Obfuscation, Learning, and the Evolution of Investor Sophistication*

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September 1, 2009

Abstract

Investor sophistication has lagged behind the growing complexity of retail financial markets. To explore this, we develop a dynamic model to study the interaction between obfuscation and investor sophistication. Taking into account different learning mechanisms within the investor population, we characterize the optimal timing of obfuscation for financial institutions who offer retail products. We show that educational initiatives that are directed to facilitate learning by investors may induce producers to increase wasteful obfuscation, further disorienting investors and decreasing overall welfare. Obfuscation decreases with competition among firms, since the information rents from obfuscation dissipate as each firm attracts a smaller market share.

*We would like to thank Nittai Bergman, Tony Bernardo, Michael Brennan, Bhagwan Chowdry, Doug Diamond, Darrell Duffie, Paolo Fulghieri, Xavier Gabaix, Rick Green, Mark Grinblatt, Joel Hasbrouck, Leonid Kogan, David Laibson, Francis Longstaff, Holger Mueller, Lubos Pastor, Dick Roll, Andrei Shleifer, Eduardo Schwartz, Avanihdar Subrahmanyam, Sheridan Titman, Pietro Veronesi, Pierre-Olivier Weill, Bilge Yilmaz, and seminar participants at the University of Chicago, NYU, MIT, Columbia University, University of Washington, Northwestern University, University of Southern California, Yale University, UCLA, the Texas Finance Festival, and the annual Netspar Pension Management Conference at Tilburg University.

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1 Introduction

The menu of offerings for retail financial products has grown steadily over the last twenty years, and the shear number of choices is now daunting.[1] While such proliferation may add value in completing markets, it may also adversely affect consumer sophistication.[2] Newcomers in the market have more to learn when they make their initial choices, and incumbent participants bear a higher burden to keep up with developments in the product market. Moreover, it remains unclear whether having access to more options leads to better decisions, as participants often make suboptimal choices in the face of too much information (e.g. Iyengar, Huberman, and Jiang, 2004; Salgado, 2006; Iyengar and Kamenica, 2008).

The interaction between the number/attributes of product offerings and the evolution of consumer sophistication, therefore, introduces an externality that new products and changing product mixes slow learning and may preserve industry rents for producers. This practice can be used strategically by producers and has been termed obfuscation by Ellison and Ellison (2008).[3] Indeed, many new products do not depart much from old ones, and are redundant even within the lines of specific producers. There are straightforward strategic considerations at play: as Christoffersen and Musto (2002) point out, financial institutions often offer several classes of products to price discriminate among investors of varied levels of sophistication. Discrimination through such purposeful distortions in transparency is an important source of firm value.

The purpose of this paper, then, is to explore the dynamic relationship between obfuscation and sophistication in retail financial markets, taking into account that learning mechanisms within the investor population play an important role. We specifically address the following questions: How often do firms optimally practice obfuscation, given that consumers learn over time? How do specific learning processes (e.g. learning from others, learning from periodicals) affect these dynamics? What effect do competition and participation have on obfuscation? What are the policy implications for educational initiatives and regulation in financial markets?

To address these questions, we begin by analyzing a retail market in continuous time in which a monopolist markets a class of financial products (say, a class of mutual funds or a line of credit cards) to a heterogeneous group of consumers. Consumers are either experts, in which case they

[1] For example, as of 2007, there were 8,029 mutual funds to choose from and 21,631 different share classes (Investment Company Institute 2008)

[2] Many participants in the market have limited sophistication regarding the products in the market (e.g. NASD Literacy Survey 2003). See also Capon, Fitzsimons, and Prince 1996; Alexander, Jones, and Nigro 1998; Barber, Odean, and Zheng 2005; Agnew and Szykman 2005.

[3] For static theoretical models of obfuscation, see Ellison and Wolitzky (2008) and Carlin (2009).

always choose the optimal product within the offering, or they are non-experts, in which case their sophistication waxes and wanes based on learning mechanisms and changes in the product market. When non-experts are informed (i.e., sophisticated), either through access to experts or public signals, they get the best deal. However, when they are uninformed, they may either pay too much or not get the best quality. As such, the monopolist earns higher rents from consumers who are unsophisticated.

Sophistication evolves according to a general learning process, which is a differential equation with commonly known initial conditions. When the monopolist changes his product mix, the population is "refreshed" so that sophistication returns to its initial level and learning begins again. In essence, then, there are three groups of consumers in the market: experts who are always sophisticated, non-experts who become sophisticated transiently, and non-experts who remain unsophisticated. Controlling the balance of sophisticated and unsophisticated non-experts is at the heart of the monopolist's problem. As such, the monopolist maximizes profits by deciding how frequently to alter his product mix, affecting the learning process.

The problem that the monopolist faces is stationary and therefore, in equilibrium there exists a unique optimal time to change the product mix. The optimal time is strictly decreasing in the extra rents gained from unsophisticated consumers (compared to monopoly rents) and strictly increasing in the cost incurred in doing so. The intuition is that the more the monopolist gains from unsophisticated consumers, the higher is the benefit from refreshing the population and keeping consumers in the dark. On the other hand, the more costly it is for the monopolist to do this, the lower is the benefit from refreshing the population. These comparative statics have straightforward cross-sectional empirical implications. For example, our analysis predicts that we should observe more product changes and redundancy among classes of mutual funds with higher price dispersion.[4] To our knowledge, this prediction is novel and has yet to be tested.[5]

The relationship between the frequency of obfuscation and the speed at which non-experts become sophisticated is non-monotonic, however. The intuition is as follows. If non-experts learn very quickly, then the gains to refreshing the population are short-lived. Given that there is a fixed cost of changing product attributes, it may not be worthwhile to change the product mix as frequently. Moreover, when non-experts are very slow to learn and the extra rents gained from

[4] As Hortacsu and Syverson (2004) note, the amount of price dispersion varies among different classes of funds, which likely indicates a difference in the sophistication (i.e. search costs) among consumers within each asset class. See Table 1 in Hortacsu and Syverson (2004).

[5] Testing this prediction might involve correlating price dispersion within groups of homogeneous offerings with number of share classes offered, either cross-sectionally or in a time-series.

unsophistication are long-lived, the monopolist will optimally choose to refresh the population less frequently, again because there is a cost to doing so. This phenomenon has welfare implications for educational initiatives that the government may undertake. If the population learns relatively slowly, improving the learning process marginally will actually decrease welfare, as it increases the frequency of wasteful obfuscation.[6] Small educational initiatives that increase the speed of learning are only likely to increase welfare when non-experts already learn sufficiently fast.

The type of learning that takes place in the market also affects obfuscation and the policies that are set. Based on the information percolation model of Duffie and Manso (2007), we analyze two specific settings: one in which consumers learn by themselves (e.g. by reading periodicals) and one in which they learn from each other. In the former setting, as the initial set of experts rises, the frequency of obfuscation by the monopolist decreases. This occurs because there is less to gain from refreshing the population. Educational initiatives that improve the level of expertise in the market are always welfare enhancing in this case because increasing expertise lowers obfuscation. When investors learn from each other, however, we obtain different results. The comparative statics and policy considerations are very similar to the effect of learning speed. This non-monotonic relationship can be appreciated as follows. When no one is an expert initially, there is no one to learn from, and therefore the monopolist never changes their product offerings. When everyone is an expert initially, then there is no gain to refreshing the population. Therefore, obfuscation only takes place when a fraction of the population has expertise and there is a non-monotonic relationship between expertise and the frequency of obfuscation.

We extend our analysis to consider other welfare effects of obfuscation. Indeed, in the base model, the only cost to society is the cost that the monopolist incurs when he refreshes. There are many other potential costs to society such as non-participation, the cost of learning, or the misallocation of resources. In the paper, we extend the model to consider another source of welfare loss: the opportunity cost of becoming an expert in the first place. We analyze how expertise arises endogenously in the market, given that the monopolist has an incentive to obfuscate and maximize profits, and we quantify the welfare loss that arises due to obfuscation.

Finally, we consider the effect of competition on obfuscation. We show that increased competition should slow obfuscation. The reason for this is that there is less to gain for each firm when they refresh the population. In essence, the information rents that firms gain by refreshing dissipate

[6] Policies designed to assist consumers have the adverse effect to induce the monopolist to refresh more frequently. This causes consumers to fall further behind, making the task of keeping up more costly. Absent such marginal initiatives, the monopolist would also save on the costs of wasteful innovation.

with more competition. This improves welfare as the side effects from obfuscation decrease: fewer consumers pay the opportunity cost of becoming an expert, more consumers participate, and the misallocation of resources decreases.

Lack of sophistication regarding financial decisions degrades personal welfare as well as economic growth. For example, lack of sophistication is frequently cited as a cause of the recent financial crisis: many home owners did not appreciate the variable-rate clauses in their mortgages and their explicit exposure to interest rate risk. Likewise, failure of many home owners to appreciate the fees and interest rate schedules used commonly in credit cards has led to a record-setting amount of household debt and a growing number of personal defaults in the U.S.

Recently, the U.S. government has seriously considered credit card reform that limits the ability of credit card companies to dynamically change interest rates and fees. For example, it has been proposed that interest rates not be raised without 45 days notice, consumers be given time to approve or reject interest rate changes, double-cycle billing be banned, and fees for paying bills on-line or by phone be outlawed. However, many experts expect that if such reform takes place, credit card companies will respond in kind by shortening grace periods and increasing annual fees to maintain industry profits. That is, despite the restrictions that would be set to protect consumers going forward, the firms would have other dimensions that may be used to preserve industry profits.

Other solutions to protecting consumers in retail financial markets have also recently received much attention, though the optimal solution remains hotly debated. While Lusardi and Mitchell (2007) argue that education improves consumer welfare, Choi, Laibson, and Madrian (2008) show that consumers appear to ignore fees when making decisions, even when they are given salient information about the importance of taking fees into consideration. In lieu of a large-scale educational effort, there is now growing support for the use of default options to assist retail consumers and improve welfare (e.g. Choi, Madrian, Laibson, and Metrick 2004). While not specifically modeled in our paper, default options would in essence make more consumers experts (by proxy) and may decrease obfuscation, especially when used on a grand scale or in markets in which people learn on their own. Libertarian paternalism as posed by Thaler and Sunstein (2003) makes sense in our model, as it would slow obfuscation and encourage participation. Investigating this debate is the subject of future research.

Finally, our paper is also of general economic interest as it adds a new and important dimension to an already extensive literature on oligopoly competition with consumer search (e.g. Diamond (1971), Salop and Stiglitz (1977), Varian (1980), Stahl (1989), Gabaix and Laibson (2006)). Indeed, in many existing models, consumers search for the best alternative, but the firms are unable to

affect the search environment except through the prices they choose. Few notable exceptions are papers by Robert and Stahl (1993), Carlin (2008), and Ellison and Wolitzky (2008). Robert and Stahl (1993) analyze a model of sequential search in which firms may advertise to consumers in the population. Carlin (2008) and Ellison and Wolitzky (2008) analyze a static model in which firms simultaneously choose whether to add complexity to their pricing schedules. The analysis that we consider here departs from the above papers in several ways. First, our model is fully dynamic so that we can characterize how obfuscation evolves over time. Second, our model is general in that it can account for innovation in both prices and product characteristics. Finally, in our analysis we are able to characterize how different types of learning affects both obfuscation and policies that govern these markets.

The rest of the paper is organized as follows. In Section 2, we pose a dynamic model of obfuscation and consumer sophistication, given that the producer in the market is a monopolist. In Section 3, we characterize optimal obfuscation by the monopolist, and evaluate the effect that different learning models have on welfare and policy considerations. In Section 4, we analyze how expertise arises endogenously. In Section 5, we consider the effect that competition has on obfuscation. Section 6 concludes. All of the proofs are contained in the Appendix.

2 The Model

Consider a market in which a monopolist sells a class of retail financial products to a unit mass of household consumers. Time evolves continuously, and future cash flows are discounted at an interest rate r. The products could be a group of instruments used to finance the purchase of consumption goods (for example, a line of credit cards) or a set of investment vehicles that are available to maximize lifetime utility (for example, a family of mutual funds).

Consumers in the market have unit demand for the product. Based on the information that they have, each consumer chooses within the class of offerings to maximize their expected payoff. At $t = 0$, consumers are divided into two groups: experts x_0 and non-experts $y_0 = 1 - x_0$. Experts costlessly acquire information about the products in the market, and are able to quickly adapt to changes in the market and the product line. At any time t, experts always make the best choice. In contrast, non-experts are less discriminating. When they are uninformed, they purchase one item in the product line randomly.[7] When they become informed transiently through learning,

[7]Random purchase by non-experts is standard in the literature on all-or-nothing search (e.g. Salop and Stiglitz, 1979; Varian, 1980), but is not necessary to derive the results that follow in this paper. For example, using a sequential search model as in Stahl (1989) would generate the same lack of discriminating behavior.

they mimic the experts and choose the optimal good. This knowledge is fragile, however, as non-experts are unable to keep up with changes that occur in the market or product line. As such, the sophistication level of non-experts may wax and wane, whereas experts are always sophisticated. For now, we take the values of x_0 and y_0 as given exogenously, but we consider their evolution endogenously in Section 4.

As time evolves, non-experts may learn either through interaction with experts or through access to public signals (e.g., reading periodicals), which we specify shortly. At any time t, the fraction of sophisticated consumers x_t in the population is composed of the initial fraction of experts plus non-experts who have become informed through interaction. The remaining population y_t is the group of non-experts who remain unsophisticated. The monopolist earns profits $\pi(x_t, y_t)$ from its consumers at each instant in time. Because unsophisticated consumers are less able to choose the most advantageous product, the rents that the monopolist captures are larger for these consumers.[8] To capture this, we consider that at any instant, the firm's profit is

$$\pi(x_t, y_t) = ax_t + by_t,$$

where $b > a$. The profit a represents the rent that is gained from selling the product to informed consumers, whereas $b - a$ is the added gain from selling to unsophisticated consumers.[9] As sophistication increases, the added gains to the monopolist diminish.

Without monopolist intervention, the fraction x_t of sophisticated consumers in the population evolves according to the differential equation

$$dx_t = f(\lambda, x_t)dt, \tag{1}$$

with initial values x_0 and y_0. By construction, $dy_t = -dx_t$. We assume that $f(\lambda, x_t)$ is continuous, strictly positive, and increasing in λ. Therefore, without a change in the product offerings, x_t increases over time while y_t decreases.

[8] For example, within a group of homogenous funds (e.g. money funds), experts will always choose the one with the lowest fees, whereas non-experts may sometimes invest in funds with higher costs, and therefore not get the best deal. This holds generally in our model on many product dimensions. For example, within a group of funds that are heterogeneous across multiple dimensions, sophisticated consumers will choose the one that offers the most value, whereas unsophisticated consumers may not.

[9] In Section 5, when we study competition, the rents obtained from sophisticated and unsophisticated consumers arise endogenously. The assumption that $b > a$ can also be considered a Nash bargaining solution in which the monopolist gains a larger fraction of the trade surplus from unsophisticated consumers than from sophisticated consumers. In this sense, both experts and non-experts are rationally willing to participate in the market. This assumption is consistent with the search literature (e.g. Varian, 1980; Stahl, 1989) and the literature in which the timing of sales is used to discriminate between well-informed and ill-informed consumers (Salop, 1977; Conlisk, Gerstner, and Sobel, 1984; Rosenthal, 1982; Sobel, 1984).

The evolution equation (1) is flexible enough to incorporate several important forms of learning.[10] At each instant in time, the rate at which investors become informed is a function of the fraction x_t of informed investors in the population at time t, and of λ, which parameterizes how easy it is for investors to access public information or learn from each other and thus measures the speed of learning. For now, we consider a general learning process and characterize the optimal behavior by the monopolist. Later, in Section 3.2, we consider more specific models of learning and contrast the effects that they have on the monopolist's behavior and economic welfare.

The monopolist may alter learning by changing their product attributes (i.e. obfuscation). Specifically, we assume that the monopolist can at any time t refresh population learning by returning the fraction of sophisticated and unsophisticated consumers to their initial levels x_0 and y_0. The monopolist pays a cost c to do so, and chooses their timing optimally to maximize lifetime profits. As such, the times in which the monopolist changes their product mix or their prices are given by the vector $T = (t_1, t_2, t_3, \ldots)$. For any $t \in [t_i, t_{i+1}]$, the dynamics of sophistication evolves according to the differential equation (1) with $x_{t_i} = x_0$ and $y_{t_i} = y_0$. The fraction x_0 catches on quickly when the monopolist makes changes, but the fraction y_0 does not: whereas non-experts may become informed for a while, their sophistication is fragile as they depend on the learning process in (1).

The problem that the monopolist faces may thus be posed as

$$\sup_{T \in \mathcal{T}} \int_0^\infty e^{-rt} \pi(x_t, y_t) dt - C(r, T) \tag{2}$$

where $C(r, T)$ sums the discounted lifetime costs of obfuscation according to the plan T. We characterize the solution to this problem in the next section.

Before doing so, though, it is instructive to consider a practical example of obfuscation. Consider a firm that offers two homogeneous funds with different fee structures. Sophisticated consumers invest in the one with lower fees, whereas unsophisticated consumers choose one of the them randomly. As time progresses and unsophisticated investors learn about the fund with lower fees, fewer investors demand the fund with higher fees. The firm may then add a third fund of the same type (or a new class of shares), that have low fees and progressively raise the fees of the "old" low-fee fund. Such pricing behavior has been documented previously in money funds by Christoffersen and Musto (2002). The result is that expert investors will catch on quickly to this strategy and switch funds, while a new breed of unsophisticated customers evolves. What our model captures, therefore, is the idea that consumers need to keep up with innovation in prices and quality in order

[10] See Duffie and Manso (2007) for a model of information percolation in consumer markets.

to continue to get the best deal. Other examples might include different funds (or share classes) that sell for the same price, but have quality differences that require different investment on the part of the firm. The model as posed is general to consider heterogeneity on multiple dimensions.

3 Obfuscation and Sophistication

We begin by solving the monopolist's obfuscation problem and characterize its solution in generality, given that learning proceeds according to (1). Then, we consider specific examples of learning processes and contrast the obfuscation that takes place in each case. Following this, we discuss several welfare and policy considerations that arise based on the type of learning that takes place in the market.

3.1 Optimal Obfuscation

The monopolist's problem is stationary. That is, after the monopolist obfuscates, he faces a problem that is isomorphic to the one he faced at $t = 0$. The following proposition relies on dynamic programming techniques to simplify the monopolist's problem.

Proposition 1. *An optimal $T = (t_1, t_2, t_3, \dots)$ that solves (2) is such $t_{i+1} - t_i = t_i - t_{i-1}$ for any i. Therefore, the monopolist's problem reduces to choosing the duration t^* of each cycle:*

$$\max_t \frac{\int_0^t e^{-rs}\{ax_s + b(1 - x_s)\}ds - e^{-rt}c}{1 - e^{-rt}}. \tag{3}$$

where x_s evolves according to (1), the learning process without obfuscation.

According to Proposition 1, we may focus on the solution to the problem in (3) to derive the optimal plan $T^* = (t^*, 2t^*, 3t^*, \dots)$, which is stationary. As such, the population of sophisticated consumers evolves according to (1) from x_0 to x_{t^*}, until the monopolist refreshes and the process begins again. In the proof of Proposition 1 in the appendix, we show that such a stationary plan is superior to any arbitrary control sequence that changes over time (non-stationary).

The next proposition characterizes the solution to (3).

Proposition 2. *(Optimal Obfuscation) There exists a unique optimal stopping time $t^* > 0$ that solves the monopolist problem. If*

$$c < \bar{c}(\lambda, x_0) \equiv \lim_{t\to\infty} \frac{r\int_0^t e^{-rs}\{ax_s + b(1 - x_s)\}ds - \{ax_t + b(1 - x_t)\}}{r} \tag{4}$$

then the optimal stopping time t^* is finite and solves

$$(1-e^{-rt})\{ax_t + b(1-x_t)\} - r\int_0^t e^{-rs}\{ax_s + b(1-x_s)\}ds + rc = 0. \tag{5}$$

Otherwise, $t^* = \infty$. Moreover, for $t^* < \infty$,

(i) $\frac{\partial t^*}{\partial b} < 0$

(ii) $\frac{\partial t^*}{\partial a} > 0$

(iii) $\frac{\partial t^*}{\partial c} > 0$

According to Proposition 2, obfuscation takes place more frequently when the additional rents that are gained from unsophisticated consumers are higher ($b-a$ high) and when the cost is lower. That is, if resetting the learning process is more valuable because unsophisticated consumers forfeit significant surplus, then the monopolist will wish to capture these rents as frequently as possible. Since $c > 0$, though, they will not optimally do this continuously, that is, $t_{i+1} - t_i > 0$ for all i. When the cost is higher, the monopolist will wait longer before he incurs this cost, *ceteris paribus*. If the cost is sufficiently high, that is, if $c > \bar{c}$, the monopolist will never change their prices or their product mix.

The relationship between t^* and λ is a bit trickier. We first state the proposition that characterizes this relationship and then describe it intuitively.

Proposition 3. *Suppose that $c < \sup_\lambda \bar{c}(\lambda, x_0)$. If*

$$\lim_{\lambda\to 0} x_t = x_0 \quad \textit{and} \quad \lim_{\lambda\to\infty} x_t = 1 \quad \forall t > 0, \tag{6}$$

then

$$\lim_{\lambda\to 0} t^*(\lambda) = \infty \quad \textit{and} \quad \lim_{\lambda\to\infty} t^*(\lambda) = \infty, \tag{7}$$

and the function $t^*(\lambda)$ is non-monotone in λ.

Proposition 3 can be appreciated as follows. The conditions about a particular learning process in (6) are sufficient for $t^*(\lambda)$ to be non-monotone in λ. The first condition says that as the rate of learning converges to zero, then the fraction of sophisticated consumers remains the same (the initial level) for any fixed time. The second condition says that as the rate of learning converges to infinity, the entire population becomes sophisticated for any arbitrarily small time.

If these two conditions hold for a particular learning process, then $t^*(\lambda)$ is non-monotone in λ. Intuitively, if learning occurs very slowly, the monopolist will make changes to their product line infrequently. In fact, if $\lambda \to 0$, then the monopolist will never make any changes because there is no benefit to paying the cost c. As learning occurs more frequently (higher λ), then the monopolist might want to make changes more quickly to keep resetting the process and stay ahead of consumer sophistication. However, as learning becomes sufficiently fast, there is a diminishing benefit to obfuscation. To see this, consider a limiting case when $\lambda \to \infty$, that is when sophistication evolves instantaneously. In this case, the rents that are gained compared to the monopoly rent a are negligible because they are short-lived. Since, $c > 0$, the optimal strategy for the monopolist is never to make changes. Therefore, when $\lambda \to 0$ and when $\lambda \to \infty$, we expect $t^* = \infty$, whereas for values of λ in between we may observe a finite t^*. Thus, there exists a non-monotonic relationship between λ and t^*.

Now, we consider the relationship between t^* and x_0.

Proposition 4. *Suppose that* $c < \sup_{x_0} \bar{c}(\lambda, x_0)$. *If*

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0, \tag{8}$$

then $\frac{\partial t^*}{\partial x_0} > 0$. *If*

$$\lim_{x_0 \to 0} x_t = 0 \quad \forall t > 0 \tag{9}$$

then

$$\lim_{x_0 \to 0} t^*(x_0) = \infty \quad \textit{and} \quad \lim_{x_0 \to 1} t^*(x_0) = \infty, \tag{10}$$

and the function $t^*(x_0)$ *is non-monotone in* x_0.

According to Proposition 4, if the learning process has the property in (8), then as x_0 increases, the monopolist obfuscates less frequently. This means that for higher x_0, the rents that are collected from unsophisticated consumers are strictly less, and therefore the benefits to paying the cost c are decreased. The higher x_0 is, the more time passes between cycles. As we will soon illustrate by example, the condition in (8) may exist when consumers learn independently from each other, but does not hold generally for group learning processes.

If the condition in (9) holds, however, then the relationship between t^* and x_0 is non-monotonic. Intuitively, the limit in (9) says that if the fraction of experts tends to zero, that no matter how much time passes, all consumers remain unsophisticated. This condition will hold generally when

consumers learn from each other. If no one is sophisticated in the first place, then there is no one to learn from. In contrast, if consumers learn on their own from accessing information from outside sources, it may be that x_t will increase over time, despite the fact that $x_0 = 0$ initially.

To better appreciate how the conditions in Propositions 3 and 4 affect obfuscation, it is instructive to consider a few examples.

3.2 Learning and Information Percolation

Based on the information percolation model of Duffie and Manso (2007), we analyze two specific settings: one in which consumers learn by themselves and one in which they learn from each other.

Let us first consider a learning process in which consumers learn from their own research. This may occur by reading periodicals, accessing news through the media, or reading a prospectus. More specifically, we assume that an uninformed non-expert learns what is the optimal product choice of a particular product line at a Poisson arrival time with a mean arrival rate (intensity) λ, which is common across uninformed consumers. After this time, the consumer becomes transiently informed and is able to choose the optimal good until there are further changes in the product line.

Relying formally on the law of large numbers, (1) takes the form

$$dx_t = \lambda(1 - x_t)dt. \tag{11}$$

In this process, a fixed proportion λ of unsophisticated consumers become sophisticated at each point in time. Integrating and using the initial condition x_0 yields the solution

$$x_t = 1 - (1 - x_0)e^{-\lambda t}. \tag{12}$$

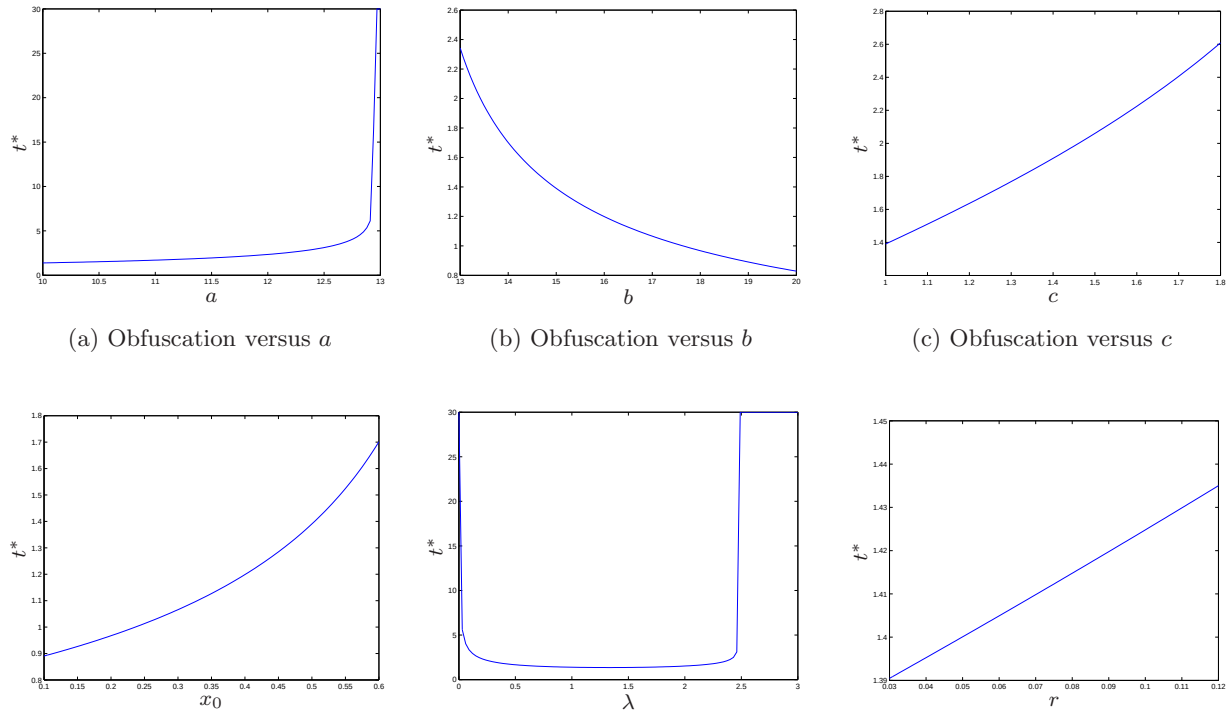
Now, we can consider (12) in terms of the conditions in Propositions 3 and 4. For this process,

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} = -e^{-\lambda t}\lambda, \tag{13}$$

which is always negative. From Proposition 4, we have that the optimal time to obfuscate is increasing in x_0. This makes intuitive sense, since in this example the ability to learn is independent of other consumers' sophistication, so that as x_0 rises the rents available to the monopolist decrease. Obfuscation is less attractive and occurs with decreased frequency.

To show that the relationship between t^* and λ is non-monotone, we compute that

$$\lim_{\lambda \to 0} x_t = x_0$$



(a) Obfuscation versus a (b) Obfuscation versus b (c) Obfuscation versus c

(d) Obfuscation versus x_0 (e) Obfuscation versus λ (f) Obfuscation versus r

Figure 1: Learning On Your Own: The series of figures (a)-(f) plot t^* versus the fundamental parameters in the model, when the learning process involves learning about products through the use of periodicals or media. The time t^* is monotonically decreasing in b and increasing in a, c, x_0, and r. The relationship between t^* and λ is U-shaped. Parameters, when held fixed, are $r = 0.03$, $\lambda = 1$, $x_0 = 0.5$, $a = 10$, $b = 15$, and $c = 1$.

and

$$\lim_{\lambda \to \infty} x_t = 1.$$

Consequently, $t^* = \infty$ when λ approaches 0 or ∞. Intuitively, this implies that if there are no sources of information and consumers do not learn, then there is no incentive for the monopolist to refresh the consumer population. In contrast, if access to media or periodicals allows consumers to educate themselves quickly, then the monopolist will avoid obfuscation because refreshing the consumer population is a futile effort.

Consider the example in Figure 1, in which consumers learn without the help of others. The series of subfigures plot t^* versus the underlying parameters, while holding all else fixed. As predicted by Proposition 2, t^* is strictly decreasing in b and increasing in a and c. As discussed, t^* is strictly increasing in x_0 and is non-monotonic (U-shaped) in λ.

Now, let us consider an alternative learning process in which consumers learn from each other. Indeed, as unsophisticated consumers meet those who are informed, sophistication within the population rises. Such meetings may occur via friends, relatives, co-workers, or advisors. The key factor in these types of learning processes is that the chance that an unsophisticated consumer becomes sophisticated depends directly on the fraction of consumers who are already knowledgeable.

Let us consider a particular example in which consumers meet each other in bilateral meetings. More specifically, we assume that any particular consumer is matched to another consumer at each of a sequence of Poisson arrival times with a mean arrival rate (intensity) λ, which is common across consumers. At each meeting time, another consumer (the counterparty in a meeting) is randomly selected from the population. An uninformed consumer that meets an informed consumer becomes transiently informed about the product line and can choose the optimal good until there are further changes in the product line.

Relying formally on the law of large numbers, (1) takes the form

$$dx_t = \lambda x_t(1 - x_t)dt. \tag{14}$$

This process differs from (11) in that the rate at which market participants become informed depends on the proportion of sophisticated consumers in the market. Integrating and using the initial condition x_0 yields the solution

$$x_t = \frac{x_0 e^{\lambda t}}{(1 - x_0) + x_0 e^{\lambda t}}. \tag{15}$$

Again, we can consider (15) in terms of the conditions in Propositions 3 and 4. With this process,

$$\frac{\partial^2 x_t}{\partial x_0 \partial t} = \frac{e^{\lambda t}\lambda(1 - x_0 + e^{\lambda t}x_0)}{(1 - (1 - e^{\lambda t})x_0)^3}, \tag{16}$$

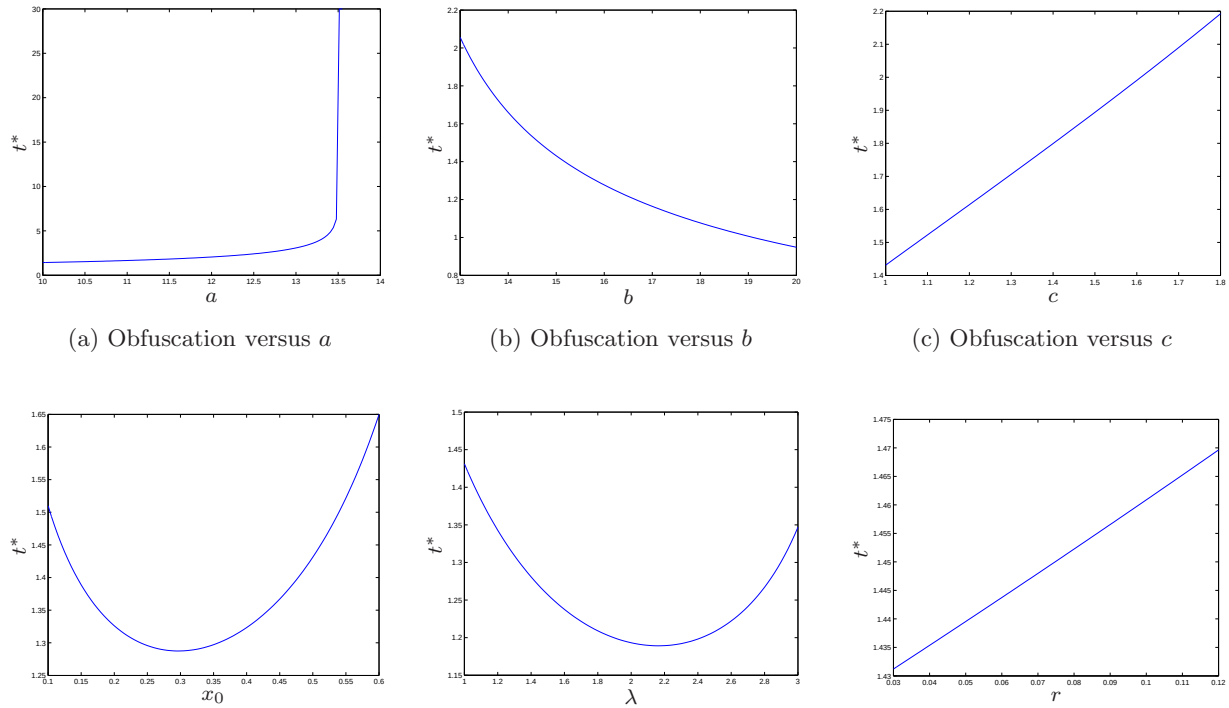
which can be positive or negative depending on t. Therefore, based on Proposition 4, we are not guaranteed that that the optimal time to obfuscate is monotonically increasing in x_0. In fact, we can use the condition in (9) to show that the relation between t^* and x_0 is non-monotone. Specifically,

$$\lim_{x_0 \to 0} x_t = 0. \tag{17}$$

This follows from the fact that if no one is an expert ($x_0 = 0$), there is no one to learn from. It then follows from Proposition 4 that the relationship between x_0 and t^* is non-monotone.

As before, using conditions in (6), we can show that the relation between t^* and λ is also non-monotone. For that, it is enough to note that

$$\lim_{\lambda \to 0} x_t = x_0 \tag{18}$$



(a) Obfuscation versus a (b) Obfuscation versus b (c) Obfuscation versus c

(d) Obfuscation versus x_0 (e) Obfuscation versus λ (f) Obfuscation versus r

Figure 2: Learning From Others: The series of figures (a)-(f) plot t^* versus the fundamental parameters in the model, when the learning process involves learning about products through bilateral meetings between consumers. The time t^* is monotonically decreasing in b and increasing in a, c, and r. The relationship between t^* and λ is U-shaped, as is the relationship between t^* and x_0. Parameters, when held fixed, are $r = 0.03$, $\lambda = 1$, $x_0 = 0.5$, $a = 10$, $b = 15$, and $c = 1$.

and

$$\lim_{\lambda \to \infty} x_t = 1. \tag{19}$$

Consequently, t^* approaches infinity when λ approaches 0 or ∞.

Consider the example in Figure 2, in which consumers learn from each other. The series of subfigures plot t^* versus the underlying parameters, while holding all else fixed. Again, as predicted by Propositions 2-4, t^* is strictly decreasing in b, increasing in a and c, and non-monotonic (U-shaped) in λ and x_0.

Now, we consider how these different learning process may affect welfare in the market and the efficacy of educational initiatives.

3.3 Welfare and Policy Implications

So far in the model, the only source of welfare loss is the cost that the monopolist incurs when he changes his product line. The prices that consumers pay and the extra rents gained from unsophisticated consumers are transfers between the parties to the transaction. A social planner who wishes to maximize welfare in this market, therefore, seeks to minimize the quantity

$$\mathcal{L} = C(r, T),$$

where we recall that $C(r, T)$ is the lifetime costs of obfuscation, given the plan T.

The social planner may consider undertaking initiatives to raise the rate of learning λ or alter the fraction of experts in the market x_0. For example, subsidizing websites to enhance consumer education or legislating initiatives to enhance disclosure might increase the ability for people to learn about the market (i.e. increase λ). Requiring that financial education be an integral part of secondary education would be likely to increase the fraction of experts in the first place (i.e. raise x_0).

The discussion in Section 3.2, though, implies that optimal intervention through policies needs to take into account the way in which people learn. Likewise, the magnitude of intervention is equally important as small scale programs might actually decrease welfare. For example, consider in the two examples discussed that the relationship between t^* and λ is non-monotonic. For low λ, small increases in the speed of learning will decrease the time to obfuscation and will thus decrease welfare.

The key here is that when a social planner considers an initiative to improve consumer sophistication, they need to consider the natural response on the part of the monopolist to maximize rents, given the initiative that is undertaken. In this way, for any λ on the decreasing portion of the curve in Figure 1e or 2e, a small supplement to λ will lead to more obfuscation by the monopolist, which destroys value. Only if the magnitude of intervention is large enough will the market reach the upward sloping portion in which increased access to information leads to lower obfuscation. For low values of λ it may be more effective for the social planner to make learning more difficult, decreasing λ.

The differences in the relationship between t^* and x_0 among the two examples of learning also highlights that the social planner needs to take into account the mechanism by which people learn when they set policy. If people learn from periodicals, increasing education is always welfare enhancing (see Figure 1d) no matter how unsophisticated the population is. This does not hold for learning processes in which people learn from each other (see Figure 2e). If x_0 is low, small

positive increments will induce the monopolist to obfuscate more frequently and destroy welfare. Only larger scale initiatives are able to overcome this loss in value.

So far, in the model, the only loss from obfuscation arises from the cost the monopolist pays to "refresh" the product line. Realistically, though, there are other costs of obfuscation in the market that a social planner needs to consider. First, consumers might incur costs to learn or keep up with changes in the market. That is, the learning process in (1) might proceed with a cost to non-expert consumers. Any policy that would increase obfuscation would increase consumers' reliance on (1), which would cause mounting welfare losses. Likewise, becoming an expert represents an important opportunity cost to society. The more that firms practice obfuscation, the more consumers will rationally choose to pay an opportunity cost (of time and resources) to make sure they get a good deal. We characterize this deadweight loss in the next section on endogenous expertise.

Obfuscation also has an adverse effect on the willingness of consumers to participate in the market. If the market is too confusing, some consumers may choose to just drop out, which represents an important deadweight loss. We previously addressed this issue analytically and refer the reader to a previous version of this paper (cite omitted for now to maintain anonymity of the peer review process). There we consider settings in which participation is exogenously given and those in which participation evolves endogenously. Here, we briefly mention one interesting finding that arises when participation is given exogenously, that is, the fraction ϕ of the potential consumers who participate in the market is known a priori. In this case, the monopolist solves a scaled version of our base problem

$$\max_t \frac{\int_0^t e^{-rs}\phi\{ax_s + b(1-x_s)\}ds - e^{-rt}c}{1-e^{-rt}}. \tag{20}$$

The key result is that for $t^* < \infty$, $\frac{\partial t^*}{\partial \phi} < 0$. That is, participation increases obfuscation. This occurs because there is more value at play in the market and the monopolist has greater incentives to obfuscate more frequently. This result, then, might give a policy maker a reason to pause when encouraging consumers to participate in the market, since they can expect a profit maximizing firm to increase obfuscation as a best response.

Finally, an important cost to society might be that non-experts incur a cost when they misallocate resources when they do not choose products optimally. Indeed, non-participation is a severe form of such misallocation. Our base model is flexible to consider such welfare losses. Specifically, if misallocation increases with obfuscation, then any policy that would increase the incentives for the monopolist to change the landscape more frequently would lead to further welfare loss. As such, an alternative model might include such costs, but this would not change the analysis here

as obfuscation would magnify these welfare losses.

4 Endogenous Expertise

Now, we consider that the fraction of expert consumers x_0 arises endogenously. The timing of the game is as follows. First, each consumer $i \in I$ chooses whether to pay a cost k_i to become an expert and join the x_0 population. Following this, the rest of the game follows in the same fashion as in Section 2. The cost k_i is a one-time cost, and could be considered to be the decision whether to obtain a financial education. Alternatively, it could represent the decision whether to become familiar with a particular sector of the financial system.[11] Consumers in the x_0 pool are experts and know to keep up with developments as time goes on. Those in the y_0 pool may learn about particular products over time, but do not have higher levels of sophistication that are required to make sure that they always get the best deal. Learning takes place as before according to the differential equation in (1).

Suppose that consumers are heterogeneous and the costs to become an expert are distributed over the support $[0, \bar{k}]$ according to a twice continuously differentiable function $G(\cdot)$. Define B as the expected benefit of becoming an expert given the actions of other consumers and the expected actions of the monopolist. Therefore, if $k_i \leq B$, consumer i becomes sophisticated. It follows then that

$$x_0 = G(B).$$

The value of B will depend on x_0 and on the t^* that is chosen by the monopolist based on x_0 and the underlying parameters. Going forward, we define $\overrightarrow{v} \equiv (a, b, c, \lambda)$ to be the parameters that are given exogenously in the model, and keep t^* and x_0 separate since they are determined endogenously. As such, we express the expected benefit to becoming an expert as $B(x_0, t^*, \overrightarrow{v})$. The function B is decreasing in t^* since the rents that the unsophisticated pay decrease when less obfuscation takes place. Also, $\frac{\partial B}{\partial b} > 0$ and $\frac{\partial B}{\partial a} < 0$. Define $\underline{B}$ as the lower limit of B, that is, the benefit to becoming an expert when $t^* = \infty$. We assume that $\underline{B} < \bar{k}$; otherwise, all consumers would become experts leading to an uninteresting interaction.

In any equilibrium of this game, the fraction of expert consumers, denoted x_0^*, is implicitly

[11] An alternative specification of the model would be to allow consumers to make this choice each time the monopolist refreshes their product line. Focusing on Markov Perfect Equilibria, the results would be qualitatively similar to what we derive here. Of course, in such a model, other Nash Equilibria that are not stationary might arise, but this is beyond the scope of our analysis.

defined by

$$x_0^* = G\Big(B\Big(x_0^*, t^*, \overrightarrow{v}\Big)\Big) \equiv H(x_0^*). \tag{21}$$

Going forward, we follow the standard approach of Debreu (1970) and Mas-Colell (1985) and focus on the "regular" equilibria of the game.[12] Such equilibria are robust to small perturbations of the set of parameters, and are therefore locally unique, which allows for meaningful comparative statics. Technically, in our setting, a regular equilibrium is defined as a fraction $x_0^* > 0$ such that $h(x_0^*) \equiv H(x_0^*) - x_0^* = 0$ and $\frac{\partial h}{\partial x_0^*} \neq 0$. By straightforward application of Sard's Theorem and Mas-Colell (1985), other pathologic equilibria may be ruled-out as non-generic. Specifically, it is straightforward to show that, except for a set of parameters having zero measure in the general parameter space, the equilibria that will arise will be regular.[13]

The following proposition proves existence of a fixed point and characterizes the regular equilibria of the game. In doing so, we distinguish equilibria based on their stability in the sense of Vives (1990, 1999, 2005). Specifically, an equilibrium is said to be locally stable at a point x_0^* if there exists a neighborhood around it such that for any initial position x_0 within that neighborhood, the system converges to the point x_0^* according to the function $H(x_0)$.

Proposition 5. *A solution $x_0^* \in (0,1)$ exists for the expression in (21) for any $G(\cdot)$. If $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, there exists a unique $x_0^* > 0$ with the following properties:*

(i) $\frac{\partial x_0^*}{\partial b} > 0$

(ii) $\frac{\partial x_0^*}{\partial a} < 0$

(iii) $\frac{\partial x_0^*}{\partial c} < 0$

If $\frac{\partial^2 x_t}{\partial x_0 \partial t} \geq 0$, there may exist multiple regular equilibria. However, in any stable equilibrium, the same properties hold.

According to Proposition 5, if $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$ then there exists only one equilibrium for the game. This will generally be the case when people learn on their own from periodicals, which was the

[12] Originally, Debreu (1970) and Mas-Colell (1985) focused on "regular" equilibria to characterize general equilibria in exchange economies. Indeed, there are pathologic situations in which the excess demand function $z(p)$ might lead to an infinite number of equilibria, preventing comparative statics exercises. By limiting the focus to regular equilibria and proving that such pathologic cases are non-generic, local uniqueness and differentiability of the equilibria is guaranteed, thereby allowing for comparative statics to be generated.

[13] See Chapter 8 in Mas-Colell (1985) for a thorough discussion of genericity analysis and Carlin, Dorobantu, and Viswanathan (2008) for an application of this in finance. A proof of this statement would follow from the same arguments in the proof of Proposition 2 in Carlin, Dorobantu, and Viswanathan (2008).

case when $f(\lambda, x_t)$ took the form in (11). In other cases, where this condition does not hold (e.g. the process in (14)), we are not guaranteed to have a unique equilibrium. Indeed, two classes of regular equilibria may form: a stable one with the properties specified in Proposition 5 and an unstable variant that has the opposite comparative statics. The proof of Proposition 5 details this distinction.

The comparative statics in Proposition 5 imply that as the rents to the monopolist $b-a$ increase, more consumers will become experts in the first place. This occurs through two channels. First, as $b-a$ increases, unsophisticated consumers will pay higher rents over time, so they are more willing to invest in education. Second, and equally important, rising $b-a$ induces the monopolist to increase the frequency with which they change their product mix, which drives more consumers to gain expertise in the first place. The same relationship holds with regard to the cost c.

Based on this analysis, there is another cost of obfuscation to society. Specifically, a proportion of consumers will have to allocate resources to gain expertise, which represents an opportunity cost. This cost may then be computed as

$$\mathcal{K} = \int_0^B k dG(k).$$

As in Section 3.3, this has important policy implications when a social planner considers initiatives to change learning through λ. As shown there, a change in λ may induce the monopolist to change their product mix more frequently. Looking forward, more consumers will expend resources to gain expertise, which destroys value. If the magnitude of intervention is large enough, though, increased access to information will lead to lower obfuscation and higher welfare.

Of course, throughout the paper so far, we have considered the effects of obfuscation when a monopolist produces in the market. Now, we turn our attention to the effects of competition on obfuscation and welfare.

5 Competition and Obfuscation

Consider now that n homogeneous firms, indexed by $j \in N = \{1, ..., n\}$, each sell a continuum (i.e., a line) of retail financial products. Each product in the market is denoted by $w_{j,k}$, where j designates the provider and $k \in [0, 1]$ is their k^{th} option. There is a unit mass of consumers with unit demand who are willing to forfeit rents of $v > 0$ to purchase a product.[14]

Experts are able to identify the best available products among all the product lines being offered,

[14] For example, if the goods in each product line are homogenous so that the only differentiating feature is price, v is each consumer's willingness to pay for the good.

and they efficiently update their purchase decisions as firms change their product lines over time. Each expert at any time thus demands the best available product as long as the forfeited rents are less than v. If more than one firm offers the best deal, experts will choose randomly among these options. Non-experts can become sophisticated transiently through learning according to the process in (1). When they do so, they learn about the best available products in the market. Each transiently sophisticated investor gets the best deal in the market, and purchases in the same fashion as experts. However, when non-experts are unsophisticated, they purchase randomly from one of the product lines as long as the expected forfeited rents from purchasing randomly from one of the product lines is less than v. As already mentioned, this approach to modeling unsophisticated consumers is standard in both the literature on consumer search theory (e.g. Salop and Stiglitz, 1977, Varian, 1980, and Stahl, 1989) and household finance (e.g. Carlin 2008).[15]

The firms, therefore, design product lines and choose refreshing policies to maximize their profits. The following proposition characterizes an equilibrium for this game.

Proposition 6. *(Competition and Obfuscation) There exists a symmetric equilibrium in which at any time t each firm extracts zero rents from sophisticated consumers and rents v from unsophisticated consumers. Moreover, for any n, there exists a unique optimal stopping time $t^*(n) > 0$ that solves each firm's refreshing problem*

$$\max_t \frac{\int_0^t e^{-rs} v \frac{y_s}{n} ds - e^{-rt} c}{1 - e^{-rt}}. \tag{22}$$

For finite n, $\frac{\partial t^*(n)}{\partial n} > 0$, and as $n \to \infty$, $t^*(n) \to \infty$.

Proposition 6 can be appreciated as follows. Because every firm offers a product line with a continuum of products, they can design product lines to perfectly discriminate between sophisticated and unsophisticated consumers. By introducing a measure zero set of products that yields rents a, and a measure one set of products that yields expected rents b, a firm can assure that only sophisticated consumers will be ever able to find its best available products, forfeiting rents a. With such a product line, unsophisticated investors, who purchase randomly, will forfeit rents b

[15] For example, in models of "all-or-nothing" search (e.g. Salop and Stiglitz 1977 and Varian 1980), unsophisticated consumers are explicitly assumed to choose randomly among firms. In sequential search models, unsophisticated consumers are randomly assigned to their first firm and then choose whether to continue searching for the best alternative. In equilibrium, unsophisticated consumers stop at the first firm, so that they in essence make purchases randomly from the firms. See either Stahl (1989) or Baye, Morgan, and Scholten (2006) for a complete review of consumer search theory.

almost surely.[16]

Competition drives the rents a earned from sophisticated investors to zero. Suppose that in equilibrium the best available product in the market yields rents $a > 0$. One firm could profitably deviate from the equilibrium and produce an even better product that yields a rent of $a - \epsilon$ per consumer, with a small $\epsilon > 0$, capturing the demand of all sophisticated consumers. This might involve decreasing the price of the product, improving quality, or adding an attractive dimension. In classic Bertrand fashion, the ensuing equilibrium involves each firm offering the same superior product at the same level of value, earning zero rents ($a = 0$) from sophisticated consumers.

On the other hand, at each time t, each firm faces a captive demand y_t/n of unsophisticated consumers. To maximize profits without violating the participation constraint of unsophisticated consumers, it is optimal for firms to set $b = v$.

With regard to refreshing policies, each firm's optimal choice is thus qualitatively similar to that when they are a monopolist, except that their problem is a scaled down version of the stationary problem in Section 2. Each firm solves the problem in (22). As compared to the problem in (3), firms obtain zero rents from the sophisticated consumers. Because unsophisticated consumers purchase their goods randomly from any of the n firms, at any time t each firm receives an equal share of demand from unsophisticated consumers, $\frac{y_t}{n}$. Moreover, the rents forfeited by an unsophisticated consumer are equal to v, their reservation value for the product.

As in our previous analysis, there exists an optimal solution (i.e., time to obfuscate) $t^*(n)$ that is strictly increasing in n. Under perfect competition, obfuscation disappears altogether.

One concern that may arise is that the results of Proposition 6 depend on the assumption that obfuscation involves only fixed costs c to each firm, while the rents earned by each firm are decreasing in the number n of firms. However, the results of Proposition 6 are robust to an alternative formulation in which the obfuscation cost incurred by firms each time they refresh their product line is $c(\mu) = c_1 + c_2\mu$, where μ is the market share of the firm. In this case, as long as $c_1 > 0$, obfuscation is decreasing in n and disappears altogether when all consumers become sophisticated as in the benchmark case studied above.

The decrease in obfuscation associated with competition has straightforward welfare implications based on the discussion in previous sections of the paper. Clearly, as obfuscation decreases the incentive to become an expert (an x_0-type) decreases, so that consumers in aggregate incur

[16] Alternatively, the firms could each choose a measure one distribution of rents according to a continuous function $F(\cdot)$ with expected rent of b. Since consumers in this model are risk-neutral, such equilibria are payoff-equivalent to the one described here.

lower costs when they participate in the market. It remains ambiguous, however, the effect that lower obfuscation has on the aggregate costs that firms incur. On an individual basis, $C(r, T)$ is lower when there is more competition, but it is unclear whether $nC(r, T)$ is lower in aggregate.

The results here, though, must be taken with some degree of caution as the model does not admit generality to cover all market settings and conditions. It is intuitive that the information rents that accrue due to obfuscation dissipate when there are more firms. This result is consistent with Robert and Stahl (1993), who show that informative advertising increases with competition. However, it is also possible to consider alternative models in which competition may lead to greater obfuscation. Indeed, Carlin (2009) presents a static model in which complexity rises with competition. In the same way, if λ or x_0 were to decrease with competition because learning is more arduous (i.e., consistent with Carlin, 2009), then obfuscation might increase. Therefore, while the results in Proposition 6 are plausible and intuitive, we do not assert that they are general to all market settings. This is the subject of future research.

6 Concluding Remarks

Many retail investors lack sophistication regarding financial products, but choose to participate in the market. Over time, many learn but are required to keep abreast of developments in the market as they occur. Such changes are endogenously induced by producers in the financial market, and must be taken into account when government-sponsored educational initiatives are implemented.

In this paper, we study the interaction between obfuscation and consumer sophistication in a dynamic setting. We characterize optimal cycles of obfuscation and demonstrate how they change based on primitives in the market: the extra rents available from unsophisticated investors, the baseline financial education that consumers possess, the speed at which learning takes place, and the underlying mechanism in which sophistication evolves. Strikingly, we show that small educational initiatives may induce further obfuscation by producers, which destroys economic surplus. Such wasteful obfuscation is enhanced as more consumers participate in the market. On the other hand, major educational initiatives are effective in protecting consumers and reducing wasteful obfuscation, but may entail high implementation costs. Our results suggest that an alternative way to reduce obfuscation and increase welfare is to increase competition among producers.

The analysis in this paper supports the view that education may not be an effective solution in retail financial markets. As Choi, Laibson, and Madrian (2008) show, retail investors do not make improved investment choices when they have better information about the market. There is

now growing support for the use of default options to assist retail investors and improve welfare (e.g. Choi, Madrian, Laibson, and Metrick 2004). While not specifically modeled in our paper, default options would in essence make more consumers experts (by proxy) and may decrease obfuscation, especially when used on a grand scale or in markets in which people learn on their own. Such libertarian paternalism makes sense in our model, as it would slow obfuscation and encourage participation. Given the welfare impact of such policies, continued exploration appears warranted.

Appendix

Proof of Proposition 1

Let t^* solve (3). Then the discounted profits achieved by the monopolist under the optimal policy are given by:

$$V \equiv \frac{\int_0^{t^*} e^{-rs}\{ax_s + b(1-x_s)\}ds - e^{-rt}c}{1-e^{-rt^*}}. \tag{23}$$

We need to show that there does not exist another policy that achieves a higher discounted profits. Let $T = (t_1, t_2, \ldots)$ be an arbitrary policy. Then, by the above definition of t^*,

$$V \geq \int_{t_i}^{t_{i+1}} e^{-rs}\{ax_s + b(1-x_s)\}ds - e^{-r(t_{i+1}-t_i)}c + e^{-r(t_{i+1}-t_i)}V, \tag{24}$$

for $i = 0, 1, \ldots$ and $t_0 = 0$. Multiplying both sides of the above inequalities by e^{-rt_i} we have

$$e^{-rt_i}V \geq e^{-rt_i}\int_{t_i}^{t_{i+1}} e^{-rs}\{ax_s + b(1-x_s)\}ds - e^{-rt_{i+1}}c + e^{-rt_{i+1}}V \tag{25}$$

Summing the inequalities from $i = 0$ to $i = I$ causes telescopic cancellation on the left-hand side, leaving only

$$V - e^{-rt_{I+1}}V \geq \sum_{i=0}^{I}\int_{t_i}^{t_{i+1}} e^{-rs}\{ax_s + b(1-x_s)\}ds - e^{-rt_{i+1}}c. \tag{26}$$

Taking the limit as I goes to infinity yields the result. ■

Proof of Proposition 2

The derivative of the objective function in (3) with respect to t is:

$$\frac{e^{-rt}\{ax_t + b(1-x_t)\} + re^{-rt}c}{1-e^{-rt}} - \frac{re^{-rt}(\int_0^t e^{-rs}\{ax_s + b(1-x_s)\}ds - e^{-rt}c)}{(1-e^{-rt})^2}. \tag{27}$$

The first order condition is given by:

$$rc + (1-e^{-rt})\{ax_t + b(1-x_t)\} - r\int_0^t e^{-rs}\{ax_s + b(1-x_s)\}ds = 0. \tag{28}$$

The derivative of the left-hand side of (28) with respect to t is equal to

$$-(1-e^{-rt})(b-a)f(\lambda, x_t) \tag{29}$$

which is strictly negative since $b > a$ and $f(\lambda, x_t)$ is strictly positive. Therefore, if there exists a t^* that solves (28), it is unique. Moreover, because (29) is negative, the left-hand side of (28) is positive for $t < t^*$ and negative for $t > t^*$. Consequently, t^* is a global maximum.

If, on the other hand, there does not exist a t^* that solves (28), then the derivative of the objective function with respect to t is always positive, since the left-hand side of (28) is positive for $t = 0$. Therefore, the maximum is at infinity, meaning that it is optimal for the monopolist never to innovate.

Because the expression in (29) is positive and $f(\lambda, x_t)$ is continuous, we can apply the implicit function theorem to prove comparative statics.[17] First, we take the derivative of the left-hand side of (28) with respect to b:

$$(1 - e^{-rt})e^{-rt}(1 - x_t) - re^{-rt}\left(\int_0^t e^{-rs}(1 - x_s)ds\right) \tag{30}$$

which is negative if x_t is increasing in t. Therefore, the optimal time t^* to innovate is decreasing in b. A similar calculation for a shows that the optimal time t^* to innovate is increasing in a.

Next, we take the derivative of the left-hand side of (28) with respect to c:

$$(1 - e^{-rt})re^{-rt} + re^{-2rt} \tag{31}$$

which is always positive. Therefore, the optimal time t^* to innovate is increasing in c. ■

Proof of Proposition 3

If the first limit condition in (6) holds, it is easy to see that there is a λ sufficiently small such that $\bar{c}$ as defined in 4 is lower than c and therefore $t^*(\lambda) = \infty$. Using the same argument and the second limit condition in (6), we have that for λ sufficiently large $t^*(\lambda) = \infty$. ■

Proof of Proposition 4

For the first part of the proposition, we take the derivative of (28) with respect to x_0 to obtain:

$$(1 - e^{-rt})e^{-rt}(a - b)\frac{dx_t(\lambda, x_0)}{dx_0} - re^{-rt}\left(\int_0^t e^{-rs}(a - b)\frac{dx_s(\lambda, x_0)}{dx_0}ds\right) \tag{32}$$

which is negative if $\frac{dx_t(\lambda,x_0)}{dx_0}$ is increasing in t and positive if $\frac{dx_t(\lambda,x_0)}{dx_0}$ is decreasing in t. Therefore, the optimal time t^* to innovate is decreasing (increasing) in x_0 if $\frac{dx_t(\lambda,x_0)}{dx_0 dt}$ is positive (negative).

The second part of the proposition is proved with a similar argument as in the previous proposition. ■

[17] See, for example, Rudin (1976, p. 224).

Proof of Proposition 5

First, we show that if a solution x_0^* exists, it must be that $x_0^* \in (0,1)$. Since $\lambda < \infty$ and $b > a$, it must be that $\underline{B} > 0$. There must exist a fraction of consumers $G(\underline{B})$ such that $k_i \leq \underline{B}$, which implies that x_0^* cannot be zero. Now, suppose that $x_0^* = 1$. Then, $t^* = \infty$ and $B = \underline{B}$. Since $\bar{k} > \underline{B}$, there exists an $i \in I$ such that $k_i > \underline{B}$. Specifically, a fraction $1 - G(\underline{B})$ will not pay the cost k_i. Therefore, it cannot be that $x_0^* = 1$.
Now, we can prove existence of an equilibrium. We know that $H(x_0) > 0$ when $x_0 = 0$ and that $H(x_0) < 1$ when $x_0 = 1$. Therefore, the function $H(\cdot)$ must cross the 45-degree line at least once. Given the continuity of H, there must exist at least one point x_0^* at which (21) holds with equality.

According to Proposition 4, if $c < \sup_{x_0} \bar{c}(\lambda, x_0)$ and $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, then we know that $\frac{\partial t^*}{\partial x_0}$ is positive. Since $H(x_0)$ is decreasing in t^*, this implies that once $H(x_0)$ crosses the 45-degree line from above and never crosses again. Therefore, when $\frac{\partial^2 x_t}{\partial x_0 \partial t} < 0$, the fixed point at x_0^* is unique. Comparative statics follow from using the implicit function theorem.

For convenience, we define the function $\omega(t^*, \overrightarrow{v})$ as

$$\omega(t^*(x_0), \overrightarrow{v}) = H(x_0) - x_0,$$

so that in any equilibrium $\omega(t^*(x_0^*), \overrightarrow{v}) = 0$. If $\frac{\partial^2 x_t}{\partial x_0 \partial t} \geq 0$, then $H(x_0)$ may cross the 45-degree line multiple times, sometimes from above and sometimes from below. In such case, there will not exist a unique x_0^*, but rather two classes of equilibria (Class 1 and Class 2). For those that cross from above (Class 1),

(i) $\frac{\partial x_0^*}{\partial b} > 0$

(ii) $\frac{\partial x_0^*}{\partial a} < 0$

(iii) $\frac{\partial x_0^*}{\partial c} < 0$

This follows from the implicit function theorem and the fact that $\frac{\partial \omega(t^*(x_0^*), \overrightarrow{v})}{\partial x_0} < 0$ at the point of equilibrium. For those that cross from below (Class 2),

(i) $\frac{\partial x_0^*}{\partial b} < 0$

(ii) $\frac{\partial x_0^*}{\partial a} > 0$

(iii) $\frac{\partial x_0^*}{\partial c} > 0$

This follows from the implicit function theorem and the fact that $\frac{\partial \omega(t^*(x_0^*), \overrightarrow{v})}{\partial x_0} > 0$ at the point of equilibrium.

Now, we follow the discussion of Vives (2005, pages 440-445) and show that Class 1 equilibria are stable and Class 2 equilibria are unstable. The function $H(x_0) = G(B(x_0))$ is an aggregate best response function. Note that the aggregate best-response function as defined by Vives (2005) is $r(\tilde{a}) = F(g(\tilde{a}))$, where F is our function G, g is our benefit function B, and $\tilde{a}$ is the fraction of players who take a particular binary action.

Consider a particular Class 1 equilibrium x_0^* and the neighborhood $\mathcal{N}_1 = [x_0^* - \epsilon, x_0^* + \epsilon]$. By the definition of this class of equilibrium, $H'(x_0) < 1$ at $x_0 = x_0^*$. First, choose an arbitrary $x_0 \in \mathcal{N}_1$ such that $x_0 - x_0^* = \delta > 0$. By the definition of a Class 1 equilibrium, it follows that $H(x_0) - H(x_0^*) < \delta$. Since $x_0^* = H(x_0^*)$, we know that $x_0^* + \delta > H(x_0^* + \delta)$ or $x_0 > H(x_0)$, which implies that the cost of becoming an expert exceeds the benefit of doing so for the marginal consumer. Converging toward equilibrium implies that $x_0 \to x_0^*$. In words, since the benefit to becoming an expert is lower than x_0, fewer consumers will become experts. The same can be shown for an arbitrary $x_0 \in \mathcal{N}_1$ such that $x_0 - x_0^* = -\delta < 0$: the system will again converge toward the equilibrium point x_0^*. These two observations together assure that Class 1 equilibria are locally stable (Vives 1999).

Now, we show the opposite for a Class 2 equilibrium. Consider a particular Class 2 equilibrium x_0^* and the neighborhood $\mathcal{N}_2 = [x_0^* - \epsilon, x_0^* + \epsilon]$. By the definition of this class of equilibrium, $H'(x_0) > 1$ at $x_0 = x_0^*$. First, choose an arbitrary $x_0 \in \mathcal{N}_2$ such that $x_0 - x_0^* = \delta > 0$. By the definition of a Class 2 equilibrium, it follows that $H(x_0) - H(x_0^*) > \delta$. Since $x_0^* = H(x_0^*)$, we know that $x_0^* + \delta < H(x_0^* + \delta)$ or $x_0 < H(x_0)$, which implies that the cost of becoming an expert is less than the benefit of doing so for the marginal consumer. In words, more consumers will have an incentive to become an expert. Therefore, the system does not converge back to x_0^*. The same can be shown for an arbitrary $x_0 \in \mathcal{N}_2$ such that $x_0 - x_0^* = -\delta < 0$: the system will again fail to converge toward the equilibrium point x_0^*. Either of these two observations assure us that Class 2 equilibria are not locally stable (Vives 1999). ■

Proof of Proposition 6

Suppose that indeed all firms play the strategy outlined in the statement of the proposition. We will show that there is no profitable deviation available to a firm j.

First, the rents a extracted from sophisticated investors must indeed be zero at any point in time. If a firm j attempted to extract rents $a > 0$, no sophisticated investor would purchase the

inferior good from firm j. Instead they would just purchase it from another firm that offers $a = 0$. Therefore, this is not a profitable deviation for firm j.

Moreover, a firm j cannot extract rents higher than v from unsophisticated investors at any point in time. If that is the case, unsophisticated investors would opt for the outside option in which case each of them only forfeits rents v. Also, attempting to extract rents lower than v from unsophisticated investors can only reduce the profits for firm j, since the demand from unsophisticated consumers for firm j product at time t is fixed at y_t/n as long as the rents unsophisticated consumers forfeit in the market are less than or equal to v.

The problem of setting an optimal obfuscation schedule is symmetric since all n firms earn zero rents from sophisticated consumers and face a captive demand from unsophisticated consumers. As such, the firms all choose an obfuscation schedule given that they receive demand $\frac{x_t}{n}$ from sophisticated consumers and demand $\frac{y_t}{n}$ from unsophisticated consumers. The rents to sophisticated consumers are zero ($a = 0$) and the rents to unsophisticated consumers are positive ($b = v$).

Stationarity of the problem follows in the same fashion as in the proof of Proposition 1. Each firm therefore solves

$$\max_t \frac{\int_0^t e^{-rs} v \frac{y_s}{n} ds - e^{-rt} c}{1 - e^{-rt}}. \tag{33}$$

Uniqueness of a solution $t^*(n)$ is established using the same logic as in the proof of Proposition 2.

Finally, we take the cross-derivative of the objective function with respect to n and t:

$$\left(-(1 - e^{-rt}) e^{-rt} (1 - x_t) + r e^{-rt} \left(\int_0^t e^{-rs} (1 - x_s) ds \right) \right) \times \left(\frac{1}{n^2} \right) \tag{34}$$

which is positive if x_t is increasing in t. Therefore, the optimal time to innovate is increasing in n. Taking the limit as $n \to \infty$ yields the result that $t^* \to \infty$. ■

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