Vertical Differentiation with Variety-Seeking Consumers

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Abstract

We analyze price and quality competition in a vertically differentiated duopoly in which consumers have a preference for variety. The variety-seeking preferences are a consequence of diminishing marginal utility for repeated consumption experiences of the same product. We find that variety-seeking preferences can either soften or intensify price competition, depending on the range of feasible qualities and the strength of consumer preference for variety. When the range of feasible qualities is small enough, prices are higher than would be obtained in the absence of variety seeking—leading to higher profits—and competing firms choose to minimally differentiate themselves from each other. On the other hand, if qualities are exogenously set to be different enough from each other then stronger preferences for variety are associated with more-intense price competition and lower profits.

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1. Introduction

In several product categories, consumers want to sample many different experiences over multiple consumption occasions. For example, most people enjoy eating at multiple restaurants in a given month, both to sample different cuisines and also simply to go somewhere different each time. Skiers enjoy going to multiple resorts during a single trip to the mountains, even when some of the resorts are objectively better in terms of size, service, and the quality of the runs. Art lovers on a weekend trip to New York may all prefer the larger Metropolitan Museum of Art when they visit town for a single day, but most would also head to another museum if they visited for two weekend days, perhaps to the Museum of Modern Art. Tourists to Orlando typically go to multiple parks during a single trip. Consumers "prefer variety" in such experience goods because they get diminishing marginal utility over multiple units of the same experience. Extensive research has documented the presence of variety-seeking behavior and described its psychological antecedents (Givon 1984; McAlister and Pessemier 1982; Kahn, Kalwani, and Morrison 1986; McAlister 1982; Ratner, Kahn, and Kahneman 1999).

We examine how preferences for variety affect competition between two vertically differentiated firms. Each firm produces multiple units of its product. Consumers have two single-unit consumption occasions and vary in how much they value quality. If they consume one unit of each of the two products then they obtain utility in a manner that is standard in vertically differentiated models. On the other hand, if a consumer chooses to consume two units from the same firm then she experiences less utility from the second consumption experience than from the first one due to satiation. Consumers who have already consumed the higher-quality good thus face a tradeoff: should they consume a second unit of the high-quality product even though they will obtain a discounted utility from that second unit, or should they consume one unit of the lower-quality good? We explore how consumers solve this tradeoff, and how the resulting behavior affects price and quality competition between the firms.

The consumer behavior involves variety seeking. Consider the problem of an avid weekend skier planning a visit to a mountain town near two competing ski resorts that happen to charge the same price but differ in quality. The skier cares about the quality of the skiing, so he will go to the better resort for one of his two days, putting downward pressure on the price of the inferior resort. However, the skier anticipates that a second day at the better resort may not be as enjoyable as visiting the inferior resort for a day instead. If the qualities of the two resorts are not

too different, the skier visits both resorts during his ski trip, and this "variety seeking" relaxes price competition because the inferior resort will be able to sell one ticket to most consumers even if it charges a similar price as the better resort.

A question thus arises regarding the extent to which the two resorts wish to be vertically differentiated. Should the inferior resort remain differentiated or should it improve its quality to be more like the superior resort? We show that when the range of qualities available to firms is narrow enough (relative to the extent of preferences for variety), both resorts want to minimize differentiation in order to benefit from the variety-seeking behavior described in the previous paragraph. This result contrasts with the basic result in markets without a taste for variety, where Shaked and Sutton (1982) show that firms differentiate themselves to soften price competition. The reason behind our reversal of the basic principle is that variety seeking softens competition more when the two firms' products are less differentiated. Consider the skier: the more similar the quality of the inferior resort is to that of the superior resort, the more appealing switching resorts becomes after the skier tires of the first resort, and the more the higher-quality resort would have to cut its price to induce the skier to ski there both days.

We find that the softening of price competition depends on the extent of quality differentiation available to firms. If the inferior resort is restricted to be abysmal perhaps because of its low elevation and less snowfall, it attracts only price-sensitive consumers who do not care much about quality, whereas our avid skier skis at the better resort for the entire weekend. In an extension to our main model, we consider a situation with the firms exogenously differentiated enough, and we show that when enough differentiation exists then greater preferences for variety intensify price competition. The intuition for this result is that when the level of differentiation is large enough, the two firms now compete for two marginal consumers, with the high-quality resort trying to convince avid skiers to stay for both days while simultaneously convincing novice skiers to come for at least one day. The competition to keep the price-sensitive consumers at the high-quality resort for the second day, in particular, becomes more intense as the taste for variety increases, leading to lower prices and profits.

Having outlined our two main results, we now explain how they relate to the existing literature. Feinberg, Kahn, and McAlister (1992) pioneered research on the supply-side implications of consumer variety seeking. Their model does not consider price competition, which we find to be a significant factor in location choice. Two recent papers have examined the

competitive implications of variety seeking with endogenous pricing, both in the context of horizontal differentiation. Seetharaman and Che (2009) consider how two exogenously horizontally differentiated firms set prices to consumers with a preference for variety, and find that equilibrium prices are higher than would be realized in markets without variety-seeking consumers. This finding is consistent with our finding that preferences for variety softens competition when the range of qualities firms can offer is limited enough. However, we show that when the products are differentiated enough, then the opposite happens: preferences for variety cause prices and profits to fall. Sajeesh and Raju (2010) build on Seetharaman and Che (2009) by examining the impact of variety seeking on location competition in a two-period horizontal Hotelling model. They show that the presence of a variety-seeking consumer segment leads to less product differentiation than would be optimal if no consumers were variety seeking. In contrast to our results, they explicitly rule out minimum differentiation in equilibrium. Sajeesh and Raju also find that the presence of variety-seeking consumers reduces profits, whereas we find that preferences for variety can either reduce or increase profits depending on the extent of differentiation between the products.

One of the key reasons why our results differ from those in Seetharaman and Che or Sajeesh and Raju is that under vertical differentiation, there is an asymmetry that arises between the firms. We show that the interaction between this asymmetry and preferences in variety lead to qualitatively different outcomes than those that would occur in horizontal models.

We organize the rest of the paper as follows. Section 2 presents our main model, demonstrates that greater preference for variety leads to higher prices when the range of feasible qualities is limited. Section 2 also shows our main result that firms will minimize the level of product differentiation. In section 3, we present an extension in which product qualities are exogenously set such that there is enough product differentiation, and demonstrate that in such a context, preferences for variety can instead intensify price competition. Section 4 concludes.

2. Model

We model the competition between two firms as a two-stage game. In the first stage, firms simultaneously and costlessly select quality q from an interval of feasible qualities. We omit the costs of setting quality in order to highlight the purely competitive effects of quality differentiation. However, in section 2.5, we show that our results are robust to the presence of

costs of increasing quality. In the second stage, the firms set prices simultaneously, and consumer demand is realized. We now outline the notation of our model before discussing our assumptions in detail.

Firms: Two firms exist with identities j =1,2. In the first stage, firm j selects quality $q_j \in \left[\underline{q}, \overline{q}\right]$, where $\underline{q} \geq 0$. In the second stage, each firm j charges a price p_j for each unit of its good. We assume marginal costs of production are constant, and then normalize these cost without loss of generality to be zero. When the two firms select different qualities $L \equiv q_j < q_{-j} \equiv H$ in the first stage, we also sometimes label each firm L (for "low") and H (for "high") according to the relative magnitude of its quality. This conflation of identity and quality into a single index will clarify our exposition of the pricing game.

Consumers: Consumers differ in their marginal utility for quality θ: consumer type θ is distributed across the population uniformly on the [0,1] interval. The consumers have utility for two units of the good, and they care not only about quality but also about which product they consume. When a consumer buys a second unit of the good from the same firm, he experiences the second unit as if its quality were diminished due to satiation: the indirect utility of consumer θ who buys two units from firm j is $U_j = \beta + (1+\delta)\theta q_j - 2p_j$, where δ <1 reflects the rate at which utility is diminishing across units of the same firm's product. Alternatively, the consumer can buy one unit from each firm; such a choice yields $U_V = \beta + \theta \sum q_j - \sum p_j$. When the two firms select different qualities $L \equiv q_j < q_{-j} \equiv H$ in the first stage, the variety bundle has utility $U_V = \beta + \theta(L+H) - (p_L + p_H)$. As is common in these types of models, we assume β is large enough that no consumer buys the outside good in equilibrium. Further, because β appears additively in each of the cases, we drop β from any utility calculations in the rest of the paper (except for our analysis of monopoly choices, where β affects the monopolist's price).

Having outlined the model, we now discuss its key assumptions. The key departure of our model from standard models of vertical differentiation following Shaked and Sutton (1982) is the diminishing marginal utility for multiple units of the same firm's product (δ<1). Although the quality differentiation is the only dimension of product differentiation we model, our model is consistent with the two products exogenously differing in other attributes. For example, two sushi restaurants probably share a similar menu but different qualities and different decors. Our

assumption is that the variance in consumer preferences for quality is much higher than the variance in consumer preferences for décor, and consumers satiate of repeated experiences of the same décor as in McAlister (1982).[1]

Another assumption that departs from existing models in the literature is that the firms set a price that does not change between each of the consumers' consumption experiences. This "stable price" assumption stands in contrast with the assumptions in Seetharaman and Che (2009) and Sajeesh and Raju (2010). Having firms charge the same price in both consumption occasions is a reasonable assumption if the firms have menu costs that only allow price adjustments less frequently than the time between two consumption occasions. For example, amusement parks and ski resorts may have to print prices for brochures for a given season, or they may have to run television ads that announce prices are held steady for some length of time. Restaurants do not usually print different menus every day. Alternatively, one can think of our model as applying to scenarios in which consumers plan and purchase their consumption occasions ahead of the actual experiences. Ultimately, we note that in the industries we consider—ski resorts, amusement parks, and restaurants—prices do not tend to vary on a consumption-experience basis within the time frames we consider, such as a weekend. Also, whereas Sajeesh and Raju (2010) find that prices fall in the two periods of their model, Seetharaman and Che (2009) find that prices can rise or fall. Thus our assumption represents somewhat of a middle ground.

Another difference between our assumptions and those of prior work is that the extent to which a consumer discounts repetitive consumption experiences is proportional to her valuation of quality. We believe this assumption is reasonable for vertically differentiated markets. For example, we propose that foodies would pay more to try a new restaurant in town in order to obtain variety than other people even though foodies care more about quality and the new restaurant may not be as good as an established one. Similarly, avid skiers generally care more about variety than casual skiers. By contrast, the conventional assumption used in horizontal models would state that people who care a lot about food lose the same amount of utility from visiting the same restaurant twice as people who do not care about the quality of food.

[1] Mathematically, suppose firm attributes are defined on two dimensions—a quality dimension and a horizontal Hotelling line with the two firms located at opposite ends of the line. Our model is isomorphic to a scenario in which all consumers are located at the center of the Hotelling line and consumer preferences for quality are distributed $\theta \sim U[0,1]$ (as above). The firms are thus differentiated horizontally (allowing for the satiation) and vertically, but mathematically the travel costs between the consumers and the firms is rolled into the β term of the utility function.

Finally, we make an assumption about the relationship between δ and the range of feasible qualities:

Assumption 1: The firms select qualities from a range $\left[\underline{q}, \overline{q}\right]$ that satisfies $\underline{q} \geq \frac{1+4\delta}{4+\delta}\overline{q}$.

The fact that $\overline{q}$ cannot be infinity is a standard assumption in models of vertical differentiation (Shaked & Sutton 1982). The lower limit ensures that the price equilibrium in the second stage of the game will be tractable. From a consumer perspective, the lower limit implies that the qualities of the two products must be relatively similar to each other relative to the consumers' preferences for variety. Specifically, Assumption 1 implies that $L>\delta H$, so the variety bundle is the highest-utility consumption alternative: all consumers would choose to purchase one unit from each firm if the two firms charged the same price. Restricting the qualities firms can offer is realistic in many contexts because technology or government regulations often constrain the quality choices available to the firms in the real world. For example, health standards in the restaurant industry set a minimum allowable quality level. In resort industries that also motivate our work, natural conditions such as snowfall and sunshine dictate a large component of the quality of the entire region, so nature effectively constrains qualities to be similar akin to the restriction in Assumption 1. A restriction on the set of potential locations is consistent with the style of other papers on variety-seeking behavior in differentiated industries: For example, Seetharaman and Che (2009) limit the positions of the firms to be at the end points of a Hotelling line, and Sajeesh and Raju (2010) limit firms to choose symmetric locations.

Although we believe Assumption 1 is reasonable in many contexts, one of its disadvantages is that the range of qualities allowed under the assumption diminishes as $\delta \to 1$ (as $\delta \to 1$, $\underline{q} \to \overline{q}$) Note that we find that both firms choose the maximum quality in equilibrium, so the range restriction does not dictate the minimum differentiation result directly for any δ <1. In section 3, we consider how the price equilibrium changes when Assumption 1 does not hold and the firms offer exogenously determined qualities that differ from each other. Note that the assumption intertwines the range of qualities $\overline{q} - \underline{q}$ with the strength of preference for variety: the inequality can be rearranged as $\delta \leq \frac{4\underline{q} - \overline{q}}{3\overline{q}}$. Therefore, an alternative interpretation of Assumption 1 is that the preference for variety needs to be strong enough for a given a range of qualities.

2.1. Consumer behavior

We start by considering consumer behavior when the two firms select different qualities in the first stage, and we adopt the aforementioned notation of L being the lower-quality firm and H being the higher-quality firm. Consumers can make three possible choices (2 units of H, 2 units of L, or one of each), and the following inequalities determine consumer preference for all pairs of choices in terms of the consumer's preference for quality θ:

$$U_V > U_L \Leftrightarrow (L+H)\theta - p_H - p_L > (1+\delta)L\theta - 2p_L \Leftrightarrow \theta > \frac{p_H - p_L}{H - \delta L} \tag{1}$$

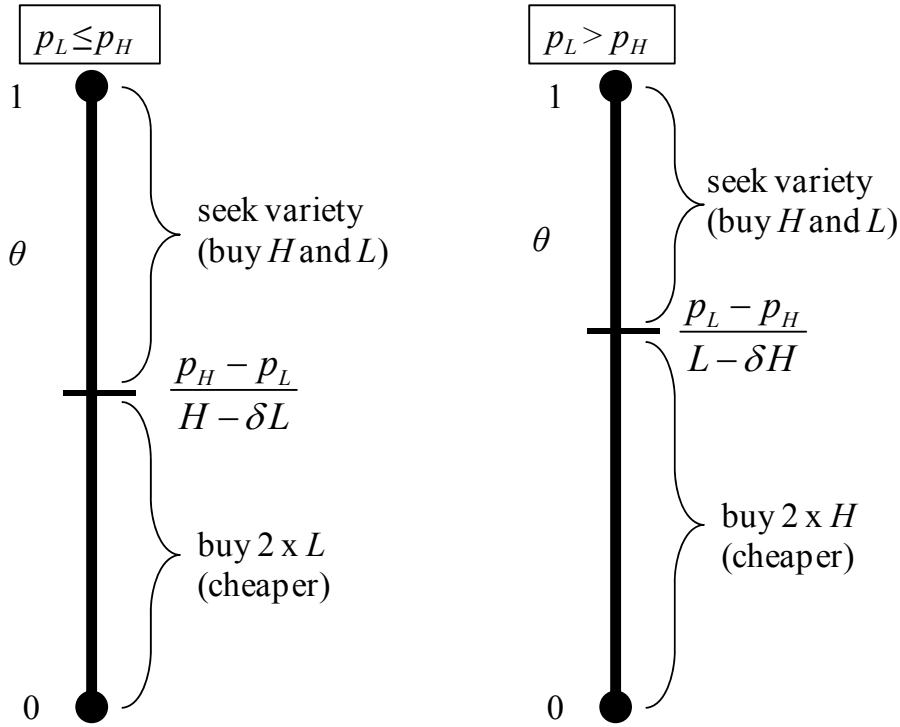
$$U_H > U_L \Leftrightarrow (1+\delta)(H-L)\theta > 2(p_H - p_L) \Leftrightarrow \theta > \frac{2(p_H - p_L)}{(1+\delta)(H-L)} \tag{2}$$

$$U_V > U_H \Leftrightarrow (L+H)\theta - p_H - p_L > (1+\delta)H\theta - 2p_H \Leftrightarrow \theta > \frac{p_L - p_H}{L - \delta H}. \tag{3}$$

For any set of prices, only high-θ consumers will buy one unit of each good; the rest of the consumers will buy two units of whichever good is cheaper. To see this behavioral regularity, note that when the superior good is cheaper ($p_H < p_L$), nobody buys two low-quality units and consumers choose between variety seeking and buying two high-quality units. On the other hand, when the superior good is more expensive ($p_H > p_L$), nobody buys two units of it: whoever prefers two units of the superior good $\{H, \delta H\}$ to two units of the inferior good $\{L, \delta L\}$ will also prefer the variety bundle $\{L, H\}$ to $\{H, \delta H\}$ because variety seeking gives more quality ($L>\delta H$) for less money ($p_H + p_L < 2p_H$). Therefore, $p_H>p_L$ makes the consumers choose between variety seeking and buying two low-quality units.

Figure 1 summarizes the consumer behavior under the two possible price orderings. Note that equal prices ($p_H = p_L$) make all consumers seek variety, so consumer behavior transitions continuously from one ordering of prices into the other. In other words, the demand functions of both firms are continuous at $p_H = p_L$, but the slopes of the demand functions change at that point. Also note that if the differences in prices are high enough, perhaps none of the consumers will choose to seek variety, and the more expensive product can be foreclosed out of the market.

Figure 1: Consumer behavior when firms have similar qualities



When both firms offer the same quality q, the two graphs in Figure 1 coincide because $L=H=q$. Therefore, high-θ consumers still seek variety to gain the benefit of $2q$, whereas low-θ consumers buy two units of the cheaper good. The equal-quality case illustrates the importance of consumer preference for different firm identities in addition to quality: if consumers care only about quality (i.e., if $\delta = 1$) then price-competition wipes out profits in the absence of product differentiation. With diminishing marginal utility for the same product, each firm can profitably deviate from $p_1 = p_2 =$ *marginal cost* $= 0$ and gain the variety-seeking consumers who care enough about quality.

2.2 Monopoly: benchmark case

Before presenting the duopoly analysis, we first consider what quality levels a monopolist offering two products with different identities would offer. We assume the utilities for consumers match those presented in section 2.1. Suppose the monopolist associates each product with a different quality level $q_1 \equiv L < H \equiv q_2$, where qualities are selected from a range $\left[\underline{q}, \overline{q}\right]$ as defined in Assumption 1. Assume β is high enough that the market is covered; that is, excluding any customers from buying two units is not profitable. In such a case, the price for the low-

quality good must be at most $p_L=\beta$ to ensure that even consumers with $\theta=0$ buy two units. Assuming the market is covered, the monopolist's profits are $\Pi(p_H, p_L) = p_H\left(1-\frac{p_H - p_L}{H-\delta L}\right) + p_L\left(1+\frac{p_H - p_L}{H-\delta L}\right)$, so the optimal price to charge for the high-quality good is $p_H = p_L + \frac{H-\delta L}{2}$. Plugging this equality, along with $p_L=\beta$, into the profit function yields $\Pi = \frac{H-\delta L}{4} + 2\beta$, which is maximized by selecting $L = \underline{q}$ and $H = \overline{q}$.[2] Therefore, the monopolist wants to select maximally different qualities to benefit from price discrimination. Note that the equilibrium prices vary with β, which we show in section 2.3 is competed away in a duopoly, so the pricing results are not directly comparable across the monopoly and duopoly settings. However, we can compare the qualities that emerge in equilibrium. We turn to the duopoly analysis next.

2.3 Duopoly pricing analysis

We solve the game through backward induction, starting with the second stage, in which the firms set prices conditional on the qualities decided in the first stage. We first show that if the two firms select different qualities, a tractable mixed-strategy equilibrium but no pure-strategy pricing equilibrium exists. Second, we show that if the firms choose identical qualities, a pure-strategy price equilibrium exists to which our mixed-strategy equilibrium converges as the qualities approach each other. Consider, first, the case in which the two firms select different qualities $L<H$ in the first stage.

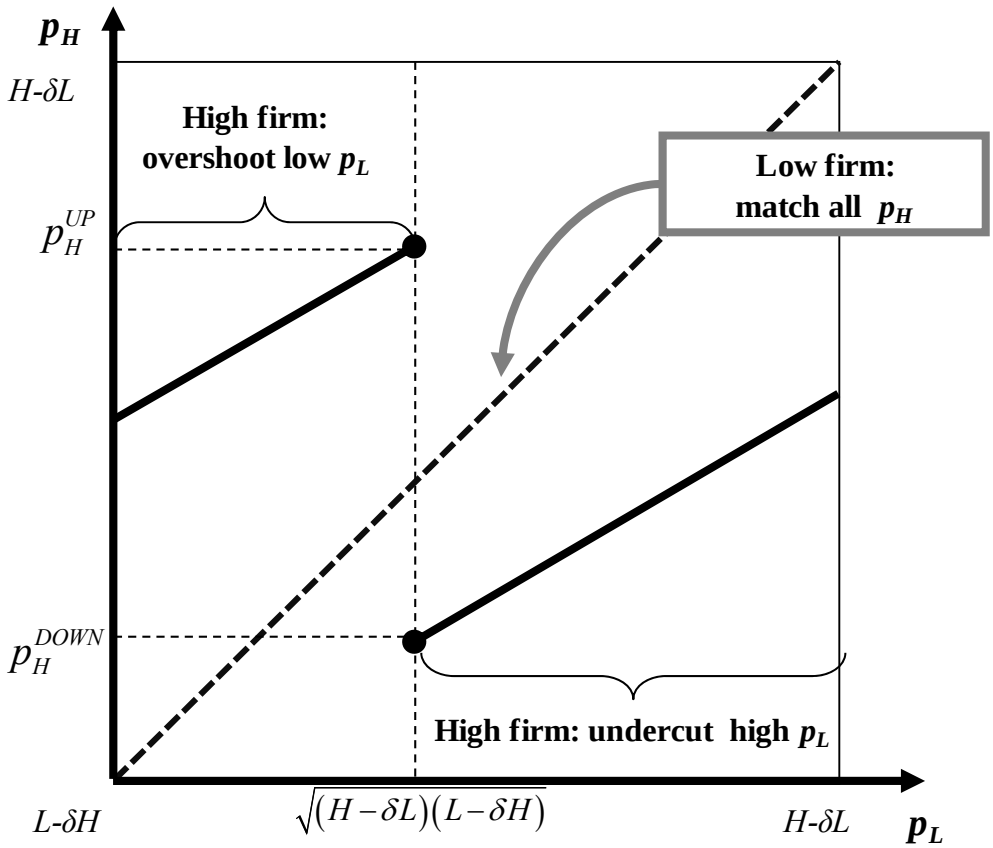
Proposition 1: When qualities of the two firms are similar but not identical, no pure-strategy pricing equilibrium exists.

Please see the Appendix for detailed proofs of all propositions. Intuitively, one might expect a pure-strategy equilibrium with $p_L < p_H$ based on the results of Shaked and Sutton (1982). However, when $p_L < p_H$, no one buys two high-quality units and the low-quality firm enjoys a lot of demand. Thanks to this abundant demand, the low-quality firm's profits are increasing in

[2] A monopolist would maximize the difference in qualities even if the range of qualities available to firms was extended beyond the range in Assumption 1, assuming the market is covered, although demand would take a different form. If β is small enough then the monopolist would not set prices to serve all customers.

p_L for $p_L < p_H < H\text{-}\delta L$. The low-quality firm thus has an incentive to match p_H and suggests $p_L = p_H$ as a candidate for a pure-strategy equilibrium. However, the high-quality firm always deviates from equal pricing because it has a quality advantage: when p_L is high, the high-quality firm's incentives are to undercut p_L and sell two units to some consumers. On the other hand, when p_L is low, the high-quality firm's incentive is to charge a much higher price and extract profit from the quality-sensitive consumers. The high-quality firm is indifferent between undercutting and overshooting when $p_L = \sqrt{(H-\delta L)(L-\delta H)}$. Figure 2 illustrates the best-response curves.

Figure 2: The best-response functions of both firms



Note: The dashed diagonal line is the low-quality firm's best response to p_H. The solid line is the high-quality firm's best response to p_L. The solid line is discontinuous at $p_L = \sqrt{(H-\delta L)(L-\delta H)}$. See Proposition 2 for the definitions of p_H^{UP} and p_H^{DOWN}.

The key to the non-existence of the pure-strategy equilibrium in Proposition 1 is that the high-quality firm's demand curve is kinked at p_L. It is not uncommon for demand curves in product-differentiation models to be kinked, but in this case, the demand curve is kinked the "wrong way": when $p_H > p_L$, the slope of the demand curve is $\frac{-1}{H-\delta L}$ because the indifferent consumer is deciding between variety seeking and buying two units of the cheaper, low-quality good. However, when $p_H < p_L$, the high-quality good becomes cheaper, so the indifferent consumer is deciding between variety seeking and buying two units of the high-quality good. Because of the high firm's quality advantage, the demand curve is flatter, with a slope of $\frac{-1}{L-\delta H}$. Figure 3 in the Appendix presents a graph of the kinked demand curve. Although no pure-strategy equilibrium exists, the game does have a partial mixed-strategy equilibrium, which we present in Proposition 2.

Proposition 2: When the firms select different qualities $L<H$ in the first stage, a partially mixed equilibrium exists in which the low-quality firm sets a price of $p_L = \sqrt{(H-\delta L)(L-\delta H)}$ and the high-quality firm mixes between two prices $\{p_H^{DOWN}, p_H^{UP}\}$ where $p_H^{UP} = \frac{H-\delta L+\sqrt{(H-\delta L)(L-\delta H)}}{2} > p_H^{DOWN} = \frac{L-\delta H+\sqrt{(H-\delta L)(L-\delta H)}}{2}$. The mixing probability is $\Pr\left(p_H^{UP}\right) = \frac{\sqrt{H-\delta L}}{\sqrt{(H+L)(1-\delta)}}$. The equilibrium expected profits are $\Pi_H = \left(\frac{\sqrt{H-\delta L}+\sqrt{L-\delta H}}{2}\right)^2 > \Pi_L = \sqrt{(H-\delta L)(L-\delta H)}$, and profits for both firms increase as δ decreases, that is, as preference for variety increases.

The intuition behind this equilibrium is that $p_L = \sqrt{(H-\delta L)(L-\delta H)}$, which is between $L-\delta H$ and $H-\delta L$, makes the high-quality firm indifferent between undercutting p_L halfway toward $L-\delta H$ and overshooting p_L halfway toward $H-\delta L$. The low-quality firm's demand in the face of a high-quality firm mixing between two points is concave, and the probability of the

high-quality firm undercutting the low-quality firm is set such that $\sqrt{(H-\delta L)(L-\delta H)}$ is the low-quality firm's best response to the high-quality firm's mixed strategy. Assumption 1 ensures that the low-quality firm does not deviate to matching p_H^{UP} and leave the market entirely to the high firm whenever it plays p_H^{DOWN}.

The equilibrium of Proposition 2 has intuitive comparative statics: the high firm earns greater equilibrium profits than the low firm, and it is more likely to overshoot than undercut the low firm's pure strategy. The finding that greater preferences for variety increase both firms' profits echoes similar findings in the literature that variety-seeking softens price competition (Seetharaman and Che 2009 and Sajeesh and Raju 2010). Intuitively, variety-seeking softens competition by ensuring that a higher-priced firm gets positive demand, albeit only for a single unit of its good. In the extension in section 3, we show that this intuition requires the firm qualities to be similar to each other in the sense that $L>\delta H$.

We now consider the price competition when both firms select the same quality q in the first stage. Proposition 2 suggests a candidate equilibrium when we set $q=H$ and then let L approach H: all three prices $\left(p_L, p_H^{DOWN} \text{ and } p_H^{UP}\right)$ converge to $H(1-\delta)$. It turns out this symmetric behavior is indeed a pure-strategy equilibrium of the game.

Proposition 3: When both firms select the same quality q in the first stage then both firms setting a price of $q(1-\delta)$ forms a Nash equilibrium. All consumers buy one unit from each firm, and profits are $q(1-\delta)$ for each firm.

Equalizing the firm qualities smoothes all of the kinks in the demand functions, allowing for a pure-strategy equilibrium. Importantly for the next section that describes quality selection by the firms, profits are thus continuous as one firm's quality approaches the others.

The equal-quality case illustrates the importance of consumer preference for experiencing different products in addition to their preference for quality: if consumers care only about quality, and the firms offered products with identical qualities, price-competition would wipe out profits. In our equilibrium, all consumers seek variety, and the products are thus ex-post differentiated in utility for the second unit, which in turn allows the firms to charge positive profit margins even when they both offer the same quality.

2.4 Duopoly: Equilibrium quality choice

Having analyzed the pricing equilibrium of the second stage, we analyze the first stage of the game, where each firm chooses their quality. We find that both firms offer the highest possible quality.

Proposition 4 (Minimum Differentiation): For every δ and $\overline{q} > 0$, a unique pure-strategy quality selection equilibrium exists. In the equilibrium, both firms select the highest possible quality $\overline{q}$.

The proof of Proposition 4 is straightforward. Suppose firm 1 selects $q_1=L<q_2=H$. Differentiating Π_L from Proposition 2 with respect to L shows that firm 1's profits are increasing in quality: $\frac{\partial \Pi_L}{\partial L} = \frac{(H-\delta L)-\delta(L-\delta H)}{2\sqrt{(H-\delta L)(L-\delta H)}} > 0$. Therefore, the lower-quality firm would always prefer to raise its quality up to H, where Proposition 3 with $q=H$ describes the pricing behavior and equilibrium profits. Each firm, in turn, wants to raise its quality level from any $q_1 = q_2 < \overline{q}$ configuration and become the higher-quality firm: $\left.\frac{\partial \Pi_H}{\partial H}\right|_{L=H=q} = \frac{1-\delta}{2} > 0$, so the unique equilibrium is for both firms to set $q_j = \overline{q}$.

Proposition 4 contrasts with the basic result in vertically differentiated markets without a taste for variety, where Shaked and Sutton (1982) show that firms want to differentiate themselves to soften price competition. The result is similar to the findings of Sajeesh and Raju (2010), who find that variety-seeking preferences reduce the level of differentiation in equilibrium in a horizontal-differentiation model, although they do not find minimum differentiation. Further, under Sajeesh and Raju's model, margins for each firm would shrink to zero under minimum differentiation, so the underlying mechanics driving the results are different, and variety seeking appears to soften location competition more in vertical settings than in horizontal ones.

The intuition behind Proposition 4 is that variety seeking softens price competition more than vertical differentiation, and the equilibrium extent of variety seeking is negatively correlated with the amount of vertical differentiation. Variety seeking softens price competition most when the firms have the same qualities, because when qualities are identical, the firms have difficulty

convincing many consumers to purchase two units of their products. On the other hand, as the quality of the lower-quality firm decreases, competition over the provision of the consumers' second unit becomes more intense: consumers who purchase one unit of the high-quality good then have the choice between a discounted high-quality unit and a full-utility low-quality unit. When the quality of the low-quality product is high, this comparison is not very close for most consumers, with variety seeking dominating in terms of overall utility. As the quality of the low-quality unit falls, consumers have a tougher choice to make and competition is more intense.

The reduction in price competition from bringing the firm qualities closer together is so strong that even the higher-quality firm benefits as L approaches H: $\frac{\partial \Pi_H}{\partial L} = \frac{\sqrt{H-\delta L} - \delta\sqrt{L-\delta H}}{4\sqrt{(H-\delta L)(L-\delta H)}}\left(\sqrt{H-\delta L} + \sqrt{L-\delta H}\right) > 0$. Thus both firms are better off under minimum differentiation. Put another way, both firms providing an equal level of quality is not just an equilibrium outcome but also a win-win scenario in which both firms are better off providing equal qualities than either firm would be if the firms provided different qualities.

We summarize the profits from Propositions 2 and 3 in Figure 3 as a function of the level of product differentiation (the quality level for the lower-quality firm conditional on the higher quality firm selecting quality $\bar{q}$) and the preference for variety (1-δ). The figure demonstrates that both firms earn the greatest profits under minimum differentiation and that the profits decrease as the preference for variety seeking decreases (or, equivalently, as δ increases).

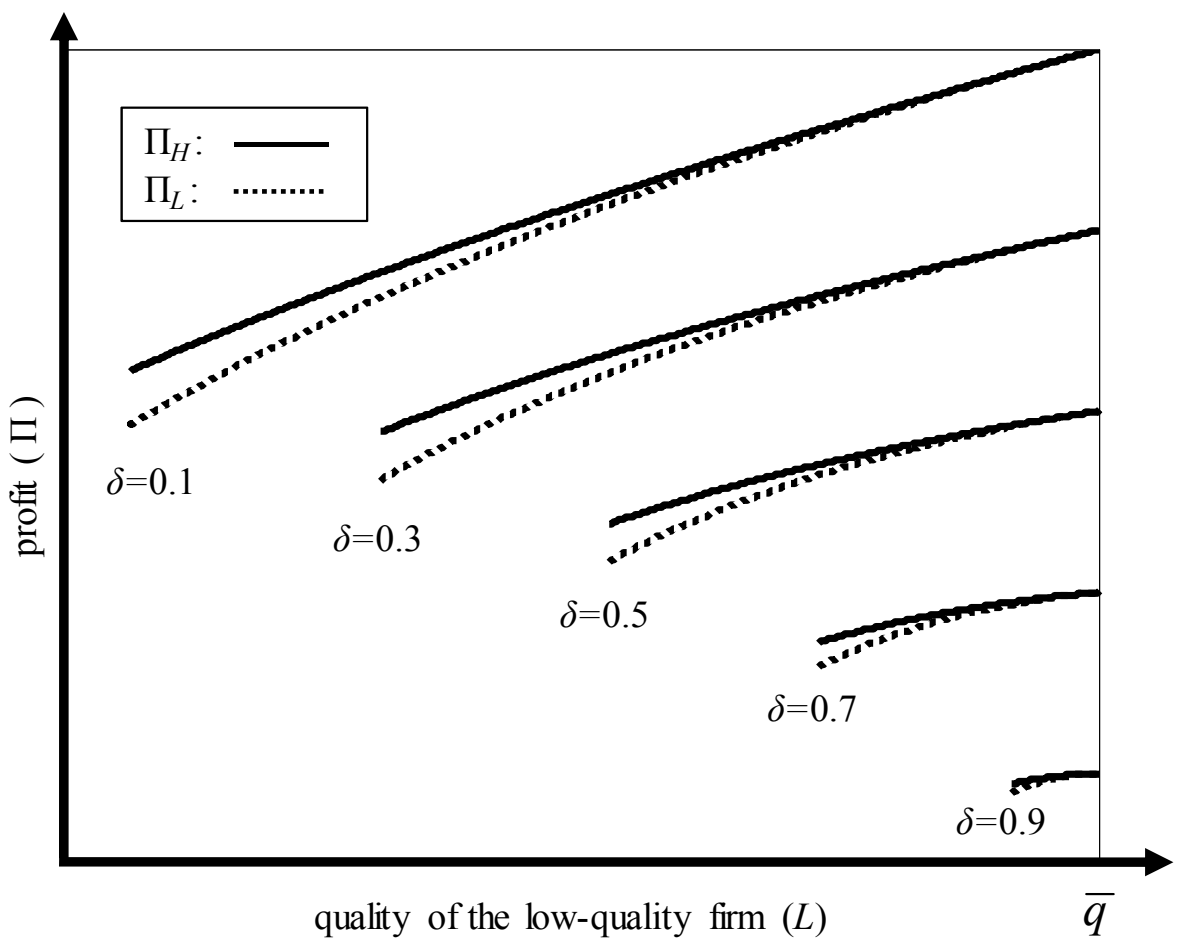
A comparison with the monopoly benchmark further clarifies the intuition by showing that minimum differentiation arises from equilibrium considerations rather than from the structure of demand. The monopolist selects maximally different qualities to price discriminate, whereas two competing firms benefit more from creating increased consumer value.

2.5 Quality choices with costs of quality

In previous sections, we assume that there are no costs associated with setting different qualities. We now consider how introducing costs of increasing quality change the analysis. Specifically, suppose a fixed cost exists for providing increased quality, and this cost is increasing and convex ($C'(q) > 0, C''(q) > 0$). In such a case, the equilibrium will always involve both firms offering the same quality, which will be $\min(\bar{q}, \tilde{q})$, where $\tilde{q}$ is the level where $C'(\tilde{q}) = \frac{1-\delta}{2}$.

To see that $q_1 = q_2 = \min(\bar{q}, \tilde{q})$, note that Π_L and Π_H are both concave, and that $\left.\frac{\partial \Pi_L}{\partial L}\right|_{L<H} > \frac{1-\delta}{2} = \left.\frac{\partial \Pi_L}{\partial L}\right|_{L=H} = \left.\frac{\partial \Pi_H}{\partial H}\right|_{L=H}$. No equilibrium can involve one firm offering a lower quality, because L's marginal benefit of increasing its quality would always be larger than the marginal cost of additional quality. Similarly, the equilibrium cannot involve firms offering a higher quality than $\tilde{q}$, since the higher-quality firm (or each firm, if the two qualities are equal) would have an incentive to offer a lower quality. Thus the firms will always choose minimum differentiation if the range of qualities available to them is limited enough, although the quality might not occur at the highest possible level if such quality is expensive enough.

Figure 3: Equilibrium profits under similar qualities



Note: Each line shows the relationship between the low firm's quality (conditional on $H = \bar{q}$) and profits for different δ. Each line starts at the lowest L possible under Assumption 1.

3. Extension: Price Competition with Exogenously Dissimilar Qualities

The preceding analysis is based on the assumption that the range of qualities firms can offer is sufficiently narrow given consumers' preferences for variety (i.e., Assumption 1). In such a case, we find that firms choose to minimize the level of product differentiation, and that preference for variety softens price competition. In this section, we demonstrate that the impact of variety seeking on price competition can reverse when the two firms happen to be differentiated enough. This latter finding may be counterintuitive, especially in light of Seetharaman and Che (2009), who find that variety seeking softens price competition in a horizontally differentiated industry.

Instead of considering the two-stage game from section 2, we assume a firm's qualities are set exogenously consistent with Assumption 2:

Assumption 2: Firm qualities are exogenously set to $L<H$ such that $0 \le L \le \frac{\delta-\alpha}{1-\alpha\delta}H$, where $\alpha \equiv \frac{8\sqrt{10}-17}{39}$.

We limit our analysis to exogenously set levels of product differentiation because an "intermediate" range of relative qualities $\frac{L}{H} \in \left[\frac{\delta-\alpha}{1-\alpha\delta}, \frac{1+4\delta}{4+\delta}\right]$ exists with no pure-strategy pricing equilibria, and no mixed strategy equilibria tractable to us. We can show that for part of the range, the equilibria involve both firms pricing at a set of many discrete points, where the points for the two firms differ. Assumption 2 delineates a range of relative qualities differentiated beyond Assumption 1 but with a tractable pricing equilibrium. Note that Assumption 2 implies $L< \delta H$; that is, the variety bundle $\{L,H\}$ is inferior to two H units ($H+\delta H>H+L$).

Just as we motivated examples in section 2 in which the range of qualities firms could choose could be limited to a narrow range, we note that in some cases, qualities could be exogenously limited to greater degrees of differentiation. For example, characteristics of nature largely determine the quality of ski resorts. In other industries, some competitors might not have the skills to produce products with the highest quality: a chef at one restaurant might not be as good as a chef at another restaurant, or one company might not be as good at designing theme parks as its rival.

We begin our analysis of price competition under Assumption 2 by summarizing consumer behavior. The threshold parameters defining the cutoff between when consumers seek variety instead of two L units remains the same as in equation (1), and the cutoff of when consumers prefer two H units over two L units remains the same as in equation (2). The only cutoff that changes under Assumption 2 is the one between variety seeking and consuming two units of the high-quality product:

$$U_V > U_H \Leftrightarrow (L+H)\theta - p_H - p_L > (1+\delta)H\theta - 2p_H \Leftrightarrow \theta < \frac{p_H - p_L}{\delta H - L}. \tag{4}$$

Comparing equation (4) with equation (3), we see a key difference between Assumptions 1 and 2: in "similar quality" settings (Assumption1), the variety bundle $\{L,H\}$ is the highest-quality consumption alternative, so variety seeking occurs "at the top" in that the consumers with high willingness to pay for quality (θ) buy one unit of each product; the remaining consumers buy two units of the cheaper product. In "dissimilar quality" settings (Assumption 2), the variety bundle $\{L,H\}$ is inferior to buying two H units (because $L < \delta H$), so the consumers with high θ buy two H units. Variety seeking instead emerges "in the middle" of the θ spectrum because the variety bundle becomes a middle-quality option. As long as the inferior good is cheaper, the middle-quality option also carries a middle price. Therefore, unlike in the case of similar qualities, dissimilar qualities allow all three consumption choices to occur given a single pair of prices. Specifically, equations (1), (2), and (4) together imply that as long as $p_L < p_H$, consumers with

$\theta \in \left[0, \frac{p_H - p_L}{H - \delta L}\right]$ consume 2 units of L,

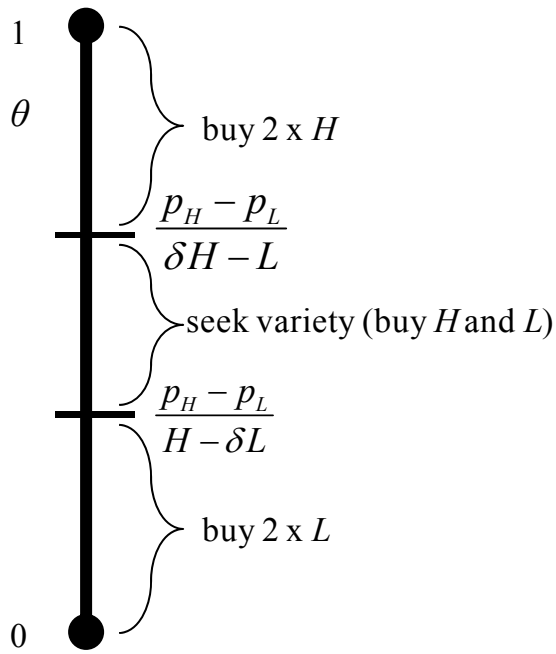
$\theta \in \left[\frac{p_H - p_L}{H - \delta L}, \frac{p_H - p_L}{\delta H - L}\right]$ seek variety (i.e., consume one unit of H and one unit of L),

$\theta \in \left[\frac{p_H - p_L}{\delta H - L}, 1\right]$ consume 2 units of H.

Please see Figure 4 for an illustration of the consumer behavior under dissimilar qualities.

When $p_L > p_H$ then all of the thresholds in (1), (2), and (4) are negative and all consumers consume two units of the high-quality good. This price ordering obviously cannot occur in a pure-strategy pricing equilibrium because the low-quality firm can do better by lowering its price.

Figure 4: Consumer behavior when $L < \delta H$ and $p_L < p_H$



Under Assumption 2, a pure-strategy pricing equilibrium exists. Because the calculations behind the equilibrium are standard, we present the equilibrium in Proposition 5 below and prove the result in the appendix.

Proposition 5: Suppose the qualities of the two firms are exogenously set such that $0 \le L \le \frac{\delta - \alpha}{1 - \alpha\delta} H$, where $\alpha = \frac{8\sqrt{10} - 17}{39}$. Then a unique pure-strategy Nash equilibrium exists with prices

$$\left\{ p_L = \frac{2(\delta H - L)(H - \delta L)}{3(1+\delta)(H - L)} < p_H = \frac{4(\delta H - L)(H - \delta L)}{3(1+\delta)(H - L)} \right\} .$$

Firm profits are $\Pi_L = \frac{4(\delta H - L)(H - \delta L)}{9(1+\delta)(H - L)} < \Pi_H = \frac{16(\delta H - L)(H - \delta L)}{9(1+\delta)(H - L)}$. Both firms' profits decrease as δ decreases, that is, as preference for variety increases.

The intuition for why increased preference for variety reduces profits comes from the variety bundle effectively becoming a "middle" option and the firms having to compete for two

sets of marginal consumers: high θ consumers decide between variety seeking and two H units, and low θ consumers decide between variety seeking and two L units. In other words, the firms compete intensely to sell the consumer the second unit: when δ is low, consumers find the quality difference between a second unit of H and a first unit of L is smaller, so a given-sized change in price leads to a larger jump along the quality-sensitivity parameters; that is, variety-seeking consumers are more price sensitive.

The equilibrium in Proposition 5 possesses many of the standard properties of price-competition equilibria in vertically differentiated markets as originally described by Shaked and Sutton (1982): the high-quality firm charges a higher price and earns higher profits than the low-quality firm. The parallel is more than qualitative—the profit functions presented in Proposition 5 reduce to (twice) the profit functions from Shaked and Sutton when $\delta = 1$.

Note, too, that if the firms are differentiated in a manner consistent with Assumption 2 then the low firm would want to differentiate further from the high firm (i.e., voluntarily reduce its quality), consistent with Shaked and Sutton. The intuition for this reversal relative to the minimum differentiation found in Proposition 4 is that if the products are differentiated enough then increasing the quality differences increases the amount of differentiation for both the first and the second unit of the good; in contrast, in section 2, when the products had similar qualities, greater quality differences led to lower differentiation between a second unit of H and a first unit of L.

4. Discussion

We examine how consumer preferences for variety affect competition between firms. When the range of qualities firms can offer is narrow enough, we find that greater tastes for variety lead to softer price competition. To benefit from this softening, both firms want to offer the same quality, minimizing product differentiation. The resulting minimum differentiation contrasts with the basic result in vertically differentiated markets without a taste for variety, where Shaked and Sutton (1982) show that firms will differentiate themselves to soften price competition. The intuition for this result is that when firms produce products of the same quality, they essentially give up trying to sell any customers a second unit of the good, and thus this accommodation of their rival leads to soft price competition. On the other hand, when the firms offer different qualities, more intense competition occurs over the sale of the second unit of the high-quality

good any time H undercuts L, and the softening due to differentiation does not compensate for the lost profits.

When producing higher quality involves no costs, the firms offer the highest feasible quality level. When increasing quality is costly, both firms still offer the same quality, but the quality they produce may be below the maximum feasible quality. This main result of our paper predicts that co-located competitors in experience goods where customers have strong preferences for variety, such as restaurants or resorts, are likely to offer similar quality rather than differentiating their qualities. For example, imagine two restaurants in an isolated city where tourists often spend a weekend. Variety seeking is strong in the restaurant industry in that tourists would rather visit two restaurants than the same restaurant twice. We explain the seeming puzzle that co-located restaurants often offer a similar quality of food instead of vertically differentiating as motels have been shown to do (Mazzeo 2002). Neither restaurant benefits from reducing its quality and differentiating, because consumers value visiting both restaurants more than going to the cheaper restaurant twice.

Because minimizing product differentiation softens price competition, a superior firm would have an incentive to help the other firm improve its quality, which means a superior firm would be willing to expend resources to improve its rival's products. The additional result of the superior firm helping the inferior firm improve also has immediate implications: suppose one of the two chefs is excellent and the other is good but not great. Our model demonstrates that the excellent chef has an incentive to help the weaker chef become a better cook, as long as the weaker chef has similar-enough initial quality. In equilibrium, the restaurants offer the same quality, charge the same price, and all consumers seek variety.

Although greater preferences for variety can soften price competition when technology or regulations restrict qualities to a narrow-enough range, we also show that variety seeking can intensify price competition if the firms offer products that are exogenously differentiated enough. With enough quality differentiation, the variety bundle becomes a middle-quality option that moderately quality-sensitive consumers choose. The high-end consumers buy two units of the high-quality good, paying a premium to the firm that produces it, and the low-end consumers buy two units of the cheaper product. The resulting price competition is stronger than in the absence of a preference for variety because both firms try to secure more consumers to buy two

units, but the consumers retain more flexibility through an option to seek variety and avoid satiation.

The result that the sign of the impact of preferences for variety changes with the level of differentiation between the firms contrasts with what would occur in the presence of switching costs. To see the contrast, note that one can say customers exhibit such switching costs similar to Klemperer (1987) or Dubé, Hitsch, and Rossi (2009) when $\delta > 1$. In such a case, the model is similar to Shaked and Sutton (1982), and prices and profits both increase with δ, that is, as firms exhibit greater switching costs. Although the full exposition of this case is beyond the scope of this paper, this observation demonstrates that the results we show, and the analysis of variety-seeking behavior, cannot be summarized as the negative extension of results for models with switching costs.

References

Che, Hai., K. Sudhir, and P.B. Seetharaman. 2007. "Bounded Rationality in Pricing under State Dependent Demand: Do Firms Look Ahead? How Far Ahead?", *Journal of Marketing Research* 44(3): 434–49.

Chintagunta, Pradeep. 1998. "Inertia and Variety Seeking in a Model of Brand Purchase Timing." *Marketing Science* 17(3): 253–70.

Dubé, Jean-Pierre, Günter Hitsch, and Peter Rossi. 2009. "Do Switching Costs Make Markets Less Competitive." *Journal of Marketing Research* 46(4): 435–45.

Feinberg, F. M., B. E. Kahn, and L. McAlister. 1992. "Market Share Response When Consumers Seek Variety." *Journal of Marketing Research* 29(2): 227–37.

Givon, Moshe. 1984. "Variety Seeking through Brand Switching" *Marketing Science* 3(1): 1–22.

Kahn, B.E., M.U. Kalwani, and D.G. Morrison. 1986. "Measuring Variety-seeking and Reinforcement Behaviors Using Panel Data." *Journal of Marketing Research* 23(2): 89–100.

Klemperer, Paul. 1987. "Markets with Consumer Switching Costs." *Quarterly Journal of Economics* 102(1): 375–94.

Mazzeo, Michael. 2002. "Product Choice and Oligopoly Market Structure." *RAND Journal of Economics* 33(2): 221-242

McAlister, Leigh. 1982. "A Dynamic Attribute Satiation Model of Variety-Seeking Behavior." *Journal of Consumer Research* 9(3): 141–50.

McAlister, Leigh, and Edgar. A. Pessemier. 1982. "Variety Seeking Behavior: An Interdisciplinary Review." *Journal of Consumer Research* 9(3): 311–22.

Ratner, Rebecca K., Barbara E. Kahn, and Daniel Kahneman. 1999. "Choosing Less-Preferred Experiences for the Sake of Variety." *Journal of Consumer Research* 26(1): 1–15.

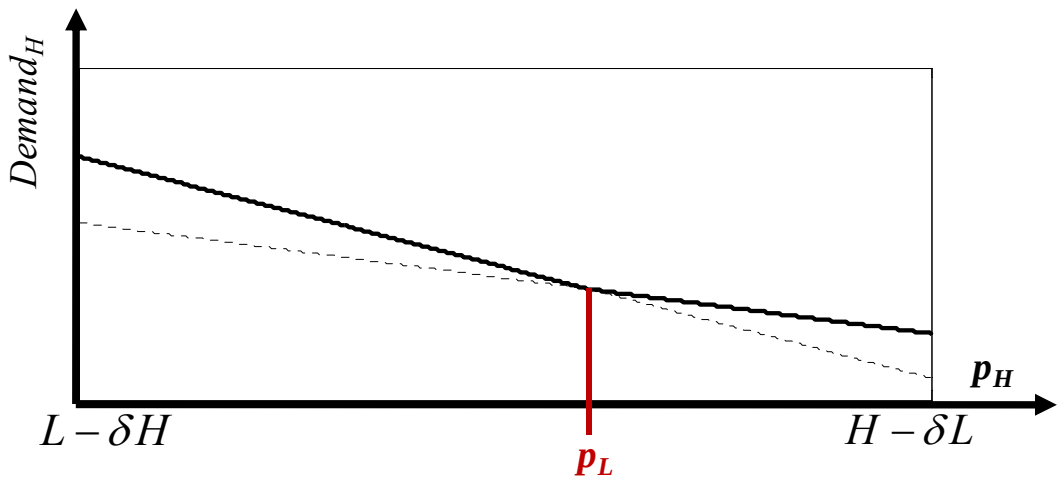
Sajeesh, S., and Jagmohan S. Raju. 2010. "Positioning and Pricing in a Variety Seeking Market." *Management Science* 56(6): 949–61.

Seetharaman, P.B., and Hai Che. 2009. "Price Competition in Markets with Consumer Variety Seeking." *Marketing Science* 28(3): 516–25.

Shaked, Avner, and John Sutton. 1982. "Relaxing Price Competition through Product Differentiation." *Review of Economic Studies* 49(1): 3–13.

Appendix: Additional figure and proofs of all propositions

Figure 3: The high-quality firm's demand is kinked at $p_H = p_L$



Note: The solid line is the demand curve. To visually emphasize the kink, the dashed lines continue the demand above and beyond $p_H = p_L$, respectively.

Proof of Proposition 1: First suppose $p_L < p_H$. Then no one buys two units of the H good because $U_H > U_L$ implies $U_V > U_H$. Consumers thus only choose between two L units and variety seeking (see left side of Figure 1). The cutoff is $U_V > U_L \Leftrightarrow \theta > \frac{p_H - p_L}{H - \delta L}$. The two firms thus make the following profits: $\Pi_L(p_L) = p_L\left(1 + \frac{p_H - p_L}{H - \delta L}\right)$ and $\Pi_H^{up}(p_H) = p_H\left(1 - \frac{p_H - p_L}{H - \delta L}\right) = p_H\left(1 + \frac{p_L - p_H}{H - \delta L}\right)$. The symmetry of the profit functions implies only $p_L = p_H$ can be a pure-strategy equilibrium. The best-response functions are $p_L = \frac{H - \delta L + p_H}{2}, p_H^{up} = \frac{H - \delta L + p_L}{2}$, and the candidate equilibrium prices are $p_L = p_H = H - \delta L$. Thus no pure-strategy equilibrium exists with $p_L < p_H$.

Second, $p_L > p_H$ cannot support a pure-strategy equilibrium either, because nobody would buy two L units (inferior to two H and also more expensive). Consumers thus only choose between two H units and variety seeking (see right side of Figure 1). The cutoff is $U_V > U_H \Leftrightarrow \theta > \frac{p_L - p_H}{L - \delta H}$. The profits are

$\Pi_L(p_L) = p_L\left(1 - \frac{p_L - p_H}{L - \delta H}\right) = p_L\left(1 + \frac{p_H - p_L}{L - \delta H}\right)$ and $\Pi_H^{down}(p_H) = p_H\left(1 + \frac{p_L - p_H}{L - \delta H}\right)$. Again, the symmetry of the profit functions implies only $p_L = p_H$ can be a pure-strategy equilibrium. The best-response functions are $p_L = \frac{L - \delta H + p_H}{2}, p_H^{down} = \frac{L - \delta H + p_L}{2}$, and the candidate equilibrium prices are $p_L = p_H = L - \delta H$. Thus no pure-strategy equilibrium exists with $p_L > p_H$.

It remains to be shown that no equilibrium exists with $p_L = p_H$. We show the high-quality firm will deviate from any $p_H = p_L$, undercutting large p_L and overshooting small p_L. We now analyze both possible directions of the deviation. By undercutting p_L charged by the low firm, the high firm can secure some consumers to buy two H units, resulting in profit Π_H^{down}. The first derivative of Π_H^{down} is $\left.\frac{d\Pi_H^{down}}{dp_H}\right|_{p_H = p_L} = 1 - \frac{p_L}{L - \delta H} < 0 \Leftrightarrow p_L > L - \delta H$, so undercutting a small amount is profitable for all p_L large enough. By overshooting p_L charged by the low firm, the high firm faces a profit Π_H^{up}, with the first derivative at $p_H = p_L$ of $\left.\frac{d\Pi_H^{up}}{dp_H}\right|_{p_H = p_L} = 1 - \frac{p_L}{H - \delta L} > 0 \Leftrightarrow p_L < H - \delta L$, so overshooting a small amount is profitable for all p_L small enough. Since $L - \delta H < H - \delta L$, at least one of the deviations is profitable for all p_L. QED Proposition 1.

Proof of Proposition 2: The equilibrium can be exposed most clearly in terms of the following quantities: $D \equiv H - \delta L, \mathrm{C} \equiv L - \delta H, B \equiv \sqrt{L - \delta H} = \sqrt{C}, A \equiv \sqrt{H - \delta L} = \sqrt{D}$. Under this notation, Assumption 1 is equivalent to $A < 2B \Leftrightarrow D < 4C$. Let $p_L = AB$, and note that $C < p_L < D$ because AB is the geometric mean of C and D. We first show the high firm is indifferent between undercutting and overshooting this p_L: the optimal undercutting deviation is $p_H^{down} = \frac{B(A+B)}{2}$ (see proof of Proposition 1) and yields a profit of $\Pi_H^{down}\left(p_H^{down}\right) = \frac{(C + p_L)^2}{4C}$. The undercut to p_H^{down} does not drive the low firm out of the market because

$Demand_L\left(p_L \mid p_H = p_H^{down}\right) = 1 - \frac{p_L - p_H^{down}}{C} > 0 \Leftrightarrow AB - \frac{B\left(A+B\right)}{2} < B^2 \Leftrightarrow A < 3B$, which is in turn implied by Assumption 1. The optimal overshooting deviation is $p_H^{up} = \frac{A\left(A+B\right)}{2}$ (see proof of Proposition 1), which yields a profit $\Pi_H^{up} = \frac{\left(D+p_L\right)^2}{4D}$. Comparing the two profits, we find that $\Pi_H^{up} = \Pi_H^{down} \Leftrightarrow C\left(D^2 + 2Dp_L + p_L^2\right) = D\left(C^2 + 2Cp_L + p_L^2\right) \Leftrightarrow DC = p_L^2 \Leftrightarrow p_L = AB$. Since the high firm is indifferent between undercutting and overshooting $p_L = AB$, any mixed strategy that plays p_H^{up} with probability ρ and p_H^{down} with probability 1-ρ is a high firm's best response to $p_L = AB$.

To close the equilibrium construction, we now find ρ such that $p_L = AB$ is indeed the best response of the low firm. When the high firm plays a mixed strategy with a support of two points $\left\{p_H^{DOWN}, p_H^{UP}\right\}$, the profit of the low firm is

$$\Pi_L\left(p_L\right) = \begin{cases} p_L < p_H^{down} : p_L\left(1 + \frac{\rho p_H^{up} + \left(1-\rho\right) p_H^{down} - p_L}{D}\right) \\ p_H^{down} < p_L < \min\left(p_H^{up}, p_H^{down} + C\right) : p_L\left(1 + \rho\frac{p_H^{up} - p_L}{D} + \left(1-\rho\right)\frac{p_H^{down} - p_L}{C}\right) \\ p_H^{down} + C < p_L < p_H^{up} : \rho p_L\left(1 + \frac{p_H^{up} - p_L}{D}\right) \\ p_L > p_H^{up} : p_L\left(1 + \frac{\rho p_H^{up} + \left(1-\rho\right) p_H^{down} - p_L}{C}\right) \end{cases}.$$

We first show that only $p_H^{down} \le p_L \le p_H^{up}$ can maximize profit for any $\left\{p_H^{down}, p_H^{up}\right\}$. The intuition is the same as that for the kinked Π_L profit function in the above search for a pure-strategy equilibrium: $\Pi_L\left(p_L\right)$ is increasing for all $p_L < p_H^{down}$ and decreasing for all $p_L > p_H^{up}$. To see the first claim, note $\Pi\left(p\right) = p\left(1 + \frac{x-p}{y}\right) \Rightarrow \Pi'\left(p\right) \propto \frac{x+y}{2} - p$. Since

$x = \rho p_H^{up} + (1-\rho) p_H^{down} > p_H^{down}$ and $y = H - \delta L > p_H^{down}$, it follows that $\frac{x+y}{2} > p$ for all $p_L < p_H^{down}$. The proof of the second claim is analogous.

Since the profit function is increasing for small prices ($p_L < p_H^{down}$), decreasing for large prices ($p_L > p_H^{up}$), and continuous at $p_L = p_H^{up}$ and $p_L = p_H^{down}$, the maximum must be somewhere in the $\left[p_H^{down}, p_H^{up}\right]$ interval. Since $p_H^{down} + C < p_L < p_H^{up}$ imply an increasing Π_L, the maximum must be either at $p_L = p_H^{up}$ or somewhere in the $\left[p_H^{down}, p_H^{down} + C\right]$ interval. To find the best price in $\left[p_H^{down}, p_H^{down} + C\right]$, let $p_H^{down} = \frac{B(A+B)}{2}, p_H^{up} = \frac{A(A+B)}{2}$ and note that the profit function for $p_H^{down} + C < p_L < p_H^{up}$ simplifies to $\Pi_L = p_L\left[1 + \left(\frac{A+B}{2}\right)\left(\frac{\rho}{A} + \frac{(1-\rho)}{B}\right) - p_L\left(\frac{\rho}{A^2} + \frac{(1-\rho)}{B^2}\right)\right]$, which is concave. The first-order condition is $p_L = \left(\frac{AB}{4}\right)\left(1 + \frac{3AB}{(1-\rho)A^2 + \rho B^2}\right)$. Solving for the mixing probability for $p_L = AB$ in turn yields $\rho^* = \frac{A}{A+B}$. Plugging ρ^* back into the profit function yields $\Pi_L(AB \mid \rho^*) = AB$. When the high firm plays ρ^*, the first-order condition for the low firm has an interior solution inside the $\left[p_H^{down}, p_H^{down} + C\right]$ interval: $\frac{B(A+B)}{2} < AB$ is obvious, and $AB < \frac{B(A+B)}{2} + B^2 \Leftrightarrow A < 3B$, which is implied by Assumption 1. It remains to be confirmed that the low firm indeed prefers to play $p_L = AB$ to $p_L = p_H^{up}$.

$\Pi_L(AB \mid \rho^*) = AB > \Pi_L\left(p_H^{up}\right) = \rho p_H^{up} = \frac{A^2}{2} \Leftrightarrow A < 2B$, which is exactly Assumption 1.

In equilibrium, the high firm makes $\Pi_H = \left(\frac{A+B}{2}\right)^2$, which exceeds the equilibrium profit of the low firm because the arithmetic mean always exceeds the geometric mean (Cauchy): $\Pi_H = \left(\frac{A+B}{2}\right)^2 > AB = \Pi_L \Leftrightarrow \frac{A+B}{2} > \sqrt{AB}$. Finally, the $\frac{\partial \Pi_H}{\partial \delta} < 0$ and $\frac{\partial \Pi_L}{\partial \delta} < 0$ comparative statics are obvious. *QED* Proposition 2.

Proof of Proposition 3: The proof is immediate when one substitutes $L=H=q$ in the profit functions analyzed in the proof of Proposition 1 and keeps track of firm identities:

$$\Pi_1(p_1) = p_1\left(1+\frac{p_2-p_1}{q(1-\delta)}\right) \text{ and } \Pi_2^{up}(p_2) = \Pi_2^{down}(p_2) = p_2\left(1+\frac{p_1-p_2}{q(1-\delta)}\right).$$

Neither firm's demand function has a kink now, and the profit functions are the same. Therefore, only $p_1 = p_2$ can be a pure-strategy equilibrium. The profit functions are also concave, so the unique candidate prices $p_1 = p_2 = q(1-\delta)$ are indeed equilibrium prices. *QED* Proposition 3.

Proof of Proposition 5: The demand functions consumer behavior implies are

$$Demand_L = \frac{p_H - p_L}{H-\delta L} + \frac{p_H - p_L}{\delta H - L} = (p_H - p_L)\frac{(1+\delta)(H-L)}{(\delta H - L)(H-\delta L)} = \frac{p_H - p_L}{R}$$

$$Demand_H = 2 - \left(\frac{p_H - p_L}{H-\delta L} + \frac{p_H - p_L}{\delta H - L}\right) = 2 - (p_H - p_L)\frac{(1+\delta)(H-L)}{(\delta H - L)(H-\delta L)} = \frac{2R + p_L - p_H}{R},$$

where $R = \dfrac{(\delta H - L)(H-\delta L)}{(1+\delta)(H-L)}$.

Therefore, the profit functions are

$$\Pi_L(p_L) = \frac{p_L(p_H - p_L)}{R} \text{ and } \Pi_H(p_H) = \frac{p_H}{R}\left[(p_L - p_H) + 2R\right]$$

The local (within the ordering of the cutoffs) best-response functions are therefore

$p_L = \dfrac{p_H}{2}, p_H = \dfrac{p_L}{2} + R$, and candidate prices for equilibrium are $p_L = \dfrac{2}{3}R < p_H = \dfrac{4}{3}R$.

It is easy to check that both marginal consumers lie within the support of θ:

$$0 < \frac{2(\delta H - L)}{3(1+\delta)(H-L)} = \frac{p_H - p_L}{H-\delta L} \text{ and } \frac{p_H - p_L}{\delta H - L} = \frac{2(H-\delta L)}{3(1+\delta)(H-L)} < 1.$$

The second constraint holds because $2(H-\delta L) < 3(1+\delta)(H-L) \Leftrightarrow (H-\delta L) + 3(\delta H - L) > 0$

.

The profits in the candidate equilibrium are $\Pi_L = \dfrac{4}{9}R, \Pi_H(p_H) = \dfrac{16}{9}\mathrm{R}$.

The above situation is an equilibrium whenever the high firm does not want to deviate up to make money only on the variety seekers. The consumer behavior then becomes $\left[0, \frac{p_H - p_L}{H - \delta L}\right]$: L both days and $\left[\frac{p_H - p_L}{H - \delta L}, 1\right]$: variety seek. The best such deviation is

$p_H = \frac{H - \delta L + p_L}{2} = \frac{3D^2 + 5CD}{6(D + C)}$, where $D = H - \delta L$ and $C = \delta H - L$.

The best possible deviation yields a profit of $D\left(\frac{(3D + 5C)}{6(D + C)}\right)^2$, so it is profitable when

$$D\left(\frac{(3D + 5C)}{6(D + C)}\right)^2 > \frac{16CD}{9(D + C)} \Leftrightarrow 9D^2 - 34CD - 39C^2 > 0,$$

which holds for C=0 and is decreasing for positive C in C, so a cutoff $C^*<D$ exists, beyond which this deviation is not profitable. For all $C>C^*$, the above candidate equilibrium is an equilibrium. The cutoff is $C = \frac{8\sqrt{10} - 17}{39} D \approx 0.21D$.

To prove the comparative static in δ, it is sufficient to show that $\frac{\partial R}{\partial \delta} > 0$, because all prices and profits are linear in R. R increases in δ because

$$\frac{\partial R}{\partial \delta} = \frac{H^2 - (\delta^2 + 2\delta - 1)HL + L^2}{(1 + \delta)^2 (H - L)} = \frac{(H - L)^2 + (3 - 2\delta - \delta^2)HL}{(1 + \delta)^2 (H - L)} > 0.$$

QED Proposition 5.