Incentivizing Impact Investing*

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Abstract

The impact investing asset class is gaining popularity among investors. However, it is not well understood what market mechanisms and which financial securities will attract investors and provide incentives for managers to pursue projects that create social impact. We propose that investors who choose investments in social projects purchase Social Impact Guarantees (SIGs). SIGs are debt-like securities with par values endogenously determined by realized social output. The repayment design aligns the incentives of commercial and socially-conscious investors, allowing joint investment in social projects. SIGs create a social market within standard market frameworks and exploit well-established market mechanisms to increase social investment efficiency. Furthermore, the pricing of SIGs and residual claims provide valuable information to decision makers.

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1 Introduction

The advent of impact investing as an accepted asset class has been met with mixed reactions. Unlike traditional asset classes in which measured return is a straightforward calculation, the relevant returns for impact investments are less definitive. Specifically, there are opposing views regarding whether or not social good (i.e., social return) is an acceptable substitute for financial returns.\(^1\) Despite a lack of consensus regarding the objectives of an impact investment, the asset class is attracting sizable interest. Estimates of the total size of impact investments vary from $50 billion (Monitor Institute, 2009) to nearly $1 trillion (J.P. Morgan, 2010).\(^2\) Many questions are yet unanswered: Is there a mechanism that enables joint investment by those that value social good and those that do not? In businesses in which managers are rewarded with equity-based compensation, is there a mechanism that mitigates the natural tendency for those managers to pursue profit-maximization at the expense of social objectives? What mechanism allows impact investment dollars to be leveraged and have the largest reach?

In this paper, we answer these questions. We show that it is possible to jointly mobilize socially-conscious and commercial investment in businesses that pursue social objectives via a novel security design. Furthermore, the security we propose allows investors to write equity-based compensation contracts for managers that induce the managers to pursue well-defined social objectives and at the same time employ scarce resources efficiently. Our design also allows social investors to leverage their social capital and our security construct can be applied in both the public and private sector, unlike others that have been recently proposed, i.e., Social Impact Bonds (SIBs). SIBs, also called pay-for-success bonds, attract private investment to fund social projects. If the project achieves predefined output targets, the private investors are rewarded with market returns. The SIB design is insufficient for the private sector because it does not provide incentive alignment to the project managers. Our design does.

A candidate project for an impact investment must, by definition, produce both a financial

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\(^1\)Milton Friedman (1970), and more recently Aneel Karnani (2010), argue that businesses have a fiduciary duty to maximize financial returns for shareholders. Businesses must therefore refrain from engaging in any activities that might be perceived as socially desirable if such activities would be detrimental to those returns. Social causes are best left for governments, philanthropic individuals, and organizations, they contend. Conversely, Muhammad Yunus, the recipient of the Nobel Peace Prize in 2006, has contended that free market capitalism is not capable of addressing the important social problems facing the world today. Yunus (2008) argues that those businesses with the ability to undertake an impact investment be organized as social entities in which financial returns must be sacrificed in the pursuit of maximizing social goals.

\(^2\)It is important, however, to point out that the relative size of impact investments relative to other asset classes is quite small: the pool of dollars directed towards impact investments pale in comparison to the amount of funds commercially available for investments that generate market returns.
return and a social good. Many of these projects do not produce sufficient risk-adjusted financial returns to cover their investment costs and are not attractive to for-profit investors. Instead, these projects must be funded by social investors that view social good as an acceptable substitute for financial returns. Naturally, this could lead to a segmented capital market in which one segment — the for-profit segment — funds positive net present value projects and the other segment — the social segment — funds negative net present value projects that also generate ample social good. If the capital providers in the social segment were unconstrained a segmented market would not be problematic. However, social capital is scarce.

Naturally, if one were able to design a market in which for-profit and social investors could jointly invest in social projects, the problem could be remedied. For example, in a world without financial constraints or agency conflicts, social investors could commit to pay the cash value of realized social output (assuming the social good can be measured in equivalent cash units). As such, the project would be considered profitable and it would be undertaken. However, as previously mentioned, social capital is scarce: social investors cannot afford extremely successful social outcomes and the ability of governments to impose taxes on citizens is limited and inefficient. Indeed, this too could be remedied if social investors subsidized the project’s upfront investment and let the project owner undertake it. In reality, however, agents, e.g., project managers, are limited in their ability to commit: granting ex ante subsidies does not provide incentives to maintain social goals through the project’s life. Indeed, the agency conflict associated with limited commitment is at its greatest when social goals are at odds with financial profit maximization.

In this paper, we provide the design of a new security, coined a Social Impact Guarantee (SIG), that mitigates both frictions scarce social capital and limited commitment. A SIG is a security that is sold by the project owner, an agent that cares only about financial returns, to social investors. The proceeds from the security’s sale subsidize the project’s upfront cost sufficiently to make the investment profitable for commercial investors and the security’s contractual payments guarantee that project owner and social investors’ incentives are aligned throughout the project’s life.

SIG’s repayments have a debt-like structure. The project owner must pay the project’s cash profits to the SIG holders up to some par value. We emphasize par value because, unlike standard debt contracts, it is endogenously determined by the realized social output. If social output is large, the par value is small and the project owner pays only this amount while retaining all residual cash generated from the project and the security’s sale. On the other hand, if observable and verifiable output measures suggest that the social project was underfunded, the par value is large which means that the project owner must surrender the project’s cash profits to the SIG
holders.\footnote{Chan (2011) develops a model with a similar insight.} The design implies that the project owner’s incentive to generate social good coincides with profit maximization. That in of itself is valuable, as it indicates that social projects do not need to be managed by both skilled and socially conscious managers. Instead, social projects may be managed by the most competent managers even if they do not value the produced social good.

We also show that our security design is not perfect: a SIG may involve some inefficiency and perfect incentive alignment may not always be possible. The optimal design trades off inefficiency caused by differences in preferences between the project owner and social investors and the limited availability of funds from social investors. Despite potential inefficiencies, SIG’s contractual payments are continuous in both outputs, and, as such, minimize the incentive to manipulate observed outputs for financial gain. We also show that allowing social investors to make ex post payments to the project manager, rather than paying an upfront price, does not trivialize our analysis; the optimal contract in that setting looks similar and balances the same tensions.

From a practical perspective, the design we propose is more widely applicable than Social Impact Bonds (SIBs), which have been proposed and introduced in the U.K., the U.S., and Australia. SIBs involve three parties: investors, guarantors, and project managers. A SIB is sold to an investor and the funds raised are used by the project manager to accomplish some well-defined social objective. If the objective is met, the guarantor (e.g., the government) — who values the social benefit — pays the investors after the success has been demonstrated and independently verified (see Liebman, 2011). This has the advantage of attracting upfront investment from commercial investors with deep-pockets who will get paid back — by the guarantor — only if the social goal is met. This might also elicit effective monitoring by the investor holding the SIB to ensure that the project manager does pursue policies that generate social impact. Although there has been optimistic clamor regarding SIBs, the actual use of the instruments has been underwhelming (Warner 2013). Ultimately, the failure of SIBs to take hold is because their design is effective for only a niche of projects.

The SIB design is not widely applicable for one fundamental reason — the security does not create incentive alignment between all involved parties. Specifically, the SIB does nothing to ensure that the project manager internalizes the impact of his actions. Consequently, SIBs can only be implemented in settings in which the project manager is naturally aligned with the guarantor, e.g., public works projects in which the project managers and guarantors are from the same body. This greatly limits the use of SIBs to only public works projects and excludes worthwhile projects in the private sector. Indeed, SIBs are not suited for the private sector in any regard as the motives
of the SIB investors will be in conflict with other commercial investors who would rather have the firm pursue profit-maximizing opportunities. Thus, a SIB may be useful for raising funds but it does not resolve the conflict of interest among different investors. Our security design, however, does resolve that conflict and is also effective at raising funds.

2 Related literature

Our paper provides a novel addition to the security design literature by considering how to incentivize an agent to invest in a non-rival, public good that is only valued by a subset of investors. Innes (1990) provides the most similar framework to ours. In his model, a financially constrained entrepreneur sells a security to cover a project’s upfront cost and the entrepreneur subsequently makes an unobservable effort choice. The entrepreneur’s effort choice directly influences the project’s cash output which is observable and contractible. Innes shows that the optimal security construct, with the constraint that the repayments be non-decreasing, is a debt contract. We add to his findings by considering that output has two dimensions: cash profit and social good. Furthermore, we demonstrate that the par value of the debt contract is endogenously determined by the realized social output.

Baron (2008) considers a setting similar to ours in which social investors rely on a manager to undertake costly investment in a social good on their behalf. Naturally, the delegation from social investors to the manager creates an agency conflict which is subsequently remedied through corporate governance. Namely, the social investors use their ownership to elicit the desired social investment (and managerial effort) by constructing a compensation contract that maximizes profits and social good production. Our solution, however, does not rely on corporate governance influence or a complex compensation contract. Instead, we only require that the firm’s capital structure contains a Social Impact Guarantee which creates alignment between the social investors’ desires and profit maximization. This simplification allows us to leverage the extensive literature regarding optimal managerial compensation, e.g., equity-based compensation, without reinventing the wheel.

The design we propose necessarily partitions the project’s cash flows between contractual claim holders and residual claim holders. Boot and Thakor (1993) suggest that the partitioning of a firm’s cash flow is attributed to creating an “informationally sensitive” and an “informationally insensitive” component. The two distinct components encourage investors to acquire information, which enhances firm value. In our setup, the partitioning of cash flows induces incentive alignment between those that hold the contractual claim and those that hold the residual. Furthermore, the
partitioning of cash flows in our setup allows the project’s cash production to be financed mostly by for-profit investors allowing social investors to target their limited capital at social good production. Allen and Gale (1988) consider a setup in which security issuance is costly and different groups of investors assess different values to the same security. They show that the optimal security allocates each state-contingent cash flow according to which investors value the proceeds in that state. Our model, however, assumes that all investors value cash profit in the same manner in every state. Investors do differ with regard to whether or not they value social good.\footnote{Madan and Soubra (1991) consider the model of Allen and Gale (1988) with the addition of marketing costs. Chowdhry, Grinblatt and Levine (2005) consider the case in which cash-flow components are denominated in different currencies.}

3 The Model

3.1 The project, the owner, and the social investor

Investment in social projects often produces some financial return alongside achievement of a social good. Impact investing is concerned with funding such investment projects; however, the capital available from socially conscious investors is limited. It is natural then to consider the possibility of financing part of the investment cost from the relatively deep pockets of traditional profit-motivated investors. Including profit-motivated investors in the ownership of the project, however, brings about a conflict of interests because these investors do not weigh social good in realized returns.

In particular, if the impact investment generates sufficient financial return (i.e., it is positive net present value without consideration of the social good), the investment may be funded entirely by those investors that are solely profit-motivated.\footnote{However, as a caveat, the chosen level of investment may be lower than what a social investor would invest.} But, if the impact investment does not generate sufficient financial return it will not attract any investment from for-profit investors without the involvement of social investors that see social good as a sufficient substitute for financial return. In what follows, we model exactly this type of impact investment, which is uniquely positioned for joint investment by both profit-motivated and social investors.

Consider a social project that meets the necessary criteria to be considered an impact investment, namely it produces both cash profit and social good. Undertaking the project requires a fixed upfront cost equal to $k$. If the project is undertaken, for some additional level of investment $i \geq 0$ over the upfront cost $k$, the cash profit is a random variable $x \geq 0$ with conditional density $f(x|i)$, and the social good produced is a random variable $s \geq 0$ with conditional density $g(s|i)$. 
We assume that conditional on investment, cash profit and social good are independent so that the joint density function is simply the product of the two marginal densities $f(x|i)g(s|i)$.

The project’s ownership rights are endowed to a regular profit-maximizing agent who values only the cash profit (we refer to the agent as the “owner” hereafter and we will use superscript $\pi$ to characterize him). That the owner is concerned only with maximizing wealth is designed to model the notion that the number of agents who are both talented managers and socially conscious, may be limited. We assume that the project owner has unlimited access to capital from for-profit investors at the gross market rate of return $\rho$, which we normalize to one. In effect, the project owner can always choose to invest in an alternative project that is zero net present value and has unlimited investment capacity. This feature captures the idea that profit-maximizing agents possess an alternative use for capital outside of the social project. That is, they will be tempted to invest in these profit-maximizing projects instead if the financial return on the social project does not exceed its opportunity cost. We assume the owner operates in a competitive market and expects to earn zero profits.\(^6\)

There also exists a social investor who values both cash profit and social good (we will use superscript $\psi$ to characterize this investor). The social investor, however, is constrained and has a maximum capital budget $\beta > 0$ where $\beta$ is less than the fixed cost of the investment $k$.\(^7\) This assumption is made to capture an important feature of impact investing — the funds available from socially-minded investors are limited so that many social projects are forgone because they cannot be solely funded by investors who are willing to accept a smaller rate of cash return on their invested capital. Investment occurs in period 1, and the payoffs — which are assumed to be observable — occur in period 2.

Both $f(x|i)$ and $g(s|i)$ satisfy a monotone likelihood ratio property (MLRP):

\[
\frac{\partial}{\partial x} \left[ \frac{f_i(x|i)}{f(x|i)} \right] > 0, \tag{1}
\]

and

\[
\frac{\partial}{\partial s} \left[ \frac{g_i(s|i)}{g(s|i)} \right] > 0, \tag{2}
\]

where $f_i(x|i) \equiv \frac{\partial f(x|i)}{\partial i}$ and $g_i(s|i) \equiv \frac{\partial g(s|i)}{\partial i}$. These conditions characterize a necessary feature of

\(^6\)Shortly, we will introduce a security that is traded between the project owner and the social investor. The assumption that the project owner is competitive is natural and allows us to pin down a unique price for the security.

\(^7\)The units of social good $s$ are denominated in the cash currency. One can think of the value as being ascribed to the social investor’s shadow price for additional investment.
impact investments: greater investment is associated with higher outputs in that social project and vice versa.

Our setup so far is redolent of Innes (1990). As such, we make the following assumption in the same spirit,

**Assumption 1.** There exists a finite \( i^{max} \) such that

\[
\lim_{i \to i^{max}} \int_{0}^{\infty} x f(x|i) \, dx + \int_{0}^{\infty} s g(s|i) \, ds - i - k < 0.
\]

(3)

Assumption 1 implicitly requires that the cash profit and social good returns on invested capital are finite and that they are negative as \( i \) approaches \( i^{max} \). The assumption allows us to focus on the investment choice set \([0, i^{max}]\) without loss of generality. We also make an assumption for expected output with respect to \( i \),

**Assumption 2.** Both \( \int_{0}^{\infty} x f(x|i) \, dx \) and \( \int_{0}^{\infty} s g(s|i) \, ds \) are concave in \( i \).

Assumption 2 is a regularity condition that allows us to focus on unique solutions to the project owner’s investment choice problems we consider hereafter. The assumption is natural as well — the social project demonstrates diminishing returns to investment.

We begin by considering the project owner’s and investor’s joint surplus maximizing level of investment in a frictionless economy, that is, no agency conflicts or financial constraints exist. In this case, the joint surplus function is given by,

\[
W(i) \equiv E_x [x|i] + E_s [s|i] - i - k,
\]

(4)

and the surplus maximization problem is,

\[
\max_i W(i).
\]

(5)

The solution, \( i^{FB} \), to the “first-best” choice problem is characterized by the first-order condition

\[
0 = \int_{0}^{\infty} x f_i(x|i^{FB}) \, dx + \int_{0}^{\infty} s g_i(s|i^{FB}) \, ds - 1,
\]

(6)

so long as \( W(i^{FB}) \geq 0 \). We assume that the \( W(i^{FB}) \geq 0 \) hereafter to focus on the case in which the social project should be undertaken.\(^8\)

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\(^8\)We emphasize joint surplus, as opposed to welfare, intentionally. Because the social project produces a public good, the relevant welfare function would require additional assumptions regarding the economic environment. We do not consider the standard political economy questions regarding the provision of a public good. Instead, we are concerned with the optimal bilateral contract between an agent that values the social good and one that does not.
Now, recall that the project owner cares only about cash profit. His level of investment is the solution to the following optimization,

$$\max_{i \geq 0} E_x[x|i] - i - k$$

s.t. $$E_x[x|i] - i - k \geq 0.$$  \hspace{1cm} (7) \hspace{1cm} (7.1)

Let $$i^{\pi}$$ denote the solution if the participation constraint is slack. Then $$i^{\pi}$$ is implicitly defined by,

$$\int_{0}^{\infty} x f_i(x|i^{\pi}) = 1.$$  \hspace{1cm} (8)

Assumption 2 implies that $$i^{\pi} < i^{FB}$$. In other words, the project owner will underinvest in this social project. We further assume that $$E_x[x|i^{\pi}] - i^{\pi} - k < 0$$ so that he will choose not to invest in the social project at all. This captures the central problem with impact investing — regular profit-maximizing agents do not find it profitable to invest in projects that may be socially beneficial because they are not profitable enough to recoup their cost of capital. This allows us to focus on the set of impact investments that are not undertaken at all.\(^9\)

Recall, both the project owner and the social investor enjoy cash returns; however, only the social investor cares about the social good. Intuitively, if one were able to bifurcate the projects outputs and the investment costs, it would be sensible to have the profit-motivated agent fund the cash-producing segment as much as possible. This would allow the social investor to focus more of his limited capital directly towards the social good-producing segment. However, the social project we consider is not divisible. Instead, we consider a security construct that harnesses the above intuition and creates a payoff structure that returns much of the project’s cash to the project owner. The security, which is sold by the project owner to the social investor, allows the impact investment to be undertaken via joint investment by the project owner and the social investor. In addition to dividing the project’s payoffs, the security accomplishes a dual role: first, the payoff structure improves the ex ante incentives of the project owner to invest in the social project, and, second, the security’s upfront price is affordable for the social investor, i.e., the security leverages the social investor’s limited capital.

\(^9\)Albeit, there may exist projects that generate sufficient financial returns and would be undertaken by regular profit-maximizing agents. Those projects, however, may be underfunded as compared to the levels of investment desired by social investors. Our analysis extends easily to such projects and the security construct we prescribe later in this paper would lead to an increase in investment.
3.2 The Social Impact Guarantee

The investment above and beyond the upfront fixed cost is unobservable and thereby non-contractible. Both outputs and the initial capital outlay $k$, however, are observable and contractible.\(^\text{10}\) Consider a traded security offered by the project owner to the social investor. The security has price $p^\psi$ and pays $y^\psi(x, s)$ in period 2. It is important that both variables, $x$ and $s$, be ex post observable and contractible.

The ex ante payoff to the project owner is equal to the expected cash from for-profit investment minus the expected security repayment to the social investor plus the cash raised from the initial sale of the security,

$$E_x[x|i] - i - k - E_{x,s}[y^\psi(x, s)|i] + p^\psi. \quad (9)$$

When considering the design of the security, we assume (i) $y^\psi(x, s) \leq x$: the project owner cannot be required to pay more than the cash profits produced to the social investor;\(^\text{11}\) (ii) $y^\psi(x, s) \geq 0$: the social investor’s liability is limited to his initial investment;\(^\text{12}\) and (iii) $y^\psi(x, s) \leq y^\psi(\hat{x}, s)$ for all $x \leq \hat{x}$: the security payoff is non-decreasing in cash profit.

Both (i) and (ii) are standard limited liability constraints. We motivate (iii) by considering the possibility for the project owner to report greater earnings than those produced by the investment project.\(^\text{13}\) Given our assumption on the cost of the capital, the project owner can always supplement cash output with costless borrowing to make it appear as if he had undertaken more upfront investment. A non-decreasing contract eliminates any incentive for this type of manipulation. Since social output is measured by an independent third party, we do not restrict the contract to be non-decreasing in $s$. However, as we will show shortly in Lemma 2, the optimal contract is continuous in social output. Any gains from a slight manipulation in social output will therefore be relatively small.

\(^\text{10}\)The ability to observe the initial capital outlay $k$ and the inability to observe incremental investment $i$ captures the notion that undertaking a project is observable but investment in the project is not (e.g., one can observe if a factory was built, but cannot observe how much investment has been made into it).

\(^\text{11}\)If the cash produced by traditional for-profit projects is observably different from the cash produced by social projects, this assumption can be relaxed to include a setting in which the social project is one of several projects in a firm’s portfolio. In this case, the security design depends on both the realized outputs from the social project and the output from the firms’ other projects. In particular, the total combination of firm cash can be used to help guarantee the payment to SIG investors.

\(^\text{12}\)The model could be generalized to consider the case in which social investors are able to commit to payments after observing the social output in period 2. Because social capital is limited and commercial capital is not, this will enhance welfare, but the basic tradeoffs analyzed in our simple setup will remain as long as social investors’ ability to pay is limited. See Appendix B.2 for a characterization of the optimal contract with pledgable assets.

\(^\text{13}\)The addition of a monotonicity constraint is standard in the security design literature, e.g., Innes (1990) and Gangopadhyay et al. (2005).
The investment decision is made after the security has been sold to the social investor. Thus, the project owner does not internalize the impact of his investment choice on the price of the security. Consequently, the level of investment chosen by the project owner is the solution to the following optimization,

$$\max_i E_x[i] - E_{x,s}[\psi(x,s) | i] - i.$$ (10)

Since the social investor benefits from the investment in the social project, he internalizes the impact of the security on the expected level of social output. Thus, the ex ante payoff from owning the security is equal to the expected security payoff plus the expected social good minus the upfront cost of the security,

$$E_{x,s}[\psi(x,s) | i] + E_s[i] - p^\psi.$$ (11)

Recall that the project owner is competitive, and, necessarily, his individual rationality constraint binds at zero, i.e., his expected payoff (9) is equal to zero. The binding constraint implies the security’s price is,

$$p^\psi = i + k - E_x[i] + E_{x,s}[\psi(x,s) | i],$$ (12)

where $i$ is the equilibrium level of investment. A substitution of this explicit form of the price into the social investor’s payoff yields,

$$E_x[i] + E_s[i] - i - k,$$ (13)

which is equal to the joint surplus function from (4).

All surplus generated from the interaction between the project owner and the social investor accrues to the social investor. This is because the project owner is competitive. The contract that maximizes the social investor’s expected payoff, and as result the joint surplus, solves the following optimization problem,

$$\max_{\{i,y^\psi()\}} E_x[i] + E_s[i] - i - k$$ (14)

s.t. $E_x[i] + E_s[i] - i - k \geq 0$ (14.1)

$$i \in \arg \max_{i'} E_x[i'] - E_{x,s}[\psi(x,s) | i'] - i'$$ (14.2)

$$y^\psi(x,s) \geq 0 \ \forall \ (x,s)$$ (14.3)

$$y^\psi(x,s) \leq x \ \forall \ (x,s),$$ (14.4)

$$p^\psi \leq \beta ,$$ (14.5)

$$y^\psi(x,s) \leq y^\psi(\hat{x},s) \ \forall \ x \leq \hat{x},$$ (14.6)

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where (14.1) is the social investor’s individual rationality constraint, (14.2) is the incentive compatibility constraint of the project owner, (14.3) is the social investor’s limited liability constraint, (14.4) is the project owner’s limited liability constraint, (14.5) requires that the security’s price is less than $\beta$, and (14.6) requires that the security’s payoff is non-decreasing in realized profit.

Before we proceed to solving the social investor’s optimization in (14) we define $h(x,s,i)$ as,

$$h(x,s,i) \equiv \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)},$$

which is the likelihood function for joint output. The assumption that the marginal distributions satisfy MLRP implies that $h()$ is an increasing function in $x$ and $s$ for all $i$. Furthermore, MLRP dictates that the distribution $f(x|\tilde{i})$ first-order stochastically dominates $f(x|\hat{i})$ for all $\hat{i} > \tilde{i}$, i.e., density is shifted from lower states to higher states. It is also true that $\int_0^\infty f_i(x|i) \, dx = 0$ for all $i$; that is, shifting the density is a zero-sum game. The expression we call $h(x,s,i)$ in (15) is a normalization: the changes in density are divided by the densities themselves, $f_i(x|i)/f(x|i)$ and $g_i(s|i)/g(s|i)$. The normalization provides a measurement of relative density increases (or decreases) across states, where each state is a unique combination of $x$ and $s$. A positive value of $h(x,s,i)$ implies a relative increase in density in that state. From an incentive perspective, providing the entire cash flow, $x$, to the project owner when $h(x,s,i)$ is positive and forcing him to pay it out when the function is negative maximizes ex ante investment incentives. That is, an all-or-nothing security construct with a threshold of $h = 0$ maximizes the project owner’s incentive to invest in the social project. An all-or-nothing security construct, however, violates our requirement that the security’s repayment be non-decreasing in $x$. We refer the reader to Appendix B, where we consider the optimal security construct in the absence of that constraint. The following two lemmas characterize the optimal security construct that solves the social investor’s optimization in (14).

**Lemma 1.** For any given $s$, the security which maximizes the joint surplus (given constraints 14.1-14.6) and which is non-decreasing in $x$ is a debt contract with respect to $x$.

**Lemma 2.** The optimal security which satisfies constraints 14.1-14.6 and which is non-decreasing in $x$ is a debt-like security in which the par value of debt decreases in $s$. This security is continuous in $s$, and the par value of debt, $\overline{\pi}(s,i,h^-)$, in any cross section of $s$ is implicitly defined by,

$$h(x,s,i)|_{x=\overline{\pi}(s,i,h^-)} = h^-,$$

for some $h^- < 0$. 

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According to Lemma 1, for any cross section in \((x, s)\) in which \(s\) is fixed, the optimal contract resembles a debt contract. That is, for low values of \(x\) below some threshold (the so called “par value” of debt which depends on \(s\)), the security pays all cash flows to the investor. Above this threshold, the security pays the par value to the investor with the owner retaining the residual cash flow. In Appendix B we consider the optimal non-monotonic security and demonstrate that optimal investment incentives are achieved by rewarding the project owner for output realizations which are indicative of high initial investment and punishing the project owner otherwise. The same intuition leads debt to be optimal here. For a given realization of \(s\), low realizations of \(x\) indicate low initial investment. Given limited liability, the debt contract penalizes the project owner as much as possible when \(x\) is below the par value by transferring all project profits to the social investor. When \(x\) is above the par value, the optimal security rewards the project owner as much as possible without violating monotonicity. That is, the security repays the social investor just the par value and assigns all residual cash flows to the project owner.\(^{14}\) Furthermore, because our non-decreasing constraint applies only to \(x\), the par value of debt for each cross section of \(s\) can be evaluated in pointwise fashion.

Additionally, Lemma 2 dictates that the threshold is non-positive. A lower threshold makes the contract more affordable by allowing the project owner to keep a greater share of \(x\) in more states. This is valuable due to the social investor’s budget constraint, as it necessarily lowers the security’s upfront price. However, lowering the threshold may also have an adverse effect on ex ante investment incentives: it allows the project owner to keep a greater share of the project’s realized cash, even if outputs are suggestive of low investment. As the threshold decreases, the contract becomes increasingly affordable, but investment incentives may be tapered.

**Definition 1.** We define the security characterized in Lemmas 1 and 2 as a Social Impact Guarantee (SIG). That is, a SIG is a debt-like contract sold by a project owner to a social investor. The security’s repayment schedule requires that the par value of the contract decrease with the measured social output.

Definition 1 is the heart of our paper: a Social Impact Guarantee, as defined, better aligns the incentives of the project owner and social investors, and mobilizes social capital. The security

\[^{14}\text{In general, any non-debt-like contract which satisfies limited liability, monotonicity, and is a revenue neutral (i.e., the upfront price is unchanged) necessarily includes a smaller penalty for sufficiently low realizations of } x \text{ and a smaller reward for sufficiently high realizations of } x \text{ when compared with our debt-like contract. Innes (1990) considers a one dimensional setup where the project’s only output is cash. He demonstrates that any revenue neutral deviation (i.e., equal expected repayment to investors) from a debt contract to another monotonic security induces lower ex ante incentives.}\]
Figure 1: The Social Impact Guarantee is non-decreasing in project profit and is a debt-like security in which the par value of debt declines as measured social output increases.

is non-decreasing in cash profit $x$, and, as mentioned previously, is continuous in realized social output $s$. As such, the incentive to manipulate measured cash profit is suppressed and any small manipulation of realized social production has a small effect on repayment. It is also important that a SIG’s par value decrease with the realized social output. If it did not, that is, if it resembled a standard debt contract with a fixed par value, the contract would adversely affect incentives. In fact, the project owner’s incentive to invest in the project is lower with a standard debt contract than in the absence of it. A visual depiction of the SIG is illustrated in Figure 1: for each cross section of $s$, the contract resembles a debt contract and the par value is decreasing with $s$.

4 Implementing SIGs

4.1 SIGs and measurement of social output

SIG’s contractual payments rely crucially on well-defined measurable benchmarks. In addition to cash flow performance measures that are more readily available, we also require reliable indicators of social performance. There are several recent developments that make this possible. For example, the Benefit Corporation requires “reports on its overall social and environmental performance using recognized third party standards.” (See benefitcorp.net). Impact Reporting & Investment
Standards (IRIS : http://iris.thegiin.org/) and GIIRS Ratings and Analytics for Impact Investing (http://giirs.org/) are examples of organizations that also help establish independent standards similar to the role played by credit rating agencies in providing useful default information on corporate bonds.

4.2 SIGs trading

We also consider that a SIG may trade in the secondary market. The prices of the SIG and other firm securities will aggregate investor information that might be useful both for managers as well as investors.\(^{15}\) For example, a rise in the secondary price of the SIG would indicate that the firm is less likely to meet its social objective benchmark. If the SIG is senior to a firm’s other claims, the SIG does not need to trade to provide useful information via prices: the prices of all junior claims will provide indirect information about the SIG’s performance. Furthermore, in a dynamic model in which the firm repeatedly raises funds, secondary pricing of securities provides useful information to managers about which social objectives will likely be valued more highly by investors (and society) in the future. This allows them to build capacities for future expansion in desirable social activities.\(^{16}\)

4.3 SIGs and atomistic Investors

Social good is a non-rival public good, and, as such, free-riding potentially impedes individual social investors from participating in the sale of a SIG. In our earlier analysis, we informally treat social utility as if there is a single social investor whose value for social output is equal to the total social surplus generated. In practice, this simplification is problematic since individual social investors do not generally internalize the benefit of social output to all other investors. Albeit, the contract derived in Section 3.2 may still incrementally improve social welfare if social investors are willing to pay, at least in part, for the increase in social investment that results from their security. A social investor is only willing to pay if he believes that the equilibrium level of social investment depends critically on his purchase of the SIG. This, however, prohibits the sale of SIGs to small social investors.

If investors are indeed atomistic, each investor recognizes that his purchase of the SIG has a negligible impact on the project owner’s investment incentives. Since each individual contract does not improve incentives for social investment, each investor is unwilling to pay upfront for any

\(^{15}\)See Subrahmanyam and Titman (1999).
\(^{16}\)See Subrahmanyam and Titman (2001) for an argument on how stock prices provide useful information on firms' future cash flows.
expected increase in social output. Consequently, the contracts will fail to transfer any social utility from social investors to the project owner, prohibiting the project owner’s zero-profit constraint from being satisfied. As a result, for the security to successfully improve welfare, it must be sold to large social block holders who internalize (and pay for) their direct impact on social investment, e.g., the Bill and Melinda Gates Foundation and the J. Paul Getty Trust. It is worthwhile to mention that the case for large social block holders is further motivated by our analysis in Appendix B. In that Appendix, we allow the SIG holder to pledge his residual social capital, $\beta - p$, as an ex post payment to the project owner. We show that ex post payments from the investor to the project owner increase investment incentives and leads to a security construct that weakly dominates our design from Section 3.2.

### 4.4 SIGs and renegotiation

The existence of a secondary market for trading SIGs critically depends on whether or not the contract is renegotiable. If the project owner’s incentives remain unchanged when the security is resold, i.e., the contract cannot be legally renegotiated or investment has already been made (and is consequently fixed), a secondary market can exist without compromising the security’s intent. This is because the original social investor continues to enjoy the benefits of greater social investment even if he resells the security. Furthermore, the secondary market value of the security to those that value social good and to those that do not is the same (the expected cash flow). If, however, the security is not renegotiation proof, i.e., either investment has not been made or investment can be liquidated at low enough cost, secondary market trading may be limited. This is evident by considering a sale to a different investor that only cares about cash profit: once the security is sold to this investor, the expected utility from owning that security is $E_{x,s}[y(x,s)|i]$. Recall that the expected profit from investment for the project owner is $E_x[x|i] - E_{x,s}[y(x,s)|i] - i - k$. Thus, the sum of utilities to the project owner and security holder is equal to $E_x[x|i] - i - k$, which is the project owner’s profit maximization problem under sole-ownership in (7). Here, total profit is maximized by letting $i = i^\pi$, which can be obtained by renegotiating the security to the null contract. However, if the initial social investor anticipates that the contract will be renegotiated and that social investment will subsequently be reduced, the security’s intent unravels. Thus, resale of the security to investors that do not value social good is not possible.
4.5 SIGs and the prices of residual securities

The sale of a SIG may provide useful information to market participants through prices, even if a secondary market for the SIG does not exist. Security prices play an important role in aggregating investors’ diverse private information (Hayek 1945). In this way, the introduction of a new security with cash flows tied directly to achievement of social output provides a new channel for firms, investors, and policy makers to acquire information related to the value of social good production. If the SIG is not renegotiable and trading is liquid, its market price provides information directly. Conversely, if a secondary market is not supported for reasons previously discussed or the SIG’s price is considered stale due to infrequent trade, information may still be available. In particular, if the social project is held by a publicly traded firm, the firm’s stock price provides an indirect measure of social good production: recall that the residual profit which flows to project owners is net of the repayment of the SIG. Furthermore, analyst reports on earnings forecasts and target prices may provide additional resources for teasing out the portion of firm value which reflects social good versus expected earnings.
Appendix A

Proof of Lemma 1:

Similar to Innes (1990), we consider a non-debt security \( y_{ND}(x, s) \) which is non-decreasing in \( x \) and show that there exists an affordable debt-like contract \( y_D(x, s) \) which generates at least as much surplus as \( y_{ND}(x, s) \).

Consider an affordable contract \( y_{ND}(x, s) \) (i.e., the upfront price \( p_{\psi, ND} \) is less than \( \beta \)) which is offered by the social investor to the project owner and which is non-decreasing in \( x \) for all \( s \). Suppose that for \( s \in \bar{S} \) the contract is not a debt contract with respect to \( x \) (where \( \bar{S} \) is a positive measure subset in the domain of \( s \)). And suppose that for \( s \not\in \bar{S} \), the contract is a debt contract (where \( s \not\in \bar{S} \) is potentially empty). Denote by \( i_{ND} \) the level of investment the project owner chooses when faced with the contract \( y_{ND}(x, s) \). Then \( i_{ND} \) is implicitly defined by,

\[
\int_0^\infty \int_0^\infty (x - y_{ND}(x, s))h(x, s, i_{ND})f(x | i_{ND})g(x | i_{ND}) \, ds \, dx = 1.
\] (A1)

We now consider two cases: (i) \( i_{ND} < i_{FB} \) and (ii) \( i_{ND} = i_{FB} \).

Case (i): \( i_{ND} < i_{FB} \)

Suppose \( y_{ND}(x, s) \) is a solution to the optimal contract problem outlined in (14), and consider \( s' \in \bar{S} \). Then \( y_{ND}(x, s') \) is not a debt contract with respect to \( x \) (on a subset of positive measure of \( x \)). Since the expected value of a standard debt contract is monotonic in the face value of debt, it is straightforward to show that there exists a unique contract \( y_D(x, s = s') \) which is a debt contract with respect to \( x \) and which satisfies,

\[
\int_0^\infty y_D(x, s = s')f(x | i_{ND}) \, dx = \int_0^\infty y_{ND}(x, s = s')f(x | i_{ND}) \, dx.
\] (A2)

We denote the par value of \( y_D(x, s = s') \) by \( \bar{X}(s') \). Since a debt contract entails maximal repayment to the social investor for \( x \leq \bar{X}(s') \), there exists a threshold \( x_B(s') \geq \bar{X}(s') \) such that

\[
y_{ND}(x, s = s') \leq y_D(x, s = s') \text{ for all } x < x_B(s'),
\] (A3)

\[
y_{ND}(x, s = s') \geq y_D(x, s = s') \text{ for all } x \geq x_B(s'),
\] (A4)

with strict inequality on some set of positive measure. That is, \( y_{ND}(\cdot) \) entails higher repayment for high profit states and lower repayment for low profit states when compared to \( y_{D}(\cdot) \). From MLRP, this implies,

\[
\int_0^\infty (x - y_D(x, s'))h(x, s', i_{ND})f(x | i_{ND}) \, dx > \int_0^\infty (x - y_{ND}(x, s'))h(x, s', i_{ND})f(x | i_{ND}) \, dx.
\] (A5)
The preceding analysis shows that for \( s' \in \tilde{S} \) there exists a unique debt contract which has the same expected repayment to the social owner for \( i = i^{ND} \). We can replicate this strategy state-by-state for all \( s \in \tilde{S} \) so that,

\[
\int_0^\infty y^D(x, \hat{s}) f(x|i^{ND}) \, dx = \int_0^\infty y^{ND}(x, \hat{s}) f(x|i^{ND}) \, dx \text{ for all } \hat{s} \in \tilde{S}.
\]

(A6)

Then \( y^D(x, s) = y^{ND}(x, s) \) for all \( s \notin \tilde{S} \) and \( y^D(x, s) \) is given by the above transformation for all \( s \in \tilde{S} \). Then by definition, \( y^D(x, s) \) is a debt contract with respect to \( x \) for all \( s \) and yields the same expected repayment,

\[
\int_0^\infty \int_0^\infty (x - y^D(x, s)) f(x|i^{ND}) g(x|i^{ND}) \, ds \, dx = \int_0^\infty \int_0^\infty (x - y^{ND}(x, s)) f(x|i^{ND}) g(x|i^{ND}) \, ds \, dx.
\]

(A7)

And from MLRP,

\[
1 = \int_0^\infty \int_0^\infty (x - y^{ND}(x, s)) h(x, s, i^{ND}) f(x|i^{ND}) g(x|i^{ND}) \, ds \, dx
\]

\[
< \int_0^\infty \int_0^\infty (x - y^D(x, s)) h(x, s, i^{ND}) f(x|i^{ND}) g(x|i^{ND}) \, ds \, dx.
\]

(A8)

Now let \( i^D \) denote the level of investment the project owner chooses in response to the contract \( y^D(x, s) \). This implies that \( i^D \) is implicitly defined by,

\[
\int_0^\infty \int_0^\infty (x - y^D(x, s)) h(x, s, i^D) f(x|i^D) g(x|i^D) \, ds \, dx = 1,
\]

(A9)

and from the concavity of the objective function, \( i^D > i^{ND} \). Importantly, the contract \( y^D(x, s) \) is also affordable. By construction,

\[
- \int_0^\infty \int_0^\infty (x - y^D(x, s)) f(x|i^{ND}) g(x|i^{ND}) \, ds \, dx + i^{ND} + k
\]

\[
= - \int_0^\infty \int_0^\infty (x - y^{ND}(x, s)) f(x|i^{ND}) g(x|i^{ND}) \, ds \, dx + i^{ND} + k
\]

\[
= p^{\psi,ND}
\]

(A10)

\[
\leq \beta.
\]

(A11)

(A12)
Furthermore, since $i^D = \arg\max_{i'} \int_0^\infty \int_0^\infty (x - y^D(x,s)) f(x|i') g(x|i') \, ds \, dx - i' - k$, we have,

$$p^\psi,D = -\int_0^\infty \int_0^\infty (x - y^D(x,s)) f(x|i^D) g(x|i^D) \, ds \, dx + i^D + k$$  \hspace{1cm} (A13)

$$\leq -\int_0^\infty \int_0^\infty (x - y^D(x,s)) f(x|i^{ND}) g(x|i^{ND}) \, ds \, dx + i^{ND} + k$$  \hspace{1cm} (A14)

$$\leq \beta.$$  \hspace{1cm} (A15)

So the debt-like security $y^D()$ with price $p^\psi,D$ is affordable and induces $i^D > i^{ND}$.

If $i^D \leq i^{FB}$, then $i^{ND} < i^D \leq i^{FB}$, and from the concavity of the joint surplus function in (4), total surplus and the social investor’s payoff are strictly greater under the debt-like contract $y^D()$ than under the non-debt contract $y^{ND}()$. This contradicts the optimality of $y^{ND}()$.

Alternatively if $i^D > i^{FB}$ there exists a debt-like contract $y^\overline{D}()$ such that $y^\overline{D}()$ is affordable and $i^\overline{D} = i^{FB}$. Since $y^D(x,s)$ is a debt-like security, it is characterized by a series of par values $\pi(s)$ for all $s$. Now consider an alternative security $y^\overline{D}(x,s,\alpha)$ such that the par values for this security are equal to $\alpha \pi(s)$ for all $s$. Then $y^\overline{D}()$ is a debt-like contract for all $\alpha \geq 0$. Furthermore, the contract induces a level of investment $i^\overline{D}$,

$$i^\overline{D} = \arg\max_{i'} \int_0^\infty \int_0^\infty (x - y^\overline{D}(x,s,\alpha)) f(x|i') g(x|i') \, ds \, dx - i' - k,$$  \hspace{1cm} (A16)

and his choice is continuous in $\alpha$.

Now let $\alpha = 1$, then $i^\overline{D} = i^D > i^{FB}$. Alternatively let $\alpha = 0$, then $i^\overline{D} = 0 < i^\pi < i^{FB}$. By the intermediate value theorem, there exists $\alpha^* \in [0,1]$ such that $i^\overline{D} = i^{FB}$.

All that remains is to show that $y^\overline{D}(x,s,\alpha^*)$ is affordable. Since $y^\overline{D}(x,s,\alpha^*)$ has a lower par value than $y^D(x,s)$ for all $s$, it is clear that,

$$\int_0^\infty \int_0^\infty (x - y^\overline{D}(x,s,\alpha^*)) f(x|i^D) g(x|i^D) \, ds \, dx - i^D \geq \int_0^\infty \int_0^\infty (x - y^D(x,s)) f(x|i^D) g(x|i^D) \, ds \, dx - i^D.$$

(A17)
Then, since \( i^{FB} \) minimizes the project owner’s loss from incremental investment given \( \alpha^* \), we have,

\[
p^{\psi,D} = -\int_0^\infty \int_0^\infty (x - y(x,s,\alpha^*)) f(x|i^{FB}) g(x|i^{FB}) \, ds \, dx + i^{FB} + k
\]

\[
\leq \int_0^\infty \int_0^\infty (x - y(x,s,\alpha^*)) f(x|i^{D}) g(x|i^{D}) \, ds \, dx + i^{D} + k
\]

\[
\leq p^{\psi,D}
\]

\[
\leq \beta.
\]

So there exists a debt-like contract \( y^D(x,s,\alpha^*) \) which is affordable and implements the first-best level of investment. This contradicts the optimality of \( y^{ND}(x,s) \).

Case (ii): \( i^{ND} = i^{FB} \)

In the proof of case (i), we show that there exists a debt-like security \( y^D(x,s) \) which is affordable and which entails \( i^D > i^{ND} = i^{FB} \). Furthermore, we show that whenever there is a debt-like security with \( i^D \geq i^{FB} \), then there exists an alternative debt-like security \( y^D(x,s,\alpha^*) \), which is affordable and which implements \( i^D = i^{FB} \). Thus, the set of optimal securities includes a debt-like contract.

Proof of Lemma 2:

In the proof of Lemma 1, we show that the optimal security is a debt-like contract with respect to \( x \) for each \( s \). For each \( s \), debt is optimal because any non-debt contract contract necessarily entails higher repayment in high \( h(x,s,i) \) states and lower repayment in low \( h(x,s,i) \) state when compared with a debt-like contract that has the same expected repayment (i.e., upfront price) for a given level of investment.

A complete proof of Lemma 2 includes a similar construction: present a contract which is non-decreasing or discontinuous in \( s \) and show that there exists a revenue-neutral deviation to a continuous contract that is decreasing in \( s \) which induces greater investment in the social project. Since the formal steps mimic those applied in Lemma 1, we omit them here and instead describe the relevant deviations.

Let \( \bar{\pi}(s,i,\hat{h}) \) denote the par value of debt for a given \( s \), level of investment \( i \), and threshold \( \hat{h} \), and consider a contract such that \( \bar{\pi}(s_1,i,\hat{h}) - \bar{\pi}(s_2,i,\hat{h}) > 0 \) for some \( s_1 > s_2 \). Given this contract, repayment is strictly higher for all \( x > \bar{\pi}(s_2) \) when \( s = s_1 \) than when \( s = s_2 \). However \( h(x,s_1,i) > h(x,s_2,i) \) \( \forall x \), so following the intuition in Lemma 1, investment incentives can be improved without impacting affordability by lowering the \( \bar{\pi}(s_1,i,\hat{h}) \) and increasing \( \bar{\pi}(s_2,i,\hat{h}) \) such that expected repayment remains the same. Thus any security in which the par value of debt
increases in social output is weakly dominated by an alternative contract in which the par value of debt is decreasing in $s$.

The next step is to show that the monotonic (in $x$) security is continuous in $s$. First note that it suffices to show that the par value of debt is continuous in $s$. Since we have already established that the par value is decreasing in $s$, we must rule out the possibility of a discontinuous downward jump in $\pi(s, i, \hat{h})$ for some $s$, investment $i$, and threshold $\hat{h}$. Consider a security with such a discontinuity at $s^*$. Since $h(x, s, i)$ is continuous in $x$ and $s$ for any $i$, there exists $\epsilon > 0, x^+$, and $x^-$ such that $y(x_1, s^* + \epsilon) < y(x_2, s^*)$ and $h(x_1, s^* + \epsilon, i) < h(x_2, s^*, i)$ for all $x_1 < x^+$ and all $x_2 > x^-$. Thus, there exists a revenue neutral deviation which improves investment incentives by lowering the par value of debt for $s = s^*$ and increasing the par value for $s = s^* + \epsilon$. 

$\square$
Appendix B

B.1 Optimal Non-Monotonic Security

In this subsection we consider the setup from Section 3.2 and we solve for the optimal non-monotonic security to establish intuition for the proofs in Appendix A. Consider the social investor’s problem outlined in (14) without the requirement that the payoff be non-decreasing in \( x \). The social investor’s problem is,

\[
\max_{\{i, y^\psi(\cdot)\}} E_x[x|i] + E_s[s|i] - i - k \tag{B1}
\]

s.t.

\[
E_x[x|i] + E_s[s|i] - i - k \geq 0 \tag{B1.1}
\]

\[
i \in \arg \max_{i'} E_x[x|i'] - E_{x,s}[y^\psi(x,s)|i'] - i' \tag{B1.2}
\]

\[
y^\psi(x,s) \geq 0 \quad \forall (x,s) \tag{B1.3}
\]

\[
y^\psi(x,s) \leq x \quad \forall (x,s), \tag{B1.4}
\]

\[
p^\psi \leq \beta, \tag{B1.5}
\]

where (B1.1) is the social investor’s individual rationality constraint, (B1.2) is the incentive compatibility constraint of the project owner, (B1.3) is the investor’s limited liability constraint, (B1.4) is the owner’s limited liability constraint, and (B1.5) requires that the security’s price is less than \( \beta \).

Lemma B1. The security which maximizes the joint surplus (given constraints B1.1-B1.5) is an all-or-nothing contract of the form

\[
y^\psi(x,s; h^-) = \begin{cases} 
 x & \forall h(x,s,i) < h^- \\
 0 & \forall h(x,s,i) \geq h^- 
\end{cases} \tag{B2}
\]

where \( h^- \leq 0 \) and \( h(x,s,i) \) is an increasing function of \( x \) and \( s \) for any \( i \).

Proof of Lemma B1:

Consider the constrained optimization and replace the project owner’s incentive compatibility constraint with the first-order condition. The constraints and objective function are combined to
form the following Lagrangian,

\[
L(y^\psi, i, \kappa, \mu, \theta, \eta, \lambda) = \int_0^\infty \int_0^\infty \left[ (1 + \kappa)(x + s - i - k) \right. \\
+ \mu \left( (x - y^\psi(x, s)) \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - 1 \right) \right] f(x|i)g(s|i) \, ds \, dx \\
+ \int_0^\infty \int_0^\infty \left[ \theta(x, s) y^\psi(x, s) + \eta(x, s)(x - y^\psi(x, s)) \right] f(x|i)g(s|i) \, ds \, dx \\
+ \lambda \left( \beta - \int_0^\infty \int_0^\infty y^\psi(x, s) - x \right) f(x|i)g(s|i) \, ds \, dx - i - k \right].
\] (B3)

We now focus on solving for the optimal security construct with point-wise maximization. The first-order conditions with respect to the security repayment are

\[
\left[ -\mu \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \lambda \right] + \theta(x, s) - \eta(x, s) = 0 \quad \forall (x, s) \quad (B3.1)
\]

Due to non-negativity and complementary slackness conditions for \( \theta(x, s) \) and \( \eta(x, s) \), condition (B16.1) yields

\[
-\mu \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \lambda > 0 \quad \implies \quad y^\psi(x, s) = x \quad (B3.1)
\]

\[
-\mu \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \lambda \leq 0 \quad \implies \quad y^\psi(x, s) = 0 \quad (B3.2)
\]

Since \( i^* < i^{FB} \), project payoffs are improved by incentivizing additional social investment in excess to that which the project owner undertakes on his own. As a result, the multiplier \( \mu \) is strictly positive. Let \( h(x, s, i) \equiv \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \). Then,

\[
-\mu \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \lambda \leq 0 \iff h(x, s, i) \geq \frac{-\lambda}{\mu} \equiv h^-
\] (B4)

and by MLRP, \( h(x, s, i) \) is increasing in \( x \) and \( s \) for all \( i \).

\[\blacksquare\]

According to Lemma B1, all produced cash flows remain with the project owner when the function \( h(x, s, i) \) exceeds the threshold \( h^- \). Conversely, when \( x \) and \( s \) are small enough such that \( h(x, s, i) \) falls below \( h^- \), all cash flows are paid to the social investor.
Figure 2: Optimal security is an all-or-nothing contract which pays all cash flows to the investor when profit and social output are low and retains all cash flows with the project owner when output is high.

B.2 Pledgable Assets

Our base model in Section 3.1 assumes $0 \leq y^\alpha(x, s) \leq x$: the project owner cannot be required to pay more than the profits produced and the social investor’s liability is limited to his initial investment. However as discussed in Section 4, implementation of a SIG might require the social investor to be sufficiently large so that his investment has a direct (and non-trivial) impact on the project owner’s incentive for social investment. When SIGs are sold primarily to large social block holders, limited commitment from the point of view of social investors need not apply. That is, large foundations and investment trusts likely have adequate reputational concerns (and sufficient seizable financial assets) such that they can be expected not to renege on payments promised after the output realizations. With this in mind, we relax the social investor’s liability constraint in this section, and allow him to pledge any residual social capital, $\beta - p^\alpha$, as a contractual payment to the project owner,

$$p^\alpha - \beta \leq y^\alpha(x, s) \leq x.$$  \hspace{1cm} (B5)

We do not allow the security price to fall below zero ($p^\alpha \geq 0$). Indeed, if the price was negative, the interpretation would be that the security provides a single period loan from the project owner to the social investor.

With the exception of the conditions just outlined, we adopt the model assumptions from Section
3.1. The project owner’s individual rationality constraint binds and the security’s price is given by,

\[ p^\alpha = i + k - E_x[x|x\mid i] + E_{x,s}[y^\alpha(x, s)\mid i], \tag{B6} \]

and the social investor’s expected payoff is,

\[ E_x[x|x\mid i] + E_s[s\mid i] - i - k, \tag{B7} \]

The contract that maximizes the social investor’s expected payoff, and as result the joint surplus, solves the following optimization problem,

\[
\begin{align*}
\max_{\{i,y^\alpha(.)\}} & \quad E_x[x|x\mid i] + E_s[s\mid i] - i - k \\
\text{s.t.} & \quad E_x[x|x\mid i] + E_s[s\mid i] - i - k \geq 0 \\
& \quad i \in \arg \max_{i'} E_x[x|x\mid i'] - E_{x,s}[y^\alpha(x, s)\mid i'] - i' \tag{B8.1} \\
& \quad y^\alpha(x, s) \geq p^\alpha - \beta \quad \forall (x, s) \tag{B8.2} \\
& \quad y^\alpha(x, s) \leq x \quad \forall (x, s), \tag{B8.3} \\
& \quad y^\alpha(x, s) \leq y^\alpha(\hat{x}, s) \quad \forall x \leq \hat{x}, \tag{B8.4} \\
& \quad p^\alpha \leq \beta, \tag{B8.5} \\
& \quad p^\alpha \geq 0, \tag{B8.6}
\end{align*}
\]

here (B8.1) is the social investor’s individual rationality constraint, (B8.2) is the incentive compatibility constraint of the project owner, (B8.3) is the investor’s limited liability constraint, (B8.4) is the owner’s limited liability constraint, (B8.5) requires that the security’s repayment is non-decreasing with \( x \), (B8.6) requires that the security’s price is less than \( \beta \), and (B8.7) requires that the security’s price is weakly greater than zero.

**Lemma B2.** If the social investor is permitted to pledge residual capital as payment to the project owner,

\[ y^\alpha(x, s) \geq p^\alpha - \beta, \tag{B9} \]

then the incentives induced by the contract \( y^\psi \) derived in Lemma 1 can be replicated with a zero price contract.

**Proof of Lemma B2:** Consider an alternative problem to the one outlined in (14) such that the social investor is compelled to make a payment \( p^\psi \) (the price of the security constructed in Lemma B1) for all \( (x, s) \) to the project owner, while the project owner’s payment to the social investor remains based on observed output. Consequently, the security’s net payment \( y(x, s) \) contains two
components: a payment \( \hat{y}(x, s) \) from the project owner and the compulsory payment \( p^\psi \) from social investor,

\[
y(x, s) = \hat{y}(x, s) - p^\psi. \tag{B10}
\]

Now consider the contract which adds the compulsory payment to the optimal contract derived in Lemma B1. Then \( y(x, s) = \hat{y}(x, s) - p^\psi \). The price of this security is given from the regular owner’s zero-profit constraint,

\[
p = i + k - E_x[x|i] + E_{x,s}[y(x, s)|i]
= i + k - E_x[x|i] + E_{x,s}[\hat{y}(x, s) - p^\psi|i]
= i + k - E_x[x|i] + E_{x,s}[\hat{y}(x, s) - p^\psi]
= 0, \tag{B11}
\]

where the last equality comes from (12). In addition, the level of investment chosen by the project owner is given by

\[
i \in \arg \max_{i'} E_x[x|i'] - E_{x,s}[\hat{y}(x, s)|i'] - i' - p^\psi. \tag{B12}
\]

Since the compulsory payment \( p^\psi \) does not depend on the choice of \( i \), the equilibrium level of investment is equal to that chosen under contract \( \hat{y}(x, s) \). Finally, we show that the contract \( y(x, s) \) satisfies the social investor’s relaxed liability constraint,

\[
y(x, s) = \hat{y}(x, s) - p^\psi
\geq \hat{y}(x, s) - \beta
\geq -\beta
= p - \beta, \tag{B13}
\]

where the last equality comes from \( p = 0 \).

\[\blacksquare\]

From Lemma B2 any contract which solves (14) can be replicated with a related contract \( y^\alpha(x, s) \) which satisfies (B10) and in which \( p^\alpha = 0 \). A similar logic extends this claim to any contract with a positive price and that satisfies (B10). As such, when allowing for ex post payments by social investors, we can restrict attention to only those contracts with a zero price, since any contract with a positive price can be replicated by a zero-price contract with a compulsory payment equal to the previous price.

**Lemma B3.** If the social investor is permitted to pledge residual capital as payment to the project owner, there is a zero price contract, \( y^\alpha(x, s; h^\alpha) \), that is a debt-like security with a par value that
is continuous and decreasing in $s$. The par value for a given $s$ and level of investment $i$ is given by,

$$\pi(s, i, h^\alpha) - \beta,$$

(B14)

and it can be negative for some values of $s$. Furthermore, the zero price contract weakly dominates $y^\psi(x, s; h^-)$ outlined in Lemma 1.

**Proof of Lemma B3:** The zero price contract allows the social investor to pledge his entire capital budget $\beta$ as a contractual payment. The zero price contract requires the addition of an individual rationality constraint for the project owner (in Section 3.2 the project owner’s individual rationality constraint was redundant). Therefore, the social investor’s problem is,

$$\max_{\{i, y^\alpha(\cdot)\}} \mathbb{E}[x|i] + \mathbb{E}[s|i] - i - k$$

(B15)

s.t. $\mathbb{E}[x|i] + \mathbb{E}[s|i] - i - k \geq 0$

(B15.1)

$$\mathbb{E}[x|i] - \mathbb{E}[x,s][y^\alpha(x, s)|i] - i - k \geq 0$$

(B15.2)

$$i \in \arg\max_{i'} \mathbb{E}[x|i'] - \mathbb{E}[x,s][y^\alpha(x, s)|i'] - i'$$

(B15.3)

$$y^\alpha(x, s) \leq y^\alpha(\hat{x}, s) \quad \forall \ x \leq \hat{x},$$

(B15.4)

$$y^\alpha(x, s) \geq -\beta \quad \forall \ (x, s)$$

(B15.5)

$$y^\alpha(x, s) \leq x \quad \forall \ (x, s),$$

(B15.6)

where (B15.1) is the social investor’s individual rationality constraint, (B15.2) is the project owner’s individual rationality constraint, (B15.3) is the incentive compatibility constraint of the project owner, (B15.4) requires that the security’s repayment is non-decreasing with $x$, (B15.5) is the investor’s limited liability constraint, and (B15.6) is the owner’s limited liability constraint. It is helpful to note that the constraint in (B15.2) almost surely binds because of the project owner is competitive.

In the same way that it was useful to understand the optimal non-monotonic contract for the proof of Lemma 1, consider the constrained optimization outlined in (B15) without the constraint in (B15.4). A replacement of the project owner’s incentive compatibility constraint with the first-order condition allows the constraints and objective function to be combined to form the following
Lagrangian,

\[
L(y^\alpha, i, \kappa, \mu, \theta, \eta, \nu) = \int_0^\infty \int_0^\infty \left[ (1 + \kappa) \left( x + s - i - k \right) \right.
\]

\[
+ \mu \left( x - y^\alpha(x, s) \right) \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - 1 \right] f(x|i) g(s|i) \, ds \, dx
\]

\[
+ \int_0^\infty \int_0^\infty \left[ \nu \left( x - y^\alpha(x, s) - i - k \right) \right] f(x|i) g(s|i) \, ds \, dx
\]

\[
+ \int_0^\infty \int_0^\infty \left[ \theta(x, s) \left( y^\alpha(x, s) + \beta \right) + \eta(x, s)(x - y^\alpha(x, s)) \right] f(x|i) g(s|i) \, ds \, dx
\]

(B16)

We now focus on solving for the optimal security construct with point-wise maximization. The first-order conditions with respect to the security repayment are

\[
\left[ -\mu \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) \right] - \nu + \theta(x, s) - \eta(x, s) = 0 \quad \forall (x, s) \quad (B16.1)
\]

Due to non-negativity and complementary slackness conditions for \( \theta(x, s) \) and \( \eta(x, s) \), condition (B16.1) yields

\[
-\mu \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \nu > 0 \quad \Longrightarrow \quad y^\psi(x, s) = x \quad (B16.1)
\]

\[
-\mu \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \nu \leq 0 \quad \Longrightarrow \quad y^\psi(x, s) = -\beta \quad (B16.2)
\]

Since \( \pi^\pi < \pi^{FB} \), project payoffs are improved by incentivizing additional social investment in excess to that which the project owner undertakes on his own. As a result, the multiplier \( \mu \) is strictly positive. Let \( h(x, s, i) \equiv \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \). Then,

\[
-\mu \left( \frac{f_i(x|i)}{f(x|i)} + \frac{g_i(s|i)}{g(s|i)} \right) - \nu \leq 0 \quad \iff \quad h(x, s, i) \geq \frac{-\nu}{\mu} \equiv h^\alpha \quad (B17)
\]

and by MLRP, \( h(x, s, i) \) is increasing in \( x \) and \( s \) for all \( i \).

A similar discussion to the proofs of Lemma 1 and Lemma 2 imply that the optimal non-decreasing security is a debt-like contract where the par value in any cross section of \( s \) is given by \( \pi(s, i, h^\alpha) - a \) for some \( h^\alpha \). Furthermore, \( \pi(s, i, h^\alpha) - a \) is continuous in \( s \).

\[ \blacksquare \]

The optimal contract with the relaxed social investor liability constraint is again a debt-like contract. However in this case, \( p^\alpha = 0 \), and the social investor subsidizes high output realizations
by paying the project owner the full social budget when \( h^\alpha(x, s, i) > h^\alpha \) and he receives the par value of the contract. It is important to note that the net payment may be negative,

\[
\bar{x}(s, i, h^\alpha) - a < 0 \text{ for some values of } s. \tag{B18}
\]

Consequently, one may think of the zero price contract as a Social Impact Forward Contract (SIFC).

By subsidizing high output, the social investor is able to elicit even greater incentives for social investment. Since the span of contracts considered in the relaxed problem nests those considered in (14), investment incentives are at least as great when the social investor can commit to ex post subsidies as they are when liability is limited to upfront investment.
References


