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Leverage and Market Stability: The Role of Margin Rules and Price Limits*

I. Introduction

Market trading rules—particularly those related to issues of levered investing—have been a topic of much public policy debate since the stock market crash of October 1987.¹ A problem confounding the debate is the lack of a framework in which to analyze the effect of rules designed to improve market stability since there is little formal analysis of the source or the mechanisms whereby instability may arise.² The general presumption in the academic finance literature has been that many of the observed rules that organized trading markets operate under, particularly those that have been driven by a need to “stabilize” the market, are either ineffectual or may in fact have

We show that when some investors hold levered portfolios by engaging in margin borrowing, repeated rounds of trading can result in market instability—in the sense that prices can move rationally—even in the absence of any change in fundamentals. We show this with a simple model in which all agents are rational and symmetrically informed. We discuss welfare implications of price stability and explore the effects of market composition and market trading rules on the stability of the market. A major result of this article is that price limits might enhance market stability by excluding potentially destabilizing market prices.

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1. See Greenwald and Stein (1988); the Brady Report (1989); and Miller (1989).

2. Garbade (1982) summarizes some arguments that have been made that suggest how credit-financed speculation might affect stock prices.

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a deleterious effect on the efficiency of prices.³ We propose a simple model in which borrowing on the margin by some investors can result in price instability as a rational outcome when there is more than one round of trading. The model allows us to explore the role of market composition and market trading rules—such as margin requirements and price limits—on the stability of the market.

It may help to make our notion of instability clear from the outset. We consider the market to be unstable if prices can move rationally even in the absence of changes in economic fundamentals. The economic environment we consider is one in which investors are symmetrically informed and the trading process does not generate new information either about asset fundamentals or about market composition. Hence, we do not take the approach of some papers in which it is the trading process that reveals new information, which is then reflected in market prices. This approach has been used in Grundy and McNichols (1989), Genotte and Leland (1990), Jacklin, Kleidon, and Pfleiderer (1992), and Romer (1993) for modeling the role of trading in revealing information to market participants. It is also used in Kraus and Smith (1989) to explain why small amounts of information may lead to large changes in prices. In all these models, information possessed by market participants changes as a consequence of trading itself. This is not the case in our model.

The basic story in our model is as follows. We analyze an economy in which investors are heterogeneous in terms of risk preferences, with some investors unable to afford their desired investment portfolios. These investors are able to engage in margin borrowing for investment purposes. We show that, even if the borrowing is risk-free and there is no arrival of new information, price movements may still occur if there is more than one round of trading.⁴

The instability arises from the possibility that there may be multiple prices at which the market can clear. This is analogous to the instability arising from multiple equilibria in the bank-run model in Diamond and Dybvig (1983). The intuition for the multiplicity of market clearing prices is as follows. Suppose that a substantial fraction of investors have engaged in margin borrowing in order to make their investments. Once these investors have taken such portfolio positions, their ability to borrow additional amounts or calls on their margins will affect the extent to which they are able to hold risky assets. In other words, there

3. See Grossman and Miller (1986); Miller (1989); Grossman (1990); Hsieh and Miller (1990); Seguin (1990); and Seguin and Jarrell (1993). An exception to this view is Hardouvelis (1990).

4. The assumption that margin borrowing is risk-free appears to be consistent with the fact that defaults on margin borrowing are extremely rare; see Miller (1989). This may reflect a public policy concern with ensuring the integrity and smooth functioning of securities markets by preventing widespread defaults and subsequent exchange failures.

is a link between the capacity to invest in risky assets and the price of the risky assets that serve as collateral. As a result, it is possible for there to be “rational” price rises or price falls in which the gains or losses sustained by some investors are such as to make these price changes self-fulfilling in equilibrium. A fall in price results in wealth losses and diminishes the capacity of levered investors to purchase their desired quantity of risky assets. This in turn makes the price fall rational since risky assets will now be held by fewer risk-averse investors who demand a higher risk premium to hold a larger quantity of risky assets. Precisely the opposite types of effects can result if the price rises.⁵ This is what we mean by price instability: the fact that it may be possible under some conditions for the prices of risky assets to change without any change in the underlying fundamentals.⁶

We argue that at the root of such price instability is the fact that margin requirements cannot readily be made contingent on whether the price changes are caused by fundamental or nonfundamental factors. While it may be appropriate to have margin rules that are proportional to the value of the asset to take care of fundamental risks, these may be clearly inappropriate for the case of nonfundamental changes in prices. A fixed margin requirement, following a nonfundamental price change, becomes either less or more onerous depending on whether the price rises or falls. A drop in price, in this sense, is equivalent to an increase in the borrowing constraints while a price increase is equivalent to an easing of the borrowing constraints. The rigidity of margin requirements is precisely what leads to price instability.⁷

We examine consequences of price instability and discuss why stability may be desirable. The model is analyzed to develop an under-

5. While our focus is on the role of market rules such as margin requirements, any model in which a decrease in wealth, caused by a fall in prices, results in a decreased demand for the risky asset (and vice versa) by some investors can make the price changes self-fulfilling and thus generate price instability. For instance, an anonymous referee has pointed out that price instability could also arise in a model in which risk aversion is decreasing in investor wealth. In such a model, a rise in price increases the wealth of investors making them more risk-tolerant and thus increasing their demand for the risky asset—which can make the price rise self-fulfilling in equilibrium.

6. All agents in our model are rational, unlike in the models, such as the one in DeLong et al. (1990), in which the presence of “noise traders” is crucial. There are some similarities between our argument and the “pyramiding-depyramiding” arguments summarized in Garbade (1982) and with some recent papers in the macroeconomics literature that analyze how credit constraints interact with aggregate economic activity over the business cycle; see, e.g., Bernanke and Gertler (1989); Gertler (1992); and Kiyotaki and Moore (1993).

7. It is worth emphasizing here that rigidity of margin rules is not necessarily caused by inflexible regulation. Even in an environment in which lending terms were totally unregulated, private lenders may not be in a position to determine whether a given price change is due to risks that are fundamental or nonfundamental. This inability to distinguish fundamental price changes from nonfundamental ones will result in margin requirements that are somewhat inflexible.

standing of the role of various factors—such as dispersion in risk preferences of investors and market trading rules such as margin requirements and price limits—on market stability. We show that there may be a region of values of margin requirements that results in stability and a region that does not. We show that sufficiently large margin requirements are always associated with stability. We also show that decreasing the margin requirements sufficiently—that is, relaxing the borrowing constraints—may also ensure stability. The effect of other factors on stability is also analyzed. In general, the market is stable when the aggregate borrowing by leveraged investors is small and investors are not too heterogeneous in their risk preferences.

The model elucidates the welfare implications involved in the choice of appropriate margin rules. Suppose that the objective of the regulator is to choose margin requirements that ensure market stability as well as produce the highest achievable social welfare. We know that, by tightening margin requirements, price stability can always be achieved. It may, however, be socially optimal to choose the lowest feasible margin requirement that preserves stability.

A major result of this article is a demonstration of how price limits might enhance the stability of the market. Price limits constrain the feasible set of prices at which trading is allowed to occur at any moment in time. When more than one market clearing price is possible in equilibrium—as is the case in the model we analyze—an appropriate choice of price limits may be a useful device to rule out potentially destabilizing prices. This method of achieving stability is analogous to the use of “suspension of convertibility” in the context of preventing bank runs (see Diamond and Dybvig 1983). However, there are costs associated with instituting price limits as these may result in market shutdowns even when the change in price reflects a change in the underlying fundamentals. Since margin requirements and price limits may very well act as substitutes, we analyze the trade-offs involved in choosing an optimal combination of price limits and margin requirements to achieve market stability. We also discuss the possible contribution of specialists and price continuity rules in enhancing market stability.

II. The Model

We consider an economy in which there are two types of investors that differ in terms of their risk preferences. For analytical simplicity, one type is assumed to be risk-neutral and the other risk-averse.⁸ We nor-

8. The results generalize to the case where all investors are risk-averse, with one type more risk-averse than the other.

malize the number of total investors in the economy—at least the ones that invest in risky assets—to one. Risk-averse investors account for a fraction μ of all investors.

At date $t = 0$ there is an initial round of trading as investors form their investment portfolios. All consumption takes place at $t = 2$. There are two types of investment technologies in this economy. One is basically a storage technology or a risk-free asset such that an investment of one consumption unit at $t = 0$ yields one unit of the consumption good at $t = 2$. There is no limit on the extent of investment that can be made into this storage technology. The other investment opportunity is a risky project that will yield a value of V units of the consumption good at $t = 2$ and in which investors can buy shares at $t = 0$. The total number of shares is normalized to be one. Without loss of generality, we assume that all these shares are initially held by the risk-averse investors prior to trading at $t = 0$. Each risk-neutral investor is endowed with W units of the consumption good.

While for the sake of the analysis we need to examine only the aggregate demand for assets by these two types of investors, it is helpful to assume that there also exists another class of extremely risk-averse “fringe” investors. These investors provide funds for borrowing, in addition to funds available from the risk-averse investors. Another benefit of assuming such investors is that their presence provides a well defined lower bound on the price of the risky asset in equilibrium.

In addition to the initial round of trading at $t = 0$, we introduce another round of trading at $t = 1$, prior to the date at which consumption takes place. There is no information revelation at $t = 1$ that affects investor expectations regarding the payoff of the risky project at $t = 2$ or about fundamentals of the economic environment. However, as we shall see later, the possibility of an intermediate round of trading at $t = 1$ can make a great deal of difference, even in the absence of any new information.

The risky asset is assumed to have an expected payoff of \bar{V} and a variance of σ_V^2 . We assume that the lowest possible realization of V is given by $V_L > 0$.

The risky asset is held by investors according to their risk preferences and wealth endowments at $t = 0$. For analytical ease, we assume the risk-averse investors to be mean-variance optimizers. This assumption greatly simplifies the analysis of the model since it delivers linear demand functions. Such linearity is not, however, crucial to the results that, as we show in the appendix, continue to hold for a more general class of investor demand functions. Specifically, we assume that risk-averse investors maximize the following function:

$$E(C) - \frac{\gamma}{2} \text{var}(C),$$

where $E(C)$ is the expected consumption at $t = 2$, and $\text{var}(C)$ represents the variance of consumption; γ represents investor risk aversion.

The risk-neutral investors, in addition to their endowment of W , can borrow funds for the purchase of shares of the risky asset using their ownership of the risky asset as collateral (we are ruling out other forms of borrowing by assuming that it is sufficiently costly to verify and appropriate other assets of any individual investor). With such borrowing, the risk-neutral investors can lever up to W/α to purchase shares of the risky asset, where $\alpha \leq 1$ denotes the amount of "margin" required to be able to purchase shares worth \$1. We assume that margin rules are set prior to trading at $t = 0$ and are not contingent on, say, some price realization. For analytical tractability, we assume that the margin rules are set so as to ensure that the borrowing is always risk-free.⁹ Let α_{\min} denote the smallest margin requirement that is consistent with risk-free borrowing. Thus, feasible margin requirements are characterized by $\alpha \in [\alpha_{\min}, 1]$.

A. Market Instability as a Result of Rigid Margin Requirements

We now demonstrate that, for some parameter values, the equilibrium price of the risky asset at $t = 1$ could be different from the equilibrium price at $t = 0$.

To solve for the equilibrium, we solve the dynamic programming problem backward by first obtaining the equilibrium prices and quantities at $t = 1$. Let P_1 denote the equilibrium price at $t = 1$ of the risky asset. So long as $\bar{V} > P_1$, the risk-neutral investors would like to hold as large a quantity as possible of the risky asset at $t = 1$. Assuming that the trading price at $t = 0$ is P_0 and the quantity of the risky asset held by a risk-neutral investor is Q'_0 , his wealth at date 1 will equal $W + (P_1 - P_0)Q'_0$. Given the margin rules, risk-neutral investors are thus constrained, in the aggregate, to hold $(1 - \mu)[W + (P_1 - P_0)Q'_0] / \alpha P_1$ units of the risky asset if $\bar{V} > P_1$. The equilibrium price P_1 is characterized by the following supply constraint:

$$\mu Q_1 + (1 - \mu)Q'_1 = 1, \quad (1)$$

9. While it may be interesting to introduce the possibility of default and then explicitly derive optimal risk-sharing rules, formally modeling the possibility of default makes the model sufficiently complex to be intractable. The reason is that the presence of risky debt introduces another risky security. Solving for the equilibrium with multiple risky securities, one of them being in zero net-supply in equilibrium, poses a nontrivial analytical challenge. Our intuition is that price instability is caused by the fact that, because borrowing terms cannot be made contingent on price changes, a nonfundamental drop in prices makes these terms more onerous, and a nonfundamental rise in prices makes these terms less onerous. Hence, as long as the default premium charged is not made contingent on price realization (because of the inability to distinguish fundamental price changes from nonfundamental ones), the central reason for price instability in our model will be present even in a more general model with risky borrowing.

where

$$Q'_1 \equiv \frac{W + (P_1 - P_0)Q'_0}{\alpha P_1}$$

denotes the quantity of the risky asset held by each risk-neutral investor, and Q_1 denotes the optimal quantity of the risky asset held by each risk-averse investor. Since there is no trading between $t = 1$ and the date when consumption takes place, mean-variance optimization by risk-averse investors implies that

$$Q_1 = \frac{\bar{V} - P_1}{\lambda}, \quad (2)$$

where

$$\lambda \equiv \gamma \sigma_V^2.$$

We first observe that, for P_1 to be an equilibrium price, it must lie within certain bounds. It is easy to see what the largest value of P_1 can be—this is the value if all investors were risk-neutral. In such a case, an upper bound on P_1 is

$$P_{\max} = \bar{V}.$$

There is a similar lower bound on the value of P_1 . This lower bound could arise from two possible sources. First, if the shares of the risky asset were held entirely by μ risk-averse investors, then $\mu Q_1 = 1$, which, on substituting in equation (2), gives $P_1 = \bar{V} - \lambda/\mu$. Second, if the price P_1 falls to the lowest possible value that V can take, that is, V_L , the large number of “fringe” risk-averse investors will “flood in” and demand shares of the risky asset. Hence, a lower bound on P_1 can be defined as

$$P_{\min} = \max \left[\bar{V} - \frac{\lambda}{\mu}, V_L \right].$$

Therefore, a price P_1 will be “infeasible” in equilibrium if

$$P_1 \notin [P_{\min}, P_{\max}].$$

The lowest feasible margin requirement that is consistent with risk-free borrowing is thus given by

$$\alpha_{\min} = 1 - \frac{P_{\min}}{P_0}.$$

To solve for P_1 , we substitute equation (2) in (1), which yields a quadratic equation in P_1 . The two roots of the quadratic equation are

$$P_1^H = X + \sqrt{Y}$$

and

$$P_1^L = X - \sqrt{Y},$$

where

$$X \equiv \frac{1}{2} \left[\left(\bar{V} - \frac{\lambda}{\mu} \right) + \frac{\lambda}{\alpha} \frac{(1 - \mu)}{\mu} Q'_0 \right]$$

and

$$Y \equiv X^2 - \frac{\lambda}{\alpha} \frac{(1 - \mu)}{\mu} (P_0 Q'_0 - W).$$

If $P_1^H \neq P_1^L$ and these prices are not "infeasible," then let $\pi \in (0, 1)$ denote the probability that $P_1 = P_1^H$. Clearly, if either of these two prices were infeasible, π would equal either zero or one.

Consider the case (which exists, as we shall later see) in which the marginal investor determining the equilibrium price at date $t = 0$ is the risk-neutral investor. The quantity of risky asset held by each risk-neutral investor is

$$Q'_1 \equiv \frac{W + Q'_0(P_1 - P_0)}{\alpha P_1} = \frac{1}{\alpha} \left[Q'_0 + \frac{W - Q'_0 P_0}{P_1} \right]. \quad (3)$$

We know if $P_1^H > P_1^L$ then, from equation (2), we have $Q_1^H < Q_1^L$. It follows then from equation (1) that Q'_1 must be increasing in P_1 . From equation (3), this can only happen if $W < Q'_0 P_0$, which implies that risk-neutral investors must have levered positions in the risky asset at $t = 0$.

Each risk-neutral investor chooses Q'_0 to maximize his expected consumption at date $t = 2$, which can be expressed as

$$E(Q'_1 V) - \left[\frac{1}{\alpha} - 1 \right] \{W + Q'_0 [E(P_1) - P_0]\},$$

where the second term represents the expected payment on the margin borrowing. Since P_1 and V are uncorrelated, the first-order condition for risk-neutral investors' maximization problem is

$$\frac{\bar{V}}{\alpha} \left[1 - P_0 E\left(\frac{1}{P_1}\right) \right] - \left[\frac{1}{\alpha} - 1 \right] [E(P_1) - P_0] = 0.$$

Therefore,

$$P_0 = \frac{\frac{\bar{V}}{\alpha} - \left[\frac{1}{\alpha} - 1 \right] E(P_1)}{\frac{\bar{V}}{\alpha} E\left(\frac{1}{P_1}\right) - \left[\frac{1}{\alpha} - 1 \right]}.$$

Notice that so long as P_0 satisfies the above equation, risk-neutral investors are indifferent as to the quantity of the risky asset they hold between dates 0 and 1. In equilibrium, Q_0' will be determined by the supply constraint and the optimal quantity demanded by the risk-averse investors. It follows, from Jensen's inequality, that $P_0 < E(P_1)$.

We now analyze the portfolio optimization problem solved by risk-averse investors. The consumption of a risk-averse investor at $t = 2$ is given by

$$C = W^R + Q_0(P_1 - P_0) + Q_1(V - P_1),$$

where W^R denotes risk-averse investor's wealth at date 0. Given the equilibrium price P_0 , the optimal quantity Q_0 chosen by the risk-averse investor maximizes

$$E(C) - \frac{\gamma}{2} \text{var}(C).$$

The optimal quantity Q_0 (after algebraic manipulations) can be shown to be¹⁰

$$Q_0 = \frac{E(P_1) - P_0}{\gamma \text{var}(P_1)} - \frac{\lambda \text{cov}[P_1, Q_1^2]}{\text{var}(P_1)}. \quad (4)$$

10. To see this, notice that in the last term in the expression for C ,

$$(V - P_1) = (\bar{V} - P_1) + (V - \bar{V}) = \lambda Q_1 + \epsilon,$$

where $\epsilon \equiv (V - \bar{V})$ has mean zero and is uncorrelated with P_1 and Q_1 . Making this substitution in the expression for C we get

$$E(C) = Q_0[E(P_1) - P_0] + (\text{terms not involving } Q_0)$$

and

$$\text{var}(C) = Q_0^2 \text{var}(P_1) + 2Q_0\lambda \text{cov}[P_1, Q_1^2] + (\text{terms not involving } Q_0).$$

On substituting the above expressions for $E(C)$ and $\text{var}(C)$ in the risk-averse investor's objective function, differentiating with respect to Q_0 and simplifying, the first-order condition is obtained.

This expression simplifies to¹¹

$$Q_0 = \frac{E(P_1) - P_0}{\gamma \text{var}(P_1)} + Q_1^H + Q_1^L,$$

where

$$Q_1^H \equiv (\bar{V} - P_1^H)/\lambda \text{ and } Q_1^L \equiv (\bar{V} - P_1^L)/\lambda.$$

The intuition for the above expression for Q_0 is as follows. The first term simply reflects the expected payoff and risk associated with holding the risky asset between dates 0 and 1. The second term in equation (4) reflects a hedging motive. Notice that the quantity of the risky asset Q_1 that risk-averse investors acquire at date 1 (and hence the corresponding consumption at date 2) is inversely related to the price change between dates 0 and 1. This suggests that risk-averse investors can hedge this consumption risk at date 2 through their holding of the risky asset between dates 0 and 1. Observe that the covariance between P_1 and Q_1^2 is negative and, therefore, contributes to an increase in Q_0 .

Given Q_0 , the quantity held by risk-averse investors, Q_0' , is obtained from the following supply constraint:

$$\mu Q_0 + (1 - \mu)Q_0' = 1.$$

PROPOSITION 1. There exists a set of parameter values for which the market for the risky asset is unstable.

Proof. We prove this by providing an example in which, in equilibrium, $P_1^H \neq P_1^L$, these prices are not infeasible and both prices can occur with strictly positive probability.

Let $\bar{V} = 100$, $\gamma = 2$, $\sigma_V^2 = 20$, $\alpha = .5$, $\pi = .5$, $\mu = 0.168$, $W = 13.14$. With this set of parameter values the equilibrium prices are $P_1^H = 38$, $P_1^L = 26$, and $P_0 = 30.67$. Q.E.D.

This model described above illustrates how instability in prices and "excess trading," as investors readjust portfolio holdings at $t = 1$, can occur in a rational framework. The intuition for the price instability is as follows. So long as $\bar{V} > P_1$, risk-neutral investors will borrow the

11. To see this, substitute $Q_1 = (\bar{V} - P_1)/\lambda$ into the covariance term in eq. (4) above and simplify to get

$$\text{cov}[P_1, Q_1^2] = \frac{1}{\lambda^2} [-2\bar{V} \text{var}(P_1) + \text{cov}[P_1, P_1^2]].$$

Now,

$$\text{cov}[P_1, P_1^2] = \pi[P_1^H - E(P_1)][(P_1^H)^2 - E(P_1^2)] + (1 - \pi)[P_1^L - E(P_1)][(P_1^L)^2 - E(P_1^2)].$$

Substituting $E(P_1) = \pi P_1^H + (1 - \pi)P_1^L$ and $E(P_1^2) = \pi(P_1^H)^2 + (1 - \pi)(P_1^L)^2$ above, simplifying, and further using $\text{var}(P_1) = \pi(1 - \pi)(P_1^H - P_1^L)^2$, we can obtain

$$\text{cov}[P_1, P_1^2] = \text{var}(P_1)(P_1^H + P_1^L).$$

Substituting these expressions and simplifying, we obtain the simplified expression for Q_0 .

maximum amount allowed and invest everything in the risky asset at $t = 1$. Since they hold levered positions in the risky asset at $t = 0$, a rise in the price results in their being able to purchase a larger quantity of the risky asset at $t = 1$. This increase in demand can make the price increase self-sustaining in equilibrium. Similarly, a fall in price will result in their being able to purchase a smaller quantity—resulting in a self-sustaining price fall. The anticipated price instability at $t = 1$ is rationally taken into account by investors in choosing their portfolio holdings at $t = 0$.

It may seem strange that the price of the risky asset changes in equilibrium though nothing fundamental has changed. This reason is that margin requirements do not change as a result of the price move. If one knew that the price drop occurred for reasons not related to fundamentals, one would relax margin requirements from, say, 50% to a sufficiently lower level. Similarly, if one knew that the price increase occurred for reasons not related to fundamentals, margin requirements would be tightened from 50% to a sufficiently higher level.

The instability then is caused by the rigidity of the margin rules. This, of course, raises the question why margin rules need be rigid. For instance, why not allow margin rules to be contingent on prices themselves so that a price drop would automatically trigger less onerous margin requirements while a price increase would trigger tighter margin requirements? The problem with implementing such a mechanism is that it is not easy to determine if a price move has occurred as a result of change in fundamentals or not. If the price change is indeed caused by a change in fundamentals, then a mechanical rule that relaxes the margin requirements when prices fall, and vice versa, would not be appropriate. The inability to set rules that can determine whether or not a price change is due to fundamental reasons is at the heart of the problem that we are addressing in this article.

It should be pointed out that, in practice, margin requirements are not completely rigid. First, brokerage houses are free to increase margin requirements beyond the minimum required levels when prices rise. Second, maintenance margins are typically smaller than initial margins. This has the effect of easing the effective margin requirements when prices fall.

Further, regulators do intervene in the markets to alter the terms for borrowing from time to time. For instance, after the stock market crash of October 1987, the Federal Reserve in the United States did intervene in an attempt to enhance market liquidity.¹² Similarly, there are in-

12. Chairman Alan Greenspan released the following statement on the morning of October 20: "The Federal Reserve, consistent with its responsibilities as the nation's central bank, affirmed today its readiness to serve as a source of liquidity to support the economic and financial system" (Brady Report 1988, p. 183).

stances in which regulators tighten margin requirements following large stock price rises.¹³ Hsieh and Miller (1990, p. 6) document evidence that is consistent with the observation that "the Fed apparently raised margins when the stock market was booming and cut them after it fell."

We are not suggesting that regulators are indeed able to discern when the price moves are for fundamental reasons and when they are not. We are rather providing a rational framework in which price moves unrelated to fundamentals are indeed possible, and a rationale for why regulatory interventions may help in such cases.

B. Equilibrium When Market Is Stable

Having shown that there exists a set of parameter values for which market instability can arise in equilibrium, we now determine sufficient conditions under which the market for the risky asset is stable. Simply put, our notion of stability is that the equilibrium price of the risky asset remains unchanged between dates 0 and 1. Characterizing the sufficient conditions for the stable equilibrium will allow us to analyze the effect of market composition and trading rules on market stability.

A sufficient condition for stability is that there be only one feasible price of the risky asset at $t = 1$. This ensures that the price at $t = 0$, in equilibrium, will have to equal the price at $t = 1$. Let P_0 denote the equilibrium price of the risky asset at $t = 0$. Under the assumption that the equilibrium is stable, the equilibrium price P_0 is characterized by the following equation, where the terms Q_0 and $W/\alpha P_0$ represent the quantities of the risky asset held by the two investor types:¹⁴

$$\mu Q_0 + (1 - \mu) \frac{W}{\alpha P_0} = 1, \quad (5)$$

where

$$Q_0 \equiv \frac{\bar{V} - P_0}{\lambda}. \quad (6)$$

We now characterize conditions under which any candidate equilibrium price at $t = 1$ that is not equal to P_0 lies outside the feasible bounds for P_1 .

If the equilibrium price at $t = 1$ is P_1 , the wealth of each risk-neutral investor at $t = 1$ is given by $W + (P_1 - P_0)W/\alpha P_0$. Given the margin

13. For instance, the Japanese Economic Newswire reported on March 8, 1986, that "the (Tokyo) exchange may tighten its controls on margin trading in the near future to stem an 'overheating' of the market."

14. In a stable equilibrium, the price does not change between dates 0 and 1, and therefore, the quantity held at $t = 0$ is indeterminate. We assume that in this case investors weakly prefer to hold the same quantity at $t = 0$ as they would at $t = 1$.

rules, the total quantity of the risky asset held by this group of investors is given by $(1 - \mu)[W + (P_1 - P_0)W/\alpha P_0]/\alpha P_1$. Hence, the equilibrium price P_1 is characterized by the following equation:

$$\mu Q_1 + (1 - \mu) \frac{W + (P_1 - P_0)W/\alpha P_0}{\alpha P_1} = 1, \quad (7)$$

where

$$Q_1 = \frac{\bar{V} - P_1}{\lambda}. \quad (8)$$

From equations (5)–(7), we obtain a quadratic equation in P_1 that can be solved for P_1 as a function of P_0 . By inspection, we can see that $P_1 = P_0$ is one solution. The second possible solution to the quadratic equation is

$$\hat{P}_1 = \lambda \frac{1 - \mu}{\mu} \frac{1 - \alpha}{\alpha} \frac{W}{\alpha P_0}. \quad (9)$$

In general, this second solution for \hat{P}_1 is not equal to P_0 .¹⁵ If \hat{P}_1 is infeasible—that is, $\hat{P}_1 \notin [P_{\min}, P_{\max}]$ —then the only feasible value for P_1 is equal to P_0 . The market is stable in this case since the equilibrium price does not change between dates 0 and 1.

To obtain the solution for P_0 , we solve equation (5), which is quadratic in P_0 . The only positive root for this equation represents the solution for P_0 and is given by

$$P_0 = \frac{1}{2} \left(\bar{V} - \frac{\lambda}{\mu} \right) + \left\{ \left[\frac{1}{2} \left(\bar{V} - \frac{\lambda}{\mu} \right) \right]^2 + \lambda \frac{1 - \mu}{\mu} \frac{W}{\alpha} \right\}^{1/2}. \quad (10)$$

C. Comparing Unstable and Stable Equilibria

We have seen that, when investors can trade at an intermediate date between the initial trading date 0 and date 2, when final consumption takes place, there can be unstable price movements on date 1 even in the absence of new information about fundamentals. If no trading were allowed on date 1, then, by definition, the stable equilibrium defined in Section IIB will obtain. The question we explore in this subsection is whether instability arising from the trading in the intermediate period is desirable or undesirable in terms of ex ante investor welfare. Intuitively, the effect of price instability on investor welfare is as follows. Since, in equilibrium, risk-neutral investors hold levered portfolios at date 0, an increase in price on date 1 amounts to a relaxation of the

15. We show in the appendix that the linearity of demand functions is not necessary to guarantee that there is a unique solution for \hat{P}_1 ; this property holds for a more general class of demand functions.

margin requirements and benefits risk-neutral traders, allowing them to increase their holding of the risky asset. Conversely, a decrease in price on date 1 amounts to a tightening in margin requirements, which hurts risk-neutral traders. So, in principle, the overall effect on risk-neutral traders' ex ante welfare may be positive or negative with unstable prices. For risk-averse traders, price instability will affect their ex ante utility through its effect on expected consumption as well as the variance of consumption on date 2. As we have seen, risk-averse investors will hold some quantity of the risky asset between dates 0 and 1 in order to hedge the effects of price movements between dates 0 and 1.

In order to explore these questions within our framework, we compare the welfare of investors by examining their ex ante utility under the two scenarios: when trading on date 1 is not allowed and when trading is allowed and prices are unstable, keeping various parameters unchanged across the two scenarios. Let u and u' denote expected utilities of date 2 consumption of risk-averse and risk-neutral investors, respectively. We will use the superscript S to denote the case with stable prices and superscript U to denote the case with unstable prices.

For risk-neutral traders, their expected utilities under the two scenarios are as follows:

$$u^{S'} = \frac{W}{\alpha P_0^S} \bar{V} - \left(\frac{1}{\alpha} - 1 \right) W$$

and

$$u^{U'} = \frac{W}{\alpha \tilde{P}_1} \bar{V} - \left(\frac{1}{\alpha} - 1 \right) W,$$

where

$$\tilde{P}_1 \equiv 1/E(1/P_1).$$

The expression for $u^{S'}$ represents expected consumption at date 2 for risk-neutral traders who borrow $(1/\alpha - 1)W$ at date 0 to purchase $W/\alpha P_0^S$ units of the risky security at date 0. Recall that, in the unstable equilibrium, the price of the security at date 0 is such that risk-neutral investors are indifferent as to the quantity of the risky asset they hold between dates 0 and 1. So, in particular, if a risk-neutral investor were to hold no risky asset at date 0, his wealth at date 1 will equal W , and his expected utility will be identical to the case in which he holds the equilibrium quantity Q_0' of the risky security between dates 0 and 1. The expression for $u^{U'}$ represents expected consumption at date 2 for a risk-neutral trader who holds no risky asset between date 0 and 1 and borrows $(1/\alpha - 1)W$ at date 1 to purchase $W/\alpha \tilde{P}_1$ units of the risky security at date 1.

Clearly then, the comparison of utilities for risk-neutral investors corresponds to simply comparing P_0^S and \bar{P}_1 .

For risk-averse traders, their expected utilities under the two scenarios are as follows:

$$u^S = W^R + Q_0^S(\bar{V} - P_0^S) - \frac{\gamma}{2} \text{var}(Q_0^S V) = W^R + \frac{(\bar{V} - P_0^S)^2}{2\lambda}$$

and

$$u^U = E(C^U) - \frac{\gamma}{2} \text{var}(C^U).$$

It can be shown that

$$E(C^U) = W^R + Q_0^U(E(P_1) - P_0^U) + E\left[\frac{(\bar{V} - P_1)^2}{\lambda}\right],$$

$$\frac{\gamma}{2} \text{var}(C^U) = \frac{(E(P_1) - P_0^U)^2}{2\gamma \text{var}(P_1)} + \frac{(\bar{V} - E(P_1))^2}{2\lambda} + \frac{\text{var}(P_1)}{2\lambda}.$$

Unfortunately the problem is highly nonlinear, and we do not have analytical results that characterize the welfare of investors under the two scenarios. However, numerical simulations suggest the following: (a) risk-neutral investors are generally worse off with unstable prices;¹⁶ (b) there are instances when risk-averse investors are better off with unstable prices while in other instances they are worse off; (c) if one constructs a social welfare function that places weights on the utility of risk-neutral and risk-averse investors in proportion to their numbers—that is, μ and $(1 - \mu)$ —the ex ante comparison between the two scenarios is ambiguous—there are instances when social welfare is higher with unstable prices as well as instances when it is lower.¹⁷ Hence, based on these results, it is not obvious that price stability is desirable in all instances.

It should be recognized, however, that welfare comparisons based on our simple model cannot address many important considerations. For instance, policy makers may legitimately be concerned about the effect of price instability on the cost of capital and its effect on real investments in the economy. To see what light our model sheds on the effect of price instability on the required rate of return, consider the following result that describes the relation between the equilibrium price of the risky security at date 0 when the market is stable to when it is unstable.

16. In fact, we failed to generate a single example in which risk-neutral investors are better off with unstable prices.

17. The numerical results are available from us on request.

LEMMA 1.

$$P_0^S > P_0^U.$$

Proof. See the appendix.

An interpretation of the above result is that investors require a higher expected rate of return to hold risky assets in the presence of market instability. This suggests that, in a more complete model in which entrepreneurs sought financing for new risky investments, the cost of capital may be higher with market instability. This could result in fewer new investments being undertaken, possibly resulting in welfare loss for entrepreneurs and other agents in the economy.

Other important considerations that our model does not explicitly take into account, and which may make price stability desirable, include the informational role of prices in resource allocation decisions, the effect of excess volatility on investor confidence, and so forth. We will assume for the rest of our analysis that market stability is desirable from a policy perspective. In the subsequent sections, we will attempt to characterize market stability and examine policy instruments that can be used to enhance stability.

D. Characterizing Market Stability

We now characterize the set of parameter values for which the market for the risky asset is stable. We first prove the following lemma.

LEMMA 2. The price P_0 is increasing in \bar{V} , W , and μ and is decreasing in α and λ . The price \hat{P}_1 is increasing in W and λ and is decreasing in \bar{V} , α , and μ .

Proof. See the appendix.

The results in lemma 2 allow us to obtain the following result.

LEMMA 3. The closure of the set of parameter values for which $\hat{P}_1 \in [P_{\min}, P_{\max}]$ is compact and convex and is bounded away from $\alpha = 0$, $\alpha = 1$, $\mu = 0$, $\mu = 1$, $W = 0$, and $\lambda = 0$.¹⁸

Proof. See the appendix.

We now discuss the implications of the result in lemma 3.

Leverage and market stability. The concern about the effect of leverage on market stability is raised quite frequently in public policy debates.¹⁹ What is lacking in all these debates is a coherent and rational argument for why and how leverage might affect market stability. The analysis in our model provides a framework and a step toward understanding this complex issue.

18. The set of parameter values for which both solutions for P_1 are identical is also associated with stability. However, this set has measure zero and therefore does not affect our analysis.

19. For instance, the Securities and Exchange Commission chairman Richard Breeden is reported to have said that "rules should be to prevent excessive leverage and encourage overall market stability" (Breeden 1992).

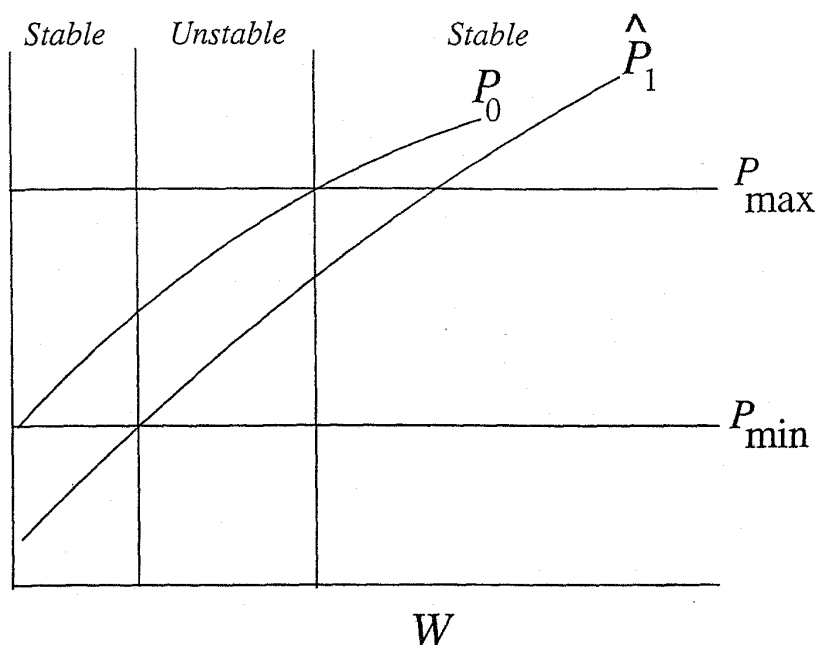


FIG. 1.—The relation between market stability and W

The result in lemma 3 shows that the market is stable for low values of W (see fig. 1). A low value of W suggests that the aggregate level of borrowing by the risk-neutral investors will not be very large since the amount each of these investors is able to borrow is related to his or her level of wealth. We thus obtain the following implications.

Implication 1. The market is stable if the aggregate borrowing by leveraged investors is small.

Consistent with this implication, those who do not believe that leverage in the stock market is responsible for instability in any significant way have argued: “By the early 1970’s, the total stock market credit, which, even at the height of the 1929 boom had never amounted to more than 10 percent of the value of listed equities was down to only 2 percent of market value” (Hsieh and Miller 1990, p. 4).

The following implication shows that it is not just the total amount of leverage but also the relative importance of levered and unlevered investors in the market that can affect stability.

Implication 2. The market is stable if the proportion of the risky asset held by investors with leveraged positions is sufficiently low. The market is also stable if the proportion of the risky asset held by investors with leveraged positions is sufficiently high.

The intuition for this implication is straightforward. The proportion of the risky asset held by investors with leveraged positions depends on the fraction of such investors in the market as well as their capacity to invest in the risky asset. The result in lemma 3 shows that the market

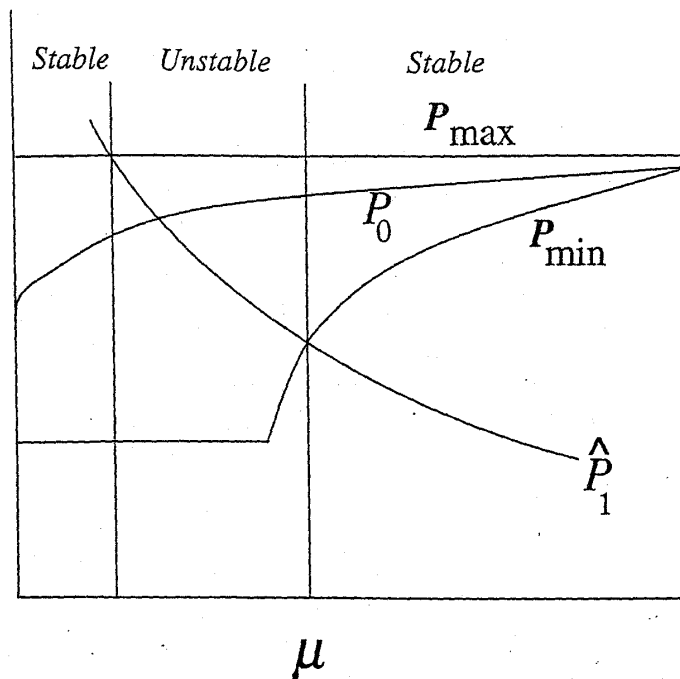


FIG. 2.—The relation between market stability and μ

is stable for low as well as high values of W (see fig. 1). Also, the market is stable for low as well as high values of μ (see fig. 2). If the quantity of the risky asset held by investors with leveraged positions is very large, the price change required for the risk-averse to absorb the portfolio rebalancing of leveraged investors would have to be so large as to be outside the feasible range. This ensures stability. Similarly, if the quantity of the risky asset held by investors with leveraged positions is very small, gains and losses incurred by these investors will be relatively small, and the price changes could not be self-sustaining. This also ensures stability.

This implication implies that heterogeneity in risk preferences across investors can affect market stability. Note that, if the value of the parameter μ is either zero or one, the market is always stable. This suggests that a lower dispersion in risk preferences across investors enhances market stability. This also suggests a possible reasoning behind the claim that is often voiced by market professionals that a significant presence of small investors in the stock market not only raises the level of stock prices but also increases the fragility of the market. These observations are consistent with our model. A broader participation by investors not only implies a larger value of P_0 but also an increase in dispersion of the types of investors, which may increase the possibility of market instability.

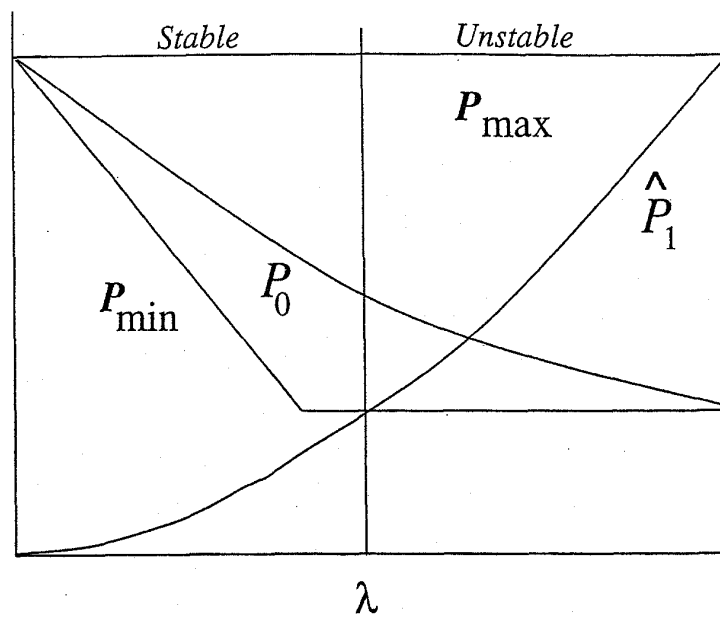


FIG. 3.—The relation between market stability and λ

Risk and market stability. Asset risk premium is affected by both the risk-aversion of investors as well as the riskiness of asset payout. In our discussion, the effect of both these factors is captured in λ ($\equiv \gamma\sigma_v^2$). From lemma 3, the relation between market stability and λ can be stated as follows.

Implication 3. The market is stable if either the risk-aversion of investors or the riskiness of the asset is sufficiently low. The market is also stable if either the risk-aversion of investors or the riskiness of the asset is sufficiently high.

Figure 3 depicts the relation between λ and market stability. The intuition underlying implication 3 is as follows. When λ is small, risk-averse investors are close to being risk-neutral and a small price decrease causes these investors to increase substantially their holdings of the risky asset. This has the effect of dampening possible price movements and making market stability more likely. The opposite effect takes place when λ is high. The initial prices are low and have to change by large amounts in order to induce significant changes in the portfolio holdings of risk-averse investors. Such large prices may not lie within the feasible set of prices, thereby resulting in market stability.

It is clear from the discussion that different risk environments may require different trading rules in order to ensure stability. In practice, margin requirements, price limits, and other trading rules are often changed in response to changed conditions.²⁰ Our model suggests that

20. See Garbade (1982); Hardouvelis (1990); and Hsieh and Miller (1990).

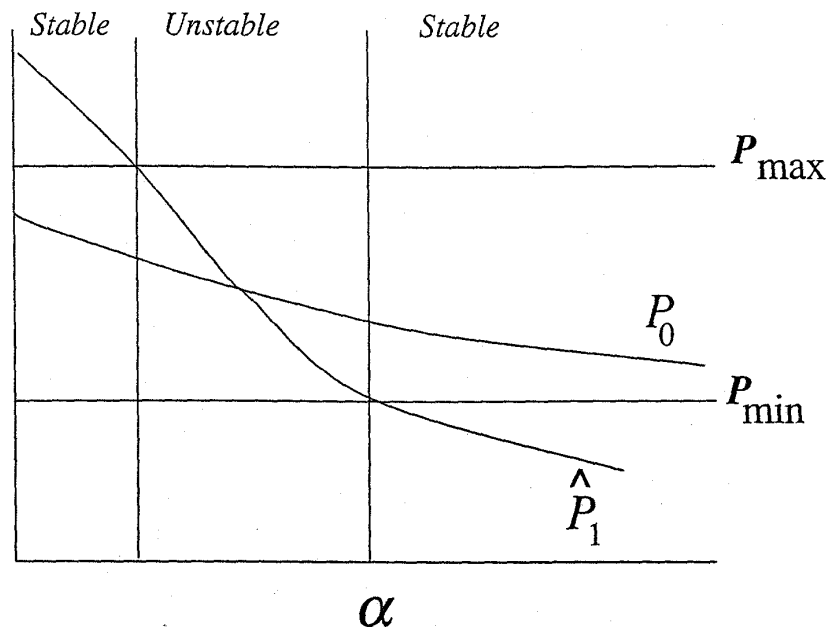


FIG. 4.—The relation between market stability and margin requirements, α

changes in the riskiness of assets and in the composition of investors may need to be accommodated by changing trading rules. For example, the presence of many small skittish investors may well require a change in margin requirements. A problem, of course, is that it is not obvious what the effect of anticipated rule changes is on market stability. It is conceivable that the very possibility of rule changes may result in market instability.

Margin requirements and market stability. As margin requirements are raised (i.e., α is increased), it is clear that the aggregate amount of borrowing will decrease.²¹ We know, therefore, from the discussion above (implication 1) that for a large enough value of α the market must be stable. From lemma 3 it follows that there may also exist small values of α for which the market is stable. The market may be unstable for intermediate values of α (see fig. 4). The intuition for this is that the loosening of margin requirements may sometimes bolster stability by increasing the capacity of risk-neutral investors to invest in the risky assets. Similarly, tightening the margin requirements may increase the possibility of the market becoming unstable. We summarize this in the following implication.

Implication 4. The market is stable for margin requirements that are sufficiently large. The market may also be stable for margin requirements that are sufficiently small.

21. Hsieh and Miller (1990) document evidence that is consistent with this observation.

From a social welfare perspective, it is clear that a lower α allows risk-neutral investors to hold portfolios closer to their desired optimal portfolios. This is reflected in equation (10) by the fact that P_0 is decreasing in α .²² Given a policy to ensure market stability, social welfare will be maximized if α is set at the lowest value consistent with market stability.

E. Price Limits and Market Stability

In this section we argue that price limits may potentially serve as an effective substitute for margin requirements in terms of their effect on market stability. We believe that our argument provides a somewhat novel justification for price limit rules that are implemented in various financial markets.

Price limits have usually not been regarded favorably in the academic literature. By forcing a shutdown in trading, price limits prevent investors from reallocating portfolios in response to developments in the economic environment. Further, they disrupt the role of market prices in aggregating information (see Grossman 1990, for instance). Arguments have, however, been made in defense of price limits. It has been argued that, in particularly volatile economic environments, trading halts, triggered by price limits, may provide market participants the opportunity to assess the situation. Market makers may be able to infer market conditions more accurately once less active investors have had an opportunity to react to the new developments. The notion that such trading halts may be beneficial in ensuring "fairness" in trading prices underlies some of the recommendations in the Brady commission report on "circuit breakers" and trading halts (see Greenwald and Stein 1988, 1991). Kodres and O'Brien (1994) argue that, during periods of volatile price movements, delays in initiating and executing orders give rise to implementation risk that cannot be transferred optimally among investors: price limits, in such cases, may be Pareto superior to unconstrained trade. Price limits also help maintain the integrity of markets by giving the exchanges an opportunity to ascertain the solvency of investors. Brennan (1986) has argued that price limits, by preventing investors from realizing the magnitude of their losses in futures markets, may, on occasion, reduce investor incentive to default. Our argument, however, is that price limits may also serve to ensure market stability when there is more than one feasible price at which the market can clear.

Price limits constrain the feasible set of prices at which trading is allowed to occur at any moment in time. If no such price is to be found, then the limits are either raised or lowered in the following period,

22. Sequin (1990) documents evidence that margin eligibility leads to an increase in stock price.

depending on whether there was excess supply or demand at the limit prices. The discussion in the literature has mainly been concerned with the effect of price limits when there are fundamental economic factors causing the change in price movements. In this case, whether a market clearing price can be found within the price limits is determined by the information investors have regarding fundamental factors. When, however, there is the possibility of more than one market clearing price—as in the model we have analyzed—price limits may be a useful device to rule out potentially destabilizing prices by an appropriate choice of price limits.

In the context of our model, price limits tighten the feasible region of prices from $[P_{\min}, P_{\max}]$ to a smaller set of feasible prices defined by the price limits. Since there is no change in fundamental economic factors, price P_0 remains a feasible equilibrium price at $t = 1$. In this context it is clear that tight price limits could be used to rule out other possible market clearing prices, thereby assuring market stability. Since the market price does not change at $t = 1$, neither do the limits—as they would have had there been fundamental economic factors behind the price moves. The argument generalizes to multiple trading rounds, in the absence of changes in fundamentals, since the market clearing price and the price limits are unchanged each period.

Given the restrictive assumptions invoked in our model, one should view the conclusions we make based on our analysis above as suggestive. In a more complete model that allows for price movements because of changes in fundamentals, some of the conclusions regarding the effectiveness of price limits in ruling out unstable equilibria may not be robust. Though price limits impose no costs in the context of our model, the existence of price limits may result in significant costs to investors when the price changes are driven by changes in economic fundamentals. Hence, in setting price limit rules, we would have to weigh their potential social cost against that of alternative mechanisms in ensuring market stability. The trade-off between margin rules and price limits as alternative means of achieving market stability is discussed next.

F. Margin Requirements versus Price Limits

We have discussed above the role that both margin rules and price limits can play in ensuring market stability. Rather than use one of these mechanisms exclusively, it appears that using these mechanisms in some combination may provide a less onerous means of achieving market stability—which we assume to be a policy objective. It is beyond the scope of this article to attempt to model formally the social costs associated with these alternative mechanisms. What we show is that market stability may be achieved by various choices of margin requirements in combination with price limit rules; the precise choice

of rules would, of course, be determined by the costs and benefits associated with the alternative choices. As discussed above, the cost of imposing margin requirements is that it restricts investor portfolio choices and lowers the market price of the risky asset. The cost of price limits is that they may result in trading halts even when price moves are appropriate given changes in the economic environment.

We have assumed that it is a policy imperative to ensure market stability. The rules are set before the start of trading, and there is uncertainty regarding the realization of the various parameters that characterize the market. We note that price limit rules by themselves, set (say, in percentage terms) before the market opens for trading, may not be sufficient to ensure market stability. The reason is that no matter how tight the price limits (assuming that they do not completely restrict all price moves), there will exist parameter values for which the market may be unstable. This suggests that ensuring market stability may require that at least some margin restrictions be used.

The trade-off between price limits and margin rules can be characterized as follows.

PROPOSITION 2. If margin requirements necessary for stability are such that they are larger than those required to ensure that the borrowing is risk-free (i.e., if $\alpha > \alpha_{\min}$ for all P_0), then stability can also be ensured with tighter price limits and lower margin requirements.

Proof. We know that for any given P_0 , there is a unique value for \hat{P}_1 , which could be either larger than P_0 or smaller than P_0 . The reason why price limits will take the form of a price band is that price limits must be set before observing the value of P_0 .

We first make an important observation that an efficient choice of feasible margin requirements and a band of price limits that ensures stability must be such that it is the lower price limit that is binding. To see this, suppose it is not the case. Since \hat{P}_1 is decreasing in α , it will be possible to decrease the margin requirement α by at least a small amount without the \hat{P}_1 s (associated with different P_0 s arising from different realizations of exogenous parameters) hitting the lower price limit. For \hat{P}_1 s that were larger than the upper price limit to begin with, the slack will increase even more. That means that the original margin requirements α (given price limits) were not efficient to begin with since these could be lowered without sacrificing stability.

Now the trade-off between an efficient level of margin requirements and price limits is easy to see. Suppose we make the price limits tighter. Since the initial lower price limit was binding, using an argument similar to the one above, we can see that margin requirements can be lowered without sacrificing stability. Q.E.D.

It is worth noting that the futures markets have used low margin requirements with price limits for stock index futures contracts, while the stock market has relied mainly on margin rules that are higher, at

least for small investors. One explanation for these differences may be that the stock market margin requirements, set by the Securities and Exchange Commission rather than by exchanges on individual stocks may well be demanding enough to obviate the need for price limits. Our model suggests an alternative explanation. Since fundamental price volatility of individual stocks is likely to be higher than that of a well diversified index, the potential cost of price limits associated with trading halts may be higher for individual stocks than for an index. This may account for the trading of stock market indexes in the form of futures contracts with price limits.

G. Alternative Institutional Mechanisms and Stability

In addition to margin rules and price limits, other mechanisms exist that may contribute to market stability. One such institutional arrangement may be the requirement that specialists on major exchanges maintain continuity in market clearing prices. Black (1971) has suggested a role for price continuity rules by arguing that there may be a trade-off between price stability and market efficiency.²³

In our framework, if the market specialist is required to search for market prices in the region of the last trading price, this may result in the specialist discouraging nonfundamental price moves. In a sense, if the specialist is able to "lean against the wind," this may help avoid nonfundamental price moves. In game theory jargon, the specialist may play a role in ensuring that the stable equilibrium is a "focal point."

The limited positions that specialists are required to assume in order to meet their obligations under price continuity requirements may, however, make this mechanism ineffective in ensuring stability. To the extent that the price continuity rules assist in furthering market stability, they may justify the retention of specialists in the trading process—as opposed to moving entirely toward computer-based trading that may be more susceptible to price instability. It is also possible that the specialist may be more favorably situated than other market participants in being able to distinguish fundamental from nonfundamental price moves and to call for trading halts in such situations.

H. Implications of Multimarket Trading of Securities

After the stock market crash of October 1987, there emerged considerable discussion about the possible consequences of inconsistency in margin requirements across the futures and cash markets for stocks. It was claimed by some that the margin requirements in the futures markets, being lower than those in the cash markets, ought to be raised

23. See Dutta and Madhavan (1995) for an analysis of the effect of price continuity rules on price dynamics and possible rationales for their existence.

to the same level (see, e.g., Brady Report 1989).²⁴ By extending our discussion to the case when an equivalent security trades in more than one market, we argue that it may instead make sense to have *inconsistent* margin requirements across the markets.

Suppose that a security trades in two different markets, which we will label the spot and the futures market, with the futures contract on the security being a zero net-supply contract. We further assume that margin requirements are indeed lower in the futures market than in the spot market for the security; that is,

$$\alpha^f < \alpha^s,$$

where α^f and α^s denote margin requirements in the futures and spot market, respectively. We let variables with superscripts f and s indicate variables corresponding to the futures and spot markets, respectively.²⁵

Arbitrage between the spot and the futures markets ensures that the prices of the security in the two markets are equal; that is,

$$P^f = P^s. \quad (11)$$

Therefore, the $(1 - \mu)$ risk-neutral investors will prefer to be entirely in the futures market (because $\alpha^f < \alpha^s$) and take as large a long position as margin requirements will allow (so long as $\bar{V} > P^f$). Formally,

$$Q^f = \frac{W}{\alpha^f P^f}. \quad (12)$$

The μ risk-averse investors will hold a quantity Q^s of the risky asset in the spot market and take a position Q^f in futures contracts such that their net exposure to the risky asset, based on their mean-variance optimization, is as follows:

$$Q^s + Q^f = \frac{\bar{V} - P^s}{\lambda}. \quad (13)$$

Note that the risk-averse investors also act as arbitrageurs in the model.²⁶ The risk-averse investors take the short positions in the futures market to offset the long positions of the risk-neutral investors and also hold the risky asset in the spot market.

The supply constraints for the security in the two markets are

24. Miller (1989) and others have challenged the presumption that *effective* margin requirements are lower in the futures markets than those in the cash markets for *equivalent* securities.

25. In this section, we drop the time subscripts to avoid notational clutter, keeping in mind that we are examining an equilibrium where exogenous parameters are such that prices are stable.

26. The fringe (highly risk-averse) investors in the model can also act as arbitrageurs, ensuring that the prices of the security are indeed equal across the two markets.

$$\mu Q^f + (1 - \mu)Q^f = 0 \quad (14)$$

and

$$\mu Q^s = 1. \quad (15)$$

The resulting net exposure of the risk-averse investors, based on their mean-variance optimization, is given by (13). Substituting into (13) for Q^s from (15), for Q^f from (12) and (14), and imposing the no-arbitrage condition (11), we obtain a quadratic equation in the equilibrium price whose solution is identical to the one in Section IIB with α replaced by α^f .

Hence, the equilibrium in this case with multimarket trading and different margin requirements is equivalent to the one with a single market with a margin requirement of α^f , the smaller of the two margin requirements in this case. All other implications related to the stability of the market can also be extended to this case.

Notice that the equilibrium is not affected by the margin requirements in the market with the more stringent requirements. Effective margin requirements are dictated by the market with the lowest margin requirements. Therefore, if the regulators must intervene to set margin requirements that ensure stability, it is the market that has the least onerous margin requirements that needs regulatory oversight.

III. Conclusion

We presented a simple model to show that, when some investors hold levered portfolios by engaging in margin borrowing, repeated rounds of trading can result in market instability—in the sense that prices can move rationally, even in the absence of any change in fundamentals. The analytical framework we developed allowed us to explore the effects of market composition and market trading rules on the stability of the market.

We argued that a fixed margin requirement, following a nonfundamental price change, becomes either less or more onerous depending on whether the price rises or falls. A drop in price, in this sense, is equivalent to an increase in borrowing constraints—while a price increase is equivalent to an easing of borrowing constraints. The rigidity of margin requirements is precisely what makes these price changes self-fulfilling and causes price instability. The rigidity arises from the fact that margin requirements cannot readily be made contingent on whether the price changes are caused by fundamental or nonfundamental factors. We showed that sufficiently large margin requirements are always associated with stability. But interestingly, and perhaps more important, decreasing margin requirements sufficiently may also ensure stability.

A major result of this article is that price limits might enhance market stability by excluding potentially destabilizing market prices. We also discussed the possible role of specialists and price continuity rules in enhancing market stability.

The analysis of the trade-offs involved in choosing an optimal combination of margin requirements, price limits, and price continuity rules to achieve market stability allowed us to speculate why the futures markets have used low margin requirements with price limits for stock index futures contracts while the stock market had relied mainly on margin rules that were typically higher, at least for small investors.

First, since fundamental price volatility of individual stocks is likely to be higher than that of a well-diversified index, the potential cost of price limits associated with trading halts may be higher for individual stocks than for an index. Thus, alternative methods of achieving price stability, such as higher margins and price continuity rules, may be more efficient for assets such as stocks. Second, we argued that, when equivalent securities trade on multiple markets, effective margin requirements are dictated by the market with the lowest margin requirements.

An interesting issue is whether large-scale intervention by the government or some other large organization helps in ensuring stability. Again, this hinges entirely on whether or not rules can be written such that bureaucrats are able to intervene when there are nonfundamental as opposed to fundamental factors at work. If there are fundamental factors at work, then intervention may simply be a waste of resources. If, however, there is a risk to market stability due to nonfundamental factors, timely interventions may enhance stability. The real benefit of such interventions may be that it assures market participants that such interventions would indeed take place when necessary. There are numerous examples of such interventions; the extent to which interventions have proven helpful may, of course, be a matter of dispute.

We now offer some concluding remarks about our analysis in relation to the literature.

First, our analysis of market stability hinges on the ability of some investors to borrow in order to buy risky securities. The prediction of our model is that aggregate levered investing in the risky securities should diminish following a decline in stock prices. Perhaps, a stronger prediction is that aggregate levered investing should increase following an increase in stock prices. Other informational models of price instability (mentioned in the introduction) do not predict any association between prices and levered investing.

Further, in our story, margin restrictions and other regulatory measures such as price limits do not have any effect unless they are simultaneously imposed either on a large number of risky securities or on a basket of securities such as a stock index containing a wide range of

securities. This is because a substantial change in the wealth of levered investors is critical for the price changes to be self-fulfilling in equilibrium. The rationale for price limits offered in Brennan (1986) and Kildes and O'Brien (1994), however, continues to be valid even for an individual stock.

Finally, in Brennan (1986) price limits are desirable only if they are imposed in all markets from which levered investors can infer the true market price of the security. In our story, however, information transmission across markets is not critical. To the extent price limits prevent levered investors from realizing large gains and losses that make unstable price movements self-fulfilling, price limits may only be required in markets in which levered investors are largely present while trading continues in other markets. This leads to an intriguing conjecture: in the case of a single market, price limits reduce welfare because they may result in trading halts even when price movements are appropriate given changes in economic fundamentals. Furthermore, the absence of a market price may hinder the efficient allocation of resources. However, with multiple markets, contrary to the recommendation in the Brady Report (1989) that margin requirements across markets be made homogeneous, it may actually make sense to keep margin requirements unequal. This will result in a type of market segmentation, attracting levered investing to the market with lower margin requirements and in which price limits can be imposed to ensure stability. Trading can continue uninterrupted in markets with stricter margin requirements but no price limits. We leave it to future research to explore this possibility rigorously.

Appendix

Conditions and Proofs

Sufficient Conditions for a Unique Solution for $P_1 \neq P_0$

Let $Q_0(P_0)$ and $Q_1(P_1)$ denote the demand functions of risk-averse investors. Rewriting the equilibrium pricing conditions, we get

$$\mu Q_0(P_0) + (1 - \mu) \frac{W}{\alpha P_0} = 1, \quad (A1)$$

$$\mu Q_1(P_1) + (1 - \mu) \frac{W + (P_1 - P_0)W/\alpha P_0}{\alpha P_1} = 1. \quad (A2)$$

Subtracting (A1) from (A2) and rearranging, we get

$$\frac{1}{P_1} - KQ_1(P_1) = \frac{1}{P_0} - KQ_0(P_0), \quad (A3)$$

where

$$K \equiv \frac{\mu}{1-\mu} \frac{\alpha}{1-\alpha} \frac{\alpha}{W} > 0.$$

For there to be a unique solution of $P_1 \neq P_0$ as a function of P_0 , the left-hand side of equation (A3) must be quadratic in P_1 , which implies that

$$\frac{2}{P_1^3} - KQ_1''(P_1)$$

must be either positive or negative but should not reverse its sign. Therefore, the sufficient conditions are that either

$$Q_1''(P_1) < \frac{2}{KP_1^3} \quad \forall P_1 \in [P_{\min}, P_{\max}] \quad (\text{A4})$$

or that

$$Q_1''(P_1) > \frac{2}{KP_1^3} \quad \forall P_1 \in [P_{\min}, P_{\max}]. \quad (\text{A5})$$

For the case analyzed in this article, the demand functions are linear so that $Q_1''(P_1) = 0$, and therefore condition (A4) is satisfied.

Proof of Lemma 1

Define

$$\begin{aligned} A &\equiv \bar{V} - \frac{\lambda}{\mu}, \\ B &\equiv \frac{\lambda}{\alpha} \frac{(1-\mu)}{\mu} Q_0', \end{aligned} \quad (\text{A6})$$

and

$$C \equiv \frac{\lambda}{\alpha} \frac{(1-\mu)}{\mu} W.$$

This implies that X and Y defined in Section IIA can be expressed as

$$X = \frac{1}{2}(A + B) \quad (\text{A7})$$

and

$$Y = X^2 - BP_0^U + C. \quad (\text{A8})$$

From (10),

$$P_0^S = \frac{1}{2}A + \left[\left(\frac{1}{2}A \right)^2 + C \right]^{1/2}. \quad (\text{A9})$$

Notice that the above implies that

$$P_0^S > A. \quad (\text{A10})$$

Equation (A9), after rearranging yields

$$P_0^{S^2} - AP_0^S = C. \quad (\text{A11})$$

Substituting from (A7) and (A11) into (A8), we get

$$Y = X^2 - (2X - A)P_0^U + [P_0^{S^2} - AP_0^S]. \quad (\text{A12})$$

Adding $P_0^{U^2} - AP_0^U$ on both sides of (A12) and rearranging, we get

$$(P_0^{U^2} - P_0^{S^2}) - A(P_0^U - P_0^S) = (X - P_0^U)^2 - Y. \quad (\text{A13})$$

Factoring on both sides of (A13), we get

$$(P_0^U - P_0^S)(P_0^U + P_0^S - A) = (X + \sqrt{Y} - P_0^U)(X + \sqrt{Y} - P_0^U). \quad (\text{A14})$$

Recall that $P_1^H = X + \sqrt{Y}$ and $P_1^L = X - \sqrt{Y}$, and since $P_1^L < P_0^U < P_1^H$, the right-hand side of (A14) must be negative, which implies that

$$P_0^U - P_0^S < 0,$$

as $(P_0^U + P_0^S - A) > 0$ from (A10). Q.E.D.

Proof of Lemma 2

That P_0 is increasing in \bar{V} , W , and μ and is decreasing in α follows directly by inspecting equation (10). To see that P_0 is decreasing in λ , we partially differentiate (10) and simplify to obtain

$$\frac{\partial P_0}{\partial \lambda} = \frac{P_0/2\mu[(1-\mu)W/\alpha P_0 - 1]}{[P_0 - \frac{1}{2}(\bar{V} - \lambda/\mu)]}.$$

The denominator of the above expression is positive, and the numerator is negative since we know that the fraction of shares held by risk-averse investors, $(1-\mu)W/\alpha P_0$, must be less than one.

That \hat{P}_1 is decreasing in \bar{V} and μ and increasing in λ follows directly from equation (9) and from observing that P_0 is increasing in \bar{V} and μ and decreasing in λ .

To see that \hat{P}_1 is increasing in W , we proceed as follows. First notice that

$$\frac{\partial P_0}{\partial W} = \frac{1}{2 \left\{ \left[\frac{1}{2} \left(\bar{V} - \frac{\lambda}{\mu} \right) \right]^2 + \lambda \frac{1-\mu}{\mu} \frac{W}{\alpha} \right\}^{1/2}} \lambda \frac{1-\mu}{\mu} \frac{1}{\alpha} < \frac{1}{P_0} \lambda \frac{1-\mu}{\mu} \frac{1}{\alpha}.$$

Therefore,

$$\frac{W \partial P_0}{P_0 \partial W} < \frac{\lambda \frac{1 - \mu W}{\mu \alpha}}{P_0^2} < 1.$$

This implies that

$$\frac{\partial}{\partial W} \left[\frac{W}{P_0} \right] = \frac{1}{P_0} \left[1 - \frac{W \partial P_0}{P_0 \partial W} \right] > 0.$$

From equation (9) then, it follows that \hat{P}_1 is increasing in W .

Now consider

$$\begin{aligned} \frac{\partial}{\partial \alpha} [\alpha P_0] &= P_0 + \alpha \frac{\partial P_0}{\partial \alpha} \\ &= P_0 - \frac{1}{2 \left\{ \left[\frac{1}{2} \left(\bar{V} - \frac{\lambda}{\mu} \right) \right]^2 + \lambda \frac{1 - \mu W}{\mu \alpha} \right\}^{1/2}} \lambda \frac{1 - \mu W}{\mu \alpha} \\ &> P_0 - \frac{1}{P_0} \lambda \frac{1 - \mu W}{\mu \alpha} \\ &= P_0 \left[1 - \frac{\lambda \frac{1 - \mu W}{\mu \alpha}}{P_0^2} \right] > 0. \end{aligned}$$

From equation (9) then it follows that \hat{P}_1 is decreasing in α . Q.E.D.

Proof of Lemma 3

We first observe that \hat{P}_1 intersects with the bounds for feasibility P_{\min} and P_{\max} at at most one point. To see this, notice that P_{\min} and P_{\max} are both nonincreasing in λ and \hat{P}_1 is increasing in λ . The values P_{\min} and P_{\max} are both nondecreasing in μ , and \hat{P}_1 is decreasing in μ . Last, P_{\min} and P_{\max} do not change with α and W , whereas \hat{P}_1 is monotonic in these parameters. This proves convexity. (See figs. 1-4.)

Now, notice that, if $\alpha = 1$ or $\mu = 1$ or $W = 0$ or $\lambda = 0$, then $\hat{P}_1 = 0$, which is not feasible, and therefore these parameter values are associated with market stability. Similarly, if $\alpha = 0$ or $\mu = 0$, then \hat{P}_1 is unboundedly large and is therefore also infeasible representing market stability. Finally, notice that, since \hat{P}_1 is monotonically increasing in W as well as λ , for sufficiently large values of these parameters, \hat{P}_1 becomes sufficiently large to be infeasible and therefore is associated with market stability. This completes the proof that the set of parameters for which \hat{P}_1 is in the feasible range is closed and bounded and therefore compact. Q.E.D.

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