B E^B(\theta),$$

less the value of payments to senior cash flow claimants (given the firm's information),

$$C = E^{NB}(F) - \pi^B E^B[f_X(s)[(x+z_y) + sy] + [1 - f_X(s)][x + s(y+z_y)]],$$

which, after recognition that $\pi^{NB} + \pi^{B} = 1$, simplifies to

$$W = P - I + J - C$$

= $E(A) - I - \pi^B E^B [\theta + f_X(s)s(y - \bar{y}) + [1 - f_X(s)](x - \bar{x})].$

Observe that the first two terms, E(A) and I, are unaffected by security design and that the third term captures both the deadweight cost of bankruptcy θ and a pair of terms representing adverse selection costs faced by firms with low reorganization costs. Thus, for such firms, equity value is increased whenever a security design reduces the value of the deadweight loss from bankruptcy's effect on operations $\pi^B E^B(\theta)$ or reduces the adverse selection penalty

$$\pi^{B}E^{B}[f_{x}(s)s(y-\bar{y})+[1-f_{x}(s)](x-\bar{x})].$$

Thus there are two relatively independent means for increasing equity value by security design. Effectively lowering the number of bankrupt states by reducing F(s) for some states (still raising at least I) reduces both the deadweight loss and the adverse selection component. The solution to the optimal F(s) is distribution dependent and generally intractable. The second instrument for increasing equity value is the sharing rule for the fixed-income claimants, which affects the conditional expectation multiplicand

$$E^{B}[f_{X}(s)s(y-\bar{y}) + [1-f_{X}(s)](x-\bar{x})]$$

 $^{^{\}rm 8}\,\rm Notice$ that the transfer payments do not affect the aggregate proceeds from security issuance.

but does not affect the probability of bankruptcy, π^B . In the following proposition, we show that this conditional expectation is minimized, and thus the maximum equity value is attained with a security design that has the sharing rule described below.

Proposition 1. The maximum equity value of a firm with low reorganization costs is achieved for a security design with $f_X(s) = 1$ in states in which $x - \bar{x} > s(y - \bar{y})$ and $f_X(s) = 0$ otherwise.

Proof. First, hold F fixed, which determines the set of bankrupt states. Equity value W is then affected only by the conditional expectation of the adverse selection penalty

$$E^{B}[f_{X}(s)s(y-\bar{y}) + [1-f_{X}(s)](x-\bar{x})],$$

which is minimized on a state-by-state basis by the security design with

$$f_X(s) = \begin{cases} 1 & \text{when } x - \bar{x} > s(y - \bar{y}) \\ 0 & \text{otherwise.} \end{cases}$$

Since this feature maximizes equity value for each candidate F(s), it maximizes equity value at the optimal F(s) as well. Q.E.D.

Firms with high reorganization costs will mimic the security designs of firms with low reorganization costs lest they be identified as firms with high reorganization costs. Such firms sell their securities at a higher price by being in a pool with firms that have low reorganization costs.⁹

Proposition 1 shows that the proceeds from bankrupt states are maximized by minimizing adverse selection on a state-by-state basis, where states are defined by the realized exchange rate. Alternatively, this can be viewed as writing a state-contingent contract that allocates each state-contingent cash flow to claimants who value it the most. Obviously, two pari passu debt contracts (e.g., foreign and domestic debt) cannot be optimal because bankruptcy proceeds are shared. Also, senior and junior debts are suboptimal because they cannot reverse priority contingent on s.

D. Security Design and Currency Swaps

In this subsection, we analyze simplified representations of currency swaps. A currency swap to domestic investors contractually obligates the firm to pay n_x units of domestic currency to them, and in return they are obligated to pay $r_x n_x$ in foreign currency to the firm.¹⁰ Clearly, if

⁹ Note that all firms in the pool have the same probability of bankruptcy because their aggregate cash flows are identical and they issue identical securities.

¹⁰ A plain vanilla currency swap involves the exchange of the cash flows of a domestic bond for the cash flows of a foreign bond. The bonds may be fixed or floating. Both interest and principal are typically exchanged. There may even be an exchange of cash for foreign currency at the initiation of the swap.

 $r_X = 0$, the instrument is domestic debt with face value n_X . Similarly, a currency swap issued to foreign investors contractually obligates the firm to pay n_Y units of foreign currency to them, and in return they are obligated to pay $r_Y n_Y$ units of domestic currency to the firm. Here, $r_Y = 0$ represents foreign debt. In general, for positive values of r_X and r_Y , these instruments represent currency swap contracts. In bankruptcy, each swap has a *pari passu* claim in proportion to its contractually promised payment.¹¹

With the notation of the previous subsection, this means

$$F_X(s) \equiv n_X(1 - sr_X),$$

$$F_Y(s) \equiv n_Y(s-r_Y).$$

For any given domestic and foreign cash flow pair, $x + z_x$ and $y + z_y$, let x^* and y^* denote the reservation values of domestic and foreign assets in bankruptcy by foreign and domestic investors, respectively, for the firm with the lowest reorganization costs, that is, the pool-leading firm.

Proposition 2. Two currency swaps with n_X and n_Y positive, one issued to domestic investors and the other to foreign investors, with

$$\frac{1}{r_X} = r_Y = \frac{x^* - \bar{x}}{y^* - \bar{y}} \equiv r^*,$$

implement a security design with the feature

$$f_X(s) = \begin{cases} 1 & \text{when } x^* - \bar{x} > s(y^* - \bar{y}) \\ 0 & \text{otherwise.} \end{cases}$$

Proof. If $s < r_Y = (x^* - \bar{x})/(y^* - \bar{y})$, $F_X(s) > 0$ and $F_Y(s) < 0$ and in bank-ruptcy $f_X(s) = 1$ and $x^* - \bar{x} > y^* - \bar{y}$. The complementary case is symmetric. Q.E.D.

Propositions 1 and 2 describe securities that minimize adverse selection for the pool-leading firm. Optimally designed securities, according to proposition 1, have the feature

$$f_X(s) = \begin{cases} 1 & \text{when } x^* - \bar{x} > s(y^* - \bar{y}) \\ 0 & \text{otherwise,} \end{cases}$$

¹¹ Traditional analyses have ignored the default risk inherent in these contracts (although Cooper and Mello [1991] and Litzenberger [1992] are notable exceptions). Solnik (1990) notes that firms with default risk are charged a markup over the market swap prices that are determined (and quoted) using traditional methods that ignore default risk considerations. Because default risk is critical to our motivation for swaps, the model in this paper is designed to explain the currency swap market between corporations and banks, as opposed to the marked-to-market swap market, which operates more like a futures market, that almost exclusively involves money center banks and investment banks.

which, trivially, is generated by promised claims for which

$$f_X(s) = \begin{cases} \text{a positive number} & \text{when } x^* - \bar{x} > s(y^* - \bar{y}) \\ \text{a nonpositive number} & \text{otherwise,} \end{cases}$$

$$F_Y(s) = \begin{cases} \text{a nonpositive number} & \text{when } x^* - \bar{x} > s(y^* - \bar{y}) \\ \text{a positive number} & \text{otherwise.} \end{cases}$$

Proposition 2 illustrates that this feature of an optimal security is shared by two properly designed currency swaps. Since n_x and n_y are positive, these swaps pay domestic currency to domestic investors in exchange for foreign currency and vice versa for foreign investors. Moreover, since $1/r_x = r_y = r^*$, the swaps' rates of currency exchange at settlement are identical. This implies that the firm receives cash for entering into one of the two swaps and pays cash for entering into the other. The scale of the swaps is set so as to generate the required financing I. Such currency swaps possess this optimality feature irrespective of the ratio of the notional amounts of the two swaps n_x/n_y . The ratio n_x/n_y does, however, affect the probability of bankruptcy.

In general, optimal security design must simultaneously consider the probability of bankruptcy and the cash flow allocation rules in bankruptcy. The bankruptcy boundary is determined by the promised aggregate contractual payment $F = F_X(s) + F_Y(s)$, and a detailed description of optimal securities is not possible without making specific assumptions about the joint distribution of s and θ . Consequently, we cannot assert that the optimal F can be implemented with a pair of currency swaps. However, it is possible to show that currency swaps dominate debt without making further distributional assumptions.

Proposition 3. Any pair of debt contracts and any pairing of a currency swap with debt are dominated by a pair of currency swaps.

Proof. See the Appendix.

A general conclusion we draw from the model is that securities should be designed so as to minimize the penalty from adverse selection. As seen in figure 1, the cost of adverse selection is a weighted average of two lines—the horizontal line represents $x^* - \bar{x}$ and the 45-degree line represents $s(y^* - \bar{y})$ —corresponding to whether foreign or domestic investors, respectively, own the firm's assets in bankruptcy. It is obvious from figure 1 that two debt contracts, represented by the curved line in figure 1, do not minimize the cost of adverse selection. However, the currency swaps described in proposition 3 generate adverse selection costs at the minimum of the horizontal and the 45-degree lines. The exchange rate at which these two lines cross represents $(x^* - \bar{x})/(y^* - \bar{y}) \equiv r^*$.

IV. Related Literature

The role of standard securities such as debt and equity in the financing of real investments has been explored in a rapidly burgeoning literature on security design.¹² However, only recently has research in financial economics begun to address why seemingly trivial packagings of securities are so popular. One of the earliest papers on security design is by Allen and Gale (1988), who show that when it is costly to issue securities and when different groups of investors place different values on the same security, optimal securities split up the firm's state-contingent cash flows, allocating all cash flow in a given state to the investor who values it the most. Madan and Soubra (1991) introduce marketing costs into the Allen and Gale model and show that the sharing of cash flow in several states may be optimal in the presence of marketing costs. Ross (1989) also explores the implications of marketing costs and shows that financial innovation can reduce the costs of marketing securities; Pesendorfer (1995) generalizes this result to a general equilibrium framework. Boot and Thakor (1993) argue that selling multiple financial claims partitions a firm's total cash flow into "informationally sensitive" and "informationally insensitive" components. This encourages the acquisition of information by investors, which enhances firm revenue.

Similarly to our paper, Ohashi (1995) also considers a situation in which different investors have private information on different sources of uncertainty and argues that the introduction of properly designed securities may symmetrize the investors' information. Whether, in equilibrium, such securities are issued or not depends on the objective of the issuer. If the issuer is a volume-maximizing futures exchange, it may choose not to issue these securities or, as in Marin and Rahi (1996), limit the number of such securities issued. Consequently, the market may remain incomplete and equilibrium prices may not be fully revealing. In our model, it is the firm that decides to issue securities to minimize the impact of adverse selection on revenues raised.

Most related to our work are the models of Rahi (1996) and DeMarzo and Duffie (1999), which analyze the effect of adverse selection when issuing firms possess superior information. Rahi uses an exponential-normal rational expectations framework to study security design. He shows that within the joint normal class (which rules out risky debt securities), firms prefer to issue an information-free security, such as pure equity, that does not exploit the firm's information advantage. DeMarzo and Duffie study the impact of adverse selection on security design in a setting in which agents are risk-neutral. In their model, the signal from the quantity issued generates a downward-sloping convex

¹² See Allen and Gale (1994), Allen and Winton (1995), and Duffie and Rahi (1995) for recent surveys of literature on security design.

demand curve for the security. They then show that the design of securities such as collateralized mortgage obligations allows intermediaries to retain the portion of the security's return for which adverse selection, due to private information, is greatest, thereby mitigating the effect of adverse selection on revenues collected.

While prior research has developed general principles of security design and a handful of papers have tried to broadly link theoretical results on design to the existence of derivatives, we are not aware of any paper that specifically motivates currency swaps as an outcome of optimal security design.

V. Conclusion

This paper has argued that different groups of investors may be asymmetrically informed about different components of the cash flows generated by firms. For instance, a bank in a given country may be as informed as the firm about a multinational firm's costs and revenues in that country. However, the bank may be less informed than the firm about its costs and revenues from operations in other countries. In this case, firms face an adverse selection problem if foreign cash flows affect the value of the bank's claims on the firm. We have shown that issuing two distinct fixed-income securities to domestic and foreign investors allows investors to credibly transmit their private information to each other. While this facilitates efficient financing, it may not resolve all of the information advantage the firm has. To mitigate the adverse selection problem caused by unresolved information asymmetries, it is optimal to issue swap-like derivatives that have the feature that, in each bankrupt state, only one type of cash flow claimant—the one facing the least amount of adverse selection given the exchange rate—owns all of the firm's assets. Properly designed currency swaps ensure that, in bankruptcy, only one investor type—domestic or foreign—owes money when the other is owed money and that the investor that is owed money is precisely the one that faces the smaller adverse selection costs given the exchange rate. It is this feature of switching the ordering of priority, depending on the realized exchange rate in bankruptcy, that allows swaps to minimize adverse selection costs and dominate seemingly identical debt contracts.

Our model did not make use of risk aversion and hedging needs to motivate the purchase of derivative securities. This does not fundamentally alter our results and indeed strengthens many of them since we are able to show that firms may issue securities such as currency swaps purely on the basis of issue cost minimization considerations. We believe that this also sheds light on the behavior of many corporations that issue derivative securities of various types when a motive based purely on risk management considerations seems implausible.

Appendix

Proof of Remark 1

Substituting the given g functions in the pricing relations

$$P_X = g_X(P_Y|x + z_x),$$

$$P_{Y} = g_{Y}(P_{X}|y+z_{y}),$$

we obtain

$$P_{X} = \alpha_{X}(x + z_{x}) + \beta_{X}\bar{y} \left[\frac{P_{Y} - \alpha_{Y}\bar{x}(x + z_{x})}{\beta_{Y}} \right]$$
(A1)

and

$$P_{Y} = \alpha_{Y} \bar{x} \left[\frac{P_{X} - \bar{y}(y + z_{y})}{\alpha_{Y}} \right] + \beta_{Y}(y + z_{y}). \tag{A2}$$

Substituting (A2) into (A1) yields

$$P_{X} = \alpha_{X}(x+z_{x}) + \beta_{X}\bar{y}\left(y+z_{y} + \frac{\alpha_{Y}}{\beta_{Y}}\left\{\bar{x}\left[\frac{P_{X} - \beta_{X}\bar{y}\left(y+z_{y}\right)}{\alpha_{Y}}\right] - \bar{x}(x+z_{x})\right\}\right).$$

Subtracting $\alpha_X(x+z_x) + \beta_X \bar{y}(y+z_y)$ from both sides, we get

$$P_{X} - \left[\alpha_{X}(x+z_{x}) + \beta_{X}\bar{y}(y+z_{y})\right]$$

$$\begin{split} &=\beta_{X}\bigg[\bar{y}\bigg(y+z_{y}+\frac{\alpha_{Y}}{\beta_{Y}}\bigg\{\bar{x}\bigg[\frac{P_{X}-\beta_{X}\bar{y}\left(y+z_{y}\right)}{\alpha_{X}}\bigg]-\bar{x}(x+z_{y})\bigg\}\bigg)-\bar{y}(y+z_{y})\bigg]\\ &=\beta_{X}m_{Y}\frac{\alpha_{Y}}{\beta_{Y}}\bigg\{\bar{x}\bigg[\frac{P_{X}-\beta_{X}\bar{y}\left(y+z_{y}\right)}{\alpha_{X}}\bigg]-\bar{x}(x+z_{x})\bigg\}\\ &=\frac{\alpha_{Y}/\beta_{Y}}{\alpha_{X}/\beta_{X}}m_{X}m_{Y}\{P_{X}-\left[\alpha_{X}(x+z_{y})+\beta_{X}\bar{y}\left(y+z_{y}\right)\right]\}, \end{split}$$

where m_X and m_Y are given by the mean value theorem. Rearranging, we get

$$(1 - \phi')\{P_{x} - [\alpha_{x}(x + z_{y}) + \beta_{x}\bar{y}(y + z_{y})]\} = 0,$$

where

$$\phi' \equiv \frac{\alpha_{Y}/\beta_{Y}}{\alpha_{Y}/\beta_{Y}} m_{X} m_{Y}.$$

For nondegenerate cases, $\phi' \neq 1$. Therefore,

$$P_{X} = \alpha_{X}(x + z_{x}) + \beta_{X}\bar{y}(y + z_{y}),$$

which, upon substitution into (A2), implies that

$$P_{Y} = \alpha_{Y} \bar{x}(x + z_{x}) + \beta_{Y}(y + z_{y}).$$

Q.E.D.

Proof of Remark 2

We write down, as before, the individual rationality conditions,

$$K_X + \alpha_X(x + z_x) + \beta_X \bar{y}(y + z_y) = g_X(K_Y + \alpha_Y \bar{x}(x + z_x) + \beta_Y(y + z_y)|x, z_x),$$

$$K_{y} + \alpha_{y}\bar{x}(x+z_{y}) + \beta_{y}(y+z_{y}) = g_{y}(K_{x} + \alpha_{x}(x+z_{y}) + \beta_{x}\bar{y}(y+z_{y})|y, z_{y}),$$

for which the only solution is

$$g_X(P_Y|x+z_x) = K_X + \alpha_X(x+z_x) + \beta_X \bar{y} \left[\frac{P_Y - K_Y - \alpha_Y \bar{x}(x+z_x)}{\beta_Y} \right],$$

$$g_{Y}(P_{X}|y+z_{y}) = K_{Y} + \alpha_{y}\bar{x} \left[\frac{P_{X} - K_{X} - \beta_{x}\bar{y}(y+z_{y})}{\alpha_{y}} \right] + \beta_{Y}(y+z_{y}).$$

Substituting the given g functions in the pricing relations

$$P_X = g_X(P_Y|x + z_x),$$

$$P_{y} = g_{y}(P_{x}|y+z_{y})$$

and solving and simplifying as in the proof of remark 1, we obtain the desired result. Q.E.D.

Proof of Proposition 3

We show this by proving that a pair of currency swaps alone (not necessarily the pair described in proposition 1) can achieve the same bankruptcy boundary as the design involving debt and yet realize lower adverse selection costs for each realization of *s*.

We first note that a pair of swaps promising $n_x(1-sr_x)$ to domestic investors and $n_y(s-r_y)$ to foreign investors with the swap rate r_x or r_y or both set to zero (i.e., debt) has the same s-contingent bankruptcy boundary for any realization of θ as a pair of swaps with swap rates R_x and R_y that are closer to $1/r^*$ and r^* , respectively. The respective notional amounts for the alternative design that achieves this are

$$N_X = \frac{n_X (1 - r_X R_Y) + n_Y (R_Y - r_Y)}{1 - R_X R_Y}$$

and

$$N_{Y} = \frac{n_{Y}(1 - R_{X}r_{Y}) + n_{X}(R_{X} - r_{X})}{1 - R_{X}R_{Y}}.$$

There is now complete freedom to select R_x and R_y to reduce adverse selection. It follows that the equity value from bankrupt states is higher and the equity value from nonbankrupt states is the same. Hence, equity value is larger with the alternative design than with the proposed debt-based design.

For example, if $r_x = r_y = 0$ (two debt contracts), then, letting all expectations be conditional on bankruptcy, if

$$E\left[\frac{r^*-s}{n_x+n_ys}\right] > 0,$$

set $R_X = 0$ and $R_Y > 0$ but small. In this case, the portion of equity value that is affected by security design is

$$\begin{split} E[f_X(s)(r^*-s)] &= \Pr\left(s \leq R_y\right) E_{s \leq R_y}[r^*-s] \\ &+ \Pr\left(s > R_y\right) E_{s > R_y} \left[\frac{r^*-s}{n_x + n_y s}\right] (n_x + n_y R_y), \end{split}$$

which is increasing in R_y for small R_y . For the complementary case, set $R_y = 0$ and $R_x > 0$ but small. In this case,

$$E[f_X(s)(r^*-s)] = \Pr\left(s \le \frac{1}{R_y}\right) E_{s \le 1/R_X} \left[\frac{r^*-s}{n_y + n_y s}\right] [n_X(1-sR_y)],$$

which is increasing in R_x for small R_x . Q.E.D.

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