Incentivizing Impact Investing*

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Abstract

Impact investing is gaining popularity. However, it is not well understood which financial securities provide managerial incentives to pursue social projects efficiently. We derive optimal securities when social projects are funded jointly between agents that value social good and traditional for-profit investors that do not. If the impact investment is a public works opportunity, the security features a pay-for-success structure, rationalizing the use of Social Impact Bonds in practice. If the opportunity is in the private sector, the security features a pay-for-failure structure. We coin this security design a Social Impact Guarantee (SIG).

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1 Introduction

> "We realized that an investment that offers returns while delivering crucial social services was a potential game changer." Andrea Phillips, Goldman Sachs.

On January 29, 2014, the Commonwealth of Massachusetts announced the largest pay-for-success initiative in history. The contract, also referred to as a Social Impact Bond (SIB), raised \$12 million from private investors to provide life skills and employment training to young, at-risk men in the Boston area with the objectives of reducing recidivism and generating taxpayer savings. If the project is successful in reducing re-conviction rates by at least 40% over 5 years, the contract requires up to \$16 million in success payments to investors.

Social Impact Bonds, like that in Massachusetts, have been hailed as a financial engineering triumph. In these contracts, private investors provide upfront investment for a social program that may also generate cost savings — an "impact investment." The project manager allocates the investment proceeds with these two objectives in mind: maximize social value and maximize cost savings. In time, if the project succeeds at generating the desired outcomes (measured and verified by independent parties), an outcome payer (e.g., the Commonwealth of Massachusetts in this case) returns capital to the private investors. The more successful the project along social dimensions, the higher the investment return. While the ability to leverage for-profit capital in social projects is inviting, several questions remain: How does the pay-for-success design facilitate private investment in social services? Can this design be used to support impact investments within traditional for-profit entities?

In this paper, we derive the optimal security for funding an impact investment when the project owner must allocate scarce resources between the competing goals of cost savings and improved social outcomes. When this decision is unobservable, the contract design depends critically on whether the impact investment is in the public or private sector. For public works projects, such as reducing recidivism, the optimal security exhibits the pay-for-success feature that is the hallmark of SIBs. By forcing a greater security repayment following a successful social outcome, the security design discourages service providers from over emphasizing social goals and from under emphasizing cost savings. To the best of our knowledge, the analysis provides the first theory of the SIB design. Alternatively, when the impact investment is in the private sector, the optimal contract possesses a pay-for-failure feature. That is, the security encourages pro-social investment within for-profit firms by rewarding the firm owner with a low repayment following strong social performance. We coin this new security a Social Impact Guarantee (SIG), since it promises SIG investors a greater financial return when their desired social goals are not attained.

Importantly, both SIBs and SIGs allow for the joint funding of impact investments by investors that value social good and for-profit investors that do not. While estimates of the total size of social investor capital vary from $50 billion (Monitor Institute, 2009) to as much as $1 trillion (J.P. Morgan, 2010), the pool of dollars directed towards social projects is limited in comparison to the amount of funds commercially available for investments that generate market returns. Each contract therefore provides a crucial mechanism for leveraging for-profit capital in the pursuit of social value.

In the model, we consider a project that produces both cash profit (equivalently cost savings) and social good (i.e. an impact investment), and we assume that the economy is populated with two investors: a regular profit maximizing investor that values cash only and a social investor that values both cash and social good. The project is endowed to one of the investors. We describe an impact investment as a public works opportunity if it is endowed to the social investor (i.e., a *public works project* managed by a *service provider*), and as a private sector opportunity if it is endowed to the for-profit investor (i.e., a *firm* managed by a *firm owner*). A distinguishing feature of these projects is that the balance between generating social good and maximizing profit is chosen by the manager. Namely, the project owner allocates a scarce resource — effort, attention, or investment — over the cash production and the social good technologies separately. While the owner's choices are unobservable, outputs are observable and contractible.

In the case of the Massachusetts pay-for-success initiative, the public works project to reduce recidivism generates two outputs: (i) the social good associated with reformed citizens and (ii) cost savings to tax payers. While a reduction in re-conviction rates may generate both the desired social good and taxpayer savings through a decrease in the number of resources expended in the criminal justice system, these cost savings may not be genuine if individuals kept out of prison end up instead in the welfare system. Thus, it is not obvious that a reduction in recidivism leads to an overall reduction in government spending, despite the social benefits. In administering the project, there is therefore a balance between focussing on morality training which has been shown to be effective at reducing recidivism versus employment training which may lead to greater long-term cost savings to taxpayers.

In the private sector, the impact investment may be a financial literacy program to foster proper retirement planning among a firm's employees. The two outputs from the project are (i) cost savings to the firm when employees save and retire on schedule (referred to as "on time retirement") and (ii) the social good associated with individuals that make sound financial decisions in preparation for and after retirement. A firm may stress on time retirement by emphasizing the benefits of employer matching in defined contribution retirement plans, but do nothing to ensure that employees are equipped to manage consumption and savings in retirement. Conversely, the firm could stress financial planning and education to foster appropriate savings and consumption in retirement but do little, on average, to affect employees' decisions over when to retire. Again, there is a balance between social "impact" and financial "investment."

We first consider the optimal contract when the project is a public works opportunity administered by the service provider. Without financial constraints, the service provider chooses the welfare maximizing levels of focus in achieving both cost savings and social output. In reality, this case is rare because social capital is scarce and the ability of governments to impose taxes on citizens is limited and inefficient. It may appear that this capital constraint could be mitigated by pre-selling the project's entire cost savings to the for-profit investor and using the proceeds to fund the project. However, doing so distorts the service provider's incentive to balance cost savings and social goals, leading the service provider to over emphasize social output relative to first-best. Nevertheless, we derive the optimal security that can be sold to the for-profit investor to fund the project. The security assigns a greater repayment to the investor when social output measurements are high, that is, a pay-for-success contract. The intuition for the design is straightforward: forcing a greater security repayment when social output is high tapers the pressure to overweight social value. Thus, a pay-for-success contract (or SIB) is optimal whenever the public works opportunity relies on external financing.

We then consider the setting in which the impact investment is in the private sector — the project is administered by a for-profit firm owner and partially funded by a social investor. All else equal, the for-profit firm owner under emphasizes social value. Furthermore, because the firm owner's choice is unobservable, the social investor cannot simply provide an ex ante subsidy or grant to induce greater emphasis on social goals. Instead, we propose a Social Impact Guarantee that is sold by the firm owner to the social investor. The security features a greater security repayment when social output measurements are low. By punishing the firm owner with a high repayment when social output is low and rewarding him with a low repayment when social output is high, the security encourages pro-social behavior by the for-profit firm owner.

Currently, most socially-conscious investments in private sector opportunities are made via direct investment, e.g., purchasing a business' debt or equity;[1] yet these investments are unlikely to materially affect firms' attention to social output, especially when social impact and profit maxi-

[1]See J.P. Morgan and Global Impact Investing Network (2014).

mization are at odds.[2] This conflict of interest between socially-minded investors and for-profit firm owners is mitigated by the SIG design, since profit-maximization by the residual claimant coincides with greater social investment. As such, our analysis provides a normative recommendation for impact investing — investors who choose to make impact investments in the private sector should purchase SIGs rather than traditional debt or equity securities.

2 Related literature

Our analysis adds to a growing body of literature that considers socially conscious investing, more commonly known as socially responsible investing (SRI), and its effects in financial markets.[3] Typically, socially responsible investing is pursued via exclusionary investment — a proactive approach to avoid companies, industries or nations that engage in behaviors deemed socially irresponsible. While there is likely a warm-glow associated with such strategies, they are not costless. Hong and Kacperczyk (2009) show that socially conscious investors pay a financial cost in avoiding stocks that are not SRI acceptable, i.e., "sin stocks." Sin stocks have lower price-to-book ratios and higher expected returns than comparable stocks. Geczy, Stambaugh, and Levin (2005) provide similar evidence; investors that adhere to socially responsible investments add an additional constraint to their portfolio selection problem and under-perform relative to an unconstrained portfolio by as much as 30 bps per month. Our analysis is consistent with these empirical findings. For impact investments in which the expected financial return is less than the market rate of return on other investments, service providers subsidize the returns of for-profit investors through a SIB's repayment design and social foundations subsidize the returns of for-profit firms through the upfront price of a SIG. Nevertheless, socially conscious parties are willing to accept below market returns because they see social good, i.e., social return, as an acceptable substitute for financial returns.

Our analysis also provides an alternative mechanism for socially conscious investing in the private sector. While most social investment in the private sector is done via direct investment (or an analogue strategy of divestment), there is little evidence to suggest that such strategies achieve the desired outcomes. Teoh, Welch, and Wazzan (1999) provide empirical evidence that the South African divestment campaign to end apartheid had no discernible effect on the valuation of companies with ties to South Africa. Furthermore, Davies and Van Wesep (2015) provide a

[2] Heinkel, Kraus, and Zechner (2001) show that direct investment (and divesture) may affect a firm's cost of capital. The result requires that a large fraction of investors adopts the behavior, which is unlikely to be the case in our setting with limited social capital.

[3] For a survey of the SRI literature see Renneboog, Ter Horst, and Zhang (2008).

model in which divestment campaigns are not only ineffective but also counterproductive in the sense that they incent managers to pursue the very behaviors that social investors disdain. Given this evidence, it is difficult to see how direct investment or divestment invokes the desired goals that social investors campaign for, especially when these goals are at odds with profit maximization.[4] We provide an alternative prescription: pro-social investment ought be done via the purchase of a SIG which aligns the interests of for-profit managers and social investors via the security's repayment schedule.

The designs we propose build on and add to the existing literature that considers security design as a means to incentivize a residual claimant to take certain unobservable actions. Most notably, Innes (1990) considers a model in which a financially constrained entrepreneur must raise external finances. Because the entrepreneur must share the profits he produces with an investor, he does not fully internalize the effect of his effort. The optimal contract between the entrepreneur and investor, given standard limited liability constraints, calls for the manager to surrender all profits when outputs are a signal of low effort and keep greater profits when outputs are a signal of high effort. Our designs provide similar insights. For public works opportunities, the service provider must surrender profits when outputs are indicative of over investment in the social technology and keep them otherwise. In private sector opportunities, the firm owner surrenders profits when outputs are indicative of under investment in the social technology and keeps a greater share otherwise.

Finally, the optimal security constructs in our model rely on both realized profits and realized social good. Although both outputs are valued by at least one agent in our model, such a condition is unnecessary for the optimal contract. As noted by Holmstrom (1979), any signal that reveals information about the project manager's choices should be included in the contract.[5] Thus, any outputs that are indicative of the project manager's choices, should be considered in the optimal security design, even if no agent values them.

3 Model

Consider a project which may produce both cash profit (cost savings) and social good — an "impact investment." The project requires an upfront investment equal to I. The project manager then

[4]Baron (2008) provides a model in which socially responsible investors induce pro-social preferences through corporate governance. While this may be possible in theory, the relatively limited capital of social investors implies that it is unlikely that they will gain the clout necessary to directly affect managerial compensation.

[5]See also Harris and Raviv (1979) and Milgrom (1981).

allocates an unobservable scarce resource between two production technologies, one which aids in the production of social good and another that aids in the production of cash output. The unobservable scarce resource could be unobservable investment, effort, or attention. Without loss of generality, we refer to the unobservable resource as investment hereafter. Denote by i_s the level of investment in the social technology and by i_x the level of investment in the traditional for-profit technology, where $I = i_s + i_x$.[6]

The cash profit generated by the project is a random variable $x \in \{0, 1\}$. When $x = 1$, the project succeeds in producing cash output, and when $x = 0$ the project fails to generate cash output. The probability of success is given by $Pr\,(x = 1 \mid i_x, i_s) = f(i_x)$, which satisfies $f'(\cdot) > 0$ and $f''(\cdot) < 0$. The conditions on $f(\cdot)$ are natural: greater investment in the for-profit technology increases the probability of successfully producing cash output, but at a diminishing rate. In addition, the project may either succeed ($s = 1$) or fail ($s = 0$) in its production of social output s. The probability of success is given by $Pr\,(s = 1 \mid i_x, i_s) = g(i_s)$, which satisfies $g'(\cdot) > 0$ and $g''(\cdot) < 0$. For simplicity, we assume that, conditional on the chosen levels of investment, the probabilities of successfully producing cash output and social output are independent. The distinguishing feature between x and s is that s is non-excludable. Consequently, s cannot be sold after it is produced.

In the Massachusetts example, the public works project produces cost savings to taxpayers if lower incarceration rates lead to a reduction in total government expenditures. The present value of these savings is the cash component x in the model. The impact investment also educates an at-risk group of individuals, enabling them to create a better future while reducing criminal activity. This represents the social good component s in the model. While undertaking the impact investment positively affects both tax-payer savings and social good, the service provider must choose how much to invest in each output; some interventions may lead to greater taxpayer savings (i.e. employment training) while other interventions may provide a greater reduction in criminal behavior and re-conviction rates (i.e. morality training).[7] In our private sector example, financial literacy training for employees promotes cost savings due to on time retirement x as well as greater financial acumen and financial planning among employees and retirees s. Generous employer matching in the firm's defined contribution plan may nudge senior employees towards on time retirement, but do nothing

[6]While the upfront investment I is necessary for our model, the bifurcation of I across the two technologies is not. The qualitative implications would be the same if the project required an upfront investment I, and the manager allocated some attention or effort budget B across the two technologies.

[7]While we focus on the example of recidivism, there are numerous pay-for-success contracts globally that fund projects to increase childhood literacy, reduce homelessness, and provide services to the mentally ill.

to ensure they use their nest eggs appropriately in retirement. Conversely, extensive training on post-retirement budgeting may improve the well-being of retirees, but do little to achieve on time retirement.

The economy is populated with two risk-neutral investors: a commercial investor who derives value only from cash and a social investor who values both cash profit and social good. We use the superscript π to denote the regular commercial investor and the superscript ψ to denote the social investor. The regular investor is endowed with initial wealth $\beta^{\pi} > I$ and is therefore able to fully fund the impact investment. The social investor, however, is financially constrained and has a maximum capital budget $\beta^{\psi} < I$. Thus, the social investor must raise additional funds from the regular investor in order to undertake the impact investment. This assumption is made to capture an important feature of impact investing — the funds available from socially-minded investors are limited so that many social projects are forgone because they cannot be solely funded by investors who are willing to accept a smaller rate of cash return on their invested capital. The gross market rate of return on alternative investments is $\rho \geq 1$. Investment occurs in period 1 and the payoffs occur in period 2. In addition, we assume $g(0) > \rho\beta^{\psi}$ and $f(0) > \rho(I - \beta^{\psi})$. The first assumption guarantees that expected social output covers the social investor's *maximum* contribution to the upfront investment cost I while the latter assumption guarantees that expected cash output covers the regular investor's *minimum* contribution.[8] Furthermore, these assumptions highlight the distinguishing feature of impact investments as projects capable of producing both social value and cash output. However, within this context, there is a tradeoff regarding which output to emphasize.

We begin by considering the levels of investment which maximize the joint surplus across investors in a frictionless economy, that is an economy without agency conflicts or financial constraints.[9] Making the substitution $i_x = I - i_s$, the surplus maximizing share of investment directed towards the social technology solves

$$\max_{i_s} f(I - i_s) + g(i_s). \tag{1}$$

We make the following assumption to guarantee an interior solution to the "first-best" choice problem,

Assumption 1. $\lim_{i_s \to 0} \frac{g'(i_s)}{f'(I - i_s)} = \infty$ *and* $\lim_{i_s \to I} \frac{f'(I - i_s)}{g'(i_s)} = \infty$.

[8] These assumptions provide sufficient conditions for the existence of an optimal security in sections 3.1 and 3.2.

[9] We emphasize joint surplus, as opposed to welfare, intentionally. We do not consider the standard political economy questions regarding the provision of a public good. Instead, we are concerned with the optimal bilateral contract between an agent that values the social good and one that does not.

Assumption 1 ensures that the solution to the first-best problem involves investment in both the for-profit and social components. The solution i_s^{FB} which maximizes joint surplus is therefore characterized by the first-order condition,

$$f'\left(I - i_s^{FB}\right) = g'\left(i_s^{FB}\right). \tag{2}$$

The expression in (2) implies that surplus is maximized when the marginal benefits of cost savings and social good are equated. What we have in mind is that there is a wide (and in the model continuous) menu of choices a manager could make; while each choice generates both profit and social good with positive probability, some tilt more heavily towards cash production and others more heavily towards social good production. Returning to our examples, the optimal choice i_s^{FB} in the Massachusetts initiative equates the marginal benefit of taxpayer savings to the marginal social benefit of a reduction in criminal activity (as valued by the Commonwealth of Massachusetts). In the private sector example, i_s^{FB} equates the marginal benefit of a reduction in the firm's compensation expenses from on time retirement to the marginal social benefit of reducing the income gap for retirees (as valued by the social investor).[10]

3.1 Social Impact Bond

Consider first the setting in which the social investor owns and operates the project. We term the project a public works opportunity and the social investor a service provider. The service provider's wealth is limited and additional funds must be raised from the commercial investor in order to undertake the impact investment. The service providers's investment choice is not observable, however, outputs x and s are observable and contractible. Consider a traded security offered by the service provider to the commercial investor. The security has price p^π and pays $y^\pi(x, s)$ in period 2. The superscript π is used to denote a security which is purchased by the commercial investor. When considering the design of the security, we assume standard limited liability constraints; (i) $y^\pi(x, s) \leq x$: the service provider cannot be required to repay more than the cash profit produced by the project;[11] and (ii) $y^\pi(x, s) \geq 0$: the investor's liability is limited to the initial level of investment p^π. Since no repayment is possible when the project fails to produce

[10]While we focus on a single social investor in the model, in Section 4 we discuss implementation in a market with multiple social investors.

[11]A recidivism impact investment in New York City was abandoned in 2015 after it was deemed to not be cost effective, implying that limited liability with respect to cash production is reasonable (Barron's 2015). Furthermore, many local governments face large budget deficits with as many as 8 general-purpose local governments filing for bankruptcy protection between 2010 and 2014.

cash output, limited liability requires $y^\pi(0, s) = 0$. For ease of notation, we therefore express the repayment $y^\pi(1, s)$ as $y^\pi(s)$.

In the case of the Massachusetts initiative, the project is administered by Rocca, a non-profit organization which focuses on reducing incarceration and poverty among high-risk youth, while success payments are made by the Commonwealth of Massachusetts. In the model, we consider both the service provider and outcome payer to be the same agent, i.e., the social investor who owns the project. The primary for-profit investor is Goldman Sachs. The contracting problem is to design a security that can be sold by the Commonwealth of Massachusetts to Goldman Sachs in order to help fund the upfront program costs while providing a fair market return to the for-profit investor.

Because the service provider values both cash output and social good, the expected future value of investment to the service provider is equal to the sum of cash and social outputs minus the expected security repayment to the commercial investor minus the service provider's contribution to the upfront investment cost,

$$f(I - i_s) + g(i_s) - f(I - i_s)\left[g(i_s)y^\pi(1) + (1 - g(i_s))y^\pi(0)\right] - \rho\left(I - p^\pi\right). \tag{3}$$

The investment decision is made after the security has been sold to the commercial investor. Since the share of investment allocated to the social technology is unobservable, the service provider does not internalize the impact of this investment choice on the price of the security. Consequently, the level of social investment chosen by the service provider is the solution to the following optimization,

$$\max_{i_s \in [0,I]} f(I - i_s) + g(i_s) - f(I - i_s)\left[g(i_s)y^\pi(1) + (1 - g(i_s))y^\pi(0)\right]. \tag{4}$$

When $i_s \in (0, I)$, the chosen level of social investment is defined implicitly by

$$\begin{aligned} 0 = &- f'(I - i_s) + g'(i_s) + \left[f'(I - i_s)g(i_s) - g'(i_s)f(I - i_s)\right] y^\pi(1) \\ &+ \left[f'(I - i_s)(1 - g(i_s)) + g'(i_s)f(I - i_s)\right] y^\pi(0). \end{aligned} \tag{5}$$

The equation in (5) describes the effect of $y^\pi(1)$ and $y^\pi(0)$ on the service provider's incentive to invest in the social technology. Define,

$$\kappa_0(i_s) \equiv f'(I - i_s)(1 - g(i_s)) + g'(i_s)f(I - i_s), \tag{6}$$

and,

$$\kappa_1(i_s) \equiv f'(I - i_s)g(i_s) - g'(i_s)f(I - i_s). \tag{7}$$

Using the preceding notation, (5) is rewritten as,

$$0 = -f'(I - i_s) + g'(i_s) + \kappa_1 y^\pi(1) + \kappa_0 y^\pi(0). \tag{8}$$

The functions $\kappa_0(i_s)$ and $\kappa_1(i_s)$ represent incentive weights associated with the security repayment schedule. Note that $\kappa_0(i_s) > 0$ for all $i_s \in [0, I]$. Thus, an increase in $y^\pi(0)$ compels the service provider to increases the share of investment directed toward the social technology. The intuition is straightforward; increasing $y^\pi(0)$ penalizes the service provider with a high security repayment when the project successfully produces cash output but fails to achieve its social objectives. This increases the ex ante incentive for socially directed investment. The sign on $\kappa_1(i_s)$ is ambiguous. When the project succeeds along both the cash production and social value dimensions, it is not generally clear whether investment tilted in favor of the cash or social technologies. Thus, the effect of an increase in $y^\pi(1)$ on the service providers incentive for social investment is equivocal. When $\kappa_1(i_s)$ is positive, an increase in $y^\pi(1)$ increases the share of investment in the social technology. The opposite relationship holds when the term is negative.

In what follows, we consider the security design which maximizes joint welfare when investment is unobservable.[12] Without loss of generality, we assume that all surplus from investment accrues to the service provider.[13,14] As such, maximizing the service provider's payoff is equivalent to maximizing joint surplus. Given this assumption, the equilibrium security price is,

$$p^\pi = \frac{f(I - i_s)\left[g(i_s) y^\pi(1) + (1 - g(i_s)) y^\pi(0)\right]}{\rho}, \tag{9}$$

implying that the commercial investor is willing to pay a price exactly equal to the expected security payment, discounted by the gross market rate of return. At this price, the portion of the project cost coming from the service provider is minimized. Since social capital is limited, this price provides the greatest (weakly) opportunity for the funding of valuable social projects. The contract that

[12]Since the service provider and commercial investor have linear utility, the share of investment which maximizes joint welfare is often the unique Pareto optimal level of investment. However, when the service provider's budget constraint binds, other levels of investment may also be Pareto efficient.

[13]In the analysis that follows, if the security achieves the first-best investment level, the surplus may be divided between the service provider and commercial investor via a constant term in the security's price. If the service provider's budget constraint binds and first-best is not achieved, accruing all surplus to the service provider is equivalent to maximizing the joint surplus.

[14]One interpretation is that commercial investors are competitive and therefore earn zero profits in expectation.

maximizes joint surplus solves

$$\max_{\{i_s, y^\pi(0), y^\pi(1)\}} f(I - i_s) + g(i_s) \tag{10}$$

$$\text{s.t. } i_s \in \arg\max_{i'_s \in [0,I]} f(I - i'_s) + g(i'_s) - f(I - i'_s)\left[g(i'_s)y^\pi(1) + (1 - g(i'_s))y^\pi(0)\right] \tag{10.1}$$

$$0 \leq y^\pi(0) \leq 1 \tag{10.2}$$

$$0 \leq y^\pi(1) \leq 1 \tag{10.3}$$

$$I - \frac{f(I - i_s)\left[g(i_s)y^\pi(1) + (1 - g(i_s))y^\pi(0)\right]}{\rho} \leq \beta^\psi. \tag{10.4}$$

Inequality (10.4) is the service provider's capital budget constraint, where we have substituted for p^π from (9). Holding fixed the levels of investment, the security which raises the most money in period 1 and which most easily satisfies the service provider's budget constraint sets $y^\pi(0) = y^\pi(1) = 1$. By repaying all realized cash flow to the commercial investor in period 2 the security's price is maximized. However, under the full repayment contract, constraint (10.1) simplifies to

$$i_s \in \arg\max_{i'_s \in [0,I]} g(i'_s). \tag{11}$$

In this case, the service provider overinvests in the social technology, choosing $i_s = 1$ to maximize social output only. When external financing is raised from the commercial investor, the service provider must repay the commercial investor part of the cash profits generated by the project. While this diminishes the service provider's own benefit from cash output, the service provider's utility from social output is unaffected by the existence of the security (because social output is non-excludable). Under the full repayment contract, the service provider retains no cash in period 2 and therefore benefits only from the social value produced by the project. Therefore, to reduce the service provider's incentive to overinvest in the social technology, either $y^\pi(0)$, $y^\pi(1)$, or both must be reduced below one.

The incentive effect of reducing $y^\pi(0)$ and $y^\pi(1)$ is captured by incentive weights $\kappa_0(i_s)$ and $\kappa_1(i_s)$ described with the service provider's incentive compatibility constraint in (8). The cost of reducing $y^\pi(0)$ and $y^\pi(1)$ is reflected in the service provider's budget constraint in (10.4). Since the security repayments enter linearly into each constraint, we can, roughly speaking, capture the benefit-to-cost ratio of adjusting $y^\pi(0)$ and $y^\pi(1)$ by scaling the marginal effect on incentives by the marginal cost of a smaller expected repayment. For $y^\pi(0)$ this ratio is

$$\frac{\kappa_0(i_s)}{f(I - i_s)(1 - g(i_s))} = \frac{f'(I - i_s)}{f(I - i_s)} + \frac{g'(i_s)}{1 - g(i_s)}, \tag{12}$$

and for $y^\pi(1)$ this ratio is

$$\frac{\kappa_1(i_s)}{f(I - i_s)g(i_s)} = \frac{f'(I - i_s)}{f(I - i_s)} - \frac{g'(i_s)}{g(i_s)}. \tag{13}$$

Clearly, the preceding equations imply,

$$\frac{\kappa_0(i_s)}{f(I-i_s)(1-g(i_s))} \geq \frac{\kappa_1(i_s)}{f(I-i_s)g(i_s)}, \tag{14}$$

because $\frac{g'(i_s)}{1-g(i_s)} \geq 0$ and $\frac{g'(i_s)}{g(i_s)} \geq 0$. Thus for all $i_s \in [0, I]$, reducing $y^\pi(0)$ provides a more efficient reduction in social investment. This leads to the following result.

Lemma 1. *Any security which maximizes joint surplus subject to constraints (10.1)-(10.4) has the feature* $y^\pi(1) \geq y^\pi(0)$.

From Lemma 1, any optimal security features greater repayment from the service provider to the commercial investor following a successful social outcome. The result reconciles what may seem a particularly puzzling contract design — pay-for-success contracts, including the Massachusetts initiative, appear to penalize governments for doing the "right" thing. Our analysis provides justification for the design of Social Impact Bonds. Since the social benefit that accrues to the service provider cannot be captured by the commercial investor, the service provider has an incentive to pursue social goals at the expense of cash output. The pay-for-performance feature arises in order to dull this tendency to overinvest in the social technology. Pay-for-success bonds are therefore optimal when social projects are operated by agents with pro-social preferences and funded in-part by commercial investors.

While, in the preceding analysis, we do not restrict the project's maximum expected cash output, many impact investments likely deliver below market rates of return even when project managers fully emphasize the cash production technology ($f(I) < \rho I$). Thus, many social projects do not provide sufficient financial returns to justify sole funding by for-profit investors and instead require joint funding between for-profits investors and social investors who accept a smaller rate of cash return on their invested capital. In the model, the security price p^π reflects the for-profit investor's upfront investment, and when $p^\pi < I$, the service provider covers the remaining $I - p^\pi$ in project costs. It is easy to see from (9) that, for $f(I) < \rho I$,

$$p^\pi \leq \frac{f(I-i_s)}{\rho} < I,$$

meaning that the project involves joint funding from the commercial investor and the service provider. Furthermore, since the commercial investor earns exactly the market rate of return in expectation, the residual cash which flows to the service provider after repayment of the SIB must deliver less than the market return on the service provider's upfront investment. Because the service provider also values the project's social output, he is willing to subsidize the cash return to the commercial investor, enabling joint investment in an impact investment that neither that service provider nor commercial investor could fund independently.

3.2 Social Impact Guarantee

In the previous section, we analyze the case in which the social investor owns and operates the project and argue that this setting is most reflective of public sector projects. We now turn our attention to impact investments in the private sector where the commercial investor owns and operates the project. We interpret this setting as a private sector opportunity. We term the project a firm and the commercial investor the firm owner. For clarity, we term the social investor a social foundation.

Because the firm owner is concerned with only maximizing profit, he chooses $i_s = 0$, all else equal. In the context of our example of a financial literacy program, the firm owner undertakes the impact investment only to the extent that it is profitable. This leads to less financial acumen among retirees than the social foundation prefers. To further illustrate this point, consider as well an impact investment to develop new products (e.g. orphan drugs) for the diagnosis and treatment of a rare disease. In absence of a contract, the firm owner focusses research and development opportunities on only the medical interventions that are least costly to produce and administer so that the profit from sale of the drug is greatest. However, this is unlikely to achieve as great an improvement in long-term health outcomes as desired by the social foundation.

In order to increase the firm owner's attention to social value, we consider a traded security offered by the firm owner to the social foundation. The superscript ψ is used to denote the security which is purchased by the social foundation. The security has price p^{ψ} and pays $y^{\psi}(1)$ in period 2 when the project succeeds in producing both cash output and social output and pays $y^{\psi}(0)$ when the project succeeds in producing cash output but fails to achieve its social objective. We again restrict attention to securities in which the firm owner cannot be required to repay more than the cash profit produced by the project and the social foundation's liability is limited to the initial level of investment.

Given security repayments $y^{\psi}(0)$ and $y^{\psi}(1)$, the level of social investment chosen by the firm owner is the solution to the following optimization,

$$\max_{i_s \in [0,I]} f(I - i_s) - f(I - i_s)\left[g(i_s)y^{\psi}(1) + (1 - g(i_s))y^{\psi}(0)\right]. \tag{15}$$

When $i_s \in (0, I)$, the chosen level of investment is defined implicitly by

$$\begin{aligned}0 =& - f'(I - i_s) + \left[f'(I - i_s)g(i_s) - g(i_s)'f(I - i_s)\right] y^\psi(1) \\ &+ \left[f'(I - i_s)(1 - g(i_s)) + g'(i_s)f(I - i_s)\right] y^\psi(0). \end{aligned} \tag{16}$$

Again, we interpret $\kappa_0(i_s)$ and $\kappa_1(i_s)$ as the incentive weights associated with the security repayment schedule and rewrite (16) as,

$$0 = -f'(I - i_s) + \kappa_1(i_s)y^\psi(1) + \kappa_0(i_s)y^\psi(0). \tag{17}$$

This implies, as in the previous section, that penalizing the firm owner with a high security repayment when the project fails to produce social output increases the incentive for socially directed investment. We maintain the assumption that all surplus accrues to the social foundation. Consequently, the price of the security is,

$$p^\psi = I - \frac{f(I - i_s) - f(I - i_s)\left[g(i_s)y^\psi(1) + (1 - g(i_s))y^\psi(0)\right]}{\rho}. \tag{18}$$

As discussed in the previous section on SIBs, if the project's maximum expected cash return is less than the market rate, then $p^\psi > 0$, implying that the project must be jointly funded by the firm owner and social foundation. In this case, the social foundation earns a below market financial return in expectation and therefore subsidizes the financial return for the firm owner.

The security design which maximizes joint welfare when investment by the firm owner is unobservable solves

$$\max_{\{i_s, y^\psi(0), y^\psi(1)\}} f(I - i_s) + g(i_s) \tag{19}$$

$$\text{s.t. } i_s \in \arg\max_{i'_s \in [0,I]} f(I - i'_s) - f(I - i'_s)\left[g(i'_s)y^\psi(1) + (1 - g(i'_s))y^\psi(0)\right] \tag{19.1}$$

$$0 \leq y^\psi(0) \leq 1 \tag{19.2}$$

$$0 \leq y^\psi(1) \leq 1 \tag{19.3}$$

$$I - \frac{f(I - i_s) - f(I - i_s)\left[g(i_s)y^\psi(1) + (1 - g(i_s))y^\psi(0)\right]}{\rho} \leq \beta^\psi \tag{19.4}$$

The inequality in (19.4) is the social foundation's capital budget constraint, where we substitute the explicit form of the security's price into the constraint. Holding fixed the levels of investment, the security which is least costly to the social foundation and which most easily satisfies the social foundation's budget constraint sets $y^\psi(0) = y^\psi(1) = 0$. However, under a no repayment contract, constraint (19.1) simplifies to,

$$i_s \in \arg\max_{i'_s \in [0,I]} f(I - i'_s). \tag{20}$$

Thus, when all cash output remains with the firm owner, the owner places no value on social output. The firm owner under invests in the social technology, choosing $i_s = 0$ to maximize cash output only. To increase the firm owner's incentive to invest in the social technology, either $y^{\psi}(0)$, $y^{\psi}(1)$, or both must be raised above zero. As in Section 3.1, the incentive effect of increasing $y^{\psi}(0)$ and $y^{\psi}(1)$ is captured by the firm owner's incentive compatibility constraint in (17) while the cost of increasing $y^{\psi}(0)$ and $y^{\psi}(1)$ is reflected in the social foundation's budget constraint in (19.4). Since the security repayments enter linearly into each constraint, a relevant ratio is the marginal effect on incentives scaled by the marginal increase in the security cost. For $y^{\psi}(0)$ this ratio is again,

$$\frac{\kappa_0(i_s)}{f(I-i_s)(1-g(i_s))} = \frac{f'(I-i_s)}{f(I-i_s)} + \frac{g'(i_s)}{1-g(i_s)}, \tag{21}$$

and for $y^{\psi}(1)$ this ratio is

$$\frac{\kappa_0(i_s)}{f(I-i_s)g(i_s)} = \frac{f'(I-i_s)}{f(I-i_s)} - \frac{g'(i_s)}{g(i_s)}, \tag{22}$$

with $\frac{\kappa_0(i_s)}{f(I-i_s)(1-g(i_s))} \geq \frac{\kappa_0(i_s)}{f(I-i_s)g(i_s)}$. Thus for all $i_s \in [0, I]$, increasing $y^{\psi}(0)$ provides a more efficient increase in social investment. This leads to the following result.

Lemma 2. *Any security which maximizes joint surplus subject to constraints (19.1)-(19.4) has the feature* $y^{\psi}(0) \geq y^{\psi}(1)$.

From Lemma 2, any optimal security features greater repayment to the social foundation when the project fails to achieve its social objective. We coin this security a Social Impact Guarantee (SIG), since it insures the social foundation with a greater security repayment when social value is low. Like Social Impact Bonds, SIGs assign a greater fraction of project cash to profit-maximizing agents when social value is high, in this case the firm owner. By rewarding the firm owner with a low security repayment when the project succeeds in producing social output, the security aligns the incentives of the firm owner and the social foundation in pursuit of social value.

The results in Lemma 1 in Section 3.1 and Lemma 2 provide an obvious contrast. While there has been much focus on the use of Social Impact Bonds to fund impact investments in social services, pay-for-success bonds in their current form have little use in the private sector. Not surprisingly, the popular pay-for-success design provides no incentive for investment in social outcomes when the project manager is profit maximizing. Perhaps for this reason, impact investments in the private sector have largely involved direct purchases of a business's debt or equity when the firm is deemed to be socially responsible. However, it is unlikely for debt or equity purchases to incent pro-social investment especially when social capital is limited. Our analysis provides an alternative recommendation for impact investing in the private sector through the introduction of Social Impact Guarantees.

4 Concluding Discussion

Social output is a non-excludable public good, and, as such, free-riding potentially impedes individual social investors from participating in the markets for SIBs and SIGs. In our analysis, we treat social utility as if there is a single social investor whose value for social output is equal to the total social value generated by the impact investment. For impact investments in the public sector, it seems reasonable to approximate the project's total social value by the social preferences of the local government or non-profit organization operating the investment. However, for impact investments in the private sector, it is unlikely for any individual social investor to internalize the benefit of social output to all other social investors. That being said, a SIG will still incrementally improve social welfare if social investors are willing to pay, at least in part, for the increase in social investment that results from their contribution to the funding of the impact investment. Thus in practice, SIGs are required to be sold to large social block holders who internalize (and pay for) their direct impact on social investment, e.g., the Bill and Melinda Gates Foundation and the J. Paul Getty Trust.

A separate issue impacting the implementation of SIBs and SIGs is that the contractual payments rely crucially on well-defined measurable benchmarks of social value. This concern has been somewhat addressed for public works opportunities funded by SIBs through the use of third-party verification and often times randomized experiments to measure program outcomes. While additional steps are necessary to penetrate private sector opportunities, several recent developments make social-output-contingent contracts possible in the private sector. For example, Impact Reporting & Investment Standards (IRIS : http://iris.thegiin.org/) and GIIRS Ratings and Analytics for Impact Investing (http://giirs.org/) provide independent evaluations of social value similar to the role credit rating agencies play in providing default information on corporate bonds.

Because repayment to SIB and SIG investors depends on realized social output, secondary market prices for these, as well as residual securities, aggregate investor information that might be useful both for managers and investors when pursuing social impact investments.[15] For example, a rise in the secondary price of a SIG indicates that the for-profit firm is less likely to meet its social objective benchmark. Furthermore, since the SIG is senior to the firm's other claims, the

[15] See Subrahmanyam and Titman (1999).

SIG does not need to trade to provide useful information via prices: the prices of all junior claims will provide indirect information about the firm's social performance, as well. Furthermore, in a dynamic model in which the firm repeatedly raises funds, secondary pricing of securities provides useful information to managers about which social objectives are likely to be valued more highly by investors (and society) in the future. This allows project managers to build capacities for future expansion in desirable social activities.[16]

While SIBs may trade freely in a secondary market among for-profit investors, the existence of a secondary market for SIGs depends critically on whether the contract is renegotiable. If the firm owner's incentive for social investment necessarily remains unchanged after the SIG is resold (i.e. the contract cannot be legally renegotiated or investment has already been made and is consequently fixed), a secondary market can exist without compromising the security's intent. This is because the original social investor continues to enjoy the benefits of greater social investment even if he resells the security. Furthermore, the secondary market value of the security to those that value social good and to those that do not is the same (the expected cash flow). If, however, the security is not renegotiation proof (i.e., either investment has not been made or investment can be liquidated at low enough cost), secondary market trading may be limited. This is evident by considering the sale of a SIG security a regular investor that values only cash profit: once the security is sold to a regular investor, the expected utility from owning that security is $E_{x,s}[y(x,s)|i_x,i_s]$. Recall that the expected profit from investment for the firm's shareholders is $E_x[x|i_x] - E_{x,s}[y(x,s)|i_x,i_s]$. Thus, the sum of utilities to the firm's shareholders and SIG holder is equal to $E_x[x|i_x]$, which is simply the for-profit firm owner's maximization problem without a SIG. Hence, total profit is maximized by letting $i_x = I$ $(i_s = 0)$, which can be obtained by renegotiating the security to a null contract. However, if the initial social investor anticipates that the contract will be renegotiated and that social investment will subsequently be reduced, the security's intent unravels. Thus, resale of a SIG security to investors that do not value social good is not possible

[16] See Subrahmanyam and Titman (2001) for an argument on how stock prices provide useful information on firms' future cash flows.

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Appendix A

Proof of Lemma 1: The proof is constructed as a proof by contradiction: *There exists a security which maximizes joint surplus subject to constraints (10.1)-(10.4) and has the feature* $\hat{y^\pi}(1) < \hat{y^\pi}(0) \leq 1$.

The service provider's optimal choice for $\hat{i_s}$ is in $(0, I]$ under any contract. We start by assuming that $\hat{i_s} < I$ and then consider the case in which $\hat{i_s} = I$. With an internal solution for $\hat{i_s}$, the service provider's incentive compatible first-order condition in (10.1) is satisfied with equality,

$$\begin{aligned} 0 = &- f'(I - \hat{i_s}) + g'(\hat{i_s}) + \Big[f'(I - \hat{i_s}) g(\hat{i_s}) - g'(\hat{i_s}) f(I - \hat{i_s}) \Big] \hat{y^\pi}(1) \\ &+ \Big[f'(I - \hat{i_s})(1 - g(\hat{i_s})) + g'(\hat{i_s}) f(I - \hat{i_s}) \Big] \hat{y^\pi}(0). \end{aligned} \tag{A1}$$

The preceding equation is rewritten as,

$$\begin{aligned} 0 = &- f'(I - \hat{i_s}) + g'(\hat{i_s}) \\ &+ \underbrace{f'(I - \hat{i_s}) \hat{y^\pi}(1)}_{+} + \underbrace{\Big(f'(I - \hat{i_s})(1 - g(\hat{i_s})) + g'(\hat{i_s}) f(I - \hat{i_s}) \Big)}_{+} \underbrace{(\hat{y^\pi}(0) - \hat{y^\pi}(1))}_{+ \text{ by assumption}}. \end{aligned} \tag{A2}$$

The first part of the right-hand side, $-f'(I-i)+g'(i)$ equals zero at $i = i_s^{FB}$, however the remaining terms are strictly positive. Therefore, by the concavity of $f(\cdot)$ and $g(\cdot)$, $\hat{i_s} > i_s^{FB}$. There are two cases to consider,

(i) $\Big[f'(I - \hat{i_s}) g(\hat{i_s}) - g'(\hat{i_s}) f(I - \hat{i_s}) \Big] \leq 0$,

(ii) and $\Big[f'(I - \hat{i_s}) g(\hat{i_s}) - g'(\hat{i_s}) f(I - \hat{i_s}) \Big] > 0$.

In case (i), it is straightforward that there exists $\epsilon > 0$ such that an alternative contract of the form

$$\dot{y^\pi}(1) = \hat{y^\pi}(1) + \epsilon, \tag{A3}$$

$$\dot{y^\pi}(0) = \hat{y^\pi}(0), \tag{A4}$$

dominates. This alternative contract delivers $\hat{i_s} > \dot{i_s} \geq i_s^{FB}$ and is also more affordable for the service provider. Thus, by concavity of the production technologies, $\{\dot{y^\pi}(0), \dot{y^\pi}(1)\}$ delivers greater surplus which contradicts the optimality of $\{\hat{y^\pi}(0), y^\pi \hat{(1)}\}$.

Now, consider case (ii). There exists a contract of the form,

$$\tilde{y^\pi}(0) = \hat{y^\pi}(0) - \delta \frac{f'(I - \hat{i_s}) g(\hat{i_s}) - g'(\hat{i_s}) f(I - \hat{i_s})}{f'(I - \hat{i_s})(1 - g(\hat{i_s})) + g'(\hat{i_s}) f(I - \hat{i_s})} \tag{A5}$$

$$\tilde{y^\pi}(1) = \hat{y^\pi}(1) + \delta. \tag{A6}$$

The alternative contract $\{\tilde{y^\pi}(0), \tilde{y^\pi}(1)\}$ maintains the same incentives as the contract $\{\hat{y^\pi}(0), \hat{y^\pi}(1)\}$. The alternative contract satisfies (10.4) and is more affordable than $\{\hat{y^\pi}(0), \hat{y^\pi}(1)\}$,

$$(I - \beta^\psi)\rho \leq f(I - \hat{i_s}) \left(g(\hat{i_s})\tilde{y^\pi}(1) + (1 - g(\hat{i_s}))\tilde{y^\pi}(0) \right) \tag{A7}$$

$$= f(I - \hat{i_s}) \left(g(\hat{i_s}) \left(\hat{y^\pi}(1) + \delta \right) + (1 - g(\hat{i_s})) \left(\hat{y^\pi}(0) - \delta \frac{f'(I - \hat{i_s})g(\hat{i_s}) - g'(\hat{i_s})f(I - \hat{i_s})}{f'(I - \hat{i_s})(1 - g(\hat{i_s})) + g'(\hat{i_s})f(I - \hat{i_s})} \right) \right) \tag{A8}$$

$$= f(I - \hat{i_s}) \left(g(\hat{i_s})\hat{y^\pi}(1) + (1 - g(\hat{i_s}))\hat{y^\pi}(0) \right)$$
$$+ \delta f(I - \hat{i_s}) \left(g(\hat{i_s}) - \frac{(1 - g(\hat{i_s})) \left(f'(I - \hat{i_s})g(\hat{i_s}) - g'(\hat{i_s})f(I - \hat{i_s}) \right)}{f'(I - \hat{i_s})(1 - g(\hat{i_s})) + g'(\hat{i_s})f(I - \hat{i_s})} \right) \tag{A9}$$

$$= f(I - \hat{i_s}) \left(g(\hat{i_s})\hat{y^\pi}(1) + (1 - g(\hat{i_s}))\hat{y^\pi}(0) \right) + \underbrace{\frac{\delta f(I - \hat{i_s})^2 g'(\hat{i_s})}{f'(I - \hat{i_s})(1 - g(\hat{i_s})) + g'(\hat{i_s})f(I - \hat{i_s})}}_{+}. \tag{A10}$$

Therefore, there exists a contract,

$$\acute{y^\pi}(0) = \tilde{y^\pi}(0) - \epsilon, \tag{A11}$$

$$\acute{y^\pi}(1) = \tilde{y^\pi}(1), \tag{A12}$$

for some small $\epsilon > 0$. By continuity, the contract $\{\acute{y^\pi}(0), \acute{y^\pi}(1)\}$ is affordable, and by concavity, delivers $\hat{i_s} > \acute{i_s} \geq i_s^{FB}$. Thus $\acute{i_s}$ delivers greater surplus which contradicts the optimality of $\{\hat{y^\pi}(0), \hat{y^\pi(1)}\}$.

For the analysis thus far in this proof, it is assumed that $\{\hat{y^\pi}(0), \hat{y^\pi(1)}\}$ delivers an internal choice $\hat{i_s} \in (0, I)$. It is trivial to show that $\hat{i_s} > 0$ with any contract due to Assumption 1 and the fact that s is non-excludable. Consider the remaining corner solution possibility, $\hat{i_s} = I$. In this case, an alternative security,

$$\bar{y^\pi}(0) = 1 - \epsilon, \tag{A13}$$

$$\bar{y^\pi}(1) = 1, \tag{A14}$$

can be constructed for some $\epsilon > 0$. This security leads to an internal choice of $\bar{i_s}$,

$$0 = g'(\bar{i_s}) - \epsilon \left[f'(I - \bar{i_s})(1 - g(\bar{i_s})) + g'(\bar{i_s})f(I - \bar{i_s}) \right], \tag{A15}$$

$$= g'(\bar{i_s})(1 - \epsilon f(I - \bar{i_s})) - \epsilon f'(I - \bar{i_s})(1 - g(\bar{i_s})), \tag{A16}$$

because of Assumption 1. Therefore, the contract $\{\bar{y^\pi}(0), \bar{y^\pi}(1)\}$ delivers $I > \bar{i_s} > i_s^{FB}$, and by continuity, $\bar{i_s}$ delivers greater surplus which contradicts the optimality of $\{\hat{y^\pi}(0), \hat{y^\pi(1)}\}$.

■

Proof of Lemma 2: The proof is constructed as a proof by contradiction: *There exists a security which maximizes joint surplus subject to constraints (19.1)-(19.4) and has the feature* $\hat{y^\psi}(0) < \hat{y^\psi}(1)$.

The firm owners's incentive compatible first-order condition in (19.1) is given by,

$$0 \geq -f'(I-\hat{i_s}) + \Big[f'(I-\hat{i_s})g(\hat{i_s}) - g'(\hat{i_s})f(I-\hat{i_s})\Big]\hat{y^\psi}(1) \\ + \Big[f'(I-\hat{i_s})(1-g(\hat{i_s})) + g'(\hat{i_s})f(I-\hat{i_s})\Big]\hat{y^\psi}(0). \tag{A17}$$

It is straightforward to see that the preceding expression is strictly negative,

$$0 > \underbrace{-f'(I-\hat{i_s})(1-\hat{y^\psi}(1))}_{-} - \underbrace{\Big[f'(I-\hat{i_s})(1-g(\hat{i_s})) + g'(\hat{i_s})f(I-\hat{i_s})\Big]}_{+}\underbrace{(\hat{y^\psi}(1)-\hat{y^\psi}(0))}_{+\text{ by assumption}}, \tag{A18}$$

because the first term is negative and the subtracted second term is positive. Therefore, the firm owner's choice is $\hat{i_s} = 0$ under the contract $\{\hat{y^\psi}(0), \hat{y^\psi}(1)\}$. There exists an an alternative contract design,

$$\tilde{y^\psi}(0) = \epsilon, \tag{A19}$$

$$\tilde{y^\psi}(1) = 0, \tag{A20}$$

for some small $\epsilon > 0$. The firm owner's choice under the alternative contract is determined by,

$$0 \geq -f'(I-\hat{i_s}) + \Big[f'(I-\hat{i_s})(1-g(\hat{i_s})) + g'(\hat{i_s})f(I-\hat{i_s})\Big]\epsilon. \tag{A21}$$

The preceding first-order condition is satisfied with equality with an internal value $\tilde{i_s} \in (0, I)$ due to Assumption 1. Therefore, the contract $\{\tilde{y^\psi}(0), \tilde{y^\psi}(1)\}$ delivers $0 < \tilde{i_s} \leq i_s^{FB}$, and by continuity, $\tilde{i_s}$ delivers greater surplus which contradicts the optimality of $\{\hat{y^\psi}(0), y^\psi\hat{(}1)\}$.

■