The Regulation of Financial Products

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Abstract

We explore a theoretical model of product regulation in which the social planner chooses an optimal level of market complexity, given that people have varied sophistication. We study whether we can depend on the most qualified planners to implement regulation. To do so, we characterize the effect of lobbying and voting behavior on product regulation, and show that it is usually the case that both sophisticated and unsophisticated market participants have incentives to elect the least informed and educated planners. Based on this, well-intended regulation may not achieve first-best. Finally, we show that improving clarity in the market (e.g., more disclosure or better education) and enforcing simplicity (i.e., limiting differentiation) are strict substitutes from a welfare standpoint. Based on this, a fully informed social planner optimally trades off between better decision support and standardizing products in the market.

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1 Introduction

Complexity has outpaced sophistication in both retail and institutional financial markets.[1] This mismatch appears to affect social welfare and has been cited as a contributor and catalyst of the recent financial crisis.[2] The traditional approach to this problem allows markets to be free and then provides assistance to people ex post to help them make good decisions. In consumer markets, this might involve improved education (e.g., Mandell, 2009)[3], better timely decision support (e.g., Bertrand and Morse 2009; Lynch, 2009), or a policy of libertarian paternalism (e.g., Thaler and Sunstein, 2003; Choi, Madrian, Laibson, and Metrick, 2009; Carlin, Gervais, and Manso, 2011). In institutional markets, this might involve better disclosure or improved market transparency. In either case, these types of policies aim to help market participants protect themselves, while still implicitly allowing markets to be as complete as possible (and thereby complex).

Newer proposals, however, call for limiting the types of products that can be offered or traded in markets. Such policies aim to protect people from themselves. Possibilities include, for example, limiting the types of mortgages available to consumers or controlling the types of derivatives that can be used by traders. The idea here is that, instead of improving sophistication, simplicity or standardization is enforced. In retail markets, such intervention is meant to avoid information overload problems (e.g., Iyengar, Huberman, and Jiang, 2004; Salgado, 2006; Iyengar and Kamenica, 2008; Heidhues and Koszegi, 2010). In institutional markets, it is meant to decrease valuation errors and induce responsible asset management.

In this paper, we provide a theoretical model of product differentiation to explore these new policies and address several important questions: What is the socially optimal level of collective complexity in the market, given that people have varied sophistication? Can we depend on qualified planners to get elected and implement regulation optimally? When should regulators limit what is offered in the market to protect market participants from themselves? When should they increase access to information to assist participants in protecting themselves? Are these two initiatives

[1] For example, in retail settings the menu of offerings is now daunting, but financial literacy remains in short supply (Lusardi and Mitchell, 2007). Many participants in the market have limited sophistication regarding the products in the market (e.g. NASD Literacy Survey, Associated Press, 2003). See also Capon, Fitzsimons, and Prince (1996), Alexander, Jones, and Nigro (1998), Barber, Odean, and Zheng (2005), and Agnew and Szykman (2005). In institutional markets, the use of exotic derivatives and synthetic products has grown exponentially, yet our ability to accurately value many of these products lags behind.

[2] In retail markets, many home owners did not appreciate the variable-rate clauses in their mortgages and their explicit exposure to interest rate risk. Many individuals failed to appreciate the fees and interest rate schedules used commonly in credit cards, which exacerbated the amount of household debt and number of personal defaults in the United States (Campbell, 2006). In institutional markets, as securities were serially repackaged (e.g. collateralized debt obligations), market participants lost sight of the value underlying these instruments.

[3] See also Bernheim, Garrett, and Maki (2001), Bernheim and Garrett (2003), and Carlin and Robinson (2010)

(i.e., simplicity and clarity) mutually exclusive or complementary?

In our base model, a continuum of products are offered in the market. The upper bound of the continuum represents the extent of the market, that is, how complete the market is. A unit mass of agents who use these products are divided into two groups: sophisticated agents who identify the product in the market that is best suited for their needs and unsophisticated agents that cannot do this. As such, unsophisticated agents make errors because they choose randomly among the products in the market. This sets up a natural tension between the two groups. Sophisticated agents desire the market to be as complete as possible, so they have more to choose from and can identify the product that is best tailored for their needs. In contrast, unsophisticated agents are ambivalent about market complexity: while they enjoy the benefits of having more choices in aggregate, they incur the cost of making inappropriate choices. Therefore, unsophisticated agents desire less market completion than people who are sophisticated.

We solve for the optimal level of market completion that maximizes aggregate welfare. The upper bound of the continuum is strictly increasing in both the needs of the sophisticated and unsophisticated agents, increasing in the fraction of sophisticated agents, and decreasing in the fraction of unsophisticated agents. Importantly, we show in equilibrium that unsophisticated consumers actually desire *less* market completion than meets their needs in aggregate.

We then study the quality of regulation by reconsidering the model when the social planner is uninformed about the specific needs of her constituents. Before regulating the market, the planner listens to the recommendations of two advocates, each lobbying on behalf of one of the groups of constituents. Based on these reports, the planner chooses and enforces a level of market complexity that best represents the wishes of her constituents, but is regrettably second best. We consider two types of uninformed planners in this fashion. The first is naive in that she accepts and uses reports by the lobbyists at face value. The second is savvy (i.e., rational) and uses her consistent beliefs about the lobbyists' incentives to misreport and therefore unwind their messages to better appreciate their information. We use a cheap-talk framework to analyze this latter setting (e.g., Crawford and Sobel, 1982).

Not surprisingly, both advocates do in fact misreport their private information with both types of planners. Indeed, it is a weakly dominant strategy for the sophisticated agents' advocate to lobby for full market completion (i.e., a libertarian platform). Taking this into account, the advocate for the unsophisticated also shades down, but his optimal strategy depends on the parameters in the model. In many cases, he successfully lobbies the planner to set a level of complexity that exactly optimizes the utility of the unsophisticated people. In other cases, he is less successful and receives

an inferior outcome.

In equilibrium, both types of uninformed planners only implement second-best regulation. However, the punchline is that the least qualified planner usually gets elected to implement regulation. We model an election in which a perfectly informed planner, a rational uninformed planner, and a naive uninformed planner run for office. We show that when a supermajority exists, the most naive planner usually gets elected, even when sophisticated agents dominate the market. This places a bound on how effective regulation can be in the market. When unsophisticated agents have the supermajority, their advocate can get their needs met perfectly. When sophisticated agents dominate, they will often support the less qualified planner so their lobbyist can maximize the benefit of exaggerating his recommendations. This is most likely to occur when the potential extent of the market exceeds their actual needs.

Based on this, we add an important aspect to the debate on product regulation. Specifically, even though there are two groups whose wishes need to be balanced, it is the incentive problems that arise in the market that predispose the planner to be less qualified. That is, quality suffers endogenously when two groups compete for their interests, which causes welfare to deviate from first best.

Finally, we consider an additional channel for intervention in the market: increasing access to information. Specifically, we allow the planner to choose a fraction of unsophisticated agents to educate about their needs. As such, the social planner now has the choice of decreasing the scope of the market (i.e, simplifying it), enhancing decision support for unsophisticated agents (i.e., clarifying it), or both. One of the main messages that arises from this analysis is that when a social planner wishes to maximize welfare, decision support mechanisms and enforcing simplicity (i.e., limiting market completion) are strategic substitutes for each other. Any intervention that increases decision support decreases the benefit of standardization in the market, and vice versa.

The loss function that we use in our analysis is the absolute distance between an agents type and the product that he chooses. As such, the social planner regulates the market to minimize the sum of these individual losses across the population. In studying this problem, we have considered other welfare functions to investigate the robustness of our results. One modification that we included in the paper is worth mentioning. We consider an alternative welfare function in which disparity between losses of different agents are valued as well. That is, we study the tradeoff between adequacy and equality in the market. Interestingly, our analysis demonstrates that if equality is sufficiently important (e.g., represents $\frac{2}{3}$ of aggregate utility in the welfare function), the optimal regulation involves having one, and only one, product in the market. As such, a sufficiently

socialistic perspective necessarily results in making the market as incomplete as possible, even though all agents end up hurt in the effort to maintain equality.

While we study optimal regulation in this paper, one might wonder whether regulation is even necessary in financial or consumer markets. Previous work suggests that it is: competition in financial markets may be an unreliable driver of market transparency and education may not improve social welfare. As Carlin, Davies, and Iannaccone (2010) show, when providers compete for attention in the market, competition actually makes it less likely that they disclose private information. Likewise, Carlin (2009) shows that when competition increases, providers actually have an increased non-cooperative incentive to add complexity to their offerings. Both of these papers make a strong argument for the value of market intervention. Carlin and Manso (2010) show that educational initiatives may be welfare decreasing in some circumstances since it induces providers of financial products to decrease clarity in the market via obfuscation. In such cases, regualtion may be important to increase welfare.

Last, our paper is of general economic interest as it adds to a large literature on product differentiation and efficiency in exchange economies. Starting with, Hotelling (1929), Chamberlin (1933), and Lancaster (1966), economists have focused on oligopoly behavior when there is a demand for differentiated products and consumers are heterogeneous.[4] In this literature, location games using linear city and circular city models are commonplace. Our work here departs in that we model the needs of two subsets of people by superimposing two (possibly different) linear city distributions and considering that one group is less sophisticated in assessing their needs for products in the market. We abstract away from the oligopoly behavior that might lead to the evolution of such markets and focus instead on regulating such markets. Considering the two groups of consumers simultaneously is novel, and to our knowledge represents a new contribution to the general economic literature on product differentiation.

The rest of the paper is organized as follows. In Section 2, we pose and characterize our base model, and derive the socially optimal product regulation. In Section 3, we evaluate the quality of regulation by considering imperfectly informed planners and the implications of lobbying and elections. In Section 4, we characterize the tradeoff between clarity and simplicity in the market. Section 5 concludes. All of the proofs are in Appendix A. In Appendix B, we consider an alternative setup where the planner chooses both the lower and upper bounds of the market's completeness.

[4]See Lancaster (1990) or Anderson, de Palma, and Thisse (1992) for a thorough survey of this literature.

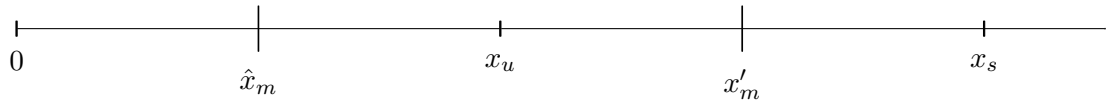


Figure 1: The product market. The types for unsophisticated agents are uniformly distributed along the interval $[0, x_u]$ and the types for sophisticated agents are uniformly distributed on $[0, x_s]$. Products in the market are distributed on $[0, x_m]$. The bound $\hat{x}_m$ is an example of an incomplete market for both sophisticated and unsophisticated agents. The bound x'_m is a market that is complete for unsophisticated agents, but is incomplete for sophisticated agents.

2 Products and regulation

Consider the market in Figure 1 in which a continuum of products $[0, x_m]$ are offered for use, where x_m measures the extent of the market. We interpret x_m as a measure of market completeness. For example, in Figure 1, the market with x'_m is more complete than the one with $\hat{x}_m$. Since people have more choices to consider with x'_m than with $\hat{x}_m$, we also consider x_m to be a measure of collective complexity in the market. The purpose of this paper is to study the socially optimal level of x_m. As such, we remain silent about what oligopoly behavior actually leads to any particular x_m.

There exists a unit mass of agents who participate in the market. A fraction λ_s of the agents are sophisticated and have a type $\tilde{t_s}$, which is uniformly distributed over $[0, x_s]$. These agents know their type exactly and maximize their payoff (to be described shortly) by choosing the product closest to their type. For example, in a product market where $x_m = 0.55$, a sophisticated agent with type $\tilde{t_s} = 0.5$ would choose $x = 0.5$. However, if the product market was more limited in scope such that $x_m = 0.4$, the same sophisticated agent with type $\tilde{t_s} = 0.5$ would choose $x = 0.4$, which is the best alternative available.

The remaining agents, $\lambda_u = 1 - \lambda_s$, are unsophisticated and have a type $\tilde{t_u}$ distributed uniformly over $[0, x_u]$. We assume that unsophisticated agents do not know their own type and choose a random product $\tilde{x}$ in $[0, x_m]$. By construction, unsophisticated agents make errors. This is standard in both the literature on consumer search theory (e.g., Salop and Stiglitz, 1977; Varian, 1980; and Stahl, 1989) and household finance (e.g., Carlin, 2009; Carlin and Manso, 2010).[5] An alternative

[5]For example, in models of "all-or-nothing" search (e.g., Salop and Stiglitz, 1977; and Varian, 1980), unsophisticated consumers are explicitly assumed to choose randomly among firms. In sequential search models, unsophisticated consumers are randomly assigned to their first firm and then choose whether to continue searching for the best alternative. In equilibrium, unsophisticated consumers stop at the first firm, so that they in essence make purchases randomly from the firms. See either Stahl (1989) or Baye, Morgan, and Scholten (2006) for a complete review of

specification might allow them to see a noisy signal about their type, but the economics would be qualitatively similar to what follows here.

Sophisticated agents maximize their expected utility from participation. In this setting, this involves minimizing a loss function in which an agent is better off the closer they are to their true type. Since sophisticated agents know their type, they solve

$$\min_{x \in [0, x_m]} \quad L(x|t_s, x_m) = |x - t_s|. \tag{1}$$

Unsophisticated agents, on the other hand, choose randomly from the menu of products offered.

In this model, we assume that $x_u \leq x_s$ to capture the idea that sophisticated agents may have use for more exotic products. For example, in a finance context, there may be a subset of traders who have need for a complicated derivative product and know how to use such a product responsibly in the market. Alternatively, sophisticated home owners may demand a mortgage that amortizes in a particular way that is not appropriate for most home buyers. As we will see shortly, this induces a natural tension in the model: whereas sophisticated agents desire a more complete market, unsophisticated agents would prefer more standardization to avoid making errors and suffering losses. As we analyze in Section 2.3, the goal of the social planner is to set x_m to maximize welfare in the presence of this tension. It is important to point out, though, that this assumption is made for analytic convenience and is not necessary for the tension to be present: in Lemma 2, we show that the tension exists as long as $x_s > \frac{3}{4}x_u$.

We begin by showing some intuitive results about agent behavior that will be useful in Section 2.3 when we consider the social planner's problem.

2.1 Sophisticated Agents

Since each sophisticated agent knows their type, we can write the aggregate loss to these agents $\mathcal{L}_s$ as

$$\mathcal{L}_s = \int_0^{x_m} 0 \frac{d\tilde{t}_s}{x_s} + \int_{x_m}^{x_s} |x_m - \tilde{t}_s| \frac{d\tilde{t}_s}{x_s}. \tag{2}$$

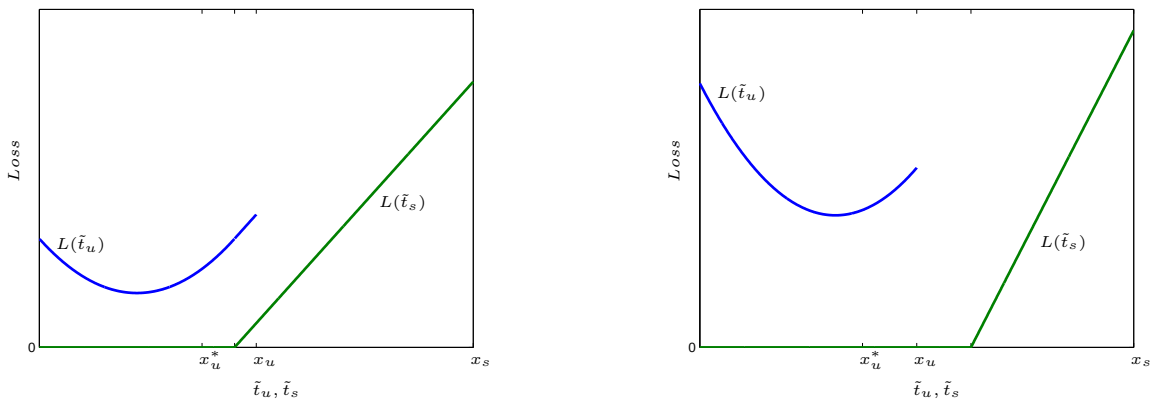
The following lemma establishes some useful results.

Lemma 1. *If $x_m < x_s$, the aggregate loss for sophisticated agents is*

$$\mathcal{L}_s = \frac{x_s^2 - 2x_m x_s + {x_m}^2}{2x_s}, \tag{3}$$

which is decreasing and convex in x_m. If $x_m \geq x_s$, then $\mathcal{L}_s = 0$.

consumer search theory.



(a) $x_m \leq x_u$

(b) $x_m > x_u$

Figure 2: Individual Losses Given x_m

According to Lemma 1, for $x_m < x_s$, some sophisticated agents are able to use a product that is identically perfect for their needs. Others, however, have to settle for a suboptimal choice. Figures 2(a) and 2(b) provide examples of this. The "hockey-stick"-shaped figured labeled $L(\tilde{t}_s)$ plots the individual losses that sophisticated agents experience. The aggregate loss for the group is the triangular area bounded by the kink in the curve to the left and x_s to the right. This area is calculated analytically by the expression in (3).

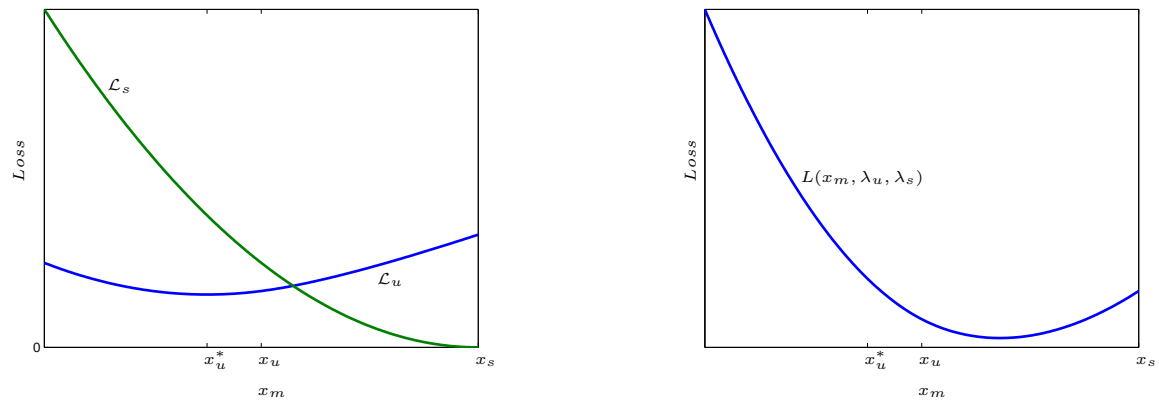
As x_m rises, this lowers the aggregate loss to sophisticated agents. This is best seen in Figure 3(a): the curve labeled $\mathcal{L}_s$ is downward sloping as a function of x_m. In the limit, when $x_m \to x_s$, $\mathcal{L}_s \to 0$. This implies that any market expansion beyond x_s adds no value to sophisticated agents. We define

$$x_s^* \equiv x_s \tag{4}$$

to be the point at which the aggregate loss to sophisticated agents is minimized. As we will see shortly when we consider the social planner's problem, we will be able to allow x_s^* to serve as an upper bound for x_m without loss of generality.

2.2 Unsophisticated Agents

Unsophisticated agents have no information regarding their type $\tilde{t_u}$ and choose randomly from the products in the market. The expected aggregate loss for unsophisticated agents in a product market



(a) Unsophisticated and Sophisticated Losses

(b) Aggregate Expected Loss

Figure 3: Ex ante expected losses as a function of x_m

characterized by x_m is

$$\mathcal{L}_u \equiv E[L(\tilde{t_u}, \tilde{x}|x_m)] = \int_0^{x_u} \int_0^{x_m} |\tilde{x} - \tilde{t_u}| \frac{d\tilde{x}}{x_m} \frac{d\tilde{t_u}}{x_u}. \tag{5}$$

We can now make some statements regarding the unsophisticated agents' preferences for x_m.

Lemma 2. *The level of x_m that maximizes the expected aggregate welfare of unsophisticated agents is given by*

$$x_u^* \equiv \frac{3}{4} x_u. \tag{6}$$

When $x_m > \frac{3}{4}x_u$, $\mathcal{L}_u$ is increasing and convex in x_m.

Lemma 2 tells us that unsophisticated agents are worse off when products are offered in the market that exceed their particular needs. More interestingly, they not only have an aversion to such products, but actually prefer the market to be *less* complete than meets their needs in aggregate. The intuition behind the result lies in the tradeoff between providing access to more products in the market and the cost of introducing more room for error. The highest types of unsophisticated agents benefit from more products because it improves the chances that they randomly select a product near their type. Conversely, lower types suffer as it becomes more likely that they choose wrongly. The loss for the mass of unsophisticated agents is minimized at $\frac{3}{4}x_u$, which results from the triangular nature of the loss function induced by the uniform distribution.

These results can be appreciated visually. The individual losses for the unsophisticated agents are plotted on the curves labeled $L(\tilde{t}_u)$ in Figures 2(a) and 2(b). By inspection, the loss function is convex with a minimum strictly less than x_u. The aggregate loss function to the unsophisticated agents $\mathcal{L}_u$ is plotted in Figure 3(a) as a function of x_m. Again, by inspection, it is clear that an x_m strictly less than x_u maximizes welfare for unsophisticated agents.

Since $x_u^* < x_u$, this implies a natural tension between sophisticated and unsophisticated agents, even if $x_u = x_s$. The social optimal level of x_m will take these forces into account, and we derive x_m^* next.

2.3 Optimal Product Market Complexity

The aggregate loss to all agents is given as

$$L(x_m, \lambda_u, \lambda_s) = \lambda_u \mathcal{L}_u + \lambda_s \mathcal{L}_s. \tag{7}$$

The aggregate loss has the form

$$L(x_m, \lambda_u, \lambda_s) = \begin{cases} \lambda_u \left[\frac{{x_m}^2}{3x_u} + \frac{-x_m + x_u}{2} \right] + \lambda_s \left[\frac{x_s^2 - 2x_m x_s + {x_m}^2}{2x_s} \right] & x_m \leq x_u \\ \lambda_u \left[\frac{{x_m}^2 - x_m x_u + (2/3)x_u^2}{2x_m} \right] + \lambda_s \left[\frac{x_s^2 - 2x_m x_s + {x_m}^2}{2x_s} \right] & x_m > x_u. \end{cases} \tag{8}$$

The analytic expressions for $\mathcal{L}_u$ in (8) are derived in the proof of Lemma 2. One concern about $L(x_m, \lambda_u, \lambda_s)$ might be whether it is discontinuous at x_u. As we show in Lemma A1 in the appendix, this function is indeed continuously differentiable at the point x_u.

The social planner solves the following problem

$$\min_{x_m \in [0, x_s]} \quad L(x_m, \lambda_u, \lambda_s). \tag{9}$$

As noted before, the social planner can restrict her attention to $x_m \leq x_s$ because once $x_m \geq x_s$, increasing it further makes the sophisticated agents no better off, but hurts the unsophisticated. The following proposition characterizes the unique socially optimal level of x_m^*.

Proposition 1. *There exists a unique optimal $x_m^* \in [x_u^*, x_s^*]$ that minimizes $L(x_m, \lambda_u, \lambda_s)$. If*

$$\frac{\lambda_u}{\lambda_s} < 6 \left(1 - \frac{x_u}{x_s} \right), \tag{10}$$

then $x_m^* > x_u$.

The optimal x_m^* is

(i) increasing in x_u and x_s;

(ii) decreasing in the mass of unsophisticated agents, λ_u;

(iii) increasing in the mass of sophisticated agents, λ_s.

According to Proposition 1, if the needs of either type of agent increase ceteris paribus, the optimal scope of the market is higher. However, x_m is determined based on the proportion of types in the market. As the fraction of sophisticated agents rises, x_m^* is higher. As λ_u increases, x_m^* is lower. These comparative statics are a direct result of the natural tension between the two groups.

It is important to note that the market does not need to have the particular structure that we consider here to have the same comparative statics hold. For example, in Appendix B we consider an alternative specification in which the lower bound of the market is not tethered to zero. As we show, sophisticated consumers still prefer the market to be as complete as possible, whereas unsophisticated consumers desire there to be one product that is the median of their needs. We solve for the planner's optimal choice of lower and upper bounds, and show that such bounds change monotonically in the underlying parameters of the model.

Before closing this section, let us consider a special case that we will use periodically in the rest of the paper. Specifically, let

$$x_u = x_s \equiv x_p. \tag{11}$$

From Proposition 1, we know that $\frac{\lambda_u}{\lambda_s} \geq 6\left(1 - \frac{x_u}{x_s}\right) = 0$, so that $x_m^* \leq x_p$. Taking first-order conditions with respect to the aggregate loss function, we obtain

$$0 = \frac{\partial}{\partial x_m} L(x_m, \lambda_u, \lambda_s | x_m \leq x_u) \tag{12}$$

$$= \lambda_u \left[\frac{2x_m}{3x_p} - \frac{1}{2}\right] + \lambda_s \left[-1 + \frac{x_m}{x_p}\right]. \tag{13}$$

Solving for x_m yields

$$x_m^* = \frac{3}{2}\frac{x_p(2-\lambda_u)}{(3-\lambda_u)}. \tag{14}$$

Note that when $\lambda_u = 0$ we obtain that $x_m^* = x_p$, which is intuitive since all agents are sophisticated. Likewise when all agents are unsophisticated, $\lambda_u = 1$, we obtain $x_m^* = \frac{3}{4}x_p$, which is consistent with their ideal point. If $\lambda_u = \lambda_s = \frac{1}{2}$, we obtain $x_m^* = \frac{9}{10}x_p$. Thus, the expression in (14) confirms our previous claim that even when all agents are distributed uniformly on identical supports, the presence of unsophisticated agents yields an internal optimum (i.e., $x_m^* < x_p$).

2.4 Welfare: Equality versus adequacy

In the previous section, the social planner chose the optimal level of market sophistication, x_m^*, by minimizing the total aggregate loss. The comparative statics of x_m^* in Proposition 1 reveal that the difference in expected loss for unsophisticated and sophisticated agents can be large when one group is much smaller than the other and when the upper bounds on the groups' needs are greatly dissimilar. That setup assumes that total aggregate loss is the only concern of the social planner. Here we extend the analysis to a scenario where the social planner is interested in not only the total loss, but also the expected degree of disparity between sophisticated and unsophisticated agents[6]. We assume that $x_u = x_s = x_p$ and $\lambda_u \in (0, 1)$, and we define

$$D(x_m, \lambda_u, \lambda_s) \equiv |\mathcal{L}_u - \mathcal{L}_s|. \tag{15}$$

Equation 15 and Equation 8 from Section 2.3 can be combined to produce a welfare equation that incorporates both concerns,

$$W = \kappa L(x_m, \lambda_u, \lambda_s) + (1 - \kappa) D(x_m, \lambda_u, \lambda_s), \tag{16}$$

where $\kappa \in [0, 1]$. We assume that κ is exogenously given and it represents the social planner's preferences over the two matters.

Proposition 2. *The optimal level of market sophistication when the social planner is concerned with both total aggregate loss and the disparity between sophisticated and unsophisticated agents is given by*

$$x_m^* = \begin{cases} 0 & \text{if } \kappa < \frac{1}{3-\lambda_u} \\ \frac{3}{2}\frac{1-\kappa(3-\lambda_u)}{1-\kappa(4-\lambda_u)} x_p & \text{if } \kappa \geq \frac{1}{3-\lambda_u}. \end{cases} \tag{17}$$

If $\kappa \geq \frac{1}{3-\lambda_u}$, then x_m^* is strictly increasing in κ and strictly decreasing in λ_u.

The comparative statics in κ and λ_u are straightforward. That is, as the importance of aggregate social loss increases (higher κ), the optimal market completeness rises. Likewise, as the number of unsophisticated agents increases, the lower x_m^* will be.

What is interesting is that as long as $\kappa < \frac{1}{3}$, the social planner optimally chooses to have a one-product market. This means that if equality is most important, as it would be in a socialistic

[6] Given that there are continua of sophisticated and unsophisticated agents, the law of large numbers makes our analysis here apply to ex post dispersion as well.

society, no differentiation is allowed in the market. Of course, as λ_u rises, this bound becomes larger. As $\lambda_u \to 1$, if $\kappa < \frac{1}{2}$, then a one product market is optimal.

Proposition 2 implies that there is a tradeoff between a market that provides adequate products to its constituents and the equality that people experience when they make choices. As κ decreases and equality is more important, x_m decreases and deviates more from the optimum derived in Section 2.3. This means that the more equality is weighted, the less adequate is the market, especially for sophisticated agents. As such, Proposition 2 captures the idea that aggregate losses may increase as equality concerns are introduced. When $\kappa < \frac{1}{3}$, equality is indeed achieved, but unfortunately both sophisticated and unsophisticated agents are equally worse off. This captures one of the potential drawbacks of a socialistic agenda (e.g., Stiglitz, 1994).

We complete this section with the following example.

Example 1. *Suppose that* $\kappa = \frac{1}{2-\lambda_u}$. *Then, plugging into (17) yields* $x_m^* = \frac{3}{4}x_u$.

The significance of Example 1 is that unsophisticated agents may benefit substantially at the expense of sophisticated agents. In this case, κ is such that $x_m = x_u^*$. Of course, if κ were to decrease further, the aggregate loss for both groups would rise, even as the two aggregate losses converged more.

Up to this point, we have assumed that the social planner is completely informed. Also, we have assumed that unsophisticated agents cannot learn and that the social planner cannot educate participants in the market. These are the issues that are central to the ongoing policy debate described in the introduction. We turn to these issues in the next two sections.

3 The quality of product regulation

The quality of regulation depends, among other things, on the ability of elected officials to understand markets and their knowledge regarding the needs of their constituents. In this section we explore how these considerations affect product regulation. So far, we have assumed that the social planner perfectly observes x_u and x_s. But in reality, this is far from true. Policy makers, as benevolent as their motives might be, cannot know everything about their constituents and often resort to listening to the opinions of lobbyists and advocates before setting policy. Therefore, in this section, we consider a setting of imperfect information and study the distortions that may arise from both lobbying and voting behavior.

We consider two types of social planners. The first is *naive* in that she does not have information about x_u or x_s, and blindly accepts recommendations from advocates who represent the

sophisticated and unsophisticated agents' interests. The second planner is *savvy* in that while she does not have information about market participant needs, she rationally understands the incentives of the advocates to misreport, and therefore unwinds such reports to refine her beliefs before making a final decision regarding regulation.

Compared to a social planner with perfect information, these two types of leaders do not achieve first best regulation. However, we complete the analysis by studying the voting behavior of sophisticated and unsophisticated agents, given that they participate in an election in which all three types of planners run for office: the naive uninformed, the savvy uninformed, and the perfectly informed. We determine how qualified the regulator is that gets elected and how this affects the quality of regulation.

3.1 Lobbying efforts - Naive Social Planner

Consider the model setup from Section 2, except that the upper bounds x_u and x_s are unobservable to the social planner. Instead, there are two advocates that represent each group: A_u lobbies for unsophisticated agents and A_s lobbies for the sophisticated. Each advocate makes a report, r_u and r_s, about their respective bound. Since it is prohibitively costly to canvass the population to assess each person's needs, the reports are not verifiable ex post. As in Section 2, $x_u \leq x_s$, but we now assume that there is a finite bound on the sophisticated agents' needs (i.e., $x_s \leq \overline{x}$).

The social planner faithfully accepts the values provided by the lobbyists and uses them to choose an $x_m(r_u, r_s)$. However, the planner does not attempt to unwind the true observations of x_u and x_s from the reports, using the incentives that each advocate has to lobby for their constituents. We explore that consideration in the next subsection.

The following proposition characterizes optimal reporting behavior by the advocates.

Proposition 3. *It is a weakly dominant strategy for A_s to always report $r_s = \overline{x}$. If, and only if,*

$$x_u \geq \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right), \tag{18}$$

A_u reports $r_u > 0$ such that $x_u^* = x_m(r_u, \overline{x})$. Otherwise, A_u reports $r_u = 0$ and the planner sets

$$x_m = \bar{x}\Big[1 - \frac{\lambda_u}{2\lambda_s}\Big]. \tag{19}$$

The intuition behind the first part of Proposition 3 rests on the observation that sophisticated agents are only hurt when they are underserved. If a social planner sets x_m below x_s, the sophisticated agents with needs in $[x_m, x_s]$ are unable to find a product with perfect fit. When x_m is set

equal to or above x_s, all sophisticated agents are able to find the product they need. With this, and knowing that it is the social planner's objective to balance the needs of the two groups, the best response for A_s is to maximally exaggerate the sophisticated agents' needs. The strategy is costless and shifts x_m upward, ceteris paribus.

The advocate for unsophisticated agents takes this into account and reports $r_u < x_u$ (i.e., shades down). According to Proposition 3, if the condition in (18) holds, A_u will make a positive report and successfully lobby the planner to restrict the market to $x_m = x_u^*$. If (18) does not hold, then A_u cannot push the planner to choose the unsophisticated ideal point with any report. In such case, A_u will claim that unsophisticated agents need one, and only one, product.

The condition in (18) provides some natural economic insights. First, the closer x_u is to $\overline{x}$, the more likely it is that unsophisticated agents are able to obtain their ideal point. The more interesting result is expressed in the following corollary.

Corollary 3.1. *When the planner depends on lobbyist reports, the unsophisticated agents obtain $x_m = x_u^*$ if $\lambda_u \geq 2\lambda_s$.*

Corollary 3.1 tells us that if unsophisticated agents make up two thirds of the population, they will be able to persuade an imperfect social planner to choose $x_m = x_u^*$. Importantly, this holds true no matter how large $\overline{x}$ is and does not depend on any other exogenous political pressures (i.e., getting re-elected or monetary payoffs). So, as long as unsophisticated agents make up at least $\frac{2}{3}$ of the population, they get their way by lobbying a naive social planner without full information.

3.2 Lobbying Efforts - Savvy Social Planner

Consider now that the social planner anticipates that the two advocates have the incentive to misreport their group's needs. Again, the social planner does not observe x_s and x_u but instead has to rely on the advocates' reports and her prior beliefs. For analytic ease, we assume that the social planner believes that x_u and x_s are distributed uniformly over $[0, \bar{x}]$, with $x_u \leq x_s$.

Lemma 3. *The social planner's unconditional beliefs for x_u and x_s are given by,*

$$E[x_u] = \frac{\bar{x}}{3} \tag{20}$$

$$E[x_s] = \frac{2\bar{x}}{3}. \tag{21}$$

The two agents again have the incentive to misreport their needs because the planner is imperfectly informed. Furthermore, since there is no punishment for lying and reporting is costless, a

truth-telling, incentive compatible mechanism is elusive. Since this is a cheap-talk game, messages that are sent to the planner will be insubstantial if the advocates are restricted to perfectly reliable, noiseless communication channels (e.g., Farrell, 1998; Forges, 1986 and 1988). Therefore, in the spirit of Crawford and Sobel (1982), we model the described lobbying problem as a cheap-talk game in which the advocates and the planner adhere to an equilibrium message strategy and a corresponding action function.

Each of the agents independently observes their group's needs and sends the planner a report, r_i with $i \in \{u, s\}$. Correspondingly, the planner processes r_u and r_s and then determines an x_m conditional on the messages and her prior beliefs. Denote $x_m^a(r_u, r_s)$ to be the decision made by the social planner. The planner's problem therefore is to minimize,

$$\min_{x_m^a \in [0,\bar{x}]} |x_m^a(r_u, r_s) - x_m^*|, \tag{22}$$

where x_m^* is the optimal level of sophistication if the planner was perfectly informed. Similarly, denote each advocates's objective function as,

$$\min_{r_i \in [0,\bar{x}] | r_{-i}} |x_m^a(r_u, r_s) - (x_m^* - b_i)|, \tag{23}$$

where b_i represents the advocate's bias from the first-best solution. For unsophisticated agents, the bias is given by $b_u = x_m^* - x_u^*$, whereas for sophisticated agents it is $b_s = x_m^* - x_s^*$.

Definition 1. *An equilibrium in the cheap-talk lobbying problem consists of*

(i) a message strategy for each advocate, $q_i(r_i|x_i)$, such that $\int_0^{\bar{x}} q_i(r_i|x_i)\, \mathrm{d}r_i = 1$ for all $x_i \in [0, \bar{x}]$,

(ii) a choice function for the principal, $x_m^a(r_u, r_s)$ such that

(a) for each $x_u, x_s \in [0, \bar{x}]$ if $q_i(r_i'|x_i) > 0$ then it must be that

$$r_i' \in \arg\min_{r_i} |x_m^a(r_u, r_s) - (x_m^* - b_i)|$$

,

(b) and $x_m^a(r_u, r_s) \in \arg\min_x \int_0^{\bar{x}} |x - x_m^*| p(x_m^*|r_u, r_s)\, \mathrm{d}x_m^*$ where

$$p(x_m^*|r_u, r_s) = \frac{q_u(r_u|x_u, r_s) q_s(r_s|x_s)}{\int_0^{\bar{x}} \int_0^{\bar{x}} q_u(r_u|\mu_u, r_s) q_s(r_s|\mu_s)\, \mathrm{d}\mu_u\, \mathrm{d}\mu_s}$$

Definition 1 essentially says that for any realization of x_i, advocate i will mix over a set of messages, $\{\vec{r}_i\}$, such that the sum of probabilities on each possible message add to one. Furthermore,

given the planner's decision rule, $x_m^a(r_u, r_s)$, any message r_i', sent with positive probability, must imply that $x_m^a(r_i'|r_{-i})$ results in an outcome that is no worse than the outcome that would have resulted from sending any other message $r_i'' \in [0, \bar{x}]$. Additionally, given the family of message rules for advocates A_u and A_s, the choice $x_m^a(r_u, r_s)$ must be a solution to the social planner's problem.

We now appeal to the result of Crawford and Sobel (1982) that, for any $b_i > 0$, there exists at least one "partition" equilibrium, with specific properties to be discussed momentarily, such that each advocate reports in which partition of $[0, \bar{x}]$ their realization lies. Correspondingly, the planner takes the midpoint of the partition as a "noisy" estimate of the true realization.[7]

Lemma 4. *It is a weakly dominant strategy for A_s to always report $r_s = \bar{x}$. Furthermore, this implies that $q_s(\bar{x}|x_s) = 1$ and $q_s(r_s'|x_s) = 0$ for all $x_s \in [0, \bar{x}]$ and $r_s' \in [0, \bar{x})$ and that*

$$p(x_m^*|r_u, \bar{x}) = \frac{q_u(r_u|x_u)}{\int_0^{\bar{x}} q_u(r_u|\mu_u, \bar{x})\, \mathrm{d}\mu_u}.$$

Lemma 4 greatly reduces the complexity of our lobbying problem since we now only need to concern ourselves with the message strategy of A_u. The properties of the "partition" equilibrium follow as,

(i) there is a positive integer, N, such that one can define a set of $N+1$ real numbers, generically denoted $\{r_u^0, r_u^1, \ldots r_u^N\}$ with $r_u^0 < r_u^1 < \ldots < r_u^N$,

(ii)
$$x_m^a(r_u^j) = \begin{cases} x_m\left(\frac{r_u^{j+1}+r_u^j}{2}, E\left[x_s \middle| x_s \geq \frac{r_u^{j+1}+r_u^j}{2}\right]\right) & N > 1 \\ x_m\left(\frac{\bar{x}}{3}, \frac{2\bar{x}}{3}\right) & N = 1, \end{cases}$$
where $x_m(\cdot,\cdot)$ is the optimal x_m given a noisy indication of x_u and x_s.

(iii) $q_u(r_u|x_u)$ is uniform over $[r_u^i, r_u^{i+1}]$ if $x_u \in [r_u^i, r_u^{i+1}]$.

Now, we direct our attention to ex ante efficient message strategies and action rules, since x_u is unobservable and the advocate's true bias is unknown to the social planner. The following proposition addresses the maximum number of partitions that can be supported in equilibrium.

Proposition 4. *An equilibrium exists in which the maximum number of partitions, $\overline{N}$, that can be supported is*

$$\left\langle -\frac{1}{2} + \frac{1}{2}\sqrt{\frac{32 - 3\lambda_s}{5\lambda_s}} \right\rangle \tag{24}$$

[7]For any $b_i > 0$ it is essential that each partition is "noisy", meaning that it has positive mass, for substantive communication to occur. Farrell (1988) and Forges (1986, 1988) demonstrate that messages in sender-receiver games are insubstantial if players are restricted to perfectly reliable, noiseless communication channels.

where $\langle z \rangle$ denotes the smallest integer greater than or equal to z. Additionally, only a babbling equilibrium, $\overline{N} = 1$, exists if $\lambda_s > \frac{2}{3}$, while a perfectly informative equilibrium exists if $\lambda_s = 0$.

According to Proposition 4, a babbling equilibrium exists as long as λ_u is less than 1/3, in which the report is completely uninformative. Indeed, Crawford and Sobel (1982) show that a babbling equilibrium, $N = 1$, always exists. In such an equilibrium, the planner would be left to choose x_m based solely on her unconditional beliefs outlined in Lemma 3.

Proposition 4 tells us that the informativeness of agent A_u's report depends on the proportion of unsophisticated agents in the economy. Interestingly, because A_s exaggerates his report, equilibrium messages from A_u do not contain much information unless unsophisticated agents make up a substantial proportion of the population. According to (24), $\overline{N} \geq 3$ can only be supported if $\lambda_u > 3/4$ and $\overline{N} \geq 4$ can only be supported if $\lambda_u > 27/31$ (i.e., 87% of the population). This means that the savvy planner only partitions the message space into quartiles if the unsophisticated agents comprise roughly 87% of the population. This severely limits the planner's ability to conduct inference. Of course, as $\lambda_u \to 1$, the number of partitions goes to infinity and the planner receives a perfectly informative signal. This is not surprising because when $\lambda_u \to 1$, there is no longer any conflict and A_u simply tells the truth. Practically speaking, however, this is usually not the case, and we often have to settle with a savvy planner that cannot learn much from lobbying efforts.

Voting Behavior

We now consider what type of social planner is elected to regulate markets, which will greatly impact the quality of such regulation. We assume that all three types of planners run for office prior to the implementation of any financial policy. We assume all agents participate in the election, so outcomes will be based on their respective proportions within the population.

Let the optimal levels of market sophistication under each type of planner be denoted as,

$$\begin{cases} x_m^P & \text{Perfectly Informed} \\ x_m^V & \text{Savvy} \\ x_m^N & \text{Naive} \end{cases}$$

where $x_m^P \neq x_m^V \neq x_m^N$.

Proposition 5. *In any election, it is impossible for a unanimous decision to take place.*

Sophisticated and unsophisticated agents never agree. The reason for this is that they always have conflicting preferences as to which planner is optimal. For example, when the condition in (18) holds, unsophisticated agents are satisfied perfectly with an imperfect planner, but sophisticated agents would rather elect a fully informed planner to drive x_m upward. However, if (18) is not satisfied, then $r_u = 0$ and the two groups' preferences diverge based on the two differences, $\bar{x} - x_s$ and $\bar{x} - x_u$. If these differences are large, sophisticated agents prefer the uninformed planner because they gain from their exaggerated report; unsophisticated agents would rather have an informed planner to minimize the cost of A_s's exaggeration. If these differences are small, the opposite results hold.

Going forward, we analyze the election results given that there exists a supermajority of one type of agent in the population. Specifically, we consider either that $\lambda_u \geq 2/3$ or that $\lambda_s \geq 2/3$. This is for mathematical convenience, but supermajority voting requirements are commonplace in U.S. legislative procedures and other political arenas.

Proposition 6. *When a supermajority of $\frac{2}{3}$ exists, a savvy, uninformed social planner never gets elected. When $\lambda_u \geq 2/3$, a naive social planner always gets elected. When $\lambda_s \geq 2/3$, the planner with the higher x_m gets elected, that is $\max(x_m^P, x_m^N)$. If $\lambda_s \geq 2/3$ and $\bar{x} \geq \frac{4}{3}x_s$, a naive social planner always gets elected.*

According to Proposition 6, the least qualified social planner often gets elected by whomever has a supermajority. Per Corollary 3.1, if $\lambda_u \geq 2/3$, A_u always gets his way: $x_m = x_u^*$ is always chosen. If $\lambda_s \geq 2/3$ and $\bar{x} \geq \frac{4}{3}x_s$, the sophisticated agents again elect the least qualified social planner. This implies that if the potential extent of the market ($\bar{x}$) is sufficiently high compared to the actual needs of the sophisticated agents (x_s), the sophisticated advocate is able to use misreporting to his advantage to better satisfy the needs of the sophisticated agents. It is also important to note that $\bar{x} \geq \frac{4}{3}x_s$ is a sufficient condition, but is not necessary for the sophisticated agents to elect the least qualified planner. That is, there are other parameters for which a naive planner gets elected when $\lambda_s \geq 2/3$. It is only when $\bar{x}$ is close to x_s that a perfectly informed planner gets elected. This occurs because the ability of A_s to misreport is lower than that for A_u. As such, the sophisticated agents elect someone who is knowledgeable.

This analysis has several qualitative welfare consequences. Proposition 6 tells us that even if we have the ability to regulate markets, this may not be desired by market participants. Indeed, the least qualified person gets elected in many cases, and due to lobbying behavior, x_m is not set at the first best level set in Section 2. This implies that proponents of product regulation need to

take into account how qualified the leaders are who implement such policies, and incentives within the system to preclude knowledgeable people from attaining such roles.

4 Regulation and Consumer Support

The heart of the debate over regulation in financial markets is whether the social planner should limit the scope of the market to protect people who are less sophisticated versus educating them to help them protect themselves. Such education may take the form of improved literacy training, timely decision support, access to intermediaries or other resources that provide guidance, or screening mechanisms.

In our model, the key question of interest is what should a social planner do when they identify that the market is operating away from the optimum. Do they improve access to information, require standardization of products (i.e., force simplicity), or both? To address this, we need to enhance the model to allow unsophisticated agents to access information about their choice. That is, while the fractions λ_u and λ_s are still exogenously given, we need to allow unsophisticated agents to learn. Once that channel is present, the social planner can potentially intervene in two ways: via x_m and via learning.

Suppose that the social planner can exert effort to educate α fraction of the unsophisticated agents. This effort is costly and the social planner incurs $\frac{1}{2}k\alpha^2$ for some $k > 0$. If an unsophisticated agent becomes educated and acquires information, they are essentially the same as a sophisticated consumer. Going forward, we assume that types for all agents are uniformly distributed over $[0, x_p]$, where $x_p = x_s = x_u$.

Once the social planner chooses α, the expected aggregate loss for unsophisticated agents is

$$L_u \equiv \lambda_u \Big\{ (1-\alpha) E[L(\tilde{t}_u, x|x_m)] + \alpha E[L(x|x_m, \tilde{t}_u)] \Big\}. \tag{25}$$

The educated fraction, α, identify the products closest to their types, which precludes them from making mistakes. As a result, their preferences mirror those of sophisticated agents in that they want markets to be complete. Given such assistance, we can now compute the aggregate loss to all agents in the market and education costs as

$$L(x_m, \lambda_u, \lambda_s, \alpha) = L_u + \lambda_s E[L(\tilde{t}_s, x|x_m)] + \frac{1}{2}k\alpha^2. \tag{26}$$

This can be written as

$$L(x_m, \lambda_u, \lambda_s, \alpha) = \lambda_u \Big\{ (1-\alpha) \Big[\frac{x_m^2}{3x_p} + \frac{x_p - x_m}{2} \Big] + \alpha \Big[\frac{x_p^2 - 2x_m x_p + x_m^2}{2x_p} \Big] \Big\} + \lambda_s \Big[\frac{x_p^2 - 2x_m x_p + x_m^2}{2x_p} \Big] + \frac{1}{2}k\alpha^2. \tag{27}$$

The next proposition calculates the marginal effects of changing x_m and altering α.

Proposition 7. *The marginal effect on the aggregate loss function from increasing x_m is*

$$\mathcal{L}_{x_m} \equiv \lambda_u(1-\alpha)\Big[\frac{2x_m}{3x_p} - \frac{1}{2}\Big] + (\lambda_s + \alpha\lambda_u)\Big[\frac{x_m - x_p}{x_p}\Big]. \tag{28}$$

The marginal effect on the aggregate loss function from educating is

$$\mathcal{L}_{\alpha} \equiv \lambda_u\Big[\frac{x_m^2 - 3x_m x_p}{6x_p}\Big] + k\alpha. \tag{29}$$

Education and standardization in the product market are strict substitutes, i.e., $\frac{\partial^2[L(x_m,\lambda_u,\lambda_s,\alpha)]}{\partial x_m \partial \alpha} < 0$.

By inspection of Equation 28 it is clear that the expression is negative for $x_m < \frac{3}{4}x_p$, meaning that increasing x_m strictly enhances welfare (decreases expected losses). Conversely, the first term, which is the effect of increasing x_m on uneducated and unsophisticated agents, is strictly positive for $x_m > \frac{3}{4}x_p$. This is consistent with our base model where unsophisticated agents prefer simpler markets when x_m exceeds three quarters of their aggregate needs. The second term however, which represents the marginal effect on both sophisticated agents *and* the educated unsophisticated fraction, is negative for all $x_m < x_p$. This follows from their ability to perfectly identify optimal products, i.e. they only incur losses when underserved. This tension, between uninformed and informed agents' preferences, suggests that a planner's optimal choice of x_m falls in the interval $\Big[\frac{3}{4}x_p, x_p\Big]$. In fact, because a fraction $\alpha\lambda_u$ essentially become sophisticated, a planner's optimal choice of x_m is likely to be higher with the ability to educate than without.

Costly education increases aggregate welfare when Equation 29 is negative. This arises when the first term, which is the expected reduction in losses from providing unsophisticated individuals full information, offsets the marginal cost of educating. As we show in the next corollary, this is likely to be the case for small values of k and when the fraction of unsophisticated agents is large.

The concluding finding of Proposition 7 is of central importance to determining optimal regulation because it tells us about the interplay between decreasing x_m and increasing α. The negative sign on the cross-derivative of Equation 27 implies that clarity and simplicity are strict substitutes. That is, when the market is not welfare optimal, if the social planner chooses one type of intervention, it makes the value of the other decline. For example, if the social planner subsidizes information acquisition, i.e. *clarifying*, the benefit to limiting the scope of products in the market drops. Likewise, if the social planner enforces simplicity, the benefit of increasing access to information decreases. This relationship holds for all parameter choices, the only differences are in the magnitude.

Corollary 7.1. *In equilibrium, the optimal level of education* α^*

(i) is increasing in λ_u*,* x_p*, and* x_m

(ii) is decreasing in k*.*

Corollary 7.1 contains an intuitive message for planners considering the extent to educate. First, when the fraction of unsophisticated agents is large, educating becomes more efficient. A fraction $\alpha\lambda_u$ of the entire population reaps the benefit of education. Because α is independent of how many unsophisticated agents participate, education is going to have the largest impact when λ_u is large. Additionally, when the cost of educating is low, a planner finds it advantageous to provide more learning. In fact, as k approaches zero, all agents in the economy are provided with their type and x_m approaches x_p. Finally, as the extent of the peoples' needs increases or as the market is more complete, the optimal α^* increases ceteris paribus.

5 Concluding Remarks

The model we have explored provides an innovative framework to explore the tension that exists in offering products to agents with heterogeneous levels of sophistication. There is a natural inclination to think markets should be complete so that participants are free to make choices that best fit their needs. Skeptics, on the other hand, think such a paradigm is too idealistic. They believe that agents are prone to make mistakes and that offering too many products introduces room for error. We have characterized this friction by modeling a market where perfect and imperfect agents jointly participate. Our main contribution, in this parsimonious model, is the characterization of the optimal level of collective complexity with respect to each group's size and needs.

We also have explored the relationship between simplicity and clarity. When a social planner has both the abilities to standardize and to educate the market, the two are strict substitutes from a welfare standpoint. Policy makers should be aware of this relationship when implementing regulation. That is, policies aimed at accomplishing both may be effectively impotent since the two are *not* complementary.

Finally, we contrasted the effects of lobbying and voting behavior on product regulation when the social planner is imperfectly informed. An immediate implication of our analysis is that it is always a weakly dominant strategy for sophisticated agents to claim ex ante a need for the most advanced products, even if they will not utilize them ex post. To counter this claim, unsophisticated agents understate their needs and can achieve their bliss point with a simple two-thirds majority.

Uninformed planners, whether naive or rational, will only achieve second best regulation. As we show, the problem that arises is that it is often the case that the least qualified, most naive official that gets elected, which endogenously affects the quality of regulation. This implies that incentive problems may erode well intended regulation.

In the end, our analysis provides a new dimension to consider in the debate of financial product regulation. Policy makers should carefully consider the tradeoff between satisfying agents' needs and introducing the possibility of blunders. Furthermore, a robust policy is one that balances decision support and simplicity.

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Appendix A

Proof of Lemma 1

First, consider that $x_m \geq x_s$. The loss for a given sophisticated consumer with type $\tilde{t_s}$ is given by

$$L(x|\tilde{t_s}, x_m) = |x - \tilde{t_s}|. \tag{A1}$$

Because the consumer knows their type perfectly and their type is available in the set of financial products $[0, x_m]$, the agent will choose $x = \tilde{t_s}$. Thus, $|\tilde{t_s} - \tilde{t_s}| = 0$ for all types of sophisticated agents. Therefore, $\mathcal{L}_s = 0$.

Now consider that $x_m < x_s$. By the same reasoning, for any sophisticated agent with type $\tilde{t_s} \in [0, x_m]$, their loss is zero. However, the sophisticated agents with type $\tilde{t_s} \in [x_m, x_s]$ will choose $x = x_m$ to minimize their losses. Since sophisticated consumers are distributed uniformly, the aggregate expected loss in this population is given by

$$\begin{aligned}
\mathcal{L}_s &= \int_0^{x_m} 0 \frac{d\tilde{t_s}}{x_s} + \int_{x_m}^{x_s} |x_m - \tilde{t_s}| \frac{d\tilde{t_s}}{x_s} \\
&= \int_{x_m}^{x_s} (\tilde{t_s} - x_m) \frac{d\tilde{t_s}}{x_s} \\
&= \frac{\tilde{t_s}^2}{2x_s} - \frac{x_m \tilde{t_s}}{x_s} \Bigg|_{x_m}^{x_s} \\
&= \frac{x_s^2 - 2x_m x_s + {x_m}^2}{2x_s}.
\end{aligned}$$

The first-order derivative of the expected loss with respect to x_m is

$$\frac{\partial}{\partial x_m} E[L(\tilde{t_s}, x|x_m)] = \frac{-x_s + x_m}{x_s},$$

The which is strictly negative because $x_m < x_s$ by assumption.

The second-order derivative of the expected loss with respect to x_m is given by,

$$\frac{\partial^2}{\partial {x_m}^2} E[L(\tilde{t_s}, x|x_m)] = \frac{1}{x_s} > 0,$$

which tells us that the loss function is convex in x_m. ■

Proof of Lemma 2

Suppose first that $x_m > x_u$. Using the expected loss for a given unsophisticated consumer in Equation 5, we can compute the expected loss for the group as a whole. The expected loss is,

$$\begin{aligned}
E[L(\tilde{t}_u, x|x_m)] &= \int_0^{x_u} \left(\frac{(x_m - \tilde{t}_u)^3}{2x_m|x_m - \tilde{t}_u|} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_u} \left(\frac{(x_m - \tilde{t}_u)^3}{2x_m(x_m - \tilde{t}_u)} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_u} \left(\frac{(x_m - \tilde{t}_u)^2}{2x_m} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_u} \left(\frac{{x_m}^2 - 2x_m\tilde{t}_u + 2\tilde{t_u}^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \left(\frac{{x_m}^2\tilde{t}_u - x_m\tilde{t}_u^2 + (2/3)\tilde{t_u}^3}{2x_m x_u} \right) \Bigg|_0^{x_u} \\
&= \frac{{x_m}^2 x_u - x_m x_u^2 + (2/3)x_u^3}{2x_m x_u} \\
&= \frac{{x_m}^2 - x_m x_u + (2/3)x_u^2}{2x_m}.
\end{aligned}$$

For convenience we note the following computation that was used in the last calculation:

$$\int |(ax+b)^n| dx = \frac{(ax+b)^{n+2}}{a(n+1)|ax+b|} + C,$$

where n is odd and $n \neq -1$.

The first-order derivative of the expected loss with respect to x_m is given by,

$$\frac{\partial}{\partial x_m} E[L(\tilde{t}_u, x|x_m)] = \frac{1}{2} - \frac{x_u^2}{3{x_m}^2},$$

which is strictly positive because $x_m > x_u$ by assumption. Additionally, the loss function is convex. By second-order conditions we obtain,

$$\frac{\partial^2}{\partial {x_m}^2} E[L(\tilde{t}_u, x|x_m)] = \frac{2x_u^2}{3{x_m}^3} > 0.$$

Note that we did not solve for the optimal x_m for unsophisticated consumers using this first-order equation because our assumption that $x_m \geq x_u$ allowed us a simplifying step that $|x_m - \tilde{t}_u| = (x_m - \tilde{t}_u)$. We address the optimal x_m shortly. It is useful to note here, that had we solved for the optimum using this first-order condition we would have obtained,

$$x_m^* \equiv x_u \sqrt{\frac{2}{3}}. \tag{A2}$$

In this case, $x_m^* < x_u$, violating our assumption that $x_m \geq x_u$.

Now, we consider the case when now we look at $x_m < x_u$. Using the expected loss for a given unsophisticated consumer in Equation 5, we can compute the expected loss for the group as a whole. The expected loss is,

$$\begin{aligned}
E[L(\tilde{t}_u, x|x_m)] &= \int_0^{x_u} \left(\frac{(x_m - \tilde{t}_u)^3}{2x_m|x_m - \tilde{t}_u|} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_m} \left(\frac{(x_m - \tilde{t}_u)^3}{2x_m|x_m - \tilde{t}_u|} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} + \int_{x_m}^{x_u} \left(\frac{(x_m - \tilde{t}_u)^3}{2x_m|x_m - \tilde{t}_u|} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_m} \left(\frac{(x_m - \tilde{t}_u)^3}{2x_m(x_m - \tilde{t}_u)} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} + \int_{x_m}^{x_u} \left(\frac{(x_m - \tilde{t}_u)^3}{2x_m(\tilde{t}_u - x_m)} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_m} \left(\frac{(x_m - \tilde{t}_u)^2}{2x_m} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} + \int_{x_m}^{x_u} \left(\frac{-(x_m - \tilde{t}_u)^2}{2x_m} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_m} \left(\frac{{x_m}^2 - 2x_m\tilde{t}_u + 2\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} + \int_{x_m}^{x_u} \left(\frac{-{x_m}^2 + 2\tilde{t}_u x_m}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \left(\frac{{x_m}^2\tilde{t}_u - x_m\tilde{t}_u^2 + (2/3)\tilde{t}_u^3}{2x_m x_u} \right) \Bigg|_0^{x_m} + \left(\frac{-{x_m}^2\tilde{t}_u + \tilde{t}_u^2 x_m}{2x_m x_u} \right) \Bigg|_{x_m}^{x_u} \\
&= \frac{{x_m}^3 - {x_m}^3 + (2/3){x_m}^3}{2x_m x_u} - 0 + \frac{-{x_m}^2 x_u + x_m x_u^2}{2x_m x_u} - \frac{-{x_m}^3 + {x_m}^3}{2x_m x_u} \\
&= \frac{{x_m}^2}{3x_u} + \frac{-x_m + x_u}{2}.
\end{aligned}$$

First-order conditions with respect to x_m yield the ideal level of market sophistication for unsophisticated consumers,

$$\begin{aligned}
0 &= \frac{\partial}{\partial x_m} E[L(\tilde{t}_u, x|x_m)] \\
&= \frac{2x_m}{3x_u} - \frac{1}{2}.
\end{aligned}$$

Additionally, the loss function is convex. The second-order condition is given as,

$$\frac{\partial^2}{\partial {x_m}^2} E[L(\tilde{t}_u, x|x_m)] = \frac{2}{3x_u} > 0.$$

Therefore, the unsophisticated consumers' losses are minimized when

$$x_u^* \equiv \frac{3}{4} x_u < x_u. \tag{A3}$$

■

Lemma A1. *The function $L(x_m, \lambda_u, \lambda_s)$ is continuously differentiable at x_u.*

Proof: Consider the following:

$$
\begin{aligned}
E[L(\tilde{t}_u, x|x_m \leq x_u)] &= \frac{{x_m}^2}{3x_u} + \frac{-x_m + x_u}{2}\bigg|_{x_m = x_u} \\
&= \frac{x_u}{3} \\
E[L(\tilde{t}_u, x|x_m > x_u)] &= \frac{{x_m}^2 - x_m x_u + (2/3)x_u^2}{2x_m}\bigg|_{x_m = x_u} \\
&= \frac{x_u}{3} \\
\frac{\partial}{\partial x_m} E[L(\tilde{t}_u, x|x_m \leq x_u)] &= \frac{2x_m}{3x_u} - \frac{1}{2}\bigg|_{x_m = x_u} \\
&= \frac{1}{6} \\
\frac{\partial}{\partial x_m} E[L(\tilde{t}_u, x|x_m > x_u)] &= \frac{1}{2} - \frac{x_u^2}{3{x_m}^2}\bigg|_{x_m = x_u} \\
&= \frac{1}{6} \\
\frac{\partial^2}{\partial {x_m}^2} E[L(\tilde{t}_u, x|x_m \leq x_u)] &= \frac{2}{3x_u}\bigg|_{x_m = x_u} \\
&= \frac{2}{3x_u} \\
\frac{\partial^2}{\partial {x_m}^2} E[L(\tilde{t}_u, x|x_m > x_u)] &= \frac{2x_u^2}{3{x_m}^3}\bigg|_{x_m = x_u} \\
&= \frac{2}{3x_u}
\end{aligned}
$$

■

Proof of Proposition 1

Both of the loss functions of the sophisticated and unsophisticated agents are strictly convex by Lemmas 1 and 2. Since the sum of two convex functions is also convex, $L(x_m, \lambda_u, \lambda_s)$ is strictly convex. By Lemma A1, $L(x_m, \lambda_u, \lambda_s)$ is continuously differentiable. Finally, since $L(x_m, \lambda_u, \lambda_s)$ evaluated on the compact set $[0, x_s]$, we know that a unique global minimum exists.

Now, we proceed to the claim in (10). By Lemmas 1 and 2, the loss function for unsophisticated agents is strictly increasing for any $x_m > x_u^*$ and the loss function for sophisticated consumers is strictly decreasing for any $x_m < x_s$. If the marginal benefit of increasing x_m to sophisticated consumers is greater than the marginal cost to the unsophisticated, the first-order condition of the

aggregate loss function will be negative. Evaluating the first-order condition of the aggregate loss function at $x_m = x_u$ yields,

$$\frac{\partial}{\partial x_m} L(x_u, \lambda_u, \lambda_s) = \frac{\lambda_u}{6} + \lambda_s \left(-1 + \frac{x_u}{x_s} \right),$$

which is negative if and only if $\frac{\lambda_u}{\lambda_s} < 6\left(1 - \frac{x_u}{x_s}\right)$, implying that the $x_m^* > x_u$.

Clearly, since $\frac{\partial \mathcal{L}_u}{\partial x_m} > 0$ for all $x_m > x_u^*$ and $\frac{\partial \mathcal{L}_s}{\partial x_m} < 0$ for all $x_m < x_s^*$, the solution to the social planner's problem lies in the interval $[x_u^*, x_s^*]$. Moreover, it is straightforward to show that there exists an internal solution. The derivative $\frac{\partial \mathcal{L}_s}{\partial x_m}$ is zero at x_s, whereas $\frac{\partial \mathcal{L}_u}{\partial x_m} = \frac{2x_s}{3x_u} - \frac{1}{2}$. Likewise, The derivative $\frac{\partial \mathcal{L}_u}{\partial x_m}$ is zero at x_u^*, whereas $\frac{\partial \mathcal{L}_s}{\partial x_m} = \frac{x_u}{x_s} - 1 < 0$.

We now proceed to write an expression for x_m^* so that we may proceed with comparative statics exercises. Using the condition in (10), we consider the two possible cases for optimums. First, suppose that $\frac{\lambda_u}{\lambda_s} \geq 6\left(1 - \frac{x_u}{x_s}\right)$. First-order conditions with respect to the aggregate loss function yield

$$0 = \frac{\partial}{\partial x_m} L(x_m, \lambda_u, \lambda_s | x_m \leq x_u) \tag{A4}$$

$$= \lambda_u \left[\frac{2x_m}{3x_u} - \frac{1}{2} \right] + \lambda_s \left[-1 + \frac{x_m}{x_s} \right]. \tag{A5}$$

Recall that the ideal point for sophisticated consumers is $x_s^* = x_s$, and the ideal point for unsophisticated consumers, given that $x_m \leq x_u$, is $x_u^* = \frac{3}{4} x_u$. Substituting in the ideal points yields

$$0 = \lambda_u \left[\frac{2x_m}{4x_u^*} - \frac{1}{2} \right] + \lambda_s \left[-1 + \frac{x_m}{x_s^*} \right]. \tag{A6}$$

Solving for x_m yields

$$x_m^* = \frac{x_u^* x_s^* (\lambda_u + 2\lambda_s)}{\lambda_u x_s^* + 2\lambda_s x_u^*}. \tag{A7}$$

Now, consider that $\frac{\lambda_u}{\lambda_s} < 6\left(1 - \frac{x_u}{x_s}\right)$. We define the function $g(x_m)$ to be the first-order conditions with respect to the aggregate loss function, i.e.

$$g(x_m) \equiv \frac{\partial}{\partial x_m} L(x_m, \lambda_u, \lambda_s | x_m > x_u)$$

$$= \lambda_u \left[\frac{1}{2} - \frac{x_u^2}{3{x_m}^2} \right] + \lambda_s \left[-1 + \frac{x_m}{x_s} \right]. \tag{A8}$$

Since, solving for an explicit solution to $g(x_m) = 0$ is analytically intractable, we leave x_m^* as implicitly defined by $g(x_m^*) = 0$, and use the implicit function theorem to derive comparative statics.

Because of the piecewise construction of the aggregate loss function, we need to determine the four comparative statics for both the case when $x_m^* \leq x_u$ and when $x_m^* > x_u$. First we consider the former.

The optimal level of sophistication is given by Equation A7. We start by considering the comparative static of x_m^* with respect to λ_u and note that $\lambda_u + \lambda_s = 1$,

$$\begin{aligned}
\frac{\partial x_m^*}{\partial \lambda_u} &= \frac{\partial}{\partial \lambda_u} \frac{x_u^* x_s^* (\lambda_u + 2\lambda_s)}{\lambda_u x_s^* + 2\lambda_s x_u^*} \\
&= \frac{-x_u^* x_s^* (\lambda_u x_s^* + 2\lambda_s x_u^*) - x_u^* x_s^* (\lambda_u + 2\lambda_s)(x_s^* - 2x_u^*)}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{-\lambda_u x_u^* x_s^{*2} - 2\lambda_s x_u^{*2} x_s^* - \lambda_u x_u^* x_s^{*2} + 2\lambda_u x_u^{*2} x_s^* - 2\lambda_s x_u^* x_s^{*2} + 4\lambda_s x_u^{*2} x_s^*}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{-2\lambda_u x_u^* x_s^{*2} + 2\lambda_s x_u^{*2} x_s^* + 2\lambda_u x_u^{*2} x_s^* - 2\lambda_s x_u^* x_s^{*2}}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{-2\lambda_u x_u^* x_s^{*2} + 2(1-\lambda_u) x_u^{*2} x_s^* + 2\lambda_u x_u^{*2} x_s^* - 2(1-\lambda_u) x_u^* x_s^{*2}}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{2x_u^{*2} x_s^* - 2x_u^* x_s^{*2}}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{2x_u^* x_s^* (x_u^* - x_s^*)}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2}
\end{aligned}$$

and because $x_u^* < x_s$,

$$< 0.$$

The comparative static of x_m^* with respect to λ_s,

$$\frac{\partial x_m^*}{\partial \lambda_s} = \frac{\partial x_m^*}{\partial \lambda_u} \frac{\partial \lambda_u}{\partial \lambda_s},$$

and because $\frac{\partial \lambda_u}{\partial \lambda_s} = -1$,

$$> 0.$$

Now we consider the comparative static of x_m^* with respect to x_u. For analytic ease, we substitute out the ideal points for both sophisticated and unsophisticated consumers, i.e. $x_s^* = x_s$, and given that $x_m \leq x_u$, $x_u^* = \frac{3}{4}x_u$.

$$\begin{aligned}
\frac{\partial x_m^*}{\partial x_u} &= \frac{\partial}{\partial x_u}\frac{x_u x_s(3\lambda_u + 6\lambda_s)}{4\lambda_u x_s + 6\lambda_s x_u} \\
&= \frac{(3\lambda_u x_s + 6\lambda_s x_s)(4\lambda_u x_s + 6\lambda_s x_u) - 6\lambda_s(3\lambda_u x_u x_s + 6\lambda_s x_u x_s)}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&= \frac{12\lambda_u^2 x_s^2 + 18\lambda_u\lambda_s x_u x_s + 24\lambda_u\lambda_s x_s^2 + 36\lambda_s^2 x_u x_s - 18\lambda_u\lambda_s x_u x_s - 36\lambda_s^2 x_u x_s}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&= \frac{12\lambda_u^2 x_s^2 + 24\lambda_u\lambda_s x_s^2}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&> 0.
\end{aligned}$$

And lastly with the comparative static of x_m^* with respect to x_s,

$$\begin{aligned}
\frac{\partial x_m^*}{\partial x_s} &= \frac{\partial}{\partial x_s}\frac{x_u x_s(3\lambda_u + 6\lambda_s)}{4\lambda_u x_s + 6\lambda_s x_u} \\
&= \frac{(3\lambda_u x_u + 6\lambda_s x_u)(4\lambda_u x_s + 6\lambda_s x_u) - 4\lambda_u(3\lambda_u x_u x_s + 6\lambda_s x_u x_s)}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&= \frac{12\lambda_u^2 x_u x_s + 18\lambda_u\lambda_s x_u^2 + 24\lambda_u\lambda_s x_u x_s + 36\lambda_s^2 x_u^2 - 12\lambda_u^2 x_u x_s - 24\lambda_u\lambda_s x_u x_s}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&= \frac{18\lambda_u\lambda_s x_u^2 + 36\lambda_s^2 x_u^2}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&> 0.
\end{aligned}$$

Now we consider when $x_m^* > x_u$. Because we did not solve for an explicit solution for x_m^*, we utilize the Implicit Function Theorem with our characteristic equation, Equation A8, that implicitly defines x_m^*, i.e. $g(x_m^*) = 0$. The Implicit Function Theorem tells us how the optimal level of sophistication changes with the parameter values. For each parameter $\theta \in \{\lambda_u, \lambda_s, x_u, x_s\}$, the IFT gives

$$\frac{\partial x_m^*}{\partial \theta} \equiv \frac{-\left.\frac{\partial g(x_m)}{\partial \theta}\right|_{x_m = x_m^*}}{\left.\frac{\partial g(x_m)}{\partial x_m}\right|_{x_m = x_m^*}} \tag{A9}$$

We begin by showing that $\frac{\partial g(x_m)}{\partial x_m} > 0$,

$$\begin{aligned}
\frac{\partial g(x_m)}{\partial x_m} &= \lambda_u \frac{2x_u^2}{3x_m^3} + \frac{\lambda_s}{x_s} \\
&> 0.
\end{aligned}$$

Differentiating with respect to each of the parameters and recalling that $\lambda_u + \lambda_s = 1$ yields

$$\begin{aligned}
\frac{\partial g(x_m)}{\partial \lambda_u} &= \underbrace{\frac{1}{2} - \frac{x_u^2}{3x_m^2}}_{>0 \text{ since } x_u < x_m} + \underbrace{1 - \frac{x_m}{x_s}}_{\geq 0 \text{ since } x_m \leq x_s} && > 0 \\
\frac{\partial g(x_m)}{\partial \lambda_s} &= \underbrace{-\frac{1}{2} + \frac{x_u^2}{3x_m^2}}_{<0 \text{ since } x_u < x_m} + \underbrace{-1 + \frac{x_m}{x_s}}_{\leq 0 \text{ since } x_m \leq x_s} && < 0 \\
\frac{\partial g(x_m)}{\partial x_u} &= \frac{-2\lambda_u x_u^2}{3x_m^2} && < 0 \\
\frac{\partial g(x_m)}{\partial x_s} &= \frac{-\lambda_s x_m}{x_s^2} && < 0
\end{aligned}$$

■

Proof of Proposition 2

To solve for the level of market sophistication that minimizes the weighted sum of aggregate loss and agent dispersion, we begin by showing that $|L_u - L_s| = (L_u - L_s)$ when $x_u = x_s = x_p$.

$$\begin{aligned}
|L_u - L_s| &= \left| \left[\frac{x_m^2}{3x_p} + \frac{-x_m + x_p}{2} \right] - \left[\frac{x_p^2 - 2x_m x_p + x_m^2}{2x_p} \right] \right| \\
&= \left| \left[\frac{2x_m^2}{6x_p} + \frac{-3x_m x_p + 3x_p^2}{6x_p} \right] - \left[\frac{3x_p^2 - 6x_m x_p + 3x_m^2}{6x_p} \right] \right| \\
&= \left| \frac{2x_m^2 - 3x_m x_p + 3x_p^2 - 3x_p^2 + 6x_m x_p - 3x_m^2}{6x_p} \right| \\
&= \left| \frac{3x_m x_p - x_m^2}{6x_p} \right| \\
&= \left| \frac{x_m(3x_p - x_m)}{6x_p} \right| \\
&= \frac{x_m(3x_p - x_m)}{6x_p},
\end{aligned}$$

which is always positive since $x_m \in [0, x_p]$.

To find an internal solution that minimizes the aggregate loss and the disparity between agents, we need Equation 16 to be convex. To determine the conditions under which the function is convex in x_m, we take the second derivative of Equation 16,

$$
\begin{aligned}
\frac{\partial^2 W}{\partial x_m^2} &= \frac{\partial^2}{\partial x_m^2}\left[\kappa L(x_m,\lambda_u,\lambda_s) + (1-\kappa)D(x_m,\lambda_u,\lambda_s)\right] \\
&= \kappa\left[\lambda_u \frac{2}{3x_p} + \lambda_s \frac{1}{x_p}\right] + (1-\kappa)\left[\frac{2}{3x_p} - \frac{1}{x_p}\right] \\
&= \kappa\left[\frac{2\lambda_u + 3(1-\lambda_u)}{3x_p}\right] - (1-\kappa)\frac{1}{3x_p} \\
&= \frac{\kappa(3-\lambda_u) - (1-\kappa)}{3x_p}, \\
&= \frac{\kappa(4-\lambda_u) - 1}{3x_p},
\end{aligned}
$$

which is postive so long as the numerator is positve. Therefore, the welfare function is convex so long as,

$$
\kappa \geq \frac{1}{4-\lambda_u}. \tag{A10}
$$

When κ is smaller than the condition stated in (A10), the welfare function is strictly concave in x_m over $[0, x_p]$. This means that the optimal level of market sophistication is a corner solution; either $x_m^* = 0$ or $x_m^* = x_p$. We now show that losses are monotonically increasing in x_m, indicating that $x_m^* = 0$ when $\kappa < \frac{1}{4-\lambda_u}$. The first derivative of the welfare function with respect to x_m is

$$
\begin{aligned}
\frac{\partial W}{\partial x_m} &= \frac{\partial}{\partial x_m}\left[\kappa L(x_m,\lambda_u,\lambda_s) + (1-\kappa)D(x_m,\lambda_u,\lambda_s)\right] \\
&= \kappa\left[\lambda_u\left(\frac{2x_m}{3x_p} - \frac{1}{2}\right) + \lambda_s\left(\frac{-x_u + x_m}{x_u}\right)\right] + (1-\kappa)\left[\left(\frac{2x_m}{3x_p} - \frac{1}{2}\right) - \left(\frac{-x_u + x_m}{x_u}\right)\right] \\
&= \kappa\left[\lambda_u\left(\frac{4x_m - 3x_p}{6x_p}\right) + \lambda_s\left(\frac{-6x_p + 6x_m}{6x_p}\right)\right] + (1-\kappa)\left[\frac{4x_m - 3x_p + 6x_p - 6x_m}{6x_p}\right] \\
&= \frac{\kappa[\lambda_u(4x_m - 3x_p + 6x_p - 6x_m) - 6x_p + 6x_m] + (1-\kappa)[3x_p - 2x_m]}{6x_p} \\
&= \frac{\kappa[\lambda_u(3x_p - 2x_m) - 6x_p + 6x_m] + (1-\kappa)[3x_p - 2x_m]}{6x_p} \\
&= \frac{\kappa[\lambda_u(3x_p - 2x_m) - 9x_p + 8x_m] + 3x_p - 2x_m}{6x_p}.
\end{aligned} \tag{A11}
$$

It follows that $\frac{\partial W}{\partial x_m} > 0$ if

$$
\kappa[\lambda_u(3x_p - 2x_m) - 9x_p + 8x_m] + 3x_p - 2x_m > 0
$$

or

$$
x_m \leq \frac{3}{2}\frac{1 - k(3-\lambda_u)}{1 - k(4-\lambda_u)} x_p, \tag{A12}
$$

which is always the case when $\kappa < \frac{1}{4-\lambda_u}$. Therefore, $x_m^* = 0$ when $\kappa < \frac{1}{4-\lambda_u}$.

When $\kappa \geq \frac{1}{4-\lambda_u}$, we can utilize the first-order condition of Equation 16 with respect to x_m since the welfare loss function is convex:

$$
\begin{aligned}
0 &= \frac{\partial W}{\partial x_m} \\
&= \frac{\partial}{\partial x_m}\left[\kappa L(x_m, \lambda_u, \lambda_s) + (1-\kappa) D(x_m, \lambda_u, \lambda_s)\right] \\
&= \kappa\left[\lambda_u\left(\frac{2x_m}{3x_p} - \frac{1}{2}\right) + \lambda_s\left(\frac{-x_u + x_m}{x_u}\right)\right] + (1-\kappa)\left[\left(\frac{2x_m}{3x_p} - \frac{1}{2}\right) - \left(\frac{-x_u + x_m}{x_u}\right)\right] \\
&= \kappa\left[\lambda_u\left(\frac{4x_m - 3x_p}{6x_p}\right) + \lambda_s\left(\frac{-6x_p + 6x_m}{6x_p}\right)\right] + (1-\kappa)\left[\frac{4x_m - 3x_p + 6x_p - 6x_m}{6x_p}\right] \\
&= \frac{\kappa[\lambda_u(4x_m - 3x_p + 6x_p - 6x_m) - 6x_p + 6x_m] + (1-\kappa)[3x_p - 2x_m]}{6x_p} \\
&= \frac{\kappa[\lambda_u(3x_p - 2x_m) - 6x_p + 6x_m] + (1-\kappa)[3x_p - 2x_m]}{6x_p} \\
&= \kappa[\lambda_u(3x_p - 2x_m) - 9x_p + 8x_m] + 3x_p - 2x_m \\
&= 3\kappa\lambda_u x_p - 2\kappa\lambda_u x_m - 9\kappa x_p + 8\kappa x_m + 3x_p - 2x_m \\
&= -3x_p(3\kappa - \kappa\lambda_u - 1) + x_m(-2\kappa\lambda_u + 8\kappa - 2) \\
x_m^* &= \frac{3}{2}\frac{1 - k(3-\lambda_u)}{1 - k(4-\lambda_u)} x_p
\end{aligned}
\tag{A13}
$$

The comparative statics of x_m^* with respect to κ and λ_u are determined by simple differentiation. ■

Lemma A2. *The function $x_m^*(x_u, x_s, \lambda_u, \lambda_s)$ is continuously differentiable at x_u.*

Proof: The proof follows directly from Lemma A1 because x_m^* is determined by the first-order condition of the aggregate loss function with respect to x_m. We know that the first and second derivatives of the aggregate loss function are continuous, therefore $x_m^*(x_u, x_s, \lambda_u, \lambda_s)$ is continuous. ■

Proof of Proposition 3

We begin the proof by showing that it is a weakly dominant strategy to choose $r_s = \overline{x}$. The aggregate loss to sophisticated agents, $\mathcal{L}_s$, is a function of the true upper bound x_s and the social planner's choice of x_m based on the reports r_s and r_u. The change in the aggregate loss to sophisticated agents with respect to r_s is given by,

$$\frac{\partial}{\partial r_s}\mathcal{L}_s(x_s, x_m(r_s, r_u)) = \frac{\partial \mathcal{L}_s}{\partial x_m}\frac{\partial x_m}{r_s}$$

From Lemma 1 we know that $\frac{\partial \mathcal{L}_s}{\partial x_m} \leq 0$. Furthermore, because the social planner takes A_s at their word, we know from Proposition 1 that $\frac{\partial x_m}{r_s} > 0$. Therefore, $\frac{\partial}{\partial r_s}\mathcal{L}_s(x_s, x_m(r_s, r_u)) \leq 0$ which yields the desired result that $r_s = \bar{x}$

Now we examine the strategy of A_u. By Lemma A2 and Proposition 1 we know that $x_m(r_u, r_s)$ is continuously differentiable and strictly increasing in r_u. Because $[0, \overline{x}]$ is compact, if $x_m(0, \overline{x}) \leq x_u^* \leq x_m(\overline{x}, \overline{x})$ we know by the Intermediate Value Theorem that there exists an $\hat{r}_u \in [0, \overline{x}]$ such that $x_u^* = x_m(\hat{r}_u, \overline{x})$. Furthermore, because x_m is strictly increasing in r_u, $\hat{r}_u$ is unique.

When $x_u^* \notin [x_m(0, \overline{x}), x_m(\overline{x}, \overline{x})]$ it must be the case that either $x_u^* < x_m(0, \overline{x})$ or $x_m(\overline{x}, \overline{x}) < x_u^*$. However, since $x_m(\bar{x}, \bar{x}) \in [\frac{3}{4}\bar{x}, \bar{x}]$ and $x_u^* \leq \frac{3}{4}\bar{x}$, then it cannot be that $x_m(\overline{x}, \overline{x}) < x_u^*$. We know from Lemma 2 that the losses for unsophisticated agents, $\mathcal{L}_u$, are increasing for all $x_m > x_u^*$. Therefore, when $x_u^* \notin [x_m(0, \overline{x}), x_m(\overline{x}, \overline{x})]$, it is a dominant strategy for A_u to report the lowest possible value for x_u, that is $r_u = 0$.

Consequently, the reporting strategy of A_u is segregated into two cases; those when he makes a positive report and those when he reports $r_u = 0$. We first consider the former, when there exists an $r_u > 0$ such that $x_u^* = x_m(r_u, \overline{x})$.

Because the naive planner takes r_u to be the true value of x_u, we know from Proposition 1 that the planner's choice of x_m is set according to a piecewise function. In fact, $x_m \leq r_u$ so long as $\frac{\lambda_u}{\lambda_s} > 6\left(1 - \frac{r_u}{\overline{x}}\right)$, and $x_m > r_u$ otherwise. (A7) provides the planner's rule for setting x_m when it falls below r_u. The advocate chooses r_u such that it results in the planner setting x_m equal to the unsophisticated bliss point,

$$\begin{aligned} x_u^* &= x_m(r_u, \overline{x}) \\ &= \frac{r_u r_s(3\lambda_u + 6\lambda_s)}{4\lambda_u r_s + 6\lambda_s r_u} \\ &= \frac{r_u \overline{x}(3\lambda_u + 6\lambda_s)}{4\lambda_u \overline{x} + 6\lambda_s r_u}. \end{aligned}$$

A rearrangement yields,

$$4\lambda_u \overline{x} x_u^* = r_u \overline{x}(3\lambda_u + 6\lambda_s) - 6\lambda_s r_u x_u^*.$$

Solving for r_u we obtain,

$$r_u = \frac{4\lambda_u \overline{x} x_u^*}{\overline{x}(3\lambda_u + 6\lambda_s) - 6\lambda_s x_u^*},$$

and recall that $x_u^* = (3/4)x_u$

$$r_u = x_u \left(\frac{3\lambda_u \overline{x}}{3\lambda_u \overline{x} + 6\lambda_s(\overline{x} - (3/4)x_u)} \right). \tag{A14}$$

Clearly from (A14), conditional on $x_m \leq r_u$ and $\overline{x}$ being finite, there always exists a report r_u such that $x_u^* = x_m(r_u, \overline{x})$. Thus, r_u cannot be 0 if $x_m \leq r_u$.

Consider now the planner's rule for $x_m > r_u$. Although we do not have a closed-form solution, the planner chooses x_m so that it satisfies (A8). Again, the advocate will choose r_u such that $x_u^* = x_m(r_u, \overline{x})$,

$$\begin{aligned} 0 &= \lambda_u \left[\frac{1}{2} - \frac{r_u^2}{3x_m^2} \right] + \lambda_s \left[-1 + \frac{x_m}{r_s} \right] \\ &= \lambda_u \left[\frac{1}{2} - \frac{r_u^2}{3x_u^{*2}} \right] + \lambda_s \left[-1 + \frac{x_u^*}{\overline{x}} \right]. \end{aligned}$$

A rearrangement yields,

$$\begin{aligned} \frac{\lambda_u r_u^2}{3x_u^{*2}} &= \frac{\lambda_u}{2} + \lambda_s \left[-1 + \frac{x_u^*}{\overline{x}} \right] \\ r_u^2 &= \frac{3x_u^{*2}}{\lambda_u} \left(\frac{\lambda_u}{2} + \lambda_s \left[-1 + \frac{x_u^*}{\overline{x}} \right] \right) \\ &= x_u^{*2} \left(\frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[-3 + \frac{3x_u^*}{\overline{x}} \right] \right) \\ r_u &= x_u^* \left(\frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[-3 + \frac{3x_u^*}{\overline{x}} \right] \right)^{1/2}, \end{aligned}$$

and recall that $x_u^* = (3/4)x_u$

$$r_u = x_u \frac{3}{4} \left(\frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[\frac{9x_u}{4\overline{x}} - 3 \right] \right)^{1/2}. \tag{A15}$$

Equation (A15) is real valued so long as $\frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[\frac{9x_u}{4\overline{x}} - 3 \right] \geq 0$, or

$$x_u \geq \overline{x} \left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right) \tag{A16}$$

The unsophisticated advocate's report is characterized by either (A14) or (A15), depending on whether the planner's choice of x_m is greater or less than r_u. The specific condition is pinned down by solving for $r_u = x_m$. Utilizing Equation A8 yields,

$$\begin{aligned} 0 &= \lambda_u \left[\frac{1}{2} - \frac{r_u^2}{3x_m^2}\right] + \lambda_s \left[-1 + \frac{x_m}{r_s}\right] \\ &= \lambda_u \left[\frac{1}{2} - \frac{1}{3}\right] + \lambda_s \left[-1 + \frac{r_u}{\overline{x}}\right] \\ &= \frac{\lambda_u}{6} + \lambda_s \left[-1 + \frac{x_u^*}{\overline{x}}\right]. \end{aligned}$$

A rearrangement of this expression yields,

$$x_u^* = \overline{x}\left(\frac{6\lambda_s - \lambda_u}{6\lambda_s}\right), \tag{A17}$$

or,

$$x_u = \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{9\lambda_s}\right). \tag{A18}$$

We have now fully characterized when A_u reports a value greater than zero. For $x_u \geq \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{9\lambda_s}\right)$, the advocate adheres to (A14) for his report. When $x_u \in \left[\overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right), \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{9\lambda_s}\right)\right]$ the advocate will choose r_u according to (A15). An advocate is unable to make a positive report that results in $x_m = x_u^*$ for values of $x_u < \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right)$. Instead, the advocate will make the smallest possible report, $r_u = 0$. Mathematically we define this reporting strategy, $r_u = \sigma(x_u)$, as

$$\sigma(x_u) = \begin{cases} 0 & \text{for } x_u < \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right) \\ x_u\left(\frac{3}{4}\sqrt{\frac{3}{2} + \frac{\lambda_s}{\lambda_u}\left[\frac{9x_u}{4\overline{x}} - 3\right]}\right) & \text{for } \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right) \leq x_u < \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{9\lambda_s}\right) \\ x_u\left(\frac{3\lambda_u\overline{x}}{3\lambda_u\overline{x} + 6\lambda_s(\overline{x} - (3/4)x_u)}\right) & \text{for } \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{9\lambda_s}\right) \leq x_u. \end{cases}$$

Therefore, the advocate will make a report greater than zero if, and only if, $x_u \geq \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right)$. Finally, when $r_u = 0$, we know that $x_m > r_u$. Substituting into (A8) yields

$$0 = \lambda_u[\frac{1}{2} - 0] + \lambda_s[-1 + \frac{x_m}{\bar{x}}].$$

Solving for x_m yields the desired result. ■

Proof of Corollary 3.1

Proposition 3 tells us that there exists a report, $r_u > 0$, such that $x_u^* = x_m(r_u, \overline{x})$ if and only if $x_u \geq \overline{x}\left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right)$. We direct our attention to Equation 18. Because both x_u and $\overline{x}$ are greater than or equal to zero by definition, if $0 > \left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right)$ the condition is satisfied.

$$\begin{aligned} 0 &> \left(\frac{4}{3} - \frac{2\lambda_u}{3\lambda_s}\right) \\ \frac{2\lambda_u}{3\lambda_s} &> \frac{4}{3} \\ \lambda_u &> 2\lambda_s \end{aligned}$$

■

Proof of Lemma 3

The conditional expectation of x_u, given x_s and that it is distributed uniformly over $[0, \bar{x}]$, can be written as,

$$E[x_u | x_u \leq x_s] = \frac{x_s}{2}. \tag{A19}$$

Similarly, the conditional expectation of x_s, given x_u, is

$$E[x_s | x_s \geq x_u] = \frac{x_u + \bar{x}}{2} \tag{A20}$$

Taking expectations of both equations gives,

$$E[x_u] = \frac{E[x_s]}{2} \tag{A21}$$

$$E[x_s] = \frac{E[x_u] + \bar{x}}{2} \tag{A22}$$

A rearrangement yields,

$$E[x_u] = \frac{\bar{x}}{3} \tag{A23}$$

$$E[x_s] = \frac{2\bar{x}}{3} \tag{A24}$$

■

Proof of Lemma 4

Because the planner takes the midpoint of the partition, it is always a weakly dominant strategy for the sophisticated advocate to report that x_s lies in the partition that contains $\bar{x}$. This follows from the fact that sophisticated agents are never harmed by having an $x_m > x_s$, but do incur losses if $x_m < x_s$. Therefore, for any realization of x_s, the probability that $r_s = \bar{x}$ equals one. ■

Proof of Proposition 4

An equilibrium of this game is guaranteed by Theorem 1 of Crawford and Sobel (1982), since the utility functions of the social planner and A_u satisfy those listed on page 1433 of Crawford and Sobel (1982) for $U^R(y, r)$ and $U^S(y, r, b)$ respectively.

Equation 24 is derived in similar fashion as Equations (20)-(22) in from Section 4 of Crawford and Sobel (1982). We first derive the expected bias for A_u. Using Equation A7 and the results of Lemma 3, we compute

$$\begin{aligned} E[b_u] &= E[x_m^*] - E[x_u^*] \\ &= \frac{E[x_u^*]E[x_s^*](\lambda_u + 2\lambda_s)}{\lambda_u E[x_s^*] + 2\lambda_s E[x_u^*]} - E[x_u^*] \\ &= \frac{\frac{\bar{x}^2}{6}(\lambda_u + 2\lambda_s)}{\lambda_u \frac{2\bar{x}}{3} + 2\lambda_s \frac{\bar{x}}{4}} - \frac{\bar{x}}{4} \\ &= \frac{5\lambda_s \bar{x}}{16 - 4\lambda_s} \end{aligned} \tag{A25}$$

Evaluating Equation 24 with the unconditional expectation for A_u's bias yields the maximum number of partitions that can be supported, which is computed as

$$\begin{aligned} \overline{N} &= \left\langle -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2\bar{x}}{E[b]}} \right\rangle \\ &= \left\langle -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{32 - 8\lambda_s}{5\lambda_s}} \right\rangle \\ &= \left\langle -\frac{1}{2} + \frac{1}{2}\sqrt{\frac{32 - 3\lambda_s}{5\lambda_s}} \right\rangle \end{aligned} \tag{A26}$$

It follows that $\overline{N}$ is less than or equal to 1 if $\frac{32-3\lambda_s}{5\lambda_s} \leq 9$. Thus, if $\lambda_s > \frac{2}{3}$ only a babbling equilibrium exists. Conversely, as $\lambda_s \to 0$, the number of partitions goes to infinity. ■

Proof of Proposition 5

The optimal levels of market sophistication under each type of planner is denoted as

$$\begin{cases} x_m^P & \text{Perfectly Informed} \\ x_m^V & \text{Savvy} \\ x_m^N & \text{Naive,} \end{cases}$$

where $x_m^P \neq x_m^V \neq x_m^N$.

We begin by considering only x_m^P and x_m^N. According to Proposition 1, a perfectly informed social planner will choose $x_m^P \in [x_u^*, x_s^*]$. Furthermore, we know that $\frac{\partial \mathcal{L}_u}{\partial x_m} > 0$ for all $x_m \geq x_u^*$ and $\frac{\partial \mathcal{L}_s}{\partial x_m} < 0$ for all $x_m \leq x_s^*$. Therefore, the unsophisticated agents always prefer an $x_m < x_m^P$ and sophisticated agents always prefer an $x_m > x_m^P$.

From Proposition 3 we know that $r_s = \overline{x}$ and $r_u > 0$ if $x_m^N(r_u, \overline{x}) = x_u^*$ and 0 otherwise. Substituting these reports into the social planner's equation for x_m^N yields two possible values,

$$x_m^N = \begin{cases} x_u^* & r_u > 0 \\ \overline{x}\left(1 - \frac{\lambda_u}{2\lambda_s}\right) & r_u = 0. \end{cases} \tag{A27}$$

It follows then that if $x_m^N < x_m^P$ then unsophisticated agents will prefer the imperfect social planner. Conversely, if $x_m^P < x_m^N$, sophisticated agents will prefer the imperfect social planner. Specifically, the unsophisticated agents always prefer $\min(x_m^P, x_m^N)$ and the sophisticated prefer $\max(x_m^P, x_m^N)$.

It is straightforward to show that adding a third choice does not induce unanimity. If $x_m^V < x_u^*$ and is preferred by the unsophisticated agents, it will not be preferred by sophisticated agents because $x_m^P > x_m^V$. If $x_m^V > x_s^*$ and is (weakly) preferred by the sophisticated agents, it will not be preferred by unsophisticated agents because $x_m^P < x_m^V$. If $x_m^V \in [x_u^*, x_s]$, the argument against unanimity follows as in the two-option case above. ■

Proof of Proposition 6:

The result for $\lambda_u \geq \frac{2}{3}$ follows from Corollary 3.1. For $\lambda_s \geq \frac{2}{3}$, with a savvy social planner, the optimal level of market sophistication will never exceed her beliefs of x_s. That is, when sophisticated agents have a supermajority, the planner's unconditional expectation of x_s serves as an upper bound for x_m^V, i.e. $x_m^V \leq \frac{2\bar{x}}{3}$.

However, sophisticated agents with a supermajority can obtain a higher level of market sophistication under a naive social planner. According to Proposition 3, unsophisticated agents will report

$x_u = 0$ if $\lambda_u \leq 1/3$. This leads to $x_m^N = \bar{x}\left[1 - \frac{\lambda_u}{2\lambda_s}\right]$, which obtains its minimum at $\lambda_s = 2/3$ since it is increasing in λ_s. Evaluating at 2/3 yields $x_m^N = \frac{3\bar{x}}{4}$, which is strictly greater than x_m^V.

Without any further information regarding the true values of x_u and x_s, we cannot say whether sophisticated agents prefer x_m^P or x_m^N. Their choice of planner will be governed by $\max(x_m^P, x_m^N)$. However, the sufficient condition in the proposition can be derived as follows. We can examine whether $x_m^N > x_m^P$ for values of $\lambda_s \geq \frac{2}{3}$. As mentioned above, x_m^N reaches its minimum of $\frac{3}{4}\bar{x}$ at $\lambda_s = \frac{2}{3}$ and is increasing in λ_s. Assessing x_m^P, if $x_u = x_s = x_p$, the perfectly informed planner sets $x_m^P = \frac{15}{16}x_p$. If $x_u < x_s$, then $x_m^P < \frac{15}{16}x_p$. Therefore, at $\lambda_s = \frac{2}{3}$, a sufficient condition for $x_m^N > x_m^P$ is $\bar{x} > \frac{5}{4}x_p$. However, as λ_s increases, x_m^P also rises. When $\lambda_s \to 1$, $x_m^P \to x_p$. Therefore, the sufficient condition that ensures $x_m^N > x_m^P$ is $\bar{x} > \frac{4}{3}x_p$ as desired. ■

Proof of Proposition 7

We first consider the marginal effect with regard to x_m. Taking the derivative of (27) with respect to x_m yields

$$\lambda_u\Big\{(1-\alpha)\Big[\frac{2x_m}{3x_p} - \frac{1}{2}\Big] + \alpha\Big[\frac{-x_p + x_m}{x_p}\Big]\Big\} + \lambda_s\Big[\frac{-x_p + x_m}{x_p}\Big].$$

re-arranging this yields the expression in (28).

Now, we consider the marginal effect with regard to α. Taking the derivative of (27) with respect to α yields

$$-\lambda_u\Big\{\Big[\frac{x_m^2}{3x_p} + \frac{-x_m + x_p}{2} - \frac{x_p^2 - 2x_m x_p + x_m^2}{2x_p}\Big]\Big\} + k\alpha.$$

re-arranging this yields the expression in (29).

Taking the derivative of (29) with respect to x_m yields

$$\frac{\lambda_u}{6x_p}(2x_m - 3x_p) < 0,$$

which implies that $\frac{\partial^2[L(x_m,\lambda_u,\lambda_s,\alpha)]}{\partial x_m \partial \alpha} < 0$. ■

Proof of Corollary 7.1

First-order conditions of Equation 27 with respect to x_m and α yields the following system of equations,

$$0 = -\lambda_u \Big\{ \Big[\frac{x_m^2}{3x_p} + \frac{-x_m + x_p}{2} - \frac{x_p^2 - 2x_m x_p + 2x_m^2}{2x_p} \Big] \Big\} + k\alpha \tag{A28}$$

$$0 = \lambda_u \Big\{ (1-\alpha) \Big[\frac{2x_m}{3x_p} - \frac{1}{2} \Big] + \alpha \Big[\frac{-x_p + x_m}{x_p} \Big] \Big\} + \lambda_s \Big[\frac{-x_p + x_m}{x_p} \Big]. \tag{A29}$$

Define x_m^* and α^* to be the optimal levels of sophistication and education that satisfy Equations A28 and A29. Because Equation 27 is strictly convex in both x_m and α, we know that the set x_m^* and α^* is unique. We now re-write α^* in terms of x_m^*,

$$\alpha^* = \frac{\lambda_u}{6kx_p} \left(3x_m^* x_p - {x_m^*}^2 \right). \tag{A30}$$

The partial derivatives of Equation A30 with respect to k, λ_u, x_p, and x_m^* are,

$$\begin{aligned}
\frac{\partial \alpha^*}{\partial k} &= -\frac{\lambda_u}{6x_p k^2} \left(3x_m^* x_p - {x_m^*}^2 \right) && \leq 0 \\
\frac{\partial \alpha^*}{\partial \lambda_u} &= \frac{1}{6kx_p} \left(3x_m^* x_p - {x_m^*}^2 \right) && \geq 0 \\
\frac{\partial \alpha^*}{\partial x_p} &= \frac{\lambda_u x_m^2}{6kx_p^2} && \geq 0 \\
\frac{\partial \alpha^*}{\partial x_m^*} &= \frac{\lambda_u}{6kx_p} \left(3x_p - 2x_m^* \right) && \geq 0
\end{aligned} \tag{A31}$$

■

Appendix B

In this appendix, we explore an alternative model to the one in Section 2. There, we tethered the lower bound of the market to be zero and studied how a social planner optimal chose the upper bound. As such, we interpreted the size of $[0, x_m]$ to be the extent of the market, where the planner could regulate how complete the market is. Here, we consider that the planner can choose both the lower and upper bounds of this continuum and show that the planner still faces the same tensions from unsophisticated and sophisticated agents. As such, the planner balances both groups' needs and chooses an internal level of market completeness.

Define $x_{m,l} \geq 0$ as the least sophisticated product that is offered in the market and define $x_{m,u} \geq x_{m,l}$ to be the most sophisticated product. The planner's problem is to balance the demands of the two groups to minimize the aggregate loss,

$$\min_{x_{m,l}, x_{m,u} \in [0, x_p]} L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s), \tag{B1}$$

where

$$L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s) = \lambda_u E[L(\tilde{t}_u, x | x_{m,l}, x_{m,u})] + \lambda_s E[L(\tilde{t}_s, x | x_{m,l}, x_{m,u})]. \tag{B2}$$

The following proposition is the analog of Lemmas 1 and 2, which evaluates the loss to the two types of market participants.

Proposition B1. *The aggregate loss for sophisticated agents is*

$$\mathcal{L}_s = \frac{x_{m,l}^2 + x_p^2 + x_{m,u}^2 - 2x_m}{2x_p}, \tag{B3}$$

which is increasing in $x_{m,l}$, decreasing in $x_{m,u}$ and convex in both parameters respectively.

The aggregate loss for unsophisticated agents is

$$\mathcal{L}_u = \frac{3x_{m,l}x_{m,u} + 2(x_{m,u} - x_{m,l})^2 + 3(x_{m,u} - x_p)(x_{m,l} - x_p)}{6x_p}, \tag{B4}$$

which reaches a minimum at $x_{m,u} = x_{m,l} = \frac{1}{2}x_p$.

Proof of Proposition B1

The loss for a given sophisticated agent with type $\tilde{t}_s$ is given by

$$L(x | \tilde{t}_s, x_{m,l}, x_{m,u}) = |x - \tilde{t}_s| \tag{B5}$$

Because the agent knows his type perfectly he will choose the closest product to his type. When $\tilde{t}_s < x_{m,l}$ the agent chooses $x = x_{m,l}$. Similarly, when $\tilde{t}_s > x_{m,u}$ the agent chooses $x = x_{m,u}$. An

agent with type $\tilde{t}_s \in [x_{m,l}, x_{m,u}]$ does not incur a loss since he will choose $x = \tilde{t}_s$. Since sophisticated agents are distributed uniformly, the aggregate expected loss in the population is given by,

$$\begin{aligned}
\mathcal{L}_s &= \int_0^{x_{m,l}} |x_{m,l} - \tilde{t}_s| \frac{d\tilde{t}_s}{x_p} + \int_{x_{m,l}}^{x_{m,u}} 0 \frac{d\tilde{t}_s}{x_p} + \int_{x_{m,u}}^{x_p} |x_{m,u} - \tilde{t}_s| \frac{d\tilde{t}_s}{x_p} \\
&= \int_0^{x_{m,l}} (x_{m,l} - \tilde{t}_s) \frac{d\tilde{t}_s}{x_p} + \int_{x_{m,u}}^{x_p} (\tilde{t}_s - x_{m,u}) \frac{d\tilde{t}_s}{x_p} \\
&= \left[\frac{x_{m,l}\tilde{t}_s}{x_p} - \frac{\tilde{t}_s^2}{2x_p} \Big|_0^{x_{m,l}} \right] + \left[\frac{\tilde{t}_s^2}{2x_p} - \frac{x_{m,u}\tilde{t}_s}{x_p} \Big|_{x_{m,u}}^{x_p} \right] \\
&= \left[\frac{x_{m,l}^2}{x_p} - \frac{x_{m,l}^2}{2x_p} \right] + \left[\frac{x_p^2}{2x_p} - \frac{x_{m,u}x_p}{x_p} - \frac{x_{m,u}^2}{2x_p} + \frac{x_{m,u}^2}{x_p} \right] \\
&= \frac{x_{m,l}^2 + x_p^2 - 2x_{m,u}x_p + x_{m,u}^2}{2x_p} \qquad \text{(B6)}
\end{aligned}$$

The first-order derivative of the expected loss with respect to $x_{m,l}$ is

$$\frac{\partial}{\partial x_{m,l}} E[L(\tilde{t}_s, x | x_{m,l}, x_{m,u})] = \frac{x_{m,l}}{x_p}, \qquad \text{(B7)}$$

which is strictly positive. The first-order derivative of the expected loss with respect to $x_{m,u}$ is

$$\frac{\partial}{\partial x_{m,u}} E[L(\tilde{t}_s, x | x_{m,l}, x_{m,u})] = \frac{x_{m,u} - x_p}{x_p}, \qquad \text{(B8)}$$

which is negative since x_p serves as the maximum possible level of market sophistication. The second-order derivative of the expected loss with respect to $x_{m,l}$ is

$$\frac{\partial^2}{\partial x_{m,l}^2} E[L(\tilde{t}_s, x | x_{m,l}, x_{m,u})] = \frac{1}{x_p}, \qquad \text{(B9)}$$

which tells us the function is convex in $x_{m,l}$ since it is positive. Similarly, the second-order derivative of the expected loss with respect to $x_{m,u}$ is

$$\frac{\partial^2}{\partial x_{m,u}^2} E[L(\tilde{t}_s, x | x_{m,l}, x_{m,u})] = \frac{1}{x_p}, \qquad \text{(B10)}$$

which is also strictly positive.

The loss for a given unsophisticated agent with type $\tilde{t}_u$ is given by

$$L(x | \tilde{t}_u, x_{m,l}, x_{m,u}) = \int_{x_{m,l}}^{x_{m,u}} |\tilde{x} - \tilde{t}_u| \frac{d\tilde{x}}{x_{m,u} - x_{m,l}}, \qquad \text{(B11)}$$

since the unsophisticated agent randomly selects a product on the continuum $[x_{m,l}, x_{m,u}]$. Since unsophisticated agents are distributed uniformly, the aggregate expected loss in the population is

given by

$$
\begin{aligned}
\mathcal{L}_u =& \int_0^{x_p} \int_{x_{m,l}}^{x_{m,u}} |\tilde{x} - \tilde{t}_u| \frac{d\tilde{x}}{x_{m,u} - x_{m,l}} \frac{d\tilde{t}_u}{x_p} \\
=& \int_0^{x_p} \frac{(\tilde{x} - \tilde{t}_u)^3}{2|\tilde{x} - \tilde{t}_u|(x_{m,u} - x_{m,l})} \Bigg|_{x_{m,l}}^{x_{m,u}} \frac{d\tilde{t}_u}{x_p} \\
=& \frac{1}{(x_{m,u} - x_{m,l})x_p} \int_0^{x_p} \left[\frac{(x_{m,u} - \tilde{t}_u)^3}{2|x_{m,u} - \tilde{t}_u|} - \frac{(x_{m,l} - \tilde{t}_u)^3}{2|x_{m,l} - \tilde{t}_u|} \right] d\tilde{t}_u,
\end{aligned}
$$

which expands to,

$$
\begin{aligned}
=& \frac{1}{(x_{m,u} - x_{m,l})x_p} \Bigg[\int_0^{x_{m,l}} \left[\frac{(x_{m,u} - \tilde{t}_u)^3}{2|x_{m,u} - \tilde{t}_u|} - \frac{(x_{m,l} - \tilde{t}_u)^3}{2|x_{m,l} - \tilde{t}_u|} \right] d\tilde{t}_u \\
&+ \int_{x_{m,l}}^{x_{m,u}} \left[\frac{(x_{m,u} - \tilde{t}_u)^3}{2|x_{m,u} - \tilde{t}_u|} - \frac{(x_{m,l} - \tilde{t}_u)^3}{2|x_{m,l} - \tilde{t}_u|} \right] d\tilde{t}_u + \int_{x_{m,u}}^{x_p} \left[\frac{(x_{m,u} - \tilde{t}_u)^3}{2|x_{m,u} - \tilde{t}_u|} - \frac{(x_{m,l} - \tilde{t}_u)^3}{2|x_{m,l} - \tilde{t}_u|} \right] d\tilde{t}_u \Bigg] \\
=& \frac{1}{2(x_{m,u} - x_{m,l})x_p} \Bigg[\int_0^{x_{m,l}} \left[(x_{m,u} - \tilde{t}_u)^2 - (x_{m,l} - \tilde{t}_u)^2 \right] d\tilde{t}_u \\
&+ \int_{x_{m,l}}^{x_{m,u}} \left[(x_{m,u} - \tilde{t}_u)^2 + (x_{m,l} - \tilde{t}_u)^2 \right] d\tilde{t}_u + \int_{x_{m,u}}^{x_p} \left[-(x_{m,u} - \tilde{t}_u)^2 + (x_{m,l} - \tilde{t}_u)^2 \right] d\tilde{t}_u \Bigg].
\end{aligned}
$$

An evaluation of the integrals yields,

$$
\begin{aligned}
=& \frac{1}{2(x_{m,u} - x_{m,l})x_p} \Bigg[\Bigg[x_{m,u}^2 \tilde{t}_u + \frac{\tilde{t}_u^3}{3} - x_{m,u}\tilde{t}_u^2 - x_{m,l}^2 \tilde{t}_u - \frac{\tilde{t}_u^3}{3} + x_{m,l}\tilde{t}_u^2 \Bigg|_0^{x_{m,l}} \Bigg] \\
&+ \left[x_{m,u}^2 \tilde{t}_u + \frac{\tilde{t}_u^3}{3} - x_{m,u}\tilde{t}_u^2 + x_{m,l}^2 \tilde{t}_u + \frac{\tilde{t}_u^3}{3} - x_{m,l}\tilde{t}_u^2 \Bigg|_{x_{m,l}}^{x_{m,u}} \right] \\
&+ \left[-x_{m,u}^2 \tilde{t}_u - \frac{\tilde{t}_u^3}{3} + x_{m,u}\tilde{t}_u^2 + x_{m,l}^2 \tilde{t}_u + \frac{\tilde{t}_u^3}{3} - x_{m,l}\tilde{t}_u^2 \Bigg|_{x_{m,u}}^{x_p} \right] \Bigg] \\
=& \frac{x_{m,l}x_{m,u}}{2x_p} + \frac{(x_{m,u} - x_{m,l})^2}{3x_p} + \frac{(x_{m,u} - x_p)(x_{m,l} - x_p)}{2x_p} \\
=& \frac{3x_{m,l}x_{m,u} + 2(x_{m,u} - x_{m,l})^2 + 3(x_{m,u} - x_p)(x_{m,l} - x_p)}{6x_p}.
\end{aligned} \tag{B12}
$$

The first-order derivatives of the expected loss with respect to $x_{m,l}$ and $x_{m,u}$ are

$$
\frac{\partial}{\partial x_{m,l}} E[L(\tilde{t}_u, x | x_{m,l}, x_{m,u})] = \frac{4x_{m,l} + 2x_{m,u} - 3x_p}{6x_p} \tag{B13}
$$

and

$$
\frac{\partial}{\partial x_{m,u}} E[L(\tilde{t}_u, x | x_{m,l}, x_{m,u})] = \frac{2x_{m,l} + 4x_{m,u} - 3x_p}{6x_p}. \tag{B14}
$$

At optimality (i.e., setting (B13) and (B14) equal to zero), $x_{m,l} = x_{m,u}$, which implies that $x_{m,l} = x_{m,u} = \frac{1}{2}x_p$. It is straightforward to verify that the second-order condition is satisfied so that a minimum is attained when $x_{m,l} = x_{m,u} = \frac{1}{2}x_p$. ■

From Proposition B1, sophisticated agents prefer the market to be complete as possible at both ends of the spectrum, whereas unsophisticated agents want the market to contract towards their median needs. The planner's problem is to balance the demands of the two groups and the following proposition solves for a socially optimal $x^*_{m,l}$ and $x^*_{m,u}$.

Proposition B2. *There exists a unique optimal set $\{x^*_{m,l}, x^*_{m,u}\}$, with $0 \leq x^*_{m,l} \leq x^*_{m,u} \leq x_p$, that minimizes $L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s)$,*

$$\{x^*_{m,l}, x^*_{m,u}\} = \left\{ \frac{\lambda_u x_p}{6 - 4\lambda_u}, x_p - \frac{\lambda_u x_p}{6 - 4\lambda_u} \right\}. \tag{B15}$$

The optimal $x^*_{m,l}$ is

(i) increasing in x_p

(ii) increasing in the mass of unsophisticated agents.

The optimal $x^*_{m,u}$ is

(i) increasing in x_p

(ii) decreasing in the mass of unsophisticated agents.

Proof of Proposition B2

Both sophisticated and unsophisticated agents' losses are convex and continuous in both $x_{m,l}$ and $x_{m,u}$ which means the aggregate loss function is also convex and continuous in these parameters. The first-order condition of Equation B2 with respect to $x_{m,l}$ is,

$$\begin{aligned} 0 &= \frac{\partial}{\partial x_{m,l}} L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s) \\ &= \lambda_u \left[\frac{4x_{m,l} + 2x_{m,u} - 3x_p}{6x_p} \right] + \lambda_s \left[\frac{x_{m,l}}{x_p} \right] \\ &= \lambda_u [4x_{m,l} + 2x_{m,u} - 3x_p] + 6\lambda_s x_{m,l} \\ x_{m,l} &= \frac{3\lambda_u x_p - 2\lambda_u x_{m,u}}{4\lambda_u + 6\lambda_s}. \end{aligned} \tag{B16}$$

The first-order condition with respect to $x_{m,u}$ is,

$$
\begin{aligned}
0 &= \frac{\partial}{\partial x_{m,u}} L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s) \\
&= \lambda_u \left[\frac{2x_{m,l} + 4x_{m,u} - 3x_p}{6x_p} \right] + \lambda_s \left[\frac{x_{m,u} - x_p}{x_p} \right] \\
&= \lambda_u [2x_{m,l} + 4x_{m,u} - 3x_p] + 6\lambda_s (x_{m,u} - x_p) \\
x_{m,u} &= \frac{3\lambda_u x_p - 2\lambda_u x_{m,l} + 6\lambda_s x_p}{4\lambda_u + 6\lambda_s}. \qquad \text{(B17)}
\end{aligned}
$$

Combining Equations B16 and B17 we can solve for $x^*_{m,l}$,

$$
\begin{aligned}
x_{m,l} &= \frac{3\lambda_u x_p}{4\lambda_u + 6\lambda_s} - \frac{2\lambda_u (3\lambda_u x_p - 2\lambda_u x_{m,l} + 6\lambda_s x_p)}{(4\lambda_u + 6\lambda_s)^2} \\
x_{m,l}(4\lambda_u + 6\lambda_s)^2 &= 3\lambda_u x_p (4\lambda_u + 6\lambda_s) - 2\lambda_u (3\lambda_u x_p - 2\lambda_u x_{m,l} + 6\lambda_s x_p).
\end{aligned}
$$

Recall that $\lambda_s = 1 - \lambda_u$,

$$
\begin{aligned}
x_{m,l}(4\lambda_u + 6(1-\lambda_u))^2 &= 3\lambda_u x_p (4\lambda_u + 6(1-\lambda_u)) - 2\lambda_u (3\lambda_u x_p - 2\lambda_u x_{m,l} + 6(1-\lambda_u) x_p) \\
x_{m,l}(36 - 24\lambda_u) &= 6\lambda_u x_p \\
x^*_{m,l} &= \frac{\lambda_u x_p}{6 - 4\lambda_u}. \qquad \text{(B18)}
\end{aligned}
$$

Now $x^*_{m,u}$ is given by,

$$
\begin{aligned}
x_{m,u} &= \frac{3\lambda_u x_p - 2\lambda_u \left(\frac{\lambda_u x_p}{6 - 4\lambda_u} \right) + 6\lambda_s x_p}{4\lambda_u + 6\lambda_s} \\
&= \frac{3\lambda_u x_p - 2\lambda_u \left(\frac{\lambda_u x_p}{6 - 4\lambda_u} \right) + 6\lambda_s x_p}{4\lambda_u + 6\lambda_s} \\
&= \frac{(6 - 5\lambda_u) x_p}{6 - 4\lambda_u} \\
x^*_{m,u} &= x_p - \frac{\lambda_u x_p}{6 - 4\lambda_u} \qquad \text{(B19)}
\end{aligned}
$$

The comparative statics of $x^*_{m,l}$ and $x^*_{m,u}$ with respect to λ_u and x_p are,

$$
\begin{aligned}
\frac{\partial x^*_{m,l}}{\partial \lambda_u} &= \frac{x_p(6-4\lambda_u)-\lambda_u x_p}{(6-4\lambda_u)^2} \\
&= \frac{x_p(6-5\lambda_u)}{(6-4\lambda_u)^2} && \geq 0 \\
\frac{\partial x^*_{m,l}}{\partial x_p} &= \frac{\lambda_u}{6-4\lambda_u} && \geq 0 \\
\frac{\partial x^*_{m,u}}{\partial \lambda_u} &= -\left(\frac{x_p(6-4\lambda_u)-\lambda_u x_p}{(6-4\lambda_u)^2}\right) \\
&= -\left(\frac{x_p(6-5\lambda_u)}{(6-4\lambda_u)^2}\right) && \leq 0 \\
\frac{\partial x^*_{m,u}}{\partial x_p} &= 1-\frac{\lambda_u}{6-4\lambda_u} \\
&= \frac{6-5\lambda_u}{6-4\lambda_u} && \geq 0 \qquad \text{(B20)}
\end{aligned}
$$

■