

POSSIBILITY OF DYING AS A UNIFIED EXPLANATION OF WHY WE DISCOUNT THE FUTURE, GET WEAKER WITH AGE, AND DISPLAY RISK-AVERSION

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I formulate a simple and parsimonious evolutionary model that shows that because most species face a possibility of dying because of external factors, called extrinsic mortality in the biology literature, it can simultaneously explain (a) why we discount the future, (b) get weaker with age, and (c) display risk-aversion. The paper suggests that testable restrictions—across species, across time, or across genders—among time preference, aging, and risk-aversion could be analyzed in a simple framework (JEL A10, D90).

I. INTRODUCTION

A number of evolutionary theories have been proposed to explain the phenomenon of aging or senescence or why we get weak as we get older (see the review article by Gavrilov and Gavrilova (2002)). Economists have also explored the biological basis of preferences, such as discounting of future consumption (Rogers 1994), and risk-aversion (McAfee 1984), that are usually taken as primitive (see a comprehensive article by Robson (2002)). In this paper, I formulate a simple and parsimonious evolutionary model that shows that because most species face a possibility of dying because of external factors, called extrinsic mortality in the biology literature, it can simultaneously explain (a) why we discount the future, (b) get weaker with age,¹ and (c) display risk-aversion.

The central idea is as follows. I consider an organism that in principle can live and reproduce forever, that is, it faces no intrinsic

mortality. The organism does, however, face external threats, such as predation, disease, and cataclysmic events that can potentially kill the organism because it does not possess infinite amounts of resources to fight the external threats. Possibility of death directly implies that organism will discount the future. Moreover, if the organism has a flexibility in allocating some of its limited resources over time, then following an argument proposed in the Antagonistic Pleiotropy Theory of Williams (1957), the organism will disproportionately allocate resources to when it is young at the expense of having fewer resources when it is old. This is because it is more important to be alive when young, as being alive when old is pre-conditioned on also being alive when young, whereas the converse is not true. Thus the implication suggesting why we are strong when young is nicely captured in Williams (1957):

...natural selection will frequently maximize vigor in youth at the expense of vigor later on and thereby produce a declining vigor (senescence) during adult life.

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1. Sozou and Symour (2003) analyze interaction between aging and time preference. See Robson and Kaplan (2007) for a sketch of a model of aging that is developed in Robson and Kaplan (2006).

I show that the possibility of death also suggests that the organism will prefer a sure endowment of resources to an uncertain endowment of resources that has the same expected value. The intuition is that a decrease in resources

ABBREVIATIONS

RHS: Right-Hand Side

has a disproportionately larger negative impact compared with an equal amount of increase in resources because the decrease is more likely to cause the organism to die² and thereby permanently forgo the possibility of producing offspring in the future that will contribute to its genetic fitness. This intuition is similar to results in models of financial distress and bankruptcy in which firms with large amounts of intangible assets that might disappear in bankruptcy will tend to avoid financial distress by avoiding risky outcomes and by hedging.

My central contribution is in demonstrating with a simple model that although the possibility of death induces preference for expending physical body resources on maintaining health in youth, the fact that vigor declines smoothly and not abruptly (i.e., aging) suggests that preference for smoothing (and perhaps risk-aversion) might also arise endogenously and testable restrictions—across species, across time, or across genders—among time preference, aging, and risk-aversion could be analyzed in a simple framework. Acharya and Balvers (2004) derive not only time preference with endogenous survival but also other properties of intertemporal preferences such as time-consistent discounting, risk, intertemporal substitution, and non-separability of utility function in an evolutionary model similar in spirit to this paper though they apply the model to study life-cycle consumption choices among individuals.

II. THE MODEL

Consider a living organism that maximizes its expected lifetime genetic contribution or genetic fitness:

$$F_0 \equiv \max_{x_t} E \left[\int_0^{\infty} f_t dt \right],$$

where x_t denotes the rate at which resources are expended by the organism to produce offspring and f_t represents the consequent rate of organism's genetic contribution at any time $t \geq 0$. I want to make sure that risk-aversion is not induced by concavity in the production function and therefore I posit a simplest linear production function in which f_t depends only on the rate of expenditure x_t , and is equal to the rate of expenditure, that is,

$$(1) \quad f_t = f(x_t) = x_t.$$

2. Using an evolutionary framework, this intuition is also captured in a result in McAfee (1984).

Suppose the organism is endowed with some initial stock of resources Z_0 to allocate over time. I do not, in this paper, analyze the mechanism by which the initial stock of resources is acquired. Robson and Kaplan (2006) develop a model in which resources are built up in early ages and are depleted later in life. In my formulation, $t = 0$ represents the time at which the organism can start reproducing. Essentially, what I am trying to formalize here is the notion that the organism has some flexibility, induced perhaps by genetic mutations over time, in allocating some body resources over time.

Notice that I have assumed no explicit discounting of future fitness in the organism's objective function. However, I assume that the organism can die at any instant when it meets exogenous external threats (disease, predators, cataclysmic events, etc.) that arrive randomly. Clearly, $f_t = x_t = 0$, if the organism is dead. The fact that the organism can die at any instant causes the organism to discount the future. With a linear production function posited in Equation (1), it will be optimal for the organism to expend all its initial resources Z_0 immediately to produce offspring unless being alive in the future conferred some additional advantage in terms of producing offspring in the future. I assume that in addition to the initial stock, resources are replenished continuously at a constant endowment rate of η per unit of time. Thus, the level of resources at any given time, Z_t , follow the following equation:

$$(2) \quad dZ_t = (\eta - x_t)dt.$$

The idea here is that as the organism consumes food and drink over time, it resuscitates and repairs the body, engenders cell growth, and so forth.

Let δdt denote the probability of death over an infinitesimal period dt . Thus, $1 - \delta dt$ represents the probability of surviving over the next infinitesimal period dt . I assume that δ depends on x_t . The organism is stronger if it expends resources at a greater rate and the stronger the organism is, the larger is the probability that it will survive the external threat and continue to live. Formally, I make the following assumption:

Assumption 1

$$(3) \quad \delta'(x_t) < 0$$

which implies that the organism lowers its probability of death (increases its probability of

survival) by expending resources at a greater rate x_t .

Notice that in my formulation, a higher expenditure x_t leads not only to a higher probability of survival but also to a larger number of offspring given the production function in Equation (1). One might argue that there is, in fact, a tradeoff between fertility and survival³ as in models in Shepard and Zeckhauser (1984) and Rosen (1988). I choose a simpler formulation not merely to abstract from this tradeoff but also because there is evidence in the biomedical literature that suggests that the mechanism that links expenditure of organism's resources (that cannot be transferred from one organism to another) to production of offspring is through better health of parents. When such resources are spent by the organism at greater intensity, it leads to better overall health which in turn leads to a greater number of its offspring that survive. In other words, healthier organisms are more likely to reproduce and their offspring are more likely to survive.

The organism's maximization problem can thus be expressed as the following dynamic optimization problem:⁴

$$(4) \quad \max_{x_t} E \left[\int_t^\infty x_s ds \right] \equiv F_t = \max_{x_t} x_t dt + [1 - \delta(x_t)dt](F_t + dF_t).$$

It is easy to see that the fitness function F will depend on the state variable Z_t and thus $F_t = F(Z_t)$ and⁵

$$dF_t = F'(Z_t)dZ_t = F'(Z_t)(\eta - x_t)dt.$$

Substituting in the Bellman Equation (4), we get:

$$F_t = \max_{x_t} x_t dt + [1 - \delta(x_t)dt] [F_t + F'(Z_t)(\eta - x_t)dt].$$

3. A referee pointed out that for many organisms, particularly mammals, this tradeoff can be substantial. For example, in models of weaning conflict, a female's own survival probability improves if she weans her offspring but the offspring's survival probability improves if she does not wean for a longer time.

4. A formally equivalent formulation of the problem would be to posit the following stochastic process for Z_t :

$$dZ_t = (\eta - x_t)dt - Z_t dq,$$

where dq represents a jump process that takes value 1 with probability δdt and value 0 with the probability $1 - \delta dt$. Applying Ito's Lemma for jump processes and solving is formally equivalent to the formulation presented here.

5. In general, F may also depend on time t but in our formulation the dependence on time enters only through the state variable Z_t .

Rearranging terms, we get:

$$F_t = \max_{x_t} F_t + [x_t + F'(Z_t)(\eta - x_t) - \delta(x_t)F(Z_t)] dt - [\delta(x_t)F'(Z_t)(\eta - x_t)](dt)^2$$

which, after canceling F_t from both sides and ignoring the last term

$[\delta(x_t)F'(Z_t)(\eta - x_t)](dt)^2$ which is of second order, yields

$$(5) \quad 0 = \max_{x_t} [x_t + F'(Z_t)(\eta - x_t) - \delta(x_t)F(Z_t)] dt.$$

The first-order condition for an interior maximum to the maximization problem is:

$$(6) \quad 1 - F'(Z_t) - \delta'(x_t)F(Z_t) = 0.$$

The second-order condition for an interior maximum is:

$$-\delta''(x_t)F(Z_t) < 0.$$

Since $F(Z_t) > 0$, a sufficient condition for an interior maximum to exist is:

$$(7) \quad \delta''(x_t) > 0$$

at the optimal value of x_t .

First suppose that an interior maximum to the optimization problem does not exist for all values of Z_t in which case it will be optimal for the organism to expend all its initial resources immediately and then only consume what the organism replenishes at the rate $x_t = \eta$. Clearly, this will result in a constant rate of expenditure—after the initial burst of expenditure—and thus produce a constant probability of death $\delta(\eta)$ which is inconsistent with the observed phenomenon of aging or increasing mortality rate with age.

Let us now examine the case when the above second-order condition is satisfied and an interior maximum to the optimization problem does obtain for some value of Z_t —perhaps after an initial burst of expenditure that reduces initial level of resources from Z_0 to Z_t .⁶ Let $x(Z_t)$ represent the optimal rate of expenditure in this case.

6. In a more complete model in which resources are first accumulated from 0 to Z_0 , it might not be optimal to overaccumulate resources in the first place only to consume them in a burst.

Proposition 1 *The optimal rate of expenditure $x(Z_t)$ decreases over time and the probability of death at any instant $\delta(x(Z_t))$ increases over time.*

Proof: Since $x(Z_t)$ denotes the optimal policy function that satisfies the first-order condition (6), substituting it in Equation (5), we get:

$$(8) \quad 0 = x(Z_t) + [\eta - x(Z_t)]F'(Z_t) - \delta(x(Z_t))F(Z_t).$$

Differentiating Equation (8) with respect to Z_t and using the envelope theorem:

$$(9) \quad 0 = [\eta - x(Z_t)]F''(Z_t) - \delta(x(Z_t))F'(Z_t).$$

Differentiating the first-order condition (6) with respect to Z_t gives the following comparative statics result (suppressing the dependence of δ on $x(Z_t)$ and those of functions F on Z_t for notational clarity):

$$(10) \quad x'(Z_t) = (-F'' - \delta'F')/(\delta''F).$$

Now,

$$dx/dt = x'(Z_t)(dZ_t/dt).$$

Substituting from Equations (10) and (2), we get:

$$dx/dt = (-F'' - \delta'F')/(\delta''F)(\eta - x).$$

Substituting for F'' from Equation (9), multiplying and dividing the above equation by F , we get:

$$dx/dt = -F'/(\delta''F^2)[\delta F + (\eta - x)\delta'F].$$

Since Equation (8) can be rewritten as $\delta F = \eta - [\eta - x](1 - F')$, substituting it above and simplifying, we get:

$$dx/dt = -F'/(\delta''F^2) [\eta - (\eta - x)(1 - F' - \delta'F)].$$

Substituting $1 - F' - \delta'F = 0$ from the first-order condition (6), we get:

$$dx/dt = -F'/(\delta''F^2)\eta < 0$$

since F' is positive from Equation (6) and the fact that $\delta'(x_t) < 0$ from Assumption 1, and δ'' is positive from Equation (7). As x decreases over time, it follows that δ increases over time because it is decreasing in x . ■

The result in Proposition 1 formally shows that it is optimal for the organism to expend

more resources when young and less resources as it gets older. Since the level at which resources are spent also determines the probability of death and survival, it follows that a bias toward greater expenditure of resources when young also implies that the organism is healthier when it is young and gets weaker, in the sense that it is more likely to succumb to external threats and die at any instant, as it gets older.

Proposition 2 *The optimal rate of expenditure $x(Z_t)$ exceeds the rate of replenishment η :*

$$(11) \quad \eta < x(Z_t)$$

and consequently the level of resources Z_t decreases over time.

Proof: If the rate of expenditure $x(Z_t) < \eta$, and (from Proposition 1) it continues to fall over time, it cannot possibly be a maximizing strategy as it can be shown to be inferior to a feasible strategy of choosing $x(Z) = \eta$. With $x(Z_t) < \eta$, $\delta(x(Z_t))$ will be strictly greater than $\delta(\eta)$ and thus the lifetime genetic contribution will be smaller because the rate of expenditure $x(Z_t)$ is strictly smaller than η at all times and the probability of death at any point is strictly larger.⁷ Clearly, $x(Z) = \eta$ could not be an interior optimum either for any $Z_t > 0$ because by expending a little more resources in any period, the organism can increase its lifetime production while at the same time decrease its probability of death.

Since $\eta < x(Z_t)$ it follows from Equation (2) that Z_t depletes over time. ■

Since the expected life-span, and fitness at any instant F is increasing in Z_t which depletes over time from Proposition 2, it follows that as the organism gets older, its remaining life-span decreases monotonically.

The derivation of the results in Propositions 1 and 2 allows me to interpret the observed phenomenon of aging and senescence as an interior solution to a formal dynamic maximization problem. But importantly, I now show that the interior solution also implies an additional restriction on preferences that can be interpreted as risk-aversion.

Lemma 1 *At the interior maximum, the fitness function $F(Z_t)$ is concave in Z_t and $-(F''(Z_t))/(F'(Z_t))$ increases over time.*

7. The solution in which $x(Z_t) < \eta$ will violate a technical transversality condition because Z_t will grow unboundedly large over time.

Proof: From the first-order condition (6), we know that $F'(Z_t) > 0$ which from Equation (9) implies that

$$[\eta - x(Z_t)]F''(Z) > 0.$$

From Proposition 2, since $\eta - x(Z_t) < 0$, it follows that

$$F''(Z_t) < 0.$$

Rearranging Equation (9), we get:

$$-(F''(Z_t))/(F'(Z_t)) = (\delta(x(Z_t)))/(x(Z_t) - \eta).$$

Since δ is increasing over time and x is decreasing over time from Proposition 1, it follows that $-(F''(Z_t))/(F'(Z_t))$ increases over time. ■

The intuition for this result is that an interior solution for the maximum problem obtains because probability of death decreases at a decreasing rate as resources are expended at a higher rate. This implies an asymmetry—low levels of resources are much worse than high levels are good—resulting in concavity of the fitness function.

The concavity of the fitness function $F(Z_t)$ allows me to interpret that the organism will be risk-averse. The first interpretation follows directly in a case in which the organism faced the choice of initial level of resources Z_0 . The organism, if offered with two uncertain bundles of Z_0 with the same expected values, will prefer the one with smaller variance. However, in the present formulation, once Z_0 is determined, the level of resources Z_t evolves deterministically as long as the organism is alive because the endowment is replenished at a constant rate. I now show that allowing the replenishing endowment process to evolve stochastically with a process

$$(12) \quad \eta dt + \sigma dw$$

where dw represents the standard diffusion process, and assuming a specific death probability function.

Assumption 2

$$(13) \quad \delta(x_t) = \exp(-x_t)$$

implies risk-aversion in the following formal sense:

Proposition 3 *The concavity of the fitness function $F(Z_t, \sigma^2)$ in Z_t implies that it is decreasing in σ^2 , that is,*

$$(\partial F(Z_t, \sigma^2))/(\partial \sigma^2) < 0 \quad \text{as} \quad \sigma^2 \rightarrow 0.$$

Proof: We can write an equation similar to Equation (8) for the stochastic process described in Equation (12) as (suppressing the dependence of the optimal expenditure function x and the fitness function F on Z_t and σ^2):

$$(14) \quad \delta F = \eta - [\eta - x](1 - F') + (1/2)\sigma^2 F''.$$

The first-order condition (6), using Equation (13), can be rewritten as:

$$(15) \quad 1 - F' + \delta F = 0.$$

Substituting for $(1 - F')$ from Equation (15) into Equation (14) and rearranging we get:

$$(16) \quad \delta F = (\eta + (1/2)\sigma^2 F'')/(1 + x - \eta).$$

Taking natural logs of Equation (16) and differentiating with respect to σ^2 , we get:

$$\begin{aligned} -(\partial x/\partial \sigma^2) + (1/F)(\partial F/\partial \sigma^2) &= ((1/2)F'' \\ &+ (1/2)\sigma^2(\partial F''/\partial \sigma^2))/(\eta + (1/2)\sigma^2 F'') \\ &- 1/(1 + x - \eta)(\partial x/\partial \sigma^2). \end{aligned}$$

Letting $\sigma^2 \rightarrow 0$, and rearranging terms, we get:

$$(17) \quad (1/F)(\partial F/\partial \sigma^2) = ((1/2)F'')/\eta \\ + [(x - \eta)/(1 + x - \eta)](\partial x/\partial \sigma^2).$$

It is possible to show⁸ that the right-hand side (RHS) of the above equation has an upper bound equal to $((1/2)F'')/x$ which is negative from the concavity of the function F . ■

$-(F''(Z_t))/(F'(Z_t))$ can be interpreted as coefficient of absolute risk-aversion which we show increases over time. One might have an intuition that as an organism gets older, because it has fewer resources left, it might lead to decreasing risk-aversion as it gets older. Notice that $-Z_t(F''(Z_t))/(F'(Z_t))$, the coefficient of relative risk-aversion may indeed decrease over time because as the organism gets older, the level of resources Z_t decreases over time (from Proposition 2).

III. CONCLUDING REMARKS

I proposed a simple evolutionary model in which the possibility of death precipitated by external causes such as predation, disease, or cataclysmic events can simultaneously generate time preference, senescence, and risk-aversion and provide testable restrictions among these.

8. The complete proof is available from the author upon request.

The model can be exploited, using numerical simulations perhaps, to understand variations in time-discounting, aging, and risk-aversion, across species at a given point in time and variations across time for a given species, by varying exogenous parameters of the death probability function and of the stochastic process for endowment replenishment. Furthermore, differences between genders could also be examined by positing different production functions. For instance, one might conjecture that the production function for females might be concave rather than linear resulting in greater risk-aversion and perhaps slower aging. I leave it to future research to explore these issues in greater detail.

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