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Investment in organization capital

Bruce Ian Carlin*, Bhagwan Chowdhry, Mark J. Garmaise

UCLA, Anderson School of Management, United States

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ABSTRACT

We study a firm's investment in organization capital by analyzing a dynamic model of language development and intrafirm communication. We show that firms with richer internal language (i.e., more organization capital) have lower employee turnover, and higher diversity in skill and wages among incumbents who are promoted from within the firm. Our results also suggest that firms in rapidly changing industries are less likely to invest in organization capital, and are more likely to have high managerial turnover. Finally, our model shows that employment protection regulations lead to more investment in organization capital but less innovation.

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1. Introduction

Organization capital was defined by Prescott and Visschler (1980) to be the accumulation and use of private information to enhance production efficiency within a firm. This capital can be a significant source of firm value. For example, Atkeson and Kehoe (2005) estimate that the payments that arise from organization capital are more than one-third the size of those generated by physical assets, and represent more than 40% of the cash flows generated by all intangible assets in the US National Income and Product Accounts (NIPA). Despite its importance, however, studying the dynamics of investment in organization capital has not received much attention in the academic literature.

In this paper, we seek to fill this void by analyzing a theoretical model of organization capital in which we address the following questions: How does investment in organization capital affect investment in alternative sources of value creation? How does such investment affect the dispersion of executive compensation within a firm and a firm's propensity to promote from within in its senior

* Corresponding author. Address: UCLA, Anderson Graduate School of Management, 110 Westwood Plaza, Los Angeles, CA 90095, United States.

E-mail addresses: bruce.carlin@anderson.ucla.edu (B.I. Carlin), bhagwan@anderson.ucla.edu (B. Chowdhry), mark.garmaise@anderson.ucla.edu (M.J. Garmaise).

management ranks? How do rapidly changing industries and individual productivity affect such investments? How do employment protection regulations affect the evolution of organization capital?

To address these questions, we develop a model of organization capital, viewing it as a form of intrafirm language. This captures the idea that the value of organization capital depends on its being shared across managers and that it must be transmitted to the next generation of employees to be preserved. A firm's language summarizes informal work routines, convenient technical jargons, and a vocabulary of patterns remembered from past experiences. It creates complementarities among managers because it facilitates communication and enhanced production (Crémer, 1993). Indeed, as Arrow (1974) points out, one of the advantages of an organization is its ability to economize in communication through a common code. The richness of a firm's language, then, measures the breadth of the set of tasks covered by its communications channels, and is an important input to productivity within the firm.

In the paper, we first analyze a static model in which a firm exists for two periods and is then liquidated. Subsequently, we consider a continuous-time version in which language evolves dynamically and the firm's project choice is an endogenous choice variable. The latter allows us to contemplate how investment in organization capital affects the scope of the firm's activities.

In the static model, the firm is endowed with a language that covers some of the types of business opportunities that it may face. The firm has both junior and senior managers, and the key strategic decision it faces is how many incumbent managers to promote to senior management versus hiring from the outside. *Ex ante*, internal and external managers have the same expected productive quality, but the key difference between them is that incumbent managers may produce more efficiently by employing the firm's language. In choosing whether to retain an incumbent or to hire an external manager the firm must trade off the incumbent's valuable access to the firm's language against the potentially higher personal productivity of an outside replacement.

We show that the firm's equilibrium retention decision can be expressed as a threshold policy in which a quality is set above which incumbent employees are retained. Further, we show that greater richness of the firm's language leads to a lower minimum quality requirement for retention. As a result, in a firm with a richer language, there is less turnover and a greater difference between the highest and lowest paid incumbent senior managers. Empirically, then, the model predicts that firms with a richer language are more likely to exhibit decreased employee turnover, higher incumbent wage dispersion, and more frequent promotion of senior managers from within the organization. These findings arise from the fact that the firm increases the support from which managers are drawn (by lowering the required lower bound). We also show that managerial compensation rises more quickly in firms with more organization capital, as managers learn to exploit the internal language.

These empirical predictions require a proxy for the richness of a firm's language or organization capital. Eisfeldt and Papanikolaou (2010) have used SG&A expense as a proxy for organization capital. Other proxies might include the density of social networks that exists within an organization or the quality of the actual language used within intrafirm communications. Measuring the closeness of people within an organization or the commonality of language has become increasingly feasible, for instance by exploiting new technologies for the content analysis of intrafirm e-mail communications. Our model's predictions can be tested by empirically studying the relationship, for example, between the any of these proxies and the variability of a firm's managerial compensation, employee turnover, or its tendency to promote from within. We discuss these proxies in more detail in Section 2.3.

We then extend our analysis to consider a dynamic model in which language evolves endogenously through time. In our model, an infinitely-lived firm makes retention decisions and project choices in continuous-time. The key driver in this analysis is that the firm's project choice and retention and hiring policies not only determine the current productivity of the firm, but also its ongoing stock of organization capital that it may draw upon in the future.

As with any asset that affects production, accumulating organization capital requires investment and the allocation of resources. Optimal investment may require substituting away from alternative forms of productivity. We show that initially, to accumulate a richer language, the firm retains fewer incumbents and expands the scope of the firm's projects through active exploration. Then, in steady state, we show that intrafirm language and individual productivity are substitutes. When manager's ability to produce on their own is higher, it is relatively less attractive to sacrifice worker quality to

invest in language. The firm optimally hires from the outside when the importance of personal worker quality is higher. Moreover, the firm also optimally chooses to explore new projects. This arises because the value of engaging in old business lines is lower when there are fewer incumbents. Finally, our model also implies that firm should invest less in language when there is a greater risk of technological shocks.

These findings also give rise to novel empirical implications. For example, firms in industries with less dense social networks will invest less in language and will explore new projects more. Additionally, in such industries, we should see higher managerial turnover, less wage dispersion among incumbent employees, and fewer promotions to upper level management from within the organization. Firms with higher risk of technological obsolescence (which likely includes many high growth industries) should experience more employee turnover, and less wage dispersion among incumbents and less frequent promotion to upper-level management from within its ranks.

Finally, our model also has implications for employment protection regulations (EPRs) that make it expensive to fire incumbents and hire new employees. We show that such policies lead to more employee retention, which in turn causes firms to invest more in organization capital and less in innovation (i.e., less new project exploration). As we discuss in the paper, this leads to cross-sectional implications that may be tested based on variation in the strictness of national EPRs (e.g., [Bozkaya and Kerr, 2008](#)).

Our model contributes to the literature on intrafirm communication, and is distinctive in several respects. Many studies take the information in the firm as given, and analyze the optimal way to employ that capital. For example, [Bolton and Dewatripoint \(1994\)](#) analyze the costs and benefits of centralizing information networks within firms, given an exogenous flow of new information. [Harris and Raviv \(2002\)](#) analyze the formation of optimal organization design, given that information as a scarce is exogenously given. [Crémer et al. \(2007\)](#) study the development of optimal codes within organizations to employ the capital from information in the most efficient way. In contrast, in our paper, we focus on the nature of investment in language, and study the effect that this has on the firm's labor management policy. Intrafirm communication is explicitly promoted by the firm's decisions to retain incumbent managers and these decisions also influence employee compensation. Our model therefore describes endogenous differences in quality diversity and wage dispersion in the firm.

Our work is also related to recent work by [Dessí \(2008\)](#), who explores how cultural transmission arises in societies and countries. [Dessí \(2008\)](#) characterizes the socially optimal transmission of culture (e.g. via media and/or education) and analyzes how biases evolve through time. Our focus is different: using a transmission function that depends on the presence of incumbents, we consider how firms optimally invest in language, retain employees, choose projects, and set wages.

The rest of the paper is organized as follows: Section 2 poses and solves the two-stage version of our model. There we characterize the firm's retention policies, and their ramifications for diversity and wage dispersion. In Section 3, we analyze the continuous-time model and characterize the firm's investment in language. Section 4 provides some concluding remarks and considers how our analysis might apply to merger integration. Appendix A contains all of the proofs.

2. Production and organization capital

We begin by considering a firm that employs managers for two periods and is then liquidated. We characterize the firm's hiring and firing decisions, employment compensation, investment in organization capital through its decision to retain its incumbent managers, and diversity of skill levels in the organization. In Section 3, we will embed this two-stage interaction into a continuous-time model in which an infinitely-lived firm makes employment and project choices to maximize its lifetime value.

2.1. Language and production

The timing of the game is outlined in [Fig. 1](#). In the first period ($t = 1$), a group of new managers are hired and are considered to be "junior" to an existing set of "senior" managers. The firm consists of $2N$ workers, N seniors and N juniors. During the first period, junior managers assist the seniors in

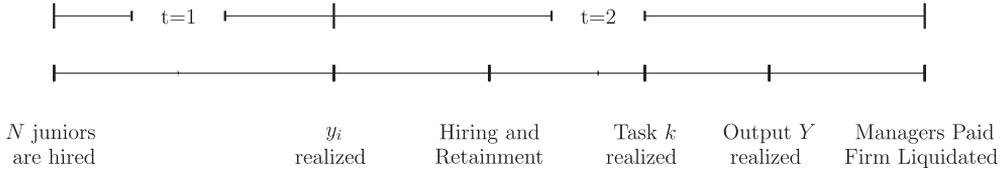


Fig. 1. At the beginning of $t = 1$, N junior managers are hired. At the beginning of $t = 2$, each incumbent manager's quality y_i is realized. Based on the firm's scope K and the firm's organization capital (language) L , the firm chooses which incumbent managers to promote and which ones to replace with outsiders. After this, the project k is realized and the senior management produces Y in aggregate based on whether $k \in \mathcal{L}$ or not. Finally, senior managers are compensated and the firm is liquidated.

producing the output of the firm, but do not create value independently from senior management. Their type (quality) \tilde{y} is unknown to both the firm and the manager, and does not affect value creation for the firm when they are junior. We assume that for each manager, \tilde{y} is distributed according to a twice continuously differentiable, strictly increasing, log-concave distribution function F over the support $[0, \mathcal{Y}]$, where $E[\tilde{y}] = \bar{y}$. As such, our scope of analysis here is fairly wide, including common distributions such as the uniform, truncated normal and exponential distributions.

At the beginning of the second period ($t = 2$), the types of all incumbent junior managers are revealed to both managers and firms. Existing senior managers retire and some junior incumbents are promoted to form the new senior management, depending on the firm's retention policy and the willingness of management to remain with the firm (to be specified shortly). Define n as the number of incumbent junior managers who are promoted to senior management, and $a = \frac{n}{N}$ as the fraction of senior managers who are promoted from within the firm. Based on this, $(1 - a)N = N - n$ is the number of outsiders hired in the second period. We assume in what follows that the firm's physical capital (real assets) is fixed, in order to analyze the firm's investment in organization capital through its retention decision.

In any given period, the firm is faced with carrying out a project $k \in \mathcal{K} = \{1, 2, \dots, K\}$ for some $K > 1$ that arises randomly according to a uniform random variable that is i.i.d. across time periods. These projects may be thought of as representing different strategic initiatives or market opportunities.

Each senior manager i produces an individual output y_i that directly depends on his own quality. Given the number of incumbent managers, all managers may increase their production through language. This type of production enhancing communication is only possible, though, when the language includes the particular project at hand. Let $\mathcal{L} \subseteq \mathcal{K}$ denote the set of all projects that are part of the firm's language and let L denote the number of such projects. As such, L provides a measure of the level of the firm's organization capital, and by construction, the probability that any task $k \in \mathcal{L}$ is $\frac{L}{K}$. We describe the evolution of \mathcal{L} and endogenize the firm's project choice in the dynamic game in Section 3.

If the given task is part of the language, then the incumbent managers will foster more communication among all senior managers. In total, language increases the total productivity of all managers and increases production by $G(a)$, where $G(\cdot)$ is an increasing function.¹

Production within the firm may then be calculated as

$$Y = \sum_{i=1}^N y_i + \Psi_{k \in \mathcal{L}} G(a),$$

where Ψ is an indicator function that is equal to one if $k \in \mathcal{L}$ and zero otherwise. The expected productivity (from real assets) of incumbent managers will depend in equilibrium on L, K , and a . We denote this dependence as $E[\tilde{y}_T | L, K, a]$ and will calculate this quantity shortly. Assuming that all outside

¹ DeMarzo et al. (2003) and Garicano (2000) provide other models of communication in organizations. Crémer et al. (2007) analyze the optimal design of a code within an organization and consider implications for integration across different groups of agents. Two specific examples of such a language based on internal jargon, shared values and common experiences are found in the workings of the consulting firm McKinsey (as described in *The McKinsey Mind* by Raisel and Friga (2002)) and the Walt Disney company (as described in Beer (1996)).

managers are randomly chosen from the same distribution F , we can compute the firm’s expected production as

$$E[Y|L, K, a] = aNE[\bar{y}_T|L, K, a] + \frac{L}{K}G(a) + (1 - a)N\bar{y}. \tag{1}$$

The sum of the first and second terms is the expected productivity that arises from incumbents seniors, whereas the third term reflects the expected contribution from new seniors from the outside.

Compensation within the firm proceeds as follows. Every senior manager has a reservation utility of \bar{u} , which does not depend on his particular realization of \bar{y} .² While we do not model the determinants of \bar{u} explicitly, its value is common to all managers and reflects conditions in the labor market such as competition, market power, and differentiation in skill. This implies that \bar{u} is expected to be higher when the skills that particular employees provide are difficult to replace. As such, participation by any particular manager will occur if and only if the firm meets their participation constraint.

Compensation for each manager is determined according to a Nash bargaining game in which the firm pays the manager a fraction $\theta \in (0, 1)$ of the value of the production in exchange for the remainder. Here, we follow Radner and Van Zandt (1992) and Garicano (2000) in that we set incentives within the firm aside and focus on the investment in language by the firm.

The payoff to a senior manager is computed as

$$\pi_i = \max\left\{\bar{u}, \theta \left[y_i + \Psi_{k \in \mathcal{L}} \frac{G(a)}{N} \right] \right\}. \tag{2}$$

As such, each manager gets a fraction of their own productivity and an equal share of the value that is created by language in the organization.³

When $\theta \left[y_i + \Psi_{k \in \mathcal{L}} \frac{G(a)}{N} \right] < \bar{u}$, the firm promises to supplement the compensation with cash to meet the participation constraint. By inspection, the higher the quality of a manager and the higher the potential for complementarities, the lower the need for cash to supplement an employee’s pay. For simplicity, we assume that $\bar{u} = 0$ so that cash is never required as a supplement and no manager would voluntarily quit if the firm promoted him. We do this, though, keeping in mind that if $\bar{u} > 0$ that an employee would quit if they were not offered enough compensation to remain at the firm. Since the employment offer, in our model, is at the firm’s discretion, it is without loss of generality to consider that $\bar{u} = 0$ and that the firm chooses whether to retain or fire certain managers.

With this in mind, we denote the decision to retain incumbent managers by $d_R \in \{0, 1\}^N$, where $d_R(i) = 0$ means that the i th incumbent is fired and $d_R(i) = 1$ means that he is retained. Thus, $n = \sum_i d_R(i)$ and the expected profit to the firm is computed as

$$\Pi(L, d_R) = (1 - \theta) \left[aNE[\bar{y}_T|L, K, a] + \frac{L}{K}G(a) + (1 - a)N\bar{y} \right]. \tag{3}$$

We are now ready to solve and characterize the two-stage game.

2.2. Equilibrium characterization

The object of interest that is determined in equilibrium is the fraction of incumbent managers a the firm wishes to promote, which will then determine the number of managers to hire from outside of the firm. At the time that this decision is made, the particular task k has not been observed, and therefore the firm does not know whether the firm’s organization capital will be put to good use in enhancing production. The firm does, however, observe the quality levels of its incumbent managers and uses

² We make this assumption for analytical simplicity. Relaxing this assumption will make the firm’s retention decision more realistic, but will also make the analysis more complicated, which will not add much to studying their investment in organization capital.

³ Our production sharing scheme describes an arrangement in which managers receive some compensation based on their own revealed quality and a fraction of firm level bonus that is divided equally among all N employees. Following Stole and Zwiebel (1996), one might argue that a manager’s compensation must depend on his or her *marginal* contribution to the firm’s total output. As long as G is concave, such a compensation scheme would be feasible in our model, and our central results would be preserved.

this to make a decision regarding retention. Not surprisingly, this leads to an optimal threshold policy in equilibrium, which we characterize in the following proposition.

Proposition 1. *There exists an optimal threshold productivity level y^* such that the firm retains all incumbent managers with $y_i \geq y^*$ and replaces the rest with outsiders. For all $L > 0$, $y^* < \bar{y}$. The threshold y^* is strictly decreasing in $\frac{L}{K}$.*

The expected quality of senior managers is strictly decreasing in $\frac{L}{K}$.

To understand the intuition of Proposition 1, consider first that $L = 0$, or that there is no potential for incumbents to have an advantage over outsiders who join the firm. In this case, since the firm can gain \bar{y} in expectation from hiring outsiders, it will only retain incumbent managers with higher quality, that is, $y_i \geq \bar{y}$. When $L > 0$, there is more to gain from keeping incumbents since there is a positive probability ($\frac{L}{K}$) that organization capital can be put to good use. When the firm considers whether to retain one more incumbent, they compare the expected productivity from the incumbent with the expected productivity from an outsider. Specifically, they retain the incumbent if and only if

$$y_i \geq \bar{y} - \frac{L}{K} \Delta G(a), \tag{4}$$

where $\Delta G(a) = G(\frac{a}{N}) - G(\frac{a-1}{N})$. By inspection, the higher the organization capital, L , relative to the span of opportunities that the firm may be confronted with, K , the smaller is the employee turnover.

Proposition 1 has several empirical implications. First, firms with larger organization capital will experience lower turnover. Indeed, with higher L , firms should be more averse to replacing managers with outsiders because employees who know the firm’s language produce effectively within the firm and provide advice for other employees. Second, firms with higher organization capital should have more frequent insider CEO succession. Denis and Denis (1995) find that only 15% of firm top executive appointments are made to external candidates, which underscores the probable importance of organization capital in most firms. Consistent with this is the observation by Parrino (1997) that outside succession occurs most frequently in commodity industries in which organization capital is likely to be less important.

Proposition 1 also implies that the average quality of managers should decrease as organization capital L increases. As L rises, the bar that must be met to be promoted decreases. We can calculate the average quality of incumbent managers as

$$E[\bar{y}_I | L, K, a] = \frac{\int_{y^*}^{\bar{y}} y dF(y)}{1 - F(y^*)},$$

which implies that the average quality of all managers in the firm is

$$E[y_i] = a \frac{\int_{y^*}^{\bar{y}} y dF(y)}{1 - F(y^*)} + (1 - a)\bar{y}.$$

As we show in Appendix A, $E[y_i]$ is decreasing in L and increasing in K .

This has two important implications. The first is that organization capital improves productivity but is also associated with lower intrinsic manager qualities. For a set of managers with given qualities, higher communication within the firm due to the presence of incumbents leads to more sharing of ideas and greater productivity. At the same time, as language becomes more important within the firm, the average quality of employees decreases because the bar that is required for promotion is lower. Therefore, when the firm operates, it must take into account both forces and weigh the tradeoffs that organization capital introduces.

The second implication is that organization capital affects the diversity of managers within the firm. As L increases, the support from which incumbents are drawn increases, which affects the difference in quality between employees. This, in turn, affects the expected amount of wage dispersion among incumbents that exists in the organization. The following proposition characterizes the relationship between language and diversity and wage dispersion.

Proposition 2. *The expected diversity and wage dispersion among incumbent senior managers is strictly increasing in $\frac{L}{K}$.*

According to Proposition 2, for incumbent senior managers, the variance of quality levels and the variance of expected wages increase as language plays a larger part within the firm. To gain intuition for this result, consider two levels of $\frac{L}{K}$, namely $\frac{L_1}{K_1}$ and $\frac{L_2}{K_2}$, such that $\frac{L_1}{K_1} > \frac{L_2}{K_2}$. By (4), it is clear that $y^*(\frac{L_1}{K_1}) < y^*(\frac{L_2}{K_2})$. The firm chooses incumbent managers from two distributions, which we can call $H_1(\tilde{y}_1)$ and $H_2(\tilde{y}_2)$, where \tilde{y}_1 and \tilde{y}_2 are random variables as defined in the text. The key observation to be made is that the distribution $H_2(y)$ is a truncation of $H_1(y)$. Therefore, it follows that $Var(\tilde{y}_2) \leq Var(\tilde{y}_1)$, or that the variance of quality among incumbents is higher when language is more important to the firm. This leads to more diversity among incumbents in the organization. Finally, since wages are linked to performance (through the fraction θ), as language becomes more important in the organization, this leads to a higher variance of wages among incumbent managers. Empirically, then, Proposition 2 implies that the difference between the top incumbent wage earner and the average incumbent in a firm should be higher when organization capital is higher.

It is important to point out that junior managers would prefer to work at firms with higher language, holding all else equal. The following proposition formalizes this result.

Proposition 3. *The expected payoff is higher for incumbents who begin the second period in firms with greater organization capital.*

Proposition 3 may be appreciated as follows. Before a junior manager becomes informed about his type, he may compute the expected wage that he will receive at the firm in the second period. If his quality turns out to be $\tilde{y} < y^*$, then he will not be retained and will earn zero. If $\tilde{y} \geq y^*$, then he will be expected to earn $E[\pi | \tilde{y} \geq y^*]$. Therefore, his expected wage is

$$E[\pi] = Pr(\tilde{y} \geq y^*)E[\pi | \tilde{y} \geq y^*] = [1 - F(y^*)] \frac{\theta \int_{y^*(L)}^y (y + \frac{L}{K}G(a(L)))dF(y)}{1 - F(y^*)}$$

or

$$E[\pi] = \theta \int_{y^*(L)}^y (y + \frac{L}{K}G(a(L)))dF(y),$$

which is increasing in L . Therefore, as the language increases within a firm, junior managers have a higher expected wage in the future.

A simple extension of our model might set wages for juniors such that the total expected two-period compensation is equal to some reservation value. In such a model, salaries for juniors would be lower in firms with large organization capital, while the seniors in these firms would be well paid. In such a model, our theory would predict that the gap in compensation between juniors and seniors would be greater in firms with large organization capital. In other words, firms with a strong organization capital would exhibit greater steepness in their managerial wage profiles.

2.3. Empirical implications

Many of the empirical predictions that follow from Propositions 1–3 are novel and have yet to be tested. It is worth discussing, however, how such implications might be analyzed. Testing our model would require a good proxy for organizational capital. One possibility is using a firm’s Selling, General, and Administrative (SG&A) expense, as in Eisfeldt and Papanikolaou (2010). Indeed, as pointed out by Lev and Radhakrishnan (2004), SG&A expense includes most of the expenditures that generate organization capital.

Another approach might be to develop a good proxies for language and scope. One candidate for language is the density of social networks and the quality of relationships within those networks (e.g. Burt and Schott, 1985; Raider and Krackhardt, 2001). Indeed, the more intertwined managers are within an organization, both at work and outside of the firm, the more readily do they engender language and observe informal work routines. Along the same lines, another direct measure of a firm’s

language is the importance and frequency of employee interactions as reported by the employees themselves. While collecting this data may be cumbersome, there is an increasing number of intrafirm studies on the value of communication (e.g. Ichniowski and Shaw, 2003). In fact, given the increased reliance of firms on written emails, this difficulty of collecting relevant data has decreased since written communication may be analyzed by content analysis (e.g. Holsti, 1969; Tetlock, 2007).

Using these proxies, then, our model predicts that a firm's SG&A and the density of intrafirm social networks should be positively correlated with wage steepness, compensation dispersion among incumbents, and internal CEO promotion, and should be negatively correlated with employee turnover. Along the same lines, firms in less volatile industries should also have more wage steepness, more dispersed wages among incumbents, and less employee turnover.

3. Investment in organization capital

In practice, firms cultivate organization capital over time, which requires investment and the allocation of resources. Moreover, firms choose the scope of their business and their activity in new ventures endogenously based on their investment in organizational capital and retention decisions. In this section, we consider a firm's project choice and optimal investment in organization capital through its employee retention policy, and characterize its evolution over time. We model the firm as an infinitely-lived entity and embed the static model of Section 1 into a continuous-time setting. The key driver in this analysis is that the firm's project choice and retention and hiring policies not only determine the current productivity of the firm, but also its ongoing stock of organization capital that it may draw upon in the future.

3.1. Employee retention and project choice

The firm makes two choices at each time $t \in [0, \infty)$, which evolves continuously. The first is the fraction of incumbents that the firm retains, $a_t \in [0, 1]$. As before, the expected productivity of all workers is \bar{y} and the firm trades off between externalities that arise because of language and the inherent productivity of workers. For analytic ease, we assume that $\bar{y} \sim U(0, 2\bar{y})$. As before, we also assume that productivity of incumbent managers is revealed when they work inside the firm. The average productivity of the retained incumbent managers is $\bar{y}(a_t) = (2 - a_t)\bar{y}$, when the fraction a_t are retained by the firm.

In essence, in this continuous-time model the length of the period in the static model is reduced to a limit of zero. The choice of a_t represents the instantaneous retention decision. As in the static model, greater retention leads to lower average productivity, as lower-quality types are kept on staff. Greater retention therefore also results in increased wage dispersion among incumbents, in the same mechanism observed in the static model.

The second choice that the firm makes is the fraction of projects it will explore that are outside of the firm's current language or expertise, $v_t \in [0, 1]$. The firm continues to choose $(1 - v_t)$ projects from the set for which it has already developed the language. When the firm explores new projects, it develops new language capabilities, but pays a price as old ones become stale and deteriorate.

The dynamic model therefore allows us to explore a new element of organization capital that is absent in the static setting: here firms choose whether or not to build greater organization capital. The benefit from choosing new projects is that the firm masters the language for new tasks. The costs are that the firm is less effective at unfamiliar projects (as in the static model) and experiences a depreciation in its existing organizational capital if it is not utilized.

Specifically, the amount of language $l_t = \frac{L}{K}$ is our state variable of interest and evolves over time based on the firm's choices of a_t and v_t . We assume that at time zero, $l_0 = 0$ and that language evolves at the following rate per unit of time:

$$g(a_t, v_t, l_t) = v_t - \{1 - a_t(1 - v_t)\}l_t. \quad (5)$$

The evolution equation captures the fact that when the firm does a larger fraction of new projects, v_t , the language increases. The language depreciates at a rate that is proportional to the level of language

l_t , and is smaller when there are more incumbents (high a_t) and when the firm takes more projects for which it has already developed a language (lower v_t). If, in fact, $a_t = 1 - v_t = 1$, the language remains intact and does not depreciate at all. If $a_t < 1$ or $(1 - v_t) < 1$, the existing language does depreciate.

At each instant, the rate at which the firm produces is

$$f(a_t, v_t, l_t) = a_t \bar{y}(a_t) + (1 - a_t) \bar{y} + a_t(1 - v_t)l_t. \tag{6}$$

The first two terms capture the expected individual productivity of their two types of managers, given the fraction of incumbents they retain. The last term is the extra benefit the firm receives from incumbent managers and language. Since the production from language is normalized to be proportional to the level of language l_t , the term \bar{y} captures the relative importance of individual productivity in relation to the productivity caused by language. So, a low value of \bar{y} represents firms where language based production is relatively more important.

The firm’s problem may then be posed as

$$V = (1 - \theta) \max_{a(l), v(l)} \int_0^\infty f(a_t, v_t, l_t) e^{-rt} dt \tag{7}$$

subject to

$$dl_t = g(a_t, v_t, L_t) dt.$$

The firm solves the following Hamilton–Jacobi–Bellman equation

$$0 = \max_{a(l), v(l)} \{f(a, v, l) + g(a, v, l)V_l - rV\}. \tag{8}$$

The variable r represents both the discount rate per unit of time and the risk to this firm that its product market will be subject to a technological shock that reduces its profits to zero. So, for firms in rapidly changing industries, the discount rate r will be greater.

3.2. Equilibrium characterization

The next proposition characterizes the firm’s initial choices of a_0 and v_0 , as well as their choices early on.

Proposition 4. *In the beginning (i.e., small t), the firm chooses $a = \frac{1}{2}$ and $v = 1$, during which time it will acquire language l_t at a fast rate. The firm’s value V_t is convex in l_t during this initial period.*

Proposition 4 implies that initially there are increasing returns to an investment in firm language. As such, the firm optimally explores the maximum number of new projects possible and only retains half of its incumbent managers. In the early stages of growth, enjoying the complementarities from having a stock of language are less important than accumulating more organizational capital.

Not surprisingly, as language grows and l_t becomes sufficiently high, the optimum value of a_t and v_t may not be as extreme. We would expect there to be an interior solution. The first order conditions for an interior maximum are

$$f_a(a, v, l) + g_a(a, v, l)V_l = (1 - 2a)\bar{y} + (1 - v)l[1 + V_l] = 0, \tag{9}$$

$$f_v(a, v, l) + g_v(a, v, l)V_l = -al + (1 - al)V_l = 0. \tag{10}$$

Notice that the second order condition for an interior maximum is also satisfied because

$$f_{aa}(a, v, l) = -2\bar{y} < 0, \\ g_{aa}(a, v, l) = g_{vv}(a, v, l) = 0.$$

Substituting the first-order conditions above in the Hamilton–Jacobi–Bellman equation results in a non-linear differential equation that cannot be characterized analytically and is complicated to solve numerically. However, we characterize the solution in steady-state as follows.

Let a^s , v^s and l^s denote the optimal values in steady state. The steady-state is reached when the language stays constant in equilibrium. This will be the case when $dl_t = 0$ or when $g(a^s, v^s) = 0$. This yields

$$a^s = \frac{1 - v^s}{1 - v^s \frac{F}{f}}. \tag{11}$$

Differentiating the Hamilton–Jacobi–Bellman equation with respect to l , and substituting the first order conditions for an interior maximum yields

$$f_l(a^s, v^s) + g_l(a^s, v^s)V_l + g(a^s, v^s)V_{ll} - rV_l = 0.$$

In steady-state, $g(a^s, v^s) = 0$, and thus

$$f_l(a^s, v^s) + g_l(a^s, v^s)V_l - rV_l = 0.$$

The equation above can be rewritten as

$$a^s(1 - v^s) - \{1 - a^s(1 - v^s)\}V_l - rV_l = 0,$$

or

$$V_l = \frac{a^s(1 - v^s)}{1 + r - a^s(1 - v^s)}. \tag{12}$$

The first order condition (10) in steady state becomes

$$V_l = \frac{a^s f^s}{1 - a^s f^s}. \tag{13}$$

Eliminating V_l from the two equations above and simplifying, we get

$$f^s = \frac{1 - v^s}{1 + r}. \tag{14}$$

Substituting V_l from (13) into (9) and rearranging, we get

$$(1 - 2a^s)\bar{y} + \frac{(1 - v^s)f^s}{(1 - a^s f^s)} = 0. \tag{15}$$

The steady-state equilibrium (a^s, v^s, f^s) is thus characterized by three Eqs. (11), (14) and (15).

Proposition 5 (*Investment in Organization Capital*). *In the steady state solution (a^s, v^s, f^s) to the problem in (8), the following comparative statics hold:*

- (i) f^s is decreasing in \bar{y} and r .
- (ii) a^s is decreasing in \bar{y} and r .
- (iii) v^s is increasing in \bar{y} .

Proposition 5 adds to our characterization of investment in organization capital in several ways. First, language and individual productivity are substitutes. As the importance of individual productivity (i.e., \bar{y}) increases, it makes it relatively less attractive to the firm to sacrifice worker quality to invest in language. For that reason, a^s is decreasing in \bar{y} : the firm is less willing to retain low quality incumbents when the relative importance of individual production is greater. As a result, there is less language retention, and the steady-state level of language f^s is also declining in \bar{y} . Moreover, the firm will also choose to explore new projects more (i.e., v^s increases) as language becomes less relevant, since the benefits from pursuing existing projects are smaller when fewer incumbents are retained.

The second central intuition is that increasing the risk of a technological shock (i.e., increasing r), leads to a reduction in the retention of incumbents. Personally inefficient incumbents may be retained to protect a firm’s language, but when future profits become less likely to be realized, the firm is unwilling to accept lower production today in exchange for uncertain future gains. This reduction in retention also leads to a lower steady-state level of language.

The impact of technical obsolescence risk on the firm’s propensity to undertake new projects is more nuanced, and this relationship cannot be signed, in general. On the one hand, higher r leads to less retention, which gives the firm less of an incentive to exploit existing projects. On the other hand, one of the advantages of exploring new business lines is that it expands the scope of the firm and builds language. This benefit, however, is less important when r is high. In some sense, therefore, both new and existing projects become less attractive when r increases, and it is not clear which becomes relatively more appealing.

The findings outlined in Proposition 5 give rise to novel cross-sectional implications. For example, firms in industries with less dense social networks will experience more managerial turnover and will explore new projects more. Additionally, in such industries, we should see less wage dispersion among incumbents, and fewer promotions to upper level management from within the organization.

It is reasonable to view r as a proxy for rates of industry change or turnover. Proposition 5 then implies that firms will invest less in language when industries are rapidly evolving. This suggests that firms in high growth industries will have higher employee turnover and less promotion to upper level management from within its ranks.

3.3. Implications for employment protection regulations

There are differences across jurisdictions in the degree to which firms are subject to employment protection regulations (EPRs). EPRs make it expensive for firms to fire workers and can thereby impede labor mobility (Blanchard and Portugal, 2001 and Autor et al., 2007). We model EPRs as a cost c associated with firing incumbent managers and replacing them with outsiders. The firm’s production is therefore modified to have an additional term as follows

$$f(a_t, v_t, l_t) = a_t \bar{y}(a_t) + (1 - a_t) \bar{y} + a_t (1 - v_t) l_t - c(1 - a_t). \tag{16}$$

The steady-state equilibrium is now characterized by (11), (14) as before but (15) is modified to

$$(1 - 2a^s) \bar{y} + \frac{(1 - v^s) \bar{l}^s}{(1 - a^s \bar{l}^s)} + c = 0. \tag{17}$$

Proposition 6. *Costly outside hiring:*

- (i) \bar{l}^s is increasing in c
- (ii) a^s is increasing in c
- (iii) v^s is decreasing in c

Proposition 6 indicates, unsurprisingly, that in jurisdictions with tougher EPRs (higher values of c) there will be more retention of incumbents. What is more interesting is that our model also shows that stronger EPRs lead to greater language and less exploration of new projects. A strong EPR regime favors the retention of existing managers. When these managers are in place, the firm does better to exploit its existing language by pursuing well-understood projects. The net result is a conservative firm with a relatively rich language that it protects by retaining its incumbents.

It is well-known that EPRs are much higher in continental Europe than in the US (Bozkaya and Kerr, 2008). From a broad perspective, therefore, Proposition 6 suggests that firms in continental Europe should tend to invest relatively more often in well-understood business lines, while US firms are predicted to invest in new projects more frequently. The model’s prediction linking tougher EPRs to less technological innovation could also be tested using more detailed country- or state-level data.

4. Discussion

Our description of organization capital as an internal language of the firm meets two important criteria. First, the firm’s language cannot be carried from the firm by departing employees. Second, the firm’s language is difficult to imitate.

It is important that organization capital be tied to the firm, for otherwise it is difficult to explain why employees and assets must stay together. A coordinated *en masse* defection by all employees

can typically be ruled out because of the coordination difficulty discussed in Klein (1988). Hart (1989) argues that a threat of simultaneous defection by all employees can be still be credible unless some physical assets are involved. In our model, the language of the firm is used to describe the firm's particular tasks and is therefore linked to the precise equipment and production arrangement used by the firm.

For organization capital to have value, it must also be costly for competitors to replicate (Rumelt, 1987). Inimitability, in our model, arises because the knowledge of a firm's language is possessed by the firm's managers and is not accessible to rivals. Moreover, the language is related to the particular way the firm is structured. In our model, learning and experience are necessary for the development of each firm's language.⁴ These features combine to make the acquisition of language within the firm time-consuming and difficult.

Based on our model, firms with richer languages retain more employees and are therefore more likely to promote senior managers from within. We demonstrate that firms with more organization capital should exhibit greater variability in the compensation levels of their managers who were promoted from within the firm. We also prove that compensation rises more quickly over time in firms with richer languages. As we highlight in the paper, our model yields novel testable implications linking the density of firms' social networks to central issues in corporate finance including firms' market values, compensation practices and merger strategies. Recent empirical evidence has bolstered the view that organization capital plays a significant role in production (Atekson and Kehoe, 2005). It is therefore important to broaden our understanding of how it creates value within the firm.

Finally, while not formally explored in the paper, our model provides a rationale for value-creating mergers. If one firm has developed a very rich language, this language may usefully be adopted by other firms performing similar tasks. Consider a merger between two firms of roughly the same size, one with significant organization capital and a second with very little organization capital. Assuming there is some overlap between the tasks of the two firms, the juniors at the newly merged firm will likely learn the rich language of the high organization capital firm.⁵ The value created by a merger is equal to the value of the merged firm minus the values of the two constituent firms. Since the organization capital of the firm whose language is not adopted is simply lost, the value created by the merger is greatest when one of the constituent firms has a lot of organization capital and the second has very little.

In general, though, mergers will indeed reduce the probability of organization capital transmission. Exporting a rich language via a merger can be beneficial, but also presents the risk of loss. It is not the case that firms with large organization capital should engage in unbridled expansion.

Appendix A

Proof of Proposition 1. The firm will retain an incumbent manager if the value they create is expected to be higher than the quality of an outside manager. The firm will choose to retain one additional incumbent if their quality satisfies

$$y_i + \frac{L}{K} \Delta G(a) \geq \bar{y},$$

or if

$$y_i \geq y^*,$$

where $y^* \equiv \bar{y} - \frac{L}{K} \Delta G(a)$. By simple differentiation, $\frac{\partial y^*}{\partial L} < 0$ and $\frac{\partial y^*}{\partial K} > 0$.

⁴ Bahk and Gort (1993) empirically document, using individual plant data for one sample of 15 industries and another sample of 41 industries, that "organization learning appears to continue over a period of at least 10 years following the birth of a plant."

⁵ Crémer et al. (2007) analyze a model in which two firms may choose to adopt a common code at some cost.

The expected quality of a senior manager is computed as

$$E[y_i] = \frac{1}{N} \left[E[n] \int_{y^*}^y y dF(y) + (N - E[n])\bar{y} \right]. \tag{A.1}$$

In our model, $E[n] = Np$ where $p = 1 - F(y^*)$. Substituting this into (A.1) yields,

$$E[y_i] = \int_{y^*}^y y dF(y) + F(y^*)\bar{y}.$$

Differentiation using Leibnitz' rule yields

$$\frac{\partial E[y_i]}{\partial y^*} = -y^*f(y^* + \bar{y}f(y^*)) > 0.$$

Since $\frac{\partial y^*}{\partial L} < 0$, $\frac{\partial E[y_i]}{\partial L} < 0$. Also, since $\frac{\partial y^*}{\partial K} > 0$, $\frac{\partial E[y_i]}{\partial K} > 0$. \square

Proof of Proposition 2. Considering two values $z_1 = \frac{L_1}{K_1}$ and $z_2 = \frac{L_2}{K_2}$, such that $z_1 > z_2$. By (4), it is clear that $y^*(z_1) < y^*(z_2)$. Define the two distributions from which the firm chooses n incumbent seniors from as $H_1(\tilde{y}_1)$ and $H_2(\tilde{y}_2)$, where \tilde{y}_1 and \tilde{y}_2 are random variables as defined in the text. By inspection, the distribution $H_2(y)$ is a truncation of $H_1(y)$. Since $F(\cdot)$ is log-concave, and that $H_1(y)$ and $H_2(y)$ are both truncations of $F(\cdot)$, it follows that $Var(\tilde{y}_2) \leq Var(\tilde{y}_1)$. (See Burdett, 1996 and An, 1998). Finally, since all incumbent managers receive $\frac{\theta G(n)}{N}$, the difference in their wages depends on their dispersion in quality. This implies that the variance of wages increases as $\frac{L}{K}$ rises. \square

Proof of Proposition 3. An incumbent who is not retained receives a payoff of $\bar{u} = 0$, so the expected payoff for an incumbent is given by

$$\theta \int_{y^*(L)}^y \left(y + \frac{L}{K} G(n(L)) \right) dF(y).$$

Proposition 1 shows that $y^*(L)$ is decreasing in L and $n(L)$ is increasing in L . It immediately follows that the incumbent payoff is increasing in L . \square

Proof of Proposition 4. Taking first order conditions of (8) with respect to our control variables yields

$$f_a(a, v, l) + g_a(a, v, l)V_l = (1 - 2a)\bar{y} + (1 - v)l[1 + V_l]. \tag{A.2}$$

Since the firm begins with $l = 0$, the expression above is equal to

$$(1 - 2a)\bar{y}$$

which is positive for $a < \frac{1}{2}$, and negative for $a > \frac{1}{2}$, so the maximizing value is $a = \frac{1}{2}$. Also,

$$f_v(a, v, l) + g_v(a, v, l)V_l = -al + (1 - al)V_l \tag{A.3}$$

which for $l = 0$ equals $V_l > 0$. This implies that firm will choose the highest possible value of $v = 1$. Inspection of (A.3) implies $v = 1$ for low levels of l . Substituting into (A.2) indicates that when $v = 1$, $a = \frac{1}{2}$ is optimal for any value of l . So at the beginning, the firm explores vigorously with $v = 1$ and $a = \frac{1}{2}$.

Let a^* and v^* denote the optimal values of a and v . The Hamilton–Jacobi–Bellman equation can then be written as:

$$f(a^*, v^*) + g(a^*, v^*)V_l - rV = 0.$$

Differentiating with respect to l , we get

$$f_l(a^*, v^*) + g_l(a^*, v^*)V_l + g(a^*, v^*)V_{ll} - rV_l = 0.$$

Notice that when a^* and v^* are at a corner solution, these do not change as l changes. The equation above can be expressed as:

$$a^*(1 - v^*) - \{1 - a^*(1 - v^*)\}V_l + [v^* - \{1 - a^*(1 - v^*)\}l]V_{ll} - rV_l = 0.$$

Since $v^* = 1$, it follows that

$$V_{ll} = \frac{(1+r)V_l}{1-l} > 0. \quad \square$$

Proof of Proposition 5. In this proof, we will suppress the superscript for steady-state for notational simplicity.

Proof that $\frac{dl}{dy} < 0$

Substituting for v from (14), and for a from (11), into (15) and rearranging, we get:

$$H(l, \bar{y}, r) \equiv -\bar{y}[(l-1)(l-s_1)(l-s_2)] + (1+r)^2 l^5 = 0, \tag{A.4}$$

where

$$s_1 = \frac{2+r+\sqrt{2+2r+r^2}}{1+r} > 1,$$

$$0 < s_2 = \frac{2+r-\sqrt{2+2r+r^2}}{1+r} < 1,$$

from which it follows that $(l-1) < 0$ and $(l-s_1) < 0$ for steady-state value of l which from (14) must be smaller than 1. For (A.4) to be satisfied,

$$[(l-1)(l-s_1)(l-s_2)] > 0,$$

which implies that in steady-state $(l-s_2) > 0$. Differentiating (A.4) with respect to \bar{y} , we get

$$H_l \frac{dl}{dy} + H_{\bar{y}} = 0.$$

Since,

$$H_{\bar{y}} = -[(l-1)(l-s_1)(l-s_2)] < 0$$

it follows that

$$H_l \frac{dl}{dy} = -H_{\bar{y}} > 0.$$

So, the sign of $\frac{dl}{dy}$ must be the same as the sign of H_l in steady-state. For $l=0$, $H(0, \bar{y}, r) = 2\bar{y} > 0$. So at the smallest value of l for which $H(l, \bar{y}, r) = 0$, the sign of H_l must be non-positive. Thus it follows that at steady-state value of l ,

$$\frac{dl}{dy} < 0. \quad \square$$

Proof that $\frac{dv}{dy} < 0$

From (14), $v = 1 - (1+r)l$, from which it immediately follows that

$$\frac{dv}{dy} = -(1+r) \frac{dl}{dy} > 0. \quad \square$$

Proof that $\frac{da}{dy} < 0$

From (11), we have

$$a(1-v) = 1 - \frac{v}{l}.$$

Differentiating both sides with respect to \bar{y} and rearranging, we get

$$(1 - v) \frac{da}{d\bar{y}} = a \frac{dv}{d\bar{y}} + \frac{v}{l^2} \frac{dl}{d\bar{y}} - \frac{1}{l} \frac{dv}{d\bar{y}} = \frac{v}{l^2} \frac{dl}{d\bar{y}} - \left(\frac{1}{l} - a \right) \frac{dv}{d\bar{y}}.$$

Since $l < 1$ in steady-state and $a < 1$, it follows that $(\frac{1}{l} - a) > 0$ and because $(1 - v) > 0$ it implies that

$$\frac{da}{d\bar{y}} < 0. \quad \square$$

Proof that $\frac{dl}{dr} < 0$

Differentiating (A.4) with respect to r , we get

$$H_l \frac{dl}{dr} + H_r = 0.$$

Since we have already shown that at steady-state value of $l, H_l < 0$, the sign of $\frac{dl}{dr}$ will be the same as the sign of H_r at that value of l . We now show that for $H = 0$ and $H_l < 0, H_r < 0$.

Substituting for \bar{y} , using $H = 0$, into the expression for H_r , we get that $H_l < 0$ is equivalent to

$$G(l, r) \equiv -10 + 2l^3 + 2l^3r - 9l^2r - 15l^2 + 24l + 8lr < 0.$$

Since $G(l, r) = 0$ is a cubic equation, it has three roots. Notice that $G(0, r) = -10 < 0$ and $G(1, r) = 1 + r > 0$. So, for at least one value of $l < 1, G(l, r) = 0$. Notice that

$$G_{ll}(l, r) = 12l + 12lr - 18r - 30 < 0$$

for all values of $l < 1$ which implies that because G is concave, there cannot be two roots of the equation $G(l, r) = 0$ that are less than 1. Let us denote the first possible root as $l_1 < 1$. So we can now write that $H_l < 0$ is equivalent to $l < l_1$.

Substituting for \bar{y} , using $H = 0$, into the expression for H_r , we get that $H_r < 0$ is equivalent to

$$-3 + l^3 + l^3r - 3l^2r - 6l^2 + 8l + 2lr < 0$$

which is equivalent to

$$(l - 1)(l - l_2)(1 - l_3) < 0$$

where

$$0 < l_2 = 1 + \frac{3}{2(1+r)} - \sqrt{1 + \left(\frac{3}{2(1+r)}\right)^2} < 1,$$

$$l_3 = 1 + \frac{3}{2(1+r)} + \sqrt{1 + \left(\frac{3}{2(1+r)}\right)^2} > 1.$$

Therefore $H_r < 0$ is equivalent to $l < l_2$ since $(l - 1) < 0$ and $(1 - l_3) < 0$.

We will now show that $l_1 < l_2$ from which it would follow that if $l < l_1$, which is the case for $H_l < 0$, then $l < l_2$ which would imply that $H_r < 0$.

The root l_1 of the cubic equation does not have a simple expression and (using Mathematica or Maple) can be shown to be given by

$$l_1 = \frac{1}{1+r} + \frac{3}{2} - \frac{q}{12(1+r)}$$

where

$$q = \frac{(D^{\frac{2}{3}} + C) - i\sqrt{3}(D^{\frac{2}{3}} - C)}{D^{\frac{1}{3}}} = D^{\frac{1}{3}}(1 - i\sqrt{3}) + \frac{4C}{D^{\frac{1}{3}}(1 - i\sqrt{3})},$$

$$C = 81 + 78r + 33r^2,$$

$$D = A + i[6(1 + r)\sqrt{B}],$$

$$B = 816r^4 + 3138r^3 + 5670r^2 + 5022r + 2106,$$

$$A = 891r + 513r^2 + 675 + 81r^3.$$

Though both q and D appear to be complex numbers, some algebraic manipulation shows that q is indeed real and thus l_1 is a real root of the cubic equation. Let, \widehat{D} denote the complex conjugate of D :

$$\widehat{D} = A - i[6(1 + r)\sqrt{B}].$$

It is easy to check that

$$(D\widehat{D})^{\frac{1}{3}} = C.$$

Then q can be expressed as:

$$q = D^{\frac{1}{3}}(1 - i\sqrt{3}) + \widehat{D}^{\frac{1}{3}}(1 + i\sqrt{3}).$$

In the expression above, q is real because it is the sum of two terms that are complex conjugates. It is easier to simplify the expression above by expressing D as follows:

$$D = h[\cos(\theta) + i \sin(\theta)],$$

where

$$\theta = \tan^{-1} \left(\frac{6(1 + r)\sqrt{B}}{A} \right),$$

$$h = \sqrt{D\widehat{D}} = C^{\frac{2}{3}}.$$

Then, using De Moivre’s Theorem,

$$D^{\frac{1}{3}} = h^{\frac{1}{3}} \left[\cos \left(\frac{\theta}{3} \right) + i \sin \left(\frac{\theta}{3} \right) \right] = \sqrt{C} \left[\cos \left(\frac{\theta}{3} \right) + i \sin \left(\frac{\theta}{3} \right) \right].$$

Now, q can be expressed as

$$q = 2\sqrt{C}V,$$

where

$$V = \cos \left(\frac{\theta}{3} \right) + i \sin \left(\frac{\theta}{3} \right) \sqrt{3}.$$

Let us compute

$$\Delta \equiv 2(1 + r)(l_1 - l_2) = r + \sqrt{4(1 + r)^2 + 9} - \frac{q}{6}$$

and we want to show that $\Delta < 0$. It will be sufficient to show that $\Delta = 0$ has no solution for any positive value of r and that $\Delta < 0$ for $r = 0$. If $\Delta = 0$, that is equivalent to

$$r + \sqrt{4(1 + r)^2 + 9} = \frac{q}{6}$$

which implies that

$$U \equiv \frac{r + \sqrt{4(1 + r)^2 + 9}}{\frac{1}{3}\sqrt{C}} = V.$$

Now,

$$V = \cos\left(\frac{\theta}{3}\right) + \sin\left(\frac{\theta}{3}\right)\sqrt{3} = 2\sin\left(\frac{\theta}{3} + \frac{\pi}{6}\right),$$

which implies that $\Delta = 0$ is equivalent to

$$\sin\left(\frac{\theta}{3} + \frac{\pi}{6}\right) = \frac{U}{2},$$

or,

$$\tan\left(\frac{\theta}{3} + \frac{\pi}{6}\right) = \frac{U}{\sqrt{4 - U^2}} \equiv x,$$

which can be shown to be equivalent to

$$\tan(\theta) = -\frac{3x^2 - 1}{x(x^2 - 3)}.$$

Both θ and x are functions of r and it can be shown analytically (using Mathematica or Maple) that the equation above has only two roots, $r = -1$ and $r = -\frac{3}{8}$. Further, Δ at $r = 0$ is equal to -0.0384710065 .

We thus have proven that at steady-state value of l ,

$$\frac{dl}{dr} < 0. \quad \square$$

Proof that $\frac{da}{dr} < 0$

Substituting for l and v from (11) and (14), into (15) and rearranging, we get:

$$F(a, \bar{y}, r) \equiv Ar^2 + Br + C = 0 \tag{A.5}$$

where,

$$\begin{aligned} A &= -\bar{y}(2a - 1), \\ B &= 1 - \bar{y}(2a - 1)(4 - 3a), \\ C &= 1 - \bar{y}(2a - 1)(1 - a)\{\bar{y}a^2(2a - 1) + 4\}. \end{aligned}$$

Differentiating (A.5) with respect to r , we get

$$F_a \frac{da}{dr} + F_r = 0.$$

It is easy to check that for $a = \frac{1}{2}$, the smallest possible equilibrium value for a , the function $F = (1 + r) > 0$. Thus the smallest value of a for which $F(a, \bar{y}, r) = 0$, the derivative $F_a \leq 0$. If F_r is negative for that value of a , then it would follow that $\frac{da}{dr}$ is also negative. From (A.5),

$$F_r = 2Ar + B.$$

We know that $A < 0$. Therefore if $B < 0$ then $F_r < 0$ follows immediately. Now consider the case when $B > 0$. This implies that

$$B = 1 - \bar{y}(2a - 1)(4 - 3a) > 0,$$

which is equivalent to

$$\bar{y}(2a - 1) < \frac{1}{4 - 3a}.$$

Substituting this in the expression for C , we get

$$C = 1 - \bar{y}(2a - 1)(1 - a)\{\bar{y}a^2(2a - 1) + 4\} > 1 - \frac{(1 - a)\{\bar{y}a^2(2a - 1) + 4\}}{4 - 3a},$$

which simplifies to

$$C > \frac{a}{4 - 3a} [1 - a\bar{y}(2a - 1)(1 - a)]. \tag{A.6}$$

For (A.5) to be satisfied and have real roots of r , it must be the case that

$$B^2 - 4AC = [1 - 4\bar{y}(2a - 1)(1 - a)](1 - \bar{y}a + 2\bar{y}a^2)^2 \geq 0,$$

which implies that

$$1 - 4\bar{y}(2a - 1)(1 - a) \geq 0,$$

which is equivalent to

$$\bar{y}(2a - 1)(1 - a) \leq \frac{1}{4}.$$

Substituting in (A.6), we get

$$C > \frac{a}{4 - 3a} \left(1 - \frac{a}{4}\right) > 0.$$

Since $A < 0$, then if $C > 0$, it follows that,

$$B < \sqrt{B^2 - 4AC}.$$

The two roots of Eq. (A.5) are

$$r = \frac{B + \sqrt{B^2 - 4AC}}{-2A} > 0,$$

and

$$r = \frac{B - \sqrt{B^2 - 4AC}}{-2A} < 0.$$

Since r must be positive, only the first root above is relevant which after rearranging implies that

$$2Ar + B = -\sqrt{B^2 - 4AC} < 0.$$

Thus we have shown that

$$F_r = 2Ar + B < 0,$$

from which the result follows that

$$\frac{da}{dr} < 0. \quad \square$$

Proof that $\frac{dv}{dr}$ cannot be signed

Suppose $\bar{y} = 2$, then for $r = 0.04$, steady-state $v = 0.3505914403$ and for $r = 0.045$, $v = 0.3493200280$.

Suppose $\bar{y} = 1.77$, then for $r = 0$, steady-state $v = 0.3298006568$ and for $r = 0.01$, $v = 0.3304330067$. \square

Proof of Proposition 6. Modifying slightly the proof of Proposition 6, the condition for steady state can be expressed as

$$\hat{H}(l, \bar{y}, r, c) = H(l, \bar{y}, r) + c = 0.$$

Differentiating with respect to c , we get

$$\hat{H}_l \frac{dl}{dc} + \hat{H}_c = H_l \frac{dl}{dc} + 1 = 0.$$

Since $H_l < 0$ from proof of Proposition 5, it follows immediately that

$$\frac{dl}{dc} > 0.$$

From (14), $v = 1 - (1+r)l$, from which it immediately follows that

$$\frac{dv}{dc} < 0.$$

From (11), we have

$$a(1-v) = 1 - \frac{v}{l}.$$

Differentiating both sides with respect to c and rearranging, we get

$$(1-v) \frac{da}{dc} = a \frac{dv}{dc} + \frac{v}{l^2} \frac{dl}{dc} - \frac{1}{l} \frac{dv}{dc} = \frac{v}{l^2} \frac{dl}{dc} - \left(\frac{1}{l} - a \right) \frac{dv}{dc}.$$

Since $l < 1$ in steady-state and $a < 1$, it follows that $(\frac{1}{l} - a) > 0$ and because $(1-v) > 0$ it implies that

$$\frac{da}{dc} > 0. \quad \square$$

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