Modeling the Sales and Customer Equity Effects of the Marketing Mix

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Abstract

This article demonstrates how the critical components of customer equity, viz. acquisition and retention rates, may be derived from readily available sales transactions data. Thus we can develop marketing-mix models that investigate the customer equity implications of product-marketing decisions. An application in the luxury-automobile category reveals that sales effectiveness and customer equity effectiveness may be quite different, and that marketing actions that are sales effective may have an adverse impact on a brand’s customer equity.
Introduction

Customer equity (CE) has emerged as a new paradigm for guiding managers to build strong and profitable relationships between the customer and the firm. The framework regards customers as a valuable asset that should be managed like any other financial assets (Blattberg and Deighton 1996). Given its emphasis on a customer-centered marketing strategy, it is not surprising that the CE paradigm has been applied successfully in relationship-marketing businesses. Examples include insurance (Jackson 1989), newspaper and magazine subscriptions (Dwyer 1997; Keane and Wang 1995), cellular phone plans (Bolton 1998), airline pilot memberships (Thomas 2001), and interactive television entertainment services (Lemon, White and Winer 2002).

However, in most industries managers make their marketing decisions based on product-performance metrics such as sales transactions and market share. As noted by Rust, Lemon and Zeithaml (2001), “Business success is based on customer relationships; however, because customer equity is difficult to measure, many companies continue to focus on metrics that capture product-based strategies rather than metrics that capture customer-based strategies.” It is not clear a priori that these product-performance metrics are aligned with customer equity development. For example, promotion campaigns that are known to be sales effective and therefore heavily utilized could erode the quality of the customer relationship over time (Jedidi, Mela and Gupta 1999). Thus a focus on product sales and profits in marketing resource allocation could inadvertently erode customer equity in the long run.

Is it possible to infer customer equity metrics from sales transaction records? Advances in information and database technology allow us to identify the customer origin of sales transactions, and therefore to derive customer loyalty metrics. For
example, since 2003, J.D. Power collects and disseminates customer loyalty data for all automobile brands, by tabulating what fraction of each brand’s sales comes from previous brand owners. Their 2004 survey revealed retention levels ranging from 5% to 60%, so the inter-brand variability is very high.

These database developments create an opportunity to assess marketing’s impact on customer equity components as distinct from product sales. When marketing programs are aimed at addressable customers (Blattberg and Deighton 1991), such as catalogues, emails, and customized coupons, it is relatively easy to examine their impact on CE-related metrics such as acquisition and retention rate. In contrast, when mass-marketing efforts (e.g., broadcast advertising) are used, their CE impact is inherently more difficult to derive.

The main objective of this paper is to propose statistical models that can measure the impact of marketing efforts on CE using traditional and readily available product-based data (e.g., sales volume). We first discuss the existing CE literature and show why the current study is important for both managers and researchers. We then develop an appropriate model to examine the dynamic relationship between marketing efforts and the components of CE. Next, we apply our model to the automobile industry as an empirical illustration. We use the empirical results to discuss managerial implications by simulating CE dynamics for two selected brands, in function of changes in their marketing mix. Conclusions, limitations, and a further research agenda are discussed in the last section.
Customer Equity and Its Components

Customer equity is generally defined as “the sum of the discounted lifetime values of all customers” (e.g., Blattberg, Getz and Thomas 2001; Rust, Zeithaml and Lemon 2000), though each element of the definition can be interpreted or measured in a somewhat different way (Berger and Nasr 1998). For instance, “all” customers could include the firm’s current customers as well as expected future customers. If the CE definition only includes already-acquired customers and ignores future acquisitions, then we obtain static customer equity (SCE) (Singh and Jain 2003).

A frequently used SCE metric is due to Blattberg and Deighton (1996). They suggested a simple formula for SCE under fixed acquisition rate \( a \), retention rate \( r \), acquisition cost \( A \), retention cost \( R \), contribution margin \( m \), and time discount rate \( \delta \) such that:

\[
SCE = am - A + a \left( m - \frac{R}{r} \right) \frac{r}{1 + \delta - r}
\]

However, without considering future acquisitions and within-customer interactions (e.g., word-of-mouth effects), customer equity may be underestimated. Therefore, some researchers include not only the value of current customers but also that of expected future customers in their CE calculation (e.g., Gupta, Lehmann and Stuart 2004; Singh and Jain 2003). In this dynamic customer equity (DCE) approach, customers are regarded as renewable resources (Drezé and Bonfrer 2003). In what follows we will use the dynamic definition of customer equity and use CE and DCE interchangeably. In
the Appendix, Equation (1) is extended to allow for future customer acquisitions, and we use this DCE equation [Equation (11)] in a numerical simulation of customer equity.

**Acquisition Rate and Retention Rate**

Among the components of customer equity, acquisition rate and retention rate are particularly important because they are *market response* variables that can be related to marketing spending. However, acquisition and retention rates are difficult to obtain in a product-marketing environment since these metrics are customer-centric as opposed to product-based.

Acquisition and retention rates have been modeled in different ways in the CE literature. They have been assumed to be fixed (e.g., Berger and Nasr 1998) or time varying (e.g., Gupta, Lehmann and Stuart 2004) metrics. Some researchers treat them as exogenous from marketing actions (e.g., Wang and Splegel 1994), while others consider them as endogenous variables affected by the firm’s marketing effort (e.g., Blattberg and Deighton 1996). We propose to model acquisition and retention rates as *time-varying endogenous variables*, since our goal is to investigate the dynamic relationship between the firm’s marketing actions and CE via acquisition and retention rates. Therefore, we regard acquisition and retention rates as metrics through which the firm’s CE performance is captured.

**The Impact of the Marketing Mix on Customer Equity**

Blattberg and Deighton (1996) measure the impact of marketing on CE by decision calculus, assuming one-to-one relationships between acquisition/retention spending and acquisition/retention rates, respectively. Their approach is subject to two important limitations. First, it does not recognize the dynamic relationship between marketing
spending and acquisition/retention. The economics and marketing literature (e.g., Heckman 1991; Seetharaman and Chintagunta 1998) show that customer purchase behavior is not zero order. Similarly, marketing spending levels may depend on past decisions and market responsiveness (Dekimpe and Hanssens 1995). Second, their approach excludes other factors that impact CE. For example, if increased customer satisfaction leads to higher retention, acquisition rates may increase as well due to enhanced word-of-mouth from satisfied customers. Ignoring these forces will likely result in an underestimation of the impact of marketing on CE.

Rust, Lemon and Zeithaml (2001; 2004) provide a more comprehensive approach to assessing the impact of the firm’s marketing efforts on CE. The firm’s CE is calculated by individual-level Markov switching matrices that are linked to each customer’s utility function. The authors measure the impact of marketing variables on CE based on stated responses in a consumer survey. This approach offers the advantage that a wide range of marketing activities can be considered (for example, significant service quality improvements). However, survey results are known to be only weakly associated with actual purchasing behavior (Morrison 1979). Furthermore, this approach is necessarily static in nature. By contrast, we will focus on using longitudinal sales transactions data that are readily available to management.

**Model Development**

We propose to link product-based metrics such as sales volume and customer-based metrics such as retention/acquisition rate by a sales decomposition. When time series of
these sales decompositions are available, the dynamic relationship between the marketing mix and CE components can be investigated by an econometric time-series model.

**Measuring Acquisition Rate and Retention Rate**

Acquisition rate can be defined as “number of acquired prospects (new customers) divided by total number of prospects,” and retention rate can be defined as “number of retained customers divided by total number of existing customers” in a fixed time interval, for example a week. In practice, these rates are not easy to obtain, particularly in product-marketing businesses such as consumer durables. First, if customers have different inter-purchase intervals, then managers cannot use calendar time as a unit for calculating these rates. Second, the denominators are difficult to observe in many cases as product-oriented companies typically do not have accurate information about their total number of existing customers and especially their prospects at any point in time.

We propose using *conditional* acquisition and retention rates to make them tractable from sales transactions data. The acquisition rate for a focal brand at time $t$ can be decomposed as:

$$a_t = \frac{N_{t}^{AP}}{N_{t-1}^{P}} = \frac{N_{t}^{AP}}{N_{t}^{PRO}} \frac{N_{t}^{PRO}}{N_{t-1}^{P}} = \frac{S_{t}^{AP}/Q_{t}^{AP}}{S_{t}^{PRO}/Q_{t}^{PRO}} \frac{N_{t}^{PRO}}{N_{t-1}^{PRO}}$$

(2)

where $N$ is the number of prospects; $S$ is sales; and $Q$ is purchase quantity. The superscript $P$ stands for total prospects, $AP$ for acquired prospects, and $PRO$ for prospects who purchase the product category at a particular point of time. Therefore, given
purchase quantity and purchase incidence ratio, we can approximate the acquisition rate 
(a) by this conditional acquisition rate (acq) such that:

\[ a_t \propto acq_t = \frac{S^A_P}{S^P} \]

By the same logic, the retention rate (r) can be approximated by the conditional retention 
rate (ret) such that:

\[ r_t \propto ret_t = \frac{S^R_C}{S^C} \]

where the superscript RC stands for retained customers, and CUS stands for customers 
who purchase the product category at a particular point of time.

The purchase quantity ratio and the incidence ratio are assumed to be constant in 
this paper, although their components are allowed to fluctuate. These are reasonable 
assumptions, particularly for consumer durables in established markets. In such 
categories, customers typically purchase only one unit at a time, therefore, the purchase 
quantity ratio will be relatively constant. Second, once a category has diffused in the 
target market, a stable fraction of customers are expected to make a category purchase in 
each time period. Therefore, the incidence ratio may also be assumed to be constant over 
time. These two constant-ratio assumptions eliminate the need to count the total number 
of prospects or customers at every point in time. These assumptions should be relaxed 
either when category demand fluctuates highly over time (e.g., in the diffusion phase of a 
new technology) or when a firm’s marketing activities have a significant impact on the
category incidence rate (e.g., when consumer promotions have strong category expansion effects).

The conditional acquisition and retention rates can be measured by examining a simple Markov switching matrix as shown in Figure 1 (see Rust, Lemon and Zeithaml 2004 for a similar approach). Each brand sales \( S^O \) can be decomposed in two parts: sales originating from retained customers \( S^{RC} \) and from acquired prospects \( S^{AP} \).

Likewise, competitors’ total sales \( S^C \) are generated from two sources: sales originating from the focal brand’s lost customers \( S^{LC} \) and from lost prospects \( S^{LP} \). With these decomposed sales, we can calculate the conditional acquisition rate and retention rate since

\[
\begin{align*}
S^\text{PRO}_t &= S^\text{AP}_t + S^\text{LP}_t \\
S^\text{CUS}_t &= S^\text{RC}_t + S^\text{LC}_t
\end{align*}
\]

(5)

In sum, given a purchase quantity and purchase incidence ratio, the acquisition rate and retention rate can be inferred by investigating sales originating from prospects and existing customers. Therefore, Equations (3) and (4) are the central connections between sales performance and the components of customer equity.

Insert Figure 1 about here

The Long-Run Impact of Marketing Efforts on Acquisition and Retention

We investigate the marketing mix effects on acquisition and retention rates by a vector-autoregressive (VAR) model (Dekimpe and Hanssens 1999), for several reasons. First,
the model can capture the long-run aspects of the relationship among these variables. Since the measurement of CE should consider future acquisition/retention processes, the model should be able to separate short-term and long-term effects. Moreover, the VAR model is capable of dealing with evolving variables that do not have fixed means, trends and variances. Second, the VAR model can also capture possible indirect effects among the variables. For example, acquisition rates may be affected not only by advertising, but also by retention rates, insofar as existing customers generate word-of-mouth to future prospects. Impulse response function (IRF) analysis quantifies such complex dynamic interactions among the variables in the VAR model. Third, due to parsimonious estimation and intuitive metrics (e.g., IRF elasticity), the VAR model can be easily understood and adapted by managers (Little 1979).

Sales Model vs. CE Model

The starting point for our CE model is the VAR sales response model proposed by Dekimpe and Hanssens (1999):

\[
\begin{bmatrix}
S_t \\
M_t \\
CM_t
\end{bmatrix} = \begin{bmatrix} c^S \\
c^M \\
c^{CM}
\end{bmatrix} + \sum_{i=1}^{k} \begin{bmatrix}
\psi_{11} \\
\psi_{21} \\
\psi_{31}
\end{bmatrix} \begin{bmatrix}
\psi_{12} \\
\psi_{22} \\
\psi_{32}
\end{bmatrix} \begin{bmatrix}
\psi_{13} \\
\psi_{23} \\
\psi_{33}
\end{bmatrix} \begin{bmatrix}
S_{t-l} \\
M_{t-l} \\
CM_{t-l}
\end{bmatrix} + \begin{bmatrix}
u^S_t \\
u^M_t \\
u^{CM}_t
\end{bmatrix}
\]

where \( S \) is sales volume (units or revenue), \( M \) is a company’s own marketing efforts, and \( CM \) is competitive marketing efforts. This model is easily extendable to multiple marketing mix variables and multiple competitors, and operates in the product-marketing domain. We replace the sales variable by acquisition rate and retention rate to investigate the impact of the marketing mix on CE components, such that:
For example, the impact of own marketing efforts $M$ on the acquisition rate $acq$ – and thereby CE – is investigated by the impulse response of acquisition rate to a shock in own marketing efforts.

We will compare the impact of the marketing mix on acquisition and retention in model (7) with its impact on sales from model (6). In this way, we can investigate to what extent the use of product-focus metrics agrees with or conflicts with the purpose of growing customer equity. In the next section, we apply the proposed model to an empirical dataset and illustrate how the marketing mix effects on CE may be measured via acquisition and retention rates.

**Empirical Illustration**

The proposed model is applied to the automobile industry, for several reasons. First, switching behavior is relatively easily observed in this industry, since most customers trade in their used cars when purchasing a new vehicle. Thus researchers can observe the origin of each transaction (i.e., from customers vs. from prospects) based on this traded-in model information. Second, unlike impulse-buying product categories, an automobile purchase requires a high level of customer involvement, so that the long-run dynamics of acquisition and retention become managerially more meaningful. Third, since automobile manufacturers have both horizontal (e.g., sedan vs. SUV) and vertical (e.g., compact vs.
luxury) product assortments, the CE framework is more relevant than in the case of the single-product firm which has limited opportunity for cross-selling and up-selling.

Data
Weekly transactions data from January 1999 to June 2002 (182 weeks) in the luxury-passenger car product category (including SUVs) are provided by the Power Information Network (PIN). The data are aggregated in terms of traded-in and currently-purchased brands. Following PIN’s definition of the luxury category results in twelve brands, however we discard three brands – Jaguar, Land Rover and Saab – due to an insufficient number of sample transactions. The remaining nine brands, however, cover 92.2% of the luxury category. The focal market consists of 26 regional sub-markets (e.g., Southern California) representing about 70 percent of the U.S. national market.

On the marketing side, the dataset includes price-related variables such as vehicle price, consumer rebates, and annual percentage rate (APR) for financing. These data are supplemented with monthly advertising expenditure data for each brand, provided by Competitive Media Reporting (CMR). The expenditures include virtually all media types including magazines, newspapers, TV, radio, and outdoor advertisements implemented by both manufacturers and dealers. We first calculate daily advertising expenditures for each brand, and then convert daily figures into a weekly data series. Finally, product quality and customer satisfaction data are obtained from three annual studies conducted by J.D. Power and Associates: the Automotive Performance Execution And Layout (APEAL) study, the Initial Quality Study (IQS), and the Vehicle Dependability Index (VDI) study. We do not incorporate distribution data in this application because, in the mature U.S. automobile market, there were no meaningful changes in dealer networks for
the luxury brands during the period under study. Some descriptive statistics of the
variables in the study are presented in Table 1.

The time-series data on acquisition and retention are constructed as follows: we first
decompose the number of transactions for each brand into two origins: purchases by
customers who traded in (1) the same manufacturer’s brand and (2) a different
manufacturer’s brand. For instance, Lexus transactions can be separated into purchases
originating from Lexus or Toyota owners and from others such as Nissan or Ford owners.
We then decompose total competitors’ sales for each brand into two similar origins. For
example, total competitors’ sales of Lexus are divided into sales originating from Lexus
or Toyota customers and from others. In sum, we construct four sales figures for each
brand, labeled as (1) sales originating from retained customers, (2) sales originating from
acquired prospects, (3) sales originating from lost customers, and (4) sales originating
from lost prospects. Figure 2 shows an example of this sales decomposition.

VAR Model Specification

VAR model specification requires a test on the stationarity of each endogenous variable.
We use both the Augmented Dickey Fuller (ADF) test with a null of non-stationarity and
the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test with a null of stationarity to verify
the presence of unit roots in the data. Only when both tests confirm the existence of a
unit root we treat the variable as evolving. The motivation for this conservative approach
is that the conventional ADF test tends to fail to reject the null of non-stationarity and
therefore over-estimates the existence of a unit root (Kwiatkowski et al. 1992). When
more than one variable in a VAR system is found to be evolving, we implement
Johansen’s cointegration test to capture a possible long-run equilibrium among the
evolving variables.

Based on the results of unit-root and cointegration tests, VAR systems are
specified with corresponding adjustments on the endogenous variables. If a variable is
found to be non-stationary, first-differenced forms are used to prevent spurious
regressions (Granger and Newbold 1974). If evolving variables are cointegrated, error-
correction terms are added to the VAR model to measure the system’s adjustment toward
the long-run equilibrium (Dekimpe and Hanssens 1999). For instance, when all variables
are non-stationary with a cointegration relationship, the CE response model can be
specified in matrix form as:

\[
\Delta Y_t = c + \sum_{i=1}^{k} \Phi_i \Delta Y_{t-i} + \Psi Z_t + \Lambda X_{t-1} + \epsilon_t
\]

where

\[
Y_t = (ACQ_t, RET_t, DIS_t^o, ADV_t^o, DIS_t^c, ADV_t^c)
\]

\(ACQ_t\): acquisition rate

\(RET_t\): retention rate

\(DIS_t^o\): own discounting index

\(ADV_t^o\): own advertising expenditure

\(DIS_t^c\): competitive discounting index

\(ADV_t^c\): competitive advertising expenditure
\[ Z_t = (SD_t, NPI_t, CS_t) \]

- \( SD_t \): seasonal dummy variable
- \( NPI_t \): new-product introduction dummy variable
- \( CS_t \): product quality and customer satisfaction index

\[ X_{t-1} = \left( e_{t-1}^{ACQ}, e_{t-1}^{RET}, e_{t-1}^{DIS}, e_{t-1}^{ADV}, e_{t-1}^{ADV^*} \right) \] error terms from the cointegrating equation

- \( \epsilon, \Phi, \Psi, \Lambda \): coefficient matrices

\[ e_t = \left( e_t^{ACQ}, e_t^{RET}, e_t^{DIS}, e_t^{ADV}, e_t^{ADV^*} \right) \sim N(0, \Sigma) \] disturbance terms

\[ \Delta \]: first-difference operator

The product-marketing model would be similarly specified, except it is a five-variable system in which sales volume \((S_t)\) replaces acquisition rate \((ACQ_t)\) and retention rate \((RET_t)\).

An appropriate lag order \((k)\) is selected by comparing Akaike’s Information Criterion (AIC)\(^6\) for each lag with a maximum of twelve\(^7\). The above VAR system is estimated for each brand, which results in nine VAR systems each for the sales-response and the CE components response model. Brand-specific VAR models are widely used in marketing (e.g., Pauwels, Hanssens and Siddarth 2002) and offer a rich set of diagnostics while avoiding over-parameterization.

**Endogenous Variables**

Following Equation (3) and (4), we define the conditional acquisition rate \((acq)\) as the ratio of brand sales from acquired prospects to category sales from total prospects, and the conditional retention rate \((ret)\) as the ratio of brand sales from retained customers to category sales from total existing customers. We then transform the acquisition rate and
retention rate with the logit operator since the variables are bounded between zero and one. Hence,

\[
ACQ_t = \ln \frac{acq_t}{1-acq_t}; \quad RET_t = \ln \frac{ret_t}{1-ret_t}
\]

A discounting index \( DIS^c \) variable is constructed as the price index variable in van Heerde, Leeflang, and Wittink (2002). We first calculate customers’ average monthly payment \( MON \) based on vehicle price \( PRI \), rebate \( REB \), and APR \( APR \) as follows:

\[
MON_t = (PRI_t - REB_t) \cdot \frac{APR_t}{1 - \frac{1}{(1 + APR_t)^\tau}}
\]

where \( \tau \) is the length of payment periods. The ratio of “undiscounted” monthly payments with zero rebate and 7% APR\(^8\) to this “discounted” monthly payments is calculated. Therefore, the lower the monthly payment is, the higher the discounting index variable.

We use each brand’s weekly advertising expenditure as the advertising \( ADV^c \) variable. All monetary values are adjusted by the consumer price index (CPI). For competitive discounting \( DIS^c \) and competitive advertising \( ADV^c \), we use the heaviest discounting and advertising by competitors at each point in time. This specification provides parsimony in both estimation and interpretation, though it lacks a detailed explanation of brand-specific competitive structure within the product category, which is beyond the scope of this paper.
Exogenous Variables

Several exogenous variables are included in the sales response and CE components response VAR system. First, seasonal dummy variables are included to control for unusually-high-demand periods: Memorial Day weekend, Labor Day weekend, and the last month of each quarter (Pauwels et al. 2004). Second, we include step dummy variables to account for the impact of new-product introductions (NPI) by brand. Following PIN’s definition of “new product”, 16 models (e.g., the Acura MDX) launched during the observation period are included in the VAR system. Third, an index variable for product quality and customer satisfaction (CS) is included by taking the average of annual z-scores from the APEAL, IQS, and VDI studies.9

Note that other marketing mix variables can be incorporated in our model without significant changes. For example, if a brand were to make a significant investment to improve its service quality, this policy can be included in the model as a step dummy variable. Alternatively, if sufficient time-series observations are available before and after the policy change, the model could be run twice and the results can be compared directly (see, e.g., Pauwels and Srinivasan 2004 for an application in the sales domain).

Impulse Response Functions

The dynamic impact of one variable on another in the VAR system is analyzed by IRFs. We follow the generalized IRF approach proposed by Pesaran and Shin (1998) and introduced in marketing by Dekimpe and Hanssens (1999). The statistical significance of the IRFs is verified by standard errors obtained by Monte Carlo simulations (e.g., Nijs et al. 2001) with 250 replications. In order to make the results comparable across brands, we calculate IRF elasticities after re-transforming to the levels of the original variables. For
example, the response elasticity of retention rate to an advertising shock means a percent change in retention rate to a percent shock in advertising expenditure.

Results

Unit-root test results

The stationarity of each endogenous variable in the VAR system is verified by the ADF test and confirmed by the KPSS test. Eleven of sixty three variables (17.5%) are found to be non-stationary, showing some interesting patterns. First, the dynamics of product sales do not necessarily coincide with those of customer equity components. For instance, while Audi’s sales volume is found to evolve, the brand’s acquisition and retention rates revert to their long-term means (i.e., they are stationary). This result implies that Audi’s observed sales increases\textsuperscript{10} do not translate to an increase in its acquisition and retention rates. Second, retention rates are found to be stationary for all brands, while some brands’ acquisition rates are evolving. In other words, in this category, some brands manage to improve (or worsen) the efficiency of their customer acquisition, but none of the brands can permanently change the efficiency of their retention.

Lastly, we find only one case of cointegration among Cadillac’s acquisition rate, discounting and advertising. Both trace and max-eigenvalue tests reveal at most one cointegration relationship among the three variables.\textsuperscript{11}

The Impact of Price Discounting on Customer Equity Components

We report the IRF elasticities of sales, acquisition rate and retention rate to a shock in the discounting index in panel A of Table 2. The table distinguishes between short-term effects (in the same week) and long-term effects\textsuperscript{12} (in 13 weeks). Note that the long-term
IRF elasticities combine short-term effects, competitive reactions, lagged response effects, and performance feedback effects. Therefore, occasional sign changes between short-term and long-term effects may occur.

Insert Table 2 about here

As expected from the extant literature (e.g., Tellis 1988), discounting (i.e., price promotion) has a significant short-term impact on sales volume. In addition, six of nine brands have significant long-term effects, and in one case (Mercedes-Benz), this long-term effect is persistent. However, these positive sales response effects may have different consequences for acquisition and retention rate. For example, Lincoln’s significant sales increases due to promotion may be attributed to a better retention of its existing customers, not a higher attraction of new customers to the brand. Therefore, Lincoln’s brand loyalty is sensitive to its discounting policies, which may erode its customer equity. Similarly, Audi’s discounting has no significant impact on sales, but increases its long-term retention rate, i.e., a higher portion of its customers are repeat buyers as a result of discounting.

Overall, some brands’ discounting is relatively effective in increasing acquisition rates (e.g., Lexus) while other brands only improve their retention rates (e.g., Cadillac). We conjecture that these important differences in discounting impact across brands are related to their levels of product quality and customer satisfaction. Figure 3 illustrates how the relative impact of price discounting on acquisition and retention is associated with both initial product quality (measured by IQS survey scores) and reliability.
(measured by VDI survey scores). We observe that discounting by higher-quality brands results in attracting more new prospects (increasing acquisition rates), while discounting by lower-quality brands results mainly in boosting retention rates. If customers are satisfied with a high-quality product, their loyalty is less affected by their current brand’s marketing interventions such as price discounting. Instead, high-quality brands can use discounting more effectively to attract new prospects, resulting in market-share gain opportunities in this mature product category. This conclusion is corroborated by the observation that three Japanese high-quality brands increased their combined annual market share in the PIN markets from 21.8% to 31.8% while two lower-quality U.S. brands decreased theirs from 23.3% to 16.6% during the observation period.

Insert Figure 3 about here

The Impact of Brand Advertising on Customer Equity Components

Panel B of Table 2 shows the impact of advertising on sales volume, acquisition rate and retention rate. Consistent with past literature (e.g., Sethuraman and Tellis 1991), advertising shocks have smaller effects on sales than discounting. Two brands enjoy a significant short-term sales impact, and four brands have a positive long-term impact. As in the case of discounting, the impact of advertising on sales volume may be different from its effect on acquisition and retention rates.

Overall, the predominant impact of advertising on customer equity components is neutral. Even sales-effective advertisers such as Lexus and Lincoln witness no changes in their acquisition and retention rates. These differences illustrate that acquisition and retention rates capture not only top-line performance (i.e., sales from acquired prospects
and retained customers) but also hidden lost opportunities (i.e., lost sales from lost prospects and customers).

In particular, advertising never increases short-term retention rate, which is consistent with Deighton, Henderson and Neslin’s findings (1994) in frequently purchased consumer products categories. In Deighton et al.’s terminology, usage dominance plays a more important role in this market than framing effects of advertising on customer. Several years of experience with a luxury automobile makes the evidence of the product quality unambiguous (Hoch and Ha 1986) so that advertising can hardly affect customer’s judgment of the product. While we find a few significant long-term effects of advertising on retention rate, these may be explained by competitive reactions, performance feedback and word-of-mouth effects.

We also observe that small competitors such as Audi and Infiniti may find their CE components diminished as a result of advertising. This result can be interpreted that for smaller brands, direct persuasive effects of own advertising are dominated by cross effects of competitive reactions. A recent study by Banerjee and Bandyopadhyay (2003) showed analytically that, given a significant portion of customers who have latent repeat-purchase inertia, advertising is not an effective competitive marketing tool for small firms. In contrast, the largest luxury brand, Mercedes-Benz, derives significant benefits from its advertising on sales as well as acquisition rate.

Using these results, one can investigate how the firm’s marketing mix affects its dynamic customer equity (DCE), which is a nontrivial combination of acquisition rate, retention rate, marketing costs, contribution margins and time discount factors. In the
next section, we numerically illustrate how DCE unfolds over time for two different brands as a result of an incremental discounting and advertising initiative.

A Numerical Simulation of Customer Equity

We first derive DCE at time $t$ as a function of conditional retention rate ($r$), conditional acquisition rate ($a$)$^{13}$, and the current number of existing customers ($N_t$), total number of customers in the product category ($N^c$), retention cost per customer ($R$), acquisition cost per prospect ($A$)$^{14}$, category sales ($S^c$), and time discount factor ($\delta$). Assumptions and derivations can be found in the Appendix. As a result, DCE evaluated at time $t$ can be expressed as:

$$DCE_t = \frac{N^c (A - R + rm - am)}{S^c \delta + N^c (1 - r + a)} \left[ N_t + S^c a \right] + \frac{S^c (am - A)}{\delta}$$

Note that this derivation is an extension of Blattberg and Deighton’s (1996) SCE formula [Equation (1)]. It incorporates the dynamic aspects of customer equity as well as a modified analytical solution to Gupta et al.’s (2004) discrete version of customer equity calculation.

We simulate how an unexpected marketing intervention such as an extra rebate or advertising campaign affects the customer equity components for two brands: Acura and Lincoln. We use each brand’s 13-week cumulative impulse responses of acquisition rate and retention rate to a shock in its marketing mix variables in order to derive the CE effects of the marketing mix.
As shown in Table 3, we first calculate weekly sales, retention rate, acquisition rate, and dynamic customer equity under status quo business conditions derived from the data and some assumptions listed in the Table. We then re-calculate the focal metrics 13 weeks after a one-time $2,000 incremental rebate (panel A) or a $1 million extra advertising (panel B) from the IRF elasticity estimates. By comparing these two scenarios (i.e. status quo vs. marketing intervention), we investigate how these aspects of the firm’s marketing mix affect customer equity.

In this numerical illustration, even though Acura has a smaller number of transactions than Lincoln, its customer equity is much higher in the status quo scenario, mainly due to a higher average retention rate (57.0% vs. 28.8%) and acquisition rate (11.6% vs. 4.6%). Implementing the incremental marketing activity affects Acura’s CE in the long run. While an extra rebate is found to decrease the brand’s CE by 1.7%, extra advertising can increase the CE by 2.0%. Note the interesting difference between sales effects and CE effects. While Acura does not increase its long-term sales volume by implementing these extra marketing activities, the brand’s advertising can raise its customer equity.

By contrast, the Lincoln brand fails to increase its CE by discounting or advertising. Thirteen weeks after a discount intervention, the brand’s customer equity is reduced by 2.1%, in spite of a 1.4% increase in sales. This result is mainly due to the negative long-run impact of discounting on acquisition rate. Though Lincoln’s extra...
rebate makes more existing customers repeat buy, it also results in less efficient new
customer acquisition in the long run. Similarly, advertising lifts Lincoln sales levels by
2.3%, but not its acquisition or retention rates, resulting in a slight combined decrease in
its customer equity.

Conclusions

In this paper, we show how product marketers can infer the impact of their marketing mix
efforts on the dynamics of customer equity via retention rate and acquisition rate. Using
a series of switching matrices, we calculate four different sales figures for each brand, viz.
sales originating from existing customers, new customers, lost customers, and lost
prospects. We then construct the time series of two core elements of CE: retention rate
and acquisition rate. The long-run impact of marketing mix efforts on these CE
components is investigated by estimating VAR models and calculating impulse responses
of retention rate and acquisition rate to shocks in the marketing mix.

The main contribution of this paper is to derive customer-equity metrics from
conventional product-based metrics such as sales, and thus to derive the long-term
customer-relationship consequences of various product-marketing actions. The proposed
model enables managers to assess the health of their brands by comparing the impact of
marketing efforts on customer equity vs. sales. By comparing these impacts, managers
can re-allocate their marketing resources to maximize the long-term value of the firm. As
an example in our empirical illustration, even though Lincoln’s discounting has a positive
impact on sales volume, it has a negative long-run impact on customer equity, and thus
should be avoided as much as possible.
The main empirical findings of this paper can be summarized as follows:

- The impact of the marketing mix on customer equity components is different from its impact on sales. In particular, positive sales effects do not necessarily translate into positive effects on acquisition or retention rates.

- Brands differ in the source of their CE change induced by price promotions. Discounting by higher-quality brands results in attracting more new prospects (increasing acquisition rates), while discounting by lower-quality brands results mainly in boosting retention rates.

- Advertising does not have a significant direct impact on retention rate.

- The impact of advertising on CE components is different across brands. Advertising by the market leader has a positive impact on its acquisition rates, while advertising by small-share brands has a negative impact. This suggests a role of advertising as an information source that increases competition among brands.

The applicability of our model is not limited to the automobile industry where, at the time of transaction, customers can be readily identified as brand loyals or brand switchers. Our approach applies equally well to any consumer durable in which brand switching vs. brand loyalty can be assessed at the time of purchase (e.g., through direct inquiry). If all transactions are databased (as done by Dell, for example), even the simple inquiry is not needed. In addition, virtually the entire business-to-business sector would apply as well. Given the additional insights managers can obtain from analyzing the sales-origin data through the proposed model, we recommend that firms invest in collecting and analyzing these data.
This research is subject to some limitations that offer opportunity for further investigation. First, other potentially endogenous drivers of customer equity were assumed to be exogenous in this paper. For example, examining how marketing activities affect future contribution margin for different customer cohorts is an interesting issue (Reinartz and Kumar 2000; 2003). Second, customer equity may be more sensitive to individualized marketing efforts (e.g., loyalty programs) than to the mass marketing efforts that were the focus of this paper (Lewis 2004). If the firm uses different marketing strategies to existing customers vs. prospects, a model may be developed that captures retention and acquisition effectiveness separately for each. Third, different product categories need to be studied. In particular, our model can be extended to fast-moving consumer products by replacing the ownership-based sales decomposition by a share-of-usage decomposition (Yoo 2004). We hope that this and future research will enhance our understanding of marketing’s role in building customer equity.
Table 1
Descriptive Statistics of Data

<table>
<thead>
<tr>
<th></th>
<th>Acura</th>
<th>Audi</th>
<th>Mercedes</th>
<th>BMW</th>
<th>Cadillac</th>
<th>Infiniti</th>
<th>Lexus</th>
<th>Lincoln</th>
<th>Volvo</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weekly Number of Transactions</strong>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>437</td>
<td>152</td>
<td>568</td>
<td>257</td>
<td>247</td>
<td>60</td>
<td>183</td>
<td>255</td>
<td>350</td>
</tr>
<tr>
<td>S.D.</td>
<td>132</td>
<td>47</td>
<td>141</td>
<td>112</td>
<td>68</td>
<td>39</td>
<td>60</td>
<td>82</td>
<td>135</td>
</tr>
<tr>
<td><strong>Vehicle Price ($)</strong>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>26,891</td>
<td>33,631</td>
<td>46,793</td>
<td>40,203</td>
<td>39,626</td>
<td>28,578</td>
<td>37,428</td>
<td>35,734</td>
<td>31,325</td>
</tr>
<tr>
<td>S.D.</td>
<td>1,535</td>
<td>1,211</td>
<td>2,207</td>
<td>1,181</td>
<td>1,927</td>
<td>1,038</td>
<td>1,315</td>
<td>1,025</td>
<td>1,639</td>
</tr>
<tr>
<td><strong>Rebate Offered ($)</strong>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>336</td>
<td>291</td>
<td>574</td>
<td>139</td>
<td>1,757</td>
<td>216</td>
<td>639</td>
<td>2,424</td>
<td>833</td>
</tr>
<tr>
<td>S.D.</td>
<td>984</td>
<td>585</td>
<td>1,256</td>
<td>586</td>
<td>414</td>
<td>418</td>
<td>205</td>
<td>864</td>
<td>595</td>
</tr>
<tr>
<td><strong>APR (%)</strong>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>7.5</td>
<td>6.9</td>
<td>9.0</td>
<td>7.8</td>
<td>5.6</td>
<td>6.6</td>
<td>7.9</td>
<td>4.2</td>
<td>5.9</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.8</td>
<td>1.5</td>
<td>0.8</td>
<td>0.9</td>
<td>1.2</td>
<td>1.0</td>
<td>0.8</td>
<td>1.5</td>
<td>1.9</td>
</tr>
<tr>
<td><strong>Weekly Advertising Expenditure (K$)</strong>**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>3,516</td>
<td>1,966</td>
<td>2,965</td>
<td>2,130</td>
<td>4,162</td>
<td>2,430</td>
<td>4,533</td>
<td>4,826</td>
<td>2,416</td>
</tr>
<tr>
<td>S.D.</td>
<td>806</td>
<td>567</td>
<td>1,277</td>
<td>708</td>
<td>1,759</td>
<td>1,047</td>
<td>1,227</td>
<td>1,522</td>
<td>894</td>
</tr>
</tbody>
</table>

*Power Information Network (PIN) dealer panel data in 26 PIN markets
**Competitive Media Reporting data in the US national market
(Note) Standard deviations (denoted by S.D.) are across weekly means
Table 2
Sales Effects vs. Customer Equity Effects of the Marketing Mix

A. Effects of Price Discounting

<table>
<thead>
<tr>
<th>Brand</th>
<th>Sales Effects</th>
<th>CE Component Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td>Acura</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>Audi</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>Mercedes</td>
<td>0.95</td>
<td>1.78*</td>
</tr>
<tr>
<td>BMW</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>Cadillac</td>
<td>2.63</td>
<td>7.09</td>
</tr>
<tr>
<td>Infiniti</td>
<td>5.47</td>
<td>22.47</td>
</tr>
<tr>
<td>Lexus</td>
<td>5.94</td>
<td>9.13</td>
</tr>
<tr>
<td>Lincoln</td>
<td>2.83</td>
<td>ns</td>
</tr>
<tr>
<td>Volvo</td>
<td>1.62</td>
<td>2.92</td>
</tr>
</tbody>
</table>

B. Effects of Advertising

<table>
<thead>
<tr>
<th>Brand</th>
<th>Sales Effects</th>
<th>CE Component Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short-Term</td>
<td>Long-Term</td>
</tr>
<tr>
<td>Acura</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>Audi</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>Mercedes</td>
<td>0.02</td>
<td>1.35*</td>
</tr>
<tr>
<td>BMW</td>
<td>ns</td>
<td>ns</td>
</tr>
<tr>
<td>Cadillac</td>
<td>0.01</td>
<td>ns</td>
</tr>
<tr>
<td>Infiniti</td>
<td>-0.65</td>
<td>ns</td>
</tr>
<tr>
<td>Lexus</td>
<td>ns</td>
<td>0.35</td>
</tr>
<tr>
<td>Lincoln</td>
<td>0.24</td>
<td>1.47</td>
</tr>
<tr>
<td>Volvo</td>
<td>ns</td>
<td>ns</td>
</tr>
</tbody>
</table>

Notes

- All entries are impulse response functions (IRFs) converted to level-form elasticities.
- “Short-term effects” are contemporaneous effects in the same week.
- “Long-term effects” combine short-term effects, competitive reactions, lagged response effects, and performance feedback effects cumulated over 13 weeks. A sign change between short-term and long-term effects is possible depending on the relative magnitude of these indirect effects.
- Only significant (| t-stat | > 1.65) intermediate IRFs are included in the calculation of long-term effects. Therefore, exact standard errors of long-term effects are not available. However, in a separate analysis using all intermediate (significant and insignificant) IRFs, the cumulative standard errors were calculated and confirmed that the results are directionally and substantively the same as reported above.
- An asterisk (*) denotes that a persistent effect exists, i.e., the percent change in the performance variable is permanent.
Table 3
Customer Equity Simulation

A. Effects of a One-Time $2,000 Incremental Rebate

<table>
<thead>
<tr>
<th></th>
<th>Status Quo</th>
<th>New</th>
<th>%Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Acura</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly Sales (unit)</td>
<td>2,869</td>
<td>2,869</td>
<td>0.0%</td>
</tr>
<tr>
<td>Retention Rate</td>
<td>57.0%</td>
<td>55.7%</td>
<td>-2.2%</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>11.6%</td>
<td>11.6%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Customer Equity (M$)</td>
<td>5,617</td>
<td>5,522</td>
<td>-1.7%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Status Quo</th>
<th>New</th>
<th>%Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lincoln</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly Sales (unit)</td>
<td>3,262</td>
<td>3,307</td>
<td>1.4%</td>
</tr>
<tr>
<td>Retention Rate</td>
<td>28.8%</td>
<td>30.5%</td>
<td>5.9%</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>4.6%</td>
<td>4.3%</td>
<td>-5.5%</td>
</tr>
<tr>
<td>Customer Equity (M$)</td>
<td>2,383</td>
<td>2,334</td>
<td>-2.1%</td>
</tr>
</tbody>
</table>

B. Effects of a One-Time $1 Million Incremental Advertising Expenditure

<table>
<thead>
<tr>
<th></th>
<th>Status Quo</th>
<th>New</th>
<th>%Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Acura</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly Sales (unit)</td>
<td>2,869</td>
<td>2,869</td>
<td>0.0%</td>
</tr>
<tr>
<td>Retention Rate</td>
<td>57.0%</td>
<td>58.3%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>11.6%</td>
<td>11.6%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Customer Equity (M$)</td>
<td>5,617</td>
<td>5,728</td>
<td>2.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Status Quo</th>
<th>New</th>
<th>%Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lincoln</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weekly Sales (unit)</td>
<td>3,262</td>
<td>3,338</td>
<td>2.3%</td>
</tr>
<tr>
<td>Retention Rate</td>
<td>28.8%</td>
<td>28.8%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Acquisition Rate</td>
<td>4.6%</td>
<td>4.6%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Customer Equity (M$)</td>
<td>2,383</td>
<td>2,381</td>
<td>-0.1%</td>
</tr>
</tbody>
</table>

Notes
- Customer Equity is calculated using Equation (11) under the following assumptions:
  - Total number of customers in the product category \(N\) is 5 million and stationary
  - Weekly category sales \(S\) are 25,000 and stationary
  - Contribution margin \(m\) is 10% of vehicle price
  - Time discount factor \(\delta\) is 0.2% per week
  - A priori acquisition cost per prospect \(A\) is $20, and a priori retention cost per customer \(R\) is $10. These costs will be different \(a posteriori\) (cost per acquired prospect and per retained customer), depending on acquisition rate and retention rate, respectively.
- Figures in the “Status Quo” column are based on sample averages in the database, except weekly sales which were adjusted to U.S. national sales levels.
- Figures in the “New” column are computed 13 weeks after the marketing interventions.
Figure 1
A Switching Matrix for Sales Decomposition

<table>
<thead>
<tr>
<th></th>
<th>Focal Brand</th>
<th>Competitors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Past Purchase</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Focal Brand</td>
<td>Retained customers</td>
<td>Lost customers (= New prospects)</td>
</tr>
<tr>
<td>Competitors</td>
<td>Acquired prospects (= New customers)</td>
<td>Lost Prospects</td>
</tr>
</tbody>
</table>

*Current Purchase*
Figure 2
Sales Decomposition Example

Total Sales

Sales from Retained Customers

Sales from Acquired Prospects

Sales from Lost Customers

Sales from Lost Prospects
Figure 3
Product Quality and the Relative Impact of Price Discounting on Acquisition and Retention*

A. Initial Quality

\[ y = 3.60x + 1.48 \]
\[ R^2 = 0.59 \]

B. Reliability

\[ y = 5.46x + 1.97 \]
\[ R^2 = 0.62 \]

* Acquisition and retention effects are long-term effects cumulated over 13 weeks. Lincoln is an outlier in terms of the relative impact of discounting (-23.09) and is excluded from the regression.
Appendix
Derivation of Dynamic Customer Equity

To specify the relationship among retention rate, acquisition rate and customer equity, we assume that:

- Category sales ($S_c$) are stationary
- Total number of customers ($N_c$) in the product category is stationary
- The number of customers who purchase in the product category is proportional to the size of the customer pool for each brand
- The average purchase quantity is unity and is constant over time

First, brand sales volume at time $t$ ($S_t$) can be expressed by retention rate and acquisition rate as

$$S_t = S_t^{RC} + S_t^{AP} = S_t^{CUS} r_t + S_t^{PRO} a_t$$

$$= S^c \frac{N_{t-1}}{N^c} r_t + S^c \left(1 - \frac{N_{t-1}}{N^c}\right) a_t$$

where

- $S_t^{RC}$: sales from retained customers
- $S_t^{AP}$: sales from acquired prospects
- $S_t^{CUS}$: category sales from existing customers
- $S_t^{PRO}$: category sales from prospects
- $r_t$: retention rate
- $a_t$: acquisition rate
- $N_t$: number of a focal brand's customers

Second, profit at time $t$ ($V_t$) can be expressed as,

$$V_t = (S_t^{RC} + S_t^{AP})m - R S_t^{CUS} - A S_t^{PRO}$$

$$= S_t^{RC} m + S_t^{AP} m - R S_t^{RC} \frac{r_t}{a_t} - A S_t^{AP}$$

$$= S^c \frac{N_0}{N^c} r_t m + S^c \frac{(N^c - N_0)}{N^c} a_t m - S^c \frac{N_0}{N^c} R - S^c \frac{(N^c - N_0)}{N^c} A$$
where

- $m$: contribution margin
- $R$: A priori retention cost per customer
- $A$: A priori acquisition cost per prospect

Third, number of customers can be measured by,

$$N_t = N_{t-1} - \frac{S^c N_{t-1}}{N^c} (1-r_t) + S^c \left(1 - \frac{N_{t-1}}{N^c}\right) a_t$$

$$= \left(1 - \frac{S^c}{N^c} (1-r_t + a_t)\right) N_{t-1} + S^c a_t$$

Fourth, customer equity under fixed retention and acquisition rate can be calculated as follows;

$$V_1 = S^c \frac{N_0}{N^c} rm + S^c \frac{(N^c - N_0)}{N^c} am - S^c \frac{N_0}{N^c} R - S^c \frac{(N^c - N_0)}{N^c} A$$

$$= \left\{ S^c \frac{N^c}{N^c} (m-R+A) - S^c \frac{N^c}{N^c} (1-r+a)m \right\} N_0 + S^c (am - A)$$

Let

$$\theta = \frac{S^c}{N^c} (m-R+A) \quad \sigma = \frac{S^c}{N^c} (1-r+a)$$

then

$$V_1 = (\theta - \sigma m)N_0 + S^c (am - A)$$

Likewise,

$$V_2 = (\theta - \sigma m)N_1 + S^c (am - A)$$

$$= (\theta - \sigma m) \left[ (1-\sigma)N_0 + S^c a \right] + S^c (am - A)$$

$$= (\theta - \sigma m)(1-\sigma)N_0 + S^c a(\theta - \sigma m) + S^c (am - A)$$

since
$$N_t = \left(1 - \frac{S^c}{N^c}(1 - r + a)\right)N_{t-1} + S^c a$$

$$= (1 - \sigma)N_{t-1} + S^c a$$

Therefore,

$$V_t = (\theta - \sigma m)(1 - \sigma)^{t-1}N_0 + S^c a(\theta - \sigma m)\left[1 + (1 - \sigma) + \ldots + (1 - \sigma)^{t-2}\right] + S^c (am - A)$$

$$= (\theta - \sigma m)(1 - \sigma)^{t-1}\left[N_0 - \frac{S^c a}{\sigma}\right] + \frac{S^c a(\theta - \sigma m)}{\sigma} + S^c (am - A)$$

$$V_\infty = \frac{S^c a(\theta - \sigma m)}{\sigma} + S^c (am - A)$$

$$= S^c a\left(\frac{S^c}{N^c}(m - R + A) - \frac{S^c}{N^c}(1 - r + a)m\right)$$

$$= \frac{S^c}{N^c}(1 - r + a) + S^c (am - A)$$

$$= S^c a\frac{m - R + A}{1 - r + a} - S^c A$$

Finally, dynamic customer equity evaluated at time $t$ with a time discount factor $\delta$ will be,

$$DCE_t = \sum_{i=1}^{\infty} \frac{V_{t+i}}{(1 + \delta)^i}$$

$$= \sum_{i=1}^{\infty} (\theta - \sigma m)(1 - \sigma)^{i-1}\left[N_i - \frac{S^c a}{\sigma}\right] + \frac{S^c a(\theta - \sigma m)}{\sigma} + S^c (am - A)$$

$$= (\theta - \sigma m)\sum_{i=1}^{\infty} \frac{(1 - \sigma)^{i-1}}{(1 + \delta)^i} + \frac{S^c a(\theta - \sigma m)}{\sigma} + S^c (am - A)\sum_{i=1}^{\infty} \frac{1}{(1 + \delta)^i}$$

$$= (\theta - \sigma m)\left[N_i - \frac{S^c a}{\sigma}\right] \frac{1}{\delta + \sigma} + \frac{S^c a(\theta - \sigma m)}{\sigma} + \frac{S^c (am - A)}{\delta}$$

$$= \frac{\theta - \sigma m}{\delta + \sigma}N_i + \frac{S^c a(\theta - \sigma m)}{\sigma\delta} - \frac{S^c a(\theta - \sigma m)}{\sigma(\delta + \sigma)} + \frac{S^c (am - A)}{\delta}$$

$$= \frac{\theta - \sigma m}{\delta + \sigma}N_i + \frac{S^c a(\theta - \sigma m)}{\delta + \sigma} + \frac{S^c (am - A)}{\delta}$$

$$= \frac{\theta - \sigma m}{\delta + \sigma}\left[N_i + S^c a\right] + \frac{S^c (am - A)}{\delta}$$

where
\[
\frac{\theta - \sigma m}{\delta + \sigma} = \frac{S^c}{N^c} \frac{(m - R + A) - S^c(1 - r + a)m}{\delta + \frac{S^c}{N^c} (1 - r + a)}
\]

\[
= \frac{m - R + A - m + rm - am}{\frac{S^c}{N^c} \delta + 1 - r + a}
\]

\[
= \frac{N^c(A - R + rm - am)}{S^c \delta + N^c(1 - r + a)}
\]

Therefore,

\[
DCE_i = \frac{N^c(A - R + rm - am)}{S^c \delta + N^c(1 - r + a)} \left[ N_i + S^c a \right] + \frac{S^c (am - A)}{\delta}
\]
Endnotes

1 Since most studies define CE as a summation of customer lifetime values (CLV), we include the studies on CLV in the CE literature.
2 Customer Retention Study by J.D. Power. For more information, see www.jdpower.com.
3 By brands we mean make-level names such as Lincoln or Lexus. The firm-level names such as Ford or Toyota will be called manufacturers, and product names within brands such as the Lincoln LS or the Lexus RX300 will be called models.
4 This manufacturer-based data construction enables us to derive CE implications at the firm level, including retention and up-selling. As an illustration, Lincoln-to-Lincoln repeat purchases as well as Ford-to-Lincoln up-sell purchases are beneficial to Ford as a manufacturer.
5 For a more detailed discussion of unit-root and cointegration tests, see Dekimpe and Hanssens (1999) or Nijs et al. (2001).
6 Lütkepohl (1985) showed that the Schwarz Criterion (SC) is superior in lag order recovery and forecast accuracy for low ordered (<3) VAR models. However, in higher-lag systems the AIC may outperform the SC in forecast precision (Lütkepohl 1993). We use the AIC in this application because the data are weekly and marketing’s impact is known to extend for more than two weeks.
7 We checked alternative model specifications with a different maximum lag order of eight and inclusion of a deterministic trend variable. As a result, the current model specification (i.e., max-lag of twelve without trend) is selected since it minimizes the Schwartz Criterion the most frequently.
8 7% APR is used as a proxy for the prime rate.
9 The average z-score is used to avoid collinearity since these three scores have inter-correlations between 0.36 and 0.58.
10 The coefficient of the trend variable in the unit-root test equation for Audi is significant and positive.
11 Detailed unit-root and cointegration test results are available from the first author upon request.
12 Note that only significant IRFs with | t-stat | > 1.65 are included in the long-term effects calculation.
13 For ease of exposition, this section uses the notation r and a for conditional retention and acquisition rate, respectively.
14 These costs are a priori and will be different a posteriori (cost per acquired prospect and per retained customer), depending on acquisition rate and retention rate, respectively.
References


