Profits and Competition:
An Analysis of Profit-Increasing Entry

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Abstract

This paper demonstrates that in differentiated industries, profits of all incumbent firms can increase with the entry of new competitors even without complementarities or externalities between the products. While profits are never above those of a monopolist, the entry of an additional competitor can increase profits when there are initially two or more firms in the market. We demonstrate that new entry can increase profits through both a direct effect causing a firm's competitors to price more softly, and an indirect effect, where competitors' incentives are not changed but the entry commits the firm to respond less aggressively in price, leading to scenarios where each firm's profits increase. We also examine the relationship between profits and the number of firms in the market, finding that profits can have a down-up-down relationship with the number of firms. Further, we show conditions under which profit-increasing entry is consistent with endogenous location choice. Finally, we examine a similar phenomenon and show that firms may profit as consumers exit the market, even if no firms exit.

1. Introduction

This paper studies conditions under which entry of a new competitor into a market can increase the profits of all firms present in the market before the entry, even when there are no complementarities or externalities between the products.

The driving force behind the result is the fact that entry of new competitors can drive up incumbent firms' prices. Chen and Riordan (2008) formally show that this result can hold as entry induces two effects: (1) a market share effect, where firms want to reduce prices in response to entry in order to regain some of their lost market share, and (2) a price sensitivity effect, where the firms want to increase prices because the customers that continue to purchase from the incumbent after the new firm's entry have a higher average valuation for the incumbent's product. While Chen and Riordan show that the sets of preferences that can allow for price-increasing entry are broad, this effect had been found earlier for a wide range of specific models of competition. Rosenthal (1980) finds this result for a model with a market with two consumer segments – one that is loyal to a firm, and another that switches to the firm with the lowest price. Hauser and Shugan (1983) find that prices can increase from entry in their defender model, while Perloff et. al. (1996) show that prices can increase in a long Hotelling line with linear travel costs. Thomadsen (2007) demonstrates that entry can induce price-increases under an empirical model of geographic competition. Empirically, Perloff et. al., Ward et. al (2002), Yamawaki (2002), Simon (2005), Goolsbee and Syverson (2008) and McCann and Vroom (2010) show that entry does sometimes increase prices in the pharmaceutical, consumer packaged goods, luxury car, magazine, airline and hotel industries, respectively. Cowan and Yin (2008) and Zacharias (2009) demonstrate that these increases in price can sometimes lead total consumer surplus to decline, a result we find as well.

The price increases in these papers have not been associated with increased profits for the incumbent firms. For example, Thomadsen (2007) shows that entry by a moderately-distant Burger King may cause a McDonald's (which was a monopolist before entry) to increase its price, but this price increase is the result of McDonald's no longer being able to attract customers located near the new Burger King outlet who had previously been obtainable when McDonald's had low prices. After entry, these consumers find the Burger King outlet to be very attractive, so McDonald's would have to drop its price substantially to keep these customers. Instead, McDonald's gives up on retaining these customers and increases its prices to extract more from the consumers located closer to the McDonald's. Despite the price increase, McDonald's is worse off because its losses too many customers to offset the higher margins it makes after the entry.

The result that a monopolist always loses profits with the entry of a second firm in the absence of some sort of complementarity or market-size externality can be stated as a theorem. The proof is simple: By revealed preference, the monopolist must make more profits by charging the monopolistic price than

they would obtain as a monopolist charging the price they would set under a duopoly.[1] Further, the monopolist must make more as a monopolist charging duopoly prices than they make as a duopolist charging duopoly prices since the firm's sales will be lower with a competitor. We know this because in the monopolistic case, the firm sells all units that generate a greater utility to consumers than the outside good, while in the duopoly case this is a necessary but not sufficient condition for a sale. Thus, profits are weakly higher under monopoly than when there are any competitors in the market.

The same reasoning, however, does not extend to entry into markets where there are two or more incumbents. In such environments each firm's optimal price is affected by their competitors' prices as well as their own. Entry by a third (or greater) competitor at the correct location can increase the incumbent firms' profits through two effects. First, the new entry can cause a competing incumbent firm to increase its prices due to the price sensitivity effect of Chen and Riordan (2008). Second, the entry by the new competitor can act as a commitment device for the incumbent firm to raise its price, leading its incumbent competitors to price less aggressively as the best response to the anticipated higher price. The first effect may be called a direct effect, while the second effect may be referred to as the indirect effect. We demonstrate that profits can increase even in the presence of only a direct or only an indirect effect.

Our paper focuses on profit-increasing entry under standard product-differentiation models. This contrasts with previous papers that obtain the result of profit-increasing entry through other mechanisms, which we now summarize. One alternative mechanism for entry to increase profits is for the entry to cause a market expansion or create a positive externality on incumbent firms: When entry increases the size of the market, entry can increase incumbents' profits by offsetting the increased intensity of price competition.[2] A similar literature on geographic agglomeration notes that firms may choose to locate near competitors due to the fact that consumers prefer shopping where there is a cluster of stores (e.g. Dudey 1990 and Gauri, Sudhir and Talukdar 2007). Thomadsen (2009) studies a similar phenomenon, and demonstrates that profits for all firms can increase when incumbent firms offer new products because the desire to avoid intra-firm cannibalization can enforce a commitment for the firm to be a softer competitor. Pazgal and Soberman (2010) show that entry by firms with high costs targeting a specific segment of consumers can also lead to increased profits for the incumbent firms under certain fairly-restrictive conditions. All of these rely on specific mechanisms, which may be appropriate in some markets.

There is one paper that we are aware of that has shown that profits can increase through a mechanism similar to the one we describe above. Chen and Riordan (2007) propose a new model of

[1] We use duopoly in this sentence, but the logic applies to any number of firms.

[2] This has been alleged to happen when Starbucks enters a market. E.g., Jim Stewart, founder of Seattle's Best Coffee, stated that when Starbucks opens nearby it does "all of your marketing for you and your sales will soar." Ward Barbee, founder of the coffee trade magazine *Fresh Cup* said that having a Starbucks open next to a coffee shop is "the best news that any local coffeehouse ever had." See "Don't Fear Starbucks: Why the franchise actually helps mom and pop coffeehouses," by Taylor Clark, *Slate*, Dec. 28, 2007, http://www.slate.com/id/2180301/.

differentiated competition, the spokes model, which is an address-based model of global competition. Chen and Riordan show that entry can increase profits in this model. While the models we study all focus on local competition, a special case of the spokes model, with exactly three spokes, does reduce to a special case of a Salop circle. Our paper examines many aspects of profit-increasing entry that Chen and Riordan do not consider. We demonstrate and separate two different effects that contribute to profit-increasing entry, and demonstrate that either one alone can increase profits. We also allow firms to endogenously choose the products they offer, and demonstrate that profit-increasing entry can be consistent with such choices. Further, we consider how prices and profits change with increasing numbers of competitors, and demonstrate a different relationship than Chen and Riordan. Finally, we examine a similar situation where consumer exit can increase profits for all firms. In contrast, Chen and Riordan do not focus much on the profit-increasing entry aspect of their model, discussing it only briefly.

The rest of the paper proceeds as follows. Section 2 lays out the basic model, which remains constant in both linear and circular markets. Section 3 presents the analysis for linear markets, which separates the two effects that can lead to increased profits. Section 4 presents the analysis for circular markets, which is used to demonstrate the impact of additional competitors on profits in Section 5, and to show that that the results are consistent with endogenous location choice in Section 6. Section 7 studies conditions that support a related phenomenon: consumer exit leading to higher prices that increase all firms' profits. Section 8 summarizes the key findings of Chen and Riordan's (2007) spokes model, and compares their findings with ours. Finally, Section 9 discusses some implications and concludes.

2. The General Model

We conduct our analysis on a Hotelling (1929) line and a Salop (1979) circle. In the case of the Hotelling line, consumers are uniformly located on an infinitely-long market with a constant density of 1, as in Perloff et. al. (1996).[3] In the circular market, consumers are located uniformly along a unit circle.

Consumer *i*'s utility from buying and consuming the product from firm *j* can be represented as

$$U_{ij} = V - p_j - d_{ij} \tag{1}$$

where p_j is the price charged by firm *j* and d_{ij} is the distance between consumer *i* and the location of firm *j*.[4] Consumers can also decide not to get the good from either firm in which case they consume only an outside good, and earn a normalized utility of zero.

[3] Alternatively, one can assume that consumers are located on a long line segment with a constant density, where the line is long enough such that neither end of the market is covered.

[4] The seemingly more general expression $U = v - \alpha P - td$, is equivalent under the normalization $V = v/t$ and $p = tP/\alpha$.

We assume that firms have constant marginal costs,[5] so profits are then given by $(p_j - c)q_j$ where q_j is the quantity sold. Without loss of generality, we set $c = 0$, with the implication that p represents the amount that the price is above marginal cost.

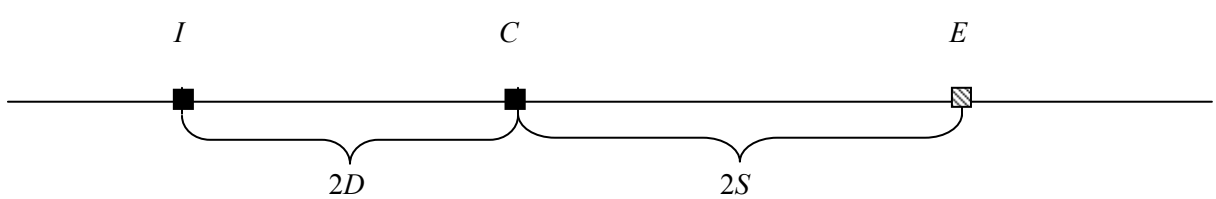
Our analysis focuses on how profits for incumbent firms change when a new competitor enters the market. In Sections 3, 4 and 7, firm locations taken to be exogenously given; we remove this assumption in the analysis in Sections 5 and 6.[6]

3. Linear Markets

In this section we show conditions under which entry can increase profits among firms competing along an infinitely-long Hotelling line. Specifically, our analysis focuses on the changes in incumbent profits under two different scenarios. In the first scenario, two firms, whose locations are given and set, are present in the market and compete via horizontally differentiated price competition. In the second scenario, the original firms retain the same locations as in the first scenario, but a third firm is added to the market at an exogenous location. These three firms then set prices and compete for customers. We examine conditions under which profits for the original two firms increase after the entry of the third firm.

Specifically, we assume that there are initially two firms exogenously located a distance $2D$ apart, which engage in a differentiated price competition. We examine how the profits for these firms change after an entrant enters the market a distance $2S$ from one of the firms (and $2D + 2S$ from the other one). Because of the asymmetry that emerges between the outlets, we index the outlets as I (incumbent), C (center) and E, entrant. Without loss of generality, we assume that the location of I is to the left of C, and that the location of E is to the right of C. Figure 1 illustrates the market.

Figure 1: The Linear Market



Before conducting any formal analysis, let us consider how the entry of E can affect the profits of C and I. Suppose that E's entry is at a location that induces firm C to increase its price. Firm I is

[5] We discuss the case of increasing marginal (convex) costs in the conclusion.

[6] We assume that once a firm's location is chosen, it cannot be changed (or repositioned). This assumption is reasonable if changing a location involves high enough costs. Examples of these costs might include the costs of reconfiguring manufacturing machines or conducting new R&D to make new products, costs of renovating new retail space if the location is geographic in nature, or the costs of advertising to change consumers' perceptions.

immediately better off because its nearest competitor now prices higher, allowing *I* to both raise its price and sell to more consumers. Is *C* also better off? The increase in *I*'s price helps offset the loss of profits from lost market share to the new entrant, firm *E*. Whether *C* is better off or not depends on the relative sizes of the price increases and the market share losses. As we show below, there are conditions under which the increased prices have the larger impact and *C*'s profits do indeed increase.

Note that firm *I* benefits from the direct effect of *E*'s entry changing *C*'s demand price-sensitivity, leading to a change in price for *C*. On the other hand, firm *I* increases its price because it anticipates that firm *C* will charge a higher price due to firm *C*'s changed incentives. Thus, firm *C* benefits from the indirect effect of *E*'s presence acting as a commitment device for *C* to raise prices, which causes firm *I* to raise its price as well.

While the logic above provides the intuition for how all incumbents' profits can increase after a new competitor enters the market, it is important to rigorously establish when such an increase can occur. We start by presenting a simple example before proceeding to a more general theorem characterizing when profits can increase after entry.

Consider the following example: suppose that *I* and *C* are located a distance $2D = ⅓$ apart, and $V = ½$. One can easily verify that in equilibrium each firm sets a price of $\frac{4}{15}$ and earns a profit of $\frac{8}{75}$. Now suppose that *E* enters a distance $2S = \frac{9}{20}$ away from firm *C*. Then the following prices form an equilibrium: $P_E = 1/4$, $P_C = 3/10$ and $P_I = 49/180$. The prices lead to firm profits of $\pi_E = 1/8$, $\pi_C = 131/1200$ and $\pi_I = 2401/21600$, respectively. Note that profits for *C* increase by 2.3% due to *E*'s entry, while *I*'s profits increase by 4%.

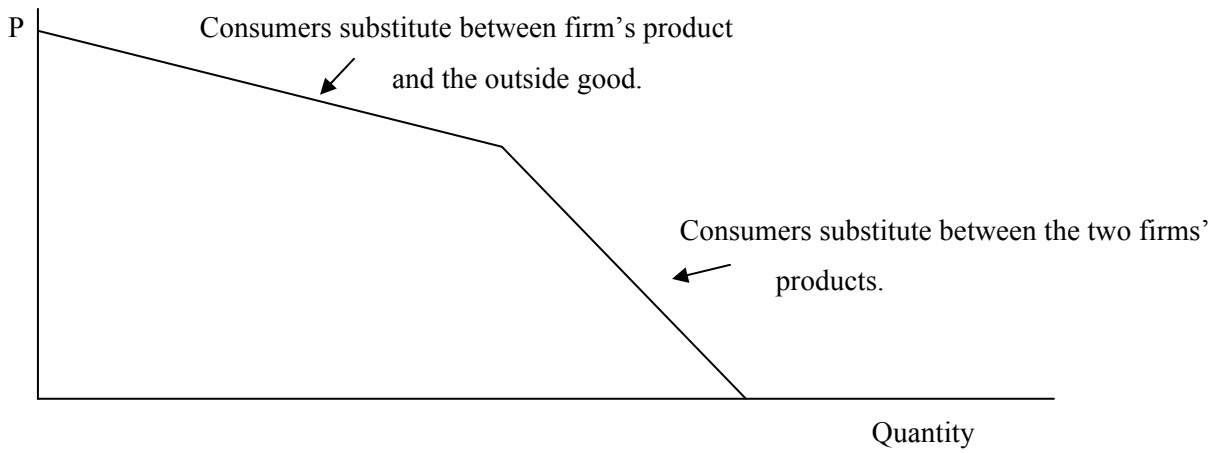
This example demonstrates that both equilibrium prices as well as equilibrium profits can increase after entry. We now present a broad set of parameters where there exist pure-strategy pricing equilibria before and after entry such that both incumbent firms are better off after the entry.

In considering the types of equilibria that support such an outcome, we note that, as observed by Salop (1979) and Perloff (1996), firms' demand curves, holding fixed their rival's prices, can have kinks in them. The kinks occur due to the fact that at high prices the firm's marginal consumer is substituting between the firm's product and the outside good; At lower prices the firm's marginal consumer is substituting between the firm's product and its rival's product. Figure 2 presents a figure of this demand.[7]

[7] When the firm has competitors on both sides, the demand curve can have two kinks – one corresponding to the price where the next-best alternative changes from the outside good to the product on the right, and one corresponding to the price where the next-best alternative changes from the outside good to the product on the left.

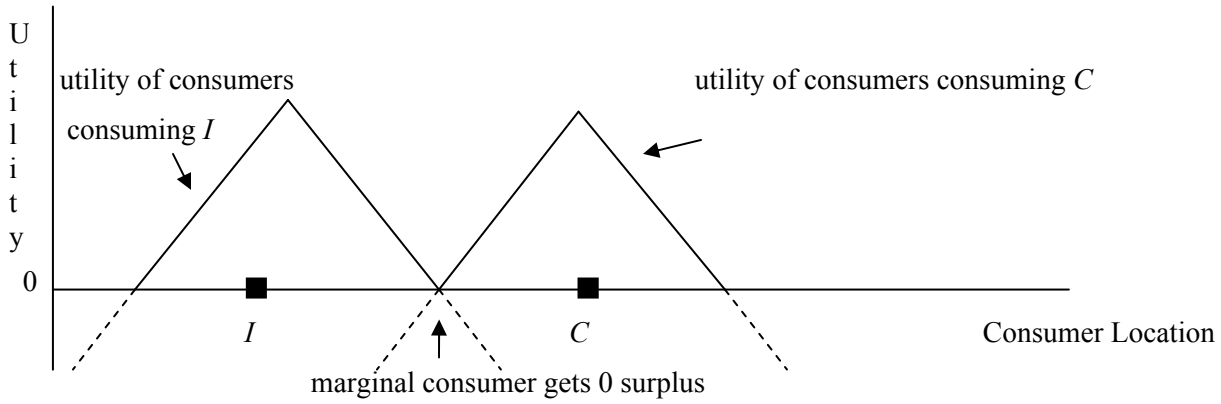
The demand curve is steeper when the prices are low because as low prices are further reduced, the consumers the firm is trying to attract like their next-best alternative, the competitor's product, even more, so it is exceedingly difficult to gain additional customers. This contrasts with what happens when the firm initially lowers a relatively high price – in such a situation, the next-best alternative for potential consumer is the outside good with a fixed utility. Of course, firms' demand curves will not always have kinks. For example, a firm's competitors may be close enough that all of its customers prefer the rival's product over the outside good. This situation is commonly depicted in many Hotelling- and Salop-style models where *V* is assumed to be large enough that the market is always completely covered.

Figure 2: Demand Curves with Kink-Points



Note that the case where the firm prices at a kink-point represents a price where the areas where adjacent firms provide positive surplus to consumers just touch, and the marginal consumers get no surplus at all. This is shown in Figure 3.[8]

Figure 3: Utilities of Consumers when *C* and *I* price at kink-points



[8] The properties of these kinks are described in-depth by Salop (1979) and Perloff et. al. (2006).

The profit-maximizing prices may occur along the linear parts, or at any of the kink-points, of a demand curve. Note that if the optimal price corresponds to a kink-point in the demand curve then there are no first-order conditions that are associated with such an optimization. Similarly, each line segment of the demand curve corresponds to a different first-order condition: the optimal price only occurs along a linear part of the demand function if the first-order condition for that line-segment can be satisfied for the range of prices corresponding to that segment. Because marginal revenues are monotonically (but discontinuously) decreasing, it must be that either the first-order conditions are satisfied on exactly one of the line segments, or the optimal price occurs at one of the kink-points, but not both.[9]

In contrast to what we find in circular markets in Section 4, we demonstrate that *I* and *C*'s profits can only increase with entry if *C* sets its price at a kink-point after *E*'s entry into the market:

Theorem 1: Any locations for *I*, *C*, and *E* that lead to (1) a pure strategy pricing equilibria for all firms before and after entry, and (2) higher profits for both *I* and *C* after *E*'s entry than each firm obtains before the entry must entail firm *C* setting prices such that its first-order conditions are satisfied along the linear portion of its demand curve where the marginal consumers to the left of *C* are substituting between *C* and *I* before *E*'s entry, and firm *C* pricing at a kink-point in its demand curve after *E*'s entry.

All proofs are presented in the Appendix. The basic intuition behind Theorem 1 is as follows. If firm *C* prices along a linear portion of its demand curve where the marginal consumers are substituting between *C* and the outside good then *C* must be charging the monopoly price and earning monopoly profits (and the outlets *I* and *C* are so far apart, that they are serving different markets); we have already shown that profits must decrease with competitive entry in such a scenario. If *C* is pricing at a kink-point in demand before entry then entry either causes *C* to raise its price, which decreases *C*'s profits (since it had the option of increasing its price before entry and still obtaining all consumers who attain a positive utility from *C*'s product), or entry causes *C* to decrease its price, which immediately means that *I*'s profits must decrease. Thus, any market where entry causes profits for both firms to increase must be characterized by *C* setting a price that corresponds to the linear part of *C*'s demand curve where the marginal consumers to the left of *C* substitute between *C* and *I* before entry. Demonstrating that when *C* prices at a kink-point after entry profits may increase relates to the relative tradeoffs between prices and demand, and is calculated in the Appendix. However, intuitively, the demand curve after *E*'s entry has 3

[9] With linear travel costs, there is a discontinuity in the demand curve at the price that makes consumers at the rival firm's location indifferent between the two products because at this price the firm "steals" all of the rival's demand. This discontinuity interferes with the monotonicity property of the marginal revenue and can prevent the existence of any pure-strategy pricing equilibrium. Thus, we confirm in each of the cases we study that no firms will completely undercut the other firms.

segments. At the lowest prices, the marginal consumers substitute between *C* and *I* on the left of *C* and between *C* and *E* on the right of *C*. In the midrange, marginal consumers substitute between *C* and *I* on the left of *C* and between *C* and the outside good on the right of *C*, while at high prices the marginal consumers substitute between *C* and the outside good, only. The demand curve becomes very steep at the bottom of the demand curve, and it turns out that in any scenario where entry increases profits, *C*'s prices must be at the kink-point just above this steepest part of the demand curve (which must also reflect a higher price than *C* charges before entry).

While Theorem 1 tells us what any scenario that leads to increasing profits for *C* and *I* must look like, it does not tell us conditions under which such profit increases occur. Theorem 2 provides a set of conditions under which profit-increasing entry is feasible.

Theorem 2: When $\max\left[\frac{24}{49}V-\frac{47}{49}D+\frac{2\sqrt{6}}{49}\sqrt{(3V+2D)(V-4D)},\frac{299}{595}V-\frac{18}{85}D,D\right]\le S\le\frac{11}{20}V-\frac{1}{5}D$ and $D\in\left(\frac{3}{7+5\sqrt{10}}V,\frac{3}{7}V\right)$ then there exist pure strategy equilibria where profits for *C* and *I* are higher after *E*'s entry than they are before it.

Intuitively, the firms must be close enough that a pure-strategy price equilibrium occurs along the correct part of the demand curve in the pre-entry (2-firm) scenario without being so close that the firms attempt to undercut each other's prices. The range on *S* ensures that the new entrant is far enough away that *C* prices at a kink-point in its demand curve after *E* enters the market, but that *E* is not so far away that its realized market no longer touches the realized market for *C*.

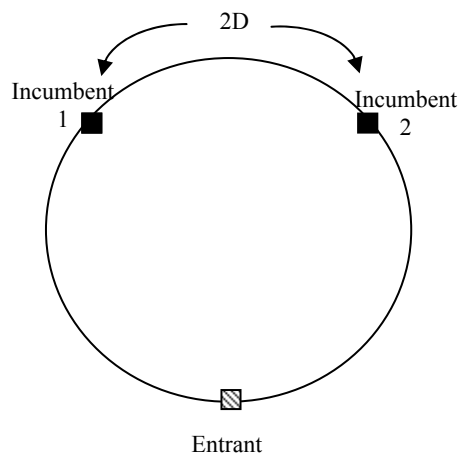
4. Circular Markets

We next consider the case of a circular model, where consumers are uniformly located along a circle with a circumference of 1. We begin by considering a market that initially has 2 firms located a distance $2D < ½$ apart from each other. A third firm is then assumed to enter at the midpoint of the long side of the market. That is, we consider the case where the entrant locates a distance $½ – D$ from each of the incumbent firms. The market is shown in Figure 4 below. In Section 5, we consider the impact of additional entrants, but understanding how entry by a third firm impacts profits in this more-limited scenario also proves to be instructive to those cases.

Before we present general conditions under which profits increase, we first present a specific example where each of the firms is located a distance ⅓ apart from each other, and show that profits

increase and consumer welfare decreases with entry. This example is relevant for the results in Section 5, as well as understanding the link between the findings of Chen and Riordan (2007) and our own results.

Figure 4: The Circular Market



Example

Consider a market where two firms, *I1* and *I2*, are located a distance $2D = ⅓$ apart, and let $V = ½$. Then the market is covered over the short arc between the two outlets, but there is a portion of the market on the long-side of the circle that is not covered. Each firm's demand is $q_j = V - p_j + D + \frac{p_{-j} - p_j}{2}$. Note that $\frac{\partial q_j}{\partial p_j} = -1.5$. Solving the first-order conditions yields prices of $\frac{4}{15}$ and profits of $\frac{8}{75}$ for both firms.

Consider what happens when a third firm, *E*, enters at a distance of ⅓ from the other two firms. Demand for the entrant is $q_E = \frac{1}{3} + \frac{p_{I1}}{2} + \frac{p_{I2}}{2} - p_E$, and demand for *I1* and *I2* are equivalent due to symmetry. Note that $\frac{\partial q_j}{\partial p_j} = -1$, so the demand is less sensitive to price than demand was before entry. As a result, prices (for the incumbents) increase, and all three firms set $p = \frac{1}{3}$. Profits are $\frac{1}{9}$, which are greater than each firm's profits when only two firms were in the market.

Not only do profits increase with entry, but one can confirm that entry causes a decrease in the total consumer welfare. Before entry, aggregate consumer surplus is $\frac{47}{450}$. After entry, some consumers are better off because the new product in the market is a better match for them than the incumbent

products. However, customers who do not switch pay more, and therefore lose consumer surplus from the entry. In total, the losses outweigh the gains, and after entry total consumer welfare is $\frac{1}{12} < \frac{47}{450}$.

We now provide general conditions under which entry of a third firm into a circular market leads to increased profits for both incumbent firms. To keep tractability, we constrain our analysis to symmetric-among-incumbents, pure-strategy pricing equilibria.

First consider the market when only two firms are present. As noted in Section 3, the demand curves for each firm when there are only two in the market can have a kink, depending on the firms' locations and rival firm's prices. Using the same logic as in Section 3, we note that competitive entry can never increase profits for any firm that prices at or above such a kink before entry. (See the paragraph after Theorem 1.) Therefore, we limit our analysis to the set of parameters (V, D) where firms price along the linear part of the demand curve below the kink, since this is the only area of the parameter space consistent with pure-strategy pricing equilibria where entry can increase profits. We first consider only the set of (V, D) where the market is not completely covered before entry. We then show that this constraint is not binding: profit-increasing entry cannot occur when the market is covered before entry.

Thus, the only part of the (V, D) parameter space that can support profit-increasing entry are the set of (V, D) where both firms price such that the marginal consumers on one side of the firm are substituting between the firm's product and the outside good, and the marginal consumers on the other side obtain a positive utility from either product. Proposition 1 provides the set of (V, D) parameters under which such a pure-strategy exists.

<u>Proposition 1:</u> There exists a pure-strategy pricing equilibrium among two firms located a distance $2D$ apart on a unit circle such that each firm's marginal consumers on the long arc between the firms are indifferent between that firm's product and the outside good, while each firm's marginal consumer on the short arc between the firms is indifferent between the 2 firms' products, but obtain a positive utility from either if and only if (i) $V \le \frac{5}{6} - D$ and (ii) $\frac{7}{3}D < V \le \max\left(\left[\frac{7+5\sqrt{10}}{3}\right]D, \frac{5}{6} - \frac{5}{6}\sqrt{\max\left(1-12D,0\right)} - D\right)$.

The proof of this proposition is in the appendix. The condition that $V \le \frac{5}{6} - D$ ensures that the market is not covered when there are only 2 firms in the market. The condition that $\frac{7}{3}D \le V$ ensures that firms price below any kink-points in their demand curve: If the firms were instead further apart, then they

would either price at the kink-point in their demand curve, as explained in Section 3, or if the firms were far enough apart, they would effectively be monopolists serving separate markets. Finally, the condition that $V \le \max\left(\left[\frac{7+5\sqrt{10}}{3}\right]D, \frac{5}{6}-\frac{5}{6}\sqrt{1-12D}-D\right)$ ensures that a pure-strategy price equilibrium exists. The absence of a pure-strategy price equilibrium due to firms locating too close together is a well-known liability of location models with linear travel costs. The restriction that $V \le \left[\frac{7}{3}+\frac{5\sqrt{10}}{3}\right]D$ is the standard restriction that one would find in a long linear model. The reason for the presence of second term $V \le \frac{5}{6}-\frac{5}{6}\sqrt{1-12D}-D$ is that for distances D below but close to $1/12$, the firms' ability to undercut a rival is somewhat limited because when a firm sets its price to undercut its rival, the price turns out to be so low that the market is covered, so the price cut garners less demand than the same price-cut would gain if the two firms were competing on an infinitely-long line.

We next show the conditions under which entry can lead to increased profits for incumbents, restricting the set of parameters to those from Proposition 1 that generate a pure-strategy equilibrium with two firms in the market. The new firm is assumed to be located at the mid-point of the long-end of the arc separating the two incumbent firms, as shown in Figure 4. To keep tractability, we constrain our analysis to symmetric-among-incumbents pure-strategy equilibria.

After entry, there are 4 types of symmetric-among-incumbents pure-strategy equilibria that can emerge: (1) the new firm can be located far enough away that it acts as a monopolist, and the incumbents are not affected by the entry, (2) the market can be covered and all 3 firms can price along the linear part of their demand curve – and below any kink in their demand curve, if such a kink exists, (3) all 3 firms can price at a kink-point in their demand curves, and (4) the incumbents can price at a kink-point in their demand curves such that the marginal consumer between the two incumbents obtains zero utility, while the entrant can price along a linear part of its demand curve. The following theorem describes which of the 4 equilibria will be realized as a function of the (V, D) parameters.

Theorem 3 Consider the set of markets where two incumbent firms are located a distance $2D$ apart, and a third firm is located a distance $\frac{1}{2} - D$ from each incumbent. Then the symmetric-among-incumbents pure-strategy pricing equilibrium for the set of (V, D) satisfying Proposition 1 can be characterized as follows:

- If $V \le \frac{5}{11}-\frac{6}{11}D$ and $\frac{7}{3}D \le V \le \left[\frac{7+5\sqrt{10}}{3}\right]D$, the entrant will price as a monopolist, and the prices and profits of the two incumbents will not be affected.

- If $\frac{7}{3}D \le V \le \min\left(\frac{15}{16}+\frac{21}{8}\mathrm{D}-\frac{9}{8}\sqrt{\frac{11}{3}\mathrm{D}^2+\frac{1}{3}\mathrm{D}+\frac{1}{4}}, \frac{7+5\sqrt{10}}{3}D\right)$ & $\frac{5-6D}{11} \le V \le \min\left(\frac{3}{5}-\frac{3}{5}D, \frac{3}{2}-6D\right)$ then the entrant prices at a kink-point such that its marginal customers earn zero surplus. Equilibrium profits for the incumbents are always higher than in the two-firm case.
- If $\frac{3}{5}-\frac{3}{5}D \le V \le \frac{5}{6}-D$ and $\frac{15\sqrt{17}-39}{256} \le D \le \frac{5}{6}V-\frac{1}{4}$ then all 3 firms price along the linear part of their demand curve (and below any kink in their demand curve, if such a kink exists). Further, equilibrium profits are $\pi_I = \frac{1}{25}\left(\frac{3}{2}+D\right)^2$ and $\pi_E = \frac{4}{25}(1-D)^2$. These profits are higher than the profits for a two-firm equilibrium when $\left(\frac{D}{5}+\frac{3}{10}\right)^2 > \frac{6}{25}(V+D)^2 \to V < \frac{D}{\sqrt{6}}-D+\frac{\sqrt{6}}{4}$.
- If $\frac{3}{2}-6D \le V \le \frac{5}{6}-D$ and $\frac{7}{3}D \le V \le \frac{6}{5}D+\frac{3}{10}$ then both incumbents set prices at kink-points in their demand curve such that marginal consumers are indifferent between the incumbents' products obtain zero utility, while the entrant prices along the linear part of its demand curve. Equilibrium profits for the incumbents are $\pi_I = \frac{1}{8}(V-D)(3-2V+4D)$. Equilibrium profits are higher post entry when $\frac{27}{98}D-\frac{5}{196}\sqrt{-2204D^2-852D+225}+\frac{75}{196} \le V \le \frac{27}{98}D+\frac{5}{196}\sqrt{-2204D^2-852D+225}+\frac{75}{196}$.
- If $\max\left(\left[\frac{7+5\sqrt{10}}{3}\right]D, \frac{5}{6}-\frac{5}{6}\sqrt{\max(1-12D,0)}-D\right) \ge V > \frac{15}{16}+\frac{21}{8}\mathrm{D}-\frac{9}{8}\sqrt{\frac{11}{3}\mathrm{D}^2+\frac{1}{3}\mathrm{D}+\frac{1}{4}}$ and $D < \frac{15\sqrt{17}-39}{256}$ then no pure-strategy symmetric-among-incumbents pricing equilibrium exists.

The areas for each of these 5 sub-cases are non empty and not overlapping; the Appendix demonstrates that the boundaries of these regions are binding. To give the reader more of a sense of what the regions for each of these areas looks like, we plot out all of the regions in Figure 5. The interior of the cone outlined by the thickest lines represents the region in (V,D) space where the results of Proposition 1 hold (i.e., this is the area where a symmetric pure-strategy pricing equilibrium exists between two firms such that both firms are pricing along the linear part of their demand curves). Within this cone there are the 5 regions specified by Theorem 3, each separated by thin solid lines. The regions are listed in Theorem 3 working from the bottom to the top of the figure, and left to right, except for the last case, where there is no pure-strategy equilibrium, which is located in the top-left side of the cone.

Figure 5: Regions of Different 3-Firm Equilibria

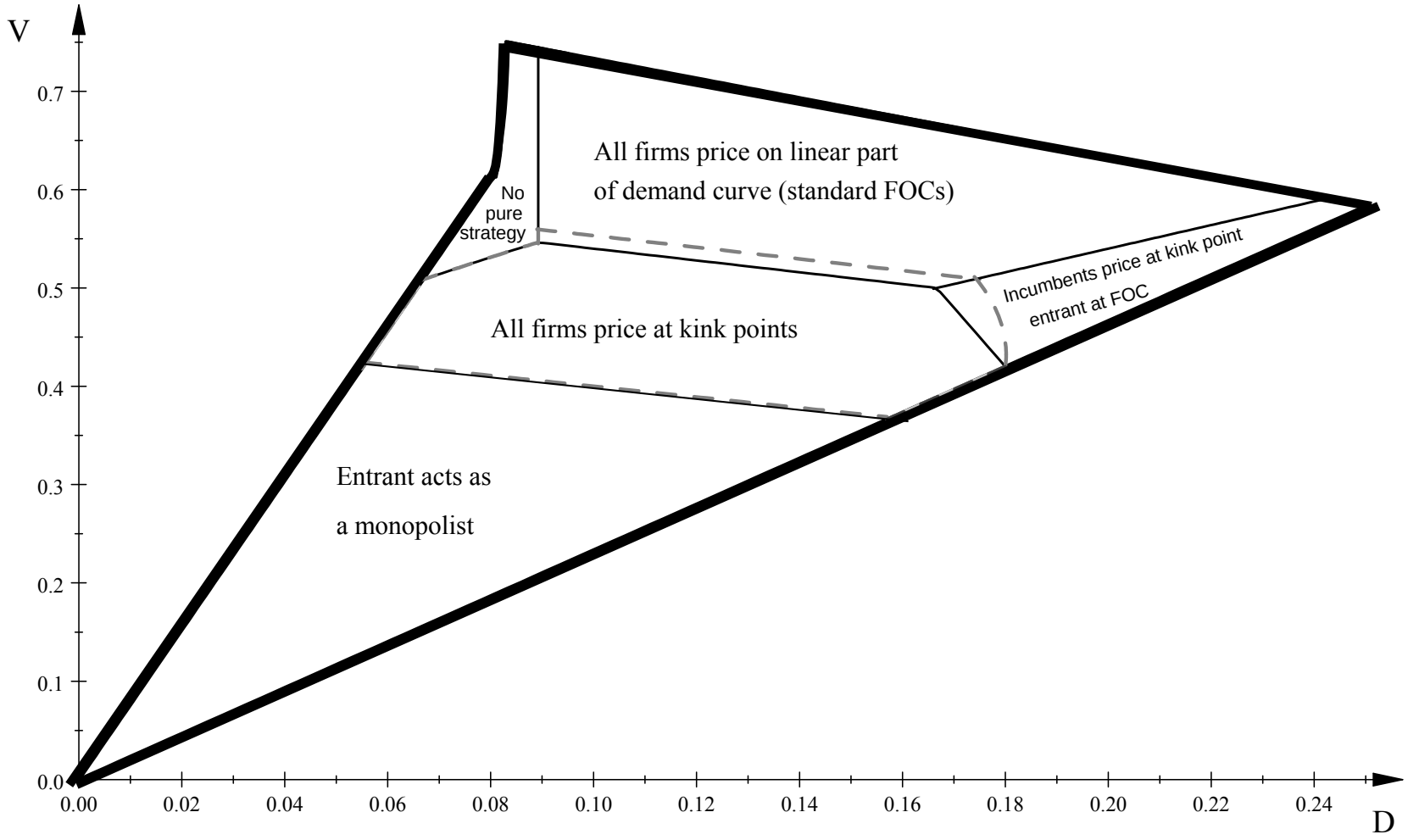


Figure 5 shows that when V is small, the new firm enters as a monopolist. This is because when V is small, most of the market is not covered by the original two firms, and the entrant fills the market hole as a monopolist. When V is a little larger, the 3 firms all compete by pricing at a kink-point. Intuitively, when V is only slightly larger than the maximum value that would support the third firm entering as a monopolist, the market for the entrant and the incumbents only slightly overlap. Once the market overlaps, the derivative of the demand with respect to price becomes closer to zero relative to what these derivatives would be if V were in the range where the entrant were a monopolist. However, because the quantity sold by each firm would be approximately unchanged, each firm now has an incentive to increase their prices. Once they increase their prices just a little bit, the realized market of the entrant no longer overlaps with the realized market of the incumbents, and the firms stop having any incentive to increase their price – and all firms price at kink-points. When V is larger, the amount that the firms would have to raise prices in order to reach a kink-point becomes higher, and the firms instead raise prices to a level where the first-order conditions for the linear portions of their demand curves are satisfied. An exception occurs when V and D are both large: In this case, prices increase to a point where the marginal customer in between the two incumbents gets zero utility. At this point, it no longer pays for the incumbents to increase their price, but the entrant still finds that the set of consumers who obtain positive utility from their product overlaps with the realized markets for the two incumbents, so it prices along the linear portion of its demand curve according to its first-order condition.

Theorem 3 also specifies conditions under which profits for both incumbents increase. This area is represented by the interior of the area outlined by the dashed gray lines in Figure 5. Note that while much of this region involves firms pricing at kink points, there is also a region where all firms price according to their standard first-order conditions and yet profits still increase, which stands in contrast to the results we found for linear markets in Section 3.

While the above analysis focuses on the case where the market is not covered before entry, it is possible to show that there are no symmetric-among-incumbents pure-strategy pricing equilibria such that profits increase with entry if the market were instead covered before entry. We provide a proof in Appendix 5. However, the intuition can be easily gleaned without a proof. When $V > \frac{5}{6} - D$, profits for the two incumbent firms before entry will be strictly greater than the profits that would be obtained by two incumbent firms with the same D but a lower V. Looking then at the 3-firm competitive analysis, the equilibria for the case where $V > \frac{5}{6} - D$ looks like an extension of the results found in Figure 5. The three-firm equilibrium for most of this area consists of the case where all firms price along the linear part of their demand curves; in such a case, the profits are dependent only on D and not on V, so profits never

increase because the 2-firm profits are higher than in any of case where the market is not covered, but the 3-firm profits remain at the same level as at the top of the cone in Figure 5. There are some instances where the market is covered before entry, but where incumbents price at kink-points after entry; however, in the appendix we prove that profits cannot increase in these cases, either.

5. Relationship between Profits and the Number of Competitors

The purpose of this section is to study the pattern that exists between profits and the number of firms in the market. Recall from Section 1 that monopoly profits are greater than profits can ever be in the presence of any number of competitors. We show in this section that profits with enough competitors must trend towards zero. Thus, the relationship between profits and the number of firms in the market will have a down-up-down pattern (if not a monotonic down pattern), rather than a down-up pattern.[10] Further, prices will generally decline with entry once there are enough competitors.

In the previous analysis, we restrict our attention to entry by a third firm. Before proceeding, we note that while the analysis in the previous sections focused on entry by a third firm leading to increased profits, one can construct examples where entry by an Nth firm for any $N \geq 3$ can increase profits.[11] Thus, there is nothing special about the third entrant as opposed to an Nth entrant.

We conduct our analysis by first presenting a theorem stating that profits trend towards zero when there are enough firms present in the market. Then we present two examples that illustrate our result.

Theorem 4: Suppose there is a market with total size M and each consumer has preferences that can be represented as $U_{ij} = V - p_j - f\left(d_{ij}\right)$, if consumer i purchases product j, where f represents a positive-valued non-decreasing function of distance, and $U_{i0} = 0$ if the consumer consumes only the outside good. Then average profits for each firm when there are N firms in the market are bounded by $\frac{VM}{N} \xrightarrow[N \to \infty]{} 0$.

[10] We cannot prove that there will not be more-complex patterns, such as down-up-down-up-down. Our objective is to show that the relationship between profits and the number of firms is not monotonic or U-shaped as in Chen and Riordan (2007).

[11] An example can be constructed with the correct market parameters under linear travel costs by placing the incumbent (N – 1) firms on a line an equal distance apart, and allowing the entrant, the Nth firm, to locate slightly less than $V/2$ units away from the edge of this realized market. Note that V must be at least 21/16 times as large as the distance between each of the incumbent outlets, and V cannot be too large or no pure-strategy pricing equilibrium will exist. The equilibrium before entry is one where all firms price on linear part of their demand function according to their first-order conditions, while after entry the entrant prices as a monopoly (which is also a kink-point), the firm closest to it prices at a kink point and the other (N-2) firms price according to their standard first-order conditions. As a specific example of profit-increasing entry by a 4th firm, let $V = 3$, the distance between firms 1 and 2 (and between firms 2 and 3) = 1, and the distance between firm 3 and the new entrant, firm 4 = 3.1. There is an equilibrium where the price for firm 4 after entry is the monopoly price of 1.5, and firm 3 prices at a kink point. In this equilibrium, profits for firm 1 increase by 0.23%, profits for firm 2 increase by 1.61%, and profits for firm 3 increase by 0.17%, where the firms are ordered from left to right on the line.

The proof of the theorem is straightforward: Average market share for each firm is bounded by $\frac{M}{N}$, and prices must be strictly below V or else the firm will make no sales.

The result of Theorem 4, along with the fact that profits will be highest when only one firm is present in the market, suggests that profits will either steadily decrease, or else decrease, then increase, and finally decrease again as more firms enter the market. In order to demonstrate this pattern, we consider a situation where firms are sequentially allowed to enter into a market. Specifically, we consider a circular market, as studied in Section 4, where there are 6 equally-spaced discrete locations that have been zoned for entry. (I.e., we allow each firm to enter at only one of these 6 locations.) We assume that there are 6 potential entrants, but in any period only one of the firms can enter into the market. In each period, the entrant chooses their location, and then firms compete through differentiated Bertrand competition. The game proceeds for 6 periods, and the objective of each entrant is to maximize the sum of profits for each of the periods where the firm is operating in the market. The solution to this game is presented in Proposition 2.

Proposition 2: Consider a market where consumers are located uniformly along a circle with circumference 1. Further, consumers have linear travel costs, as given in equation (1), and V = ½. There are 6 locations equally spaced where firms are allowed to enter the market, and there are 6 firms that can enter the market, one in each period. Then the following forms an equilibrium: the first firm enters into the market and earns a monopoly profit of 1/8 in period 1. In period 2, the second firm locates at a distance 1/3 away from the first firm, and both firms earn profits of 8/75. The third firm locates at a distance 1/3 away from each of the two incumbents, and all 3 firms earn profits of 1/9. The fourth, fifth and sixth firms locate at the remaining available locations. In such a case, the first three firms earn profits of either 1/16 or 49/576 in period 4, profits of either 100/3249 or 169/3249 in period 5, and profits of 1/36 if period 6.[12]

The proof of this proposition is in the Web Appendix. In Figure 6, we plot the average profits and prices for the first firm in the market when firms locate according to the equilibrium in Proposition 2,[13] where the average assumes that firms that are indifferent between entering in one of k locations chooses

[12] There are many ways that the firms could be determined to choose their locations whenever multiple locations provide equal profits. One way is that the firms could enter the locations randomly. Another way is that the firms could have Leontief preferences for particular locations, which only become relevant in cases of a tie. Any of these strategies yield the qualitative results above. That said, the equilibrium is not unique, and there are other strategies that would lead to different equilibria. However, these alternative equilibria generally have unusual properties (e.g., firm 4's strategy could contain the contingent plan of locating adjacent to firm 2 in the event of a tie in profits only if firm 2 located a distance1/3 from firm 1, which would lead to firm 2 locating a distance ½ from firm 1).

[13] The second firm will have the same average prices and profits (except in the monopolistic case).

each of those locations with a probability $1/k$. We observe that profits decrease, then increase, before decreasing, just as stated in the proposition. Prices increase until there are 3 firms in the market, but then decline with further entry. Prices increasing with the entry of the third firm is necessary for entry to be profit-increasing. However, prices decrease once the market becomes competitive enough.

Figure 6: Price and Profits as a function of the number of firms in the market.

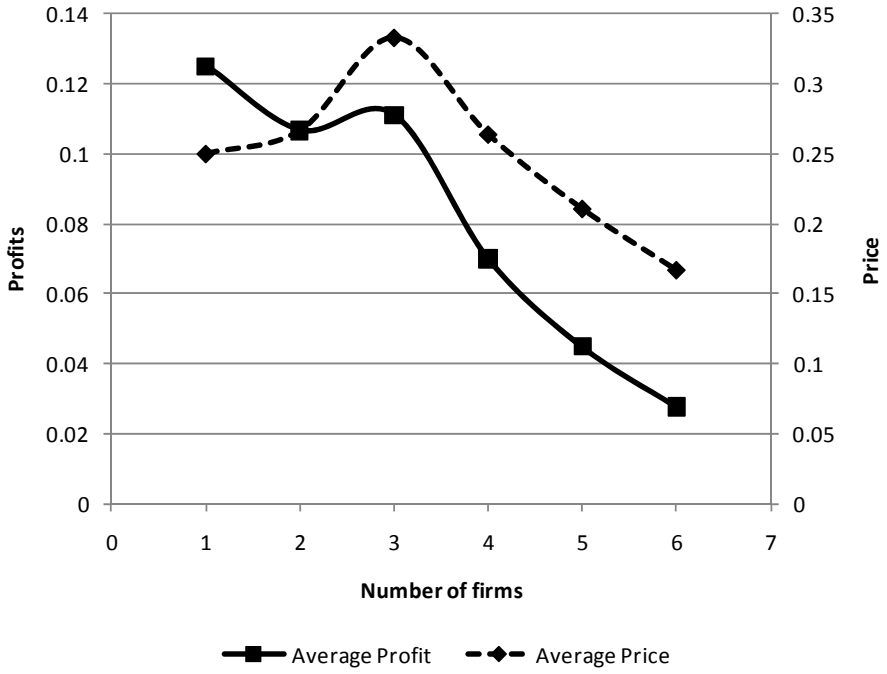
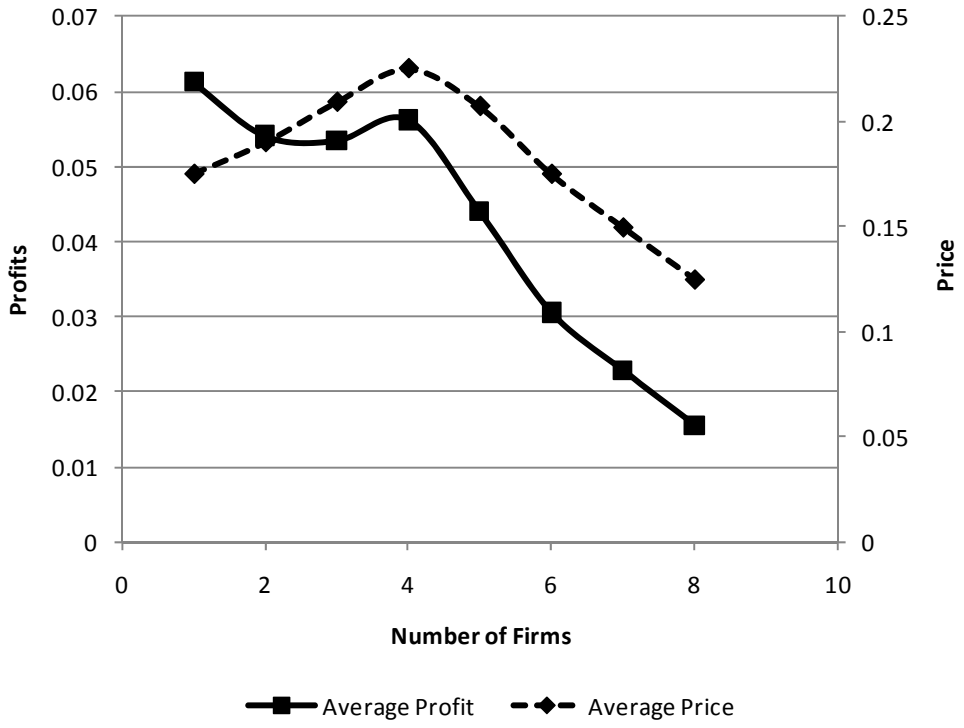


Figure 7 shows the average profits and prices for the first entrant in a similar game where V = 0.35, and where there are 8 firms that enter the market sequentially, each choosing to locate at one of 8 equally-spaced locations zoned for entry.[14] Again, we see that profits at first decrease, then increase, before decreasing again. Two things we note. First, in this case, average profits increase with the entry of the fourth, not the third, firm. In any realization of entry, one of the firm's profits increases with the entry of the third firm, and then decreases slightly with the entry of the fourth firm, although the 4-firm profits are higher than the 2-firm profits; for the other firm, profits decrease with the entry of the third firm, but increases with the entry of the fourth firm, also in such a way that the 4-firm profits are higher than the 2-firm profits. Second, we again find that prices increase with entry when there are only a few firms in the market, and then decrease once enough firms are present in the market.

[14] The 8 firm case is solved using numerical methods but verified analytically. The second firm has the same average prices and products as the first entrant (except in the monopolistic case).

Figure 7: Price and Profits when there are 8 available locations



The issue that profits do not decline monotonically in the number of competitors is not purely of theoretical interest. For example, Bresnahan and Reiss (1991) study the impact of competition on profits. They consider entry by only a small number of firms, and find that entry by the first 3 firms reduces profits, but entry by the 4th or 5th firm has almost no effect. They conclude that markets with 3 firms must have achieved a competitive state. However, our results suggest that it is possible that for some markets the entry by the 4th or 5th firm increased – or approximately left unchanged – profits, but that the market might become even more competitive if more entry occurred. Similarly, Berry (1992) studies which factors affect profitability, but uses the fairly generic-sounding assumption that profits decrease in the number of competitors. Our analysis shows that this is not necessarily an innocent identifying assumption.

Further, many recent papers that consider structural estimation of entry games rely on a reduced-form relationship between profits and the number of competitors, and analyze the structural game around this reduced-form profit function. For example, Seim (2006) assumes that profits are a linear combination of several factors, plus a linearly-decreasing function in the number of competitors, where the coefficient on the number of competitors varies by the distance between the firm and each competitor. Similarly, Ciliberto and Tamer (2009) model profits declining linearly in the number of competitors in their analysis of the airline entry, although they allow the coefficient on the effect of each airline to differ by airline.

Because the results of the analysis are dependent upon the reasonableness of the reduced form at the heart of the model, it is important to establish whether profits may be able to increase with some competition, especially when analyzing entry into markets with a small number of firms.

6. Endogenous Location Choice

While the preceding analysis demonstrates that profits can increase with entry, a natural question to ask is whether profit-increasing entry is consistent with firms choosing their locations optimally.

Section 5 presents a game where there are 6 or 8 potential locations for firms, firms enter endogenously, and profit-increasing entry occurs with the entry of the 3rd or 4th firm. This demonstrates that firms can choose locations where profit-increasing entry is observed. However, a potential critique of relying on the analysis in Section 5 is that firms are given a very limited set of locations to enter; these limitations were set for two reasons. First, and most importantly, under linear travel costs there is no pure-strategy price equilibrium if two outlets are located too close to each other. In each of the examples in Section 5, the V parameters are chosen such that there is always a pure-strategy price equilibrium, even when firms are located at the minimum distance between the outlets. Second, solving the game of where firms locate is very cumbersome if one allows for many locations and firms in the market.

To show alternative ways that profit-increasing entry can be consistent with optimal location choice, we consider a game with more location choices but fewer firms. Because we consider locations that are potentially very close to each other, we conduct our analysis using quadratic travel costs: i.e., $U_{ij} = V - p_j - d_{ij}^2$. The advantage of using quadratic travel costs is that a pure-strategy price equilibrium always exists. The disadvantage of using quadratic travel costs is that we are no longer able to solve for closed-form expressions for the equilibrium (when the market is not covered). Thus, we solve for the equilibria using computational methods.

We investigate the following entry game. The market consists of a unit circle, with consumers located uniformly around the circle with density 1. In each period, consumers make a discrete choice, consuming from one of the firms in the market in that period. Consumer i gains a utility of $U_{ij} = V - p_j - d_{ij}^2$ if they consume 1 unit of firm j's product. The consumer can alternatively choose to consume only the outside good and receive $U_{i0} = 0$.

There are N periods. In each period, exactly one firm has the opportunity to enter into the market. If the firm enters the market, it chooses a location on the circle from a set of L discrete points, which are located an equal distance apart. We emphasize that while we limit the firm's entry to a set of discrete points for computational purposes, consumers are always located on the full continuum of the circle. Once the firm chooses its location, all firms present in the market pay a fixed cost F and set prices.

Consumers then make their consumption choices, and profits are realized. The firms' objective is to maximize the sum of their profits for the remaining periods in the game. The firm may also choose not to enter, in which case it earns a profit of zero. We further assume that firms can never change their locations once they choose it, and that they cannot exit the market.

We analyze the game with the following parameter values[15]: $N = 4$, $V = 0.14$, $F = 0.02$, $L = 600$. Thus, the game has 4 periods, and the firms can locate at locations at each 1/600 of the circle (i.e., the discrete nature of the location choice is not an important constraint).

The following equilibrium can then be solved through backward induction. In the first period, the first firm locates anywhere on the circle, and earns monopoly profits in that period. In the second period, the second firm locates at a distance of 204/600 away from the first firm. Note that even by locating ½ of the circle away, it would not be possible to pre-empt entry by a third firm. Profits for each firm are approximately 0.0166 after subtracting the fixed costs. The third firm locates half-way in between the two firms (198/600 away from each firm). Each of the two incumbent firms earns approximately 0.0170 net of fixed costs in each period after entry, while the entrant earns 0.0167. Thus, the entry by the third firm increases profits for each of the incumbents. A fourth (or later, if one considered additional periods) firm will choose not to enter the market because its profits will not be high enough to offset the fixed costs.

The intuition for the above results is that when the second firm considers its optimal location, it anticipates the third firm's later entry, and thus chooses its location not just based on the second period profits, but also based on profits in future periods. Specifically, a key element of obtaining the outcome where profits increase from the entry is that the above game be played for 4 or more periods, such that there are at least two periods where the original firms reap profits after the entry of the third firm.[16] Profits for the firms in the second period are higher when the two firms are further apart, but the later periods, where the third firm is present, place a check on how far the second firm chooses to locate from the first. Note, however, that the second firm does not enter exactly 1/3 of the way apart from the first firm, so the final layout of the three firms does not have the firms spaced equidistantly. This is not only because the second firm balances their profits in the second vs. later periods; rather, the second firm's profits in later periods are higher when the firm is located a bit further than 1/3 of the way from the first firm. The reason for this is that as the second firm moves further from the first, the third firm's eventual location also shifts over. The result is that prices and profits for the second firm after all three firms have entered the market are higher when the second firm locates slightly more than 1/3 of the circle away from the first firm compared to when it locates exactly 1/3 away. The third firm's prices decrease as the distance between

[15] When fixing the values of V and F, one can verify our solution analytically.

[16] Adding moderate amounts of discounting between periods or a longer time horizon – even an infinite one – can immediately be shown to give similar results.

the first two firms grows, so the second firm's boost in prices and profits in periods 3 and 4 as it locates further from the first only occurs when the second firm locates just slightly further than 1/3 away from the first firm; after that, profits decline as the proximity to the third entrant becomes the dominant competitive threat.

Given the importance that having more than one period where all 3 firms are present has on the equilibrium outcome, one may ask whether designing such a game to have more periods than the number of entering firms is a reasonable assumption. Recent empirical work on entry supports the idea that a typical pattern for entry is that there is a relatively short period of time where significant entry occurs, followed by a long period of time where market shares remain very stable. For example, Bronnenberg, Dhar and Dubé (2009) find that market shares for a wide range of consumer packaged goods (CPG) categories remain stable for decades if not longer, and that market shares in different U.S. cities today are strongly correlated with which brand entered that city's market first, even when the entry occurred over 100 years prior. Thus, using a long-term game is the right way to model competition for many industries.

7. Profit-Increasing Consumer Exit

The mechanism behind profit-increasing entry, as demonstrated in Section 3, is that entry can lead to increased profit by increasing competitors' prices, and by acting as a commitment device by the firms to charge higher prices. The change occurs because with the entry of a competitor at an appropriate distance away, adjacent firms no longer have an incentive to charge low prices in order to attract distant consumers. This logic suggests that another way that firm profits can increase is if the marginal consumers exit the market completely. In this section, we show that this is indeed the case.

There are many combinations of customer exit that can cause profits to increase. Rather than focus on all of the combinations, we consider a market with two firms competing on a long line,[17] and consider two cases of consumer exit: consumers exiting from the edge of the realized market,[18] and consumers exiting from a section of the market between the two firms.

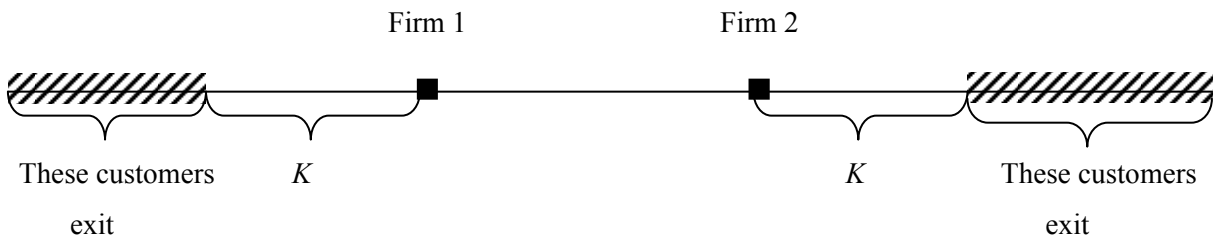
We first consider what happens to firm profits if we remove all consumers located a distance K or further from the firms on the outer edge of the market. Figure 8 illustrates such a situation. We make two comparisons: What is the range of K where profits after consumers exit the market are higher than the profits before the customers exit, and what is the range of K where incrementally more exit (i.e.,

[17] One could equivalently consider an uncovered circle.

[18] While we present a theorem where consumers exit from both edges of the market, one can find conditions under which profits for both firms increase when consumers exit from just one edge of the market. In such a case, we obtain a result analogous to the firm entry case in Section 3: A firm that does not have any of its customers exit benefits from a direct effect of having its competitor raise its price, while the firm whose customers exit benefits through the indirect effect of the exit serving as a commitment device to charge higher prices, which in turn increases its competitor's price.

incrementally smaller K) leads to greater profit. The first question is a before-after comparison, while the second question looks at the derivative of profit with respect to K.

Figure 8: Removing consumers near the edge and K or further from firms.

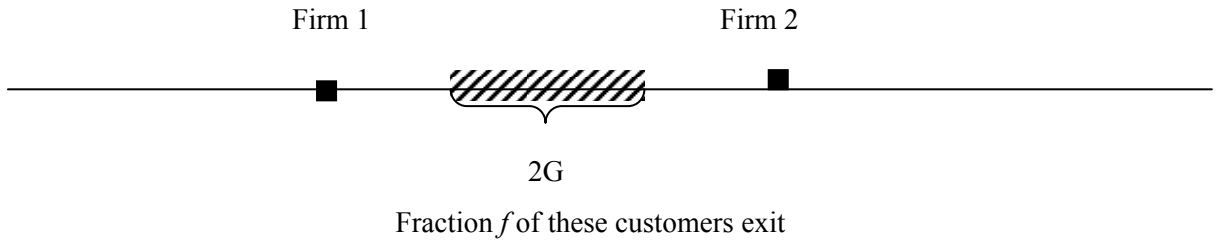


Theorem 5: Consider a linear market where consumers with preferences given in equation (1) are distributed uniformly across the market, and that there are two firms located a distance $2D$ apart from each other engaged in differentiated Bertrand competition such that there are un-served customers at each end of the market. Further, suppose that $\frac{7}{3}D \le V \le \left[\frac{7+5\sqrt{10}}{3}\right]D$. Then, (i) profits increase for both firms if all customers located at a distance greater than K from one firm, and greater than K+$2D$ from the other firm exit the market for any $K \in \left(\max\left(\frac{2V-3D}{5}, V-4D\right), \frac{3V-2D}{5}\right)$. Further, (ii) $\frac{d\pi}{dK} < 0$ after exit (that is, profits are increasing as more consumers exit the market) for $K \in \left(\frac{V-D}{2}, \frac{3V-2D}{5}\right)$.

The proof of this theorem appears in the web appendix. The upper bound in each case, $\frac{3V-2D}{5}$, reflects the marginal consumer for each firm before any consumers exit the market. Thus, profits increase as a small number of furthest previously-served consumers exit the market. The lower bound on K reflects that there is a limit on how many consumers can exit and still make the firms better off.

While the results in Theorem 5 are directly analogous to the results in the previous sections, as Theorem 6 below demonstrates, it is also possible that exit by consumers between the firms can lead to increased profits. In Theorem 6, we consider what happens when a fraction f of consumers located in an interval 2G exactly in between two competing firms exit the market. An example of such a market is shown in Figure 9 below. We show that in some cases, profits can increase. Despite the fact that the consumer exit is in the center of the market, the intuition is consistent with the results we had for competitive entry: the exit of consumers in the middle of the two firms does take away some demand, but also decreases the sensitivity of demand with respect to price, leading to higher prices, and ultimately higher profits for both firms.

Figure 9: Remove some customers from the middle of the market



Theorem 6: Consider a linear market where consumers with preferences given in equation (1) are distributed uniformly across the market, and that there are two firms located a distance 2*D* apart from each other engaged in differentiated Bertrand competition such that there are un-served customers at each end of the market. Further, suppose that $\frac{7}{3}D \le V \le \left[\frac{7+5\sqrt{10}}{3}\right]D$. Then profits increase for both firms if a fraction $0<f<1$ of consumers located in an interval of size $2G<2D$ exactly between the firms exit the market and

(i) $$\frac{(7-f)D-2fG}{3-f} \le V \le fG + (3-f)D + \frac{D\left((5-f)\sqrt{f^2-6f+10}-2\right)}{3-f}$$

(ii) $$\frac{5G\left(15-5f+(5-f)\sqrt{3(3-f)}\right)}{5-3f} \le V + D \le \frac{G\left(30+2f^2-11f+2(5-f)\sqrt{3(3-f)}\right)}{f}$$

The proof is in the web appendix.[19]

As a final note, some readers may at first think that there is a relationship between our finding that consumer exit leads to increased profits – even when no firms exit the market – and the results in the CRM literature that some customers may be so costly to serve that the firm is better off refusing to sell to them. However, there is a difference in these findings in that in our results, the benefit of consumer exit comes completely from the strategic competitive interaction between firms. By revealed preference, we know that a monopolist would never be better off if consumers exited the market, since the monopolist always has the option of increasing price and serving a narrower segment. The reason that consumer exit increases profits is because either the exit raises a rival's price by directly changing that rival's incentives, or the exit acts as a commitment device for a firm to charge a higher price, which in turn causes the rival to increase their price (or both).

[19] Note that condition (ii) requires that $f < \frac{11}{12}$, so there must always be some customers in the gap between the two firms.

8. The Spokes Model of Chen and Riordan.

Chen and Riordan (2007)'s analysis of the spokes model also finds that entry can increase profits. While they only discuss profit-increasing entry briefly, we provide a deeper understanding of the phenomenon by decomposing it into two underlying forces, demonstrating its robustness to model specifications, and extending the results to endogenous location choices and consumer exit. We also find differences in the relationship between profits and the number of competitors in the market. We now briefly discuss the spokes model, and compare our findings with theirs.

The spokes model is named as such because the market is envisioned as consisting of a fixed series of line segments, each emanating from a central hub. Firms are located at the non-hub end of the spoke, and consumers experience travel costs across the system of line segments. Consumers are exogenously constrained to consider only two products: the product at the end of their spoke, and the product at the end of one of the other spokes, where the other spoke is chosen randomly.[20]

While the constraint that consumers only consider products on two randomly selected spokes may seem arbitrary, the model is dual to a model where the market consists of the edges of an N-dimensional simplex, where N is the number of spokes. Each edge has length 1, and each of the $n \leq N$ firms operating in the market is located at one of the corners of the pyramid. Note that because there is a line between every pair of firms, all locations are equivalent. Thus, the question of endogenous product location, as explored in Section 6, cannot be studied in the spokes model.

The spokes model was not originally intended to study profit-increasing entry, so it makes some unrealistic predictions. In particular, Chen and Riordan find that for a set of parameters, prices increase with each entry, and profits at first decrease, but then always increase with additional entrants. In fact, the size of the increase in prices and profits grows with each additional entry. One reason for this result is that each new firm always fills a hole in the market, so firms do not eventually crowd out their competitors. Further, the fact that a firm competes directly with every other firm in the market means that the benefit of having all firms increase their profits is larger when there are many competitors in the market compared to when there are fewer competitors in the market. In contrast, in our context of local competition, one will not get this multiplicative effect because a firm cares about its neighbor's prices, but not (directly) the price of any other firms. Chen and Riordan's finding that prices increase with any amount of entry, and that profits uniformly increase with the number of competitors once there are enough firms in the market is at odds with what is empirically observed in some industries. In Section 9,

[20] Note that if there is no product at the end of the customer's spoke, or if the alternative spoke has no firm, then the customer only considers making a purchase from one firm, and if there is no firm at either the end of the customer's spoke or their other randomly selected spoke then the consumer makes no purchase.

we discuss empirical evidence that profits have increased in certain industries with new entry. However, one would expect that once enough firms enter into a market that prices and profits would eventually decline, and in Section 5 we show that this must be true once there is enough competition in markets with fixed sizes.

While the above discussion notes some differences between the spokes model and our analysis, which is based on local instead of global competition, there is a special case where the spokes model reduces to a model of local competition. When $N = 3$, and $V < 2$, the spokes model is dual to the special case of the circular market we show in the example at the beginning of Section 4, where the only 3 locations a firm can choose is one of the 3 locations spaced 1/3 of the circle apart, and competition plays out as we lay out in the text of that example.

9. Conclusion

This paper demonstrate that profits for incumbent firms can increase after entry in both linear and circular markets through two effects: The entry by the new competitor can increase the incumbents' profits by directly incentivizing competing outlets to raise their prices, as well as by acting as a commitment device for each firm to keep prices high in equilibrium; Profits increase as long the new entrant is located such that it does not take away too many customers from the incumbents. Because it is the effect of entry on the strategic best-response functions that leads to increased profits, profits will only increase from entry if there are initially two or more firms in the market.

We also demonstrate that profit-increasing entry can be consistent with endogenous location choices by firms, and note the rich complexity of the relationship between profits and the number of firms competing in the market. We further note that a mechanism similar to the one that supports profit-increasing entry can lead to a similar phenomenon where customer exit can increase all firms' profits, even if the customer exit is not accompanied by any firm exit.

While we assume throughout the paper that firms have constant marginal costs, the result of profit-increasing entry is robust to loosening this assumption. For example, the results are robust to models with increasing marginal costs, such as models where total costs are q_j^2.[21] We also demonstrate that the result of profit increasing entry can occur under either quadratic or linear travel costs. Thus, we show that profit increasing entry is robust to many of the aspects in the model.

[21] As an example, consider a circular market where firms are located a distance of 1/3 away from the other firms, total costs are q^2, and $V = 7/6$. Then profits for the two incumbents before entry will be 80/363, while profits after entry will be 2/9, which are higher. As in the example in Section 4, the equilibrium prices both satisfy the first-order conditions and occur at kink-points. Thus, if V is slightly higher, profits will increase even while all firms price according to first-order conditions both before and after entry. If V is slightly lower, all firms will price at kink-points after entry.

While the results we have are theoretical in nature, there is some empirical evidence that firms can be better off after entry. Pauwels and Srinivasan (2004) find that Dove, Lever 2000 and Dial soap all experienced increased profits from store-brand entry.[22] Similarly, Geroski (1989) finds that entry increases average incumbent firm's profit margins in 6% of industries. Further, Qian (2008) shows that profit margins increased for authentic shoes after the entry of counterfeiters in China, and that, on average, profits for the authentic shoes were higher two years after counterfeits were discovered for the brand. We also note that some researchers may observe markets where the presence of additional competitors leads to increased profits but ascribe the increased profits to other phenomena. For example, Vitorino (2010) finds that profits of midscale and upscale department stores both increase with the presence of more midscale stores. However, Vitorino ascribes this finding to complementarities in the market. We speculate that many of the papers that find positive relationships between the number of firms and profits search for alternative explanations for their results due to the fact that the conventional wisdom that standard competition models dictate that entry must cause profits to fall is so strong.

Our results also have clear implications for empirical researchers seeking to measure the relationship between profits and the number of firms in a market. Notwithstanding endogeneity concerns that often arise in measuring these types of relationships, our results suggest that researchers need to think carefully about the functional form relating the number of competitors to profits. In particular, there is a concern not only about the plausibility that the linear (or linear in logs) functional form is misleading, but our results suggest that researchers need to use functional forms that can account for the possibly highly non-monotonic (and non-U-shaped) relationship between the number of firms and profits. This functional form issue is not merely an issue for reduced form estimation; Rather, much of the literature on entry and entry games that is considered "structural" places the structure of a model on the game itself, but uses a reduced-form profit function. Because the results of the analysis are dependent upon the reasonableness of the reduced form at the heart of the model, it is important to establish whether profits may be able to increase with some competition, especially when analyzing entry into markets with a small number of firms – the precise situation that is studied by many of these papers.

[22] Pauwels et. al. do not directly measure profits, but the paper provides price and quantity information. We assume that higher revenues and lower unit sales (which should mean lower total costs), or higher prices and higher quantity together lead to higher profits.

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Appendix 1: Proof of Theorem 1

Proof: Before entry, *C* can (1) price along a linear segment of the demand curve where the marginal consumers are considering the outside good, (2) price at a kink-point, or (3) price along a linear segment of the demand curve with some of the marginal consumers substituting between *C*'s product and *I*'s product. If *C* is pricing on the segment corresponding to case (1), the firm must be charging the monopoly price and earning monopoly profits, in which case profits can only decrease with entry, as explained in the introduction. If before entry the firm is pricing at a kink-point (case 2), then either entry causes *C*'s prices to increase, in which case *C*'s profits decrease because before entry *C* had the option to increase its price and to sell to all consumers who would earn a positive utility at this higher price, but revealed preference demonstrates that the profits must be lower. Alternatively, entry could cause *C*'s prices to decrease, in which case firm *I* will be worse off, since it makes lower profits for any price it could set relative to profits at the same price before entry. (In the case where *C*'s prices are unchanged, *C* must sell weakly less and earn weakly lower profits.)

Therefore, any equilibrium where *C* and *I*'s profits both increase must involve case 3: *C* charging a price before entry corresponding to a linear segment of the demand curve where some of the marginal consumers substitute between *C* and *I*. The corresponding first order condition for *C* before entry is $q_C + p_C \frac{\partial q_C}{\partial p_C} = 0$, where $\frac{\partial q_C}{\partial p_C} = -1.5$. Thus, $p_{C2} = \frac{2}{3} q_{C2}$, where the 2 in the subscript denotes the total number of firms in the market. Profits are then $\pi_{C2} = \frac{2}{3} q_{C2}^2$.

When there are 3 firms in the market, either the third firm is located far enough apart that *E*'s realized market does not overlap with *C*'s market, *C* prices at a kink-point such that the marginal consumer for *C* in the direction of *E* gets zero utility, or $\frac{\partial q_C}{\partial p_C} = -1$. (Note that it is impossible that the new equilibrium will involve *C* and *I* serving separated market areas, or even having the marginal consumer between *C* and *I* obtaining zero surplus. Both of these outcomes would require firm *C* raising its price. However, the first-order condition for *C* can be written as $p_C = -\, q_c \Big/ \frac{\partial q_C}{\partial p_C}$. If *C* and *I* serve different markets but *C* is not at a kink-point where the marginal consumer between *E* and *C* gets zero utility, $\frac{\partial q_C}{\partial p_C} = -1.5$. However, at any price, q_c is lower, so the first-order condition would be satisfied at a lower price than p_{C2}, which is a contradiction.)

We now rule out that profits will increase if either *C* and *E*'s markets do not touch, or if

$\frac{\partial q_C}{\partial p_C} = -1$. In the first case, there is no overlap in the market areas, so *C* and *I*'s profits remain unchanged. Suppose instead that *C* prices according to its first-order conditions, where $\frac{\partial q_C}{\partial p_C} = -1$.The first-order conditions must satisfy $p_{C3} = q_{C3}$ and $\pi_{C3} = q_{C3}^2$. Note that $\pi_3 > \pi_2$ iff $q_{C3}^2 > \frac{2}{3} q_{C2}^2$ or $q_3 > \sqrt{2/3}\, q_2$. However, it must be that $q_{C2} - q_{C3} \geq \frac{17}{12}\left(p_{C3} - p_{C2}\right)$. The $\frac{17}{12} = 1 + \frac{1}{2} - \frac{1}{6} \cdot \frac{1}{2}$ term represents the maximum change in sales from the change in price $\left(p_{C3} - p_{C2}\right)$, and can be parsed as follows: on the side of the outlet where the new entrant enters, the quantity sold cannot be more than the firm would have sold if it had an uncontested region on that side; The rate of change in quantity on that side if it were uncontested is $\frac{\partial q_C}{\partial p_C} = -1$. On the side where the other incumbent firm is located, $\frac{\partial q_C}{\partial p_C} = -1/2$, but the best response of the other firm is to increase its price by $1/6$ times the change in its opponent's price, so the firm regains $\frac{1}{6} \cdot \frac{1}{2}$ consumers. In sum, the total effect is $\frac{dq_C}{dp_C} \leq -\frac{17}{12}$, with the even steeper decline occurring when the new firm steals even more customers from the incumbent outlet. This means that $q_{C3} \leq q_{C2} - \frac{17}{12}\left(p_{C3} - p_{C2}\right) \leq q_{C2} - \frac{17}{12}\left(q_{C3} - 2/3\, q_{C2}\right) \leq q_{C2} - \frac{17}{12}\left(\sqrt{2/3} - 2/3\right) q_{C2} < \sqrt{2/3}\, q_2$, which contradicts $q_3 \geq \sqrt{2/3}\, q_2$. □

<u>Appendix 2: Proof of Theorem 2</u>

When only *C* and *I* are present in the market, each firm's (indexed by *j*) profits are $p_j\left(V - p_j + D + \frac{p_{-j} - p_j}{2}\right)$, which yield first-order conditions: $p_j = \frac{V}{3} + \frac{D}{3} + \frac{p_{-j}}{6}$. The solution to this is that equilibrium prices are $\frac{2}{5}\left(V + D\right)$ and profits are $\frac{6}{25}\left(V + D\right)^2$. These calculations assume that the realized market for the two firms touch: this occurs when $V - p_1 + V - p_2 \geq 2D$, which becomes $\frac{7}{3} D \leq V$ after substitution and rearranging. We must also ensure that neither firm will undercut the other firm's market and steal all of their customers. A firm undercutting their competitor must charge a price of $p_{uc} = \frac{2}{5}\left(V + D\right) - 2D = \frac{2}{5} V - \frac{8}{5} D$. The undercutting firm's profits are then $2 p_{uc}\left(V - p_{uc}\right)$. The deviation

will not be profitable when $\frac{6}{25}(V+D)^2 \geq 2p_{uc}(V-p_{uc})$ which occurs when $V \leq \frac{1}{3}\left(7+5\sqrt{10}\right)D$.

After entry we need to find an equilibrium where the profit of the incumbent firms increases. It is easy to verify that the first order conditions (Best responses) for prices are given by:

$$p_I = \tfrac{1}{3}V + \tfrac{1}{3}D + \tfrac{1}{6}p_C \;;\; p_C = \tfrac{1}{4}p_I + \tfrac{1}{4}p_E + \tfrac{1}{2}S + \tfrac{1}{2}D \;;\; p_E = \tfrac{1}{3}V + \tfrac{1}{3}S + \tfrac{1}{6}p_C \qquad \text{(A1)}$$

However, we know from Theorem 2 that *C* cannot price according to the FOC condition. Hence we look for an equilibrium where firm *I* prices according to its FOC while *C* and *E* price at a kink point. Let p_C be the price charged by firm C in the new equilibrium. Then $p_E = 2V - 2S - p_C$ and $p_I = \frac{V+D}{3} + \frac{p_C}{6}$.

We must make sure that there is no separation between the set of customers who consume *C* and those who consume *E* when $p_E = ½V$ (the monopoly price). This is equivalent to the condition that

$$p_E = 2V - 2S - p_C \geq \tfrac{1}{2}V \,. \qquad \text{(A2)}$$

Furthermore $p_E = 2V - 2S - p_C$ must be below the price suggested by the FOC condition (A1). We know this because if the first-order conditions hold at a lower price than $2V - 2S - p_C$ then it must be that profits are increasing as prices decrease due to the standard optimization arguments that generally supports pricing at first-order conditions. On the other hand, if the price that satisfies condition (A1) is greater than $2V - 2S - p_C$ then equation (A1) cannot describe the optimal price because equation (A1) assumes that the realized markets for each product touch, while these markets do not touch when prices are above $2V - 2S - p_C$. Combining condition (A2) with $2V - 2S - p_C \leq \tfrac{1}{3}V + \tfrac{1}{3}S + \tfrac{1}{6}p_C$ gives us the constraint:

$$\frac{5}{7}V - \frac{p_C}{2} \leq S \leq \frac{3}{4}V - \frac{p_C}{2} \,. \qquad \text{(A3)}$$

Following the same logic as explained from *E*'s pricing, in order for firm *C* to choose a kink-point p_c as its price, this price must be lower than the price dictated by the FOC in (A1). Substituting $p_I = \tfrac{1}{3}V + \tfrac{1}{3}D + \tfrac{1}{6}p_C$ and $p_E = 2V - 2S - p_C$ into $p_C = \tfrac{1}{4}p_I + \tfrac{1}{4}p_E + \tfrac{1}{2}S + \tfrac{1}{2}D$ yields

$$p_C \leq \tfrac{14}{29}(V+D) \,. \qquad \text{(A4)}$$

Further, *C*'s profits, $\frac{1}{12}p_C\left(14V + 14D - 17p_c\right)$, must be higher than its profits before entry. This leads to

$$\frac{2}{5}(V+D) \leq p_C \leq \frac{36}{85}(V+D) \,. \qquad \text{(A5)}$$

(Note that the lower bound condition guarantees that the price charged by *C* is higher than the price before entry). Condition (A4) is not binding as condition (A5) is more restrictive. Clearly firm *I* will increase its profits and firm *E* will make positive profits in such an equilibrium. We need to check that the

markets between I and C touch: $2V - p_I - p_C \geq 2D$ or

$$p_C \leq \frac{10}{7}V - 2D. \tag{A6}$$

Combining conditions (A3), (A5) and (A6) yields: $\max\left[\frac{299}{595}V - \frac{18}{85}D, D\right] \leq S \leq \frac{11}{20}V - \frac{1}{5}D$.

Finally, we check that in equilibrium no firm will want to undercut its competitors:

<u>E will not undercut C :</u> $p_E^{dev} = p_C - 2S \leq 2p_C - 10\text{V}/7 \leq \frac{72}{85}V + \frac{72}{85}D - \frac{10}{7}V \leq$

$\frac{72}{85}V + \frac{72}{85}\cdot\frac{3}{7}V - \frac{10}{7}V = -\frac{26}{119}V < 0$

Where the first inequality is due to (A3) the second due to (A5) and the third is due to the range for D.

<u>C will not undercut E</u>: $p_C^{dev} = 2V - 2S - p_C - 2S$

$\leq 2V - p_C - \frac{20}{7}V + 2p_C \leq \frac{-6}{7}V + \frac{36}{85}V + \frac{36}{85}D \leq \frac{-6}{7}V + \frac{36}{85}V + \frac{36}{85}\frac{3}{7}V = -\frac{30}{119}V < 0.$

Where the first inequality is due to (A3) the second due to (A5) and the third is due to the range for D.

<u>C will not undercut I</u>: $p_C^{dev} = \frac{V+D}{3} + \frac{p_C}{6} - 2D$. The condition for lower profits at the new lower price is

$$p_c > \frac{14}{31}V - \frac{6}{31}D - \frac{4}{31}\sqrt{-11V^2 + 67VD + 196D^2} \equiv LB \text{ and}$$

$$p_c < \frac{14}{31}V - \frac{6}{31}D + \frac{4}{31}\sqrt{-11V^2 + 67VD + 196D^2} \equiv UB. \tag{A7}$$

It is easy to check that $\frac{2}{5}(V+D) \geq LB$ and $UB \geq \frac{36}{85}(V+D)$ so constraint (A7) is not binding and is subsumed by (A5)

<u>I will not undercut C</u>: $p_I^{dev} = p_C - 2D$. The condition for lower profits at the new lower price are

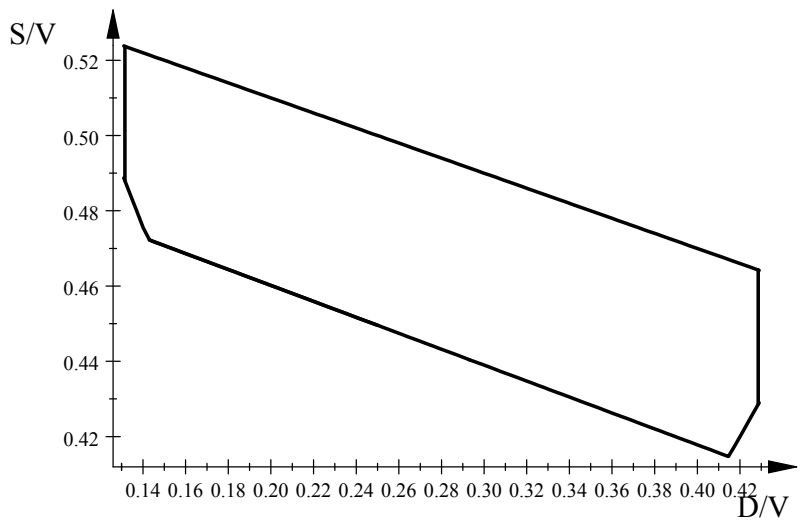
$$p_c < \frac{22}{49}V + \frac{94}{49}D - \frac{4\sqrt{6}}{49}\sqrt{(3V+2D)(V-4D)} \text{ or } p_c > \frac{22}{49}V + \frac{94}{49}D + \frac{4\sqrt{6}}{49}\sqrt{(3V+2D)(V-4D)}. \tag{A8}$$

Clearly when $D \geq \frac{1}{4}V$ the conditions hold. However when $D \leq \frac{1}{4}V$ the condition may bind.

In summery the following is the complete characterization of the parameter space allowing for a pure strategy equilibrium with higher profits for the incumbents: $D \in \left(\frac{3}{7+5\sqrt{10}}V, \frac{3}{7}V\right)$ and

$$\text{Max}\left[\frac{24}{49}V - \frac{47}{49}D + \frac{2\sqrt{6}}{49}\sqrt{(3V+2D)(V-4D)}, \frac{299}{595}V - \frac{18}{85}D, D\right] \leq S \leq \frac{11}{20}V - \frac{1}{5}D.$$

The figure below presents the area where profits increase. The sufficient conditions on the parameters specified in the theorem are shown by the gray area in the figure, and are clearly a subset of the complete characterization presented here.



Appendix 3: Proof of Proposition 1

If there is a segment of customers on the long side of the market that do not buy the product, then each firm's profits can be written as $p_j\left(V - p_j + D + \frac{p_{-j} - p_j}{2}\right)$, which yields the first-order conditions $p_j = \frac{V}{3} + \frac{D}{3} + \frac{p_{-j}}{6}$. The solution to this is that equilibrium prices are $\frac{2}{5}(V + D)$, and equilibrium profits are $\frac{6}{25}(V + D)^2$. The market is not covered if and only if $V - p_1 + V - p_2 \le 1 - 2D$, which is equivalent to (a) $V \le \frac{5}{6} - D$ after substituting $p = \frac{2}{5}(V + D)$ and rearranging. Similarly, the realized market for the two firms to touch and provide the marginal consumer between the two firms positive utility if and only if $V - p_1 + V - p_2 > 2D$, which becomes (b) $\frac{7}{3}D < V$ after substitution and rearranging.

We must also ensure that neither firm will undercut the other firm's market and steal all of its customers. A firm undercutting their competitor must charge a price $p_{uc} = \frac{2}{5}(V + D) - 2D = \frac{2}{5}V - \frac{8}{5}D$. When $V - p_{uc} \le \frac{1}{2}$ (or equivalently $V \le \frac{5}{6} - \frac{8}{3}D$) the undercutting firm does not get the entire market; its profit is given by $2p_{uc}(V - p_{uc})$. The deviation will not be profitable when $\frac{6}{25}(V + D)^2 \ge 2p_{uc}(V - p_{uc})$ which occurs when (c) $V \le \frac{1}{3}\left(7 + 5\sqrt{10}\right)D$. On the other hand, when $V > \frac{5}{6} - \frac{8}{3}D$ the undercutting firm gets the entire market and its profit is just $1 \bullet p_{uc}$; this deviation is not profitable when $\frac{6}{25}(V + D)^2 \ge p_{uc}$.

This condition is always satisfied when $D > \frac{1}{12}$ or when $\frac{1}{2}\left(\sqrt{10}-3\right) \le D \le \frac{1}{12}$ and $V \le \frac{5}{6} - \frac{5}{6}\sqrt{1-12D} - D$. Combining all the conditions finishes the proof.

Finally, note that there are only two other potential pure-strategy price equilibria that could occur: firms could price along a kink-point, or they could compete in separate *de facto* monopoly markets (and thus have different demand). There are no marginal customers that get positive utility in either case.

<u>Appendix 4: Proof of Theorem 3:</u>

The new firm will price as a monopolist as long as $2\left(\frac{V}{2}\right) + 2\left[\frac{3}{5}\left(V+D\right)\right] \le 1 \rightarrow V \le \frac{5}{11} - \frac{6}{11}D$. In such a case, the market for the entrant will not affect the market of the two incumbents.

Next, consider conditions under which the entrant prices at a kink-point such that its marginal customers earn zero surplus. In such a condition, the incumbent firms must also be pricing at a kink-point on their demand curves. We can represent the prices as $p_I = V - D - \delta$ and $p_E = V - ½ + 2D + \delta$, where δ is non-negative. For this kink-point to be an equilibrium, it must be that $p_E > V/2$, the monopoly price. This constraint implies that

$$\delta \ge ½ - V/2 - 2D. \tag{A9}$$

Also, it must be that p_E less than the price that would be dictated by the first-order conditions from the lower linear part of the demand curve (where there is substitution between all of the firms). This first-order condition is $p^{EFOC} = \frac{1}{4} - \frac{D}{2} + \frac{P_I}{2} = \frac{1}{4} - D + \frac{V}{2} - \frac{\delta}{2}$. So $p_E \le p^{EFOC}$ when

$$\delta \le \frac{1}{2} - \frac{V}{3} - 2D. \tag{A10}$$

For the incumbent, it must be that p_I is less than the price that would be dictated by the first-order conditions from the lowest linear segment of the demand curve, where there is substitution between all 3 firms. This condition is equivalent to $p_I < p^{IFOCALL} = \frac{D}{4} + \frac{1}{8} + \frac{p_I + p_E}{4} = \frac{V}{2} + \frac{D}{2}$. This is satisfied when

$$\delta \ge \frac{V}{2} - \frac{3D}{2}. \tag{A11}$$

We also need p_I to be greater than the price from the linear part of the demand curve above the kink, which represents a part of the demand curve where there is substitution between the incumbents on one side of the outlets and substitution between the incumbents and the outside good on the other side. This condition turns out to be equivalent to $\frac{2}{5}\left(V+D\right) < V - D - \delta$, which is equivalent to

$$\delta \le \frac{3}{5}V - \frac{7}{5}D. \tag{A12}$$

Note that (A12) ≥ (A9) → $V \ge \frac{5}{11} - \frac{6}{11}D$.

Further, note that (A10) > (A11) → $V \le \frac{3}{5} - \frac{3}{5}D$. (A13)

Also, $\frac{3}{5}V - \frac{7}{5}D \ge 0 \to V \ge \frac{7}{3}D$, while $\frac{1}{2} - \frac{V}{3} - 2D \ge 0 \to V \le \frac{3}{2} - 6D$. (A14)

We must also ensure that none of the firms ever undercuts the other firms. The entrant would have to charge $V - ½ - \delta$ in order to undercut the incumbents. However, substituting (A14) makes this negative when $D > 1/6$, while instead substituting (A11) and (A13) makes this negative when $D < 1/6$. Similarly, the incumbent will never undercut the entrant, since doing so would require pricing at $V - 1 + 3D + \delta$, which substituting (A10) and (A14) for $D > 1/6$ or (A10) and (A13) for $D < 1/6$ show to be negative. The final case of undercutting – incumbents undercutting incumbents – can be binding. An incumbent can undercut the other incumbent by charging $V - 3D - \delta$. Profits for the undercutting incumbent are then $(V - 3D - \delta)(5D+2\delta) = \pi_I + 3DV + V\delta - 13D^2 - 8D\delta - \delta^2$, where π_I is the profits that the firm would earn under the proposed equilibrium. Thus, we need to establish when $3DV + V\delta - 13D^2 - 8D\delta - \delta^2$ is negative. The derivative with respect to δ is always negative (because $V \le \frac{7+5\sqrt{10}}{3}D$), so we get the maximal set of (V,D) by substituting in the upper bound on δ. If $V > \frac{15}{28} - \frac{9}{14}D$ the upper bound of δ is determined by (A10), and $3DV + V\delta - 13D^2 - 8D\delta - \delta^2 =$ $-\frac{4}{9}V^2 + \frac{7}{3}VD + \frac{5}{6}V - D^2 - 2D - \frac{1}{4}$. This is negative whenever $V < \frac{15}{16} + \frac{21}{8}D - \frac{9}{8}\sqrt{\frac{11}{3}D^2 + \frac{1}{3}D + \frac{1}{4}}$.[23] If $V < \frac{15}{28} - \frac{9}{14}D$, the upper bound of δ is determined by (A12), and $3DV + V\delta - 13D^2 - 8D\delta - \delta^2 =$ $\frac{6}{25}V^2 - \frac{38}{25}VD - \frac{94}{25}D^2$, which is negative whenever $V < \frac{19+5\sqrt{37}}{6}D$, which not binding.

Note that any of these equilibria where the entrant prices at a kink-point must involve higher profits for the incumbents than they received in the 2-firm equilibrium. We can see this because incumbent profits will be $\pi_I = (V - D - \delta)(2D + \delta) > \left(V - D - \frac{3}{5}V + \frac{7}{5}D\right)\left(2D + \frac{3}{5}V - \frac{7}{5}D\right)$

[23] The other alternative is not compatible with (A14).

$= \frac{6}{25}(V+D)^2$, the profits when there were two firms in the market. (The inequality uses equation A12.)

We next consider the conditions under which all 3 firms price along the linear demand curve such that the marginal consumer between any of the two firms obtains a positive surplus. In such a case, the entrant's first-order condition is $p_E = \frac{1}{4} - \frac{D}{2} + \frac{p_{I1}+p_{I2}}{4} = \frac{1}{4} - \frac{D}{2} + \frac{p_I}{2}$, with the second equality from symmetry. Incumbent's $I1$'s profits are $p_{I1} \cdot \left(\frac{1}{4} - \frac{D}{2} + \frac{p_E - p_{I1}}{2} + D + \frac{p_{I2}-p_{I1}}{2}\right)$, which yield a first-order condition of $p_{I1} = \frac{1}{8} + \frac{D}{4} + \frac{p_E + p_{I2}}{4} = \frac{1}{6} + \frac{D}{3} + \frac{p_E}{3}$, with last equality again coming from symmetry. The first-order conditions are solved when $p_I = \frac{3}{10} + \frac{D}{5}$ and $p_E = \frac{2}{5} - \frac{2}{5}D$, which leads to profits of $\pi_I = \left(\frac{3}{10} + \frac{D}{5}\right)^2$ and $\pi_E = \frac{4}{25}(1-D)^2$. These calculations assume that the market is covered, which it will be as long as $2V - \left(\frac{D}{5} + \frac{3}{10}\right) - \left(\frac{2}{5} - \frac{2}{5}D\right) \geq \frac{1}{2} - D \rightarrow V \geq \frac{3}{5} - \frac{3}{5}D$ and $V - \frac{D}{5} - \frac{3}{10} \geq D \rightarrow$ $V \geq \frac{3}{10} + \frac{6}{5}D$ (or equivalently, $D \leq \frac{5}{6}V - \frac{1}{4}$).

The equilibrium must also be robust to undercutting. Consider the entrant undercutting the incumbents and obtaining the whole market, in which case the entrant would earn $\frac{6}{5}D - \frac{1}{5}$. This is not profitable (lower than π_E) whenever $D \leq \frac{19-5\sqrt{13}}{4}$. However, $\frac{6}{5}D + \frac{3}{10} \leq V \leq \frac{5}{6} - D \rightarrow D \leq \frac{8}{33}$, so this constraint does not bind. In the relevant parameter space the entrant will never want to undercut. We now turn to an incumbent firm; it can undercut the other incumbent, the entrant or both. To undercut the other incumbent, the firm must price $p_{ucI} = \frac{D}{5} + \frac{3}{10} - 2D = \frac{3}{10} - \frac{9}{5}D$, while undercutting the entrant requires pricing at $p_{ucE} = \frac{2}{5} - \frac{2}{5}D - \left(\frac{1}{2} - D\right) = -\frac{1}{10} + \frac{3}{5}D$. $p_{ucE} < p_{ucI} \leftrightarrow D < \frac{1}{6}$. So under this condition the incumbent pricing at p_{ucE} gets the entire market and a profit of $-\frac{1}{10} + \frac{3}{5}D$. However for $D < \frac{1}{6}$ the profit is negative and the deviation non profitable. When the incumbent prices at p_{ucI} it gets the entire market share of the other incumbent but only some of the entrant's customers for a total market share of $\left(\frac{7}{5}D + \frac{3}{5}\right)$ and a profit of $\left(\frac{3}{10} - \frac{9}{5}D\right)\left(2\left(\frac{D}{5} + \frac{3}{10}\right) + D\right) = \left(\frac{3}{10} - \frac{9}{5}D\right)\left(\frac{7}{5}D + \frac{3}{5}\right)$. This is less than

$\pi_I = \left(\frac{D}{5}+\frac{3}{10}\right)^2$ when $D \geq \frac{15\sqrt{17}-39}{256}$, which is a binding constraint. If $p_{ucE} > p_{ucI}$ ($D > \frac{1}{6}$), an incumbent can undercut the entrant and earn $\pi_{dev} \leq \frac{3}{5}D - \frac{1}{10}$, which is less than π_I as long as $D \leq 6 - \frac{5\sqrt{5}}{2}$. Because $D \leq$ ¼ (by definition), this deviation is unprofitable. Also, when $D > 1/6$, then undercutting the other incumbent requires a negative price, and is therefore unprofitable.

Finally, consider the case where the incumbents price at kink-points, but where the entrant prices along the linear part of its demand curve. In this case, $p_I = V - D$, and $p_E = \frac{1}{4} - \frac{D}{2} + \frac{p_I}{2} = \frac{1}{4} - D + \frac{V}{2}$. We need the entrant's price to be below the kink-point (or equivalently, for the market to be covered between the entrant and the incumbents): $p_E = \frac{1}{4} - D + \frac{V}{2} \leq 2V - \frac{1}{2} + D - (V - D) \rightarrow V \geq \frac{3}{2} - 6D$. Also, it is necessary that the competitive first-order condition for the incumbent along the lowest part of the demand curve where consumers are substituting between all 3 firms give a higher price than the incumbent charges: $p = \frac{1}{2}\left(D + \frac{1}{4} - \frac{D}{2} + \frac{p_I + p_E}{2}\right) = \frac{1}{16}(3 - 4D + 6V) \geq V - D$ when

$$V \leq \frac{6}{5}D + \frac{3}{10}. \tag{A15}$$

It is also required that the incumbent's price be below the price that would result from the first-order condition for the part of the demand curve where the firm is competing only with the entrant (but with a gap in the realized customers between the two incumbents). This first-order condition is $p_I = \frac{V}{3} + \frac{1}{12} - \frac{D}{6} + \frac{p_E}{6} = \frac{5V}{12} + \frac{1}{8} - \frac{D}{3}$. This is less than $V - D$ when $V > \frac{8}{7}D + \frac{3}{14}$, which is not binding when $V \geq \frac{3}{2} - 6D$ and $V \geq \frac{7}{3}V$. Profits for the incumbents are $(V - D)\left(\frac{3}{8} - \frac{V}{4} + \frac{D}{2}\right)$, which is greater than $\frac{6}{25}(V + D)^2$ when

$$\frac{27}{98}D - \frac{5}{196}\sqrt{-2204D^2 - 852D + 225} + \frac{75}{196} \leq V \leq \frac{27}{98}D + \frac{5}{196}\sqrt{-2204D^2 - 852D + 225} + \frac{75}{196}.$$

<u>Appendix 5: Proof that there are no symmetric-among-incumbent Pure-Strategy Equilibria that support profit-increasing entry if the market is covered when only 2 firms are in the market.</u>

There are two pre-entry cases that could occur: first, both firms might price along the linear part of their demand curve. Second, the two firms could price at a kink-point. We examine each case in turn.

First consider what happens if firms price along the linear portion of their demand curve. Profits are then $p_j\left(\frac{1}{2}+p_{-j}-p_j\right)$. The first-order conditions are $p_j=\frac{1}{4}+\frac{p_{-j}}{2}$, which are solved when p = ½ and profits are π = ¼. In order for the market to be covered at this price, it must be that V – ½ ≥ ½ – D, or $V \geq 1 - D$. If the equilibrium after entry is also characterized by a simultaneous solution of the first-order conditions, profits after entry are $\pi_I=\left(\frac{D}{5}+\frac{3}{10}\right)^2$ as solved above. Note that because $D \leq$ ¼, $\pi_I \leq \left(\frac{7}{20}\right)^2 < \frac{1}{4}$. The other potential post-entry symmetric-among-incumbents pure-strategies are that all firms could price at kink-points, or that the incumbent firms price at a kink point while the entrant prices along a linear portion of its demand curve. In Appendix 4, we show that having all firms price at kink-points requires (A13) to hold. Note that the derivations of (A13) are not based on restricting (V,D) to the values satisfying Proposition 1. However, (A13) cannot hold when $V \geq 1 - D$ because $\frac{3}{5}-\frac{3}{5}D>1-D \Leftrightarrow D>1$, which contradicts the constraint that $D \leq$ 1/4. Similarly, if the entrant prices along a linear part of the demand curve, but the incumbents prices at a kink point, then (A15). Note, again, that (A15) does not require imposing the (V,D) satisfying Proposition 1. However, (A15) cannot hold when $V \geq 1 - D$ because $\frac{6}{5}D+\frac{3}{10}>1-D \Leftrightarrow D>\frac{7}{22}$, which is greater than the maximum D.

Next, suppose that before entry the two firms price at a kink-point such that the market is covered. The firms price such that $V-p=\frac{1}{2}-D \rightarrow p=V-\frac{1}{2}+D$. This is only an equilibrium if the price that solves the first-order conditions where the market is assumed to be covered is above this kink-point, while the price that solves the first-order conditions assuming that the market is not covered is below the kink-point. The first condition is that $p_i=\frac{1}{4}+\frac{V-\frac{1}{2}+D}{2}=\frac{V+D}{2} \geq V+D-\frac{1}{2} \rightarrow V \leq 1-D$. The second condition is that $p=\frac{V}{3}+\frac{D}{3}+\frac{V-\frac{1}{2}+D}{6}=\frac{V}{2}+\frac{D}{2}-\frac{1}{12} \leq V-\frac{1}{2}+D \rightarrow V \geq \frac{5}{6}-D$. Profits before entry are $\frac{V+D-\frac{1}{2}}{2} \geq \frac{1}{6}$ because $V \geq \frac{5}{6}-D$. Given that $\frac{5}{6}-D \leq V \leq 1-D$, profits always decrease after entry. As in the previous paragraph, there are 3 types of pure-strategy symmetric-among-incumbents equilibria that could potential arise. If after entry all firms price along the linear portion of

their demand curve, profits are $\pi_I = \left(\frac{D}{5}+\frac{3}{10}\right)^2 \le \left(\frac{7}{20}\right)^2$. If after entry, the two incumbents price at a kink-point in demand, (using the same calculations as in the last paragraph of Appendix 4) profits are $(V-D)\left(\frac{3}{8}-\frac{V}{4}+\frac{D}{2}\right)$. Note: $\frac{V+D-\frac{1}{2}}{2} > (V-D)\left(\frac{3}{8}-\frac{V}{4}+\frac{D}{2}\right) \Leftrightarrow 2V^2+V(1-6D)+7D+4D^2-2>0$. Because $V \ge \frac{5}{6}-D$, $2V^2+V(1-6D)+7D+4D^2-2 \ge \frac{2}{9}-\frac{7}{3}D+12D^2 \ge \frac{47}{432} > 0$, so post-entry profits are always lower than pre-entry profits. Therefore, there exists no symmetric-among-incumbents pure-strategy equilibrium where profits increase from entry when the market is covered before entry.

Appendix 6: Proof of Theorem 5

Before considering what happens when customers exit the market, we first note the equilibrium that occurs with no exit: $p=\frac{2}{5}(V+D); \pi=\frac{6}{25}(V+D)^2$; and each firm sells to D customers in between the two outlets, and to $\frac{3}{5}V-\frac{2}{5}D$ customers on the other side. Thus, if $K \ge \frac{3}{5}V-\frac{2}{5}D$ then there is no impact from customers exiting the market. Now consider what happens when some customers exit the market, such that $K < \frac{3}{5}V-\frac{2}{5}D$. Firm i's profits are then $p_i\left(K+D+\frac{p_{-i}-p_i}{2}\right)$, which gives first-order conditions of $p_i = K+D+\frac{p_i}{2}$. Symmetry ensures that $p=2(K+D)$. However, this solution assumes that $p=2(K+D)<V-K$. If $2(K+D)>V-K$, then firm i's profits are $p_i\left(V-p_i+D+\frac{p_{-i}-p_i}{2}\right)$, which is maximized at a price of $p=\frac{2}{5}(V+D)$. However, at this price, $V-p_i > K$. Thus, firms price at the lower of $p=2(K+D)$ or the kink-point where $p = V-K$. This can be summarized as follows:

$$p=\begin{cases} V-K & \text{if } K \ge \frac{V-2D}{3} \\ 2(K+D) & \text{if } K < \frac{V-2D}{3} \end{cases}$$

Note that $\frac{V-2D}{3} < \frac{2V-3D}{5} < \frac{V-D}{2}$, so the conditions where profits increase involve $p = V-K$. Equilibrium profits are

$$\pi=(V-K)(K+D)=VD+K(V-D)-K^2. \tag{A16}$$

We must make sure that the firms do not undercut. To undercut their rival, a firm must charge $p = V - K - 2D$, and demand will be $2K + 2D$. One can confirm that undercutting will not be profitable as long as $K > V - 4D$.

Taking the derivative of (A16) reveals that $\frac{d\pi}{dK} = V - D - 2K < 0 \Leftrightarrow K > \frac{V-D}{2}$. Since $\frac{V-D}{2} > V - 4D$ when $V \leq \left[\frac{7+5\sqrt{10}}{3}\right]D$, this proves part ii. For part i, profits will be higher after the customer exit when $VD + K(V-D) - K^2 > \frac{6}{25}(V+D)^2 \rightarrow K > \frac{2V-3D}{5}$.

Appendix 7: Proof of Theorem 6

The indifferent consumer between the outlets is located a distance $x = D - \frac{p_1}{2} + \frac{p_2}{2}$ from firm 1. Assuming that this consumer is within the central $2G$ interval demand for firm 1 is . We can solve the first order conditions to get $p_1 = p_2 = \frac{2(V+D-fG)}{5-f}$ and profits of $\frac{2(3-f)(V+D-fG)^2}{(5-f)^2}$.

This constitutes an equilibrium as long as the market is covered in the middle or $V \geq \frac{(7-f)D-2fG}{3-f}$.

Furthermore, it must be that no firm has an incentive to undercut the other. There are two types of undercutting to consider. The first is to lower the price by *2D* and get all the customers of the other firm. The new profit will not be higher as long as:

$$2\frac{(f-3)V^2 + (14D - 12fD - 2Gf^2 + 2f^2D + 6Gf)V + (G^2f^3 - 3G^2f^2 - 2Gf^3D + 12Gf^2D - 14GfD + 4f^2D^2 - 33fD^2 + 67D^2)}{(5-f)^2} \geq 0$$

This equation is a quadratic in V and will be positive between the two roots. Since one of the roots is negative we are left with $V \leq fG + D(3-f) + D\frac{(5-f)\sqrt{(f^2-6f+10)}-2}{(3-f)}$.

A second possible deviation is to lower the price to a point on the demand curve has a different slope: In particular, firm 1 could lower its price to p such that the indifferent consumer is at $y>D+G$. In such a case, $p = \left(\frac{2}{5-f}(V+D-Gf)\right) + 2D - 2y$. The profit for the deviating firm will be maximized at

$y = \frac{1}{6}\frac{30D+5Gf-5Df+Vf-2Gf^2}{5-f}$ giving it a profit of $\frac{1}{6}\frac{(-2Gf^2+Df+11Gf+Vf-6D-6V)^2}{(5-f)^2}$. This deviation will not be profitable when the new profit is lower than before or:

$$f\frac{V^2 f+\left(-60G-4Gf^2+2Df+22Gf\right)V-32G^2f^2-4DGf^2+22DGf-60DG+D^2f+85G^2f+4G^2f^3}{6\left(5-f\right)^2}\leq 0$$

This is a quadratic in *v* which will be positive between the roots. Since one root is negative we get $V\leq -D+\frac{G}{2f}\left(60+4f^2-22f+4\left(5-f\right)\sqrt{3\left(3-f\right)}\right)$.

Finally, we must ensure that the new equilibrium (after some consumers exited the market) yields higher profit to both firms. We need:

$$2\left(3-f\right)\frac{\left(V+D-Gf\right)^2}{\left(5-f\right)^2}-\frac{6}{25}\left(V+D\right)^2\geq 0$$

As before, this is a quadratic in *V*, so it will be positive outside the roots. Since one root is negative the meaningful constraint is: $V\geq -D+\frac{5G}{\left(5-3f\right)}\left(15+\left(5-f\right)\sqrt{3\left(3-f\right)}-5f\right)$.

Combining all the conditions proves the theorem. As an example that the conditions do not yield an empty set consider V=5D, G=D/10, D=1 and f=0.4. It is easy to verify that all the conditions hold and firm profits increase from 8.64 to 8.73.