Profit-Increasing Consumer Exit

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This paper examines the phenomenon of profit-increasing consumer exit, and the related phenomenon of profit-decreasing consumer entry. We demonstrate that firms can be better off in shrinking markets and worse off in growing markets, even in the absence of competitive entry or exit. Specifically, firms may benefit if a segment of consumers who are relatively indifferent about consuming any product in the category leave the market. Profits can increase for all firms even if the exiting consumers have strong preferences for only one of the products in the market. In shrinking markets, it is reasonable that the people who are likely to exit the market first are people who are "least committed" to the category. In particular, people who are the least satisfied with the existing offers are the most likely to change their behavior by finding an alternative or adopting a new technology. Similarly, in growing markets, consumers who enter the market late are generally the "least committed" to the category. Such exit can relax the competitive pressure between firms and lead to increased profitability. Our findings provide an explanation for profit growth that has been observed in categories exhibiting slow and predictable declines over time including vacuum tubes, cigarettes and carbonated sugary drinks.

1. Introduction

Would a manager rather be competing in a growing or a shrinking market? Conventional wisdom suggests that growing markets are more attractive (Keller and Kotler 2012). This is underlined by managerial tools such as the Boston Consulting grid which uses market growth as a proxy for market attractiveness when considering investments (Lilien, Kotler and Moorthy 1992). This conventional wisdom has been challenged by researchers who say that firms could be worse off in growing markets if such growth is associated with large levels of entry that lead to intense competition. For example, the cost of R&D, obtaining distribution and obtaining awareness limit the profitability of new entrants in growing markets (Gatignon and Soberman 2002).

What happens, however, when there is no entry or exit by rival firms? Can firms be more profitable in a shrinking market and less profitable in a growing one? We show that these events are quite possible with two uncontroversial assumptions: in growing markets, consumers who enter the market late are the people who are "least committed" to the category. Similarly, in shrinking markets, consumers who most likely to exit the market first are people who are "least committed" to the category.

Consider the cigarette market in the US. Since 1982, when the cigarette market peaked at 640 billion pieces, it has declined significantly. Today, the market is estimated to be less than 400 million pieces, a decrease of more than 35% in terms of volume.[1] Explanations for this decline abound: studies have demonstrated that tobacco exhibits characteristics similar to other categories of consumer goods: important factors that have had a significant effect on overall consumption are (a) average pricing; and (b) per capita income (Wilcox and Vacker 1992). In addition, the health effects of smoking have undoubtedly contributed to reduced smoking in the

[1] "US Tobacco Production, Consumption, and Export Trends" Report for Congress, The Library of Congress, Code RL30947 or at http://www.nationalaglawcenter.org/assets/crs/RL30947.pdf.

US. Despite a gradual and predictable decline in cigarette sales that started in 1982, the profits of domestic cigarette manufacturers exhibited significant growth through 1990 (Bauder 1984, Gordon 1985, Wiggins 1985, Yankeelov 1986, Ticer 1986).[2]

A key question is why did the profits of cigarette manufacturers hold up so well in a context of declining sales and reduced popularity of smoking?[3]

To find an answer to this question, we first consider the type of people who stopped buying and consuming cigarettes over the course of the decline. We abstract away from the natural evolution of tobacco consumers (people start buying cigarettes in their teens and stop buying later in life) and focus our attention on the fraction of consumers who "stopped early" or "quit". In general, these were people who wanted to reduce their tar and nicotine intakes. Many ex-smokers were smoking low tar cigarettes before they quit, and while they may have enjoyed smoking, they were perhaps unwilling to accept the level of risk associated with tobacco consumption.

The reality is that tobacco companies were unable to (or chose not to) develop tar-free cigarettes. The existence of a segment that seeks the satisfaction of smoking without the risk is underlined by the recent development of electronic cigarettes which do not deliver tar (Zezima 2009, Keeley 2012). These observations are further supported by research and surveys which show that people who were (and are) most likely to quit are smokers of low tar and nicotine cigarettes (Kozlowski et al 1999 and Kelbsch et al 2005).

[2] It is important to note that the important lawsuits undertaken by various state governments and class actions did not start until the early 1990s. While these lawsuits should reduce the profits of the tobacco companies, we restrict our attention to the "pre-litigation" period. There were private lawsuits launched against tobacco companies prior to 1990 but generally these were successfully defended or had negligible impact on tobacco company profits.

[3] The penetration of smoking reached a peak in the 1950s at about 50% of the adult population. Penetration has declined to less than 25% of the adult population today. US Center for Disease Control and Prevention, "Smoking Prevalence Among US Adults, 1955-2007" or at www.infoplease.com/ip/A0762370.html

In our model, we show that these are precisely the conditions associated with increased profits in a shrinking market. It is also important that the US tobacco market was dominated by six major cigarette manufacturers in 1982: this market structure remained stable until 1994 when Brown and Williamson merged with American Tobacco.[4] Thus, it is difficult to explain increased profitability by changes in market structure or concentration. In a nutshell, tobacco consumers who were leaving the market were less committed to the category. Their relative preference for the products on the market were small (none of the products offered what these consumers wanted, a product that delivered flavor and satisfaction without the perceived risk). It is straightforward to see how consumers like this might exacerbate competition between firms.

Other examples of market decline coupled with profit growth comes from the carbonated beverage market (Strom 2012) and vacuum tubes (Harrigan and Porter 1983). The per capita consumption of soda declined 16% between 1998 and 2011 reflecting increased levels of concern about obesity: sugary soft drinks account for a large fraction of the calories consumed in the average American's diet. This has led to a decline in overall volume of approximately 3.1% over the 13 year period.[5] Nevertheless, beverage companies have been making more money on carbonated soft drinks by raising prices and forcing die-hard drinkers to pay more to "feed their sugar habit." In fact, revenue from carbonated soft drinks reached a record high of $75.2 billion in 2011 (Strom 2012).[6]

The production of vacuum tubes in the US stopped sometime in the 1980s after decades of market decline.[7] As noted in Harrigan and Porter (1983), the discovery of the transistor in the

[4] While Big Tobacco was often referred to as the Big Seven (Philip Morris, RJ Reynolds, Brown and Williamson, Lorilland, Liggett and Myers, American Tobacco and US Tobacco), US Tobacco was primarily a manufacturer of smokeless tobacco.
[5] The overall decline in volume obtains by multiplying the per capita decline in volume by the percentage increase in the US population over the same 13 year period (12.7%) according to US Census Data.
[6] We thank the Associate Editor for suggesting this example.
[7] As of 2005, there were still manufacturers of vacuum tubes in China and Russia.

late 1940s meant that vacuum tubes were a technological anachronism; pundits predicted that vacuum tubes would disappear by the 1970s. Despite these predictions, the decline stage for vacuum tubes lasted more than 20 years. Throughout the 1960s, the profitability of the manufacturers was impressive and manufacturers maintained profitability by reducing volume and focusing on price insensitive demand associated with replacement tubes and military applications.

In all three of our examples, it is useful to highlight three commonalities. First, the decline of the category was both gradual and predictable. Second, the decline was driven by the departure of non-committed category participants who either stopped using the product (cigarettes), switched to an alternative without "negative baggage" (diet soft drinks, water or fruit juices) or adopted an alternate technology (transistors and solid state electronics). Finally, the firms responded by raising prices for customers who remained in the market.

Our objective is to show how these conditions can lead to profit increasing consumer-exit. Moreover, we demonstrate that this phenomenon is the expected outcome in a model where the firms compete vigorously with each other. That is, the prediction that incumbents can realize profit growth in declining markets does not rely on any form of collusion or coordination between the firms.

Understanding the possibility and causes of profit-increasing consumer exit has several important implications for managers. For example, our theory provides a set of conditions under which profit-increasing entry is likely to occur. Understanding that consumer exit by customers who do not easily switch between products can increase profits provides a new context for managers to better forecast how sales, prices and profits will evolve in many industries. Further, our results run counter to conventional wisdom that managers should generally limit their

investments into growing industries. Rather, in the examples we provide above, the profit-increasing decline can take a long time to play out. Thus, firms in these industries may want to reinvest in their factories, or sign long-term leases if such leases provide better terms. Our results also have implications for advertising and consumer relationship management: managers in shrinking industries may be tempted to advertise and create loyalty programs aimed at retaining customers who are leaving the market. Our results suggest that managers should follow the opposite strategy – profits are maximized by advertising and maintaining strong relationships with their most loyal customers.

It is also useful to underline the flipside of profit-increasing consumer exit: profit-decreasing consumer entry. This phenomenon can occur when the growth of a market is both gradual and driven by "doubters" who finally enter the category. A prototypical example of this phenomenon comes from the market for flatscreen televisions. From 2008 to 2009, unit sales in the US television market rose by 2%. Despite this increase in volume, revenues declined by 7% primarily driven by a drop in prices of approximately 8%.[8] While several factors might explain lower average prices, a key trend is that televisions are increasingly sold to less-affluent and more price-sensitive customers. Sony's summary of their consumer products and devices market for 2009 concludes that their decreased revenues – and profits[9] – were based on intensified price competition, among other factors, despite higher unit sales.[10] Gregg Richard, president of the prominent retailer PC Richard & Son noted that they were "selling more TVs, more units, at

[8] See "Global LCD TV Shipments Reached 146M Units in 2009, Faster Growth Than 2008," at Display Search (Feb. 22, 2010). Available at http://www.displaysearch.com/cps/rde/xchg/displaysearch/hs.xsl/100222_gobal_lcd_tv_shipments_reached_146m_units_in_2009_faster_growth_than_2008.asp. "TVs Selling Well, but Profits Remain Low" (May 28, 2011 in the Daily Yomiuri (Tokyo) provides a similar description in the Japanese market for later years.

[9] Thus, the price decreases reflected thinner margins, and not just lower costs from learning-by-doing.

[10] See p. 3 of Sony's "Consolidated Financial Results for the Fiscal Year Ended March 31, 2009," at http://www.sony.net/SonyInfo/IR/financial/fr/08q4_sony.pdf. Unit sales are from "Supplemental Information for Q1 FY2009 Earnings" at http://www.sony.net/SonyInfo/IR/financial/fr/09q1_eleki.pdf.

lower retail prices," as reported in the *New York Times*.[11] The sales of TVs in China provide an even more stark contrast – TV sales grew 70% comparing the first quarters of 2008 and 2009, but total revenues increased by only 3% in the same period, leading to profit decreases for Chinese flat-screen TV makers of as much as 90%.[12] We posit that similar effects occur in technological industries when early movers are less price-sensitive than later adopters (such as Roger's early and late majority). In many of these industries, prices fall, and total profits are lower despite higher sales. However, these same industries do experience a learning curve, where marginal costs fall, too, making it hard to tease out how much of the price cut is due to better technology and how much is due to intensified competition.[13] In the following section, we briefly review the literature related to our topic.

2. Literature Review

The literature related to "profit increasing consumer exit" is sparse as the majority of work that examines decision making in shrinking markets is focused on how to minimize damage and not on how profits can increase (Harrigan and Porter 1983, Hague 1985). However, there are three papers that relate directly to the ideas we investigate.

In Desai (2001), two firms compete for two segments of consumers with product lines targeted by segment. One segment has a high preference for quality and is sensitive to the difference between products and the other less so (the High and the Low segments respectively). The author finds that the competing firms often reduce the quality of the products designed for the Low segment in order to minimize cannibalization. The creation of low quality line

[11] See "Plummeting TV Prices Squeeze Makers and Sellers," *New York Times*, December 27, 2011.
[12] See "Flat-panel TV sales up, but profits not" by Wang Xing in the China Daily, July 13, 2009, as accessed at http://www.chinadaily.com.cn/bizchina/2009-07/13/content_8420879.htm.
[13] Liu (2010) demonstrates that firms set early prices in ways to subsidize the learning that they anticipate will occur, also loosening the link between prices and the learning curve.

extensions reduces the incentive that firms have to reduce price on high quality products by siphoning the Low segment to a lower priced, lower quality alternative. This work points to the value that can be created by eliminating price sensitive consumers from the basket of potential demand for a product. However, it is important to note that the value of the strategy highlighted by Desai also entails firms earning significant profit on the products being sold to the Low segment.

In a similar model, Coughlan and Soberman (2005) find that competing firms can increase profits by removing price sensitive switchers from a primary market with outlet malls. Price insensitive shoppers (who remain in the primary market) are assumed to place high value on in-store service while price sensitive shoppers (who are attracted to the outlet mall) place no value on in-store service. Similar to Desai (2001), this model points to the value that firms can capture by removing "bad consumers" from a market. The key mechanism that allows segmentation to work in this model is the difference between segments in terms of their appreciation of in-store service; without this difference, self-selection by the segments would not occur.

The paper that most closely relates to our work is Ishibashi and Matsushima (2009). The authors propose a model which is similar in spirit to the two models discussed above: there are two segments, one with high preference for quality and high sensitivity to product differences and the second with low preference for quality and less sensitivity to product differences. A low quality entrant attracts all the price sensitive consumers away from two incumbents and the authors find conditions where the incumbents earn higher profits by only serving the price insensitive consumers that remain.

While none of this work deals explicitly with the topic of consumer exit (or market shrinkage), it is straightforward to see how it relates to our topic. The contribution of our analysis is threefold. First, we show that two dimensions of heterogeneity are not needed for consumer-exit to be profit enhancing.[14] We demonstrate the phenomenon in a model where the *only* dimension of heterogeneity is horizontal differentiation. Second, we demonstrate that profits can increase for both firms even when the customers that exit from the market only consider consuming from one of the firms (vs. choosing the outside option). For example, a firm's profits can increase even if the only consumers that exit are those that are essentially loyal to its product. This contrasts with the other models, where the market segment that is removed is a segment that is hotly contested by both firms. Finally, we extend our results and demonstrate that a gradual reduction in the fraction of price sensitive consumers (as would be observed in conditions of gradual market decline) can lead to higher profits, even in frameworks where consumers have different sensitivities to travel costs between products.

We now move to the model and consider a market where there is but one dimension of heterogeneity: horizontal differentiation. Our focus is to understand how consumer exit affects profitability and as noted earlier, our model is based on exit by consumers who are the most likely to exit once a market starts declining.

3. Linear Model

Our base model is a Hotelling market with two firms that are exogenously located at internal points on the linear market. The objective to explain how the departure of "less committed" consumers (i.e. consumers who realize less surplus from consumption than consumers who are

[14] Desai (2001), Coughlan and Soberman (2005) and Ishibashi and Matsushima (2009) all assume two dimensions of heterogeneity: transportation costs and willingness to pay for quality (or service).

central) can lead to higher profits for competing firms. First, we analyze a situation where the departure of consumers who are less committed to the market occurs symmetrically. We then present a version of the model that more closely reflects the dynamics of the tobacco and carbonated soft drink markets discussed in the introduction.

Consumers are uniformly distributed along the line with a density of 1. We assume that consumer *i*'s utility from buying and consuming the product from firm *j* can be represented as

$$U_{ij} = V - p_j - d_{ij} \qquad (1)$$

where p_j is the price charged by firm *j* and d_{ij} is the distance between consumer *i* and the location of firm *j*.[15] Our analysis assumes linear travel costs, but the qualitative results are robust to the use of quadratic travel costs. Consumers can decide not to buy from either firm in which case they consume only an outside good, and earn a normalized utility of zero.

We assume that firms compete in prices, and have constant and equal marginal costs, so profits for each firm are equal to $(p_j - c)q_j$ where q_j is the quantity sold. Without loss of generality, we set $c = 0$, with the implication that p represents the absolute markup on the product i.e. the difference between the price and marginal cost.

Finally, we assume that the firms are located internally, away from the edge of the market, and that V is small enough such that the market is uncovered for some locations (i.e. there are consumers in the market who find that consuming the outside good is preferable to buying either of the existing products). In particular, if the distance between the firms is $2D$, we focus our attention on a market where $\frac{7}{3}D \le V \le \left[\frac{7+5\sqrt{10}}{3}\right]D$. We further assume that the firms

[15] Note that equation (1) is equivalent to $U = v - \beta P - \alpha d$ under the normalization $V = v/\alpha$ and $p = \alpha P/\beta$. In Section 4 we consider models where consumers differ in their travel costs, and in those contexts we model the transportation cost coefficient explicitly.

are located at least a distance $\frac{3V}{5}-\frac{2D}{5}$ from the edges of the market (prior to consumer exit). The basis for these conditions is explained in the appendix.

While we do not model how the firms set their locations on the line, there are many reasons why firms might locate internally. For example, the firms could have initially located at the end of the markets when the markets were new, and then the market may have grown around them (and moving costs may prevent them from changing their location).[16] Consistent with the lifecycle events noted in the introduction, the markets may have just now started their contraction. Alternatively, the firms may be limited by their technologies. Thus, they may want to locate closer to the edges of the market but be unable to produce products at that location. For example, cigarette companies sought to produce tar-free cigarettes, but were unable to do so. Finally, we note that it is possible that firms do not fully know the state of demand when they enter into a market; a lot of ongoing marketing research today is devoted to measuring consumer preferences. However, once a firm enters it can get close to optimal pricing through trial and error.

With these assumptions, the solution to the first-order pricing conditions (before consumer exit) implies an equilibrium where the firms choose prices equal to $\frac{2}{5}(V+D)$ and earn profits of $\frac{6}{25}(V+D)^2$. Note that consumers at the edges of the market do not consume either product because they obtain a negative surplus from buying. Instead, they purchase the outside good and obtain a utility of zero.

[16] An alternative assumption would be that firms locate such that their location is optimal given the long-run state of demand, as is assumed in Neven (1987). It is then trivial to support the endogenous location choices of the firms under the above conditions under a 3-period model using this criterion: In such a model, the first two periods are the before and after consumer exit scenarios described above. The third and final resting spot for the market is whatever market size is needed to support the locations of the firms.

We now consider whether profits can increase if consumers who are near the edge of the market leave (or exit) the category. Specifically, we consider what happens to firm profits if consumers located a distance *K* and further from the firms (towards the outer edges of the market) leave. Figure 1 illustrates this situation. We make two comparisons: what is the range of *K* where profits after consumers exit the market are higher than the profits before the consumers exit, and what is the range of *K* where incrementally more exit (i.e., an incrementally smaller *K*) leads to greater profit. The first question is a before and after comparison, while the second question looks at the derivative of profit with respect to *K*.

Figure 1

Removing consumers near the edge and *K* or further from firms

Theorem 1 (Exit from both ends of the market): *Profits increase for both firms if all consumers located at a distance greater than K towards the edge (from each of the firms) leave the market for any* $K \in \left(\max\left(\frac{2V-3D}{5}, V-4D \right), \frac{3V-2D}{5} \right)$. *Further, (ii)* $\frac{d\pi}{dK} < 0$ *after exit (that is, profits are increasing as more consumers exit the market) for* $K \in \left(\frac{V-D}{2}, \frac{3V-2D}{5} \right)$.

The intuition behind Theorem 1 is that consumer exit acts as a commitment device for the firms to charge high prices. After consumers leave, the firms charge a price of $V-K$ such that the consumers that are the furthest away obtain exactly zero utility from the closest firm (in equilibrium). Intuitively, once exit has taken place, a price reduction does not yield additional sales from external consumers (all external consumers are being served). As a result, the firms have a reduced incentive to lower price (compared to their incentives before consumers leave the market); a lower price can only attract consumers between the two firms. Prior to the departure of the external consumers, low prices also attract business from consumers near the edge. At the higher equilibrium price $V-K$, the additional margin earned from external and internal (consumers between Firms 1 and 2) more than offsets the profit loss associated with the consumers who left the market.

The upper bound for K in Theorem 1, $\frac{3V-2D}{5}$, reflects the marginal consumer for each firm prior to consumers leaving the market.[17] Thus, profits increase as a small number of the furthest "served" consumers exit the market. The lower bound on *K* reflects that there is a limit on how many consumers can exit such that firm profits increase. It makes intuitive sense that at some point, the increased margin that can be made on existing die-hard consumers cannot offset the loss caused by high levels of market shrinkage. Note that prices continue to rise with consumer exit even if $K<\frac{V-D}{2}$. Thus, increases in equilibrium prices with consumer exit are not are not "sufficient" for profits to increase.

[17] Consumers more distant from the firms than $\frac{3V}{5}-\frac{2D}{5}$ can leave the market but have no effect on firms' profits because consumers at such locations purchase the outside good prior to consumer exit.

Theorem 1 refers to a situation where consumers exit from both edges of the market. We now move to Theorem 2 which addresses a situation where consumers exit from just one edge of the market. The scenario associated with Theorem 2 is illustrated in Figure 2.

Figure 2

Removing consumers from only one side of the market

If one thinks of the Hotelling continuum as representing tar and nicotine levels (in the case of tobacco) or calories or sugar content (in the case of carbonated soft drinks), Theorem 2 is a closer depiction of the dynamics described in the introduction. This is because in the tobacco market, consumers who were looking for less tar and nicotine were the most likely to exit the category. Similarly, in the carbonated soft drink market, the consumers who are most likely to exit the category are those who are looking for significantly less sugar and calories.

Theorem 2 (Exit from only one side of the market): *Profits increase for both firms if all consumers located at a distance greater than K towards the edge of the market from one of the firms exit the market for any* $K \in \left(\frac{49V - 36D}{85}, \frac{3V - 2D}{5} \right)$.

As in Theorem 1, Firm 1 increases its price from $\frac{2}{5}(V+D)$ to $V-K$, the price at which the consumer at a distance K from Firm 1 towards the edge of the market receives zero utility. Firm 1 will commit to this price for a wide range of prices from Firm 2 since this price forms a kink-point in Firm 1's demand curve. If Firm 1 charges a higher price it would lose consumers both towards the edge of the market and in between the two firms, while if Firm 1 charges a lower price it does not gain additional consumers towards the edge of the market. Firm 2 knows that Firm 1 will charge a higher price and because prices are complements, it also increases its price. Ultimately, Firm 2 benefits because Firm 1 charges a higher price. Firm 1 is also better off – despite the fact that it is the only firm to lose customers – because the higher prices of Firm 2 allow Firm 1 to increase its price enough to offset the lost volume. In essence, consumer exit acts as a commitment device to keep prices high.

It is possible that Theorem 2 better represents the cigarette market because there are two components that quitters were trying to avoid, tar and nicotine; however, the measured tar and nicotine deliveries of cigarettes tend to be highly correlated. In any event, as long as one accepts that "less committed" consumers (consumers whose preferences are relatively distant from the products on offer) are the ones exiting the market, the model provides a cogent explanation for why profits might increase in a context of gradual market decline (at least in the first few years after a decline starts).[18]

Note that consumers who leave the market are the consumers who have the lowest willingness-to-pay for the products in the market. This explains why firm profits increase even though market volume drops. Since consumer entry is the opposite of consumer exit, it is straightforward to show the inverse result: profits can decrease in a market where there is

[18] As an example supporting this assumption, an advertisement for True (low tar) cigarettes declared "I decided to either quit or smoke True. I smoke True." See Pollay and Dewhirst (2002) for more.

consumer entry if the entry occurs primarily among consumers who have a low willingness-to-pay for the products. Thus, Theorems 1 and 2 are also informative of sufficient conditions for market expansion to decrease profits. If the markets initially extend only a distance K from the firms, and then consumers enter to fill in the market up to a distance more than $\frac{3V}{5}-\frac{2D}{5}$ from the firm then profits will decrease with entry.

Further evidence to support the robustness of consumer exit leading to profit growth for competing firms obtains by considering a version of the model where instead of external consumers exiting the market, internal consumers (between the two firms) leave. This situation might relate to a context where die-hard consumers are really committed to one of the two competing brands (Playstation and X-Box in the videogame console market) and new consumers are relatively indifferent between the two competitors. In many cases, this assumption seems reasonable as novice consumers often have preferences that are poorly defined. They may even have a poor understanding of how products are different.[19]

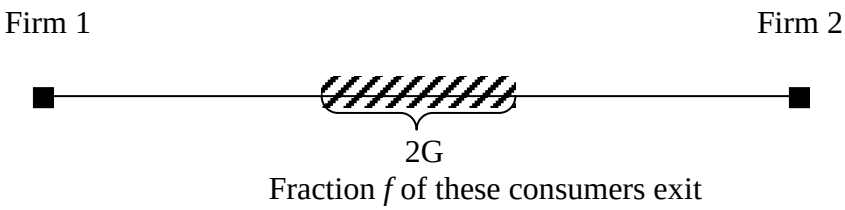
In this version of the model, we assume that the market is a line segment of unit length, consumers are uniformly distributed along the line and the two firms are located at the endpoints of the market: Firm 1 is located at point 0 and Firm 2 is located at point 1. We examine consumer exit for a fraction f of consumers in a line segment of length $2G$ (centered at 0.5) that

[19] There is evidence in the behavioral literature to demonstrate how differently novices and experts perceive and categorize choices within a category (Mitchell and Dacin 1996 and Cowley and Mitchell 2003). In a nutshell, novices are more likely to be indifferent between the available choices and sensitive to price. In fact, first time buyers are known to be heaviest users of sales staff in the retail environment because they have so little understanding (or preference) for the differences of the X-Box, the PlayStation and the Nintendo Wii. These consumers are thus more willing to switch between products than the typical consumer. We consider variation in sensitivity to the differentiation between products in Section 4.

leave the market (see Figure 3). We also restrict our attention to pure strategy outcomes where firms compete for the fraction *1-f* of price-sensitive consumers that remain in the market.[20]

Figure 3

Remove some consumers from the middle of the market



We do not present the entire analysis here but provide details in the appendix. Our analysis demonstrates that the profits of both firms increase when *f* and *G* satisfy the following conditions: $0 < f < \frac{3}{4}$ and $\frac{2-3f+2f^2-2(1-f)^{3/2}}{2f(5-8f+4f^2)} \le G < \frac{1-(1-f)^{1/2}}{2f}$. In this setup, the consumers leaving the market are clearly the most price-sensitive (i.e. the consumers who are most likely to switch from one firm to the other with small changes in price). It seems reasonable that consumers who have a strong preference for an existing product (i.e., are located adjacent to one of the products) are the least likely to leave the market, although we believe that the scenarios examined in Theorems 1 or 2 are more common.

4. Consumers with Heterogeneous Travel Costs

We now examine markets where consumers are heterogeneous in their travel costs. Mathematically, this amounts to considering a market where consumers are heterogeneous in

[20] If *f* is too high then a firm might price to extract more surplus from consumers who are nearby and ignore demand from consumers in the segment 2G as shown in Figure 3. In this situation, the equilibrium is in mixed strategies.

terms of price sensitivity.

First, we consider a model where there are two segments of consumers: one that is sensitive to the differences between products (with high travel costs) and a second segment that is less sensitive (with lower travel costs). We then extend the results to a model where the distribution of sensitivity to product differentiation is uniform, continuous and uncorrelated with the consumer's location along the line.

We assume that there are two firms, 1 and 2, located at the opposite ends of two Hotelling markets of length 1 (i.e. Firm 1 is located at 0, and Firm 2 is located at 1) and each firm offers a single product to both markets at the same price. Each market represents a segment: one market is a segment of consumers with high travel costs and the other a segment with low travel costs. We assume that the High segment consists of a mass λ of consumers and the Low segment consists of a mass 1-λ of consumers both uniformly distributed along the line implying that the total mass of consumers is one (prior to consumer exit). The consumer utility for the High segment is given by:

$$U_{ij} = V_T - p_j - \alpha_T d_{ij}, \tag{2}$$

while the Low segment consumer utility is represented as:

$$U_{ij} = V_B - p_j - \alpha_B d_{ij}.^{21} \tag{3}$$

We assume that $\alpha_T > \alpha_B$. Not that the location of the marginal consumer is unaffected by whether the utility specification is a linear or quadratic function of distance when the firms are located at the endpoints of the market. Many spatial models that employ this structure (e.g., Iyer 1998, Desai 2001, Ishibashi and Matsushima 2009) also assume that there is a service or quality component such that consumers in the first segment are sensitive to both service (or product

[21] Equations (4) – (9), and thus the results from this section, are identical if the preferences are quadratic in travel costs.

quality) and product attributes while consumers in the second segment are less sensitive to both. We assume that the competing products are of equal quality but higher sensitivity to both quality and horizontal attributes can be represented by allowing $V_T > V_B$. This assumption would have no effect on the analysis as along as V_T and V_B are high enough such that no consumer considers purchasing the outside good.

In this structure, the indifferent consumer is found at the point where the utilities offered by the competing products are identical. This implies that:

$$p_1 + x_T\alpha_T = p_2 + (1 - x_T)\alpha_T$$
$$p_1 + x_B\alpha_B = p_2 + (1 - x_B)\alpha_B$$

where x_k denotes the marginal consumer in segment k. Rearranging, we find that the marginal consumer for each segment is located at:

$$x_T = \frac{p_2 - p_1 + \alpha_T}{2\alpha_T}$$
$$x_B = \frac{p_2 - p_1 + \alpha_B}{2\alpha_B}. \tag{4}$$

Profits for each firm are then

$$\Pi_1 = p_1\left[\lambda x_T + (1-\lambda)x_B\right]$$
$$\Pi_2 = p_2\left[\lambda(1 - x_T) + (1-\lambda)(1 - x_B)\right]. \tag{5}$$

We substitute the expressions for x_T and x_B (equation 4) into equation 5 and simultaneously differentiate and then solve for the optimal prices.

$$p_1^* = p_2^* = \frac{\alpha_T\alpha_B}{(1-\lambda)\alpha_T + \lambda\alpha_B}. \tag{6}$$

Thus, the equilibrium prices are the harmonic means of the travel costs for each of the two segments. Because the market is covered and the equilibrium is symmetric, the equilibrium profits are

$$\Pi_1^* = \Pi_2^* = \frac{1}{2}\frac{\alpha_T \alpha_B}{(1-\lambda)\alpha_T + \lambda\alpha_B}. \tag{7}$$

Now suppose that a fraction (1 – ω) of the consumers in the Low segment exit (i.e. the segment of the market that has a travel cost of α_B). Analogous calculations to those above generate the following the equilibrium prices:

$$p_1^* = p_2^* = \frac{\left[\lambda + \omega(1-\lambda)\right]\alpha_T \alpha_B}{\omega(1-\lambda)\alpha_T + \lambda\alpha_B}. \tag{8}$$

Equilibrium profits are then

$$\Pi_1^* = \Pi_2^* = \frac{1}{2}\frac{\left[\lambda + \omega(1-\lambda)\right]^2 \alpha_T \alpha_B}{\omega(1-\lambda)\alpha_T + \lambda\alpha_B}. \tag{9}$$

Given these profits, Theorem 3 summarizes the conditions under which profits increase with consumer exit:

Theorem 3: *Suppose that consumer and firm behavior are given as described in equations (2)-(9). Then profits increase whenever* $\alpha_B \le \frac{\alpha_T}{2}$ *and either (1)* $\frac{\alpha_T}{2(\alpha_T - \alpha_B)} \le \lambda$ *or (2)* $\frac{\alpha_B}{\alpha_T - \alpha_B} \le \lambda \le \frac{\alpha_T}{2(\alpha_T - \alpha_B)}$ and $\omega \le \frac{\lambda^2(\alpha_T - \alpha_B) - \lambda\alpha_B}{\alpha_T + \lambda\alpha_B - 2\lambda\alpha_T + \lambda^2(\alpha_T - \alpha_B)}$.

The proof is trivial and follows directly by comparing equations (9) and (7). Note that the finding that ω must be small enough if $\frac{\alpha_B}{\alpha_T - \alpha_B} \le \lambda \le \frac{\alpha_T}{2(\alpha_T - \alpha_B)}$ means that profits do not increase when only a few Low consumers exit the market. That means that in some situations, consumer exit is profitable if *and only if* enough customers leave the market. At first, this may

seem counterintuitive – after all, it may seem that if too many consumers exit the market then profits must decrease because there are few consumers remaining to whom firms can sell. This intuition is offset by the fact that the increase in prices that occurs with exit increases convexly with the rate of exit from the market, while the rate at which consumers leave the market is directly proportional to the fraction of Low types that leave (i.e. $1 - \omega$). If only a few consumers exit, the increase in prices is negligible. However, once the fraction of consumers who exit is above a threshold, the equilibrium prices increase substantially because the firms have less incentive to fight over the reduced mass of consumers in the Low segment. Ultimately, the least restrictive range of λ where profits increase, i.e. $\lambda \geq \frac{\alpha_B}{\alpha_T - \alpha_B}$, obtains when all consumers in the Low segment exit.

The result we obtain with two distinct market segments is not idiosyncratic. Indeed, one obtains a similar result in a model with a continuum of consumer types. In particular, suppose that there exists a mass of consumers that have locations on a line uniformly distributed between 0 and 1. Further, suppose that the travel costs for consumers are distributed uniformly over the interval $[0,1]$, but that only consumers with $\alpha_i \in [\underline{\alpha},1]$ are present in the market. Note that setting the highest travel cost to 1 is a normalization that does not affect the generality of the findings. Further, we assume that distribution of consumers' locations on the line is independent of the distribution of travel costs.

The type of exit we examine is one where the most price sensitive consumers leave the market first. In this model, that means the most likely consumer to leave the market is a consumer with a travel cost of $\underline{\alpha}$ (the consumer in the market that has the lowest travel cost). Market decline is thus represented by increasing the lower limit of α of consumers that remain in

the market. If $\underline{\alpha}_1$ is the lower bound before consumer exit and $\underline{\alpha}_2$ is the lower bound after consumer exit then consumers who leave the market are all those who have travel costs in the range $\left[\underline{\alpha}_1, \underline{\alpha}_2\right]$.[22] To analyze the effect of this exit on profit, we start by writing the profit function of each firm as a function of the lower bound $\underline{\alpha}$.

Profits for each firm are given by $\int_{\underline{\alpha}}^{1} p_j \left(\frac{1}{2} + \frac{p_{-j} - p_j}{2\alpha} \right) d\alpha$. We take the first order conditions and solve simultaneously to obtain equilibrium prices of $\frac{1-\underline{\alpha}}{\left|\ln\left(\underline{\alpha}\right)\right|}$, and equilibrium profits of $\frac{\left(1-\underline{\alpha}\right)^2}{2\left|\ln\left(\underline{\alpha}\right)\right|}$. Note that the derivative of profit with respect to the lower bound, $\frac{\partial \Pi}{\partial \underline{\alpha}} = \frac{\left(1-\underline{\alpha}\right)\left(1-\underline{\alpha}+2\underline{\alpha}\ln\left(\underline{\alpha}\right)\right)}{2\underline{\alpha}\left[\ln\left(\underline{\alpha}\right)\right]^2} > 0$ as long as $1-\underline{\alpha}+2\underline{\alpha}\ln\left(\underline{\alpha}\right) > 0$, which approximately coincides with the condition that $\underline{\alpha} < 0.2847$. We summarize these findings in Theorem 4:

Theorem 4: *Suppose consumers have utility $U_{ij} = V - p_j - \alpha_i d_{ij}$, and are uniformly distributed along a linear market. Further, suppose that $\alpha_i \sim U\left[0,1\right]$ independent of the consumer's location, and that only consumers with $\alpha_i \in \left[\underline{\alpha},1\right]$ are present in the market. Finally, suppose the two firms are located at the opposite ends of line and compete via differentiated Bertrand competition. Then $\frac{\partial \Pi}{\partial \underline{\alpha}} > 0$ (consumer exit is profitable) whenever $1-\underline{\alpha}+2\underline{\alpha}\ln\left(\underline{\alpha}\right) > 0$.*[23]

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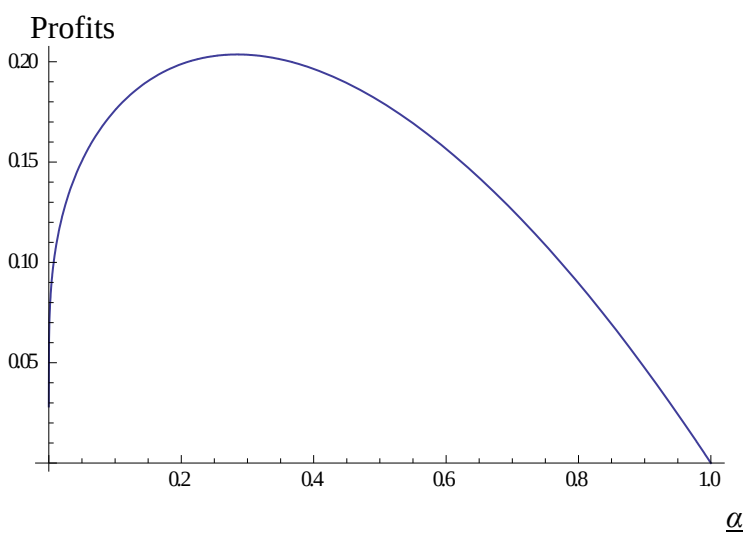
[22] Thus, the market before consumer exit consists of customers with travel costs $\left[\underline{\alpha}_1,1\right]$, but after consumer exit consists of customers with travel costs $\left[\underline{\alpha}_2,1\right]$.

[23] The results hold with the same cutoffs if consumers have quadratic travel costs: $U_{ij} = V - p_j - \alpha_i d_{ij}^2$.

Similar to Section 3, it is possible to conduct a before versus after consumer exit analysis. It is however, not possible to identify a set of closed-form conditions that fully characterize when consumer exit in the segment $[\underline{\alpha}_1, \underline{\alpha}_2]$ leads to profit increases for the firms. But we can demonstrate that profits increase from exit when the market prior to consumer exit has a sufficiently low $\underline{\alpha}$. To see this, we plot the profits for the firms against $\underline{\alpha}$ in Figure 4. It is apparent that profits increase dramatically with $\underline{\alpha}$ when $\underline{\alpha}$ is small, but decline slowly once $\underline{\alpha} > 0.2847$. With an example, we can demonstrate that profits may increase even in the face of substantial consumer exit. Profits are 0.176 when $\underline{\alpha} = 0.1$, but they are 0.180 when $\underline{\alpha} = 0.5$ (more than 44% of consumers have exited the market!).

Figure 4

Plot of profits with respect to $\underline{\alpha}$
Market Consists of Consumers with $\alpha \in [\underline{\alpha}, 1]$



5. Conclusion

The models we propose provide a simple explanation of how the profit of firms can increase in shrinking markets or decrease in growing markets even without changes in the number of competing firms or in the products they offer. Are these findings idiosyncratic because they rely on restrictive assumptions about which consumers exit the market? We think not. As explained in our discussion of the cigarette market, when a market starts to decline (and we have yet to identify a category that does not pass through the stages of the product life cycle), the consumers who are most likely to exit the market are consumers who are the most frustrated with the current offers (the consumers that realize the least amount of utility from consumption). Our model shows how their departure can help firms by reducing the competitive pressure between them. Of course, there is a limit to this dynamic. While the profits of the tobacco companies held up surprisingly well through the 1980s and into the 1990s, at some point an industry's decline hits a point where higher prices charged to the consumers who remain active do not make up for the lost volume.[24]

Another aspect of our analysis relates to recent prescriptions from the CRM literature (Reinartz and Kumar 2002). A constant refrain from this literature is that some consumers are so costly to serve that a firm may be better off refusing to serve them. This appears to echo our findings of how departing consumers can lead to higher profits for incumbent firms (even when the number of firms and their product attributes are constant). However, the finding from the CRM literature is based on firms carefully measuring the revenues and costs of serving

[24] Mergers have changed the face of tobacco in the tobacco industry in the last 15 years. Most tobacco companies are now multinational conglomerates and their profits have held up well. Declines in tobacco consumption in North America and Europe have been offset by significant growth in the rest of the world.

individual customers in their CRM system and making sure that revenues exceed the costs.[25] In contrast, the findings from our analysis emanate from the strategic interaction of competing firms: the consumers who leave the market in our models *are worth serving*. The problem for the firms is that their incentives to compete for these consumers are such that they lead to reduced profitability. By revealed preference, we know that a monopolist cannot be better off if consumers leave the market: a monopolist always has the option of increasing price and serving a narrower segment. This contrasts with the CRM example, where a monopolist simply decides not to serve certain consumers because doing so increases profit. In a market with competition however, the complementarity of prices puts firms into a Prisoners' Dilemma because they compete vigorously for consumers who are price sensitive. In this context, consumer exit increases profits because either the exit raises a rival's price by directly changing that rival's incentives, or the exit acts as a commitment device for a firm to charge a higher price, which in turn causes the rival to increase its price (or both).

Our analysis also raises a number of implications that ought to figure in the management of declining markets. First, as noted in our introduction, a fundamental tenet of the Boston Consulting Grid is that declining markets are unattractive. Our analysis shows that when markets decline but the decline is driven by the departure of consumers who are not fully satisfied with the offers in the market, declining markets might be every bit as attractive as growing markets. Why? Because the likelihood of new competitors is low and consumers who remain in the market generally have higher willingness to pay. This suggests there are often conditions where

[25] In particular, the importance of accounting for indirect costs such as advertising, service, organizational costs as well as direct product costs is emphasized in this literature (Reinartz and Kumar 2002).

the conventional wisdom related to where firms should invest is incorrect. Sometimes it pays to invest, rather than divest, in a shrinking industry.[26]

Second, the natural reaction of many managers to declining sales is to reduce price to maintain volume. Our analysis underlines the importance of not having reflexive reactions like this. In many cases, a decline in sales might be a signal to raise and not lower price.

Third, it is extremely important when a decline is encountered for managers to fully understand the segments of consumers who are driving the decline. When the decline is driven by consumers who appear to be core consumers, this is markedly different than a situation where the decline is driven by casual (or less committed) category participants.

Finally, our results have implications on which segments of consumers firms should target in declining markets. It might at first appear reasonable for a firm in a declining market to target customers who have only a marginal interest in the firm's product, and are at risk to stop their purchases. Our model shows, however, that the firms may be better off letting those customers leave the market, and instead focus on extracting more profit and sales out of its most loyal customers.

[26] In a similar vein, if a person had invested in tobacco companies that did not pursue diversifications strategies in the 1982-1990 time period (e.g. American Brands and Liggett & Myers), their returns would have been significantly higher than average market returns.

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Appendix: Proof of Theorem 1

Before considering what happens when consumers exit the market, we first note the equilibrium that occurs with no exit: $p=\frac{2}{5}(V+D);\pi=\frac{6}{25}(V+D)^2$; and each firm sells to D consumers in between the two outlets, and to $\frac{3}{5}V-\frac{2}{5}D$ consumers on the other side. Thus, if $K\geq\frac{3}{5}V-\frac{2}{5}D$ then there is no impact from consumers exiting the market. Now consider what happens when some consumers exit the market, such that $K<\frac{3}{5}V-\frac{2}{5}D$. Firm i's profits are then $p_i\left(K+D+\frac{p_{-i}-p_i}{2}\right)$, which gives first-order conditions of $p_i=K+D+\frac{p_i}{2}$. Symmetry ensures that $p=2(K+D)$. However, this solution assumes that $p=2(K+D)<V-K$. If $2(K+D)>V-K$, then firm i's profits are $p_i\left(V-p_i+D+\frac{p_{-i}-p_i}{2}\right)$, which is maximized at a price of $p=\frac{2}{5}(V+D)$. However, at this price, $V-p_i>K$. Thus, firms price at the lower of $p=2(K+D)$ or the kink-point where $p=V-K$. This can be summarized as follows:

$$p=\begin{cases}V-K & \text{if } K\geq\frac{V-2D}{3}\\ 2(K+D) & \text{if } K<\frac{V-2D}{3}\end{cases}$$

Note that $\frac{V-2D}{3}<\frac{2V-3D}{5}<\frac{V-D}{2}$, so the conditions where profits increase involve $p=V-K$. Equilibrium profits are

$$\pi=(V-K)(K+D)=VD+K(V-D)-K^2. \tag{A1}$$

Profits will be higher after the consumer exit when $VD+K(V-D)-K^2>\frac{6}{25}(V+D)^2$ $\rightarrow K>\frac{2V-3D}{5}$.

We must make sure that the firms do not undercut. To undercut their rival, a firm must charge $p=V-K-2D$, and demand will be $2K+2D$. One can confirm that undercutting will not be profitable as long as $K>V-4D$.

For part ii, taking the derivative of (A1) reveals that $\frac{d\pi}{dK} = V - D - 2K < 0 \Leftrightarrow K > \frac{V-D}{2}$. Note that $\frac{V-D}{2} > V - 4D$ when $V \leq \left[\frac{7+5\sqrt{10}}{3}\right]D$.

<u>Proof of Theorem 2</u>

The calculations for the before-exit case presented above.

Suppose that all consumers located at a distance greater than K from the closest outlet towards one end of the line exit, where $K \in \left(\frac{49V-36D}{85}, \frac{3V-2D}{5}\right)$. Without loss of generality, call the firm that is located near the exiting consumers firm A and the other firm B. Then the following prices form an equilibrium:

$$p_A = V - K;\ p_B = \frac{V+D}{3} + \frac{p_A}{6} = \frac{V}{2} + \frac{D}{3} - \frac{K}{6}. \tag{A2}$$

To see this, note that if A responded to B's price with a price above V – K, it faces the same first-order condition as it faced before entry. Its best response would thus be to price such that $p_{Dev} = \frac{V+D}{3} + \frac{p_B}{6} = \frac{5V}{12} + \frac{7D}{18} - \frac{K}{36}$. However, it is easy to check that $\frac{5V}{12} + \frac{7D}{18} - \frac{K}{36} < V - K$ whenever $K < \frac{3V-2D}{5}$, so A would never charge a higher price. Similarly, firm A's first-order condition associated with prices below $V - K$ are $p_A = V - K + \frac{p_B}{2}$. Because $p_B > 0$, the best response in this range is to raise prices back up to the limit of $V - K$.

The remaining issue is to ensure that profits rise for the two firms. Profits for A are given by $(V-K)\left(K + D + \frac{\frac{V}{2} + \frac{D}{3} - \frac{K}{6} - (V-K)}{2}\right) = \frac{(V-K)(14D+17K-3V)}{12}$, while profits for firm B are given by $\frac{(3V+2D-K)^2}{24}$. Profits for A are greater than $\frac{6}{25}(V+D)^2$, whenever

$K \in \left(\frac{49V - 36D}{85}, \frac{3V - 2D}{5} \right)$, while profits for B are greater than $\frac{6}{25}(V + D)^2$ whenever $K < \frac{3V - 2D}{5}$.

Sketch of the Analysis where a Fraction of Internal Consumers Leave the Market.

Following Figure 3 in the text, the marginal consumer in equilibrium occurs within the gap (as is the case with any symmetric equilibrium). This implies that profits for each firm after exit can be represented as $p_j \left[\frac{1}{2} - f \cdot G + (1 - f) \frac{p_{-j} - p_j}{2} \right]$. Equilibrium prices and profits are then $p_1 = p_2 = \frac{1 - 2f \cdot G}{1 - f}$ and $\Pi_1 = \Pi_2 = \frac{(1 - 2f \cdot G)^2}{2(1 - f)}$. These profits are always higher than the profits with no exit (equivalently, when $f = 0$) whenever $G < \frac{1 - (1 - f)^{1/2}}{2f}$. However, there are two non-local deviations that must be considered for this equilibrium to hold. First, one of the firms may deviate to a lower price such that the marginal consumer is located on the far side of the "exit gap" from the firm. In such a case, the profit of the deviating firm would be $p_j \left[\frac{1}{2} - 2f \cdot G + \frac{\frac{1 - 2f \cdot G}{1 - f} - p_j}{2} \right]$. Solving this reveals that the deviation prices and profits are $\frac{2 - f - 6f \cdot G + 4f^2 G}{2(1 - f)}$ and $\frac{(2 - f - 6f \cdot G + 4f^2 \cdot G)^2}{8(1 - f)^2}$, respectively. The deviation profits are greater than the equilibrium profits of $\frac{(1 - 2f \cdot G)^2}{2(1 - f)}$ whenever $G < \frac{2 - 3f + 2f^2 - 2(1 - f)^{3/2}}{2f(5 - 8f + 4f^2)}$.

However, this deviation assumes that the price is low enough that the marginal consumer is located more than a distance G from the center of the line. This occurs whenever $\frac{\frac{1 - 2f \cdot G}{1 - f} - \frac{2 - f - 6f \cdot G + 4f^2 G}{2(1 - f)}}{2} > G$, which holds whenever $G < \frac{f}{4 - 6f + 4f^2}$. Because

$\frac{2-3f+2f^2-2(1-f)^{3/2}}{2f\left(5-8f+4f^2\right)} < \frac{f}{4-6f+4f^2}$, the binding constraint for our theorem is that $G \geq \frac{2-3f+2f^2-2(1-f)^{3/2}}{2f\left(5-8f+4f^2\right)}$. In principle, the firms could also deviate non-locally by raising their prices to a level where the marginal consumer is located on the close side of the "exit gap" to them; however, such a deviation is never binding since it would earn a profit of $\frac{(2-f-2f\cdot G)^2}{8(1-f)^2} < \frac{(1-2f\cdot G)^2}{2(1-f)}$ given that $f \leq ¾$ and G < ½.