Learning and Stock Market Participation*

Juhani Linnainmaa†

The Anderson School at UCLA

November 2005

†**Correspondence Information:** Juhani Linnainmaa, The Anderson School at UCLA, Suite C4.01, 110 Westwood Plaza, CA 90095-1481, tel: (310) 909-4666, fax: (310) 206-5455, http://personal.anderson.ucla.edu/juhani.linnainmaa, email: julinnai@anderson.ucla.edu.

Learning and Stock Market Participation

Abstract

This paper examines the impact of short-sale constraints on market participation when agents learn about their investment opportunities. The possibility of binding short-sale constraints creates a feedback that can keep agents out of the market even if the risk premium is high. This effect arises with learning because the changes in investment opportunities are correlated with future realized outcomes: an agent will have a poor investment opportunity set precisely in those future states where her marginal utility is high. Non-participation arises also in an equilibrium model where agents resolve uncertainty about the cash-flow covariance between tradable and non-tradable assets. These results suggest that learning and short-sale constraints can simultaneously generate non-participation, a sizable risk premium, and insignificant contemporaneous correlation between the stock return and the income of those who do not participate in the stock market.

1 Introduction

Stock market participants have historically been a minority.[1] Despite the exceptional historical equity premium, many individuals stay out of the market. This is a puzzle because everyone should participate if the risk premium is even slightly positive and there are no frictions or incomplete markets (Arrow 1965). Many empirical determinants of participation are known. For example, education matters: 50% of individuals with a college degree own stocks while this rate is only 20% for those without a degree (Hong, Kubik, and Stein 2004). Wealth is the strongest determinant of participation: the participation rate increases from 3% to 55% from the first to the fifth wealth quintile. However, the non-participants are not only those who have nothing to invest: Mankiw and Zeldes (1991) find that even individuals with more than $100,000 in liquid assets have a participation rate of only 47.7%. The *limited participation puzzle* is our inability to understand why so many individuals choose to stay out of the market.

We propose a novel mechanism that generates non-participation in a perfectly rational setting. We first describe this mechanism with an example. Suppose an agent works in an industry sensitive to the macroeconomic conditions. If the economy stays healthy, the agent retains her job. However, the agent might lose her job in an economic downturn. If this happens, the agent's wage covaries positively with the dividends: if the economy recovers, firms pay higher dividends and the agent is rehired. However, if the economy remains weak, firms pay low dividends and the agent receives no labor income. The unemployed agent would hedge against the risk of not finding a new job by shorting the market.

Suppose now that the agent cannot open a margin account in this unemployed state. This inability to hedge has two consequences. First, the direct effect is that the agent stays out of the market after losing her job, generating a welfare loss relative to the "hedging allowed" case. Second, this potential future welfare loss is important at an earlier date for the employed agent. If the agent invested in the market today, an economic downturn would have important repercussions: the agent would not only be unemployed and restricted from hedging but she would also have high marginal utility *because* she invested. By staying out of the market today, the agent hedges against this risk.

---

[1] Only the latest Survey of Consumer Finances from 2001 finds that, for the first time in the US history, stockholders have become a majority with a 51.9% participation rate. The participation rate was 31.7% in 1989. The SCF participation rates include direct and retirement account holdings of stocks and stock mutual funds.

This paper explores this mechanism and its consequences. We first demonstrate this feedback effect from future portfolio constraints with a stylized life-cycle model where an agent learns about the risk premium over time. Because the agent learns about the risk premium, the agent faces poor investment opportunities after bad realizations. (For example, the agent revises her beliefs downwards after a low realized return.) We show that the agent may stay out of the market despite a large risk premium.[2] We also introduce *implied risk-aversion* as a measure of how much learning and portfolio constraints skew investor behavior. For example, an agent who stays out of the market despite a high risk premium appears infinitely risk-averse to an outsider who ignores the role of learning.

We then consider an equilibrium model where heterogeneous agents resolve uncertainty about the covariance between their nonfinancial income and dividends. Specifically, we consider the case where one of the agents becomes unemployed after a low dividend. We use this model to address two questions. First, does the unemployment risk together with market incompleteness generate a hedging demand that keeps the agent out of the market at an earlier date? Second, what are the consequences of this type of non-participation on the size of the risk premium? The latter question is important because the limited participation puzzle is intimately connected to the size of the equity premium.[3]

We demonstrate two results. The first result is that the *risk* of binding constraints alone may induce an agent to stay out of the market at date zero: the agent would participate today if she could hedge in the unemployed state. The second result is that the risk premium is high relative to the unconstrained economy—i.e., the same economy but without portfolio constraints—when the agent is currently out of the market, but

1. would buy a positive amount of the asset if the short-sale constraints were lifted and

2. is close to being indifferent between participating and not participating in the future.

---

[2]Learning is central to this non-participation mechanism. For example, it would not be surprising to find that an agent with currently poor investment opportunities stays out of the market. However, our finding is more surprising: the feedback from *future* trading restrictions can be so strong that an agent reduces her holdings to zero despite a high risk premium.

[3]For example, Hong, Kubik, and Stein (2004) motivate their analysis by suggesting that an understanding of what drives participation can shed light on the equity premium puzzle of Mehra and Prescott (1985). Yet, most studies that examine *why* some individuals do not participate sidestep this issue. It is not obvious what the effect on the risk premium should be. For example, if the short-sale constraints let the participating agents to hold only the entire market (instead of holding more), they only need to be compensated with a smaller risk premium.

This risk premium result holds also for agents with log-preferences. Hence, an agent may stay out at date 0 even though her labor income is constant, her preferences would generally lead to myopic behavior, and the risk premium is relatively high. This suggests that an empirical analysis of the determinants of non-participation may be difficult when agents hedge against the *future* risks of not being able to trade all the assets as smoothly as classical models presume.

The rest of the paper is organized as follows. The next section discusses related research. Section 2 solves a tractable life-cycle model that illustrates the feedback mechanism. Section 3 formulates a heterogeneous-agents equilibrium model with non-participation. Section 4 concludes.

1.1   Relation to Prior Research

1.1.1   Parameter Uncertainty and Learning

Many recent studies have examined parameter uncertainty (or ambiguity) and learning.[4] For example, Brennan (1998) assumes that agents learn about the expected return in a Merton (1969) setup; Xia's (2001) agents learn about the predictive ability of an observable state variable; Brennan and Xia (2001) show that uncertainty about dividend growth may contribute towards an explanation to the equity premium puzzle; Epstein and Miao (2003) solve an equilibrium model where agents have different prior views about the economy; and Pástor and Veronesi (2005a, 2005b) show that uncertainty about future profitability may explain both IPO waves and high Nasdaq valuations in the 1990s.

This paper is closely related to the studies on parameter uncertainty. The life-cycle model of Section 2 is a discrete-time analogue of the Brennan (1998) model where an agent accounts for the estimation uncertainty in the expected return. The pivotal difference that generates our non-participation result is that our agent faces trading frictions.

1.1.2   Labor Income, Illiquidity, and Trading Constraints

Human capital is an important component of individuals' wealth: labor income accounts for 75% of consumption (Santos and Veronesi 2005). Many studies have studied the role

---

[4]Williams (1977), Detemple (1986), and Gennotte (1986) are early contributions to asset pricing and portfolio choice under parameter uncertainty. Baron (1974) and McCardle and Winkler (1989) are portfolio choice models with learning where agents display risk-preference. Bayesian learning in a portfolio choice setup can be viewed as a particular definition of the Merton (1973) ICAPM state-variable.

of human capital in asset pricing, beginning with Mayers (1972). For example, Santos and Veronesi (2005) let agents derive income from two sources, dividends and wage. Their key assumption is that the total income (the sum of wages and dividends) grows over time while the wage share depends on economic conditions. This assumption generates return predictability and the growth/value effect. Lettau and Ludvigson (2004) examine the link between wealth (including human capital) and consumption and find that only permanent wealth changes affect consumer spending. Malloy, Moskowitz, and Vissing-Jørgensen (2005a) find asset pricing success by using firing/hiring data to measure persistent labor income shocks. These studies emphasize the potential role of labor income for explaining individuals' consumption choices and asset prices. This paper's equilibrium model is related because we let one agent face the possibility of an unemployment. The key difference to the extant studies is our focus on how the possibility of future unemployment and trading frictions generate non-participation already at earlier periods.

Numerous studies have considered the general effects of asset illiquidity and non-tradability. For example, Longstaff (2001) shows that an agent who must accumulate or unwind positions over a period of time can behave as if she faced endogenous borrowing and short-sale constraints. Liu, Longstaff, and Pan (2003) show that an agent facing jump risk is less willing to take levered or short positions. Longstaff (2005) considers a two-asset, heterogeneous agents model where one of the assets is traded initially but then enters a blackout period. He finds that this non-tradability can significantly skew agents' portfolio choices and that liquidity is an important component of an asset's equilibrium value. Pástor and Stambaugh (2003) find empirically that assets more sensitive to liquidity command a premium over low-sensitivity stocks. The common theme in this literature is that agents endowed with an illiquid asset act more cautiously than they would if the markets were complete and frictionless. In this paper, the portfolio constraints and learning skew the agents' behavior.

1.1.3 The Limited Participation Puzzle

The limited participation puzzle has attracted attention for two reasons. The first line of research examines *why* so many individuals choose to stay out of the market despite the exceptional historical equity premium. For example, Vissing-Jørgensen (2002b) suggests that the decision to stay out of the market may be optimal for about half of the non-participating

individuals even if the fixed costs are relatively modest.[5] However, the estimated costs are too high for the other half for these costs alone to be a reasonable explanation to the limited participation puzzle. Theoretical studies by Dow and Werlang (1992), Ang, Bekaert, and Liu (2005), Epstein and Schneider (2005), and Cao, Wang, and Zhang (2005) introduce non-standard utility functions to generate non-participation. Dow and Werlang (1992) (a static model), Epstein and Schneider (2005), and Cao, Wang, and Zhang (2005) (dynamic models) rely on ambiguity aversion. The latter papers are related to the present study because the agents in the paper learn over time. Ang, Bekaert, and Liu (2005) generate non-participation by assuming that investors are disappointment averse.

The second line of participation research begins with the idea that investor heterogeneity may generate a higher theoretical equity premium.[6] It is possible that if some investors are shut out of the market, their consumption processes "contaminate" aggregate consumption data. This could lead to falsely reject consumption-based asset pricing models. For example, the agents in Basak and Cuoco (1998) face frictions that shut them out of the market. In equilibrium, these agents' consumption processes do not covary with aggregate consumption. These studies argue that data on stockholders' consumption alone may fare better in asset pricing because the non-stockholders do not price the assets. Vissing-Jørgensen (2002a) estimates the bond and stock return Euler equations separately for market participants and non-participants and finds support for this idea. The problem with these studies is that even if they find success, they do not address the question of *why* non-stockholders are not pricing the assets. Cochrane (2005, pp. 61) concludes his survey of the extant literature as follows:

> "*Must we use microdata? While initially appealing, however, its not clear that the stockholder/nonstockholder distinction is vital. Are people who hold no stocks really not "marginal?" The costs of joining the stock market are trivial...Thus,*

[5]For example, suppose an individual has only $5,000 in liquid assets. If the annual stock market participation cost—e.g., brokerage fees and information costs—is, say, $50, the risk-return tradeoff from the market may not be good enough to induce the agent to participate.

[6]For example, Mankiw (1986) and others suggest that particular type of heterogeneity in individuals' marginal rate of substitution could result into higher premium but conclude that the agents still come close to complete risk-sharing even by trading just one asset in a frictionless market. Constantinides and Duffie (1996, pp. 221) note that these negative results are largely due to the assumption that each agent's labor income share is a stationary process. The paper shows how to match any historical equity premium with time-additive power utility and idiosyncratic income risk. Cochrane (2005, pp. 57) cautions that the Constantinides and Duffie solution may still require unreasonable level of variation in each individuals' consumption growth: "*Can it be true that if aggregate consumption growth is 2%, the typical person you meet either has +73% or −63% consumption growth?*"

people who do not invest at all choose not to do so in the face of trivial fixed costs."

This paper addresses Cochrane's critique: the agents in our model face no transaction costs but choose to stay out of the market in equilibrium because of the *possibility* that the short-sale constraints bind in the future. Our paper also complements earlier studies by simultaneously considering both the causes (future learning) and consequences (risk premium) of non-participation.

2   An Example of the Feedback Effect: A Life-Cycle Model

This section solves a tractable life-cycle model where a short-sale constrained agent learns about the risk premium over time. We use this model to demonstrate the feedback mechanism before turning to a more realistic equilibrium model in Section 3. We discuss two features of this model before detailing our assumptions and solving the model.

First, the agent in our model knows that the true risk premium is constant but is uncertain about its precise value in the beginning. The agent considers the possibility that the risk premium may turn out to be negative. If this happens, the asset becomes effectively useless to the agent because of the short-sale constraints. The assumption that the risk premium can become negative is a shorthand way of modeling subjective investment opportunity sets. For example, an agent who faces permanent labor income shocks (e.g., she may be hired, fired, or tenured) wants to short the market when her wage covaries sufficiently with the market returns even when the risk premium is restricted to strictly positive values.

Second, the stock price follows a binomial tree with constant up- and down-tick parameters. This means that the agent might optimally take a short position in the asset and that the assumption of short-sale constraints has repercussions. Note that if the return distribution were unbounded from above, the agent would endogenously refrain from any short positions.[7] (The continuous-time equivalent would be either jump-risk or assets that are not always tradable.) Note that our assumption is not very restrictive for two reasons. First, a binomial model converges to a diffusion process as the number of horizons increases and the length of each period shortens. Second, our emphasis is on what happens when the agent faces exogenous constraints on top of the endogenous constraints and not on the exact nature

---

[7]The agent's utility function also needs to satisfy the Inada conditions—namely that $\lim_{W \to 0} U'(W) = \infty$.

of this additional constraint.

2.1 Setup

We make the following assumptions:

- A single agent lives for $T + 1$ periods, indexed from $0, \ldots, T$.

- The agent maximizes power utility over date T wealth,

$$U_T(W_T) = \frac{W_T^{1-\gamma} - 1}{1 - \gamma}. \tag{1}$$

- There is a single risky stock and a risk-free asset. These assets are traded each period. The stock price follows a binomial tree (Cox, Ross, and Rubinstein 1979; Liu and Neis 2002):

$$S_{t+1} = \begin{cases} S_t(R_f + u) & \text{with probability } p \\ S_t(R_f - d) & \text{with probability } 1 - p \end{cases} \tag{2}$$

where $ud > 0$ to rule out arbitrage. The risk-free asset returns R_f each period for sure.

- The agent decides how much to invest in the risky asset at the beginning of each period after observing the previous period's realized return.

- The agent cannot short the stock, $\theta_t \geq 0$, where θ_t is the number of shares.

- The agent knows all the parameters of the economy precisely except for the probability p. The agent has a Beta-distributed prior about p and updates her beliefs as a Bayesian at the beginning of each period.

The wealth dynamics from these assumptions are

$$W_t = W_{t-1}(R_f + f_{t-1}\tilde{\epsilon}), \tag{3}$$

$$\text{where} \quad \tilde{\epsilon} = \begin{cases} u & \text{with probability } p \\ -d & \text{with probability } 1 - p \end{cases}$$

$$f_{t-1} = \frac{\theta_{t-1}S_{t-1}}{W_{t-1}} \quad \text{(fraction of wealth in stock)}.$$
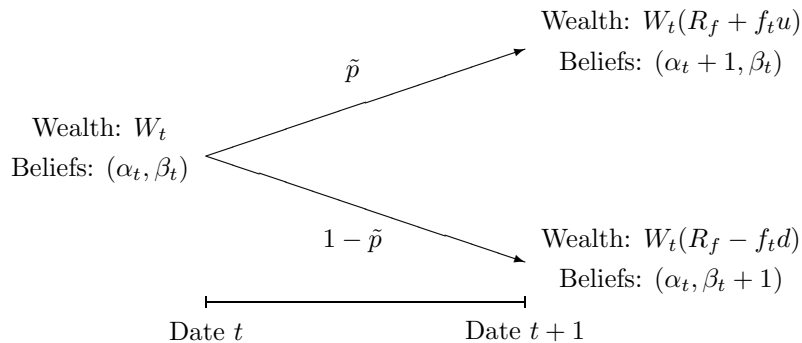
Figure 1: **The Agent's Wealth and Beliefs in the Binomial Model**

The agent's problem can be broken into two steps: an inference problem in which the agent updates her estimate of p and an optimization problem in which the agent chooses the optimal investment given the current wealth and the estimate of p (Gennotte 1986; Brennan and Xia 2001). We consider first the inference problem.

2.2 The Agent's Inference Problem

The agent has a conjugate prior distribution $\text{Beta}(\alpha_0, \beta_0)$ about p at date 0. The assumption about a binomial stock price process and a Beta-distributed prior makes the inference problem particularly tractable. The date t posterior after observing N_t positive stock price movements is $\text{Beta}(\alpha_0 + N_t, \beta_0 + t - N_t)$-distributed (see, e.g., DeGroot (1970, pp. 160)). The mean and variance of the posterior distribution are given by

$$E_t(p) = \frac{\alpha_0 + N_t}{\alpha_0 + \beta_0 + t}, \quad \text{var}_t(p) = \frac{(\alpha_0 + N_t)(\beta_0 + t - N_t)}{(\alpha_0 + \beta_0 + t)^2(\alpha_0 + \beta_0 + t + 1)}. \tag{4}$$

We let $\alpha_t \equiv \alpha_0 + N_t$ and $\beta_t \equiv \beta_0 + t - N_t$ to denote the date t belief parameters. The intuition for updating is simple: the parameters of the Beta-distribution keep track of the number of the stock price's up- and downticks. For example, if the agent starts with parameters $(1, 2)$, the parameters become $(2, 2)$ after an uptick and $(1, 3)$ after a downtick. Figure 1 illustrates how the agent's wealth and beliefs evolve in this problem.

2.3 The Agent's Optimization Problem

We solve the agent's optimization problem with dynamic programming. The agent receives utility $V_T(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}$ in the last period. (We assume that $\gamma \neq 1$.) We begin with the

9

conjecture that the date $t + 1$ Bellman equation has the form

$$V_{t+1}(W_{t+1}, (\alpha_{t+1}, \beta_{t+1})) = \frac{W_{t+1}^{1-\gamma}}{1 - \gamma} k_{t+1}(\alpha_{t+1}, \beta_{t+1}) \tag{5}$$

and then later show that it satisfies this form. With this conjecture, the date t Bellman equation solves

$$
\begin{aligned}
V_t(W_t, (\alpha_t, \beta_t)) &= \max_{f_t \geq 0} \{E_t [V_{t+1}(W_t(R_f + f_t\tilde{\epsilon}), (\alpha_{t+1}, \beta_{t+1}))]\} \\
&= \max_{f_t \geq 0} \left\{ \frac{W_t^{1-\gamma}}{1 - \gamma} \left[\frac{\alpha_t}{\alpha_t + \beta_t} (R_f + f_t u)^{1-\gamma} k_{t+1}(\alpha_t + 1, \beta_t) \right. \right. \\
&\qquad \left. \left. + \frac{\beta_t}{\alpha_t + \beta_t} (R_f - f_t d)^{1-\gamma} k_{t+1}(\alpha_t, \beta_t + 1) \right] \right\}.
\end{aligned}
\tag{6}
$$

We let $k_{t+1}^u \equiv k_{t+1}(\alpha_t + 1, \beta_t)$ and $k_{t+1}^d \equiv k_{t+1}(\alpha_t, \beta_t + 1)$ to simplify the notation. The optimal investment from the first-order condition is

$$
f_t^* = \begin{cases}
\dfrac{(\alpha_t u\, k_{t+1}^u)^{\frac{1}{\gamma}} - (\beta_t d\, k_{t+1}^d)^{\frac{1}{\gamma}}}{(\alpha_t u\, k_{t+1}^u)^{\frac{1}{\gamma}} d + (\beta_t d\, k_{t+1}^d)^{\frac{1}{\gamma}} u} R_f & \text{if } \alpha_t u\, k_{t+1}^u > \beta_t d\, k_{t+1}^d \\[4mm]
0 & \text{otherwise.}
\end{cases}
\tag{7}
$$

The following proposition gives the functional form of the coefficient $k_t(\alpha_t, \beta_t)$:

Proposition 1. *The coefficient $k_t(\alpha_t, \beta_t)$ that satisfies the Bellman equation (Eq. 6) is given recursively by*

$$
\begin{aligned}
k_t(\alpha_t, \beta_t) &= \begin{cases}
\dfrac{(u + d)^{1-\gamma}}{u\, d} \dfrac{\left((\alpha_t k_{t+1}^u\, u)^{\frac{1}{\gamma}} d + (\beta_t k_{t+1}^d d)^{\frac{1}{\gamma}} u \right)^{\gamma}}{\alpha_t + \beta_t} R_f^{1-\gamma} & \text{if } \alpha_t u\, k_{t+1}^u > \beta_t d\, k_{t+1}^d \\[4mm]
\dfrac{\alpha_t k_{t+1}^u + \beta_t k_{t+1}^d}{\alpha_t + \beta_t} R_f^{1-\gamma} & \text{otherwise}
\end{cases} \tag{8} \\[4mm]
k_T(\alpha_T, \beta_T) &= 1. \tag{9}
\end{aligned}
$$

Proof of Proposition 1. This can be proven by substituting the optimal investment (Eq. 7) into the Bellman equation (Eq. 6). The functional form of $k_t(\alpha_t, \beta_t)$ given in the proposition satisfies the resulting equation. $\square$

We note two issues before characterizing the agent's behavior in the model. First, the

optimal investment depends only on current wealth and beliefs and not on the sequence of outcomes. Second, for comparison and for future reference, we can infer the behavior of a 'no parameter uncertainty' agent from the solution. If the agent knows the parameter p precisely—or does not update her beliefs over time—the coefficient $k_t(\alpha_t, \beta_t)$ becomes a function of time alone. It follows (from Eq. 7) that the date t optimal investment of the 'no parameter uncertainty' agent is

$$
f_t^* = \begin{cases} \dfrac{(p\,u\,)^{\frac{1}{\gamma}} - ((1-p)\,d)^{\frac{1}{\gamma}}}{(p\,u\,)^{\frac{1}{\gamma}}d - ((1-p)\,d\,)^{\frac{1}{\gamma}}u} R_f & \text{if } p\,u > (1-p)\,d \\ 0 & \text{otherwise.} \end{cases} \tag{10}
$$

The agent invests a positive amount if the risk premium is positive. Otherwise the agent would short the stock.

2.4 Characterizing Optimal Behavior

We now characterize the agent's optimal behavior in the model. All proofs are in Appendix A. We begin with the key result that an agent may stay out of the market even when the risk premium is strictly positive:

Proposition 2. *The optimal investment is decreasing in the variance of the prior distribution if $\gamma > 1$ and increasing in the variance if $\gamma < 1$.*

Corollary 1. $\exists\, \delta > 0$ *such that an agent with $\gamma > 1$ does not invest when $E_0(\tilde{\epsilon}) \leq \delta$ and an agent with $\gamma < 1$ invests a strictly positive amount when $E_0(\tilde{\epsilon}) \geq -\delta$.*

These results distinguish our life-cycle model from classical models where an agent invests a positive amount if the risk premium is positive. A mildly risk-averse agent with $\gamma < 1$ prefers uncertainty: the investment is increasing in the variance of the prior. This is the same as saying that the agent wants to take an actuarially unfair gamble. The agent's willingness to pay to learn *does not* generate this behavior—note that the agent observes realized returns even without an investment. The reason is that a $\gamma < 1$ agent "weights" positive outcomes more than negative outcomes. Starting from a situation with a negative risk premium, the agent knows that the risk premium may turn out to be positive. The agent maximizes her wealth in the states with good investment opportunities by investing today.

11

An agent with $\gamma > 1$, on the other hand, needs to be compensated for the extra source of risk brought on by parameter uncertainty. This generates a non-participation region: faced with enough uncertainty, an agent with $\gamma > 1$ allocates everything in the risk-free asset even if her prior about the risk premium is strictly positive. This difference to the $\gamma < 1$ case arises because this more risk-averse agent weights negative outcomes more than positive outcomes. Starting from a situation with a positive risk premium, the agent knows that the risk premium may turn out to be negative. The agent maximizes her wealth in the states with poor investment opportunities by staying out today.[8]

It is straightforward to show that if the short-sale constraints are lifted, the agent behaves similar to the Brennan (1998) agent. The future learning still matters—e.g., an agent with risk-aversion above $\gamma > 1$ always invests proportionally less than what she would invest in absence of parameter uncertainty—but the investment is always strictly positive if the risk premium is positive.[9] The non-participation result requires that the short-sale constraints bind in some future states—i.e., that the expected risk premium can turn negative. As noted earlier, this assumption is a shorthand way of modeling subjective investment opportunity sets. For example, an agent may invest in an MBA degree and receive labor income out of this degree for the rest of her life.

The following proposition determines what beliefs an agent who knows "nothing" about the expected return must have about p to invest in the asset:

Proposition 3. *An agent with maximally dispersed prior who faces two periods of investment* $(T = 2)$ *invests if and only if*

$$E_0(p) \geq \frac{1}{1 + \dfrac{u}{d} \left(\dfrac{u+d}{d} \right)^{1-\gamma}}. \tag{11}$$

---

[8]The interpretation of differences in weighting is intuitive. From the form of the value function, the solution to the model is similar identical to the no-learning case except that the agent "weights" different outcomes using k_ts as the weights. To see how these weights change, consider the last period of investment (time $T-1$). First, if an agent with $\gamma > 1$ does not invest, $k_{T-1}(\alpha_{T-1}, \beta_{T-1}) = 1$, and if the agent invests, $k_{T-1}(\alpha_{T-1}, \beta_{T-1}) < 1$. Hence, the agent weights poor future opportunities more, or put differently, the indirect utility is convex in some regions. Second, if an agent with $\gamma < 1$ does not invest, $k_{T-1}(\alpha_{T-1}, \beta_{T-1}) = 1$, and if the agent invests, $k_{T-1}(\alpha_{T-1}, \beta_{T-1}) > 1$. Hence, the agent weights good future opportunities more—the indirect utility function is more concave in some regions.

[9]The non-participation result does not depend on the assumption that there is no intermediate consumption. The solution is nearly identical with time-separable power utility; the distinction is that each period the agent first consumes some fraction of her wealth and then allocates the rest between the assets.

This proposition assumes that the agent has a completely non-informative prior—i.e., the variance of the prior is maximized by fixing $E_0(p)$ and letting $\alpha, \beta \to 0$—and gives the boundary for the uptick probability that guarantees a positive investment. Note that because the optimal investment is decreasing in the variance of the prior for $\gamma > 1$ -agents, this proposition gives the upper-bound of what these agents require of $E_0(p)$ when their priors became completely uninformative. We can use this result show that the non-participation region can be substantial even in a three-period setting. For example, suppose that the risk premium is symmetric around zero $(u = d)$. Then, while an agent with $\gamma = 2$ may require that the probability of an uptick is $E_0(p) \geq 0.67$, an agent with $\gamma = 5$ may stay out of the market until $E_0(p) \geq 0.94$! If $u = d = 20\%$, these boundaries correspond to (expected) risk premia of 6.7% and 17.6%, respectively.

Suppose that there is an outsider who observes the agent's behavior, ignores parameter uncertainty, and backs out what the agent's behavior implies about her risk-aversion. The following proposition shows this *implied risk-aversion* has an intuitive form:

Proposition 4. *An outsider who sets $p = E_0(p)$ infers the agent's risk-aversion as being*

$$\widehat{\gamma} = \gamma \frac{\log\left(\dfrac{\alpha u}{\beta u}\right)}{\log\left(\dfrac{\alpha u}{\beta d}\right) + \log\left(\dfrac{k^u}{k^d}\right)} \quad when \ \alpha u k^u > \beta d k^- \ and \ \alpha u > \beta d. \tag{12}$$

This implied risk-aversion is strictly higher than the true risk-aversion γ if $\gamma > 1$ and strictly less if $\gamma < 1$.

This measure quantifies the impact of parameter uncertainty and short-sale restrictions on portfolio choice. We know from Proposition 2 that an agent with $\gamma > 1$ always appears strictly more risk-averse than she really is to an outsider who ignores parameter uncertainty. At the limit, an agent who stays out of the market when $E_t[\tilde{\epsilon}] > 0$ appears infinitely risk-averse.

2.5 Examples

Figure 2 illustrates the results by plotting the optimal investment for an agent with a risk-aversion of $\gamma = 2$ and an investment horizon of $T = \{10, 50\}$ periods. The optimal investment is drawn as a function of the parameters of the prior distribution, (α, β). The non-participation
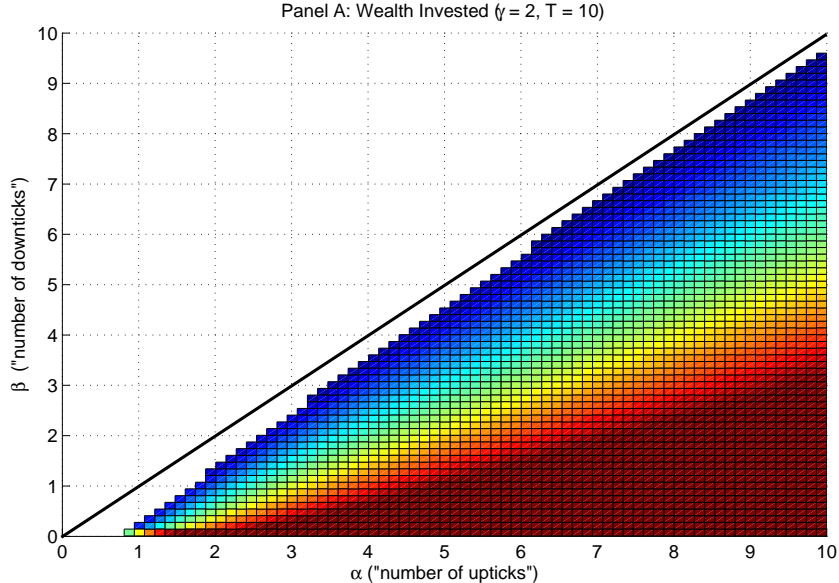
Figure 2: **Optimal Investment and Implied Risk-Aversion.** An agent with power-utility over terminal date T wealth trades a risky stock. The stock price follows a binomial process: $S_{t+1} = S_t(R_f + u)$ with probability p and $S_{t+1} = S_t(R_f - d)$ with probability $1 - p$. The agent has a Beta-distributed prior about p and updates her beliefs as a Bayesian. This figure sets $R_f = 1$, $u = d = 0.2$. Panels A and B show optimal investments for an agent with risk-aversion $\gamma = 2$ when there are $T = 10$ or $T = 50$ periods of investment. The optimal investment is drawn as a function of the parameters of the prior distribution, (α, β). The 45° line is the fair-gamble threshold; i.e., when $\alpha = \beta$, the agent's prior about the excess return is zero. The white area between the diagonal and the filled region is the non-participation region where the agent does not invest despite a positive risk premium. Panel C shows the *implied risk-aversion* for the $\gamma = 2$, $T = 10$ case. The implied risk-aversion (from Eq. 12) is the solution to an inference problem: how risk-averse does the agent appear to an outsider who ignores parameter uncertainty. The z-axis is truncated for implied risk-aversions above ten.

region is the white area between the 45° line and the filled area. Note that a Merton (1969) or Brennan (1998) agent would enter the market for all parameters to the right of the diagonal. In contrast, our agent stays out of the market for a wide range of parameters because of the risk of binding short-sale constraints.

The figure also plots the *implied risk-aversion* for the $T = 10$ period case. The implied risk-aversion increases sharply when the agent moves towards the non-participation region or when the variance of the prior increases. Note that the implied risk-aversion is significantly above the true value of $\gamma = 2$ for all parameters in the figure.[10] These life-cycle model results

---

[10]These plots are for an investor with moderate risk-aversion, $\gamma = 2$, and the results are more dramatic for more risk-averse agents. Many studies find relative risk-aversions above two. For example, Nielsen and Vissing-Jørgensen (2005) get an estimate around 5 from data on labor incomes and educational choices; Halek and Eisenhauer (2001) obtain a distribution of estimates with a median of 0.89 and a mean of 3.4 with insurance data; Bliss and Panigirtzoglou (2004) derive ("representative agent") mean estimates between 2 and 8 from the FTSE100 and S&P 500 option prices; Brennan and Xia (2001) suggest that a relative risk-aversion as high as

Panel B: Wealth Invested ($\gamma = 2$, $T = 50$)

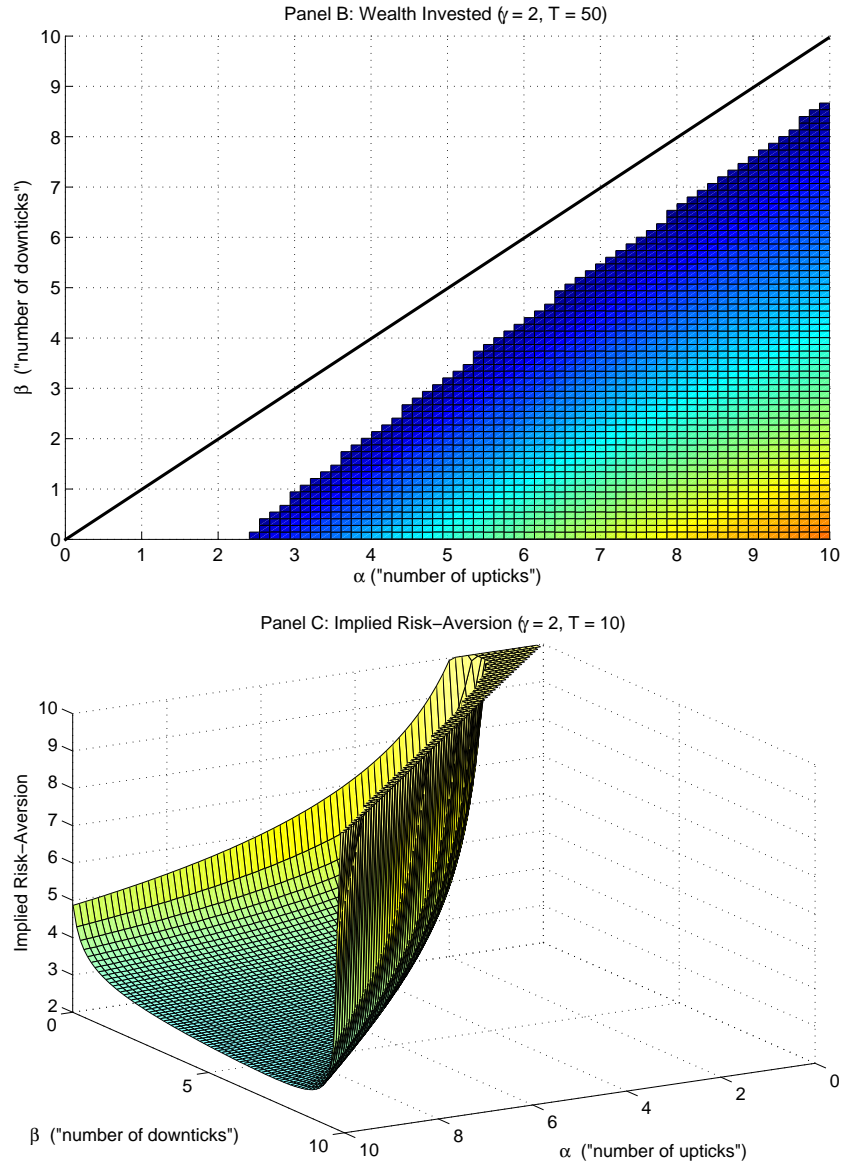Panel C: Implied Risk–Aversion ($\gamma = 2$, $T = 10$)

Figure 2: **Optimal Investment and Implied Risk-Aversion** (cont'd)

show that a short-sale constrained investor with limited information may optimally stay out of the market even if the (perceived) risk premium is sizable.

We have focused on the possibility that a short-sale constrained agent may stay completely out of the market because of the feedback from the short-sale constraints. However, more generally, the possibility of binding constraints always reduces the optimal date 0 portfolio holdings of a $\gamma > 1$ agent. For example, suppose that a $\gamma = 2$ agent with an initial wealth of

---

15 may be reasonable on theoretical grounds.

$10,000 is offered two double-or-nothing gambles and that the agent has a prior Beta$(0.23, 0.2)$ about the probability of winning ($E_0(p) = 0.535$). The optimal date 0 investment is very sensitive to learning and constraints:

- **No learning, no short-sale constraints.** The agent takes $p = 0.535$ as a fixed parameter and invests \$349.

- **Learning, no short-sale constraints.** The agent invests \$289.

- **Learning, short-sale constraints.** The agent stays out.

If we now fix the mean and decrease the variance of the prior by moving to Beta$(0.575, 0.5)$-distribution, the second scenario investment increases to \$296 and the optimal investment under short-sale constraints becomes positive but is still only \$162. We now turn to an equilibrium model that dispenses some of our unrealistic assumptions and shows that the same non-participation mechanism arises with permanent labor income shocks.

3 An Equilibrium Model with Non-Participation

This section solves an heterogeneous-agents equilibrium model where one of the agents may lose her job at a later date. The purpose of this model is two-fold. First, we show that the uncertainty about *future* labor income can significantly skew *today's* decisions when the agent is restricted from hedging with the risky asset. This generates the same type of non-participation as observed in Section 2's life-cycle model: the agent stays out only because of the *risk* of binding constraints. Second, we examine what consequences this type of non-participation has on the risk premium. For example, the first intuition would be that an introduction of short-sale constraints (if they matter at all) would reduce the risk premium: because the remaining agents only need to hold the entire market and not more, they require smaller compensation for risk.[11] However, we show that non-participation from the feedback effect can lead to a higher risk premium.

---

[11]For example, Cao, Wang, and Zhang (2005) generate non-participation with ambiguity aversion and find that the risk premium in the full economy is always higher than what it is in the limited participation economy.

3.1 The Economy

We assume the following:

- There are two agents, indexed $i \in \{A, B\}$, who live for three periods, $t = 0, 1, 2$. Trading takes place at dates 0 and 1.

- The agents maximize power utility over date 2 wealth,

$$U^i = \frac{W_2^{i^{1-\gamma}} - 1}{1 - \gamma}. \tag{13}$$

  The agents have the same risk-aversion parameter.

- There is a single risky asset in unit net supply. This asset pays a high dividend D_h with probability p and a low dividend D_l with probability $1 - p$ at dates 1 and 2, where $D_h > D_l$.

- A risk-free technology with a gross-return of R is in perfectly elastic supply.

- The agents are initially endowed shares $\bar{\theta}^i$ and consumption good $\bar{X}^i$.

- There are short-sale restrictions on the risky asset, $\theta_t^i \geq 0$, where θ_t^i is the agent i's tradable asset holdings at date t.

- The agents are endowed with a non-tradable asset (e.g., human capital) that pays income (e.g., wage) at dates 1 and 2. If the date 1 dividend is high, agent i receives a payoff of $y_1^{i,h}$ at date 1. If the dividend is low, the agent receives $y_1^{i,l}$.

- The date 2 income is *contingent* on the date 1 dividend. If the date 1 dividend is high, the date 2 income is $y_{2,h}^{i,h}$ or $y_{2,h}^{i,l}$ and if the dividend is low, the date 2 income is $y_{2,l}^{i,h}$ or $y_{2,l}^{i,l}$.

The last assumption lets agents resolve uncertainty about their future income at date 1. A natural interpretation for this date 1 signal is the risk of losing a job due to a macroeconomic shock. The key insight captured by the model is that permanent labor income shocks are positively correlated with macroeconomic shocks. We assume two states and perfect correlation
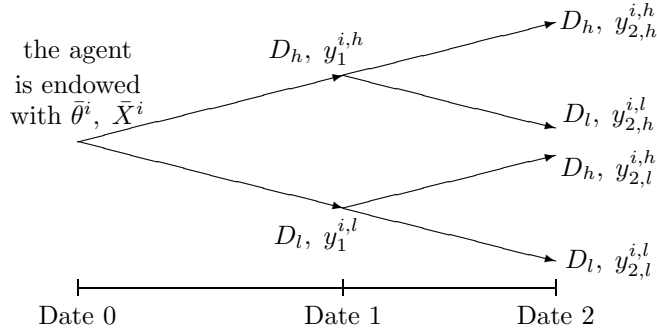
Figure 3: **The Timeline for Agent i in the Equilibrium Model**

between the assets for tractability.[12]

Figure 3 shows the timeline of the events for Agent i. The agent starts at date 0 with an endowment of shares and the consumption good. She decides how much to hold of the risky asset and puts the rest into the risk-free asset. The agent receives dividend and non-tradable asset income at date 1. She also learns the values of the date 2 incomes and then makes the date 1 investment decisions. Finally, the agent receives date 2 payoffs and consumes her terminal wealth.

We proceed as follows in the remainder of this section. First, we solve the equilibrium prices in an economy that does not have short-sale constraints. Second, we compute the equilibrium prices for a particular type of a short-sale constrained economy ("a non-participation economy") and give conditions under which these prices constitute equilibrium. Third, we show that the risk premium in the constrained economy is higher than in an unconstrained economy (i.e., an otherwise identical economy but without short-sale constraints) in particular when one of the agents is close to being indifferent between participating and not participating.

3.2   Equilibrium without Short-Sale Constraints

We first solve the date 1 problem and then move backwards to the date 0 problem. Note that the agent i's wealth constraints bind with equality:

---

[12]The assumption that labor income and dividend streams are perfectly correlated is a very particular assumption. This does, however, capture the idea that labor income shocks are affected by market conditions: when an agent's labor income stream covaries positively with the dividends, the agent is effectively already invested in the market by default, creating a hedging demand.

$$
\begin{aligned}
\text{Date 0:} \quad & \bar{\theta}^i P_0 + \bar{X}^i & = & \quad \theta_0^i P_0 + X_0^i \\
\text{Date 1:} \quad & \theta_0^i (P_1 + D_1) + X_0^i R + y_1^i & = & \quad \theta_1^i P_1 + X_1^i \\
\text{Date 2:} \quad & \theta_1^i D_2 + X_1^i R + y_2^i & = & \quad W_2^i
\end{aligned}
$$

where X_t^i is the amount borrowed or lent at the rate R. After substituting out X_t^i, we have

$$
W_2^i \;=\; W_1^i R + \theta_1^i (D_2 - P_1 R) + y_2^i, \tag{14}
$$

$$
\text{where} \quad W_1^i \;\equiv\; (\bar{X}^i + \bar{\theta}^i P_0) R + \theta_0^i (P_1 + D_1 - P_0 R) + y_1^i.
$$

3.2.1 Date 1 Decisions and Prices

Agent i's date 1 Bellman equation in state $s = \{h, l\}$ solves

$$
V_{1,s}^i (W_1^i) = \max_{\theta_1} \left\{ E_1 \left[\frac{\left(W_1^i R + \theta_1^i (\tilde{D}_2 - P_1 R) + \tilde{y}_{2,s}^i \right)^{1-\gamma}}{1-\gamma} \right] \right\}. \tag{15}
$$

The optimal demand from the first-order condition is[13]

$$
\theta_1^{i*} (W_1^i) = \frac{[p(D_h - P_1^s R)]^{-\frac{1}{\gamma}} (W_{1,s}^i R + y_{2,s}^{i,h}) - [(1-p)(P_1^s R - D_l)]^{-\frac{1}{\gamma}} (W_{1,s}^i R + y_{2,s}^{i,l})}{[p(D_h - P_1^s R)]^{-\frac{1}{\gamma}} (D_h - P_1^s R) + [(1-p)(P_1^s R - D_l)]^{-\frac{1}{\gamma}} (P_1^s R - D_l)}. \tag{16}
$$

The date 2 equilibrium price results from summing the agents' first order conditions and using the market-clearing condition, $\sum_i \bar{\theta}^i = \sum_i \theta_0^i = \sum_i \theta_1^i = 1$:

$$
P_1^{s*} \;=\; \frac{1}{R} \frac{p(\omega_{1,s}^h)^{-\gamma} D_h + (1-p)(\omega_{1,s}^l)^{-\gamma} D_l}{p(\omega_{1,s}^h)^{-\gamma} + (1-p)(\omega_{1,s}^l)^{-\gamma}}, \tag{17}
$$

$$
\text{where} \quad \omega_{1,s}^{s'} \;\equiv\; R^2 \sum_i \bar{X}^i + R \left(D_1 + \sum_i y_{1,s}^i \right) + \sum_i y_{2,s}^{i,s'} + D_{s'}.
$$

The price is a weighted average of date 2 dividends where the weights are functions of total wealth in the two states and their probabilities. The initial distribution of allocations does not matter because both agents' non-tradable asset income is perfectly correlated with dividends: the agents can use the tradable asset to hedge perfectly against the income risk.

---

[13]Substituting the optimal demand back into Eq. 15, the value function becomes $V_{1,s}^i (W_1^i) = \frac{[(D_h - D_l) W_{1,s}^i + c^{i,s}]^{1-\gamma}}{1-\gamma} k_s$. (See Eq. 18 for $c^{i,s}$ and k_s.)

3.2.2 Date 0 Decisions and Price

Agent i's date 0 Bellman equation solves

$$
\begin{aligned}
V_0^i(\bar{X}^i, \bar{\theta}^i) &= \max_{\theta_0} \left\{ E_0 \left[\frac{\left((D_h - D_l)[(\bar{X}^i + \bar{\theta}^i P_0)R^2 + \theta_0(\tilde{P}_1 + \tilde{D}_1 - P_0 R)R + \tilde{y}^i R] + \tilde{c}^i\right)^{1-\gamma}}{1-\gamma} \tilde{k} \right] \right\}, \\
\text{where} \quad c^{i,s} &\equiv y_{2,s}^{i,h}(P_1^s R - D_l) + y_{2,s}^{i,l}(D_h - P_1^s R), \\
k^s &\equiv \left(p^{\frac{1}{\gamma}}(P_1^s R - D_l)^{1-\frac{1}{\gamma}} + (1-p)^{\frac{1}{\gamma}}(D_h - P_1^s R)^{1-\frac{1}{\gamma}} \right)^{\gamma}.
\end{aligned}
\tag{18}
$$

The optimal demands can be solved from the first-order conditions. The date 0 equilibrium price follows from summing the agents' first order conditions and using the market-clearing condition:

$$
P_0^* = \frac{1}{R} \frac{p k^h \omega_{0,h}^{-\gamma}(P_1^h + D_h) + (1-p)k^l \omega_{0,l}^{-\gamma}(P_1^l + D_l)}{p k^h \omega_{0,h}^{-\gamma} + (1-p)k^l \omega_{0,l}^{-\gamma}},
\tag{19}
$$

$$
\text{where} \quad \omega_{0,s} = (D_h - D_l)\left(R^2 \sum_i \bar{X}^i + R(P_1^s + D_s + \sum_i y_{1,s}^i) \right) + \sum_i c^{i,s}.
$$

The equilibrium price is a weighted average of date 1 dividends and the prices in the two states. The weights are functions of total date 1 wealth in the two states and their probabilities. The following proposition summarizes these results:

Proposition 5. *The equilibrium prices in the unconstrained economy are given by Eqs. 17 (the date 1 prices) and 19 (the date 0 price).*

Note that the prices in the unconstrained economy do not depend on how the initial allocation is distributed between the two agents because the markets are effectively complete. Hence, the prices in the economy would be the same if there was only a representative agent who received all the endowments.

3.3 Equilibrium with Short-Sale Constraints

The non-tradable asset income in the economy can generate negative asset demands. For example, suppose that Agent A's income covaries positively with dividends after a low date 1 dividend but that Agent B's income is constant. Agent A would then short the asset after a

low dividend if the covariance were sufficiently high. The introduction of short-sale constraints has two effects. First, the direct consequence is that Agent A increases her date 1 holdings after a low date 1 dividend from negative to nothing. Second, the restriction on the agent's ability to hedge at date 1 may induce Agent A to reduce her *date 0* holdings. This effect may be significant enough to let Agent B hold the whole supply while Agent A exits the market. We now construct this equilibrium.

3.3.1 Equilibrium Prices in a Non-Participation Economy

The equilibrium prices cannot, in general, be solved in closed-form when there are short-sale constraints.[14] We focus on an exception where Agent A (1) has no initial endowment of the tradable asset, (2) optimally decides not to hold any asset at date 0 or at date 1 after a low dividend, and (3) has a holding between 0 and 1 at date 1 after a high dividend. (We later give the conditions to verify the optimality.) We also add the following assumptions for the sake of tractability:

1. The risk-free asset yields $R = 1$.

2. Agent A receives non-tradable asset income only at date 2.

3. Agent B does not receive non-tradable asset income.

4. The agents are not endowed any consumption good, $\bar{X}^A = \bar{X}^B = 0$.

   (Henceforth, we omit agent and date identifiers from y because there is no ambiguity; we write Agent B's date 2 income as $y_s^{s'}$ for $s, s' \in \{h, l\}$.) The following pricing formulas follow directly from Proposition 5.

Corollary 2. *Non-participation equilibrium is equilibrium where Agent A has zero demand for the risky asset at date 0 and at date 1 after a low dividend. Both agents have strictly*

---

[14]The difficulty is that if the date 1 constraints bind for one agent (i.e., the marginal utility at zero holdings is negative), the market-clearing conditions together with the first-order conditions for the remaining agents are not (generally) enough to solve for equilibrium prices.

positive demands at date 1 after a high dividend. The non-participation equilibrium prices are

$$P_1^h = \frac{p(2D_h + y_h^h)^{-\gamma}D_h + (1-p)(D_h + D_l + y_h^l)^{-\gamma}D_l}{p(2D_h + y_h^h)^{-\gamma} + (1-p)(D_h + D_l + y_h^l)^{-\gamma}}, \tag{20}$$

$$P_1^l = \frac{p(D_h + D_l)^{-\gamma}D_h + (1-p)(2D_l)^{-\gamma}D_l}{p(D_h + D_l)^{-\gamma} + (1-p)(2D_l)^{-\gamma}}, \tag{21}$$

$$P_0 = \frac{pk^h(P_1^h + D_h)^{1-\gamma} + (1-p)k^l(P_1^l + D_l)^{1-\gamma}}{pk^h(P_1^h + D_h)^{-\gamma} + (1-p)k^l(P_1^l + D_l)^{-\gamma}}, \tag{22}$$

$$where \quad k^s = \left(p^{\frac{1}{\gamma}}(D_h - P_1^s)^{\frac{1}{\gamma}-1} + (1-p)^{\frac{1}{\gamma}}(P_1^s - D_l)^{\frac{1}{\gamma}-1}\right)^\gamma.$$

Note that the equilibrium price after a high dividend is the same as it is in the unconstrained economy. Also, the distribution of Agent A's non-tradable asset income after a low dividend does not affect any of the prices. We now give the conditions on $\{y_l^l, \ y_l^h, \ y_h^l, \ y_h^h\}$ that guarantee that the prices in Eqs. 20, 21, and 22 constitute equilibrium. We also show that there is such an equilibrium. The proof is in the appendix.

Proposition 6. *The prices in Corollary 2 are the market-clearing prices if*

- *Conditions 1A and 1B (Agent A's optimal holding between zero and one after a high date 1 dividend):*

$$\begin{array}{rcll} \frac{y_h^h}{y_h^l} & \leq & \frac{2D_h}{D_h + D_l}, & (1A) \\[2mm] y_h^l - y_h^h & \leq & D_h - D_l. & (1B) \end{array} \tag{23}$$

- *Condition 2 (Agent A's optimal holding zero after a low date 1 dividend):*

$$\frac{y_l^h}{y_l^l} \geq \frac{D_h + D_l}{2D_l}. \tag{24}$$

- *Condition 3 (Agent A's optimal holding zero at date 0):*

$$\left[\frac{p(2D_h + y_h^h)^{-\gamma}y_h^h + (1-p)(D_h + D_l + y_h^l)^{-\gamma}y_h^l}{p(2D_h + y_h^h)^{-\gamma}2D_h + (1-p)(D_h + D_l + y_h^l)^{-\gamma}(D_h + D_l)}\right]^{-\gamma}$$
$$\leq \frac{p(y_l^h)^{-\gamma} + (1-p)(y_l^l)^{-\gamma}}{p(D_h + D_l)^{-\gamma} + (1-p)(2D_l)^{-\gamma}}. \tag{25}$$

This system of inequalities has multiple solutions $\{y_h^h, \ y_h^l, \ y_l^h, \ y_l^l\}$ for any $D_h > D_l$.

22

Conditions 1A, 1B, and 2 are intuitive. Conditions 1A and 1B state that Agent A's income cannot have a too high or too low cash-flow covariance with the tradable asset after a high date 1 dividend. Otherwise, the agent would want to short or hold more than the entire supply of the asset, respectively. Condition 2 gives the boundary for the ratio of Agent A's income after a low dividend that ensures that the agent does not want to hold any of the asset—note that this condition mirrors Condition 1A.[15]

The expression in Condition 3 is less obvious. It states that Agent A must derive higher expected utility from staying out of the market than from buying an infinitesimal amount of the asset at the equilibrium price. Note that the LHS of Condition 3 is decreasing in y_h^h and y_h^l and the RHS is decreasing in y_l^h and y_l^l. Hence, this condition says that the date 2 income following a low date 1 dividend must be small relative to the high-state income. If an agent's income is low after a low dividend, an agent ending up in this state has high marginal utility. Hence, the intuition for the condition is that the marginal utility in the upstate must be sufficiently lower than the marginal utility in the downstate. If this is the case, the agent wants to hedge against the risk of ending up in the downstate by staying out of the market at date 0. (Note that Condition 3 is always satisfied for sufficiently small y_l^h and y_l^l. In addition, agents with log-preferences can meet all the conditions. The reason why these agents do not behave myopically is that the date 1 market incompleteness creates a kink in these agents' indirect utilities.[16])

3.3.2   Example

Figure 4 shows feasible parameters for the non-tradable asset income after a low date 1 dividend, $\{y_l^h, y_l^l\}$, that generate non-participation equilibria. The figure assumes that both agents have log-preferences and that Agent A receives constant income after a high date 1 dividend. (See the figure text for the parameters of the example economy.) The x-axis is the value of

---

[15]Note that there is a minor openness consideration with Conditions 1A, 1B, and 2 in Proposition 6. These conditions must be satisfied with strict inequality for equilibrium to hold for sure. Note that if this is not the case—i.e., if one of the agents is indifferent between participating and not—the agent is precisely at the kink of her indirect utility function at date 1. Then, the date 0 zero analysis of how an infinitesimal increase in the date 0 holdings affects the agent's utility is invalid. We write these conditions with non-strict inequality to remind that the equalities denote the agents' indifference points.

[16]In related research, Cochrane, Longstaff, and Santa-Clara (2005) and Longstaff (2005) analyze "Two Lucas (1978) Trees" models and show that market-clearing and black-out (i.e., non-tradability) periods also cause log-utility investors to behave non-myopically.
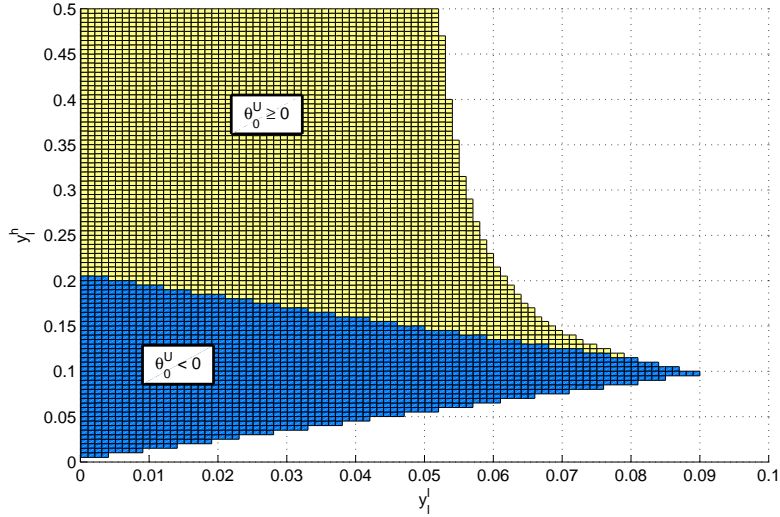
Figure 4: **Non-Participation Equilibrium.** This figure shows feasible parameters for non-tradable asset income $\{y_l^h, y_l^l\}$ that generate the non-participation equilibrium of Proposition 2. The following parameters are fixed: $D_h = 1.1$, $D_l = 1$, $\gamma = 1$, $p = 0.5$, $y_h^h = y_h^l = 0.1$ (i.e., the agents have log-preferences). The x-axis is the low-state income after a low date 1 dividend, y_l^l. The y-axis is the high-state income after a low date 1 dividend, y_l^h. The colored grid indicates the set of parameters that generate the non-participation economy. This region is divided into two sections to indicate what type of a position Agent A would take in an unconstrained economy. The darker region denotes cases where an unconstrained Agent A takes a short position in the risky asset at date 0. The lighter region denotes parameters where an unconstrained Agent A takes a long position in the risky asset at date 0.

low-state income and y-axis is the value of the high-state income. Note that the cash-flow co-variance between Agent A's non-tradable asset and the tradable asset is positive when $y_l^h > y_l^l$; hence, by keeping the x-axis value fixed and increasing the value on the y-axis, the tradable asset becomes increasingly worse to the agent.[17]

The non-participation equilibrium is divided into two areas to indicate what type of date 0 position Agent A would take in an unconstrained economy (i.e., an otherwise identical economy but without short-sale constraints). The darker area represents cases where Agent A takes a short position at date 0 in the unconstrained economy. The lighter area consists of more interesting equilibria where Agent A's strictly positive date zero holding disappears when the trading constraints are introduced. Here, the agent would take a long position if it

---

[17]Note that the y-axis in the figure is truncated at 0.5—the set of equilibria continues beyond the boundaries of the figure.

were not for the *possibility* of binding short-sale constraints.[18]

3.4   Risk Premium in the Non-Participation Economy

We now discuss how the date 0 risk premium changes when we move from the unconstrained economy to the short-sale constrained economy. The proof of the following proposition is in the appendix:

Proposition 7. *The expected date 0 payoff is always higher in the constrained economy than in an otherwise identical economy but without short-sale constraints ("the unconstrained economy"). The risk premium is higher in the constrained economy when*

1. *Agent A is close to participating at date 1 after a low dividend ($\frac{y_l^h}{y_l^l} \approx \frac{D_h+D_l}{2D_l}$; see Condition 2 of Proposition 6)*

2. *and Agent A's date 2 nonfinancial income in the upstate is small relative to the dividends ($y_h^h, y_h^l \ll D_h, D_l$).*

These are sufficient conditions.

The second part of the proposition gives sufficient conditions for the date 0 price to be strictly lower in the constrained economy. Because the expected payoff in the constrained economy is always higher than the expected payoff in the unconstrained economy, less strict conditions in practice guarantee that the difference in risk premia between the constrained and unconstrained economies is positive. The necessary condition is that the date 0 price cannot change *too much* in response to the introduction of constraints to reverse the effect of the higher expected payoff.

However, these stricter conditions have an interesting and important implication: the risk premium in the constrained economy is particularly high (relative to the unconstrained economy) when it is most puzzling that Agent A stays out—i.e., when Agent A is indifferent

---

[18]Agent A takes a long position at date 0 in the unconstrained economy if

$$\frac{p(2D_h + y_h^h)^{1-\gamma} + (1-p)(D_h + D_l + y_h^l)^{1-\gamma}}{p(2D_h + y_h^h)^{-\gamma}(2D_h) + (1-p)(D_h + D_l + y_h^l)^{-\gamma}(D_h + D_l)}$$
$$\leq \frac{p(D_h + D_l + y_l^h)^{1-\gamma} + (1-p)(2D_l + y_l^l)^{1-\gamma}}{p(D_h + D_l + y_l^h)^{-\gamma}(D_h + D_l) + (1-p)(2D_l + y_l^l)^{-\gamma}(2D_l)}. \qquad \text{(Condition 3')} \quad (26)$$
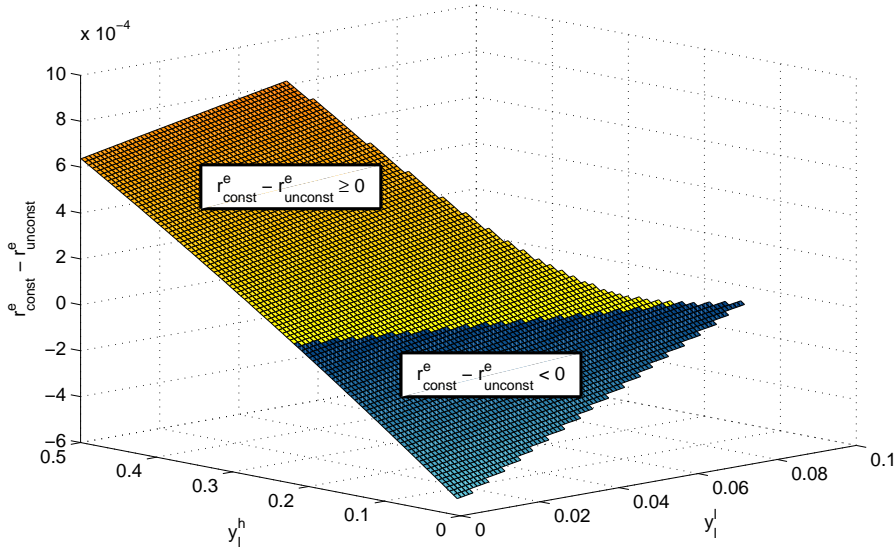
Figure 5: **Relative Risk Premium in the Short-Sale Constrained Economy.** This figure shows the relative risk premia in equilibria where Agent A does not hold any risky asset at date 0 or after a low dividend at date 1 ("non-participation equilibrium"). The relative risk premium is defined as the difference between the risk premium in the constrained economy and the risk premium in otherwise identical but unconstrained economy, $r^e_{\text{const}} - r^e_{\text{unconst}}$. The change in the shading from dark to light indicates where the risk premium difference turns positive. The following parameters are fixed: $D_h = 1.1$, $D_l = 1$, $\gamma = 1$, $p = 0.5$, $y^h_h = y^l_h = 0.1$ (i.e., the agents have log-preferences). The x-axis is the low-state income after a low date 1 dividend, y^l_l. The y-axis is the high-state income after a low date 1 dividend, y^h_l. The equity-premium in the short-sale constrained economy is constant, 0.06%, because it does not depend on parameters $\{y^h_l, y^l_l\}$.

between participating and not participating at date 1. Moreover, these necessary conditions turn out to be the same that guarantee that Agent A buys a strictly positive amount of the asset in the unconstrained economy.[19]

Figure 5 uses the same parameters as the example economy of Figure 4 and draws the difference in the risk premia between the constrained and unconstrained economies. By Corollary 2, the risk premium in the constrained economy does not depend on the income ratio after a low dividend. Hence, in the figure, the risk premium in the constrained economy is always 0.06% because everything but $\{y^h_l, y^l_l\}$ is fixed. However, the risk premium in the unconstrained economy depends on these values. In particular, note that unconstrained-economy risk premium is decreasing in both y^l_l and y^h_l. Hence, moving towards the region where the agent would participate if it were not for the short-sale constraints (i.e., towards higher values

---

[19]To see, this suppose that $y^h_l = \alpha(D_h + D_l)$ where $\alpha > 0$ and let Condition 2 first bind with equality, $y^l_l = \frac{2D_l}{D_h + D_l} y^h_l$. Then, taking the limit $y^h_h, y^l_h \to 0$ of the LHS of Condition 3′, the condition becomes $1 \leq 1 + \alpha$. Because this holds with strict inequality, we can choose $y^l_l < \frac{2D_l}{D_h + D_l} y^h_l$ and get equilibrium. This shows that Agent A would take a long position in the risky asset at date 0 if it were not for the *future* short-sale constraints.

of y_l^h), the risk premium increases. Note that in this case the area with positive (relative) risk premium almost coincides with the "positive unconstrained investment"-area in Figure 4. Although this does not always need to be the case, Proposition 7 ensures that if we move far enough into the correct direction without breaking equilibrium, the risk premium spread is eventually positive.

3.5 Implications of the Model

Our model generates non-participation by assuming that individuals may face binding trading restrictions in the future. A natural interpretation for the date 1 signal is the risk of losing a job due to a macroeconomic shock. Poor economic growth induces firms to cut workforce and dividends. If an agent loses her job because of an economic downturn (i.e., a low date 1 dividend), she faces a positive covariance between her wage and dividends; a subsequent turnaround in the economy makes it more probably that the agent is hired and dividends increase. However, a further weakness in the economy means that dividends stay low and the agent is likely to remain jobless. In this context, the short-sale constraint assumption states that the jobless agent at date 1 finds it impossible to open a margin account and short the market to hedge against the risk of not finding a new job. If there is no initial downturn at date 1 (i.e., a high date 1 dividend), the agent retains her job.[20]

There is empirical support to the idea that *learning* about labor income (and not just the contemporaneous covariance with the stock market) may affect market participation and asset prices. First, Malloy, Moskowitz, and Vissing-Jørgensen (2005b) find that stockholders' (i.e., excluding non-participants) *long-run* consumption risk does particularly well in pricing asset returns. Malloy, Moskowitz, and Vissing-Jørgensen (2005a) find asset pricing success by using firing/hiring data to measure persistent labor income shocks, consistent with our interpretation of the model's date 1 signal. Second, Guvenen (2005) considers a model where individuals enter the labor market with a prior belief about their income profiles and estimates that individuals can forecast (only) 40 percent of variation in income rates at time zero. This suggests that uncertainty about future labor income is a very real source of risk to most individuals. Finally,

---

[20]An alternative set of assumptions that generate same type of prediction involves the housing market. The housing market is illiquid and positively correlated with the stock market. The extant research has recognized the importance of homeownership on asset allocation (Cauley, Pavlov, and Schwartz 2005; Yao and Zhang 2005). Homeownership acts in the same way as human capital in the context of our model. An agent may not want to invest in the stock market because the home is already effectively such an investment.

Vissing-Jørgensen (2002b) finds that a higher *volatility* of nonfinancial income has a negative impact on the probability of market participation. This is consistent with our model where it is the uncertainty about future labor income, not the current covariance between labor income and stock returns that generates non-participation. For example, in our non-participation equilibrium, the date 0 one-period correlation between labor income and the asset return is trivially zero.

The most interesting feature of our model is not that it generates non-participation but what it implies about the risk-sharing and risk premium in the economy. The standard motivation for the limited participation puzzle is the question why many individuals choose to stay out despite very high historical equity premium. An implication of our model is that the risk premium is relatively high in the constrained economy precisely when it is most puzzling that some agents decide to stay out. For example, an agent with log-preferences would take a long position in the risky asset if it were not for the risk of facing binding constraints in the *future*. The practical implication of this result is important: an econometrician who ignores uncertainty, frictions, and the permanent shocks to investors' investment opportunities may encounter difficulties in explaining not only limited participation but also the risk premium.

4   Conclusions

This paper describes an intuitive mechanism that keeps some individuals out of the market even when there are no participation costs and when the current equity premium is high: uncertainty about investment opportunities and learning together with the possibility of binding trading restrictions. The life-cycle model illustrates this mechanism. Suppose that an agent has a prior about the risk premium and revises her beliefs after each new observation. A consequence of this setup is that the future changes in investment opportunities are positively correlated with realized returns. (For example, the agent revises her beliefs downwards after observing a low return.) Thus, a short-sale constrained agent is unable to profit from her refined information in those future states where she has learned that her investment opportunities are poor. Moreover, because of the positive correlation between realizations and expectations, these states are precisely the ones where the agent's marginal utility is high. This creates a feedback to date zero decisions: because the agent knows that the constraints

may become binding in the future, she requires a higher risk premium at date 0.

We generalize this idea with an equilibrium model where agents resolve uncertainty about the covariance between a non-tradable asset ("human capital") and a risky asset ("stock"). This setup creates a similar feedback from future short-sale constraints: the *possibility* of being constrained in the future may be enough to induce the agent to leave the market at date 0. This feedback can thus create situation where the equity premium is high yet an agent with no nonfinancial income risk today stays out of the market. Our model has the interesting implication that the equity premium is relatively high precisely when it is most puzzling that some agents choose to stay out—i.e., when the agent is currently out of the market, but

1. would buy a positive amount of the asset if the short-sale constraints were lifted and

2. is close to being indifferent between participating and not participating in the future.

The result that agents may stay out of the market because of uninsurable shocks in the future is potentially important in explaining some negative results in the participation literature. In our model, the *risk* of a high covariance generates non-participation. The forward-looking expectation effect—i.e., agents stay out of the market *before* their labor income covaries positively with the market—may make it difficult to detect the role of labor income in microdata. For example, our model's mechanism can explain why Vissing-Jørgensen (2002b, pp. 33) concludes:

> "...*the consumption growth of non-stockholders covaries substantially less with the stock return than the consumption growth of stockholders... This indicates that the primary reason for nonparticipation is not that nonstockholders are faced with nonfinancial income which is highly correlated with stock market returns.*"

Our results suggest that learning and labor income shocks driven by macroeconomic conditions may simultaneously generate non-participation, a sizable equity premium, and an insignificant contemporaneous correlation between the stock return and the income of those who do not participate.

A   Proofs

We first prove Propositions 2, 3, and 4 for the case when $T = 2$ and when $\beta d - u < \alpha u < (\beta + 1)d$ holds for the prior distribution.[21] (We omit subscripts; e.g., α denotes α_0.) Eq. 7 shows that under this assumption learning matters: the agent invests at time 1 only if the date 0 outcome is positive. (If $\alpha u < \beta d - u$, the agent never invests at date 0; if $\alpha u > (\beta + 1)d$, the agent always invests at date 0. Propositions 2 and 4 also hold for the latter case.)

The optimal date 0 investment (if any) in this case is

$$
\begin{aligned}
f_0^* &= \frac{(\alpha u)^{\frac{1}{\gamma}}(u+d)^{\frac{1}{\gamma}-1}\left(((\alpha+1)u)^{\frac{1}{\gamma}}d + (\beta d)^{\frac{1}{\gamma}}u\right) - (\beta d)^{\frac{1}{\gamma}}(ud)^{\frac{1}{\gamma}}(\alpha+\beta+1)^{\frac{1}{\gamma}}}{(\alpha u)^{\frac{1}{\gamma}}(u+d)^{\frac{1}{\gamma}-1}\left(((\alpha+1)u)^{\frac{1}{\gamma}}d + (\beta d)^{\frac{1}{\gamma}}u\right)d - (\beta d)^{\frac{1}{\gamma}}(ud)^{\frac{1}{\gamma}}(\alpha+\beta+1)^{\frac{1}{\gamma}}u}R_f \\
&= \frac{\alpha^{\frac{1}{\gamma}}(u+d)^{\frac{1}{\gamma}-1}\left(((\alpha+1)u)^{\frac{1}{\gamma}}d + (\beta d)^{\frac{1}{\gamma}}u\right) - (rd)^{\frac{1}{\gamma}}(ud)^{\frac{1}{\gamma}}(\alpha+\beta+1)^{\frac{1}{\gamma}}}{u^{\frac{1}{\gamma}}(u+d)^{\frac{1}{\gamma}-1}\left(((\alpha+1)u)^{\frac{1}{\gamma}}d + (\beta d)^{\frac{1}{\gamma}}u\right)d - (rd)^{\frac{1}{\gamma}}(ud)^{\frac{1}{\gamma}}(\alpha+\beta+1)^{\frac{1}{\gamma}}u}R_f \quad (27)
\end{aligned}
$$

where we use Eq. 7 and Proposition 1. We define $r = \frac{\beta}{\alpha}$ in the second line. With this substitution, the variance of the prior is decreasing in α while the mean stays constant.[22]

Proofs of Proposition 2 and Corollary 1. The optimal date 0 investment in Eq. 27 can be written as

$$
f_0^* = \frac{g(\alpha) - h(\alpha)}{g(\alpha)d - h(\alpha)u}. \quad (28)
$$

Hence, the optimal investment is increasing in α iff

$$
g'(\alpha)h(\alpha) > g(\alpha)h'(\alpha) \quad (29)
$$

which is a condition about the relative concavity functions g and h. This inequality reduces to

$$
\left[\frac{\alpha dr}{(\alpha+1)u}\right]^{1-\frac{1}{\gamma}} < 0. \quad (30)
$$

The fraction inside brackets is always less than unity by the assumption that $\beta d - u < \alpha u$ (see

---

[21] Proposition 2 and Corollary 1 can be proven for the general T period problem with an induction argument while the proof of Proposition 4 remains the same. The boundary in Proposition 3 is specific to the three period model.

[22] We have, after a substitution, $E_t(p) = \dfrac{1}{1+r}$ and $\text{var}_t(p) = \dfrac{r}{(1+r)^2}\dfrac{1}{\alpha(1+r)+1}$. For future reference, note that $\lim\limits_{\alpha\to 0}\text{var}_t(p) = \dfrac{r}{(1+r)^2}$.

above). Hence, the inequality is satisfied iff $\gamma > 1$. This shows that the optimal investment is increasing in the variance of the prior if an agent is more risk-averse than a log-utility investor. The proof for the $\gamma < 1$ agent is similar.

We now prove the market participation result (Corollary 1). Suppose that $E_0(\tilde{\epsilon}) = \frac{\alpha}{\alpha+\beta}u - \frac{\beta}{\alpha+\beta}d = 0$, i.e. the prior about the stock's risk premium is zero. The condition for a positive investment from Eq. 27 becomes:

$$\frac{((\alpha+1)u)^{\frac{1}{\gamma}}d + (\beta d)^{\frac{1}{\gamma}}u}{((\alpha+1)ud + \beta ud)^{\frac{1}{\gamma}}} \geq (u+d)^{1-\frac{1}{\gamma}}. \tag{31}$$

Defining $c = \frac{\beta d}{(\alpha+1)u}$, the LHS of this inequality can be written as

$$L(c) = \frac{d + c^{\frac{1}{\gamma}}u}{(d+cu)^{\frac{1}{\gamma}}}. \tag{32}$$

First, we note that $L(1) = (u+d)^{1-\frac{1}{\gamma}}$. Second, we observe that $L'(c) > 0$ if $\gamma > 1$ and $L'(c) < 0$ if $\gamma < 1$. Third, note that $c < 1$ by assumption. These imply that if $\gamma > 1$, the LHS in Eq. 31 is strictly less than the quantity on the RHS, violating the inequality. Hence, an investor more risk-averse than a log-utility investor strictly prefers not investing when the risk premium is zero. By contrast, an agent with $\gamma < 1$ makes a strictly positive investment in the same situation. Because the optimal investment is a continuous in all the parameters, an agent with $\gamma > 1$ does not invest even when $E_0(\tilde{\epsilon}) = \delta$ with $\delta > 0$ and vice versa. $\square$

Proof of Proposition 3. The condition for a positive investment from Eq. 27 is:

$$\frac{((\alpha+1)u)^{\frac{1}{\gamma}}d + (\beta d)^{\frac{1}{\gamma}}u}{((\alpha+1)ud + \beta ud)^{\frac{1}{\gamma}}} \geq (u+d)^{1-\frac{1}{\gamma}}\left(\frac{rd}{u}\right)^{\frac{1}{\gamma}}. \tag{33}$$

We let the variance of the prior to tend to its maximum ($\alpha \to 0$), $\frac{r}{(1+r)^2}$. The condition for positive investment becomes

$$r \leq \frac{u}{d}\left(\frac{u+d}{d}\right)^{1-\gamma}. \tag{34}$$

We get the boundary by writing r in terms of the mean of the prior, $r = \frac{1}{E_0(p)} - 1$. This

boundary is

$$E_0(p) \geq \frac{1}{1 + \frac{u}{d}\left(\frac{u+d}{d}\right)^{1-\gamma}}. \tag{35}$$

(The equality is reached at the limit $\alpha \to 0$.) $\qquad\square$

Proof of Proposition 4. We get the expression for the implied risk-aversion by first setting $p = \frac{\alpha}{\alpha+\beta}$ and $\gamma = \widehat{\gamma}$ in the equation for optimal investment for the 'no parameter uncertainty' agent (Eq. 10). Eq. 12 follows from equating this with the optimal investment of the 'parameter uncertainty' agent (Eq. 7) and solving for $\widehat{\gamma}$. $\qquad\square$

Proof of Proposition 6. Conditions 1A, 1B, and 2 follow after some algebra from evaluating both agents' first-order conditions at $\theta_1 = 0$ after a high dividend and from evaluating Agent A's first-order condition at $\theta_1 = 0$ after a low dividend. To get Condition 3, first write down Agent A's date 0 problem:

$$V_0^A(\bar{X}^A, \bar{\theta}^A) = \max_{\theta_0} \left\{ p \frac{\left((D_h - D_l)(\theta_0(P_1^h + D_h - P_0)) + c^{A,h}\right)^{1-\gamma}}{1-\gamma} k^h \right.$$
$$\left. + (1-p)\left(p \frac{\left(\theta_0(P_1^l + D_l - P_0) + y_l^h\right)^{1-\gamma}}{1-\gamma} + (1-p)\frac{\left(\theta_0(P_1^l + D_l - P_0) + y_l^h\right)^{1-\gamma}}{1-\gamma} \right) \right\}. \tag{36}$$

(See Equation 18 for the values of k^h and $c^{A,h}$.) This problem takes into account the postulated form of the non-participation equilibrium: the agent invests a positive amount after a high dividend (this the indirect utility on the first line) but stays out of the market after a low dividend (the indirect utility on the second line). The condition on the indirect marginal utility is then

$$pk_h(D_h - D_l)(P_1^h + D_h - P_0)c^{A,h^{-\gamma}} + (1-p)(P_1^l + D_l - P_0)\left(py_h^{h-\gamma} + (1-p)y_l^{l-\gamma}\right) \leq 0. \tag{37}$$

Condition 3 follows from substituting in the equilibrium prices from Corollary 2.

We prove the existence of a solution by constructing one. First, choose $y_l^h = k_1(D_h + D_l)$ where $k_1 < 1$ and let Condition 2 to bind with equality to get $y_l^l = 2kD_l$. Next, choose $y_h^l = D_h + D_l$ and let Condition 1A to bind with equality to get $y_h^h = 2D_h$. Now, conditions 1A and 2 are satisfied by assumption. Condition 1B is satisfied exactly and the LHS in Condition 3 is equal to one. The RHS is strictly greater than one by the assumptions about y_l^l

32

and y_l^h. It follows from the strict inequality in Condition 3 and the continuity of all conditions that the income process parameters can be perturbed while retaining equilibrium. $\square$

Proof of Proposition 7. We first show that the expected date 0 payoff is higher in the constrained economy. First, note that the date 1 price after a high dividend is the same in both economies. Second, the unconstrained and constrained date 1 prices after a low dividend can be written as

$$P_{1,\text{uc}}^l = \frac{(D_h + D_l + y_l^h)^{-\gamma} D_h + (2D_l + y_l^l)^{-\gamma} D_l}{(D_h + D_l + y_l^h)^{-\gamma} + (2D_l + y_l^l)^{-\gamma}}, \tag{38}$$

$$P_{1,\text{c}}^l = \frac{(D_h + D_l)^{-\gamma} D_h + (2D_l)^{-\gamma} D_l}{(D_h + D_l)^{-\gamma} + (2D_l)^{-\gamma}}. \tag{39}$$

Let us now define function $\lambda(x)$ as

$$\lambda(x) = \frac{(D_h + D_l + x)^{-\gamma} D_h + (2D_l + kx)^{-\gamma} D_l}{(D_h + D_l + x)^{-\gamma} + (2D_l + kx)^{-\gamma}} \tag{40}$$

and note that $\lambda(0) = P_{1,\text{c}}^l$ and $\lambda(y_l^h) = P_{1,\text{uc}}^l$ for a proper choice of k. Differentiating, the condition $\frac{\partial}{\partial x}\lambda(x) \geq 0$ can be written as

$$k \geq \frac{2D_l}{D_h + D_l}. \tag{41}$$

This is the Condition 2 of Proposition 6 and hence, satisfied in equilibrium. This shows that the date 1 price after a low dividend is decreasing in nonfinancial income. Hence, $P_{1,\text{c}}^l \geq P_{1,\text{uc}}^l$, and because the state-probabilities are the same in the constrained and unconstrained economies, the expected payoff is higher in the constrained economy.

The constrained economy risk premium is higher than the risk premium in the unconstrained economy if

$$\frac{E[\text{Constrained Payoff}]}{E[\text{Unconstrained Payoff}]} \geq \frac{P_{0,\text{c}}}{P_{0,\text{uc}}}. \tag{42}$$

Because the expected constrained payoff is always higher than the expected unconstrained payoff, this condition is a requirement that the date 0 price cannot change "too much" to compensate for this increase. We prove a slightly stronger condition by examing when the date 0 price is lower in the constrained economy. The unconstrained and constrained date 0

prices can be written as

$$P_{0,\text{uc}} = \frac{pA_1 + (1-p)B_1}{pA_2 + (1-p)B_2}, \tag{43}$$

$$P_{0,\text{c}} = \frac{pwA_1 + (1-p)B_1'}{pwA_2 + (1-p)B_2'}, \tag{44}$$

$$\text{where} \quad A_1 = p(2D_h + y_h^h)^{-\gamma}2D_h + (1-p)(D_h + D_l + y_h^l)^{-\gamma}(D_h + D_l),$$

$$A_2 = p(2D_h + y_h^h)^{-\gamma} + (1-p)(D_h + D_l + y_h^l)^{-\gamma},$$

$$B_1 = p(D_h + D_l + y_l^h)^{-\gamma}(D_h + D_l) + (1-p)(2D_l + y_l^l)^{-\gamma}(2D_l),$$

$$B_2 = p(D_h + D_l + y_l^h)^{-\gamma} + (1-p)(2D_l + y_l^l)^{-\gamma},$$

$$B_1' = p(D_h + D_l)^{1-\gamma} + (1-p)(2D_l)^{1-\gamma},$$

$$B_2' = p(D_h + D_l)^{-\gamma} + (1-p)(2D_l)^{-\gamma},$$

$$w = \frac{p(2D_h + y_h^h)^{1-\gamma} + (1-p)(D_h + D_l + y_h^l)^{1-\gamma}}{p(2D_h + y_h^h)^{-\gamma}2D_h + (1-p)(D_h + D_l + y_h^l)^{-\gamma}(D_h + D_l)}.$$

First, note that for $y_h^h, y_h^l \geq 0$, $w \geq 1$, and when $y_h^h, y_h^l \to 0, w \to 1$.[23] Second, similar to our approach above, let us define function $\lambda(x)$ as

$$\lambda(x,w) = \frac{pwA_1 + (1-p)\left\{p(D_h + D_l + x)^{-\gamma}(D_h + D_l) + (1-p)(2D_l + kx)^{-\gamma}(2D_l)\right\}}{pwA_2 + (1-p)\left\{p(D_h + D_l + x)^{-\gamma} + (1-p)(2D_l + kx)^{-\gamma}\right\}} \tag{45}$$

Note that $\lambda(0,w) = P_{0,\text{c}}$ and that for a proper choice of k, $\lambda(y_l^h, 1) = P_{0,\text{uc}}$. The condition $\frac{\partial}{\partial x}\lambda(x,w) \leq 0$ can be written after some algebra as

$$wp^2(D_h + D_l + x)^{-\gamma-1}(2D_h + y_h^h)^{-\gamma}$$

$$+ (1-p)k(2D_l + kx)^{-\gamma-1}\left\{2p(2D_h + y_h^h)^{-\gamma} + (1-p)(D_h + D_l + y_h^l)^{-\gamma}\right\}$$

$$\geq (1-p)^2(D_h + D_l + x)^{-\gamma-1}(2D_l + kx)^{-\gamma-1}(2D_l - k(D_h + D_l)). \tag{46}$$

Note that if $k = \frac{2D_l}{D_h + D_l}$, this inequality is satisfied strictly. Hence, the constrained date 0 price is strictly lower than the unconstrained price if (i) Agent A's nonfinancial income $\{y_h^h, y_h^l\}$ is small relative to the dividends (i.e., $w \approx 1$ when $y_h^h, y_h^l \to 0$ or, equivalently, $D_h, D_l \to \infty$) and (ii) Agent A is close to being indifferent between participating and not participating after a

---

[23]This is a condition about the relative size of the dividends and nonfinancial income. Alternatively, holding y_h^h and y_h^l fixed, $w \to 1$ when $D_h, D_l \to \infty$.

low dividend at date 1 (i.e., $\frac{y_l^h}{y_l^l} = \frac{2D_l}{D_h+D_l}$). Because $\lambda(x,w)$ is continuous in all the parameters of the economy, the risk premium is strictly higher also when $\frac{y_l^h}{y_l^l} > \frac{2D_l}{D_h+D_l}$, $y_h^h, y_h^l > 0$ and $D_h, D_l < \infty$.

$\square$

References

Ang, A., G. Bekaert, and J. Liu (2005). Why stocks may disappoint. *Journal of Financial Economics*, forthcoming.

Arrow, K. J. (1965). *Aspects of the Theory of Risk Bearing*. Helsinki: Yrjö Jahnsson Foundation.

Baron, D. P. (1974). Information, investment behavior, and efficient portfolios. *Journal of Financial and Quantitative Analysis 9*, 555–566.

Basak, S. and D. Cuoco (1998). An equilibrium model with restricted stock market participation. *Review of Financial Studies 11*, 309–341.

Bliss, R. R. and N. Panigirtzoglou (2004). Option-implied risk aversion estimates. *Journal of Finance 59*, 407–446.

Brennan, M. J. (1998). The role of learning in dynamic portfolio decisions. *European Economic Review, 1*, 295–306.

Brennan, M. J. and Y. Xia (2001). Stock return volatility and equity premium. *Journal of Monetary Economics 47*, 249–283.

Cao, H., T. Wang, and H. H. Zhang (2005). Model uncertainty, limited market participation, and asset prices. *Review of Financial Studies 18*, 1219–1251.

Cauley, S. D., A. D. Pavlov, and E. Schwartz (2005). Homeownership as a constraint on asset allocation. University of California, working paper.

Cochrane, J. H. (2005). Financial markets and the real economy. University of Chicago, working paper.

Cochrane, J. H., F. A. Longstaff, and P. Santa-Clara (2005). Two trees. University of Chicago, working paper.

Constantinides, G. M. and D. Duffie (1996). Asset pricing with heterogeneous consumers. *Journal of Political Economy 104*, 219–239.

Cox, J. C., S. A. Ross, and M. Rubinstein (1979). Option pricing: A simplified approach. *Journal of Financial Economics 7*, 229–263.

DeGroot, M. H. (1970). *Optimal Statistical Decisions*. New York: McGraw-Hill College.

Detemple, J. B. (1986). Asset pricing in an economy with incomplete information. *Journal of Finance 61*, 383–392.

Dow, J. and S. Werlang (1992). Uncertainty aversion, risk aversion and the optimal choice of portfolio. *Econometrica 60*, 197–204.

Epstein, L. and J. Miao (2003). A two-person dynamic equilibrium under ambiguity. *Journal of Economic Dynamics and Control 27*, 1253–1288.

Epstein, L. and M. Schneider (2005). Learning under ambiguity. University of Rochester, working paper.

Gennotte, G. (1986). Optimal portfolio choice under incomplete information. *Journal of Finance 61*, 733–749.

Guvenen, F. (2005). Learning your earning: Are labor income shocks really very persistent? University of Rochester, working paper.

Halek, M. and J. G. Eisenhauer (2001). Demography of risk aversion. *Journal of Risk and Insurance 68*, 1–24.

Hong, H., J. D. Kubik, and J. C. Stein (2004). Social interaction and stock-market participation. *Journal of Finance 59*, 137–163.

Kahl, M., J. Liu, and F. A. Longstaff (2003). Paper millionaires: how valuable is stock to a stockholder who is restricted from selling it? *Journal of Financial Economics 67*, 385–410.

Lettau, M. and S. C. Ludvigson (2004). Understanding trend and cycle in asset values: reevaluating the wealth effect on consumption. *American Economic Review 94*, 276–299.

Liu, J., F. A. Longstaff, and J. Pan (2003). Dynamic asset allocation with event risk. *Journal of Finance 58*, 231–259.

Liu, J. and E. Neis (2002). Endogenous retirement and portfolio choice. University of California, Los Angeles, working paper.

Longstaff, F. A. (2001). Optimal portfolio choice and the valuation of illiquid securities. *Review of Financial Studies 14*, 407–431.

Longstaff, F. A. (2005). Asset pricing in markets with illiquid assets. University of California, Los Angeles, working paper.

Lucas, R. (1978). Asset prices in an exchange economy. *Econometrica 46*, 1429–1446.

Malloy, C. J., T. J. Moskowitz, and A. Vissing-Jørgensen (2005a). Job risk and asset returns. University of Chicago, working paper.

Malloy, C. J., T. J. Moskowitz, and A. Vissing-Jørgensen (2005b). Long-run stockholder consumption risk and asset returns. University of Chicago, working paper.

Mankiw, G. N. (1986). The equity premium and the concentration of aggregate shocks. *Journal of Financial Economics 17*, 211–219.

Mankiw, G. N. and S. Zeldes (1991). The consumption of stockholders and non-stockholders. *Journal of Financial Economics 29*, 97–112.

Mayers, D. (1972). Non-marketable assets and capital market equilibrium under uncertainty. In M. Jensen (Ed.), *Studies in the Theory of Capital Markets*. New York: Praeger.

McCardle, K. F. and R. L. Winkler (1989). All roads lead to risk preference: A turnpike theorem for conditionally independent returns. *Journal of Financial and Quantitative Analysis 24*, 13–27.

Mehra, R. and E. C. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*, 145–161.

Merton, R. C. (1969). Lifetime portfolio selection under uncertainty: The continuous-time case. *Review of Economics and Statistics 51*, 247–257.

Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica 41*, 867–887.

Nielsen, H. S. and A. Vissing-Jørgensen (2005). The impact of labor income risk on educational choices: estimates and implied risk-aversion. Northwestern University, working paper.

Pástor, Ľ. and R. F. Stambaugh (2003). Liquidity risk and expected stock returns. *Journal of Political Economy 111*, 642–685.

Pástor, Ľ. and P. Veronesi (2005a). Rational IPO waves. *Journal of Finance 60*, 1713–1757.

Pástor, Ľ. and P. Veronesi (2005b). Was there a Nasdaq bubble in the late 1990s? *Journal of Financial Economics*, forthcoming.

Santos, T. and P. Veronesi (2005). Labor income and predictable stock returns. *Review of Financial Studies*, forthcoming.

Vissing-Jørgensen, A. (2002a). Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy 110*, 825–853.

Vissing-Jørgensen, A. (2002b). Towards an explanation of household portfolio choice heterogeneity: Nonfinancial income and participation cost structures. NBER working paper.

Williams, J. T. (1977). Capital asset prices with heterogenous beliefs. *Journal of Financial Economics 5*, 219–239.

Xia, Y. (2001). Learning about predictability: The effects of parameter uncertainty on dynamic asset allocation. *Journal of Finance 56*, 205–246.

Yao, R. and H. H. Zhang (2005). Optimal consumption and portfolio choices with risky housing and borrowing constraints. *Review of Financial Studies 18*, 197–239.