Does the term structure forecast consumption growth?

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Abstract

Relying on a simple general equilibrium model of the term structure, both nominal yields and real consumption growth rates can be shown to be affine in the unobservable state variables. We can then express real consumption growth rates in terms of nominal yields rather than the unobservable state variables with the coefficients of the resultant forecasting relation being endogenously determined by the term structure model. In this sense, we use the entire term structure to forecast real consumption growth rates and provide empirical evidence consistent with the model more accurately predicting real consumption growth rates than a regression model based on the term spread.

1 Introduction

This paper investigates the accuracy of using interest rates to forecast future consumption growth rates and demonstrates that the *entire* term structure more accurately forecasts real consumption growth than the simple term spread.

Beginning with Kessel (1965), many researchers including, among others, Harvey (1988, 1989, 1991 and 1993), Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Chapman (1997), Kamara (1997), Roma and Torous (1997), and Hamilton and Kim (2001) have demonstrated that the term spread, that is, the difference between the yields of long term and short term bonds, provides valuable predictive information about future economic growth. In particular, a positive term spread is consistent with a subsequent increase in economic activity, while a negative term spread is consistent with a subsequent recession. The intuition for this result is based on the desire of investors to smooth consumption. For example, when a recession is expected in the future, individuals will buy long term bonds and sell short term bonds to receive payoffs when their consumption level is expected to be lower. As a result, short term yields will increase while long term yields will decrease thereby inverting the yield curve in anticipation of a downturn in economic activity.

Distinct from the previous research, this paper investigates the link between interest rates and economic growth within a simple general equilibrium framework in which the behavior of *both* interest rates and real consumption are simultaneously modeled. Following Cox, Ingersoll, and Ross (1985), we construct a general equilibrium term structure model which imposes cross-equation restrictions endogenously linking the term structure of interest rates to the dynamics of real consumption. As both nominal yields and real consumption growth rates can be shown to be affine in the posited but unobservable state variables, we can then express consumption growth rates in terms of nominal yields as opposed to the unobservable state variables. The general equilibrium model, not a historically estimated regression, specifies the coefficients of this forecasting relation and consequently we use all of the information available in the term structure to forecast real consumption growth rates.

The plan of this paper is as follows. Section 2 details the general equilibrium model of the term structure and derives the endogenous relation between real consumption growth and nominal interest rates. In Section 3 we compare our forecasting model with a forecasting model based on the term spread and provide statistically reliable evidence that we more accurately forecast real consumption growth rates at horizons of one year and longer. Section 4 concludes.

2 The Model

Our theoretical framework is based on the standard general equilibrium economy of the Cox, Ingersoll and Ross (1985) type. The main underlying assumptions are:

1. A fixed number of identical individuals with rational expectations maximizing a time-additive logarithmic utility function;
2. A competitive economy with continuous trading and no transactions costs;
3. The existence of markets for contingent claims and for instantaneous borrowing and lending at the riskless interest rate;
4. Production can be allocated to consumption or investment;
5. Investment opportunities consist of a stochastic production process, a set of contingent claims and a risk-free asset.

2.1 State Variables

We assume that the economy is characterized by two latent state variables. Litterman and Scheinkman (1991) and Brown and Schaefer (1994) empirically document that the majority of the movement in the term structure of interest rates can be explained by two factors. In our case, these factors, x, follow risk-adjusted[1] uncorrelated gaussian processes:

[1] We directly specify the process for x under the risk-adjusted probability measure by assuming that the parameter ϕ includes the risk-adjustment for the market price of risk. Doing so avoids the problem of identifying the market price of risk parameter when estimating contingent claims models based on gaussian processes (see Dai and Singleton (2000)).

$$dx = (\phi + \Gamma x)\, dt + \Sigma dz \tag{1}$$

where:

$$x \equiv \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \qquad \phi \equiv \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \qquad \Gamma \equiv \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}, \qquad \Sigma \equiv \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}.$$

The solution to this stochastic differential equation for $\tau > 0$ gives:

$$E_t \{x(t+\tau)\} = a(\tau) + B(\tau)x(t) \tag{2}$$

$$Cov_t \{x(t+\tau), x(t+\tau)'\} = F(\tau) \tag{3}$$

where:

$$a(\tau) \equiv \Gamma^{-1} (B(\tau) - \mathrm{I})\, \phi, \quad B(\tau) \equiv \exp(\Gamma\tau), \quad F(\tau) \equiv -(\Psi - B(\tau)\Psi B(\tau)')$$

and the matrix Ψ is a function of the coefficients in matrices Γ and Σ.[2]

2.2 Output and Consumption

We assume that a single physical good is produced which may be allocated to consumption or investment and that a single technology exists allowing capital to be transformed into output. Let Q denote the nominal amount of the good invested in the production process and assume that it depends on both state variables. The following stochastic differential equation then describes the dynamics of nominal output in the economy:

$$\frac{dQ}{Q} = (x_1 + x_2)dt + \sigma_Q dz_Q.$$

Inflation in this economy is assumed to be non-stochastic. Let p denote the price level and assume that it evolves according to the following deterministic process:

[2]See Langetieg (1980) footnote 22 for further details on the calculation of Ψ.

$$\frac{dp}{p} = \pi dt$$

where π is the non-stochastic instantaneous expected inflation rate.[3]

Applying Ito's lemma to the expression for real output, $q = Q/p$, allows us to derive a corresponding stochastic process for q. In equilibrium, all wealth will be invested in the production process and real consumption, c, must be proportional to optimally invested wealth[4], $c = \delta W$. Therefore, a stochastic process of the following form holds for real consumption:

$$\frac{dc}{c} = (x_1 + x_2 - \pi - \delta)dt + \sigma_c dz_c$$

where $\sigma_c = \sigma_Q$ and $dz_c = dz_Q$.

Similarly, we can derive a stochastic differential equation for $\ln c$ and integrating this expression from t to $t + \tau$ gives the following expression for the growth rate in consumption over the time interval $[t, t + \tau]$:

$$E_t \left\{ \ln \frac{c(t+\tau)}{c(t)} \right\} = h(\tau; \zeta) + J(\tau; \zeta) x(t) \tag{4}$$

with:

$$\begin{aligned} h(\tau;\zeta) &= -\left(\delta + \pi + \frac{\sigma_c^2}{2}\right)\tau - \tau\iota'\Gamma^{-1}\phi + J(\tau;\zeta)\Gamma^{-1}\phi \\ J(\tau;\zeta) &= \iota'\Gamma^{-1}\left(B(\tau) - \mathrm{I}\right) \\ \iota' &= (1 \quad 1) \end{aligned}$$

where we now explicitly note the dependence upon all of the model's parameters $\zeta = (\phi_1, \phi_2, \gamma_1, \gamma_2, \sigma_1, \sigma_2, \sigma_c, \delta)$ which must be estimated.

[3]Assuming a stochastic inflation rate would result in a more complicated model which would also require us to hypothesize the nature of the interaction between inflation and output growth. Furthermore, not all of these additional parameters can be separately identified when estimating the resultant term structure model.

[4]See CIR (1985).

2.3 Term Structure of Interest Rates

In equilibrium, the current time t price of a nominal unit discount bond with maturity date $T = t + \tau$ is given by:

$$G(t;T) = E_t \left\{ \frac{Q(t)}{Q(t+\tau)} \right\}.$$

Using standard results[5], we can derive the following closed form solution for nominal bond prices:

$$G(t;T) = \exp\left[g_0(\tau;\zeta) - g'(\tau;\zeta)x(t)\right]$$

where

$$\begin{aligned} g_0(\tau;\zeta) &\equiv \tau\sigma_c^2 - \iota'\Gamma^{-1}\left[\Gamma^{-1}\left(B(\tau) - \mathrm{I}\right) - \tau\mathrm{I}\right]\phi \\ &\quad + \frac{1}{2}g'(\tau;\zeta)\Psi g(\tau;\zeta) + \frac{1}{2}\iota'\Gamma^{-1}\Sigma\Sigma(\Gamma^{-1})'\iota\tau \\ &\quad - \frac{1}{2}\iota'\left[\Gamma^{-1}\Gamma^{-1}\left(B(\tau) - \mathrm{I}\right)\Psi + \Psi\left(B(\tau) - \mathrm{I}\right)'(\Gamma^{-1})'(\Gamma^{-1})'\right]\iota \\ g'(\tau;\zeta) &\equiv \iota'\Gamma^{-1}\left(B(\tau) - \mathrm{I}\right). \end{aligned}$$

Therefore, zero coupon yields can be expressed as:

$$Y(t;T) \equiv -\frac{\ln G(t;T)}{T-t} = \kappa_0(\tau;\zeta) + \kappa(\tau;\zeta)x(t) \tag{5}$$

where:

$$\begin{aligned} \kappa_0(\tau;\zeta) &\equiv -g_0(\tau;\zeta)/\tau \\ \kappa(\tau;\zeta) &= (\kappa_1(\tau;\zeta), \kappa_2(\tau;\zeta)) \equiv g(\tau;\zeta)/\tau. \end{aligned}$$

[5] See, for example, Duffie (1996).

2.4 Implicit Relation Between Yields and Consumption

Real consumption growth rates, expression (4), and nominal yields, expression (5), are both affine in the state variables. The closed form nature of these expressions implies that we can express consumption growth rates in terms of yields rather than in terms of the unobservable latent factors. Consequently, we provide an endogenous means of exploiting the nominal term structure to forecast real consumption growth rates. Since consumption growth rates and the term structure are jointly determined in general equilibrium, the model explicitly characterizes the coefficients of this forecasting relation. In this sense, we are using all of the information available in the term structure, as opposed to just a few points, to forecast real consumption growth rates.

To fix matters, we can express the two posited state variables in terms of two distinct yields, say the yield on a short term bond, $Y_S \equiv Y(\tau_S)$, and the yield on a long term bond, $Y_L \equiv Y(\tau_L)$, $\tau_L > \tau_S$. Equivalently, to make our results comparable to previous forecasting models which rely on the spread, for example, Estrella and Hardouvelis (1991) and Harvey (1989, 1991, 1993), we can express the state variables in terms of the yield on a short term bond, Y_S, and the spread between long term and short term yields, $SP \equiv Y_S - Y_B$. From (5) we have:

$$Z(t) \equiv \begin{pmatrix} Y_S \\ SP \end{pmatrix} = \bar{\kappa}_0(\zeta) + \bar{\kappa}(\zeta)x(t)$$

where:

$$\begin{aligned} \bar{\kappa}_0(\zeta) &\equiv \begin{pmatrix} \kappa_0(\tau_S;\zeta) \\ \kappa_0(\tau_L;\zeta) - \kappa_0(\tau_S;\zeta) \end{pmatrix}, \\ \bar{\kappa}(\zeta) &\equiv \begin{pmatrix} \kappa_1(\tau_S;\zeta) & \kappa_2(\tau_S;\zeta) \\ \kappa_1(\tau_L;\zeta) - \kappa_1(\tau_S;\zeta) & \kappa_2(\tau_L;\zeta) - \kappa_2(\tau_S;\zeta) \end{pmatrix}. \end{aligned}$$

This system can be inverted with respect to the two latent state variables. Substituting the resulting expressions into expression (4) above, we can derive the endogenous relation between expected consumption growth, the spread and the short term yield :

$$
\begin{aligned}
E_t\left\{\ln\frac{c(t+\tau)}{c(t)}\right\} &= \alpha(\tau;\zeta)+\beta(\tau;\zeta)Z(t) \\
&= \alpha(\tau;\zeta)+\beta_1(\tau;\zeta)Y_S(t)+\beta_2(\tau;\zeta)SP(t) \qquad (6)
\end{aligned}
$$

where:

$$
\alpha(\tau;\zeta)\equiv h(\tau;\zeta)-\beta(\tau;\zeta)\cdot\bar{\kappa}_0(\zeta), \qquad \beta(\tau;\zeta)=(\beta_1(\tau;\zeta),\beta_2(\tau;\zeta))\equiv J(\tau;\zeta)\cdot\bar{\kappa}^{-1}(\zeta).
$$

Expression (6) gives the consumption growth forecasting relation implied by the general equilibrium term structure model. By construction, this forecasting model depends on both the short term yield and the spread. More importantly, the coefficients on the short term yield, $\beta_1(\tau;\zeta)$, and on the spread, $\beta_2(\tau;\zeta)$, are endogenously determined and depend explicitly on the model's parameters ζ. As a consequence, we do not use consumption data to construct our forecasts as all relevant information about future consumption growth is captured in general equilibrium by the term structure.

By comparison, forecasting models which rely on the spread are implemented as follows:

$$
E_t\left\{\ln\frac{c(t+\tau)}{c(t)}\right\}=\hat{a}(\tau)+\hat{b}(\tau)SP(t) \qquad (7)
$$

where the parameters $\hat{a}(\tau)$ and $\hat{b}(\tau)$ are typically estimated from an in-sample regression of realized τ-period growth rates onto past spreads. Unlike our two factor model, these parameters are not endogenously determined but rather depend on the historically estimated relation between real consumption growth rates and spreads.

3 Empirical Results

In this section the two factor model, expression (6), is compared to the spread model, expression (7), in terms of their predictive accuracy in forecasting

real consumption growth rates. To implement the two factor model requires that we fit the general equilibrium term structure model to prevailing yields and then construct forecasts at various horizons τ according to expression (6). Alternatively, the spread model uses the historically estimated relation between τ period real consumption growth rates and spreads to form τ period ahead forecasts according to (7).

Our subsequent empirical analysis relies on U.S. data drawn exclusively from the post-Volcker experiment era, 1984-2001. By doing so, we attempt to ensure that data are not sampled from differing macroeconomic regimes in which case we may erroneously attribute as forecast error a result which is due entirely to a change in macroeconomic regimes.

3.1 Data

We use monthly observations over the sample period 1984:1 to 1999:12 on the annualized zero coupon yields (the average of bid and ask yields) of U.S. Treasuries for six distinct maturities: three months and from one to five years. The three month data are taken from CRSP's Fama file while the one to five year data are taken from CRSP's Fama-Bliss file.

Our consumption data are monthly observations 1984:1 to 2001:12 on seasonally adjusted real (1996 dollars) personal expenditures on services plus non-durables from the U.S. Department of Commerce's Bureau of Economic Analysis. The corresponding deflator of personal expenditures on services plus non-durables measures the price level used to estimate the expected rate of inflation π in expression (6). In particular, for each month we use the previous ten years of monthly observations on the logarithmic change in this deflator to fit an ARIMA model and take the resultant one-step ahead forecast as our estimate of π.

3.2 Term Structure Model Estimation

We cast the estimation of the general equilibrium term structure model in a linear state-space framework.[6] Consistent with the model, the underlying

[6]See Duffee (1999) and references therein on estimating term structure models in a state-space framework.

state variable, $x(t)$, is explicitly recognized to be unobserved while observed bond yields are assumed to be a linear function of $x(t)$. While the term structure model is derived in continuous-time, its estimation will be carried out in discrete-time as yield data are only available at discrete time intervals of length $\Delta \equiv$ one month.

Suppose that at each date t we observe yields of bonds with M distinct maturity dates $T_1, T_2 \ldots T_M$ or, equivalently, M distinct terms to maturity, $\tau_1, \tau_2 \ldots \tau_M$, $Y_t = (Y(t;T_1), Y(t;T_2) \ldots Y(t;T_M))'$. Each observed yield can be expressed as the corresponding yield given by the model plus an independent, normally distributed measurement error, e_{t,τ_i}. Measurement errors in the observed bond yields reflect noise arising from, for example, the bid-ask spread or possible quotation errors. This gives the following set of measurement equations:

$$Y_t = K_0 + Kx_t + e_t \tag{8}$$

where

$$K_0 \equiv \begin{pmatrix} \kappa_0(\tau_1;\zeta) \\ \cdots \\ \kappa_0(\tau_M;\zeta) \end{pmatrix}, \quad K \equiv \begin{pmatrix} \kappa(\tau_1;\zeta) \\ \cdots \\ \kappa(\tau_M;\zeta) \end{pmatrix}, \text{ and } \; e_t \equiv \begin{pmatrix} e_{t,\tau_1} \\ \cdots \\ e_{t,\tau_M} \end{pmatrix}.$$

In addition, the state variable's transition equation in discrete-time can be written as:

$$x_{t+\Delta} = a(\Delta) + B(\Delta)x_t + \upsilon_t \tag{9}$$

where the transition errors υ_t are assumed to independently bivariate normally distributed with mean equal to the zero vector and covariance matrix given by $F(\Delta)$ from expression (3) which imposes cross-equation restrictions on the variance and covariance properties of the state variables. To complete the specification, the measurement errors e_t and the transition errors υ_t are assumed to be uncorrelated at all lags and to be uncorrelated with the initial state vector.

With these assumptions, we may use the Kalman filter to optimally predict the underlying state variable, x_t, as well as to efficiently evaluate the

corresponding likelihood function. Numerical optimization of this likelihood function over ζ gives the maximum likelihood estimator $\hat{\zeta}$ of the parameters of the general equilibrium term structure model.[7]

3.2.1 Term Structure Model Estimation Results

We recursively estimate the general equilibrium term structure model using a fixed ten year window of monthly data beginning in January 1984. That is, using one hundred and twenty months of yield data from January 1984 to December 1993, we fit the term structure model as of December 1993, obtain the corresponding maximum likelihood parameter estimates and measure the errors in pricing the sampled Treasury securities through December 1993. Subsequently, moving forward one month, the one hundred and twenty months of yield data ending in January 1994 allow us to update the maximum likelihood estimates and measure the errors in fitting the term structure through January 1994. Proceeding recursively in this fashion, we obtain maximum likelihood estimates of the term structure model's parameters at monthly intervals from December 1993 to December 1999 as well as corresponding Treasury pricing errors.

The term structure model's maximum likelihood parameter estimates from December 1993 through December 1999 are displayed graphically in Figure 1.[8] Notice that the estimated mean reversion coefficients, $\hat{\gamma_1}$ and $\hat{\gamma_2}$, are consistent with the first factor behaving like a random walk, $\hat{\gamma_1} \approx 0$, while the second factor is more stationary in its behavior, $\hat{\gamma_2} < 0$. Despite this

[7] We do not estimate the parameter δ, the rate of patience, because this parameter does not enter the closed form solution for yields given by expression (5). However, as other studies, for example, Dunn and Singleton (1986) and Ferson and Constantinides (1991), have found the intertemporal coefficient $\beta = e^{-\delta}$, a one-to-one transformation of δ, to be statistically indistinguishable from one, we estimate the remaining parameters under the equivalent restriction that δ equals zero. Our results do not change qualitatively if we set β to be less than but close to one or, equivalently, we set δ to be greater than but close to zero.

[8] Given the recursive nature of our estimation procedure, these parameter estimates are not independent. Because our focus is on investigating the predictive accuracy of consumption growth forecasts, we do not provide a statistical analysis of these parameter estimates which takes their overlapping nature into account. Any serial dependence in the resultant consumption growth forecasts, however, will be explicitly taken into account.

difference, the corresponding estimated volatility coefficients, $\hat{\sigma}_1$ and $\hat{\sigma}_2$, are remarkably similar across the entire sample period.

Summary statistics for the resultant errors in pricing the sampled zero coupon Treasury yields are provided in Table 1. An error here is defined as the fitted yield minus the actual yield and is measured in basis points. To interpret these statistics recall that in fitting the term structure model we have measured these errors by maturity for each of the preceeding one hundred and twenty months of sampled yield data. For each estimation date, we can then calculate the resultant mean errors and mean absolute errors. Table 1 provides the average of these errors across all of the estimation dates.[9] The results of Table 1 indicate that the model provides an adequate fit to the term structure although it does not appear to fit the short-end as well. In particular, the fitted yields are consistently higher than the actual three-month yields but consistently lower than the actual six-month yields.

3.3 Forecasting Results

To forecast real consumption growth using the two factor model, we use the preceeding ten years of yield data to estimate the parameters of the general equilibrium model needed in expression (6) to forecast three and six months ahead as well as one through five years ahead. Without loss of generality, we set τ_S = three months and τ_L = five years in (6) throughout. Proceeding recursively in this fashion from December 1993 through December 1999, we compute consumption growth forecasts which are then compared to realized consumption growth rates.

Alternatively, to forecast real consumption growth using the spread model, expression (7), we use the preceeding ten years of yield *and* real consumption data to fit linear regressions of realized consumption growth rates against the spread observed between five year and three month yields. Linear regressions are separately fit for each of the forecast horizons. The corresponding estimated coefficients are then used to forecast consumption growth rates over that particular horizon. Proceeding recursively from December 1993

[9] Once again, given the recursive nature of our estimation procedure, these errors are not independent and the summary statistics are provided for illustrative purposes.

through December 1999 gives competing forecasts to those produced using our general equilibrium model.

Average differences between the spread model's mean absolute errors and the two factor model's mean absolute errors in forecasting realized consumption growth rates are tabulated in Table 2. A positive difference here is then consistent with the two factor model being more accurate.[10] We also present the results of the Diebold and Mariano (1995) test of the null hypothesis of no difference in the accuracy of these competing forecasts.[11]

In Panel A we make these comparisons across all dates and all forecast horizons. In particular, the three month forecasts use term structure data through 12/99 to forecast consumption growth through 3/00 while the five year forecasts use term structure data through 12/96 to forecast consumption growth through 12/01. The corresponding mean absolute errors of the competing models are graphically displayed in Figure 2. In Panel B of Table 2, we compare these forecasts fixing the terminal calendar date (3/00) across the forecast horizons[12], while in Panel C we compare accuracy fixing the last term structure (12/97) used across the forecast horizons.[13]

The clear message that emerges from Table 2 is that the two factor model provides more accurate forecasts of real consumption growth rates at forecasting horizons of one year and longer. At short horizons, three or six

[10]Qualitatively similar results obtain for the corresponding root mean squared errors and are not reported here.

[11]Let d_t denote the difference in absolute errors between the competing forecasts at t or, in other words, the loss differential at t. Under the null hypothesis that the population mean of the loss differential series $\{d_t\}$ is zero, the statistic $\bar{d}/\sqrt{2\pi \hat{f}_d(0)T}$ is asymptotically standard normal distributed where $\hat{f}_d(0)$ is a consistent estimate of the spectral density of the loss differential at frequency $\omega = 0$. Following Newey and West (1987), a consistent estimate of $2\pi f_d(0)$ is obtained by using a Bartlett kernel with lag selected according to Newey and West's (1994) automatic bandwith selection procedure.

[12]This calendar date is the terminal date corresponding to the three month forecasts provided in Table A. We do not analyze four year or five year forecasts in Panel B because in these cases by fixing the terminal calendar date at 3/00 would result in fewer than thirty competing forecasts to compare.

[13]This is the last term structure used to forecast four year real consumption growth rates in Panel A. We do not analyze five year forecasts in Panel C because the last term structure used to forecast five year real consumption growth rates is 12/96 and terminating forecasts with this term structure would not provide as many forecasts at other horizons necessary for reliable inference.

months, we cannot reject the null hypothesis that there is no difference in the accuracy of the competing forecasts. At longer horizons, however, we see reliable evidence that the two factor model's forecasts are more accurate than the spread model's forecasts. The largest average difference in forecasting accuracy occurs at the three year horizon but statistically significant accuracy gains are also to be had at forecasting horizons of four and five years.

These conclusions are reinforced graphically in Figure 2. Notice that the forecasting accuracy of the term spread model does not appear to vary with forecasting horizon, remaining at approximately one hundred basis points throughout. By comparison, the two factor model's forecasting accuracy improves with increasing forecast horizon. In particular, at three or six month horizons, the two factor model's forecast accuracy is approximately one hundred basis points but at three, four, or five year horizons the accuracy is within fifty basis points of corresponding actual real consumption growth rates.

To further investigate the two factor model, Figure 3 graphically displays the time series properties of the model's forecasting errors, defined as predicted real consumption growth rates minus observed growth rates. Panel A of Figure 3 considers three, six, and twelve month forecasting horizons, while Panel B considers two, three, four and five year horizons. While the forecasts appear to track real consumption growth rates fairly well over the entire sample period, the model does appear to systematically under predict real consumption growth during the late 1990s, especially at short horizons (Panel A). One interpretation of this result is that actual consumption growth rates were unexpectedly high here, at least relative to what was being predicted by the term structure of interest rates, because of the significant stock market appreciation surrounding the internet bubble and the consequent effects of this increase in stock market wealth on consumer spending.[14]

4 Conclusions

In general equilibrium, investors set the yields of bonds maturing at different times by taking into account the levels of consumption expected at those

[14]See, for example, Poterba (2000) for an investigation of the effects of stock market wealth on consumption.

times. In this paper, we recover these investor expectations from a simple general equilibrium model of the term structure. By fitting this model to observed yields, we are able to exploit the entire term structure in forecasting real consumption as opposed to just two yields.

Our empirical results are consistent with the increased predictive accuracy of our general equilibrium approach. In particular, we find statistically reliable evidence that our approach provides more accurate forecasts of real consumption growth than a forecasting model based on the term spread.

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Table 1
Summary Statistics of Term Structure Model's Pricing Errors

This table summarizes the general equilibrium term structure model's error properties in fitting zero coupon Treasury yields over the sample period 1984:1 to 1999:12. An error is defined as a fitted yield minus an observed yield and is measured in basis points. Yield data, the average of bid and ask, are obtained from CRSP's Fama file (three month maturity) and CRSP's Fama-Bliss file (one year through five years). We estimate the model using maximum likelihood by casting it in a discrete-time state-space framework and evaluating the likelihood function using the Kalman filter. Proceeding recursively, we estimate the term structure model in monthly intervals from December 1993 to December 1999 and at each date measure the resultant errors in fitting the preceeding ten year's of monthly yields.

Yield maturity	Average of Mean Errors	Average of Mean Absolute Errors
3 months	10.08	19.41
1 year	-12.43	19.21
2 years	-4.31	12.22
3 years	-0.48	7.55
4 years	1.72	8.48
5 years	6.17	13.97

Table 2
Comparing Predictive Accuracy in Forecasting Real Consumption Growth Rates

This table compares the predictive accuracy of the two factor model versus the spread model in forecasting real consumption growth rates. Predictive accuracy is measured by a model's corresponding mean absolute error. We tabulate by forecast horizon the average across estimation dates of the differences between the spread model's and the two factor model's mean absolute errors. The Diebold-Mariano (1995) statistic and its asymptotic p-value in testing the null hypothesis of no difference in the accuracy of these competing forecasts are also provided.

Panel A: All estimation dates and all forecast horizons.

Time Horizon	Average of Differences in Mean Absolute Errors	Diebold-Mariano Statistic	p-value
3 months	-14.46	-1.05	29.37%
6 months	-5.87	-0.44	65.99%
1 year	22.99	1.54	12.36%
2 years	50.36	2.79	0.53%
3 years	70.16	3.71	0.02%
4 years	54.05	2.85	0.44%
5 years	47.45	2.33	1.98%

Panel B: Fixing the terminal calendar date (3/00) across all forecasts.

Time Horizon	Average of Differences in Mean Absolute Errors	Diebold-Mariano Statistic	p-value
6 months	-3.45	-0.26	79.49%
1 year	30.24	1.96	5.00%
2 years	60.26	3.23	1.24%
3 years	41.11	2.12	3.40%

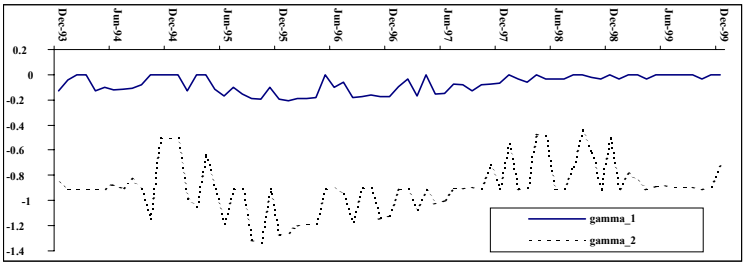
Panel C: Fixing the last term structure (12/96) across all forecasts.

Time Horizon	Average of Differences in Mean Absolute Errors	Diebold-Mariano Statistic	p-value
3 months	-4.62	-0.37	71.14%
6 months	1.27	0.10	92.00%
1 year	22.02	1.80	7.19%
2 years	53.15	3.04	0.24%
3 years	62.24	3.02	0.25%

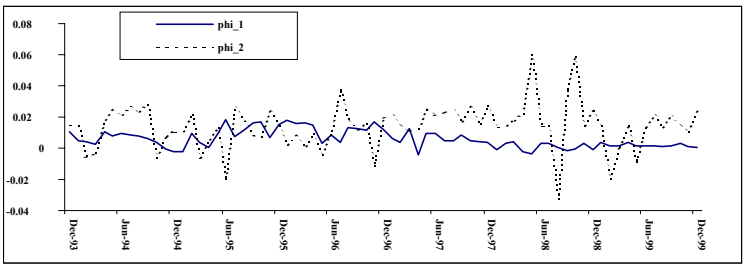
Figure 1
Term Structure Model's Parameter Estimates

This figure displays the general equilibrium term structure model's parameter estimates obtained by fitting zero coupon Treasury yields over the sample period 1984:1 to 1999:12. We estimate the model using maximum likelihood by casting it in a discrete-time state-space framework and evaluating the likelihood function using the Kalman filter. Proceeding recursively, we estimate the term structure model in monthly intervals from December 1993 to December 1999.

Mean reversion coefficients:



Risk-adjusted long-term mean coefficients:



Volatility coefficients:

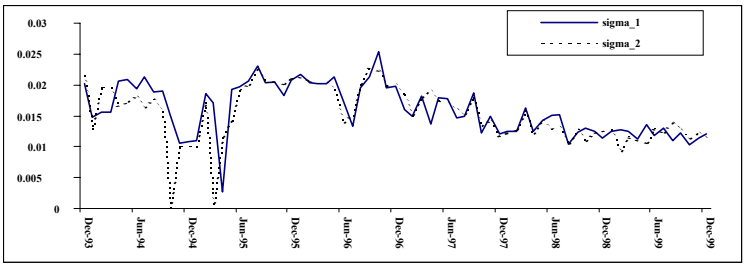


Figure 2
Mean Absolute Errors in Forecasting Consumption Growth Rates

This figure compares the predictive accuracy of the two factor term structure model with the spread model in forecasting real consumption growth rates. Predictive accuracy is measured by a model's corresponding mean absolute error and is measured in basis points. The two factor model forecasts are obtained by using the preceeding ten years of yield data to estimate the parameters needed to forecast subsequent real consumption growth rates. Alternatively, to forecast real consumption growth using the term spread model, we use the preceeding ten years of yield and real consumption data to fit linear regressions of realized consumption growth rates against the spread observed between five year and three month yields. The corresponding estimated coefficients are then used to forecast subsequent consumption growth rates over the different horizons. Proceeding recursively, we compute real consumption growth forecasts for both competing models from December 1993 to December 1999 and compare them to realized consumption growth rates.

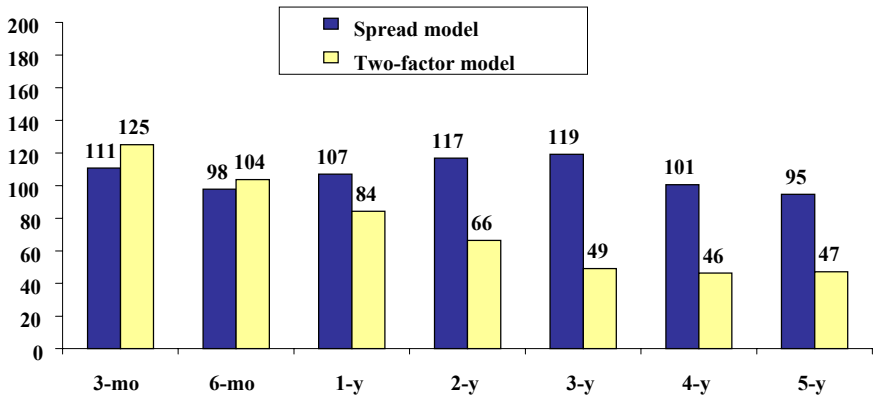
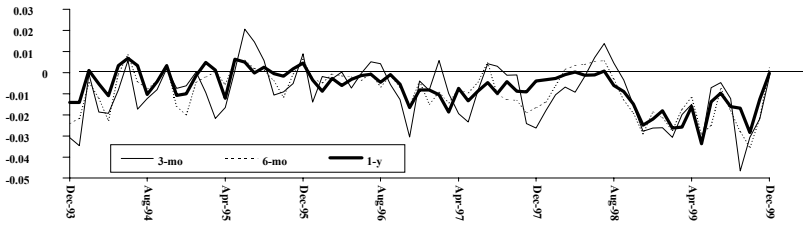


Figure 3
Two Factor Model's Consumption Forecasting Errors

This figure shows the time series properties of the two factor model's forecasting errors, defined as predicted minus observed real consumption growth rates. Model forecasts are obtained by using the preceeding ten years of yield data to estimate the parameters needed to forecast real consumption growth rates. Proceeding recursively, we compute consumption growth forecasts from December 1993 to December 1999 and compare them to realized consumption growth rates.

Panel A: Three, six and twelve month forecasting horizons:



Panel B: Two, three, four and five year forecasting horizons: