Dynamic Information Disclosure

Martin Dierker* and Avanidhar Subrahmanyam**

*Corresponding Author: KAIST Graduate School of Finance and Accounting, 83 Hoegi-ro, Hoegi-dong, Dongdaemun-gu, Seoul 130-722, Republic of Korea. Phone +82 2 9583415. Email: dierkerm@business.kaist.ac.kr.

**The Anderson School, UCLA, Los Angeles, CA 90095-1481, USA. Email: subra@anderson.ucla.edu.

We thank Tony Bernardo, Michael Brennan, Mike Fishman, Jack Hughes, Praveen Kumar, Nisan Langberg, and Bill Zame, seminar participants at the University of Arizona, UBC, University of Chicago, Concordia University, University of Houston, Melbourne Business School, University of New South Wales, Northwestern University, Simon Fraser University, UCLA, and the audience at the CEPR Summer Symposium on Financial Markets in Gerzensee for their helpful comments.
Dynamic Information Disclosure

We explore optimal disclosure timing in a dynamic setting. By postponing disclosure, the firm can enhance informational efficiency via encouraging information acquisition without exposing informed investors to the substantial risks of holding the asset until liquidation. Thus, immediate disclosure is sometimes suboptimal in a dynamic setting, contrary to the static benchmark case, where immediate disclosure leads to maximal informational efficiency. We characterize conditions under which postponing disclosure is preferable, which allows us to develop predictions on the timing of information disclosures.
The timeliness of the release of information by management is considered key to its usefulness. A U.S. Securities and Exchange Commission (SEC) proposal to improve the filing dates of 10-K annual and 10-Q quarterly reports states the following: “Shortening the due dates for quarterly, annual and transition reports would ... accelerate the delivery of information to investors and the capital markets, enabling them to make more informed investment and valuation decisions. This helps the capital markets function more efficiently, which means more efficient valuation and pricing.”

This paper examines information disclosure in a dynamic setting with noisy rational expectations where managers can time corporate disclosures. We find that firms may in fact increase the informational efficiency of their stock prices by postponing disclosure, which complements the intuition in the SEC statement. We characterize situations in which it will be optimal for firms to disclose information immediately, as well as situations in which delaying disclosure is preferable. This helps us understand differences in timing of earnings announcements and pre-announcements across firms.

In our model, a more informative stock price improves the funding of the firm’s growth opportunities, which in turn increases the net present value of its operations (as in Subrahmanyam and Titman, 1999). We assume that the firm’s manager and informed outside investors have overlapping, but not identical, information about the firm’s value. In a static setting, informed traders assimilate the risk of holding the asset until the liquidation date. This constrains their asset holdings, and thus reduces the advantages of non-disclosure. We prove that in this case informational efficiency is maximized if the firm always discloses available information.

---

2 Chambers and Penman [1984], Baginski, Hassell, and Waymire [1994], and Heflin, Subramanyam, and Zhang [2003] provide evidence that the timing of earnings announcements and pre-announcements varies considerably across US firms. Later in the paper, we explicitly discuss the implications of our work for these findings.
confirming the common intuition based on static models. In a dynamic setting, however, the manager gains the option to disclose at an intermediate date. Because the manager’s signal is correlated with informed investors’ information, this gives investors the opportunity to speculate on the information disclosure (e.g., an earnings announcement). The investors can then reverse their positions after the announcement is made, thus potentially avoiding the long-term risks of holding the asset, which stimulates ex ante information acquisition. Thus it can be optimal to the firm to postpone information disclosure, but not to withhold it indefinitely.

Our model is an extension of the multi-period rational expectations models of Hirshleifer, Subrahmanyam, and Titman (1994), and Holden and Subrahmanyam (1996). Specifically, our model consists of risk averse and competitive informed agents, a competitive, risk-neutral market making sector, and noise or liquidity trades. We augment these models to allow for managerial disclosure and also incorporate the effects of greater informational efficiency on firm valuation. Like Brown and Jennings (1989), Holden and Subrahmanyam (1996), and Wang (1994), we are unable to obtain analytic solutions for the general dynamic setting and instead rely on numerical simulations for our main results. While this limitation should be noted, we believe that our setting, by serving as an initial attempt at modeling disclosure in an intertemporal setting, might be a useful starting point for the development of other, more tractable models.

We characterize situations in which it will be optimal to disclose information immediately and situations in which delaying disclosure is preferable. Only if part of the information is unique to investors can intermediate disclosure have a positive effect. In fact, our analysis indicates that this investor-specific information has to be sufficiently important as compared to information that is manager-specific. This latter type of information discourages delayed disclosure, since it acts as a source of risk for investors and cannot affect stock price unless disclosed.
Informational overlap between manager and investors plays a key role in our setting. Only if the overlap is sufficiently large does delayed disclosure offer sufficient extra trading opportunities to investors. However, the marginal benefit of more overlap is decreasing, and at some point the “informational cost” of delaying the disclosure starts to dominate again. Thus, it is for intermediate instances of informational overlap that delayed disclosure is the most beneficial. In addition, the riskier it is to hold the asset until liquidation, or the higher investors’ risk aversion, the more beneficial it is to delay disclosure. Finally, the more costly it is to acquire information, or the less noise there is in the market, the more beneficial it is to delay disclosure.

There is a line of thinking on immediate disclosure which believes that firms should reveal their information as early as possible. This leads to the swift incorporation of the information into the share price, which in turn leads to more efficient production and investment decisions (Kunkel [1982]). In addition, public disclosure “levels the playing field” (Diamond [1985]). These arguments suggest that immediate disclosure is beneficial in a static setting because it eliminates information asymmetries by pre-empting private information acquisition.\(^3\)

We provide another perspective to the preceding arguments by analyzing disclosure in a dynamic setting. Our model implies that firms with more manager-specific information will time their disclosures earlier.\(^4\) A reasonable proxy for the amount of manager-specific information is the typical prediction error of the I/B/E/S consensus estimate. If the consensus estimate is a relatively imperfect predictor of actual earnings, manager-specific information is substantial, and we predict that these firms will have earlier earnings announcements. On the other hand, investor-specific information will be more important in young industries or growth industries, or

---

\(^3\)The static case is further analyzed in Bushman [1991], Indjejikian [1991], Lundholm [1991], Alles and Lundholm [1993], Gigler and Hemmer [2001], and Arya and Mittendorf [2007].

\(^4\) Demski and Feltham (1994) analyze a two-date model where agents’ trading behavior is influenced by the information content of an upcoming disclosure. We add to their work by considering optimal disclosure timing in a dynamic setting.
when the market is experiencing high volatility or regime shifts. In these cases, we expect firms to introduce their disclosures later. Furthermore, some companies pre-announce information before an actual earnings disclosure. Baginski, Hassell, and Waymire [1994] report that this is typically the case for a company that has important news that shareholders do not have. Consistent with this observation, our model predicts that firms that have an unusual amount of manager-specific information will want to disclose this information earlier.\(^5\)

Initial public offerings (IPOs) are good examples of our information structure. In this setting, the manager has some unique information about the firm, while large institutional investors with their expertise in valuation are required to find the market clearing price. Companies that go public are frequently cited as examples for insiders not disclosing their asymmetric information in order to expropriate uninformed market participants. In contrast, our model suggests that these firms may not want to initially disclose their information in order to attract informed investors, who will help in valuing the new issue. More generally, young or small firms who often find it hard to attract analyst coverage may be able to do so by delaying the release of information.


\(^5\) Here, an additional economic benefit may include the avoidance of possible legal action if bad news is withheld. The study mentioned above finds that two-thirds of preannouncements contained negative news.

\(^6\) Brunnermeier [2005] discusses how an agent whose information is correlated with managerial disclosure can benefit by trading both before and after the disclosure, while Indjejikian, Lu, and Yang [2011] show that an insider may wish to leak a noisy version of his information to another insider and benefit from the fact that the second insider partially trades on added noise. However, unlike us, these authors do not focus on the equilibrium managerial disclosure strategy in a dynamic setting.
firm’s disclosure today acts as a commitment to future disclosures, which in turn affects incentives today. Empirical evidence on disclosure timing is provided by Yermack [1997], Aboody and Kasnick [2000], and Brown, Christensen, and Elliott [2012]. Closely related to the focus of our paper, though not a test of it, is the study by Pawlewicz [2011], who provides evidence that firms do time their disclosure of information, and that such timing indeed affects earnings response coefficients, as our model would predict.7

This paper is organized as follows. Section 1 discusses the model. In Section 2 we show that immediate disclosure is optimal in the static setting. The steps to solving the dynamic case are outlined in Section 3, while Section 4 presents results on disclosure timing. Section 5 explores applications and empirical predictions, and Section 6 concludes. Algebraic details of derivations, and all proofs, unless otherwise stated, appear in the Appendix.

1. The Model

Consider an economy with four dates: \( t = 0, 1, 2, \) and 3. These dates represent different stages in the life cycle of the single firm in the economy. The company is set up as an equity-financed firm at date 0. Date 1 represents the early stage in the life cycle of the firm. Its shares are publicly traded, and to model the effect of informational efficiency on firm value, we use the device of Subrahmanyam and Titman (1999), and assume that the firm can expand its operations by investing in a growth opportunity. Specifically, we assume that the firm does not have the funds to invest, and therefore has to sell the growth opportunity to investors. A more informative stock price leads to a higher value of the growth opportunity, since it will enable investors to allocate

---

7The focus in Pawlewicz [2011] is on the effect of SEC Regulation G, which, in 2003, started requiring firms to report their earnings in a standardized (GAAP) format on form 8-K. His work finds that firms delay their earnings disclosures and that the announcements lead to greater investor responses (indicating greater informativeness of the announcements) following the implementation of the regulation.
capital more efficiently. Date 2 represents the intermediate stage in the firm’s life cycle. Again, shares are publicly traded, and the firm has access to another growth opportunity. Finally, date 3 represents the end of the firm’s life; operation is disbanded, assets are liquidated, and final payoffs are realized to the shareholders.

1.1 Investors and Timing

As in Vives (1995), and Hirshleifer, Subrahmanyam, and Titman (1994), the economy is populated by three types of investors: (1) a continuum of risk-averse potentially informed investors; (2) an unmodeled group of noise or liquidity traders; and (3) a competitive and risk-neutral trader who absorbs the demands of the other agents, and is termed a market maker. The continuum of risk-averse potentially informed investors can obtain information about the asset’s liquidation value at a cost. Their utility is given by:

$$U(W) = -\exp(-R \cdot W),$$  \hspace{1cm} (1)

where $W$ denotes final wealth at date $t=3$ and $R$ is the risk aversion coefficient. For simplicity, the demands of liquidity traders are assumed to be price inelastic. The risk-neutral, competitive market maker sets the price as the expectation of future payoffs conditional on his own information, which consists of publicly disclosed signals and observed net order flows to date (which, in equilibrium, are linear functions of the prices). This implies that prices are martingales [i.e., $P_1 = E(F|P_1), P_2 = E(F|P_1,P_2)$], and we can abstract from issues of discounting in order to focus on the informational role of share prices.

1.2 Informational Structure

In our model, the firm’s manager is endowed with information about the firm’s value. In addition,
information can be obtained by investors through costly research. Information can be incorporated into the share price in two different ways: the manager can choose to disclose his information, while the traders’ information is partly revealed through their trades. We assume that these two types of information are correlated, but not identical. Our reasoning is that the manager, by his informed knowledge of the firm and access to the firm’s financial situation, is likely to have information that outsiders cannot obtain, while outsiders may be more familiar with the stock market and, therefore, may have a superior understanding of the pricing implications of the firm’s available accounting information. Indeed, Dye and Sridhar [2002] and Dow, Goldstein, and Gumbel [2010] study how a firm may publicly disclose a business strategy in order to learn from the capital market’s reactions. As an example, United Airlines abandoned a plan to consolidate its airline, hotel, and rental car businesses into a planned “Allegis” conglomerate following a negative stock market reaction to the announcement.

We thus model the interaction between different sources of information, and assume that investors and managers have correlated, but non-identical, information. We write the firm’s payoff as:

$$ F = \eta + \theta + \delta + \epsilon, $$

and postulate that the manager’s signal is given as $s_m = \eta + \theta$, while the informed investors’ signal is $s_i = \theta + \delta$. Thus, $\theta$ represents the overlap between the manager’s and the investors’ information, $\eta$ is manager-specific information, and $\delta$ is investor-specific information. In addition, the asset payoff is subject to an unpredictable shock, $\epsilon$. We assume that all random variables are jointly independent and normally distributed with mean zero. The variance of a

---

8 See also Dow and Gorton [1997] for a model in which managers and shareholders have different information. In fact, in their excellent survey of the disclosure literature, Beyer et al. [2010, p. 335] explicitly suggest modeling the interaction between information possessed by outside analysts and management.

random variable $r$ is denoted by $\sigma_r^2$ (e.g., $\eta$ has variance $\sigma_\eta^2$).

We postulate that before signals are realized, the manager precommits to a dynamic policy to either disclose the signal either at date 1, or at date 2, or to not disclose the signal at all (which is equivalent to disclosure at date 3).\(^{10}\) We assume the precommitment is exogenous, as, for example, in Lanen and Verrecchia [1987]. However, a feasible precommitment device is a desire on management’s part to maintain reputation in repeated interactions with outsiders.

Besides disclosure, information can flow into share prices through informed trading.\(^{11}\) Investors can do costly research to learn about the potential payoff from a transaction. After observing the disclosure policy set by the manager, but before trading at time $t=1$, investors can obtain the signal $s_t = \theta + \delta$ at a cost $c$.\(^{12}\) Let $M$ denote the mass of informed investors. The main focus of the paper is on the case where $M$ is determined endogenously.

The independent and identically distributed liquidity shocks, $z_t \sim N(0, \sigma_z^2)$, arrive at each of the dates $t = 1, 2$. These shocks are independent of all other random variables in the model. The risk-neutral market maker sets the date 1 and date 2 prices to equal the expected values of the final payoff, conditional on all public signals and net order flows observed to date.

1.3 Growth Opportunities

The purpose of our study is to analyze the effect of a managerial disclosure on the informational

\(^{10}\)Previous drafts of this paper allowed the manager to choose to reveal a certain fraction of his information at each point in time. However, we restrict ourselves to the more simple case mentioned above, as the more complicated case leads to the same substantive insights.

\(^{11}\) We assume that the manager cannot himself acquire the information at a cost. This assumption can be motivated by the notion that opportunity costs of running the firm are high enough to preclude expertise in financial markets and valuation that outside analysts possess.

\(^{12}\) Earlier drafts also allowed the time of information arrival to depend on the amount of resources spent. Specifically, we assumed that it costs an amount $c_\text{E}$ to obtain the information early (at date 1), while it costs $c_I \leq c_\text{E}$ to obtain information at an intermediate time (at date 2). The tension between early and intermediate informed trades is an important aspect of this alternative model. However, the central insights we obtain in the simpler version continue to obtain in the more complicated alternative, so we abstract from this issue for the sake of simplicity. Furthermore, we only allow traders to acquire information before trading at time $t=1$. 

8
efficiency of the stock price in a dynamic financial market. For this purpose, we model a scenario where the manager precommits to an optimal dynamic policy that maximizes the sum of informational efficiencies of the stock price at dates 1 and 2, \( \sum_{t=1}^{2} [Var(F) - Var(F|P_t)] \). In this section, we rationalize such an objective function by modeling economic benefits from informational efficiency in such a manner that profit maximization leads to maximizing informational efficiency. In our setting, more informative stock prices lead to higher quality investment decisions (Dye [2001], Berger [2011]). As in Subrahmanyam and Titman (1999), we model the link between informational efficiency and real investment by assuming that at each of dates \( t = 1 \) and \( t = 2 \), the company has access to a growth opportunity such that the risk-neutral firm can increase its scale of operation by adding new capital. Each unit of capital increases the liquidation value by the payoff of one share, \( F = (\eta + \theta + \delta + \varepsilon) \). The dollar cost of adding \( \kappa_t \) units of capital at date \( t \) is assumed to be \( \frac{1}{2} \kappa_t^2 \). Thus, the net present value (NPV) of adding \( \kappa_t \) units of capital is given by:

\[
\kappa_t (\eta + \theta + \delta + \varepsilon) - \frac{1}{2} \kappa_t^2.
\]

Now consider the first-order condition for the optimal investment decision. Differentiating the conditional expected value of equation (3) yields:

\[
\kappa_t^* = E[F|\Phi_t],
\]

where \( \Phi_t \) represents a generic information set at date \( t \). Our focus is on information contained in the stock price. For this purpose, we model the investment decision as conditional on market prices alone. To convey this intuition, we assume that the firm does not have the resources to finance an

---

13 Conditioning on \( P_2 \) is equivalent to conditioning on both \( P_1 \) and \( P_2 \), because \( P_2 = E(F|P_1, P_2) \) contains all information embedded in \( P_1 \).

14 For empirical evidence on this notion, see Durnev, Morck, and Yeung [2004] and Chen, Goldstein, and Jiang [2007].

15 The current formulation allows the amount of capital invested to be negative. This is an undesirable but typical problem of the normal-exponential framework we use in our model. However, this problem could easily be mitigated by using distributions with positive mean payoffs instead.
expansion. Instead, we assume it spins off the growth opportunities and sells them to the market maker, who makes the real investment decision, so that $\Phi_1 = \{P_1\}$ and $\Phi_2 = \{P_1, P_2\}$. To preserve the linear structure of the model, we assume that the growth opportunities are sold ex ante, before any trade takes place, but after the manager has determined a disclosure policy. Competitive market making ensures that the company receives the ex-ante expected value of the growth opportunities under the optimal levels of investment, $\Psi C$. The manager’s objective then is to choose the disclosure policy that maximizes the sum of the ex-ante expected values of the two growth opportunities, computed at $\Psi C$ as in equation (4).

We now discuss the assumption that the real investment decision is conditional on market prices only. Our basic aim is to capture a meaningful economic tradeoff, where delaying disclosure of the manager’s information is economically costly in terms of potentially delaying the resolution of uncertainty through market prices. We then ask whether additional investor-specific information obtained from delayed disclosure is sufficient to compensate for the loss of informational efficiency at date 1. Now, if the real investment decision is publicly observed and is conditional on the managerial signal, it can directly convey information from the decision-maker’s information set to the public, possibly circumventing our prior assumption that the manager can encourage information acquisition at date 1 by precommitting to a delayed disclosure policy.

Thus, consider an alternative formulation where the manager is allowed to make the investment decision contingent on his signal and market prices. In this case, it follows from equation (4) that the optimal real investment decision, being linear in the publicly observed market prices and the manager’s signal, perfectly reveals the manager’s information to market participants, and delayed disclosure is no longer a feasible equilibrium. Selling the growth
opportunities to market makers ex-ante (with a covenant preventing the firm from reacquiring it later), however, can act as a commitment device not to convey manager’s information to the public through the investment decision. Thus, whenever delayed disclosure is optimal under the assumption that the investment decision is being made conditional on public information only, then it is indeed optimal for the firm to sell off the growth opportunities ex-ante. And, whenever delayed disclosure is not optimal, then the manager is indifferent between selling the growth opportunities to the market maker and retaining them within the firm. The reason is that the timing of the usage of the manager’s signal in the real investment decision plays a role similar to disclosure timing.\textsuperscript{16} Thus, even if the manager has the flexibility to use the managerial signal in the real investment decision, he would weakly prefer to sell the growth opportunities to the market maker after precommitting to a disclosure strategy, and before trading commences. So our assumption that the manager sells the growth opportunities to the market maker is consistent with a more general model where the manager is allowed to use the managerial signal in the real investment decision, but is also allowed to sell the growth opportunities to the market maker. Formal details are available from the authors.

\textsuperscript{16} To see this, note that whenever it is optimal to delay disclosure to date 2 in the main model, it also is optimal to delay using the managerial signal in the real investment decision till date 2. The intuition is that using the managerial signal in the real investment decision at date 1 discourages information acquisition and lowers the expected value of $GO_2$ by an amount large enough to more than offset the increase in the expected value of $GO_1$. Via similar reasoning, whenever it is optimal to delay disclosure to date 3 (i.e., to never disclose, although this option is never optimal in our numerical simulations), it is also optimal to never use the managerial signal in the real investment decision. Thus, when delayed disclosure is optimal, selling the growth opportunities to the market maker is also optimal, because it acts as a precommitment to not use the signal at date 1. Further, not delaying disclosure in the main model is equivalent to using the managerial signal in the real investment decision at date 1 (because the real decision reveals the signal to the market), and in this case, the manager realizes the same expected value whether he retains the growth opportunity within the firm and uses the managerial signal in the real decision at both dates, or sells the growth opportunities to the market maker, and has the market maker perform the investment decisions instead. While we have assumed that either both or none of the growth opportunities can be sold, even if the manager can decide whether to sell one of the growth opportunities, or both, the intuition still goes through. Thus, when delayed disclosure to date 2 is optimal, it is optimal to sell the first opportunity to the market maker (to encourage early information acquisition) and the manager is indifferent between whether the second is sold or not. When it is optimal to disclose at date 3, it is optimal to sell neither of the growth opportunities to the market maker. Finally, when early disclosure is optimal then the manager is indifferent between whether either or none of the growth opportunities are sold.
We also note that the primary role of the growth opportunities is to allow disclosure policies to endogenously depend on informational efficiency of prices at each date. Specifically, the growth opportunities in our model act as a source of economic gains from informational efficiency, since a more informative share price leads to more appropriate investment in the growth opportunities. Thus, our analysis leads to identical results as the alternative assumption where management directly maximizes the sum of the informational efficiency of prices at the two dates. To see this, note from the preceding assumptions that the optimal investment decision is given by:

$$\kappa_t^* = E[F|\Phi_t] = P_t. \quad (5)$$

Taking the ex-ante expectation yields a value of the growth opportunity of:

$$GO_t = E\left[ F \cdot P_t - \frac{1}{2}P_t^2 \right] = \frac{1}{2}(Cov(F, P_t) + Cov(F - P_t, P_t)). \quad (6)$$

However, by construction, share prices are martingales; thus $Cov(F - P_t, P_t) = 0$. It immediately follows that the NPV of the growth opportunity is as presented in the following proposition.

**Proposition 1:** The net present value of the growth opportunity at date $t$ equals to one-half times the amount of ex ante uncertainty resolved by market prices (variance of $F$ explained by $\Phi_t$):

$$GO_t = \frac{1}{2}Cov(F, P_t) = \frac{1}{2}[Var(F) - Var(F|\Phi_t)]. \quad (7)$$

Hence, maximizing the net present value of growth opportunities is equivalent to minimizing the payoff uncertainty remaining after trade, measured by $Var(F|\Phi_t)$.

In the next section, we analytically solve the static benchmark case so we can draw

---

$^{17}$ Other examples of economic benefits from informational efficiency in the literature are a mitigation of the principal-agent problem (Holmström and Tirole [1993]) and better communication of investment decisions to shareholders (Fishman and Hagerty [1989]). From a welfare perspective, we could also compare the value of growth opportunities to the cost of information production. However, since it is well-known from Diamond [1985] how disclosure can reduce costly private information production in a static setting, we largely abstract from the issue here, to focus on the aspects of the dynamic model.
meaningful comparisons with the dynamic setting to follow in Sections 3 and 4.

2. The Static Benchmark Case

The main goal of the paper is to study the effects of disclosure in a dynamic financial market. For this purpose, we first need to examine the predictions of the static benchmark case. We confirm existing notions that in a static setup, disclosure typically maximizes informational efficiency. This ensures that our new insights are driven by the dynamic feature of the model. Throughout this section only, let us assume there is only one date at which trade occurs, \( t = 1 \), and one growth opportunity, \( GO_1 \), at date 1, whose payoff takes the form in equation (3). At date \( t = 2 \), the asset is liquidated and pays:

\[
F = \eta + \theta + \delta + \epsilon, \tag{8}
\]

and we assume the asset price is a linear function of the state variables:

\[
P = a_i \cdot s_i + a_m \cdot s_m + b \cdot z_1. \tag{9}
\]

In this static model, the demand functions for the informed traders and their expected utility simplifies significantly. This makes it possible to analytically solve the model with endogenous information acquisition, and prove Proposition 2 (the proofs of this proposition and that of Proposition 3, appear in Appendix sections A.1 through A.3).

**Proposition 2:** In a static setup, there exist unique linear equilibria with and without managerial disclosure, and when information acquisition is both exogenous and endogenous. These equilibria can be solved in closed-form.

The next proposition shows that in a static setting, it is always optimal for the manager to disclose his signal.

**Proposition 3:** In a static model with endogenous information acquisition, informational
efficiency is always maximized when the manager discloses his signal, \( s_m \).

With regard to Proposition 3, there are potentially three different cases to consider (the proofs of the claims below appear in the Appendix, Section A.3):

1. The equilibrium number of informed investors is greater than zero both with and without disclosure. In this case, disclosure reduces the payoff variance remaining after a trade by:

\[
\frac{e^{2nc\sigma_n^2(\sigma_n^2 + \sigma_0^2)}}{\sigma_n^2\sigma_0^2 + \sigma_0^2(\sigma_n^2 + \sigma_0^2)}.
\]  

2. The number of informed investors is zero in the case of disclosure, and greater than zero otherwise. In this case, disclosure reduces the conditional payoff variance by at least:

\[
\frac{(\sigma_n^2 + \sigma_0^2)\sigma_0^2(\sigma_n^2 + \sigma_0^2)}{\sigma_0^2\sigma_0^2 + \sigma_0^2(\sigma_n^2 + \sigma_0^2)}.
\]  

3. The number of informed investors is zero without disclosure. In this case, disclosure reduces the conditional payoff variance by at least \( \sigma_n^2 + \sigma_0^2 \).

In the presence of manager-specific information (\( \sigma_\eta > 0 \)) disclosure is strictly beneficial.

There are two reasons for this effect. First, there is no way this information can impact share prices without disclosure. Second, delayed disclosure acts as a source of additional risk to the informed investors. Thus, in addition to maximizing informational efficiency and thus firm value, immediate disclosure reduces the risk borne by the informed and thus stimulates private information production.

3. Equilibrium in the Dynamic Model

A dynamic equilibrium with endogenous information acquisition is defined by four conditions: (1) for each competitive informed trader, his demand function at each date maximizes his expected utility conditional on available information; (2) competitive market making ensures that prices at
each date are given as expected asset payoffs conditional on all available information to market
makers at that date; (3) the certainty equivalent of an informed trader’s payoff equals the cost of
information; (4) the manager commits to either a policy of disclosing at \( t=1 \) or of disclosing at \( t=2 \),
choosing the policy that maximizes the sum of the expected values of the growth opportunities
conditional on market prices to date. In what follows, we refer to disclosure at \( t=1 \) as early or
immediate disclosure, while disclosure at \( t=2 \) is deemed delayed or intermediate disclosure.

As is common in informational models of this type, we restrict ourselves to equilibria in
which prices are given as a linear function of the underlying random variables. In particular, we
conjecture pricing functions of the type:

\[
P_1 = a_i \cdot s_i + a_\text{m} \cdot s_\text{m} + b \cdot z_1, \quad (12)
\]

\[
P_2 = d_i \cdot s_i + d_\text{m} \cdot s_\text{m} + e \cdot z_1 + f \cdot z_2. \quad (13)
\]

The more complex algebraic details for the dynamic case are presented in Appendix sections A.4
and A.5.

### 3.1 Informed Investors’ Demand

The driving force behind our results is the trading strategy of informed investors. Introducing a
dynamic feature into the model results in the presence of hedge demands at \( t = 1 \) (as explained
below), which affects the intuition obtained from static models. Let \( \Psi_t \) denote the informed
investors’ information set at date \( t \). At date \( t = 2 \), investors’ demands are simply found by
mean-variance analysis. Informed investors demand:

\[
x_2(P_2) = \frac{E(F|\Psi_2) - P_2}{\text{Var}(F|\Psi_2)}. \quad (14)
\]

This implies an indirect utility of wealth at \( t = 2 \) of:
\[ u_2(W) = E - \exp(-RW)|\Psi_2 \]
\[ = E\left[-\exp\left(-R\left(x_1(P_2 - P_1) + \frac{(E(F|\Psi_2) - P_2)(F-P_2)}{R \cdot \text{Var}(F|\Psi_2)}\right)\right)|\Psi_2\right] \]
\[ = -\exp\left(-R\left(x_1(P_2 - P_1) + \frac{(E(F|\Psi_2) - P_2)^2}{2R \cdot \text{Var}(F|\Psi_2)}\right)\right). \quad (15) \]

Thus, at \( t = 1 \), the informed investor maximizes the expectation of date 2 indirect utility. The Appendix (Section A.4) shows that the solution is of the form:

\[ x_1(P) = \frac{E(P_2|\Psi_1) - P_1}{RS} + \frac{E(F-P_2|\Psi_1)}{R \cdot \text{Var}(F|\Psi_2)} \cdot \frac{S-T}{S}. \quad (16) \]

Thus, date 1 demand is more complex than date 2 demand. It consists of two terms. The first expression is to take advantage of the expected price appreciation between dates 1 and 2, which makes up the numerator of the first term in Eq. (16). The denominator denotes effects of risk aversion, consisting of the risk aversion parameter, \( R \), and a risk measure, \( S \), that can, in general, be fairly complex. The second term denotes the hedge demand to take advantage of anticipated price changes between dates 2 and 3. The first fraction in the second term equals the expected share holdings from date 2 to date 3. However, these expected share holdings cannot be perfectly hedged. In fact, the expression \( \frac{S-T}{S} \in 0,1 \) denotes the extent to which these share holdings can be anticipated in advance.

Given normality, Section A.4 of the Appendix shows that \( S \) in (16) is determined as follows:

\[ S = \frac{-(\text{Cov}(P_2,E(F|\Psi_2)|\Psi_1))^2 + \text{Var}(P_2|\Psi_1)(\text{Var}(F|\Psi_2) + \text{Var}(E(F|\Psi_2)|\Psi_1))}{-2(\text{Cov}(P_2,E(F|\Psi_2)|\Psi_1)) + \text{Var}(F|\Psi_2) + \text{Var}(P_2|\Psi_1) + \text{Var}(E(F|\Psi_2)|\Psi_1)} \quad (17) \]

and

\[ T = \frac{(\text{Cov}(P_2,E(F|\Psi_2)|\Psi_1))(\text{Var}(F|\Psi_2) - (\text{Cov}(P_2,E(F|\Psi_2)|\Psi_1))) + \text{Var}(P_2|\Psi_1) \cdot \text{Var}(E(F|\Psi_2)|\Psi_1)}{-2(\text{Cov}(P_2,E(F|\Psi_2)|\Psi_1)) + \text{Var}(F|\Psi_2) + \text{Var}(P_2|\Psi_1) + \text{Var}(E(F|\Psi_2)|\Psi_1)}. \quad (18) \]

The various conditional moments in the expressions above can be calculated given the conjecture
of a linear equilibrium. Details are given in Section A.4 of the Appendix, which also shows that the algebra simplifies significantly if either one of the following two cases hold: (1) the manager discloses his signal at date 1; or (2) the manager discloses his information at date 2, but does not have any information beyond that of informed investors \((\sigma^2_t = 0)\). In this case, \(t = 0\), date 2 demand can be perfectly hedged, and the first date risk measure \(S\) simplifies to \(S = \left(\frac{1}{\text{Var}(F|\Psi_2)} + \frac{1}{\text{Var}(P_2|\Psi_1)}\right)^{-1}\).

3.2 Market Prices

At date 1, the market maker observes a net order flow of:

\[
\tau_1 = Mx_1(P_1) + z_1 = M \left[ \frac{E(P_2|\Psi_1) - P_1}{RS} + \frac{E(F - P_2|\Psi_1) \cdot S - T}{S} \right] + z_1. \tag{19}
\]

The date 1 price without managerial disclosure at this date is the expectation of \(F\) conditional on \(\tau_1\), and is found by regressing the payoff \(F = \eta + \theta + \delta + \varepsilon\) on \(\tau_1\). In the case of managerial disclosure at \(t = 1\), the price is found by regressing \(F\) on the vector \((\tau_1, s_m)\). These regressions can be simplified by first removing all terms involving \(P_1\) from \(\tau_1\) (which yields an observationally equivalent signal). In either case, we obtain analytical expressions for the date 1 share price as a function of the date 2 price coefficients.

At date 2, market makers observe another signal in the form of the date 2 net order flow, which is affected by the arrival of another liquidity (supply) shock, \(z_2\):

\[
\tau_2 = M \frac{E(F|\Psi_2) - P_2}{R\cdot\text{Var}(F|\Psi_2)} + z_1 + z_2. \tag{20}
\]

Again, the market maker determines the share price, \(P_2\), by regressing the payoff \(F\) on the vector of signals that is available, i.e., \((\tau_1, \tau_2)\), but also the manager’s signal, \(s_m\), if this has already been
disclosed. This leads to a system of four equations in the four unknowns: $d_i, d_m, e, \text{ and } f$. In general, this system is highly non-linear and cannot be solved analytically. However, as mentioned in Section 3.1, when the manager discloses his information before trade at date 1, the informed trader’s hedge demand is simplified. For this case, holding the number of informed investors, $M$, fixed, the solution to the equilibrium equations is in the Appendix (Section A.5).

### 3.3 Multiplicity of Equilibria

The non-linear nature of the equations for the date 2 share price implies the potential for multiple equilibria, an issue that is well known since the early work of Grundy and McNichols (GM) [1989] and Hirshleifer, Subrahmanyam, and Titman (HST) [1994]. In GM and HST, equilibria fall into one of two different categories. First, typically, there always exists an equilibrium in which prices do not move between $t = 1$ and $t = 2$. The number of other equilibria depends on the sign of the discriminant of a quadratic equation. Thus, for some combination of parameter values, it can happen that no such equilibrium exists. On the other hand, if an equilibrium does exist, there also exists a second one. Our numerical simulations indicate similar equilibrium behavior within our setting.

We feel justified in ignoring equilibria with a zero price move across dates 1 and 2, since they essentially lead to results that could also be (and have been) obtained in static models. Dierker [2003] documents the properties of the two other equilibria in HST and GM. One of the equilibria displays “reasonable” behavior in the sense that, as the number of informed agents decreases and approaches zero, price become less informative and approach the fully uninformative limit; HST mainly focus on this equilibrium. The second equilibrium, on the other hand, has the following counterintuitive feature. As the number of informed investors approaches zero, the informed trade
increasingly aggressively, so that share prices become more informative. In the limit, prices incorporate information that no investor in the economy possesses. This limit behavior is at odds with Fama [1970], who argues that informationally efficient prices should contain information possessed by at least one agent in the economy. The limit behavior also contradicts the notion of Grossman and Stiglitz [1980], who argue that investor-specific information can only flow into prices if there are agents in the economy who discover the information and trade on it. Thus, as in HST, we feel justified in ruling out this type of equilibrium in our analysis. So we focus on equilibria with a non-zero price move where price informativeness is increasing in the mass of informed agents. For a thorough discussion of the multiple equilibria, see Dierker [2003].

3.4 Expected Utility of Informed Investors

In the equilibrium we consider, both demands and prices are linear in the state variables. This implies that wealth is a quadratic function of the normally distributed random variables. We use standard results of “completing squares” for quadratics of normal random variables to compute the expected utility of informed investors (Sections A.3 and A.4 in the Appendix detail the procedure).

Expected utility is determined by the determinant of a 6x6 matrix, which is beyond analytical tractability. Thus, we have to rely on numerical simulations for our findings. The base parameter values we use are: \( R=8, \sigma_\eta = 0.2, \sigma_\theta=1, \sigma_\delta=1, \sigma_\epsilon=0.6, \sigma_\zeta=1, \) and \( c=0.39 \). Our analysis indicates that in the neighborhood of these values, equilibria with non-zero price moves across dates 1 and 2 exist, which allows us to perform comparative statics on the benefits of postponing disclosure.\(^{18}\)

\(^{18}\) We have verified that the qualitative findings we document via simulation are robust to a wide range of parameter values. Indeed, in the large parameter space we have explored, whenever equilibria of our focus exist, they exhibit behavior qualitatively identical to that presented for our chosen parameter set.
4. Optimal Disclosure Timing

In this section, we explore the factors that determine the optimal timing of managerial disclosure. Numerical simulations show that it will never be optimal to withhold information until the liquidation date, consistent with the results for static case obtained in Proposition 3. The positive effect of attracting privately informed investors by withholding information is always stronger when disclosure is delayed until an intermediate date.

4.1 Informed Trading Behavior and the Suboptimality of Late Disclosure

To appreciate our key findings, it is important to understand the trading behavior of informed investors. As in Brennan [1990], Froot, Scharstein, and Stein [1992], and Hirshleifer, Subrahmanyam, and Titman [1994], informed investors, in effect, engage in short-term profit taking. Numerical simulations show that the share price change between dates 1 and 2 is correlated positively with their date 1 asset holdings and negatively with their trades at date 2: \( \text{Cov}(x_1, P_2 - P_1) > 0, \text{Cov}(x_2 - x_1, P_2 - P_1) < 0 \). The reason for this is that information is gradually incorporated into share prices. The market maker has more information when setting the date 2 share price, since date 2 order flow provides another signal. On average, this moves the share price closer to its true fundamental value. This diminishes the returns for holding the asset another date, which in turn leads informed investors to partially reverse their risky positions. These effects do not play a role when the firm discloses information. If, however, the manager chooses to reveal his signal at the intermediate date \((t = 2)\), the news arrival at date 2 results in a large price move \(|P_2 - P_1|\) is large), which leads to a much more pronounced short-term reversal in trading by informed investors.
4.2 Early vs. Intermediate Disclosure

For the remainder of the paper, we will focus on the advantage of disclosure at \( t=2 \) relative to that at \( t=1 \), as represented by the difference in \( GO_1 + GO_2 \) (viz. Proposition 1) across these two disclosure policies. The most important and interesting determinant of disclosure timing is the informational setup. Note that if the manager and investors have identical information (\( \sigma_\eta = \sigma_\delta = 0 \)), full informational efficiency can be achieved by immediate disclosure. In the more general case where \( \eta \) and \( \delta \) have positive variances, however, investors cannot obtain manager-specific information, \( \eta \), through research efforts. Thus, delaying the disclosure of \( \eta \) has a detrimental effect. First, there is a loss in informational efficiency since there is no other way this information can flow into share prices. Second, there is no benefit for informed investors from the opportunity to trade on this information. Instead, delayed announcement acts as risk for the informed investors, therefore reducing their arbitrage positions, not increasing them.

To illustrate, we first let \( \rho = \sigma_\eta / \sigma_\theta \) denote the specialness of the manager’s information. In Figure 1, we then plot the benefits of intermediate disclosure (relative to early disclosure) as a function of specialness for various values of \( \sigma_\delta \). The Figure shows that the greater the specialness of managerial information, the less likely it is to be disclosed immediately. Figure 1 also shows that the more unique information investors can contribute, i.e., the higher is \( \sigma_\delta \), the higher are the possible advantages of delaying disclosure.

In Figure 2, we demonstrate the benefits of delaying disclosure as a function of the information overlap between managers and outsiders, measured by \( \sigma_\theta \). Note that \( \theta \) measures the covariance between the manager’s and the investors’ signals. Thus, when \( \sigma_\theta \) is higher, delayed disclosure offers more profitable, low-risk trading opportunities for informed investors. They will take on large arbitrage positions under such conditions, which they will reverse after the
news announcement, since holding an asset until liquidation is highly risky. On the other hand, delaying disclosure also has an informational cost, since $\theta$ has to be noisily inferred from net order flow instead of observed as part of the manager’s signal. Interestingly, the positive and negative effects of delaying disclosure have different marginal strengths. While the potential negative effects of delaying disclosure are somewhat linear in the amount of information, $\sigma_\theta$, the marginal positive effects are the strongest for small amounts. When $\sigma_\theta$ is small, delaying disclosure may not provide enough extra incentives to acquire information. When the informational overlap is large, however, the negative effects of delaying disclosure dominate the increase in informational trade due to disclosure. Figure 2 shows that it is for intermediate values of informational overlap that delaying disclosure is the most beneficial.

In Figure 3, we delineate the benefits from delaying disclosure by date. Specifically, we separately plot the impact of delaying disclosure on the expected values of the growth opportunities at dates 1 and 2, as a function of the specialness of managerial information. We have seen how delaying disclosure can stimulate private information acquisition, at the expense of not incorporating the manager-specific information into the share price immediately. Thus it comes as no surprise that at date 2, after the delayed disclosure, share price is typically more informative than with immediate disclosure. On the other hand, delaying disclosure typically leads to a less informative price at date 1 (unless $\sigma_\delta$ is very large). The effect of delaying disclosure at date 1 depends on the specialness of managerial information; the informational efficiency benefit at date 1 from delaying disclosure becomes increasingly negative as specialness increases. Optimal disclosure timing then depends on the comparison of loss at date 1 to gains at date 2.

Finally, Figure 4 plots the relative benefits to intermediate disclosure as a function of $\sigma_\epsilon$,

---

19In previous drafts we identified cases where it will be optimal to delay part of the information to attract outside analysts, while disclosing the remainder of the information immediately to preclude the negative effects of delay. In the present version, for brevity, we do not allow for this strategy, but instead analyze optimal disclosure timing.
the unpredictable part of the traded asset’s payoff. The higher $\sigma_e$, the more beneficial it is for informed investors to have short-term trading opportunities provided by intermediate disclosure. This simple intuition is verified in our numerical simulations and illustrated in Figure 4.

In other simulations (not reported for brevity but available on request), we analyze the effect of risk aversion, noise trade variance, and the cost of information acquisition on the dynamic disclosure policy. Increasing the risk aversion parameter, $R$, has essentially the same effect as increasing the amount of risk, $\sigma_e$. It increases the advantages of delayed disclosure to the manager. When information is disclosed early, informed investors have fewer short-term trading opportunities, and are therefore more affected by the risk of holding the asset until liquidation. When information is disclosed at an intermediate date, the opportunity to reverse their position after the information disclosure enables investors to speculate on the nature of the announcement without incurring the risk of holding the asset until liquidation. This opportunity becomes relatively more valuable as risk aversion increases.

Noise in our model, captured by the parameter $\sigma_z$, enables informed investors to disguise their trading positions, thus making share price discovery more difficult for market makers. Therefore, it comes as no surprise that more noise makes immediate managerial disclosure more valuable. In addition, noise in the second trading round also acts as a disguise for informed traders, as well as a source of risk, thus reducing the relative advantages of delayed disclosure.

Finally, informational efficiency is also affected by the cost of information acquisition. The more costly the information, the more difficult it becomes to attract informed investors without also making trading opportunities available due to delayed disclosure. In particular, when information costs are higher, investors may cease to collect information unless delayed disclosure provides them with lucrative trading opportunities. In these cases, delayed disclosure can have a
very pronounced positive effect.

To conclude, we find that low risk aversion, small payoff uncertainty, a low cost of information, a high level of noise, high levels of manager-specific information as opposed to investor-specific information, and extreme values (high or low) of informational overlap all favor early disclosure of information, while the opposite is true for intermediate disclosure.

5. Applications and Empirical Predictions

We now consider how our model can apply to issues surrounding corporate disclosures (Section 5.1) and also discuss empirical implications (Section 5.2).

5.1 Applications

In this subsection, we provide some applications of our approach to disclosure timing, initial public offerings, earnings pre-announcements, and attracting analyst coverage.

5.1.1 Disclosure Timing Regulation

Countries have different regulatory standards with respect to disclosure timing. While the U.S. and Canada give firms discretion as to when to file their disclosure reports, Australia operates under a continuous disclosure regime. We show a potential benefit in allowing firms some discretion in timing their disclosures. Specifically, delaying disclosures helps in attracting informed traders, whose information improves the efficiency of investment and valuation.

It is widely known that disclosure timing is dependent on the company’s earnings and

---

financial well-being (Givoly and Palmon [1982]; Chambers and Penman [1984]). Indeed, sub-par managers may want to conceal bad news as long as possible. Theory suggests that investors should be able to interpret this silence as a bad sign (Grossman and Hart [1980]). However, it is conceivable that some market participants cannot fully grasp the negative implications of corporate silence. In contrast to the above rationale for delayed disclosure, our arguments that temporarily (but not indefinitely) delayed disclosure can reduce the risk borne by informed agents and thus promote information acquisition, apply to both positive and negative disclosures.

5.1.2 Earnings Pre-Announcements

Firms with significant news often pre-announce this information before the scheduled announcement date (Baginski, Hassell, and Waymire [1994]). Pre-announcements are typically made when the firm has significant information unknown to shareholders, which can be interpreted as an unusually large amount of manager-specific information. In this situation, our model predicts that the firm will want to disclose its information earlier.

Before enactment of the Regulation Fair Disclosure (FD), this information could be conveyed to the market by selective communication like conference calls, which means that it was not necessary to time the public announcement earlier. Since this is no longer possible under Regulation FD, it comes as no surprise that the frequency of pre-announcements more than doubled following this regulation (e.g., Heflin, Subramanyam, and Zhang [2003]).

5.1.3 Initial Public Offerings

We believe that our correlated information structure is particularly relevant to initial public

---

21Here, economic benefits from incorporating this information into stock price include the avoidance of legal action. 22Clearly, pre-announcements depend on the news realization and often come as a surprise to the market. However, the economic rationale for timing pre-announcements seems entirely consistent with our story.
offerings (IPOs). Legal restrictions require firms that go public to disclose all relevant information to the public in the prospectus. But recent events (e.g., those surrounding the Facebook IPO) suggest that firms may not comply with these rules and withhold some information from the general public.\textsuperscript{23} One obvious reason might be that the owners want to maximize the offering price by presenting the company in a positive light. Alternatively, our model suggests that firms may delay disclosure of information to offer profitable opportunities for informed traders. This, in turn, could attract institutional investors who will be helpful in pricing the issue, provided owners do not have all of the information available for accurate valuation.

5.1.4 Firms Attracting Analyst Coverage

Attracting informed investors is important for firms even after an IPO. Young and small firms often find it very difficult or expensive to attract analyst coverage. The investor-specific information the analysts provide is important to these firms, because it increases the credibility of the share price and provides valuable monitoring services. While analysts have to recoup their research expenses,\textsuperscript{24} lowering the cost of information acquisition via selective disclosures is challenging in the current environment.\textsuperscript{25} Postponing the disclosure of information offers the firm an additional way of compensating informed agents, by allowing them to reduce the risk of holding long-term positions, and thus promote informational efficiency via more aggressive dynamic trading on acquired information.

\textsuperscript{24}Moses [2004] discusses the challenges analysts face in recouping their costs, and argues that biased analyst advice to sell-side clients partially occurs as pressure to increase revenue for the brokerage firm.
\textsuperscript{25}Bailey, Lee, Mao, and Zhong [2003] discuss how Regulation FD has increased analyst forecast dispersion, suggesting greater difficulty in forecasting earnings after its implementation.
5.2 Empirical Predictions

We now provide some cross-sectional predictions for the timing of corporate disclosures. We believe that there will always be an overlap between investors’ signals and the manager’s information (e.g., analyst forecasts do provide significant forecast power for realized earnings), so that we can focus on information that is specific either to management or investors.

Our model predicts that the degree of manager-specific information is related to disclosure timing (where timing is measured, for example, by the number of days between the pre-announcement of earnings and the date of the actual earnings release, or by the time between announcement and occurrence of significant corporate events, such as new product launches or the end of a fiscal quarter). Manager-specific information can be measured by means of analyst forecast errors. If analysts are typically able to forecast a firm’s earnings with a good degree of confidence, manager-specific information is relatively unimportant. In this case, we believe that the firm should disclose its information later. Conversely, if analysts find it hard to predict corporate earnings, then manager-specific information is pronounced, and we expect an earlier disclosure.

We also predict that firms will announce information later if investors contribute significant information. Here, we are typically thinking of valuation information, such as applicable discount rates or current market conditions. Other examples are information about demand for a specific industry or a specific product. The manager may be more familiar with the technical specifications of a product, but may be uncertain about its market appeal or future profitability. This type of information should be more pronounced in newly developing and high-growth industries, or in otherwise volatile industries. Similarly, during volatile markets or regime shifts, sophisticated investors can contribute more valuable valuation information. In these
cases, we predict the company will disclose its information later.

In terms of volume, our model implies active trading immediately following material managerial disclosures such as earnings pre-announcements, as informed agents unwind their positions to avoid holding stock for the long-term. Lastly, in our framework, higher risk (higher $\sigma_x$) of holding arbitrage positions in the asset leads to later disclosure. We thus predict that riskier firms will announce information later.

6. Conclusion

In this paper we examine how introducing a dynamic financial market changes the economics of information disclosure. Recent articles by Verrecchia [2001] and Leuz and Wysocki [2008] call for the use of dynamic models to understand the effects of disclosure. As Verrecchia [2001] writes in his survey on disclosure theory: “Assessing the effects of disclosure in the context of a single period model of trade risks comingling a host of factors that may obfuscate or obscure disclosure’s role.” We believe that we have disentangled some of these factors, and thus cast new light on the role of information disclosure. In a world in which price informativeness is desirable because it likely leads to more efficient investment decisions, we show how a firm would always want to disclose its information in a static setting. But, in a dynamic setting, in some situations, prices and investment decisions are more efficient when the firm postpones disclosure. This is because postponing disclosure allows investors to acquire information correlated with the disclosure. The subsequent disclosure further benefits investors by allowing them to unwind their position at the time of the disclosure, sparing them from the risk of holding the asset until the liquidation date, thus increasing their trading aggressiveness, and, in turn, enhancing informational efficiency.

In our paper, we abstract from other aspects of disclosure studies such as the relation to
insider trading (e.g., Fishman and Hagerty [1995]) or proprietary information (Admati and Pfleiderer [2000]). How our intuition affects this research would seem to be an important arena for future research. Promising extensions, such as to a multiple asset setting, are likewise left for future research.
References


Dierker, M., 2003, Multiple equilibria in dynamic models of informed trading, unpublished manuscript, University of California, Los Angeles.


Grundy, B., and M. McNichols, 1989, Trade and the revelation of information through prices and


Leuz, C., and P. Wysocki, 2008, Economic consequences of financial reporting and disclosure regulation: a review and suggestions for future research, working paper, University of Chicago.


Appendix - Proofs

This appendix presents the algebraic aspects of the equilibria of the static and dynamic models.

A.1 Updating for Informed Investors

When the manager discloses his private signal to the market, the investors can combine their own signal, \( s_i = \theta + \delta \), with the manager’s signal, \( s_m = \eta + \theta \). This is done by running a regression of:

\[
\nu_1 = \eta + \theta + \delta
\]  

on:

\[
\nu_2 = (s_i, s_m).
\]

Thus the new belief, \( \beta \), is given as:

\[
\beta = Cov(\nu_1, \nu_2) \cdot Var(\nu_2) \cdot \nu_2'
\]

i.e.,

\[
\beta = (\sigma_\theta^2 + \sigma_\delta^2, \sigma_\theta^2 + \sigma_\eta^2) \cdot \begin{pmatrix} \sigma_\theta^2 + \sigma_\delta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\eta^2 + \sigma_\delta^2 \end{pmatrix} \cdot (s_i, s_m)'.
\]

This yields:

\[
\beta = \frac{\sigma_\theta^2 \sigma_\delta^2 + \sigma_\delta^2 \sigma_\eta^2}{\sigma_\theta^2 + \sigma_\delta^2 + \sigma_\eta^2 + \sigma_\delta^2} \cdot s_i + \frac{\sigma_\delta^2 \sigma_\eta^2 + \sigma_\delta^2 \sigma_\eta^2}{\sigma_\theta^2 + \sigma_\delta^2 + \sigma_\eta^2 + \sigma_\delta^2} \cdot s_m.
\]

The remaining variance of \( F \) conditional on observing both \( s_i \) and \( s_m \) is found to be:

\[
V = Var(F|s_i, s_m) = Var(F|\beta) = \frac{Var(F)Var(\beta) - Cov(F, \beta)^2}{Var(\beta)} = \sigma^2 + \frac{\sigma_\delta^2 \sigma_\eta^2 \sigma_\theta^2}{\sigma_\theta^2 + \sigma_\delta^2 + \sigma_\eta^2 + \sigma_\delta^2}.
\]

A.2 Static Model with Managerial Disclosure

If the manager discloses his signal, \( s_m \), the informed investors first update their beliefs about asset
payoff as outlined in Section A.1. Standard mean variance arguments yield that the informed investors’ demand schedule is given by:

\[ x_1(P) = \frac{E(F|S_i s_m) - P}{RV \text{ar}(F|S_i s_m)} = \frac{\beta - P}{RV}. \]  

(28)

The market maker thus observes two pieces of information: \( s_m \) and net order flow \( Mx_1 + z_1 \). Thus, share price is found by regressing asset payoff \( F \) on \( s_m \) and \( Mx_1 + z_1 \). The latter is given as:

\[ M \left( \sigma_0^2 \sigma_0^2 (-P + s_i s_m) + \left( -\left( P(\sigma_0^2 + \sigma_0^2) + \sigma_0^2 s_i + \sigma_0^2 s_m \right) \sigma_0^2 \right) \right) \\
\frac{R \left( \sigma_0^2 \sigma_0^2 \sigma_0^2 + \sigma_0^2 \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \right) \right)}{+ z_1}, \]  

(29)

which, given that both \( P \) and \( s_m \) are known, is observationally equivalent to the following:

\[ \tau = \frac{M \sigma_0^2 s_i (\sigma_0^2 + \sigma_0^2)}{R \sigma_0^2 \sigma_0^2 + R (\sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \sigma_0^2)} + z_1. \]  

(30)

Thus, the share price is found to be:

\[ P = \text{Cov}(F, (s_m, \tau)) \cdot \text{Var}(s_m, \tau) \cdot (s_m, \tau)', \]  

(31)

and this yields the coefficients:

\[ a_i = \frac{M^2 \sigma_0^6 (\sigma_0^2 + \sigma_0^2)^2}{M^2 \sigma_0^4 (\sigma_0^2 + \sigma_0^2) \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \right) + R^2 \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 \left( \sigma_0^2 + \sigma_0^2 \right) \sigma_0^2 \right)^2 \sigma_0^2}, \]  

(32)

\[ a_m = 1 - \frac{M^2 \sigma_0^6 \sigma_0^2 (\sigma_0^2 + \sigma_0^2)}{M^2 \sigma_0^4 \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \sigma_0^2 \right) + R^2 \left( \sigma_0^2 \sigma_0^2 \sigma_0^2 + \sigma_0^2 \sigma_0^2 + \sigma_0^2 \left( \sigma_0^2 + \sigma_0^2 \right) \sigma_0^2 \right)^2 \sigma_0^2}, \]  

(33)

and

\[ b = \frac{M \sigma_0^2 (\sigma_0^2 + \sigma_0^2)}{R \left( \sigma_0^2 \sigma_0^2 + R \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 \sigma_0^2 + \sigma_0^2 \sigma_0^2 \right) \right) \sigma_0^2 \left( \frac{M^2 \sigma_0^6 (\sigma_0^2 + \sigma_0^2) \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \sigma_0^2 \right) + R^2 \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 \left( \sigma_0^2 + \sigma_0^2 \right) \sigma_0^2 \right)^2 \sigma_0^2 \right)}{\left( \sigma_0^2 \sigma_0^2 + R \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 \left( \sigma_0^2 + \sigma_0^2 \right) \sigma_0^2 \right) \right) \sigma_0^2 \left( \frac{M^2 \sigma_0^6 (\sigma_0^2 + \sigma_0^2) \left( \sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \sigma_0^2 \right) + R^2 \left( \sigma_0^2 \sigma_0^2 \sigma_0^2 + \sigma_0^2 \sigma_0^2 + \sigma_0^2 \left( \sigma_0^2 + \sigma_0^2 \right) \sigma_0^2 \right)^2 \sigma_0^2 \right)^2 \sigma_0^2}. \]  

(34)

The end-of-period wealth for the informed investors is thus given by:

\[ W = x_1 (F - P), \]  

(35)
which is found to be the following quadratic form of the five random variables \( \theta, \eta, \delta, \epsilon, \) and \( z_1: \)

\[
(\delta + \epsilon + \eta + \theta - a_1(\delta + \theta) - a_m(\eta + \theta) - b z_1) \\
\times \left( - (a_1(\delta + \theta) - a_m(\eta + \theta) + \frac{\sigma^2_{\omega} \sigma^2_{\rho}(\eta + \theta) + \sigma^2_{\theta}(\delta + \theta) + \sigma^2_{\theta}(\delta + \eta + 2\theta)}{\sigma^2_{\theta} + \sigma^2_{\rho}(\sigma^2_{\theta} + \sigma^2_{\rho})} - b z_1) \right) \\
\times R \left( \frac{\sigma^2_{\rho} \sigma^2_{\omega} + \sigma^2_{\rho}(\sigma^2_{\rho} + \sigma^2_{\omega})}{\sigma^2_{\theta} + \sigma^2_{\rho}(\sigma^2_{\theta} + \sigma^2_{\rho})} \right). \tag{36}
\]

Expected utility can be found by rewriting the quadratic form for wealth in matrix form \( W = (1/2)(\theta, \eta, \delta, \epsilon, z_1)' J(\theta, \eta, \delta, \epsilon, z_1), \) and applying the following well-known result about multivariate normal distributions (see, for example, Turin [1960]).

**Proposition 4:** Let \( Q(.) \) be a quadratic function of the random vector \( \chi: N(\mu, \Sigma): \)

\[
Q(\chi) = C + B' \chi - \chi' A \chi. \tag{37}
\]

Then, \( E[\exp(Q(\chi))] \) is given by:

\[
|\Sigma|^{-1/2} \cdot |2A + \Sigma^{-1}|^{-1/2} \cdot \exp(C + B' \mu + \mu' A \mu + (1/2)(B' - 2\mu' A')(2A + \Sigma^{-1})^{-1}(B - 2A \mu)). \tag{38}
\]

Applying the above result leads to:

\[
Eu(W) = E - \exp(-R * u) = -det(2R \cdot J + Cov(\theta, \eta, \delta, \epsilon, z_1))^{-1/2}, \tag{39}
\]

where \( J \) is represented by the expression in (36) above.

Calculating the determinant yields an expected utility of:

\[
- \left( \frac{(\sigma^2_{\theta} \sigma^2_{\omega} + \sigma^2_{\theta}(\sigma^2_{\theta} + \sigma^2_{\omega}))((M^2 \sigma^2_{\theta} \sigma^2_{\omega} + R^2(\sigma^2_{\theta} + \sigma^2_{\omega})(\sigma^2_{\theta} \sigma^2_{\omega} \sigma^2_{\rho} + \sigma^2_{\theta} \sigma^2_{\omega} \sigma^2_{\rho} + \sigma^2_{\theta} \sigma^2_{\omega} \sigma^2_{\rho} + \sigma^2_{\theta} \sigma^2_{\omega} \sigma^2_{\rho}))) \sigma^2_{\omega}}{M^2 \sigma^2_{\theta} \sigma^2_{\omega} + \sigma^2_{\theta}(\sigma^2_{\theta} + \sigma^2_{\omega})(\sigma^2_{\theta} \sigma^2_{\omega} \sigma^2_{\rho} + \sigma^2_{\theta} \sigma^2_{\omega} \sigma^2_{\rho} + \sigma^2_{\theta} \sigma^2_{\omega} \sigma^2_{\rho} + \sigma^2_{\theta} \sigma^2_{\omega} \sigma^2_{\rho})) \sigma^2_{\omega}} \right)^{-1/2}. \tag{40}
\]

Next we allow for endogenous information acquisition. Specifically, assume that investors can choose to collect information at a cost \( c. \) No investors will acquire information if the cost is too high, i.e., whenever:
Otherwise, some investors will find it optimal to acquire information, and their number \( M \) is given by:

\[
M = \frac{R \sigma_z \sqrt{\left(\sigma_z^2 \sigma_b^2 + \sigma_z^2 \left(\sigma_f^2 + \sigma_b^2\right)\right) \left(1 - e^{2RC} \sigma_f^2 \sigma_b^2 + \sigma_f^2 \left(\sigma_f^2 + \sigma_b^2\right) - e^{2RC - 1} \sigma_f^2 \left(\sigma_f^2 + \sigma_b^2\right)\right)}}{\sigma_b^2 \sqrt{\left(e^{2RC - 1} \left(\sigma_f^2 + \sigma_b^2\right) \left(\sigma_f^2 + \sigma_b^2\right)\right)}}. \tag{42}
\]

The equilibrium coefficients then become:

\[
a_i = \frac{e^{2RC} \left(\sigma_f^2 + \sigma_b^2\right)}{\sigma_f \sigma_b \left(\sigma_f^2 + \sigma_b^2\right)}, \tag{43}
\]

\[
a_m = \frac{\left(1 + e^{2RC}\right) \sigma_f^2 \sigma_b^2 + \sigma_f^2 \left(\sigma_f^2 + \sigma_b^2\right) + e^{2RC} \sigma_f^2 \sigma_b^2 + \sigma_f^2 \left(\sigma_f^2 + \sigma_b^2\right) - e^{2RC - 1} \sigma_f^2 \left(\sigma_f^2 + \sigma_b^2\right)}{\sigma_f^2 \sigma_b^2 \left(\sigma_f^2 + \sigma_b^2\right)}, \tag{44}
\]

and

\[
b = \frac{e^{2RC} \sigma_f^2 \sigma_b^2}{\sigma_f \sigma_b \left(\sigma_f^2 + \sigma_b^2\right)}. \tag{45}
\]

Finally, the variance of the payoff conditional on price is given by:

\[
\text{Var}(\eta + \theta + \delta + \varepsilon | P) = e^{2RC} \sigma_e^2 + \frac{e^{2RC} \sigma_f^2 \sigma_b^2}{\sigma_f \sigma_b \left(\sigma_f^2 + \sigma_b^2\right)}. \tag{46}
\]

### A.3 Static Model without Managerial Disclosure

The static case without disclosure is simply a special case of the model of Section A.2 where the informed investors know \( \theta + \delta \), whereas \( \eta + \varepsilon \) is not known to market participants till the final date. The informed demand is of the standard form \( x_i (P) = (s_i - P) / [R \text{ var}(F|s_i)] \) and the price
For brevity, we omit the intermediate steps, and just provide the results.

1. Holding the number of informed investors, $M$, fixed, the equilibrium is given by:

$$a_i = \frac{M^2(\sigma_\theta^2 + \sigma_n^2)}{M^2(\sigma_\theta^2 + \sigma_n^2) + R^2(\sigma_\theta^2 + \sigma_\eta^2)^2 \sigma_x^2}$$

and

$$b = \frac{R(\sigma_\theta^2 + \sigma_n^2)}{M}, \quad a = \frac{MR(\sigma_\theta^2 + \sigma_n^2)(\sigma_\theta^2 + \sigma_\eta^2)}{M^2(\sigma_\theta^2 + \sigma_n^2) + R^2(\sigma_\theta^2 + \sigma_\eta^2)^2 \sigma_x^2}. \quad (48)$$

The utility of informed investors is given by:

$$\left(\frac{M^2(\sigma_\theta^2 + \sigma_n^2) + R^2(\sigma_\theta^2 + \sigma_n^2)^2(\sigma_\theta^2 + \sigma_\eta^2)}{M^2(\sigma_\theta^2 + \sigma_n^2) + R^2(\sigma_\theta^2 + \sigma_\eta^2)^2 \sigma_x^2}\right)^{-1/2}. \quad (49)$$

2. Now assume that the number of informed investors in not exogenously given, but endogenously determined in the model. Specifically, assume that investors can choose to collect information at a cost $c$. Investors will not acquire private information ($M = 0$) whenever:

$$c \geq \log \left[\frac{\sigma_\theta^2 + \sigma_n^2 + \sigma_\eta^2}{\sigma_\theta^2 + \sigma_n^2} \right] \frac{1}{R}. \quad (50)$$

Otherwise, some investors will choose to collect information, and their number will be given by:

$$M = \frac{R\sigma_\theta \sqrt{\sigma_\theta^2 + \sigma_n^2} \sqrt{\sigma_\theta^2 + \sigma_\eta^2 - e^{-2Rc}(\sigma_\theta^2 + \sigma_n^2)^2}}{\sqrt{e^{-2Rc} - 1} \sqrt{\sigma_\theta^2 + \sigma_\eta^2}}. \quad (51)$$

The coefficients thus become:

$$a_i = \frac{(\sigma_\theta^2 + \sigma_n^2)^2(1 - e^{-2Rc})}{(\sigma_\theta^2 + \sigma_\eta^2)(\sigma_\theta^2 + \sigma_n^2)} \quad (52)$$

and

$$b = \frac{(e^{-2Rc} - 1) \sqrt{\sigma_\theta^2 + \sigma_n^2} \sqrt{\sigma_\theta^2 + \sigma_\eta^2 - e^{-2Rc}(\sigma_\theta^2 + \sigma_n^2)^2(\sigma_\theta^2 + \sigma_\eta^2)}}{\sqrt{1 - e^{-2Rc}} \sqrt{\sigma_\theta^2 + \sigma_\eta^2} \sigma_x}. \quad (53)$$
Finally, the conditional variance of the payoff is found to be:

\[ \text{Var}(\eta + \theta + \delta + \epsilon|P) = e^{-2Rc} \cdot (\sigma_{\epsilon}^2 + \sigma_{\eta}^2). \]  \hfill (54)

Sections A.2 and A.3 together prove existence and uniqueness of the linear equilibria in the static case (with and without managerial disclosure) and provide closed-form solutions to the equilibria, thus proving Propositions 2 and 3. Comparing the expressions (54) and (46) then yields the expressions (10) and (11) which cover the first two cases that follow Proposition 3 (the third case is trivial, because there is no informed trading).

### A.4 Dynamic Model: Informed Investors’ Demand Functions

At \( t = 2 \), investors’ demands are simply found by mean-variance analysis. Let \( \Psi_t \) denote the informed investors’ information set at time \( t \). Informed investor demand is:

\[ x_2(P_2) = \frac{E(F|\Psi_2) - P_2}{R \cdot \text{Var}(F|\Psi_2)}. \]  \hfill (55)

This implies an expected indirect utility of wealth at date 2 of:

\[
E[u_2(W)|\Psi_1] = E[-\exp(-RW)|\Psi_2] \\
= E[-\exp \left(-R \left(x_1(P_2 - P_1) + \frac{(E(F|\Psi_2) - P_2)(F - P_2)}{R \cdot \text{Var}(F|\Psi_2)} \right)\right)|\Psi_2] \\
= -\exp \left(-R \left(x_1(P_2 - P_1) + \frac{(E(F|\Psi_2) - P_2)^2}{2R \cdot \text{Var}(F|\Psi_2)} \right)\right).
\]  \hfill (56)

Thus, at date 1, the informed investor maximizes the expectation of date 2 indirect utility. We now proceed to show that date 1 demand is of the form:

\[ x_1(P) = \frac{E(P_2|\Psi_1) - P_1}{R S} + \frac{E(F-P_2|\Psi_1)}{R \cdot \text{Var}(F|\Psi_2)} \cdot \frac{S - T}{S}. \]  \hfill (57)

For this purpose, we can rewrite the expression for indirect date 2 utility \( u_2(W) \) as:

\[ u_2(W) = -\exp \left(-\frac{1}{2} \chi' A \chi + h' \chi + l \right), \]  \hfill (58)

where:
\[
\chi = (P_2 - E(P_2 | \Psi_1), E(\beta | \Psi_2) - E(\beta | \Psi_1))', \\
h = \left( R x_1 + \frac{E(P_2 | \Psi_1) - E(\beta | \Psi_1)}{\sigma^2_e}, -\frac{E(P_2 | \Psi_1) - E(P_2 | \Psi_1)}{\sigma^2_e} \right)', \\
A = \left( \Sigma^{-1} + \begin{pmatrix} \sigma^{-2}_e & -\sigma^{-2}_e \\ -\sigma^{-2}_e & \sigma^{-2}_e \end{pmatrix} \right),
\]

where \(\Sigma\) denotes the covariance matrix of \(\chi\) and \(l = R x_1 (E(P_2 | \Psi_1) - P_1) + \text{constant}\), where the constant does not depend on \(x_1\). Now we can apply the result in Proposition 4 to find:

\[
E[u_2(W) | \Psi_1] = |\Sigma|^{-1/2} \cdot |A|^{-1/2} \exp \left( \frac{1}{2} h'A^{-1}h - l \right). 
\]

Differentiation yields the first-order condition:

\[
h' \tilde{S}^{-1} \frac{\partial h}{\partial x_1} - \frac{\partial l}{\partial x_1} = 0,
\]

which yields after substitution and simplification:

\[
\chi_1(P) = \frac{E(P_2 | \Psi_1) - P_1}{RS} + \frac{E(F - P_2 | \Psi_1)}{R\text{Var}(F | \Psi_2)} \cdot \frac{S - T}{S}, 
\]

where:

\[
\tilde{S}^{-1} = \begin{pmatrix} S & T \\ T & U \end{pmatrix}.
\]

It follows that

\[
S = \frac{-(Cov(P_2, E(F | \Psi_2) | \Psi_1))^2 + Var(P_2 | \Psi_1)(Var(F | \Psi_2) + Var(E(F | \Psi_2) | \Psi_1))}{-2(Cov(P_2, E(F | \Psi_2) | \Psi_1) + Var(F | \Psi_2) + Var(P_2 | \Psi_1) + Var(E(F | \Psi_2) | \Psi_1)},
\]

as well as

\[
T = \frac{(Cov(P_2, E(F | \Psi_2) | \Psi_1))(Var(F | \Psi_2) - Cov(P_2, E(F | \Psi_2) | \Psi_1)) + Var(P_2 | \Psi_1)Var(E(F | \Psi_2) | \Psi_1)}{-2(Cov(P_2, E(F | \Psi_2) | \Psi_1) + Var(F | \Psi_2) + Var(P_2 | \Psi_1) + Var(E(F | \Psi_2) | \Psi_1)}. 
\]

In the case of immediate disclosure, the expression simplifies as given in the text. The same steps as above be used with \(Var(\beta | \Psi_2) \to 0\); alternatively, the result can be obtained via simpler
expressions for \( T \) and \( S \); specifically, using \( T = 0 \) and \( S = \left( \frac{1}{\text{Var}(F|\Psi_2)} + \frac{1}{\text{Var}(P_2|\Psi_1)} \right)^{-1} \). Note finally that since \( W \) is a quadratic form of a normal vector, the unconditional expected utility for a given \( M \) \([EU(M)]\) can also be numerically calculated by applying Proposition 4. In equilibrium, the certainty equivalent of utility \([CE(M)]\) equals the cost of acquiring information, i.e., \( CE(M) = (1/R) \log[EU(M)] = c \).

### A.5 Equilibrium Prices

The market maker sets prices as his expectation of asset payoff conditional on his information, which consists of the manager’s signal, \( s_m \), and total net order flow, \( y_1 = Mx_1 + z_1 \). Thus, prices can be found by regressing asset payoff on the vector \((s_m, y_1)\). Note that it is observationally equivalent to remove the terms involving \( s_m \) and \( P_1 \) from \( y_1 \), which yields a signal \( \tau_1 \). Denote \( v_1 = (s_m, \tau_1) \). Then the date 1 price can be found as a function of the date 2 price coefficients as:

\[
P_1 = \text{Cov}(v_1, F)\text{Var}(v_1)^{-1}v_1,
\]

which yields:

\[
a_i = \frac{M^2 \sigma_B^2 (\sigma_B^2 + \sigma_B^2) \left( \frac{\sigma_B^2 (\sigma_B^2 + \sigma_B^2)}{\sigma_B^2 \sigma_B^2 + \sigma_B^2 (\sigma_B^2 + \sigma_B^2) \sigma_B^2 + (\sigma_B^2 + \sigma_B^2) \sigma_B^2} \right) + c}{R^2 \left( \frac{M^2 (\sigma_B^2 + \sigma_B^2) \left( \frac{\sigma_B^2 (\sigma_B^2 + \sigma_B^2)}{\sigma_B^2 \sigma_B^2 + \sigma_B^2 (\sigma_B^2 + \sigma_B^2) \sigma_B^2 + (\sigma_B^2 + \sigma_B^2) \sigma_B^2} \right) + c}{R^2 \sigma_B^2} \right)^2}.
\]
and

\[ b = \frac{M \sigma_0^2 (\sigma_0^2 + \sigma_0^2)}{\sigma_0^2 \sigma_0^2 + \sigma_0^2 \sigma_0^2 + \sigma_0^2 \sigma_0^2 + \sigma_0^2 \sigma_0^2} + \frac{c}{f^2} \left( 1 + \frac{eM}{f^2 R \sigma_0^2} \right) \left( 1 + \frac{eM}{f^2 R \sigma_0^2} \right) \]

(71)

At date 2, the informed investors again submit their demand. Thus, the market maker at this second date can condition on the vector \( v_2 = (s_m, \tau_1, \tau_2) \), where \( \tau_2 \) is obtained from \( y_2 = Mx_2 + z_1 + z_2 \) by removing the \( P_2 \) and \( s_m \) terms. The date 2 price is given by:

\[ P_2 = d_l s_l + d_m s_m + e z_1 + f z_2 = Cov(v_2, F)Var(v_2)^{-1}v_2. \]

(72)

This yields a system of four equations in the four unknown price coefficients. This system is highly nonlinear, and thus problematic to solve. For the case of intermediate disclosure, share prices are determined by:

\[ P_1 = \frac{Cov(\tau_1, F)}{Var(\tau_1)} \tau_1, \]

(73)

and

\[ P_2 = Cov(v_2, F)Var(v_2)^{-1}v_2. \]

(74)

where \( v_2 = (s_m, \tau_1, \tau_2) \), as before.

For the case of immediate disclosure, the equation system above can be solved analytically. The solution is determined as follows:

\[ f = -\frac{M}{R \sigma_2^2}. \]

(75)
\[
e = \frac{E_1 + \sqrt{M^2 R^2 \sigma_2^4 \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \left(E_2 - E_3 E_4 + R^4 \sigma_2^4 \sigma_0^4 (\sigma_0^2 + \sigma_0^2) \left(\sigma_0^2 \sigma_0^2 \sigma_0^2 + \sigma_0^2 \left(\sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right)\right)^2\right)}}{2 M^2 R^2 \sigma_2^4 \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \left(\sigma_0^2 \sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right) + 2 R^4 \sigma_2^4 \sigma_0^2 \sigma_0^2 \left(\sigma_0^2 \sigma_0^2 \sigma_0^2 + \sigma_0^2 \left(\sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right)\right)^2},
\]

where

\[
E_1 = R^3 \sigma_2^4 \left(M \sigma_0^2 (\sigma_0^2 + 2 \sigma_0^2) \sigma_0^2 + M \left(\sigma_0^4 + 2 \sigma_0^2 \sigma_0^2 + 2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right) \sigma_0^2 \right),
\]

\[
E_2 = -4M^4 \sigma_0^4 \left(\sigma_0^2 + \sigma_0^2\right) \left(\sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right)^2,
\]

\[
E_3 = 4MR^2 \sigma_2 \left(M \sigma_0^2 (\sigma_0^2 + 2 \sigma_0^2) \sigma_0^2 + M \left(\sigma_0^4 + 2 \sigma_0^2 \sigma_0^2 + 2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right) \sigma_0^2 \right),
\]

and

\[
E_4 = \left(\sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right) \left(\sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right) \left(\sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right).
\]

Furthermore,

\[
d_m = \frac{D_1 + D_2 + eMR^2 \sigma_2^2 \left(eM \sigma_0^2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2) + \sigma_0^2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2) + 2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right)^2}{MR^2 \sigma_0^2 (-M + eR \sigma_2) \sigma_0^2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2) \left(\sigma_0^2 \sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right) + 2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right)^2},
\]

Where, in turn,

\[
D_1 = M^4 \sigma_0^4 \sigma_0^2 \left(\sigma_0^2 + \sigma_0^2\right) (\sigma_0^2 + \sigma_0^2) + M^2 R^2 \sigma_2 \left(\sigma_0^2 \sigma_0^2 \sigma_0^2 + \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right)^2,
\]

and

\[
D_2 = R^4 \sigma_2^2 \left(e \sigma_0^2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2) + e \sigma_0^2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right)^2.
\]

Finally,

\[
d_i = \frac{M(M^2 + e^2 R^2 \sigma_2^2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2))}{R^2 \sigma_0^2 (-M + eR \sigma_2) \sigma_0^2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2) + 2 \sigma_0^2 (\sigma_0^2 + \sigma_0^2)\right)^2}.
\]

A numerical solution to an equilibrium for a given disclosure policy involves finding
equilibrium values for $M$ and the pricing coefficients that satisfy the non-linear equation system (68) and (72) as well as the entry condition $CE(M) = c$, where $CE(M)$ represents the certainty equivalent of wealth for each informed agent at a given value of $M$. The overall equilibrium involves solving for the equilibrium with early and postponed disclosure and picking the solution that leads to a higher sum of the expected values of the growth opportunities represented by (7) in Proposition 1.
Figure 1: The Effect of Investor-Specific Information and the Specialness of the Manager’s Information

This figure illustrates how the optimal disclosure policy depends on the amount of investor-specific information, $\sigma_\delta$. The figure shows that the higher $\sigma_\delta$, the more profitable it is to delay disclosure until the intermediate date, $t=2$. On the other hand, the more special the manager’s information (i.e., the higher $\sigma_\eta/\sigma_\theta$), the more beneficial it is to disclose early (at $t=1$). In the extreme case of $\sigma_\delta = 0$, there is no investor-specific information, thus immediate disclosure is always optimal. The parameter values are: $R = 8$, $\sigma_\theta = 1$, $\sigma_\varepsilon = 0.6$, $\sigma_\zeta = 1$, and $c = 0.4$. 
Figure 2: The Effect of Information Overlap

This figure illustrates how the optimal disclosure policy depends on the amount of information overlap between informed investors and the manager. Note the non-monotonic effect: If the information overlap is small, delaying disclosure is suboptimal since doing so does not encourage sufficient additional information production by investors. On the other hand, if the degree of information overlap is too large relative to the amount of investor-specific information, delaying disclosure until $t=2$ severely decreases informational efficiency at date 1, rendering it suboptimal. The parameter values are: $R = 8$, $\sigma_\eta = 0.2$, $\sigma_\delta = 1$, $\sigma_\varepsilon = 0.6$, $\sigma_\xi = 1$, and $c = 0.4$. 

![Graph showing the relationship between information overlap and benefit from delaying disclosure. The x-axis represents information overlap $\sigma_\theta$, and the y-axis represents the benefit from delaying disclosure. The graph shows a non-monotonic curve.](image)
Figure 3: Separating the Date 1 and Date 2 Advantages of Postponing Disclosure

This figure separates the effect of delaying disclosure until $t = 2$ on the ex ante value of growth opportunities, $GO_t$, for $t \in \{1, 2\}$. Delaying disclosure typically reduces the value of $GO_t$, since manager-specific information, captured by $\sigma_\eta$, is not revealed. The benefits of delaying disclosure accrue to $GO_2$, when disclosure of manager-specific information combines with informed trading by investors to achieve a high degree of informational efficiency. The parameter values are: $R = 8$, $\sigma_\theta = 1$, $\sigma_\delta = 1$, $\sigma_\varepsilon = 0.6$, $\sigma_Z = 1$, and $c = 0.4$. 

![Graph](image-url)
Figure 4: The Effect of Risk

This figure illustrates how the optimal disclosure policy depends on the amount of risk that informed investors have to bear when holding the asset until the liquidation date, $t = 3$. The figure shows that more risk makes it less attractive for informed traders to hold the asset until $t = 3$. Thus, the partial shortening of informed investors’ horizon by delaying disclosure until $t = 2$ becomes more valuable. The marginal effect of an increase in risk is not constant, but tends to be more pronounced for higher levels of risk. The parameter values are: $R = 8$, $\sigma_\eta = 0.2$, $\sigma_\theta = 1$, $\sigma_\delta = 1$, $\sigma_z = 1$, and $c = 0.4$. 