Informed Investors and the Financing of Entrepreneurial Projects*

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Abstract

We consider a model of the financing of a small-business venture in which it is presumed that outside investors have greater expertise in project evaluation than the entrepreneur. We show that entrepreneurs and investors may restrict themselves to debt and junior equity (call-option) contracts without loss of efficiency. A "pecking order" for new ventures is demonstrated, in which entrepreneurs prefer to be financed by junior equity rather than by debt. In addition, the model correctly predicts that large and successful venture-capital firms are likelier to hold debt stakes and makes untested predictions about the lending patterns of specialist banks.

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1 Introduction

The premise of this paper is that investors lending to small businesses may well be more knowledgeable about project quality than the borrowing entrepreneurs. We argue that the empirical evidence suggests that banks and venture-capital firms often have extensive experience and accumulated data that enable them better to judge entrepreneurial ventures than the entrepreneurs themselves. That is, we invert the usual assumption that the entrepreneur has superior information about his project. In this context, we study a particular financing game and analyze the optimal financial contracts between investors and the entrepreneur.

Our first result shows that agents do not suffer economic loss if restricted to debt or junior equity (call-option) contracts. By analyzing how investors' private signals affect the types of contracts that they bid, we show the existence of a "pecking order" in securities in which firms grant an investor's request for junior equity over a second investor's request for debt. In essence, we demonstrate that optimistic investors are allocated small junior equity claims, and that pessimistic investors are given large debt claims. The pecking order proposed here is in direct contrast to the pecking order suggested by Myers (1984). We will argue that our pecking order applies to young firms, while that of Myers is appropriate for more established issuers. Our analysis suggests that the success of venture-capital firms in the U.S. may in part arise from their ability to purchase equity claims in the firms they finance, a flexibility typically not enjoyed by the banks with whom they compete. We interpret our result to suggest that firms' preferences for securities may shift over time as the balance of expertise shifts from outside investors to the entrepreneur. We also discuss an important economic implication of permitting banks to take equity claims in the firms they finance; it is argued that changing bank regulations in this way would lead banks to fund more marginally profitable, but positive-net-present-value, projects.

A second aim of this study is to contrast equilibrium financial contracts in the two cases that the entrepreneur does, or does not, acknowledge that the investors possess expert information. We demonstrate that the choice of financing is linked to the perceived expertise of investors. Specifically, we show that if entrepreneurs are optimistic (as the empirical evidence suggests), then they are more likely to provide a debt contract (and less likely to provide a junior equity contract) to a venture capitalist when they respect his opinion. We also explore a manifestation of the winner's curse in this setting; investors sometimes receive debt claims even though they are more optimistic than the entrepreneur.

We then specialize the model and consider a case in which some investors pursue a pooling strategy,

hoping to instill false optimism in the entrepreneur in order to induce the latter to undertake a marginal project. In this setting, entrepreneurs who disregard the investors' information cannot be manipulated in the way that entrepreneurs who update are. Lastly, our analysis of the loan market suggests a specific pattern in the loan-making activities of specialist and non-specialist banks; banks should make relatively more medium-to-low-interest rate loans and relatively fewer high-interest rate loans in industries to which they specialize in lending.

The notion of expert outside investors described above has been suggested several times in the literature (e.g. Admati and Pfleiderer (1994) and Allen (1993)). De Meza and Southey (1996) construct a model in which self-selecting optimistic entrepreneurs have consistently biased beliefs about project quality, in contrast to a bank, which regards all projects as of average quality. Habib and Johnsen (1997) consider a model in which outside investors provide information to a firm. They do not contrast the entrepreneur's behavior in the two cases that he does, or does not, recognize the expertise of the investors, as we do here.

There is strong empirical support for the proposition that entrepreneurs are not particularly successful at evaluating their projects. Hamilton (1993) provides evidence that entrepreneurs in business for ten years have earnings that are 32 percent lower than would have been expected had they remained in their former jobs. Cooper, Woo and Dunkelberg (1988) indicate that entrepreneurs are excessively optimistic about their prospects, with more than 30 percent of entrepreneurs regarding their future success as certain. Audretsch (1991) adduces evidence that four-year drop-out rates for entrepreneurs are on the order of thirty-five percent. There is also evidence that banks are adept at judging small-business proposals. For example, Reid (1991) shows that firms financed by banks have higher survival rates than those financed by other sources such as family investors. Small-business loans made by banks are typically profitable. However, the expertise of the outside provider of capital is often not recognized by entrepreneurs. Gompers (1994) states that entrepreneurs typically regard venture-capital firms as no more than a source of capital, while venture capitalists themselves claim that they play a strong advisory role in the firms they finance.

The investors in our model regard only the signals of other investors as informative, and ignore the signal of the entrepreneur. It is in this sense that our model includes experts who regard fellow experts as informed, and dismiss the information of non-experts.[1] This assumption is in harmony with the increased tendency on the part of banks to evaluate small-business loans using an automated scoring system.[2] These systems make use of large databases of past loans. Banks are apparently often interested in whether the databases of other banks suggest that the loan at

hand will be profitable. The recognition of the expertise of other banks may also serve as one of the economic motivations for the existence of the syndicated loan market. It may be the case that investors ignore the signals of entrepreneurs because entrepreneurs simply do not know their own type. That is, those with poor prospects will often be truly optimistic about the project. In the extreme, the attitude of the entrepreneur may be an empty signal.[3]

We suggest that the ignorance of the entrepreneur may take one of two forms. He may, as is often asserted of the uninformed in the asymmetric-information literature, be aware of his lack of information and consequently study investors' actions carefully in order to divine their information. In a sense, though, this is a somewhat odd, intermediate form of ignorance on the entrepreneur's part; the entrepreneur may well not regard himself as ignorant at all. He may well believe that his private information is a sufficient statistic for investors' information, and in this case his equilibrium actions are not affected by information that investors reveal in the course of their interactions with him. One may describe this form of ignorance as overconfidence, if in fact the investors do have information that the entrepreneur could profitably use.[4] However, settings in which the entrepreneur's signal is a sufficient statistic for that of the investors may also be accommodated in the framework of this paper.

Our model of the investment game is related to that of Broecker (1990) and Riordan (1993), who consider a first-price auction in bank loans. The focus of those papers is interbank competition, not as here the advisory role of banks; the actions of the entrepreneurs are given much less attention in their models than in the model analyzed in this paper.

In the next section we outline a model for the allocation of a security to an outside investor in a competitive financing environment. We assume, to begin with, that entrepreneurs regard their signals as sufficient statistics for project quality. In Section 3 we discuss the case of entrepreneurs who regard the signals of investors as informative. Section 4 extends the model to one with three, rather than two, investor signals. Section 5 applies the three-signal model to a setting in which entrepreneurs must make an initial investment of effort. Section 6 proposes an application of the model to loan markets. Section 7 concludes.

2 The Model

A firm with no investment capital on hand requires a cash infusion of I in order to undertake a project which will yield a stochastic return $Z \geq 0$ next period. For simplicity, we assume that the interest rate is zero. In order to finance the project, the entrepreneur who owns the firm

requires that an outside investor provide a cash infusion I in exchange for a contract C promising the investor a share of next period's cash flows. (We will presume that the investment is made by one outside investor alone.) Universal risk-neutrality is assumed. We stipulate that a contract $C : \Re_+ \to \Re_+$ is *feasible* if it is such that $C(0) = 0$, and if it possesses these properties:

Property *(i)*. C is weakly monotone increasing.

Property *(ii)*. $z - C(z)$ is weakly monotone increasing in z.

We label the set of feasible contracts γ. The restrictions $(i) - (ii)$ are fairly common in the security-design literature (e.g. Nachman and Noe (1994)), and are justified by reference to certain agency problems present in the relationship between the entrepreneur and the investor. Property *(i)* may be motivated in the following way. If the entrepreneur can make contributions to the firm's cash flows (that is, add to the realization of Z) then he will do so in such a way as to leave the investor with a payoff that is monotone in Z no matter what the original contract form (DeMarzo and Duffie (1999)). Property *(ii)* may be understood by noting that if the entrepreneur can freely dispose of the cash flows of the firm, then he will always do so in a way that makes his residual claim monotone in z. In this sense, agency considerations restrict the investor to claims satisfying the two properties.

We assume that it is common knowledge that Z is distributed according to one of two density functions, f_g or f_b. We presume further that f_g dominates f_b in the sense of the monotone likelihood ratio property (MLRP); that is, the ratio $\frac{f_g(z)}{f_b(z)}$ is well-defined and weakly increasing in z for $z \geq 0$. We further assume that $R = \{z : f_g(z) \neq f_b(z)\}$ has strictly positive Lebesgue measure.[5] The economic actors in the model have different beliefs about the relative probabilities of Z being distributed according to f_g and f_b. We will say that an agent has belief $q \in [0, 1]$ if he believes that Z is distributed according to f_g with probability q. In this case, the density function associated with his belief is given by $qf_g + (1 - q)f_b$. We will denote the expectation of any non-negative measurable function f of Z under the belief q by $E_q(f(Z))$. It is assumed that $E_1(Z) < \infty$.

We remark here that if $q \geq p$ then the density function associated with belief q dominates the density function associated with belief p in the sense of MLRP (see Technical Remark 2).

We assume that two investors bid for the financing of the entrepreneur's project. Each provides the amount I required if his bid is accepted. The entrepreneur initiates the bidding process by announcing a belief $s \in [0, 1]$; as will be discussed later, s not be equal to the entrepreneur's true belief $n \in [0, 1]$. The belief s specifies the *valuation rule* that will be used in comparing the bids of

the two investors. The investors then make simultaneous bids. A bid takes the form of a feasible contract. The bid completely specifies the investor's demand for repayment. We let C_i denote the bid of the i-th investor. Bid One is declared to be the winning bid if $E_s(C_1(Z)) < E_s(C_2(Z))$. Bid Two is declared to be the winning bid if this inequality is reversed. If the bids are valued equally under s, then a winner is selected by tossing an independent fair coin. The entrepreneur observes both bids and then decides whether or not to accept the winning bid. The entrepreneur commits to consider only the winning bid; the losing bid is viewed but discarded. That is, a first-price auction is conducted under the valuation rule, with the lowest valuation bid accepted. In this setting the entrepreneur can only gain from accepting the winning bid, so he will always do so. The valuation rule is a method for comparing contracts of different types.[6] While it is apparent that varying debt contracts may be ranked by their face values, a metric must be proposed for comparing, for example, partial equity contracts with debt contracts.

The equilibria discussed in the main body of this paper are unique in a specific sense (this question is treated explicitly in Result 1). We must introduce some technical terminology in order to make this point in a precise way. This terminology is used mainly to address the issue of uniqueness and is not critical to the other arguments in the paper. We place the uniform metric on the set γ of contracts, and thereby induce the uniform topology (Munkres (1975), p.266). The set of mixed strategies is defined to be the set of Borel probability measures over γ. Let a mixed strategy σ be given. For a given valuation rule s, we may associate a cumulative distribution function $F_{\sigma,s}$ with the strategy σ in the following way. We define

$$F_{\sigma,s}(a) = \sigma(\{\zeta \in \gamma : E_s(Z - \zeta(Z)) \leq a\}).$$

The set $\{\zeta \in \gamma : E_s(Z - \zeta(Z)) \leq a\}$ is closed in the uniform topology, so that $F_{\sigma,s}$ is well-defined. Standard arguments demonstrate that $F_{\sigma,s}$ is indeed a cumulative distribution function (CDF).

We now describe the beliefs of the agents in the model. We recall that there are two possible events regarding the distribution of Z, the good event (G) in which Z is distributed according to f_g, and the bad event (B) in which Z is distributed according to f_b. Each investor receives a noisy signal of project quality (that is, a noisy signal of the state of the world). We assume that the signal can take two values, high (H) or low (L). The event that investor $i \in \{1, 2\}$ receives the signal H is denoted H_i, and L_i is likewise defined. We posit that the signals of the two investors are independent, conditional on the state of the world. We let the distribution of states and signals be described by the probability measure Q and we assume that $Q(G) = \frac{1}{2} = Q(B)$, $1 > Q(H|G) = p > \frac{1}{2}$ and

$Q(L|B) = p$. This probabilistic structure is known to both investors. Straightforward applications of Bayes' Rule give, for $i \neq j$:

$$Q(G|H_i) = p, \qquad Q(G|H_i \cap L_j) = \tfrac{1}{2},$$
$$Q(G|H_1 \cap H_2) = \tfrac{p^2}{p^2+(1-p)^2} =: hh, \qquad Q(G|L_1 \cap L_2) = \tfrac{(1-p)^2}{p^2+(1-p)^2} =: ll,$$
$$Q(H_i|H_j) = p^2 + (1-p)^2 > \tfrac{1}{2} > 2p(1-p) = Q(L_i|H_j).$$

The entrepreneur receives a signal Y_E, a real-valued random variable. An entrepreneur whose realization y_E of Y_E is such that $P(G|Y_E = y_E) = n \in [0,1]$ is referred to as an entrepreneur of type n. We will refer to n as the entrepreneur's belief. In the basic model that we are first considering, the entrepreneur incorrectly regards his signal as a sufficient statistic for the information of investors and disregards their signals.

The analysis will proceed in steps. We will first analyze the equilibrium of the bidding game for a given announcement of s by the entrepreneur. We will then discuss the optimal choice of s.

We denote the winning bid by C. The entrepreneur expects to receive $E_n(Z - C(Z)) \geq 0$ by accepting this bid and zero otherwise, so he will always accept. Investors do not regard the entrepreneur's opinion as informative and are therefore uninterested in n. In this case, given s, n does not play a role in the bidding equilibrium. For convenience of exposition, we will call investors that have received a high signal "optimists" and investors that have received a low signal "pessimists."

For $b \in [0, \infty]$, we define B_b to be the debt contract with face value b. That is, B_b is defined by $B_b(z) = z$ for all $z \leq b$ and $B_b(z) = b$ for all $z \geq b$. For $a \in [0, \infty]$, we define J_a to be the junior equity contract with initial value a. That is, J_a is defined by $J_a(z) = 0$ for all $z \leq a$ and $J_a(z) = z - a$ for all $z \geq a$. We note that all debt contracts and equity contracts are feasible and that B_∞ and J_0 are both equivalent to a full equity contract. We suppose that $E_{ll}(Z) \geq I$; this guarantees that that the project is always financed.[7]

We have the following result:

Result 1. *For each $s \in [0,1]$ there is a symmetric Bayesian-Nash equilibrium of the bidding game. In each equilibrium, pessimists play a pure strategy and optimists play a mixed strategy.*

(i) For $s < ll$, all investors bid junior equity, and the equilibrium is identical for all s in this range.

(ii) For $s \in [ll, \frac{1}{2}]$, pessimists bid debt and optimists bid junior equity.

(iii) For $s \in (\frac{1}{2}, p)$, pessimists bid debt and the optimistic strategy includes both debt and junior

equity in its support. As s increases in this range, optimists bid debt with greater probability.

(iv) For $s \geq p$, optimists and pessimists both always bid debt, and the equilibrium is identical for all s in this range.

In every equilibrium, the bid of an optimist is accepted over that of a pessimist. Furthermore, for any s, if in the bidding equilibrium the support of the optimistic strategy includes both a debt contract B_d and a junior equity contract J_a, then $E_s(J_a(Z)) \leq E_s(B_d(Z))$.

The above equilibria are unique in the following sense. We let s be given and we denote the CDF of the above equilibrium strategy of the optimists by F^O and that of the pessimists by F^P. For every equilibrium in which an optimist plays strategy σ_1 and a pessimist plays strategy σ_2, it must be that $F_{\sigma_1,s} = F^O$ and $F_{\sigma_2,s} = F^P$.

A proof is found in the appendix. Equilibrium strategies are described in the proof of the result.

The intuition for the emergence of junior equity and debt in the investors' strategies is as follows. Suppose that in equilibrium an investor's belief, conditional on having a bid with valuation *val* win the auction, is given by r. Among all the bids with valuation *val*, the investor seeks the contract that maximizes his expected payoff; by doing so he keeps his probability of winning the auction fixed and raises his return if his bid is accepted. If $r \geq s$, then a junior equity contract maximizes $E_r(C(Z))$ subject to $E_s(C(Z)) = val$. This may be understood in the following way. The MLRP dominance of s by r implies that the high-Z states are those most expected under belief r relative to belief s. Junior equity concentrates payments in the high-Z states to the extent allowed under Property *(ii)*, and thereby maximizes the ratio of $E_r(C(Z))$ to $E_s(C(Z))$. If $r \leq s$, an analogous argument shows that the investor does best to bid debt.

We emphasize here that, in all cases, in equilibrium a junior equity contract that is bid is accepted over any debt contract that is bid. Junior equity is bid only by more optimistic investors who are willing to take smaller claims. The transition to junior equity in the optimists' strategy in *(iii)* reflects the fact that as optimists bid smaller contracts, they are increasingly confident, when their bid is accepted, that the other investor is also optimistic. This causes them to be more optimistic about their low bids, so they bid junior equity.

Now that we have determined equilibrium strategies for the investors for any given s, we can proceed to a discussion of which s the entrepreneur will announce, given his belief n. In effect, we are considering a constrained mechanism-design problem here.[8] We first assume that the entrepreneur does not regard the bids of the investors as informative. If the best bid is C, the entrepreneur's ex-

pected profit is thus $E_n(Z - C(Z))$. We define $U_F(n, s)$ to be the expected profit of an entrepreneur of type n if he declares the valuation rule s. As described in the proof of Result 1, this expected profit is uniquely defined except in the case $s = ll$. We set $U_F(n, ll)$ to be the maximal expected profit realized by an entrepreneur of type n when he sets $s = ll$ (this maximum is well-defined).

For a given s, we denote the winning bid by W_s. The equilibrium strategies described in Result 1 specify the probability distribution over γ that is associated with W_s. We denote the entrepreneur's beliefs by a probability measure Q_E on $(\Omega, \mathcal{F})$; the analysis below does not depend on the particular probability measure chosen.

We can decompose $U_F(n, s)$ as follows:

$$U_F(n, s) = E_n[Z - W_s(Z)|L_1 \cap L_2]Q_E(L_1 \cap L_2)$$

$$+E_n[Z - W_s(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)]Q_E((H_1 \cap L_2) \cup (H_2 \cap L_1))$$

$$+E_n[Z - W_s(Z)|H_1 \cap H_2]Q_E(H_1 \cap H_2).$$

We have the following result:

Result 2. *For each $n \in [0, 1]$ there is some $s \in [0, 1]$ that maximizes $U_F(n, s)$.*

(i) For $n \leq ll$, either $U_F(n, n)$ or $U_F(n, \frac{1}{2})$ is maximal.

(ii) For $n \in (ll, \frac{1}{2}]$, $U_F(n, \frac{1}{2})$ is maximal.

(iii) For $n \in (\frac{1}{2}, p)$, $U_F(n, n)$ is maximal.

(iv) For $n \geq p$, $U_F(n, n)$ is maximal.

A proof is found in the appendix.

Discussion

The main idea underlying Result 2 is that an optimal choice of s will efficiently allocate future cash flows in light of the differences of opinion between the entrepreneur and the investors. When an investor, conditional on his winning the auction, is more optimistic than the entrepreneur, then efficiency requires that the investor be promised the high-Z cash flows on which he places relatively

greater probability. If the investor, conditional on winning, is less optimistic than the entrepreneur, then it is most efficient to grant him low-Z cash flows by allocating him debt.

We now discuss the intuition for the result in greater detail. A policy $s \in (ll, \frac{1}{2})$ is always regarded by the entrepreneur as inferior to the policy $s = \frac{1}{2}$. The pessimists' strategy does not change across this range, but optimists bid smaller junior equity (in the sense of first-order stochastic dominance) as s increases. As s increases, the valuation policy causes junior equity to have a higher value relative to debt. This makes the debt bid by pessimists more competitive, forcing optimists to reduce their junior-equity requests as s increases.

Let us now consider which policies are optimal for an entrepreneur of type $n \geq \frac{1}{2}$. Any policy $s > ll$ leads to an identical debt request on the part of pessimists. For each $n \geq ll$, the entrepreneur prefers that the pessimist request debt, since the entrepreneur is more optimistic than the pessimist when the latter wins the auction. This implies that pessimistic bids generated by $s > ll$ are preferred to those generated by $s < ll$. In the range $s \in [\frac{1}{2}, p]$, changing s serves only to vary the probability with which investors bid debt, as opposed to junior equity. The entrepreneur prefers that the investor bid junior equity whenever the latter is more optimistic than the former; this divides the future cash flows in an efficient way. Declaring a policy $s = n$ accomplishes this, in the manner suggested in the discussion following Result 1. Optimistic bids under $s = n$ are therefore preferred to those under $s < ll$. The pessimists' strategy is not precisely determined if $s = ll$, but an entrepreneur of type $n \geq \frac{1}{2}$ will always prefer the contracts arising under $s = \frac{1}{2}$ to whichever contracts arise under $s = ll$; the optimistic requests are smaller and the pessimistic requests are debt under $s = \frac{1}{2}$. An entrepreneur of type $n \geq \frac{1}{2}$ therefore prefers both optimistic and pessimistic bids that arise from $s = n$ to those that arise from $s \leq ll$. We have also argued that he prefers the optimistic bids under $s = n$ to those under any other $s > ll$. This gives an indication of the arguments underlying parts *(iii)* and *(iv)* of the above result.

An entrepreneur of type $n \in (ll, \frac{1}{2})$ prefers both the optimistic and pessimistic bids that arise from declaring $s = n$, over those arising from any $s \leq ll$ (for any profile arising from $s = ll$). The argument given in the previous case indicates why policies $s > \frac{1}{2}$ will not generate preferred optimistic bids for this entrepreneur; the division of cash flows is more efficient under $s = \frac{1}{2}$. The first argument given shows that $s = \frac{1}{2}$ is also preferred to any $s \in (ll, \frac{1}{2})$. Entrepreneurs with $n \leq ll$ may prefer the pessimistic bids generated by $s < ll$, but always prefer the optimistic bids under $s = \frac{1}{2}$.

Result 2 shows that entrepreneurs who regard their signals as a sufficient statistic for project quality

can always select an optimal s such that $s \geq n$. This tendency to select optimistic valuation policies does not arise because entrepreneurs wish to induce optimism in investors. Rather, selecting a high s lowers the requests of the optimists in the sense discussed above, and generates a more attractive profile of contracts.

In light of Result 2, we can see that the assumption that the firm commits to using valuation rule s is not overly strong. Let us suppose that, for all $n \in [0,1]$, an entrepreneur of type n selects an optimal s as indicated in Result 2. If the entrepreneur declares $s < ll$ or $s \geq p$, then the commitment to valuation rule s has no effect since, in these cases, all entrepreneurs of type n agree with the ranking of contracts under s (only one contract type is bid under these valuation rules, and, for example, all types prefer to give away a smaller debt contract rather than a larger one). Result 2 shows that if a valuation rule $s \in \{ll\} \cup (\frac{1}{2}, p)$ is observed, then it must be that $n = s$. In this case, the entrepreneur obviously uses valuation s in rating contracts, irrespective of a commitment to do so. If $s = \frac{1}{2}$ is observed, it must be that $n \leq \frac{1}{2}$. The entrepreneur always prefers smaller junior-equity requests to larger junior-equity requests. The only question is whether entrepreneurs of all types $n \leq \frac{1}{2}$ want to accept a junior equity contract J_a that is bid over a debt contract B_d that is bid, as the valuation rule s prescribes. If these are the equilibrium bids, then it must be that $E_s(J_a(Z)) \leq E_s(B_d(Z))$. Technical Lemma 1 shows that $E_n(B_d(Z)) \geq E_n(J_a(Z))$. This means that for every $n \leq \frac{1}{2}$, an entrepreneur of type n will, in fact, prefer junior equity J_a over debt B_d. These remarks show that commitment to the equilibrium choice of s is ex-post optimal for the entrepreneur. Will an entrepreneur find it beneficial to deviate to a non-equilibrium choice of s and then disregard his commitment? Clearly, the option to disregard his commitment is only valuable if the entrepreneur declares $s \in [ll, p)$. Case-by-case analysis shows that no entrepreneur prefers such a course of action to his equilibrium strategy. The burden of these remarks is as follows. If we specify that the entrepreneur declares s as stipulated in Result 2 and commits to following its valuation rule (as above), and if we specify that the investors follow the strategies detailed in Result 1, then these strategies on the part of the investors and the firm represent a Bayesian-Nash equilibrium.[9]

Result 1 and the above comments suggest a pecking order in the securities used to finance a small-business venture. Entrepreneurs prefer a junior-equity request to a debt request. Junior equity requests have a lower valuation under s than do debt requests, and the entrepreneur chooses s so that he prefers the junior-equity bids to the debt bids. It should be noted that this pecking order is in marked contrast to that suggested by Myers (1984). In the Myers pecking order, debt is preferred to equity because the former minimizes the costs associated with selling a security about

which there is asymmetric information. In the pecking order proposed here, junior equity requests come from optimists who ask for smaller claims (under the valuation policy) than do pessimists. It may be that s is not equal to n, but the fact that $s \geq n$ whenever both debt and junior equity are requested guarantees that a junior-equity bid is always preferred to a debt bid.

In essence, we are proposing that firms' preferences for securities shift over time, as the entrepreneur acquires expertise and inside knowledge. When the entrepreneur seeks financing for a start-up venture, he hopes that expert outside investors are optimistic about his project so that they will offer him financing on favorable terms. The investors' optimism will be expressed in their desire to purchase the firm's equity. As the scale of the firm grows and the entrepreneur and his managerial team acquire specialized knowledge about its activities, management becomes relatively more expert than outside investors. Management's concern in issuing securities, at this point in the development of the firm, is to minimize the impact of the asymmetric information it possesses, which is best done by issuing debt as suggested by Myers. This account is consistent with the success of venture-capital firms in the U.S. These firms have been particularly successful in funding risky high-technology ventures. Projects such as these may only be funded by optimists, since the pessimistic valuation of the venture will typically be very low. For these projects, Result 1 indicates that it is often optimal for investors to be allocated junior equity securities. Banks in the U.S. are generally restricted from accepting equity claims in the firms they finance.[10] One of the competitive advantages of venture-capital firms is that they may hold equity stakes and, indeed, most of their holdings are in equity-like securities (Sahlman (1990)).

One exception to the restriction on equity investments by banks is that banks are permitted to invest five percent of their capital in Small Business Investment Companies (SBICs), federally regulated venture-capital firms. SBICs are unfettered in their selection of investment securities. Bank-owned SBICs make equity, rather than debt, investments much more frequently than other SBICs, (Brewer, Genay, Jackson, and Worthington (1996)). This fact and Result 1 indicate one effect of attenuating the current restrictions and permitting U.S. banks to engage in universal banking.[11] If banks were permitted to take equity claims in firms, the analysis above suggests that more marginally profitable, but positive-net-present-value, projects would be funded.[12] These are projects that pessimists will not finance and that are most efficiently funded by granting optimistic investors equity claims.

3 Updating Entrepreneurs

In this section we will consider an entrepreneur who does regard the signals of the investors as informative. In keeping with the theme of this paper, the expert investors regard their signals as a sufficient statistic for the information of the entrepreneur. Result 1 does not change here; the entrepreneur commits to following the evaluation rule s and the investors do not regard his signal as informative, so they bid in the same way as in the previous case. The entrepreneur's evaluation of the different contract profiles associated with various declarations of s does, however, change. If the entrepreneur observes from their strategies that both investors are optimistic, he will update his beliefs to reflect this information. In the previous analysis, the entrepreneur's belief Q_E about the distribution of the investors' signals was unspecified. In this case, the entrepreneur accepts the distribution Q as correct, except that his prior belief $Q_E(G) = Q(G|Y_E) = n$ need not equal one-half. The entrepreneur believes that his signal is conditionally independent of the signals of the investors. With this belief, Bayes' Rule yields the following probabilities:

$$\begin{array}{ll}
Q_E(H_1 \cap H_2) = p^2 n + (1-p)^2(1-n) & Q_E((H_1 \cap L_2) \cup (H_2 \cap L_1)) = 2p(1-p) \\
Q_E(L_1 \cap L_2) = (1-p)^2 n + p^2(1-n) & Q_E(G|H_1 \cap H_2) = \frac{p^2 n}{p^2 n + (1-p)^2(1-n)} =: hh(n) \\
Q_E(G|L_1 \cap L_2) = \frac{(1-p)^2 n}{(1-p)^2 n + p^2(1-n)} =: ll(n) & Q_E(G|(H_1 \cap L_2) \cup (H_2 \cap L_1)) = n
\end{array}$$

In this setting, in which the entrepreneur regards the investors as informed, he will choose s to maximize his ex-post payoff, making use of information he will acquire in the course of the bidding game. In essence, s is chosen to exploit the information contained in the bids of the investors.

Formally, we now write the entrepreneur's expected utility as

$$\tilde{U}_F(n,s) = E_{ll(n)}[Z - W_s(Z)|L_1 \cap L_2]Q_E(L_1 \cap L_2)$$

$$+E_n[Z - W_s(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)]Q_E((H_1 \cap L_2) \cup (H_2 \cap L_1))$$

$$+E_{hh(n)}[Z - W_s(Z)|H_1 \cap H_2]Q_E(H_1 \cap H_2).$$

In this setting, it is more difficult to precisely specify the optimal valuation policy for the firm. We can, however, bound the optimal policy and prove that it exists. For $n, k \in [0,1]$, we define

$$rat(n,k) = \frac{kp^2n + p(1-p)n}{k(p^2n + (1-p)^2(1-n)) + p(1-p)}.$$

We remark that $hh(n) \geq rat(n,k) \geq n$ for all $k \in [0,1]$, and that $rat(n,k)$ is increasing in n and k. We recall from the proof of Result 1 that $\hat{F}(s) = \frac{p(1-p)(2s-1)}{p^2-(p^2+(1-p)^2)s}$ for $s \in [\frac{1}{2}, p]$.

Result 3. *For each $n \in [0,1]$ there is some $s \in [0,1]$ that maximizes $\tilde{U}_F(n,s)$.*

(i) For $n \leq ll$, either $\tilde{U}_F(n,n)$ or $\tilde{U}_F(n,\frac{1}{2})$ is maximal.

(ii) For $n \in (ll, \frac{1}{2}]$, either $\tilde{U}_F(n,\frac{ll}{2})$ is maximal or $\tilde{U}_F(n,s)$ is maximal for some $s \in [\frac{1}{2}, max\{\frac{1}{2}, rat(n,1)\}]$.

(iii) For $n \in (\frac{1}{2}, p)$, $\tilde{U}_F(n,s)$ is maximal for some $s \in [min\{rat(n,\hat{F}(n)), p\}, p]$.

(iv) For $n \geq p$, $\tilde{U}_F(n,n)$ is maximal.

A proof is found in the appendix.

The entrepreneur who updates will be more pessimistic after receiving two low bids and more optimistic after receiving two high bids. This pessimism gives rise to the fact that in case *(ii)*, if there is a unique optimal s it may be strictly below n, a circumstance that cannot arise when the entrepreneur disregards the signals of the investors. However, the optimal level of s indicated above for each $n > \frac{1}{2}$ is greater than that specified in Result 2. For n in this range, the entrepreneur is more optimistic than the pessimists in the case that the pessimists win the auction, so the entrepreneur prefers that the pessimists bid debt, as they do for all $s \geq ll$. If there is one pessimistic bid and one optimistic bid, the entrepreneur retains his opinion, but if there are two optimistic bids, the entrepreneur becomes more optimistic and values junior equity more than he used to, relative to debt. This increased optimism causes an entrepreneur of type $n \in (\frac{1}{2}, p)$ to choose s at least as large as $rat(n, \hat{F}(n)) > n$. We recall from Result 1 that choosing a higher s in this range increases the probability of a debt bid on the part of the optimists.

Result 3 therefore indicates that if $n > \frac{1}{2}$, an entrepreneur who updates is more likely to receive debt requests than an entrepreneur who ignores the investors' signals. The management literature is replete with evidence that entrepreneurs are more optimistic than others (or than they should be), so the case of $n > \frac{1}{2}$ should be regarded as the most empirically relevant one (see, for example, Cooper, Woo and Dunkelberg (1988) and Kahneman and Lovallo (1993)). We suggest that large and successful venture-capital firms are much more likely to be regarded as expert by entrepreneurs than their more humble competitors. Let us suppose that for exogenous reasons entrepreneurs are randomly assigned either to the pool that receives financing from expert venture-capital firms or to

the pool that receives funding from less successful firms.[13] If we presume that entrepreneurs only update when they receive bids from expert venture capital firms, the model predicts that these firms should hold a higher proportion of their assets in debt stakes, relative to less expert venture capital firms. Brewer and Genay (1994) in their analysis of the performance of Small Business Investment Companies show that venture capital firms holding a higher proportion of debt claims have significantly higher returns on their own stock. This result is particularly striking in that one would expect equity claims to be riskier, so venture capital firms that hold a greater proportion of their assets in equity would, *prima facie*, be expected to realize higher returns. Norton and Tenenbaum (1993) show that smaller venture capital firms, measured by asset size, hold a greater proportion of their investments in equity stakes.

It is also the case that experienced entrepreneurs who have managed several start-ups are likelier to recognize the expertise of outsiders and therefore to update on their signals. If entrepreneurs are randomly allocated to venture capital firms, another implication of the model is that experienced entrepreneurs should have debt contracts with venture capital firms relatively more often than inexperienced entrepreneurs do.

Numerical Example

We will now provide a numerical example to illuminate some of the points made in the theoretical exposition. We assume that f_g is the density function of a $N(5,1)$ random variable truncated at zero, and we assume that f_b is the density of a $N(1,1)$ random variable truncated at zero. We fix the parameter values $p = \frac{3}{4}$, $n = \frac{3}{5}$ and $I = 1.65$. We have that $E_{ll}(Z) = 1.6588 \geq I$, as required for Result 1. For convenience, we refer to the setting in which the entrepreneur disregards the signals of the investors as the "confident setting," and we refer to the setting in which the entrepreneur regards those signals as informative as the "updating setting." In the updating setting, Q_E is as given in Section 3. Here we will assume that in the confident setting Q_E is also as given in Section 3 (we recall that the results in Section 2 do not depend on a precise specification of Q_E). We may now calculate $U_F(n,s) = U_F(0.6,s)$ for all $s \in [0,1]$. The graph of this function is shown in Figure 1 (all figures follow the body of the text).

We note that, as required by Result 2, $U_F(0.6,s)$ is maximized at $s = 0.6$. The function $U_F(0.6,s)$ is continuous except at $s = ll = 0.1$. This discontinuity arises from a shift in the pessimists' strategy at $s = ll$. The pessimists' strategy is constant for all $s \in [0,ll)$ and for all $s \in (ll,1]$. For $s < ll$ the pessimists' pure strategy is to bid a single debt contract, and for $s > ll$ the pessimists' pure strategy is to bid a single junior equity contract. These contracts both yield the pessimist an expected

valuation of I. The entrepreneur is significantly more optimistic than the pessimist when the latter wins the auction, so the entrepreneur prefers that the pessimist request debt (the entrepreneur's residual claim is then junior equity). The discrete increase in $U_F(0.6, s)$ at $s = ll = 0.1$ reflects the switch in the pessimists' strategy from requesting junior equity to requesting debt. The graph of $U_F(0.6, s)$ is elsewhere continuous since a change in s modifies the mixing strategy of the optimists in a continuous manner.

That $U_F(0.6, s)$ increases monotonically on the range $[0.1, 0.5]$ arises from the fact that the optimist's junior equity bids grow smaller as s increases on this range, as we remarked in the discussion following Result 2. For $s \in [0.5, 0.75]$, varying values of s only differ in the proportion of optimistic debt bids, as opposed to junior-equity bids, that they generate. Choosing $s = 0.6$ is optimal, and Figure 1 suggests that the gains to choosing the optimal s may be substantial. We observe that restricting investors to debt bids is equivalent to having the entrepreneur choose $s \geq p = 0.75$. In this example, choosing $s \geq 0.75$ results in a value for $U_F(0.6, s)$ that is 10.2 percent lower than the maximal value of $U_F(0.6, s)$ (which is achieved at $s = 0.6$). This is an indication of the efficiency gains that may be realized by permitting investors to bid junior-equity claims. It also suggests that the flexibility in contract choice enjoyed by U.S. venture capitalists may grant them a significant advantage over competing banks.

A graph of the entrepreneur's utility $\tilde{U}_F(0.6, s)$ in the updating setting is given in Figure 2. Result 3 states that the optimal s is in the range $[rat(n, \hat{F}(n)), p] = [0.6923, 0.75]$. Figure 2 indicates that the optimum is in fact achieved at $s = p = 0.75$. We observe that the discrete jump at $s = ll$ is barely perceptible in this graph. This arises from the fact that in the updating setting the entrepreneur's beliefs are quite pessimistic in the case that a pessimist wins the auction. The entrepreneur will be marginally more optimistic than the pessimists since $n > 0.5$, but the gains to efficiently dividing up the cash flows are slight in this case. Both Figure 1 and Figure 2 suggest that the entrepreneur can significantly improve his expected utility through judicious choice of s. Most of the gains arise from eliciting a superior profile of bids from the optimists; the gains from eliciting the best pessimistic bid are noticeably smaller. This is true for two reasons. Firstly, the optimists win the auction more often than the pessimists do, so the bids of the former are more important to the entrepreneur than the bids of the latter. Secondly, the pessimists always bid large contracts, so that there is little distinction between a pessimistic debt bid and a pessimistic junior equity bid; both bids are quite similar to full equity bids. Optimistic bids are much smaller. There is a substantive difference between an optimistic debt bid and an optimistic junior equity bid and the correct choice of s will induce the optimists to bid the contract that the entrepreneur prefers.

We now examine the investors' strategies under the optimal policies in the confident and updating settings. In the confident setting, as we discussed, it is optimal to set $s = 0.6$. In response to this strategy, the pessimists bid a debt contract with face value 5.9687. The optimists bid a debt contract with probability 0.2 and they bid a junior equity contract otherwise. We denote the the density function of the optimists' strategy over the space of debt contracts by $ddens(0.6, \bullet)$. Figure 3 displays this density. We note that the optimists only bid debt contracts that are strictly smaller than that bid by the pessimists. The density of the optimists' strategy over the space of junior-equity contracts, $jdens(0.6, \bullet)$, is depicted in Figure 4.

In the updating case, the policy $s = 0.75$ is optimal. We observe that under this policy the optimists bid only debt contracts. This contrasts markedly with the optimists' strategy under $s = 0.6$. The difference between the contract profiles under the two strategies provides a striking example of the fact that optimistic ($n > \frac{1}{2}$) updating entrepreneurs will receive debt bids more often than optimistic confident entrepreneurs, as discussed above. The pessimists again bid a debt contract with face value 5.9687 when the policy is $s = 0.75$. We denote the the density function of the optimists' strategy over the space of debt contracts by $ddens(0.75, \bullet)$. Figure 5 displays this density.

4 Three Types

We will now discuss an extension of the basic model in which investors belong to one of three types. This will meaningfully generalize the model from the binary type case and suggest its broader application. The three type case can also be used to explore pooling strategies not available to investors in the binary type model. Lastly, the three-type model will provide a significantly more refined result in the banking model examined in Section 6.

The basic probabilistic structure is unchanged, except that we now envision each investor as receiving two signals, rather than one as before. The investor types may now be classified as optimists (those who receive (H,H)), neutral investors ((H,L) or (L,H)) and pessimists ((L,L)). The equilibria of this model are similar in form to those of the two-type model, but the number of different cases increases.

Result 4. *For each $s \in [0, 1]$ there is a Bayesian-Nash equilibrium of the bidding game. In the separating equilibria described below, pessimists play a pure strategy and neutral investors and optimists play a mixed strategy. In the equilibria described below two types of investors play mixed strategies, but the separating structure of the binary type model is retained.*

(i) For $s < \frac{(1-p)^4}{p^4+(1-p)^4}$, all investors bid junior equity and the equilibrium is unchanged for all s in this range. An identical equilibrium arises if investors are restricted to bidding junior equity.

(ii) For $\frac{(1-p)^4}{p^4+(1-p)^4} \leq s < \frac{(1-p)^2}{p^2+(1-p)^2}$, pessimists bid debt and neutral investors and optimists bid junior equity.

(iii) For $\frac{(1-p)^2}{p^2+(1-p)^2} \leq s < \frac{1-p^2}{1+2p(1-p)}$, pessimists bid debt, neutral investors mix over debt and junior equity, and optimists bid junior equity.

(iv) For $\frac{1-p^2}{1+2p(1-p)} \leq s < \frac{p(1+p)}{2(1-p(1-p))}$, pessimists and neutral investors bid debt and optimists bid junior equity.

(v) For $\frac{p(1+p)}{2(1-p(1-p))} \leq s < \frac{p^2}{p^2+(1-p)^2}$, pessimists and neutral investors bid debt and optimists mix over debt and junior equity.

(vi) For $s \geq \frac{p^2}{p^2+(1-p)^2}$, all investors bid debt and the equilibrium is unchanged for all s in this range. An identical equilibrium arises if investors are restricted to bidding debt.

A proof is found in the appendix.

Result 4 demonstrates the robustness of the model studied in this paper. It shows that the economic implications of Result 1 are not an artifact of the assumption that there are only two types of investors, nor do they depend on the fact that only one type plays a mixed strategy. We have not demonstrated the uniqueness of the equilibria described in Result 4, but it is nonetheless indicative of the generality of the earlier results. It continues to be true that a junior equity bid is always selected over a debt bid. The transition in the investors' strategies from debt to junior equity as s rises is also present here.

We note that if $s \in [\frac{1-p^2}{1+2p(1-p)}, \frac{1}{2})$, the valuation policy is more pessimistic than the neutral investor's prior, and yet the neutral investor bids only debt for s in this range. A similar observation applies to $s \in [\frac{(1-p)^4}{p^4+(1-p)^4}, \frac{(1-p)^2}{p^2+(1-p)^2})$ and the pessimists' bidding strategy. These aspects of the equilibrium are a manifestation of the winner's curse in this setting. Neutral investors and pessimists will only win the auction if the other investor is not optimistic. This causes them to have a fairly pessimistic opinion of the project in the case that their bid is accepted, so that debt is their preferred security request. This problem is somewhat mitigated for an optimist, who will sometimes win the auction in the presence of another optimist.

5 Entrepreneurial Effort

In this section we discuss the impact of requiring entrepreneurial effort for project completion. Formally, we will require that entrepreneurs expend a non-monetary effort cost ec immediately after receiving financing for their project. We will not permit the entrepreneur to accept the financing and withhold effort (his contract with investors will require him to exert what we assume is a verifiable amount of effort). For a given random payoff Z, the attractiveness of the project is determined by the capital (I) and labour (ec) required for its undertaking. We will assume that I and ec are common knowledge; uncertainty is present in the model only with respect to the distribution of Z.

This new setting is more realistic than the basic model discussed earlier in that the entrepreneur will no longer necessarily accept the winning bid. An entrepreneur with belief n will now elect to decline the best bid C made by the investors if $E_n(Z - C(Z)) < ec$, that is, if the best bid does not allow him to recoup his effort cost. When effort is required of the entrepreneur, investors will not disregard his beliefs, as they did earlier; they must gauge the likelihood that their bids will be accepted. The entrepreneur's choice of a valuation policy will now be regarded by the investors as a signal of his type. The next two results establish the existence and the nature of the equilibria in this setting. In a later section we will make use of this framework to analyze the market for loans.

We will return now to the setting in which the entrepreneur regards his signal as a sufficient statistic for project quality and we assume that there are only two types of investors. We will first assume that the entrepreneur's belief n is known to the investors. In the result below, we will assume for each n in turn that there does not exist a feasible contract C such that $E_{(1-p)}(C(Z)) > I$ and $E_n(Z - C(Z)) \geq ec$; that is, we will assume that pessimists cannot make profitable bids that will be accepted. If they can, then the equilibria in this setting differ little from those described in Result 1. We will assume that there exists a feasible contract C such that $E_{\frac{1}{2}}(C(Z)) > I$ and $E_n(Z - C(Z)) \geq ec$, so that optimists can make profitable bids that will be accepted. For $a, d \in \Re, \alpha \in [0,1]$, we define $M_{\alpha,a,d}(x) = \alpha J_a(x) + (1-\alpha)B_d(x)$ to be a *mixed debt-equity contract.* We note that $M_{\alpha,a,d}$ is a feasible contract.

Result 5. *Suppose n is common knowledge. For every given s and n in $[0,1]$ there is a Bayesian-Nash equilibrium of the bidding game. In all equilibria optimists play a mixed strategy and pessimists bid Z.*

(i) For $s, n \leq \frac{1}{2}$, optimists bid junior equity.

(ii) For $s \geq n \geq \frac{1}{2}$, optimists mix over debt and junior equity, where the debt contracts have a higher valuation under s.

(iii) For $n > s \geq \frac{1}{2}$, optimists mix over debt, mixed debt-equity and junior equity. The debt contracts have a higher valuation under s than the mixed debt-equity, and the junior equity contracts have the lowest valuation.

(iv) For $s > \frac{1}{2} \geq n$, optimists mix over junior equity and mixed debt-equity, and may also bid debt. For some parameter values, both high and low-valuation junior equity will be bid.

(v) For $n > \frac{1}{2} \geq s$, optimists mix over debt and mixed debt-equity. For some parameter values, optimists also bid junior equity.

If $n \leq \frac{1}{2}$, the entrepreneur maximizes his expected payoff at $s \leq \frac{1}{2}$. (Choosing any such valuation policy leads to an identical equilibrium in the bidding game). If $n > \frac{1}{2}$, the entrepreneur maximizes his expected payoff at $s = n$.

A proof is found in the appendix. We will denote the equilibrium described in Result 5 for the pair (s, n) as equilibrium (s, n)

In three of the above cases, mixed debt-equity emerges as an optimal bid. For certain combinations of s and n, an investor values a debt claim more than he values a junior equity claim with the same valuation under s. It may be, however, that the debt claim would not be accepted by the entrepreneur. In this case, a mixed debt-equity claim maximizes the investor's valuation of C for a given value of $E_s(C(Z))$, subject to acceptance on the part of the entrepreneur. A similar argument applies when the investor would prefer to bid some junior equity that he knows will be rejected. The mixed debt-equity claim emerges as an optimal compromise in the case that s is chosen in a distorting way. As noted in the proof of the result, $s = n$ is an optimal choice, so mixed debt-equity will typically not appear in equilibrium. We note that $s = n$ is optimal in part because it does not give rise to the distortions that permit the emergence of mixed debt-equity bids.

In part *(iv)* of the result, we have an example for the first time in this paper of a case in which an equilibrium junior equity bid has a higher valuation than an equilibrium debt bid. The investors would prefer to bid debt contracts (since $s > \frac{1}{2}$), but large debt bids will not be accepted because they are valued highly by the entrepreneur and will not leave him with a sufficient residual. The investors are therefore forced to make their highest valuation bid junior equity. Smaller bids can be debt, since the entrepreneur will accept these. We note that the optimal division of future cash flows between the investors and the firm is much distorted by this choice of s and that the

entrepreneur will not, in general, choose s such that case *(iv)* holds.

Let us discuss an application of the idea that investors' bids can influence the entrepreneur's decision about whether to proceed with a project. Examples can be constructed in which somewhat pessimistic entrepreneurs are unwilling to undertake prospective projects because effort costs are perceived to be too high relative to project values. Well-informed investors may realize that the project is, in fact, worthwhile, but they will only be able to convince the entrepreneur to undertake the project if he updates in light of their bids. In this case, both the entrepreneur and the optimistic investors do strictly better in the updating equilibrium than in the equilibrium in which the entrepreneur disregards the investors' views and rejects the project. A Pareto improvement results from the entrepreneur's recognition of investor expertise. This is in contrast to strategic trader models in the market microstructure literature in which informed agents typically profit only at the expense of the uninformed (e.g. Kyle (1985)).

We will now consider the case in which the entrepreneur's belief is not known with certainty. We denote by $U_F(n, s, i)$ the expected payoff of an entrepreneur of type n when he declares policy s and the investors bid as in equilibrium (s, i). This is the entrepreneur's payoff if he declares policy s and is thought by investors to be of type i. There is a clear advantage for the entrepreneur in claiming to be more pessimistic than he is. This, in effect, raises the reservation price in the auction and forces the investors to offer him more appealing contracts. It is unlikely that investors would be aware of the entrepreneur's frame of mind, particularly if the entrepreneur has an interest in disguising his beliefs. However, the valuation policy s does serve as a signal of the entrepreneur's true n. Let us suppose that it is common knowledge that there are two types of entrepreneurs, n' and n such that $n' > n$. It is also common knowledge that the prior probability of a type n' entrepreneur is $e \in (0, 1)$. We recall from Result 5 that setting $i = n$ maximizes $U_F(n, i, n)$ and setting $i = n'$ maximizes $U_F(n', i, n')$. We continue to assume here that pessimists cannot make offers that will be accepted by either type of entrepreneur. That is, we assume that there does not exist a feasible contract C such that $E_{(1-p)}(C(Z)) > I$ and $E_{n'}(Z - C(Z)) \geq ec$. We also assume that there does exist a feasible contract C such that $E_{\frac{1}{2}}(C(Z)) > I$ and $E_n(Z - C(Z)) \geq ec$, so that optimists can make accepted bids.

Result 6. *A perfect Bayesian equilibrium of the above described game exists. One of the following holds in equilibrium:*

(i) Types n' and n separate by declaring policies n' and n respectively.

(ii) Types n' and n pool by both declaring policy n.

(iii) Type n' mixes between declaring n' and declaring n. Type n declares n. We refer to these equilibria as mixed separating-pooling equilibria.

If there is a mixed separating-pooling equilibrium for $e = k_1$, then there is also a mixed separating-pooling equilibrium for $e \in [k_1, 1)$.

We omit the proof, which is available upon request from the author. Here we provide a sketch of the argument. An entrepreneur of type n' faces a trade off between declaring his optimal valuation policy and being thought to be n. If the entrepreneur declares the policy $s = n$ he will receive smaller securities requests from investors who fear having their bids rejected by an entrepreneur of type n. These bids will not, however, efficiently divide future cash flows between the investors and the entrepreneur of type n'. If the distortionary costs of declaring n are relatively severe, then an entrepreneur of type of n' will elect to separate and declare the policy $s = n'$. If the benefits to declaring n are relatively large, an entrepreneur of type n' will prefer to declare $s = n$. Investors will adjust their strategies in this case to tailor some of their bids to the entrepreneurs of type n'. It may be that this adjustment will make declaring $s = n$ unattractive for entrepreneurs of type n'. The main argument of the proof is to show that the expected payoff to a type n' entrepreneur for declaring $s = n$ varies continuously with the probability of a type n' entrepreneur declaring this policy. A fixed-point argument is then adduced to show the existence of the equilibrium. It is not clear how the type of equilibrium changes as n and n' grow further apart. Both the benefits and the costs of declaring $s = n$ increase for a type n' entrepreneur as the distance between n and n' increases.

The intuition for the last remark in the result is that as e rises it is always possible to keep fixed the conditional probability cp that an entrepreneur is of type n' given that he declared a policy $s = n$. (Simply require that entrepreneurs of type n' declares $s = n'$ with appropriately increasing probability as e rises). It is the conditional probability cp that determines the strategies of the investors. Suppose that for a given value of e, there is a mixing strategy on the part of entrepreneurs of type n' that causes them to be indifferent between declaring policies $s = n$ and $s = n'$. Then for higher values of e a strategy that keeps cp fixed can be implemented in the way just suggested.

6 Banking

We will now proceed to study the model in the restricted setting in which investors can only bid debt claims. The primary argument for studying this restricted setting is that banks in the U.S.,

the main source of funding for small businesses, are typically restricted by law from holding equity claims in the firms they finance. The results in this section have testable implications for U.S. banking data.

We note that in this setting there is no role for the declaration of a valuation policy s. The auction will be conducted according to the rule that the debt contract with the lowest face value among those bid will be declared the winning bid. We continue to assume that I and ec are common knowledge and that the entrepreneur is of known type n. We will first assume that there are two types of banks. The equilibrium for $ec = 0$ is described in case *(iv)* of Result 1. We here conduct comparative statics in ec and I. We recall that we refer to the setting in which the entrepreneur regards the signals of the banks as informative as the "updating setting," and we refer to the setting in which the entrepreneur disregards those signals as the "confident setting." In all cases discussed below, pessimists bid a fixed debt contract and optimists mix over a range of smaller debt contracts.

Result 7. *We fix I. A perfect Bayesian equilibrium exists for all values of ec. There exist real numbers $d \geq c \geq b \geq a > 0$ such that:*

(i) For $a \geq ec \geq 0$, in both the confident and updating settings, all winning bids are accepted. The equilibria are identical in the two settings.

(ii) For $b > ec > a$, in the updating setting the pessimists' bids are not accepted. In the confident setting these bids are accepted. All optimistic bids are accepted in both settings and the optimistic strategies are identical in the two settings.

(iii) For $c > ec \geq b$, in both settings the pessimists' bids are rejected and the optimists' bids are accepted. The optimists' strategies are identical in the two settings.

(iv) For $d > ec \geq c$, in both settings the pessimists' bids are rejected and some of the optimists' bids are accepted. The optimists have expected payoffs of zero in the two settings. If a contract is accepted in the confident setting, a weakly larger contract will be accepted in the updating setting, but the converse is untrue.

(v) For $ec \geq d$, no bids are accepted in either setting.

A proof is found in the appendix. An analogous result holds if we fix ec and raise I.

The above result allows us to argue that if banks have a strictly positive expected payoff, then there should be a weakly greater proportion of high-interest rate loans in the confident setting relative to the updating setting. Suppose that a bank makes many loans in one sector of the economy in

which it specializes and makes a few loans in another sector, perhaps for regulatory reasons. The result above suggests that the bank should have a relatively higher proportion of high-interest rate loans in the sector in which it does not specialize. The intuition for this is that entrepreneurs will be more likely to interest themselves in the bank's opinion of their project if the bank specializes in their sector. If the entrepreneurs regard the bank's bid as informative, then a request for a high-interest loan on the part of the bank will discourage the entrepreneur for two reasons. First, the entrepreneur would obviously prefer a lower interest rate to a higher one. Second, the entrepreneur will interpret the loan request as a signal that the bank thinks his project is of low quality and the entrepreneur values the bank's opinion. If the bank is not a specialist in a given sector, then the second reason will have less weight and the entrepreneur is likelier to accept the high-interest rate loan. As a corollary to the above result, we observe that default rates will be higher in the confident setting, if banks have a strictly positive expected payoff. If the banks' signals are actually independent of project quality, it is still the case that larger loans are made in the confident setting. If, as we have assumed, the banks' signals are positively correlated with project quality then the large loans made in the confident setting will be particularly subject to default because of the banks' low signals.

Part *(iv)* of Result 7 indicates that if a bank has an expected profit of zero, then higher-interest rate-loans will be made in the updating setting. These loans have low interest rates compared to the loans described in the other three parts of the result. The intuition for this result is that in case *(iv)*, the bank is funding marginal projects. In the updating setting, two positive signals from the banks can convince the entrepreneur to undertake a project he would be unwilling to initiate on the basis of his own private signal. This suggests that expert banks should make relatively more medium-to-low-interest rates loans and relatively fewer high-interest rate loans, relative to non-specialized banks. If it is assumed that banks do make zero-expected-profit loans, then we cannot predict a clear pattern in default rates.

Result 7 extends to the three-type case, although the relationship described above changes in some respects. We assume that the separating equilibrium described in Result 4 holds.

Result 8. *We fix I. A perfect Bayesian equilibrium exists for all values of ec. There exist real numbers $d \geq c \geq b \geq a > 0$ such that:*

(i) For $a \geq ec \geq 0$, in both the confident and updating settings, all winning bids are accepted. The equilibria are identical in the two settings.

(ii) For $b > ec > a$, in the updating setting the pessimists' bids are not accepted. For low ec in

this range, in the confident setting these bids are accepted, but as ec rises the pessimists' bids are eventually rejected. In addition, as ec rises the maximal loan request from neutral investors grows smaller in both settings, but this request is always larger in the confident setting. Every bid by a neutral investor in the confident setting is accepted. All optimistic bids are accepted in both settings.

(iii) For $c > ec \geq b$, in both settings the pessimists' bids are rejected and some of the neutral investors' bids are rejected. As ec rises the maximal neutral-investor bid grows smaller. For every neutral-investor bid that is accepted in the updating setting, there is a larger neutral-investor bid that is accepted in the confident setting. All optimistic bids are accepted.

(iv) For $d > ec \geq c$, in both settings the pessimists' bids and the neutrals' bids are rejected. The maximal optimistic bid is always larger in the updating setting. Eventually, not all optimistic bids are accepted, but for every bid accepted with positive probability in the confident setting there is a larger bid accepted with positive probability in the updating setting.

(v) For $ec \geq d$, no bids are accepted in either setting.

The proof, a generalization of the proof of Result 7, is omitted and is available upon request. An analogous result holds if we fix ec and raise I.

We note that, in this case, higher-interest-rate loans are made in the confident setting if either neutral investors or pessimists make loans. If, however, only optimists make loans then higher-interest-rate loans are made in the updating setting. This suggests that if we average across different values of ec we should expect to see a greater proportion of high- and medium-to-high-interest rate loans in the confident setting, while there should be more medium-to-low-interest rate loans in the updating setting. This provides a more detailed pattern than that described in Result 7. Result 8 also shows that this general pattern does not depend on whether the banks make zero profit bids. An analysis of default patterns, as given earlier, is also possible here.

7 Conclusion

We have argued in this paper that several aspects of small-business finance may be explained by positing that banks and venture capital firms possess project evaluation skills that are superior to those of entrepreneurs. We showed that, in the context of our model, investors and entrepreneurs may restrict themselves to debt and junior equity (call-option) contracts without loss of efficiency. A pecking order of securities was proposed, in which the entrepreneur always honors an investor's request for junior equity over a request for debt; this is in direct contrast to the pecking order of

Myers (1984). We suggested that our pecking order applies to new firms, while that of Myers is appropriate for mature firms.

In a discussion of regulatory policy, it was argued that permitting banks to take equity claims in the firms they finance will lead them to fund more marginally profitable, but positive-net-present-value, projects. We further showed that if entrepreneurs are optimistic, as the empirical evidence suggests, then large and successful venture-capital firms are likelier to hold debt stakes than their less established competitors. In an application of the model to the banking industry, we showed that banks will make relatively more medium-to-low-interest rate loans and relatively fewer high-interest rate loans in industries to which they specialize in lending.

The framework suggested here may be more widely applied. Our example in which the common recognition of expertise benefits all agents is connected to the more general problem of social decision-making involving non-cooperative agents. It seems clear that it is most efficient to centralize knowledge-gathering in order to avoid redundancy, but the individual incentives for acquiring expertise when its benefits are shared should be studied more carefully. For example, one extension of the model would be to consider banking or venture-capital syndicates. One economic rationale for syndication may be the pooling of expertise. The incentive structure that gives rise to cooperation and information sharing would be a useful subject for further inquiry. Different expected patterns in the security holdings of syndicates of varying sizes might be suggested.

Comparative industry analysis could provide further details about the expertise of outside investors and suggest the sense in which their expertise is more useful in some economic contexts than in others. It would be worthwhile, as well, to devote further consideration to the argument that, in the face of an increasing trend towards securitization, banks may do well to specialize only in loan origination; we have suggested that expert project evaluation is an important function of banks, irrespective of their role in providing capital. We have proposed a role for banks and venture capital firms that goes beyond monitoring; we have suggested that bank and venture capital expertise is a resource that aspiring entrepreneurs would be foolish to squander.

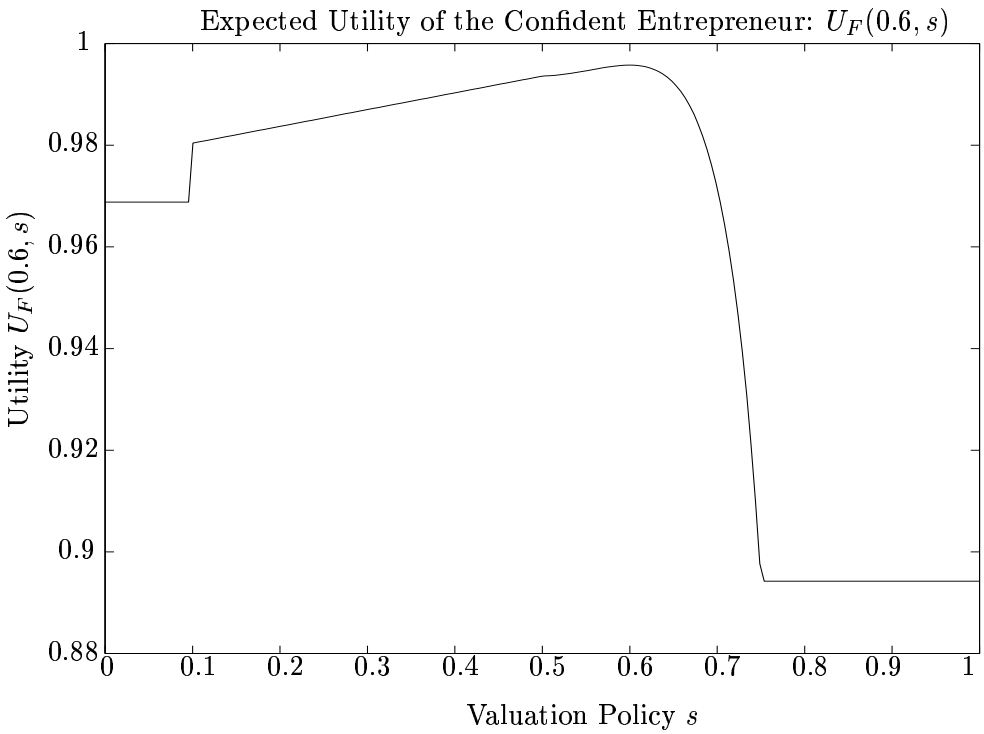

Figure 1: Expected Utility of the Confident Entrepreneur

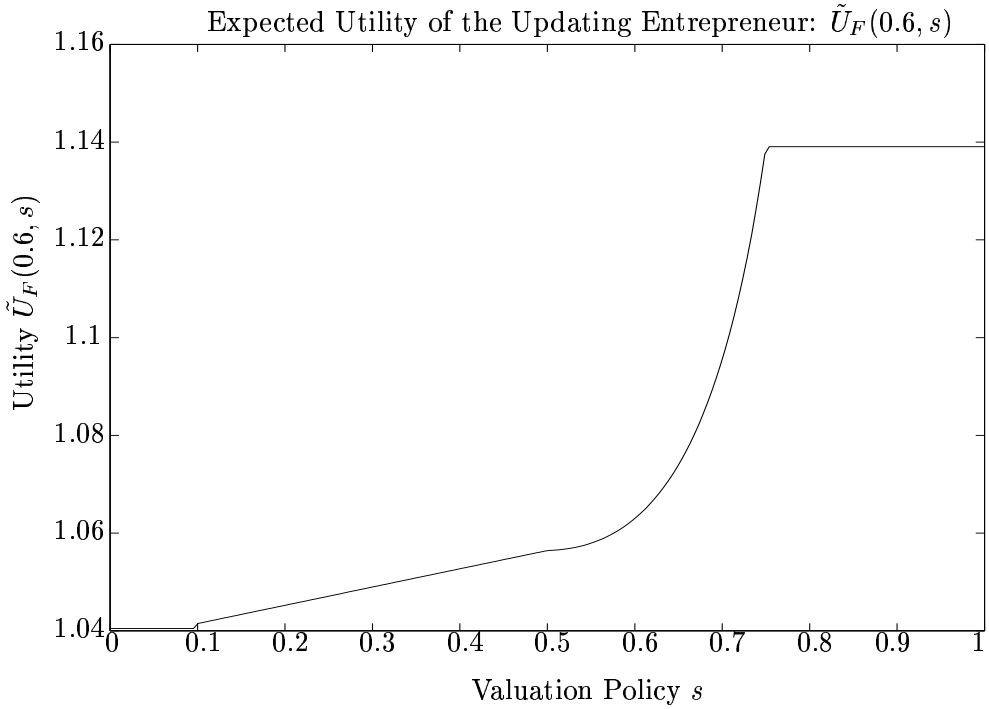

Figure 2: Expected Utility of the Updating Entrepreneur

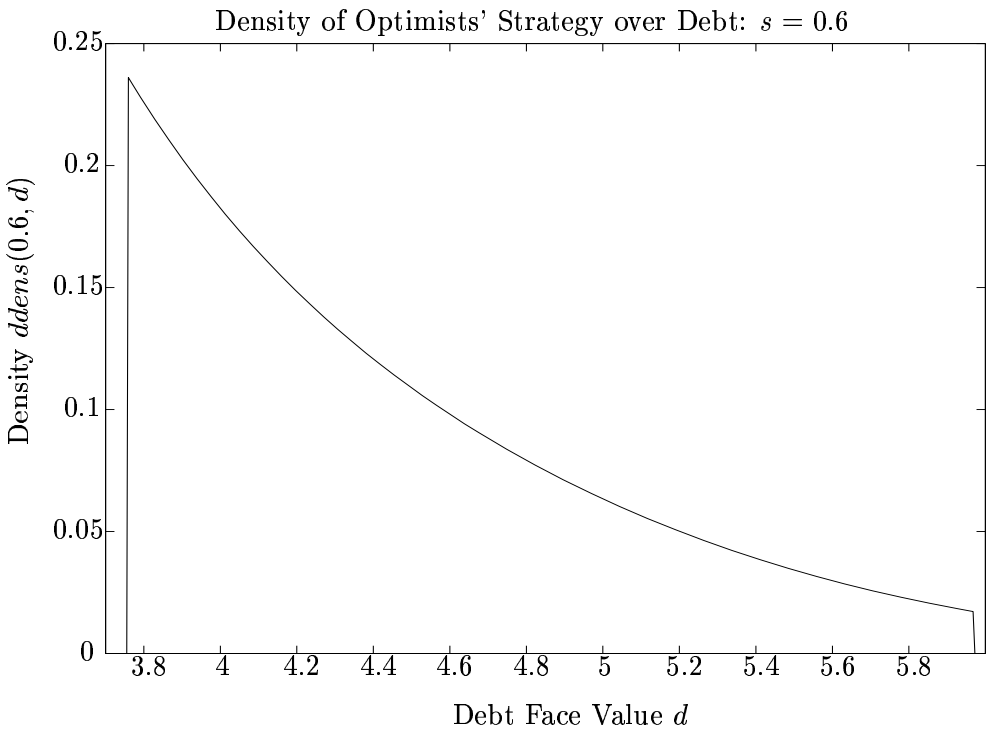

Figure 3: Density of the Optimists' Strategy over Debt: $s = 0.6$

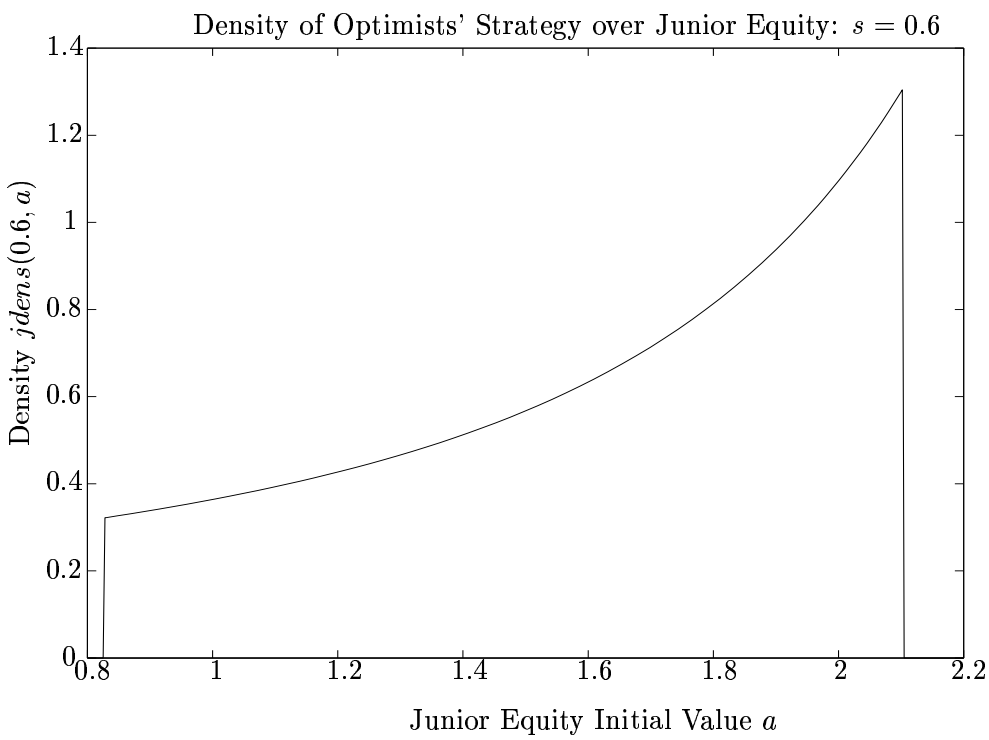

Figure 4: Density of the Optimists' Strategy over Junior Equity: $s = 0.6$

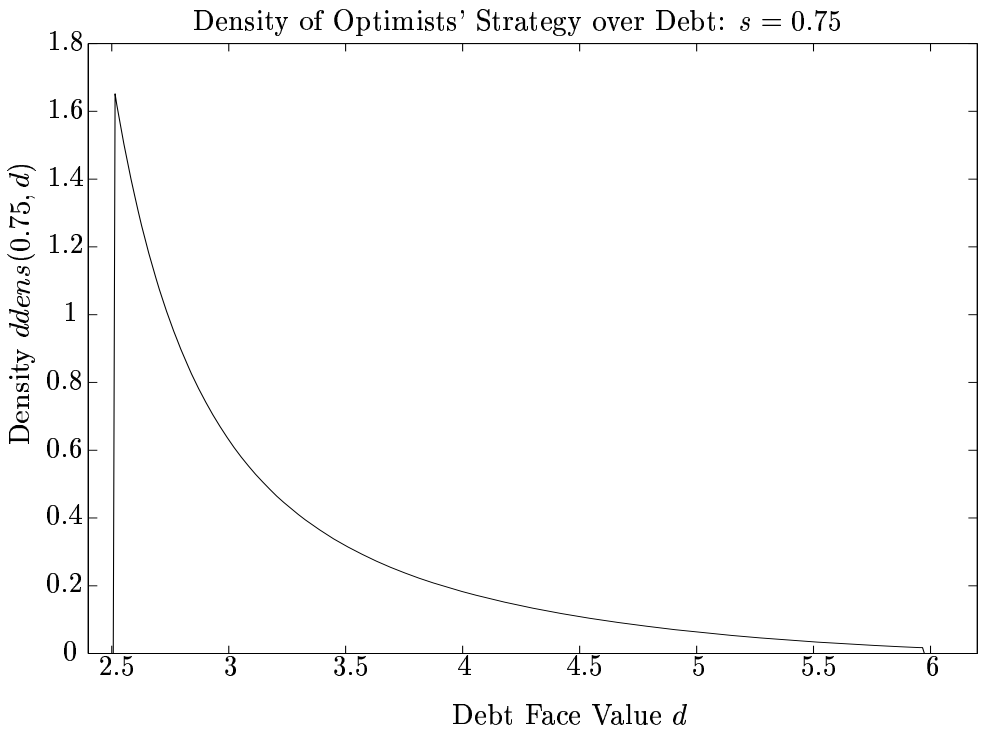

Figure 5: Density of the Optimists' Strategy over Debt: $s = 0.75$

Appendix

For the sake of brevity, we omit the proofs of the technical remarks and lemmas. Proofs are available from the author upon request.

Technical Remark 1. For any $d > 0$ let B_d be the debt contract with face value d. It is the case that $E_g(B_d) > E_b(B_d)$.

Technical Remark 2. For any $q \geq p$, the density function associated with belief q dominates the density function associated with belief p in the sense of MLRP.

Technical Lemma 1. Let f_1 and f_2 be two densities for the random variable $Z \geq 0$. Suppose that the density function f_2 dominates the density function f_1 in the sense of MLRP. Denote the expectation of a feasible contract j of Z with respect to density f_i for $i \in \{1, 2\}$ by $E_i(j(Z))$ and suppose that $E_2(Z) < \infty$. Let C be a feasible contract. Firstly, there exists a debt contract D such that $E_2(D) \leq E_2(C)$ and $E_1(D) = E_1(C)$. Secondly, there exists a debt contract D' such that $E_2(D') = E_2(C)$ and $E_1(D') \geq E_1(C)$. Thirdly, there exists a junior equity contract J such that $E_2(J) = E_2(C)$ and $E_1(J) \leq E_1(C)$. Lastly, there exists a junior equity contract J' such that $E_2(J') \geq E_2(C)$ and $E_1(J) = E_1(C)$.

The proof of Technical Lemma 1 makes use of the proof of Result 4 in Garmaise (1998).

Technical Lemma 2. Let belief $c \geq \frac{1}{2} \geq (1-c) > 0$ be given and let a feasible security X be given. Then

$$\frac{E_c(X)}{4c(1-c)E_{\frac{1}{2}}(X)} \geq \frac{E_{\frac{1}{2}}(X)}{E_{(1-c)}(X)}.$$

Furthermore, for any d, e such $1 > e \geq d > 0$, $\frac{E_e(X)}{E_d(X)} \leq \frac{e}{d}$.

Technical Lemma 3. Let f_1 and f_2 be two densities for the random variable $Z \geq 0$. Suppose that the density function f_2 dominates the density function f_1 in the sense of MLRP and that $E_2(Z) < \infty$. Let B_d and B_c be given such that $d \geq c$. Then $\frac{E_2(B_d)}{E_2(B_c)} \geq \frac{E_1(B_d)}{E_1(B_c)}$. Let J_b and J_a be given such that $b \geq a$. Then $\frac{E_2(J_a)}{E_2(J_b)} \leq \frac{E_1(J_a)}{E_1(J_b)}$. Lastly, let J be a junior equity contract and let B be a debt contract. Then $\frac{E_2(J)}{E_2(B)} \geq \frac{E_1(J)}{E_1(B)}$.

Proof of Result 1.
We will first prove the existence of symmetric equilibria in which the pessimistic banks play a pure strategy and the optimistic banks mix over bids that are all preferred to the pessimists' bid. For simplicity, we define $ll = \frac{(1-p)^2}{p^2+(1-p)^2}$ and $hh = \frac{p^2}{p^2+(1-p)^2}$. We further set $h_s(b) = E_s(B_b)$ $(s \in (0,1))$, $h_1(b) = E_{ll}(B_b)$, $h_2(b) = E_{\frac{1}{2}}(B_b)$ and $h_3(b) = E_{hh}(B_b)$. The functions g_i, $i \in \{1, 2, 3, s\}$ are defined analogously as the expectations of junior equity. We remark that the h_i are increasing and that the g_i are decreasing for $i \in \{1, 2, 3, s\}$. Lastly, we define $K = E_s(Z)$.

We provide a proof for the case $s \in (\frac{1}{2}, p)$. The arguments for the other cases are similar. We propose the following equilibrium. The pessimists will always bid $B_{\hat{d}}$, where $\hat{d}$ is the maximal d

such that $h_1(d) = I$. Define $\hat{F}(s) = \frac{p(1-p)(2s-1)}{p^2-(p^2+(1-p)^2)s}$ for $s \in [\frac{1}{2}, p]$. We note that $\hat{F}(s)$ is increasing in $s \in [\frac{1}{2}, p]$ and that $\hat{F}(\frac{1}{2}) = 0$ and $\hat{F}(p) = 1$. For convenience we define $\hat{F}(s) = 2$ for $s > p$.

The optimists will bid a mixed strategy in which they bid both debt and junior equity. The optimists will bid a mixed strategy of debt B_d on the range $(\bar{d}, \hat{d})$, according to the following distribution function

$$F(K - h_s(d)) = \frac{2p(1-p)(h_2(\hat{d}) - h_2(d))}{(p^2 + (1-p)^2)(h_3(d) - I)}.$$

We define $\bar{d}$ to be the minimal d such that $F(K - h_s(d)) = \hat{F}(s)$. We define $\bar{a}$ to be the maximal a such that $g_s(a) = h_s(\bar{d})$. The optimists will bid a mixed strategy of junior equity J_a on the range $a \in [\bar{a}, a']$ according to the following distribution function

$$F(K - g_s(a)) = \frac{2p(1-p)(h_2(\hat{d}) - g_2(a))}{(p^2 + (1-p)^2)(g_3(a) - I)}.$$

Here a' is defined to be the minimal a such that $F(K - g_s(a)) = 1$. It can be shown that the function F is well-defined, continuous and strictly increasing on the ranges $[K - g_s(\bar{a}), K - g_s(a')]$ and $(K - h_s(\hat{d}), K - h_s(\bar{d}))$. It can also be checked that $F(K - h_s(\bar{d})) = F(K - g_s(\bar{a}))$, where the first and second F functions in this equality are defined according to the first and second definitions given for F, respectively.

It is clear that bids of contracts C such that $E_s(C) < g_s(a')$ or such that $E_s(C) \geq h_s(\hat{d})$ are dominated by the equilibrium bids. We have defined F such that the payoff to every equilibrium bid is $2p(1-p)(h_2(\hat{d}) - I)$. To complete our demonstration that optimists do not deviate, we need to show that bidding any feasible contract C such that $E_s(C) \in [g_s(a'), h_s(\hat{d}))$ is dominated by bidding an equilibrium bid. We can write the optimist's payoff for any such deviation as

$$(2p(1-p) + (p^2 + (1-p)^2)F(K - E_s(C)))(E_{q'}(C) - I) \tag{1}$$

where $q' = \frac{p(1-p)+(p^2F(K-E_s(C)))}{2p(1-p)+(p^2+(1-p)^2)F(K-E_s(C))}$.
If $F(K - E_s(C)) \leq \hat{F}(s)$ then $q' \leq s$ and (1) is maximized over the set of feasible contracts with valuation $E_s(C)$ by the debt contract with that valuation. If $F(K - E_s(C)) \geq \hat{F}(s)$ then $q' \geq s$ and (1) is maximized over the set of feasible contracts with valuation $E_s(C)$ by the junior equity contract with that valuation. Both these claims follow from Technical Lemma 1. This demonstrates that the equilibrium strategies are optimal for the optimists.

The fact that $s > \frac{1}{2}$ guarantees that debt is always an optimal bid for pessimists. If the pessimists deviate by bidding B_d where $\hat{d} \geq d \geq \bar{d}$ then it is sufficient to show that

$$\frac{2p(1-p)(h_2(\hat{d}) - h_2(d))}{(p^2 + (1-p)^2)(h_3(d) - I)} \leq \frac{(p^2 + (1-p)^2)(h_1(\hat{d}) - h_1(d))}{2p(1-p)(h_2(d) - I)}, \tag{2}$$

where $h_2(d) > I$ (otherwise, deviation is clearly unprofitable). Applying Technical Lemma 2 and noting that $B_{\hat{d}} - B_d$ is a feasible security, we see that

$$\frac{(h_2(\hat{d}) - h_2(d))}{(h_1(\hat{d}) - h_1(d))} \leq \frac{(p^2 + (1-p)^2)^2 (h_3(\hat{d}) - h_3(d))}{(2p(1-p))^2 (h_2(\hat{d}) - h_2(d))}.$$

From the second part of the technical lemma it is clear that $\frac{(h_3(\hat{d})-h_3(d))}{(h_2(\hat{d})-h_2(d))} \leq \frac{2p^2}{p^2+(1-p)^2}$. We recall that $I = h_1(\hat{d}) \geq h_1(d) \geq E_b(B_d)$. We then infer that

$$\frac{h_3(d) - I}{h_2(d) - I} \geq \frac{h_3(d) - E_b(B_d)}{h_2(d) - E_b(B_d)} = \frac{\frac{p^2}{p^2+(1-p)^2}(E_g(B_d) - E_b(B_d))}{\frac{1}{2}(E_g(B_d) - E_b(B_d))} = \frac{2p^2}{p^2 + (1-p)^2}.$$

This, together with the inequalities above, shows (2). If the pessimist deviates by bidding B_d where $\bar{d} > d$ it is sufficient to show that

$$F(K - g_s(a)) \leq \frac{(p^2 + (1-p)^2)(h_1(\hat{d}) - h_1(d))}{2p(1-p)(h_2(d) - I)}, \tag{3}$$

where a is defined to be the maximal a such that $g_s(a) = h_s(d)$ (it is clearly never optimal to bid B_d such that $h_s(d) < g_s(a')$). Since $a \in (\bar{a}, a')$, we have

$$2p(1-p)(h_2(\hat{d}) - I) = 2p(1-p)(g_2(a) - I) + (p^2 + (1-p)^2)F(K - g_s(a))(g_3(a) - I)$$

$$\geq 2p(1-p)(h_2(d) - I) + (p^2 + (1-p)^2)F(K - g_s(a))(h_3(d) - I).$$

The inequality follows from the fact that $F(K - g_s(a)) \geq \hat{F}(s)$. This implies that

$$\frac{2p(1-p)(h_2(\hat{d}) - h_2(d))}{(p^2 + (1-p)^2)(h_3(d) - I)} \geq F(K - g_s(a)).$$

This means that to prove (3) it is sufficient to prove (2), which we have already done.

A proof of the uniqueness of the equilibrium in the sense described in the statement of the result is available from the author upon request. In essence, the proof generalizes the proof of the uniqueness of the equilibrium in the standard first-price auction with two bidders and two types.

Let C be a contract bid by a given investor in equilibrium and denote by $i(C) = i$ the investor's belief conditional on the contract winning the auction. In all cases, C must maximize $E_i(G)$ subject to $E_s(G) = E_s(C)$, or C is a strictly sub-optimal bid. In the equilibrium described in the result, C will be either debt or junior equity, but this need not hold for all equilibria. Let us suppose, however, that $i \neq s$ and that both C^1 and C^2 maximize $E_i(C)$ subject to $E_s(C) = E_s(C^1) = E_s(C^2)$. If $s = 0$ then $E_b(C^1) = E_b(C^2)$ and $i \neq 0$ so $E_g(C^1) = E_g(C^2)$. A similar result holds if $s = 1$, $i = 0$ or $i = 1$. Lastly, suppose that $i \neq 0, i \neq 1, s \neq 0, s \neq 1$. We then have that

$$\frac{1-s}{s} = \frac{E_g(C^1) - E_g(C^2)}{E_b(C^1) - E_b(C^2)} = \frac{1-i}{i}.$$

This implies that $s = i$, which is a contradiction. This shows that $E_b(C^1) = E_b(C^2)$ and $E_g(C^1) = E_g(C^2)$, from which we infer that $E_q(C^1) = E_q(C^2)$ for all $q \in [0,1]$.

We note that with probability one, the optimists' equilibrium bid C will be such that $i(C) \neq s$ (recall that there are no atoms in the optimists' strategy). This implies that entrepreneurs are indifferent between any two optimistic equilibrium strategies. If $s \neq ll$ then the pessimists' equilibrium bid C will be such that $i(C) \neq s$. This implies that if $s \neq ll$ then the entrepreneur will have the same expected payoff for all equilibrium strategy profiles of the investment game. If $s = ll$ then differing equilibrium strategies on the part of the pessimists will lead to varying payoffs for the entrepreneur.

Proof of Result 2.
In the proof of Result 1 it is shown that the investors' strategies are identical for all s in the range $s < ll$ and that they are likewise identical for all s in the range $s \geq p$. Let us consider the different strategies that arise for $s \in (ll, \frac{1}{2}]$. The pessimists' bids are identical on this range. The optimist bids a mixed strategy of junior equity J_a on the range $a \in (\hat{a}, \bar{a}]$ according to the distribution function

$$F_s(K_s - g_s(a)) = \frac{2p(1-p)(g_2(\hat{a}) - g_2(a))}{(p^2 + (1-p)^2)(g_3(a) - I)},$$

where $\hat{a}$ is defined to be the maximal a such that $g_s(a) = h_s(\hat{d})$ and $K_s = E_s(Z)$. We let s, s' such that $\frac{1}{2} \geq s \geq s' \geq ll$ be given. We denote by $\hat{a'}$ the maximal a such that $g_{s'}(a) = h_{s'}(\hat{d})$. Technical Lemma 3 shows that

$$1 = \frac{g_s(\hat{a})}{h_s(\hat{d})} \geq \frac{g_{s'}(\hat{a})}{h_{s'}(\hat{d})}.$$

This implies that $g_{s'}(\hat{a'}) = h_{s'}(\hat{d}) \geq g_{s'}(\hat{a})$ so that $\hat{a'} \leq \hat{a}$ and hence $g_2(\hat{a'}) \geq g_2(\hat{a'})$. We conclude, therefore, that for all $a \in (\hat{a}, \bar{a'}]$

$$F_{s'}(K_{s'} - g_{s'}(a)) \geq F_s(K_s - g_s(a)), \tag{4}$$

where $\bar{a'} \leq \bar{a}$ is defined in the usual way. Now let us consider $E_n[Z - W_i(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)]$ and for $i = s, s'$ for a given $n \in [0,1]$. For convenience, we denote the payoff of an entrepreneur of type n when he declares policy i to be η_i for $i \in \{s, s'\}$. We let $e \in [0, E_n(Z)]$ be given and we define a to be the maximal $\hat{a}$ such that $e = E_n(Z) - g_n(\hat{a})$ (note that a is independent of i). We remark that $\bar{F}_i(e) =: Prob\{\eta_i \leq e\} = F_i(K_i - g_i(a))$.

Equation (4) above shows that $\bar{F}_s$ first-order stochastically dominates $\bar{F}_{s'}$ so that

$$E_n[Z - W_s(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)] = \int_0^1 e d\bar{F}_s(e) \geq$$

$$\int_0^1 e d\bar{F}_{s'}(e) = E_n[Z - W_{s'}(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)].$$

If two H signals are received, the entrepreneur will accept the maximum of the two bids. The two optimists will mix independently. We know that if A dominates B in the sense of FOSD, C dominates D in the sense of FOSD and A, B, C and D are jointly independent then the maximum of A and C dominates the maximum of B and D in the sense of FOSD. So the argument just given shows that $E_n[Z - W_s(Z)|H_1 \cap H_2] \geq E_n[Z - W_{s'}(Z)|H_1 \cap H_2]$. In the case that $s = ll$ the pessimist's strategy is undetermined. From the standpoint of an entrepreneur of type $n \geq ll$, it is best if the pessimist bids a debt contract, as he does for $s > ll$. The argument above shows that the entrepreneur prefers the optimists' strategy under $s = \frac{1}{2}$ to that under $s = ll$. We may therefore conclude that for all $n \in [ll, 1]$, $U_F(n, \frac{1}{2}) \geq U_F(n, s)$ for all $s \in [ll, \frac{1}{2}]$.

We will now show that for all $n \geq ll$ and $s < ll$, $U_F(n, \frac{1}{2}) \geq U_F(n, s)$. If $s < ll$ then the pessimist bids $J_{\hat{a_s}}$ where $\hat{a_s}$ is defined to be the maximal a such that $g_1(a) = I$. If the valuation rule $s = \frac{1}{2}$ is selected then the pessimist bids $B_{\hat{d}}$ where $h_1(\hat{d}) = I$. Technical Lemma 3 shows that $g_n(\hat{a_s}) \geq h_n(\hat{d})$, so type n entrepreneurs prefer the pessimistic bid under the valuation regime of $\frac{1}{2}$. The optimistic bids under s and $\frac{1}{2}$ are of similar form, their distribution functions over J_a differing only in the $g_2(\hat{a})$ term. Under the $\frac{1}{2}$ valuation rule, $\hat{a}$ satisfies $g_2(\hat{a}) = h_2(\hat{d})$. We invoke Technical Lemma 3 a second time to show that $g_2(\hat{a_s}) \geq h_2(\hat{d}) = g_2(\hat{a})$. Now that we have shown that $F_s(K_s - g_s(a)) \geq F_{\frac{1}{2}}(K_{\frac{1}{2}} - g_{\frac{1}{2}}(a))$, we can proceed as in the previous case to show that for all $n \in [0, 1]$, type n entrepreneurs prefer the optimists' strategy under $s = \frac{1}{2}$ to that under $s < ll$. If $n < ll$, type n will prefer the pessimistic contracts offered under valuation rule n to those offered under valuation rule $\frac{1}{2}$, but he will prefer the optimistic contracts offered under the latter rule. It is not possible, in general, to determine which valuation rule will yield him greater utility.

We will now compare the contract profiles offered under different valuation rules for $s \geq \frac{1}{2}$. For all such s, pessimists bid $B_{\hat{d}}$. It is also the case that the contract profiles associated with these values for s differ only in the ranges in which different contract *types* are offered. That is, if for both of two different values of s, junior equity is offered with probability greater than k, then the distribution of junior equity offered under the two valuation rules will be identical for $F \geq (1 - k)$. An analogous statement holds for debt contracts.

It is thus apparent that in order to compare $U_F(n, s)$ and $U_F(n, s')$ for $s \geq s' \geq \frac{1}{2}$, we need only concern ourselves with the regions over which different contract types are offered under the two profiles. We recall that for all s the optimists' strategy F_s is continuous, so we can define $lb(s) = max\{x : F_s(x) = 0\}$ and $ub(s) = min\{x : F_s(x) = 1\}$. For every s we introduce $\zeta_s : [lb(s), ub(s)] \to \gamma$, where $\zeta_s(x)$ is defined to be the equilibrium contract C bid by the optimists such that $E_s(Z - C) = x$. We further define $W_n^s(x) = E_n(\zeta_s(x)(Z))$. We recall that the support of the optimist's strategy consists of an interval of debt and an interval of junior equity. This together with the fact that g_n and h_n are continuous shows that the function $W_n^s(x)$ is piecewise continuous. We write

$$E_n[Z - W_s(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)] = \int_{lb(s)}^{ub(s)} (E_n(Z) - W_n^s(x)) dF_s(x).$$

On the range $[lb(s), ub(s)]$ F_s is strictly increasing and continuous and it maps $[lb(s), ub(s)]$ to $[0,1]$. We define $\phi_s : [0,1] \to [lb(s), ub(s)]$ to be the inverse of F_s; ϕ_s is strictly increasing and continuous (Rudin, p.90). We may therefore write (Rudin, p.132)

$$E_n[Z - W_s(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)] = \int_0^1 (E_n(Z) - W_n^s(\phi_s(F)))dF.$$

The thrust of the comments above is that for $F \leq \hat{F}(s')$ and for $F \geq \hat{F}(s)$, $W_n^s(\phi_s(F)) = W_n^{s'}(\phi_{s'}(F))$, since the contracts associated with these values of F are identical. We then have

$$E_n[Z - W_s(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)] - E_n[Z - W_{s'}(Z)|(H_1 \cap L_2) \cup (H_2 \cap L_1)]$$

$$= \int_{\hat{F}(s')}^{\hat{F}(s)} (E_n(Z) - W_n^s(\phi_s(F)))dF - \int_{\hat{F}(s')}^{\hat{F}(s)} (E_n(Z) - W_n^{s'}(\phi_{s'}(F)))dF. \tag{5}$$

For some $y \in [\hat{F}(s'), \hat{F}(s)]$ let us consider $W_n^s(\phi_s(y))$ and $W_n^{s'}(\phi_{s'}(y))$. The construction of the optimistic strategies given in the proof of Result 1 shows that $W_n^s(\phi_s(y)) = E_n(B_d)$ and $W_n^{s'}(\phi_{s'}(y)) = E_n(J_a)$ for d and a satisfying

$$2p(1-p)(g_2(a) - I) + (p^2 + (1-p)^2)y(g_3(a) - I) = 2p(1-p)(h_2(\bar{d}) - I)$$

$$= 2p(1-p)(h_2(d) - I) + (p^2 + (1-p)^2)F_s(K_s - g_s(\bar{a}))(h_3(d) - I). \tag{6}$$

We define

$$s'' = \frac{p(1-p) + yp^2}{2p(1-p) + (p^2 + (1-p)^2)y} =: H(y).$$

We note that $H(y)$ is increasing in y and that $s = H(\hat{F}(s))$ and $s' = H(\hat{F}(s'))$. We can conclude, therefore, that $s \geq s'' \geq s'$. Equation (6) reduces to

$$(2p(1-p) + (p^2 + (1-p)^2)y)(g_{s''}(a) - I) = (2p(1-p) + (p^2 + (1-p)^2)y)(h_{s''}(d) - I)$$

which implies that $g_{s''}(a) = h_{s''}(d)$. If $n \geq s$ then Technical Lemma 3 shows that $g_n(a) \geq h_n(d)$. We then have that $W_n^{s'}(\phi_{s'}(y)) \geq W_n^s(\phi_s(y))$ so that (5) is positive. A similar proof (using $(F_s)^2$ and $(F_{s'})^2$ rather than F_s and $F_{s'}$) shows that

$$E_n[Z - W_s(Z)|H_1 \cap H_2] \geq E_n[Z - W_{s'}(Z)|H_1 \cap H_2].$$

We conclude that $U_F(n,s) \geq U_F(n,s')$. If $n \leq s'$ then

$$\frac{g_n(a)}{h_n(d)} \leq \frac{g_{s''}(a)}{h_{s''}(d)} = 1.$$

In this case, (5) is negative and analogous reasoning to that given above shows that $U_F(n,s) \leq U_F(n,s')$. So for $n \in [\frac{1}{2}, p]$, we have shown that $U_F(n,n) \geq U_F(n,s')$ and $U_F(n,n) \geq U_F(n,s)$ for all $p \geq s \geq n \geq s' \geq \frac{1}{2}$. We argued earlier that for all $n \geq ll$, $U_F(n, \frac{1}{2}) \geq U_F(n,s)$ for any $s \leq \frac{1}{2}$. We recall that valuation policies $s \geq p$ yield identical profiles of contracts. This implies that for $n \in [\frac{1}{2}, p]$, $U_F(n,n) \geq U_F(n,s)$ for all $s \in [0,1]$. For $n > p$, $U_F(n,p) \geq U_F(n,s)$ for all s. Using the first and second parts of the proof we see that for $p \in [ll, \frac{1}{2}]$, $U_F(n, \frac{1}{2}) \geq U_F(n,s)$ for all s. We also have that for $n < ll$, $U_F(n, \frac{1}{2}) \geq U_F(n,s)$ for all $s \geq \frac{1}{2}$. This completes the proof.

Proof of Result 3. We treat only the case $n \in (\frac{1}{2}, p)$; the other cases are similar. The fact that $n > \frac{1}{2}$ implies that $ll(n) > ll$, so setting $s \geq \frac{1}{2}$ will result in both pessimistic and optimistic bids that entrepreneur type n prefers to those associated with valuation policies $s < \frac{1}{2}$. We know that a policy $s \in [\frac{1}{2}, n]$ results in pessimistic bids that are identical to those generated by a policy n. However, the proof of Result 2 showed that the optimistic bids generated by n are preferred under both beliefs n and $hh(n)$. We may therefore limit our consideration to policies $s \in [n,p]$. We set $s' = min\{rat(n, \hat{F}(n)), p\} \geq s \geq n$. We then have

$$\tilde{U}_F(n,s) - \tilde{U}_F(n,s')$$

$$= \int_{\hat{F}(s)}^{\hat{F}(s')} (Q_E((H_1 \cap L_2) \cup (H_2 \cap L_1))(E_n(Z) - W_n^s(\phi_s(F)))$$

$$+Q_E(H_1 \cap H_2)(E_{hh(n)}(Z) - W_{hh(n)}^s(\phi_s(F)))2F)dF$$

$$- \int_{\hat{F}(s)}^{\hat{F}(s')} (Q_E((H_1 \cap L_2) \cup (H_2 \cap L_1))(E_n(Z) - W_n^{s'}(\phi_{s'}(F)))$$

$$+Q_E(H_1 \cap H_2)(E_{hh(n)}(Z) - W_{hh(n)}^{s'}(\phi_{s'}(F)))2F)dF$$

$$= \int_{\hat{F}(s)}^{\hat{F}(s')} (2F(p^2 n + (1-p)^2(1-n)) + 2p(1-p))(W_{rat(n,F)}^{s'}(\phi_{s'}(F)) - W_{rat(n,F)}^{s}(\phi_s(F)))dF. \quad (7)$$

For $y \in [\hat{F}(s), \hat{F}(s')]$, $W_{rat(n,y)}^{s'}(\phi_{s'}(y)) = E_{rat(n,y)}(B_d)$ and $W_{rat(n,y)}^{s}(\phi_s(y)) = E_{rat(n,y)}(J_a)$, for B_d and J_a such that $E_{s''}(B_d) = E_{s''}(J_a)$, for some $s'' \in [s, s']$, as argued in the proof of Result 2. We have $rat(n,y) \geq rat(n, \hat{F}(n)) \geq s' \geq s$. Technical Lemma 3 shows that $E_{rat(n,y)}(J_a) \geq E_{rat(n,y)}(B_d)$. This term by term argument shows that (7) is non-positive. This implies that

$\tilde{U}_F(n,s) \leq \tilde{U}_F(n, min\{rat(n, \hat{F}(n), p\})$ for all $s \leq min\{rat(n, \hat{F}(n)), p\}$. To show the existence of an optimum, it is sufficient to demonstrate that $\tilde{U}_F(n,s)$ is continuous in s on $[\frac{1}{2}, p]$. This follows from a quick argument that relies on the boundedness of all expected payoffs.

Proof of Result 4.
We sketch the proof for the case $s = \frac{1}{2}$. For ease of notation, we define $hhhh = \frac{p^4}{p^4+(1-p)^4}$ and $llll = \frac{(1-p)^4}{p^4+(1-p)^4}$ and we set $g_4(a) = E_{hhhh}(J_a)$. We propose the following equilibrium. The pessimists will always bid $B_{\hat{d}}$, where $\hat{d}$ is defined to be the minimal d such that $E_{llll}(B_d) = I$. The neutrals will bid a mixed strategy of debt B_d on the range $[\bar{d}, \hat{d})$ according to the following distribution function

$$F^2(K - h_s(d)) = \frac{(p^2 + (1-p)^2)(h_1(\hat{d}) - h_1(d))}{4p(1-p)(h_2(d) - I)}.$$

We define $\bar{d}$ to be the minimal d such that $F^2(K - h_s(d)) = 1$. We further define $\bar{a}$ to be the maximal a such that $g_s(a) = h_s(\bar{d})$. The optimists will bid a mixed strategy of junior equity J_a on the range $a \in (\bar{a}, a']$ according to the following distribution function

$$F^3(K - g_s(a)) = \frac{2(p^2 + (1-p)^2)p(1-p)(g_3(\bar{a}) - g_3(a)) + 2p^2(1-p)^2(g_2(\bar{a}) - g_2(a))}{(p^4 + (1-p)^4)(g_4(a) - I)}.$$

We define a' to be the maximal a such that $F^3(K - g_s(a)) = 1$. The proof that this is indeed an equilibrium generalizes the arguments given in Result 1.

Proof of Result 5. We give the proof for the case $n \geq s \geq \frac{1}{2}$; the other cases are similar. We propose the following equilibrium strategies. The pessimists bid Z and the optimists mix over debt, mixed debt-equity and equity. We define $\hat{d}$ to be the minimal d such that $E_n(B_d) = E_n(Z) - ec$. The optimists will bid debt B_d on the range $[\bar{d}, \hat{d}]$, according to the following distribution function

$$F(K - h_s(d)) = \frac{2p(1-p)(h_2(\hat{d}) - h_2(d))}{(p^2 + (1-p)^2)(h_3(d) - I)},$$

where we define $\bar{d}$ to be the minimal d such that $F(K - h_s(d)) = min\{1, \hat{F}(s)\}$. If $1 \leq \hat{F}(s)$ then the optimist's strategy is completely defined. Otherwise, we define $\bar{a}$ to be the minimal a such that $g_s(a) = h_s(\bar{d})$. If $E_n(J_{\bar{a}}) \leq E_n(Z) - ec$ then the optimists will bid a mixed strategy of junior equity J_a on the range $a \in (\bar{a}, a']$ according to the following distribution function

$$F^1(K - g_s(a)) = \frac{2p(1-p)(h_2(\hat{d}) - g_2(a))}{(p^2 + (1-p)^2)(g_3(a) - I)}. \tag{8}$$

Here a' is defined to be the minimal a such that $F(K - g_s(a)) = 1$. If $E_n(J_{\bar{a}}) > E_n(Z) - ec$ then the optimists will bid over mixed-debt equity. We define $\bar{a}'$ to be the minimal a such that $g_n(a) = E_n(Z) - ec$. We let $k \in (g_s(\bar{a}'), g_s(\bar{a}))$ be given and define $a(k)$ to be the maximal a such that $g_s(a) = k$ and we define $d(k)$ to be the minimal d such that $h_s(d) = k$. Mixed debt-equity M_k is defined by $M_k = \alpha(k)J_{a(k)} + (1 - \alpha(k))B_{d(k)}$, where $\alpha(k) \in [0,1]$ is defined by $\alpha(k) = \frac{E_n(Z) - ec - h_n(d(k))}{g_n(a(k)) - h_n(d(k))}$.

It is straightforward to show that $E_i(M_k)$ is continuous in k for $i \in \{1,2,3\}$. We note that M_k is constructed so that $E_n(M_k) = E_n(Z) - ec$ and $E_s(M_k) = k$ for all $k \in (g_s(\bar{a'}), g_s(\bar{a}))$. For $t \in [s,n)$, $E_t(C)$ may be written as a convex combination of $E_n(C)$ and $E_s(C)$. This shows that for $k2, k1 \in (g_s(\bar{a'}), g_s(\bar{a}))$, if $k2 > k1$ then $E_t(M_{k2}) > E_t(M_{k1})$.

We define a' to be the maximal a such that $(g_p(a) - I) = 2p(1-p)(h_2(\hat{d}) - I)$. If $\bar{a'} > a'$ then we specify that for all $x \in [x', g_s(\bar{a}))$ the optimist mixes over M_x according to the distribution function F where F is defined by

$$2p(1-p)(E_2(M_x) - I) + (p^2 + (1-p)^2)F(K-x)(E_3(M_x) - I) = 2p(1-p)(h_2(\hat{d}) - I),$$

where x' is defined to be the maximal x such that $F(K-x) = 1$. That $F(K-x)$ is continuous and strictly decreasing is established in the same way that it was in the proof of Result 1.

In the case that $\bar{a'} \leq a'$ we specify that for all $x \in (g_s(\bar{a'}), g_s(\bar{a}))$ the optimist mixes over M_x according to the distribution function F defined above.

The final part of the optimists' strategy is as follows. They bid a mixed strategy of junior equity J_a on the range $a \in [\bar{a'}, a']$ according to the distribution function F^1 given above.

We will now show that deviation to a feasible contract C is not profitable. Suppose $E_s(C) \geq E_s(B_{\hat{d}})$. If C is not accepted, it is certainly not more profitable than an equilibrium bid. If C is accepted then $E_n(C) \leq E_n(B_{\hat{d}})$ and Technical Lemma 1 shows that $E_{\frac{1}{2}}(B_{\hat{d}}) \geq E_{\frac{1}{2}}(C)$, so that $B_{\hat{d}}$ is a superior bid to C. If $E_s(C) = E_s(J_a)$ for an equilibrium bid J_a, then the proof of Result 1 shows that J_a is preferred to C.

If $E_s(C) = x \in (x', g_s(\bar{a}))$ (or $E_s(C) \in (g_s(\bar{a'}), g_s(\bar{a}))$ in the second case) and C is accepted then the mixed debt-equity M_x is a superior bid to C. This follows from the fact that for all t such that $n \geq t \geq s$, $E_t(C)$ may be written as a convex combination of $E_s(C)$ and $E_n(C)$. We know that $E_s(M_x) = x = E_s(C)$ and $E_n(M_x) = E_n(Z) - ec \geq E_n(C)$ so that $E_t(M_x) \geq E_t(C)$.

Proof of Result 7. We let $\hat{d}$ be the minimal d such that $h_1(d) = I$. If $E_{ll(n)}(Z - B_{\hat{d}}) \geq ec$ then the equilibrium described in Result 1, part (iv) holds. Namely, pessimists bid $B_{\hat{d}}$ and optimists mix over a range of B_d for $d \in [\bar{d}, \hat{d})$ according to the distribution function

$$F(-d) = \frac{2p(1-p)(h_2(\hat{d}) - h_2(d))}{(p^2 + (1-p)^2)(h_3(d) - I)}.$$

The term $\bar{d}$ is defined to be the maximal d such that $F(-d) = 1$. All winning bids are accepted.

If $E_{ll(n)}(Z - B_{\hat{d}}) < ec \leq E_n(Z - B_{\hat{d}})$ the equilibrium above still holds. In the updating setting, the entrepreneur will not accept the pessimists' bids. However, deviating yields the pessimists a payoff no higher than zero, so they play the same strategies as above.

If $ec > E_n(Z - B_{\hat{d}})$, the optimists must modify their strategies. In the updating setting, the highest face value debt contract bid by the optimists will win the auction only when the other bidder is a pessimist. This implies that the entrepreneur will retain his belief n when evaluating the proposed contract. We let d' be the maximal d such that $E_n(Z) - ec = h_n(d)$. If $h_2(d') \geq I$ then the optimists

mix over a range of B_d for $d \in [d'', d']$ according to the distribution function F where F is defined by

$$2p(1-p)(h_2(d) - I) + (p^2 + (1-p)^2)F(-d)(h_3(d) - I) = 2p(1-p)(h_2(d') - I). \tag{9}$$

The term d'' is defined in the usual way. It is clear that an identical equilibrium holds in the confident setting in which the entrepreneur always retains his belief n. All pessimistic bids are rejected.

We now suppose that $ec > E_n(Z - B_d)$ for all d such that $h_2(d) \geq I$. We let k' be the maximal d such that $h_2(k') = I$. In the confident setting equilibrium the optimists mix over a range of B_d for $d \in [k'', k']$ according to the distribution function F given in (9), replacing d' with k'.The term k'' is defined in the usual way. We note that some of the contracts bid in equilibrium will be accepted by the entrepreneur and others will not. If an optimist deviates by bidding a debt contract with a face value larger than k' it will be rejected by the entrepreneur.

The equilibrium in the updating setting depends on the parameter values. Rather than considering the cases individually, we sketch the general argument that weakly larger contracts are accepted from optimists in the updating setting. Consider any contract accepted in the confident case. The updating entrepreneur will accept this contract when bid by an optimist since the entrepreneur's belief will be at least n in this case. As ec rises, eventually all contracts are declined.

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Notes

[1]See DeMarzo, Vayanos and Zwiebel (1998) for another model in which agents update using the stated opinions of only a select group of their colleagues.

[2]See, for example, the *Wall Street Journal*, January 12, 1997, page B1 for a story on the increasing popularity of scoring systems for small-business loans.

[3]Cooper, Woo and Dunkelberg (1988) provide evidence that entrepreneurs' predictions of their probability of success are uncorrelated with objective predictors.

[4]Daniel, Hirshleifer and Subrahmanyam (1998) contains a detailed discussion of the role of overconfidence in financial markets. Welch and Bernardo (1997) analyze the social implications of having either a shortage or surplus of overconfident entrepreneurs.

[5]One implication of this assumption is described in Technical Remark 1.

[6]Athey and Levin (1998) discuss and model an auction organized by the U.S. Forest Service that bears some resemblance to this mechanism.

[7]If this assumption does not hold, the equilibrium strategies do not differ greatly. Pessimists do not bid and Optimists mix below Z. Result 5 deals with a related setting.

[8]The first-price auction in this paper is not an optimal design, for the usual reason that the entrepreneur would do better to specify a mechanism in which bids within a certain range are not accepted. That mechanism would force down the optimists' bids. However, institutional constraints may not permit entrepreneurs to extract all possible surplus from investors. Furthermore, given the large space of feasible contracts, the determination of the optimal mechanism is quite complex. Our mechanism is natural and corresponds closely to the way bank loans are made. A discussion of the application of this model to the bank-loan market is given in Section 6 below.

[9]The equilibrium would not necessarily be subgame perfect if the entrepreneur could renege on his commitment to use the valuation rule s. For example, an entrepreneur of type $n \in (ll, \frac{1}{2})$ will declare $s = \frac{1}{2}$ to minimize the junior equity that is requested of him. Nonetheless, he would prefer junior equity slightly larger than the maximal J_a bid in equilibrium to the pessimist's debt bid; by binding himself to valuation s he is making a threat that is not credible in the absence of a commitment to accept the pessimist's bid over this non-equilibrium junior-equity bid. In this setting we have assumed that agents may contract over allocations of next period's cash flows, so it is certainly reasonable to posit that fixing the choice of s is part of the general contractual agreement.

[10]The federal government restricts banks from holding non-debt claims partly to limit their power and partly in order to reduce the riskiness of federally insured banking assets. See Shull (1994) for a discussion of the disjunction between banking and commerce in the U.S.

[11]John, John, and Saunders (1994) discuss other benefits to universal banking.

[12]It should be noted that the extent of bank investments in SBICs is below the mandated limit, perhaps because of the burdensome regulations associated with this investment vehicle (Investment Advisory Council (1992)).

[13]Venture-capital firms typically receive 1000 requests for financing annually (Sahlman (1990)). Not all proposals can be closely examined, so the initial sorting process that allocates entrepreneurs to firms may be regarded as fairly arbitrary.