

# Corporate Hedging of Exchange Risk When Foreign Currency Cash Flow Is Uncertain

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**W**e analyze hedging policies for a corporation that generates a foreign currency cash flow that is not known with certainty. We obtain an intriguing result that the probability of bankruptcy for a firm that attempts to minimize this probability is *lower* when there is some uncertainty in the exchange rates than when there is no uncertainty in the exchange rates: the firm reduces the probability of bankruptcy by borrowing *more* than its financing needs through foreign currency borrowing alone and by investing the excess funds in domestic risk-free securities.

(Hedging; Exchange Risk; Corporate Risk Management; Inflation Uncertainty)

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## 1. Introduction

The volatility in foreign exchange rates increased dramatically after the breakdown of the Bretton Woods system of fixed exchange rates (see Smith, Smithson and Wilford 1990). The short term movements in exchange rates are often not accompanied by changes in all prices in the two countries. As a result, nominal changes in exchange rates do cause unexpected changes in relative prices. All firms whose value in real terms is affected by changes in relative prices thus face an exchange risk (Levi 1983). In this paper, we analyze hedging policies for a corporation that generates a foreign currency cash flow whose *real* value is affected by *nominal* changes in the exchange rate.

The exchange rate uncertainty associated with the value of a cash flow at a future date that is denominated in the foreign currency can be hedged perfectly in the forward market, provided that the foreign currency value of the cash flow is known with certainty. In this paper, we analyze hedging policies when the foreign currency cash flow is not known with certainty.

We first show that, in perfect capital markets, the profile of cash flows that accrue to the equityholders of the firm in various states is completely determined by the *contractual* value of the firm's foreign currency liabilities alone. The implication of this result is that the

optimality of any hedging policy does not depend on the seniority structure of the domestic and foreign currency debts but depends only on the contractual value of the foreign currency liabilities. This also implies that the foreign currency liability could always be chosen to be a riskless forward contract to deliver the foreign currency even though the foreign currency cash flow for the firm is uncertain (since margin requirements can be met by borrowing domestically, which does not alter the foreign currency liabilities).

Of course, in perfect capital markets, corporations need not hedge exchange risk at all since investors can do it on their own (Aliber 1978). Market imperfections, such as taxes, agency problems, and dead-weight costs associated with financial distress, however, may provide incentives for corporations to hedge the exchange risk (Dufey and Srinivasulu 1983, Stulz 1984, Shapiro and Titman 1985, and Smith and Stulz 1985). Because there are large dead-weight costs associated with bankruptcy, the firms may choose hedging policies that minimize the probability of bankruptcy (Hodder and Senbet 1990).

When the foreign currency cash flow is known with certainty, we know that the optimal hedge is to borrow an amount such that the debt repayment in foreign currency is exactly equal to the foreign currency cash flow: this implies that the foreign currency borrowing would

exceed the financing needs (for projects with positive net present values). In this paper, we show that even when the foreign currency cash flow (whose present value exceeds the financing need) is uncertain, the firm attempting to minimize the probability of bankruptcy, would borrow *more* than its debt financing needs through foreign currency borrowing alone and invest the excess funds in domestic risk-free securities, when there is some uncertainty in the exchange rate.<sup>1</sup>

We obtain an intriguing result that the probability of bankruptcy for the firm with an uncertain foreign currency cash flow, if it attempts to minimize this probability, is *lower* when there is some uncertainty in the exchange rate than when there is no uncertainty in the exchange rate. The intuition for this result is as follows. The risk faced by the firm stems from two sources, the quantity uncertainty (uncertainty in the level of foreign currency cash flow) and the price uncertainty (the uncertainty in the exchange rate). The price and the quantity are assumed to be uncorrelated. If price is certain, no hedging instrument (which can depend only on the observable and contractible variables such as the price) can be used to hedge the quantity risk. If, however, the price is variable, then the total cash flow is positively correlated with the price and an instrument whose payoff is contingent on the price, such as a forward contract, can be used to hedge the variation in the total cash flow.<sup>2</sup>

The variation in the exchange rate allows the firm to write debt contracts in such a way that it takes a gamble whose payoffs are symmetric and contingent on the value of the exchange rate in such a way that it transfers funds from highly insolvent states into slightly insolvent states and from highly solvent states into slightly solvent states. But, since a relatively small amount of domestic funds need to be infused in the slightly insolvent states to pull the firm out of bankruptcy and a relatively large amount of funds need to be extracted from highly sol-

vent states to drag the firm into bankruptcy, the overall effect of the gamble is to reduce the probability of bankruptcy.

## 2. The Model

### 2.1. Exchange Risk

We analyze a very simple one period model in which a firm based in the U.S. has a single project with positive net present value that generates a single cash flow  $x^f \geq 0$ , denominated in a foreign currency denoted FX, at the end of the period. For simplicity, let us normalize the current exchange rate to be \$1 / FX. Let  $s > 0$  denote the exchange rate, expressed in dollars for each unit of FX, at the end of the period. Let  $f(x^f)$  and  $g(s)$  denote the density functions of  $x^f$  and  $s$  respectively.

ASSUMPTION 1. *All agents care only about the real, rather than the nominal, value of the cash flows.*<sup>3</sup>

ASSUMPTION 2. *The exchange rate at the end of the period,  $s$ , is determined by the Purchasing Power Parity relationship between the general price levels in the two countries.*<sup>4</sup>

Let us also normalize the expected *real* rate of return to be zero in both countries.<sup>5</sup> For simplicity, we assume that the general price level in the U.S. stays the same. This implies that the *nominal* riskless rate of return in

<sup>3</sup> This ensures that our results are not being driven by "free lunches" due to the Siegel Paradox. See Siegel (1972), Black (1990). The fact that reasoning in real terms disposes of the Siegel Paradox has been discussed previously in the literature. For instance, see Adler and Dumas (1983), footnote 60, page 955.

<sup>4</sup> One might argue that there seems to be ample evidence the exchange rates seem to violate the Purchasing Power Parity relationship between the general price levels in the two countries (for instance, see Mussa 1979). But then, one has to model these violations in an equilibrium framework. In a simple model with perfect capital markets, such as the one we consider in this paper (and also the one in Black 1990), exchange rate movements that violate the Purchasing Power Parity relationship would be inconsistent with equilibrium and may of course imply presence of expected profit opportunities. Our goal in this paper is not to develop an equilibrium model that allows for violations of the Purchasing Power Parity but to illustrate the main insights of the paper in a simple but consistent framework. Our analysis abstracts away from issues arising from linkages between currency risk and relative price risk that are discussed in Shapiro (1984).

<sup>5</sup> Equilibrium with risk neutral agents (see Assumption 4) in financial markets ensures that expected real rates are equal across countries.

<sup>1</sup> Since the excess foreign currency borrowing is invested domestically in risk-free securities, this is equivalent to selling some foreign currency forward. However, since the firm may default on its obligations, the forward contract here, in general, is different from traded forward contracts in which the firm would be required to deposit funds for margin requirements to prevent it from defaulting on the forward contract.

<sup>2</sup> I am grateful to the referees for suggesting this intuitive explanation.

the U.S. is also zero. Since we have assumed that the price level in the U.S. does not change, variation in  $s$  is caused only by the variation in the general price level in the foreign country. This implies that the real value of any cash flow denominated in FX can be obtained by evaluating the corresponding value of the cash flow in dollars, i.e., by multiplying the FX cash flow by the prevailing \$/FX exchange rate  $s$ . Of course, by construction, the nominal and real values of cash flows in dollars are identical.<sup>6</sup> Let us also normalize the expected value of  $s$  to be equal to one. This implies that the nominal rate of return in the foreign country is also zero.

**DEFINITION 1.** A firm is said to face an exchange risk if *nominal* changes in exchange rates cause changes in *real* cash flows or *real* value of the firm.

A change in the exchange rate  $s$ , in our model, is caused only because of changes in the general price level in the foreign country to maintain the Purchasing Power Parity between the general price levels in the two countries. However, the short term movements in exchange rates are often not accompanied by changes in all prices in the two countries. As a result, nominal changes in exchange rates do cause unexpected changes in relative prices which may subject firms to exchange risks. If the FX cash flow  $x^f$  changes one to one in response to the change in general price level, then the real value of the cash flow is unchanged by the change in the exchange rate and the firm does not face any exchange risk (Cornell 1980). To ensure that the firm does face some exchange risk, we must therefore assume that the FX cash flow  $x^f$  does not change one to one with the general price level. We formalize this, in a simple way, by making the following assumption.

**ASSUMPTION 3.** *The firm's FX cash flow  $x^f$  and the exchange rate  $s$  at the end of the period are stochastically independent.*

The real value of the FX cash flow  $x^f$  is simply equal to  $sx^f$ . Intuitively, it is clear that the above assumption implies that the firm faces an exchange risk since a change in the nominal exchange rate  $s$  is not accompanied by any corresponding change in  $x^f$  and therefore the real value of the FX cash flow  $sx^f$  is affected by a

change in  $s$ .<sup>7</sup> Of course, this assumption is stronger than what is needed to ensure that the firm faces an exchange risk. All that is needed is that a change in  $s$  is not accompanied by an offsetting change in  $x^f$  such that  $sx^f$  is unaltered because then the firm does not face any exchange risk.<sup>8</sup> We make the stronger assumption only for simplicity.

## 2.2. Hedging Policies

Let  $c$  denote the part of the financing need which needs to be financed by issuing debt; the rest is financed by equity. Let  $y$  and  $y^f$  denote the proceeds from a dollar loan and an FX loan respectively. Recall that the current exchange rate is normalized to \$1/FX. The proceeds from the two loans must be at least as large as  $c$ , i.e., the following financing constraint must be satisfied:

$$y + y^f \geq c.$$

We assume that any excess funds  $y + y^f - c$  are invested in risk-free securities in the U.S. (which, we earlier assumed, pay an interest rate of zero).

Let  $z$  denote the face value (in \$) of the dollar loan and let  $z^f$  denote the face value (in FX) of the foreign currency loan to be paid at the end of the period. The real value of the cash flow (which in our case is simply the value of the cash flow in dollars) at the end of the period, then is  $sx^f + y + y^f - c$ .

The dollar value of the residual cash flow to the equityholders is

$$\text{Max}\{0, sx^f + (y + y^f - c) - (sz^f + z)\}. \quad (1)$$

Given a distribution for the exchange rate  $s$  and the FX cash flow  $x^f$ , the distribution of the residual cash flow to the equityholders, in general, depends on the levels of the two debts  $y$  and  $y^f$  and the corresponding face values  $z$  and  $z^f$  of these debts (which are determined by the seniority structure of the two debts). Since, any hedging policy essentially attempts to alter this profile of cash flows that accrue to the equityholders of the firm in various states that are characterized by different

<sup>7</sup> It is easy to check that this assumption implies that the measure of economic exposure given by  $\text{Cov}(sx^f, s)/\text{Var}(s)$  (see Hodder 1982 and Adler and Dumas 1984) is positive.

<sup>8</sup> The measure of economic exposure given by  $\text{Cov}(sx^f, s)/\text{Var}(s)$  is equal to zero in that case.

<sup>6</sup> With this normalization, one could interpret our results more generally to a case of inflation uncertainty.

realizations of  $s$  and  $x^f$ , we define a hedging policy as follows.

**DEFINITION 2.** The selection of the levels of debt in each currency  $y$  and  $y^f$  together with the seniority structure of the debts constitutes a hedging policy.

Notice that all claims are priced given a hedging policy in place. This includes firm's committing to invest any excess funds in domestic risk-free securities. We are thus assuming that the claimholders on firm's end of period cash flow are able to put covenants that rule out any ex post transfers of wealth among the various debt and equity holders of the firm.

**DEFINITION 3.** Two hedging policies are equivalent if and only if for any given realizations of  $s$  and  $x^f$ , the dollar value of the residual cash flow to the equityholders is identical for the two policies.

We now make another simplifying assumption that allows us to price these loans.

**ASSUMPTION 4.** All loans are priced competitively in a market with risk-neutral agents.

The debtholders receive either the entire contractual value of their loans  $sz^f + z$  or the entire cash flow of the firm  $sx^f + (y + y^f - c)$  if the firm cannot meet its contractual obligations fully. The expected value of the cash flow that accrues to the debtholders must equal the market value of the loans  $y + y^f$ .

$$y + y^f = E_{s,x^f}[\text{Min}\{sx^f + (y + y^f - c), sz^f + z\}]. \quad (2)$$

We now state our first important result.

**LEMMA 1.** All hedging policies with identical face values  $z^f$  of the FX loan are equivalent.

**PROOF.** Equation (2) can be rewritten as follows:

$$E_{s,x^f}[\text{Min}\{sx^f - c, sz^f - (y + y^f - z)\}] = 0.$$

Notice that choosing a value for  $z^f$  sets a corresponding value for  $y + y^f - z$  which (from (1)) completely determines the residual cash flow to the equityholders.  $\square$

This result is quite obvious for the case when the FX loan is junior to the dollar loan. In this case, given a value of  $z^f$ ,  $y - z$  as well as  $y^f$  are uniquely determined. To see this, notice that from financing constraint we know that  $y + y^f - c \geq 0$ . Suppose now that  $y + y^f - c = 0$ . Any increase in  $y$  increases the surplus cash flow  $y + y^f - c$  by exactly that amount since all excess

funds are invested in domestic risk-free securities. Since the dollar loan is senior, the face value of the dollar loan  $z$  also increases by exactly the same amount since the incremental dollar loan is riskless. So  $y - z$  remains unchanged and therefore the residual cash flow after paying the dollar loan is also unchanged. Consequently, the value of the junior FX loan  $y^f$  is also unchanged. Clearly, given a value of  $z^f$ , the residual cash flow to the equityholders is uniquely determined.

What is surprising is that this result also holds when the FX loan is senior. Consider the case when  $y + y^f = c$ . Given a value of  $z^f$ , as the junior dollar loan  $y$  is increased, since the increased surplus is used to payoff the senior FX liability first, it increases the value of the FX loan  $y^f$ . But it also increases the face value of the junior dollar loan  $z$ . What Lemma 1 guarantees is that the combined increase in the value of the two loans  $y + y^f$  is exactly offset by an equal increase in the face value of the dollar loan  $z$  such that  $y + y^f - z$  is unaltered. Consequently, the residual cash flow to the equityholders is unchanged.

The implication of this result is that the optimality of any hedging policy does not depend on the seniority structure of the domestic and foreign currency debts but depends only on the contractual value of the foreign currency liabilities. This also implies that the foreign currency liability could always be chosen to be a riskless forward contract to deliver the foreign currency even though the foreign currency cash flow for the firm is uncertain. This is because margin requirements can be met by borrowing domestically which does not alter the foreign currency liabilities.

### 2.3. Optimal Hedging Policies: Minimizing the Probability of Bankruptcy

A large body of literature in finance discusses the direct and indirect dead-weight costs associated with bankruptcy and financial distress. For instance, Shapiro and Titman (1985) argue that financial distress, or the threat of bankruptcy, affects the incentives of firm's managers, clients, suppliers, creditors and other stakeholders in such a way that it adversely affects firm's ability to generate value.

One way to avoid the possibility of bankruptcy, of course, is to finance the firm's operations entirely with equity. There are several reasons, however, why a firm

may finance its operations, at least partly (as we have assumed), with debt. One reason is the tax deductibility of interest payments (see Myers 1983). Another reason is that if an ongoing firm has to issue new securities to raise funds there may be a "pecking order" in which the firm prefers to issue debt rather than equity (see Myers and Majluf 1984). Jensen (1986) argues that debt in the capital structure provides managers with proper incentives to promote organizational efficiency. An agency theory argument for debt is that since risky debt in firm's capital structure creates a possibility of bankruptcy which is sufficiently costly for managers, they would provide an appropriate level of effort in order to minimize the probability of bankruptcy.

We assume that firm's capital structure contains risky debt and that managers structure the firm's debts in such a way so as to minimize the probability of bankruptcy. We say that the firm is bankrupt if, at the end of the period, it is not able to meet its contractual debt obligations, i.e., if

$$sx^f + (y + y^f - c) < sz^f + z.$$

Let  $\pi$  denote the probability of bankruptcy, i.e., the probability that  $sx^f + (y + y^f - c) < sz^f + z$ .

We first analyze the case when the firm has no domestic debt ( $y = z = 0$ ). We will later show that results generalize to the case when the firm does have some domestic debt as well ( $y > 0$ ). The condition for bankruptcy can now be written as:

$$x^f < z^f - \frac{y^f - c}{s}. \quad (3)$$

**LEMMA 2.** *If the firm has no domestic debt and its foreign currency loan  $y^f$  is exactly equal to its financing needs  $c$ , then a small incremental increase in the foreign currency loan amount increases the face value of the foreign currency loan by the same amount. Formally, if  $y = 0$  then  $dy^f / dz^f = 1$  for that value of  $z^f$  for which  $y^f = c$ .*

**PROOF.** Substituting  $y = z = 0$ , Equation (2) can be rewritten as:

$$y^f = z^f + E_{s,x^f} \text{Min} \{s(x^f - z^f) + y^f - c, 0\}.$$

The above equation can then be expressed as:

$$y^f = z^f + E_s \int^{z^f - (y^f - c)/s} \{s(x^f - z^f) + y^f - c\} f(x^f) dx^f.$$

Differentiating the above with respect to  $z^f$  and substituting  $y^f = c$ , we get

$$\frac{dy^f}{dz^f} = 1 - F(z^f)E_s s + \frac{dy^f}{dz^f} F(z^f),$$

where  $F(\cdot)$  denotes the distribution function. Substituting  $E_s s = 1$  and rearranging, we get

$$\frac{dy^f}{dz^f} = 1. \quad \square$$

The intuition for this result is as follows. With  $y = 0$  and  $y^f = c$ , the firm's cash flow at the end of the period is  $sx^f$ . The contractual value of the debt payment is  $sz^f$ . So, the firm is bankrupt if and only if  $x^f < z^f$ . For all states for which  $x^f > z^f$  the original debt was being paid off completely and there was some excess cash left for equityholders. For these states, a slight increase in the contractual payment could still be met completely. For all states  $x^f < z^f$  the firm was originally bankrupt, so the slight amount of excess funds that the firm now has in addition to  $c$  would go to the debtholders completely for these states. Since the debtholders are either able to recoup the entire amount of the additional loan or are paid fully the contractual amount, the marginal increase in the face value  $z^f$  is equal to the marginal increase in the value  $y^f$  of the FX loan.

We use the result in Lemma 2 to now prove the following result.

**LEMMA 3.** *If the firm has no domestic debt and its foreign currency loan  $y^f$  is exactly equal to its financing needs  $c$ , then a small incremental increase in the foreign currency loan amount decreases the probability of bankruptcy for the firm. Formally, if  $y = 0$  then  $d\pi / dz^f < 0$  for that value of  $z^f$  for which  $y^f = c$ .*

**PROOF.** The probability of bankruptcy  $\pi$  can be expressed as follows:

$$\pi = \text{Prob} \left[ x^f < z^f - \frac{y^f - c}{s} \right] = E_s \int^{z^f - (y^f - c)/s} f(x^f) dx^f.$$

Differentiating with respect to  $z^f$  and substituting  $y^f = c$ , we get

$$\frac{d\pi}{dz^f} = f(z^f)E_s \left[ 1 - \frac{1}{s} \frac{dy^f}{dz^f} \right].$$

From Lemma 2 ( $dy^f/dz^f = 1$ ), Jensen's Inequality, and the fact that  $E_s s = 1$ , we get

$$\frac{d\pi}{dz^f} = f(z^f)E_s \left[ 1 - \frac{1}{s} \right] < 0. \quad \square$$

The intuition for this result is as follows. The firm chooses a value of the FX debt  $y^f$  that is slightly larger than  $c$ , and invests the excess funds, say  $\Delta y^f$ , in U.S. riskfree securities. From Lemma 2, the increase in the face value the FX loan  $\Delta z^f$  is equal to  $\Delta y^f$  units of FX, the dollar value of which is equal to  $s\Delta y^f$ . So, essentially the firm takes an incremental gamble whose payoff, in dollars, is equal to  $\Delta y^f(1 - s)$ . The payoff from this gamble is positive if  $s < 1$ , negative if  $s > 1$  and the expected value of this gamble is zero. When  $s$  is high ( $>1$ ), the value of the surplus or the value of the deficit,  $|s(x^f - z^f)|$ , is high. When  $s$  is low ( $<1$ ) the value of the surplus or the value of the deficit,  $|s(x^f - z^f)|$ , is low. So, for any given  $x^f$ , the gamble takes funds away from high surplus (deficit) states and puts them in low surplus (deficit) states. As a result, the deficit in the high deficit states gets even larger but that does not change the probability of bankruptcy. Also, the surplus in the low surplus states gets larger and that does not alter the probability of bankruptcy either. But the deficit in the low deficit states becomes smaller, which reduces the probability of bankruptcy. However, the surplus in the high surplus states becomes smaller, which increases the probability of bankruptcy. Since a relatively small amount of domestic funds need to be infused in the low deficit states to pull the firm out of bankruptcy but a relatively large amount of funds need to be extracted from high surplus states to drag the firm into bankruptcy, the overall effect of the gamble is to reduce the probability of bankruptcy.<sup>9</sup>

<sup>9</sup> Another way to think about the intuition behind this result is as follows. The payoff from the incremental gamble expressed not in real (dollar) terms but in terms of units of foreign currency FX is  $\Delta y^f(\frac{1}{s} - 1)$ . The expected value of this payoff, in terms of FX, is positive (from Jensen's Inequality). The condition for bankruptcy if the firm borrows just enough to meet its financing need is whether or not  $x^f < z^f$ . So, the deficit, if the firm is bankrupt, or the surplus if the firm is solvent, expressed in units of FX is equal to  $(z^f - x^f)$ . Since this deficit (or surplus) in units of FX is uncorrelated with the payoff (also in units of FX) from the incremental gamble  $\Delta y^f(\frac{1}{s} - 1)$ , taking this incremental gamble reduces the probability of bankruptcy because the expected payoff of this gamble (in units of FX) is positive. I am grateful to Sheridan Titman for pointing this out to me.

What we have shown in Lemma 3 is that if the firm does not borrow domestically at all then its foreign currency borrowing *strictly* exceeds its financing needs  $c$ , if the firm attempts to minimize its probability of bankruptcy. The following proposition demonstrates that even if the firm borrows domestically also, its foreign currency borrowing alone would still exceed its financing need  $c$ .

**PROPOSITION 1.** *If the firm attempts to minimize the probability of bankruptcy, it would borrow more than its financing need  $c$  through foreign currency borrowing alone. Formally,  $y^{f*}$  is always strictly greater than  $c$  for  $c > 0$ .*

**PROOF.** In Lemma 3, we showed that if  $y = 0$  then  $\pi$  is decreasing in  $z^f$  at  $y^f = c$  which implies that the optimal level of the FX loan  $y^{f*} > c$ . Let  $z^{f*}$  denote the value of  $z^f$  for  $y^f = y^{f*}$  and  $y = 0$ . From Lemma 1 we know that any other hedging policy with the face value  $z^f = z^{f*}$  would also be optimal. Now consider if the firm also borrows domestically, i.e., let  $y > 0$ . There are two possibilities.

1. The dollar loan is senior. Since  $y^{f*} > c$  with  $y = 0$ , any additional funds raised with the dollar loan would stay in risk-free securities, so that the dollar loan could be paid off in full. Clearly for a given value of  $z^{f*}$ , the value of the FX loan would stay the same.

2. The dollar loan is junior. Any additional funds raised with the dollar loan could be used to pay off the FX loan first which reduces the riskiness of the FX loan. For a given face value  $z^f = z^{f*}$  of the FX loan, the corresponding value of  $y^f$  would increase with an increase in  $y$ . Since the value of the FX loan  $y^f$  was strictly greater than  $c$  even when  $y = 0$ , the value of  $y^f$  for  $y > 0$  would increase even further.  $\square$

Since for all optimal hedging policies  $y^{f*} > c$ , without loss of generality, we can now restrict our attention to hedging policies for which  $y = 0$ .

**LEMMA 4.** *If  $y^f = c$  then the probability of bankruptcy  $\pi$  does not depend on the variation in the end of period exchange rate  $s$ .*

**PROOF.** For  $y = 0$ , the condition for bankruptcy (3) after substituting  $y^f = c$  is  $x^f < z^f$ . Clearly, the probability that  $x^f < z^f$  does not depend on the variation in  $s$ .  $\square$

The intuition for this result is straightforward. If the firm borrows just enough to meet the costs at the be-

ginning of the period, the dollar value of the cash flow at the end of the period is  $sx^f$ . The dollar value of the contractual payment is  $sz^f$ . Any variation in  $s$  affects the cash flow  $sx^f$  and the contractual payment  $sz^f$  by the same factor so that the condition for bankruptcy ( $x^f < z^f$ ) stays unaltered regardless of any variation in  $s$ .

We now prove the main result of the paper.

**PROPOSITION 2.** *The probability of bankruptcy for the firm if it attempts to minimize this probability is lower when there is some uncertainty in the end of period exchange rate  $s$  than when there is no uncertainty in  $s$ .*

**PROOF.** We have seen in Lemma 3 and Proposition 1 that the probability of bankruptcy when there is some variation in  $s$  is smaller at the optimal value  $z^{f*}$  which implies that  $y^{f*} > c$  than for a case when  $y^f = c$ . But from Lemma 4 we know that  $y^f = c$  gives us the probability of bankruptcy when there is no variation in  $s$ .  $\square$

The result in Proposition 2 appears counterintuitive at first. But as we have seen in Lemma 3, the variation in the exchange rate allows the firm to write debt contracts in such a way that it takes a gamble whose payoffs are symmetric and contingent on the value of the exchange rate in such a way that it transfers funds from high deficit states into low deficit states and from high surplus states into low surplus states. But, since a relatively small amount of domestic funds needs to be infused in the low deficit states to pull the firm out of bankruptcy and a relatively large amount of funds needs to be extracted from high surplus states to drag the firm into bankruptcy, the overall effect of the gamble is to reduce the probability of bankruptcy.

### 3. Concluding Remarks

We analyzed hedging policies for corporations when the foreign currency cash flow is not known with certainty. We obtained an empirical implication that predicts that if there is some exchange rate variability, firms attempting to minimize the probability of bankruptcy would borrow *more* than their financing needs through foreign currency borrowing alone and invest the excess funds in domestic risk-free securities. We obtained a result that the probability of bankruptcy for a firm that attempts to minimize this probability is *lower* when there

is some uncertainty in the exchange rates than when there is no uncertainty in the exchange rates.

Our results suggest some intriguing possibilities that future research efforts could explore. Firms attempting to minimize the probability of bankruptcy may choose not to invoice their products in currencies of countries that have no inflation uncertainty. If the intuition carries over to a more general context of inflation uncertainty, our results suggest that firms attempting to minimize the probability of bankruptcy may choose to price their products in *nominal* rather than *real* terms, borrow more than the financing needs through *nominal* debt contracts and invest the excess funds in *real* assets.

We did not explore the use of instruments such as options in our analysis (Giddy 1983). One would suspect that if the firms use a richer set of instruments, they could reduce the probability of bankruptcy even further. The optimal set of contracts that minimize the probability of bankruptcy may well be non-linear contracts whose payoffs are contingent on the value of random variables, such as the exchange rate, whose values are ex post publicly observable.

We leave it to future research to explore these issues further.<sup>10</sup>

<sup>10</sup> An earlier version of this paper was titled, "Exchange Risk Management and Corporate Capital Structure." I thank Michael Brennan and Sheridan Titman for several helpful discussions. Rob Heinkel (the editor), an anonymous associate editor, two anonymous referees, and the participants at the Finance Seminars at UCLA and UC Irvine provided several useful suggestions.

### References

- Adler, Michael and Bernard Dumas, "International Portfolio Choice and Corporation Finance: A Synthesis," *J. Finance*, 38 (1983), 925-984.
- and —, "Exposure and Currency Risk: Definition and Measurement," *Financial Management*, (Summer 1984), 41-50.
- Aliber, Robert Z., *Exchange Risk and Corporate International Finance*, Halsted Press, New York, 1978.
- Black, Fischer, "Equilibrium Exchange Rate Hedging," *J. Finance*, 45 (1990), 899-908.
- Cornell, Bradford, "Inflation, Relative Price Changes and Exchange Risk," *Financial Management*, (Autumn 1980), 30-35.
- Dufey, Gunter and S. N. Srinivasulu, "The Case for Corporate Management of Foreign Exchange Risk," *Financial Management*, (Winter 1983), 54-62.
- Giddy, Ian H., "The Foreign Exchange Option as a Hedging Tool," *Midland Corporate Finance J.* (Fall 1983), 32-42.

CHOWDHRY

Corporate Hedging of Exchange Risk

---

- Hodder, James E., "Exposure to Exchange Rate Movements," *J. International Economics*, 13 (1982), 375-386.
- and Lemma W. Senbet, "Agency Problems and International Capital Structure," in S. G. Rhee and R. P. Chang (Eds.), *Pacific Basin Capital Markets Res.*, North Holland, New York, NY, 1990, 485-511.
- Jensen, Michael C., "The Takeover Controversy: Analysis and Evidence," *Midland Corporate Finance J.*, (Summer 1986), 6-32.
- Levi, Maurice, *International Finance*, McGraw-Hill, New York, NY, 1983.
- Mussa, Michael, "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market," in *Carnegie Rochester Conference Series on Public Policy*, 11, North-Holland, Amsterdam, The Netherlands, 1979, 9-58.
- Myers, Stewart C., "The Search for Optimal Capital Structure," *Midland Corporate Finance J.* (Spring 1983), 6-16.
- and Nicholas S. Majluf, "Corporate Financing and Investment Decisions when Firms Have Information that Investors do not Have," *J. Financial Economics*, 13 (1984), 187-221.
- Shapiro, Alan C., "Currency Risk and Relative Price Risk," *J. Financial and Quantitative Analysis*, 19 (1984), 365-373.
- and Sheridan Titman, "An Integrated Approach to Corporate Risk Management," *Midland Corporate Finance J.* (Summer 1985), 41-56.
- Siegel, Jeremy J., "Risk, Interest Rates and the Forward Exchange," *Quarterly J. Economics*, 86 (1972), 303-309.
- Smith, Clifford W., Jr., Charles W. Smithson, and D. Sykes Wilford, *Managing Financial Risk*, Harper Collins Publishers, 1990.
- and Renè Stulz, "The Determinants of Firms' Hedging Policies," *J. Financial and Quantitative Analysis*, 20 (1985), 391-403.
- Stulz, Renè, "Optimal Hedging Policies," *J. Financial and Quantitative Analysis*, 19 (1984), 127-140.

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