#7-97

Valuing Long Term Commodity Assets

April 1997
Revised October 1997

Eduardo Schwartz
Anderson Graduate School of Management
University of California, Los Angeles
Los Angeles, CA 90095-1481

Finance Working Paper
sponsored by

CMS CAPITAL MANAGEMENT SCIENCES

VALUING LONG TERM COMMODITY ASSETS

April 1997

Revised October 1997

Eduardo Schwartz

Anderson Graduate School of Management

University of California, Los Angeles

Valuing Long Term Commodity Assets

Abstract

In this article I develop a one factor model for the stochastic behavior of commodity prices that retains most of the characteristics of a more complex two factor stochastic convenience yield model in terms of its ability to price the term structures of futures prices and volatilities. The model is based on the pricing and volatility results of the two factor model and when applied to value long term commodity projects it gives practically the same results as the more complex model. The inputs to the model are the current prices of all existing futures contracts (and their maturities) and the estimated parameters of the two factor model. It only requires, however, the numerical solution corresponding to a simple one factor model. Existing computer programs can be easily modified to incorporate the essential elements of the new model.

Valuing Long Term Commodity Assets

1. Introduction

The application of option concepts to value real assets has been an important growth area in the theory and the practice of finance. These methods have been especially successful in the valuation of natural resource investments such as copper mines and oil deposits. The reason for this is the existence of well developed futures markets for these commodities from which essential market information can be extracted.

The traditional approach to valuing investment projects is the net present value approach which essentially consist in discounting the expected net cash flows from the project at a discount rate that reflects the risks of those cash flows (the 'risk adjusted discount rate'). In this approach the adjustment for risk is done in the discount rate. An alternative approach is to do the adjustment for risk in the cash flows and discount the certainty equivalent cash flows, instead of the expected cash flows, at the risk free rate of interest. The certainty equivalent cash flows are the certain amounts which would have the same value as the uncertain cash flows. The net present value approach is the one normally used in practice because it is felt that it is easier to estimate the risk adjusted discount rate than the certainty equivalent cash flows[1]. However, in the case of commodities the certainty equivalent cash flows are known since they can be obtained from forward (or futures[2]) prices. So, when valuing projects in which the main uncertainty is the commodity price

[1] The difficulties of accurately estimating risk adjusted discount rates, however, has been recently emphasized by Fama and French (1997).

[2] In this article I will not distinguish between forward and futures prices since the analysis will be carried in and environment with constant interest rates.

and forward prices for the commodity exist, it is much easier to use the certainty equivalent approach, since it avoids the need to compute a risk adjusted discount rate. Once the adjustment for risk has been done in the cash flows the relevant discount rate is the risk free rate of interest.

The second advantage of working in this risk neutral environment in which the relevant discount rate is the risk free rate of interest is that it is also the environment of option pricing. The multiple operating options available in a typical project can then be naturally incorporated in the analysis. These options include the optimal time to exercise the option to invest in the project, the options to stop and restart production in response to price changes and the option to abandon the project if prices are too low to justify maintaining the operations.

In the first attempt to value investment projects in natural resources using this new approach (see for example Brennan and Schwartz (1985)) the spot price of the commodity was assumed to follow a geometric Brownian motion similar to the one used for stock prices in the option pricing literature. This allowed for the extension of the option pricing framework to value real assets. Futures prices were used to determine the average convenience yield which plays the same role in the commodity spot price process as the dividend yield in the stock price process. In theory, the convenience yield is the flow of services that accrue to the holder of the spot commodity but not to the holder of a futures prices. In practice, the convenience yield is the adjustment needed in the drift of the spot price process to properly price existing futures prices.

The options approach to valuation has major advantages. First, it avoids the need to make assumptions about the trajectory of spot prices in the future since it uses the information contained in futures prices (through the convenience yield). Second, it does not require the estimation of a risk adjusted discount rate since it uses the risk free rate of interest. And finally, it

explicitly allows for managerial flexibility in the form of options in the valuation procedure. In certain situations these options can represent a significant part of the value of the project.

The assumed stochastic process for the spot commodity, however, has some drawbacks. First, since the convenience yield is assumed constant the model is unable to capture changes in the form of the term structure of futures prices (for example, from backwardation to contango or vice-versa). In reality, the convenience yield experiences significant changes through time. Second, the model implies that the volatility of all futures returns is equal to the volatility of spot returns. The data shows, however, that the volatility of futures returns decreases with the time to maturity of the futures contract. Third, the geometric Brownian motion implies that the variance of the distribution of spot prices grows linearly with time, whereas supply and demand adjustments to changing prices would suggest some type of mean reversion in spot commodity prices.

In the last few years we has seen different attempts to resolve the drawbacks of the basic model discussed above[3]. In Schwartz (1997) I compare three models of the stochastic behavior of commodity prices in terms of their ability to price the term structure of futures prices and the term structure of futures return volatility. The first model is a one factor model in which the log of the spot price of the commodity is assumed to follow a mean reverting process. The second model assumes that the convenience yield is also stochastic and follows a mean reverting process. In this model the convenience yield plays the role of a stochastic dividend in the spot price process. The third model extends the second by assuming also stochastic interest rates. For the two commercial commodities considered, copper and oil, the one factor model does a poor job in explaining the

[3] See for example Brennan (1991), Gibson and Schwartz (1990 and 1991), Ross (1995), Cortazar and Schwartz (1994) and Bessembinder, Coughenour, Seguin and Smoller (1995).

characteristics of the data. The other two models, however, are able to capture many of the characteristics of the term structure of futures prices and volatilities. Since these two models are empirically very similar, in this paper I will deal only with the two factor model which assumes constant interest rates.

A major difficulty in the application of these ideas to the valuation of projects is that commodity futures contracts traded in exchanges have maximum maturities that are typically less than two years, whereas most of the projects we want to value have much longer maturities. It is therefore very important to assess the implications of the different models with respect to futures prices beyond the existing exchange traded contracts. Fortunately, for my study Enron made available some proprietary oil forward curves with maturities up to ten years which allowed me to assess the implications of the models with respect to long term futures prices.

The two factor stochastic convenience yield model has many advantages with respect to simpler one factor models. The valuation procedure, however, is substantially more difficult especially in the presence of complex operating options since it requires the solution of a second order partial differential equation with two state variables (and time). This disadvantage can be quite important from the point of view of the practical implementation of the approach.

In this paper I develop a one factor model that retains most of the characteristics of the more complex two factor stochastic convenience yield model in terms of its ability to price the term structure of futures prices and volatilities. The model is based on the pricing and volatility results of the two factor model, but when applied to value long term commodity projects it only requires the numerical solution corresponding to a typical one factor model. The inputs to the model are the current prices of all existing futures contracts (and their maturities) and the estimated

parameters of the two factor model. Existing computer programs can be easily modified to incorporate the essential elements of the new model.

Section 2 develps the model and the main results of the paper. Section 3 shows how to implement the approach and compares the results obtained using the model with those obtained using the substantially more complicated two factor stochastic convenience yield model in the valuation of long term commodity assets. Section 4 looks at the problem of the determination of the optimal exercise criteria and Section 5 provides some concluding remarks.

2. The Model

In this section I first present the basic constant convenience yield model and the two factor stochastic convenience yield model. Then I show that simple modifications of the basic model allow for taking into account many of the important features of the two factor model, especially when applied to value longer term commodity assets. For this reason, I call the model developed in this article the Long Term Model.

Since the interest here is on the valuation of commodity contingent claims I will define all the stochastic processes under the equivalent martingale measure (i.e. risk neutrality). Also, since I assume that interest rates are constant, there will be no distinction between forward and futures prices.

2.1 The Basic Model

Assuming constant convenience yield, c, interest rate, r, and volatility, σ, for the rate of return on the spot commodity price, S, the stochastic process for the spot price under the equivalent martingale measure is given by:

$$\frac{dS}{S} = (r-c)dt + \sigma dz \tag{1}$$

where dz is an increment to a standard Brownian motion.

In this model, the futures price, F, with maturity T for a spot price S is given by:

$$F(S,T) = Se^{(r-c)T} \tag{2}$$

Applying Ito's Lemma to (2) we see that the volatility of futures return (dF/F) is also given by σ.

In the basic model the value of any contingent claim on the commodity, V(S,T), must satisfy the partial differential equation

$$\tfrac{1}{2}\sigma^2 S^2 V_{SS} + (r-c)SV_S - V_T - rV = 0 \tag{3}$$

subject to the appropriate boundary conditions. If the contingent claim is a project we also have to add the cash flows on the project, CF(T), in equation (3).

2.2 The Two Factor Model

In the two factor model the convenience yield, denoted by δ to distinguish it from the constant convenience yield in the basic model, is assumed to be stochastic and to follow a mean reverting process. The joint stochastic process for the spot price and the convenience yield under the equivalent martingale measure is:

$$dS = (r-\delta)Sdt + \sigma_1 S dz_1 \tag{4}$$

$$d\delta = \kappa(\alpha - \delta)dt + \sigma_2 dz_2 \tag{5}$$

$$dz_1 dz_2 = \rho dt \tag{6}$$

The magnitude of the speed of adjustment $\kappa > 0$ measures the degree of mean reversion to the long run mean convenience yield α, and ρ is the correlation between the two processes.

Futures prices in this model, F(S,δ,T), are given by:

$$F(S,\delta,T) = S\exp[-\delta\frac{1-e^{-\kappa T}}{\kappa} + A(T)] \tag{7}$$

where

$$A(T) = (r - \hat{\alpha} + \frac{1}{2}\frac{\sigma_2^2}{\kappa^2} - \frac{\sigma_1\sigma_2\rho}{\kappa})T + \frac{1}{4}\sigma_2^2\frac{1-e^{-2\kappa T}}{\kappa^3} + (\hat{\alpha}\kappa + \sigma_1\sigma_2\rho - \frac{\sigma_2^2}{\kappa})\frac{1-e^{-\kappa T}}{\kappa^2} \tag{8}$$

and the risk adjusted long run mean of the convenience yield process is given by

$$\hat{\alpha} = \alpha - \frac{\lambda}{\kappa} \tag{9}$$

where λ is the market price of convenience yield risk, assumed to be constant.

Applying Ito's Lemma to (7) it can be seen that the variance of futures returns is independent of the level of the state variables and depends only on time to maturity of the futures contract:

$$\sigma_F^2(T) = \sigma_1^2 + \sigma_2^2 \frac{(1-e^{-\kappa T})^2}{\kappa^2} - 2\rho\sigma_1\sigma_2 \frac{(1-e^{-\kappa T})}{\kappa} \tag{10}$$

Note that this variance converges to a fix value when maturity of the futures contract tends to infinity:

$$\sigma_F^2(\infty) = \sigma_1^2 + \frac{\sigma_2^2}{\kappa^2} - \frac{2\rho\sigma_1\sigma_2}{\kappa} \tag{11}$$

In the two factor model the value of any commodity contingent claim must satisfy the partial differential equation

$$\frac{1}{2}\sigma_1^2 S^2 V_{SS} + \sigma_1\sigma_2\rho S V_{S\delta} + \frac{1}{2}\sigma_2^2 V_{\delta\delta} + (r-\delta) S V_S + \kappa(\alpha - \delta) V_\delta - V_T - rV = 0 \tag{12}$$

subject to the appropriate boundary conditions. If the contingent claim is a project we must add the cash flows to equation (12).

2.3 The Long Term Model

In Schwartz (1997) I show that the two factor model has implications with respect to the term structure of futures prices and volatilities which are consistent with the data for the two commodities I consider (copper and oil). These two term structures determine the risk neutral distribution of spot prices. Given stochastic processes (4)-(6), the risk neutral distribution of spot prices is log-normal with mean equal to the forward price (7) and a variance (of the log price) which can be obtained integrating the variance of the forward price (10) as I show later in (22). My

objective here is to develop a model which is simpler than the two factor model, but which matches as close as possible the term structure of futures prices and the term structure of futures volatilities implied by the two factor model. As I show below I accomplish the first objective very accurately for long term futures prices and the second objective exactly for all futures prices.

In the two factor model as the maturity of the futures contract increases the rate of change in the futures price converges to a fix rate that is independent of the initial value of the state variables:

$$\frac{1}{F}\frac{\partial F}{\partial T}(T \to \infty) = r - \hat{\alpha} + \frac{\sigma_2^2}{2\kappa^2} - \frac{\rho\sigma_1\sigma_2}{\kappa} \tag{13}$$

As we shall see in Section 3, for the oil and copper data available, close convergence can be obtained in approximately three years. Note that in the basic model (2) the rate of change in the futures price is constant and equal to

$$\frac{1}{F}\frac{\partial F}{\partial T} = r - c \tag{14}$$

So, if in the long term model we define the constant convenience yield as

$$c = \hat{\alpha} - \frac{\sigma_2^2}{2\kappa^2} + \frac{\rho\sigma_1\sigma_2}{\kappa} \tag{15}$$

it will have the same rate of change in futures prices as the two factor model when (13) is a good approximation.

Our objective, however, is to match the futures prices. So, in addition to having the

correct rate of change in futures prices we need to start from an appropriate spot price to give the futures prices in (7) when a constant convenience yield (15) is applied. We define this price as the shadow spot price, Z, and it is given by:

$$Z(S,\delta) = \lim_{T\to\infty} e^{(-(r-c)T)} F(S,\delta,T) \tag{16}$$

which after simplifying gives:

$$Z(S,\delta) = S\, e^{\left(\frac{c-\delta}{\kappa} - \frac{\sigma_2^2}{4\kappa^2}\right)} \tag{17}$$

In other words, given the state variables of the two factor model, S and δ, and the parameters of the model, from (17) we can obtain the shadow spot price Z. This price is such that when used as a single state variable in a model with constant convenience yield c form (15) will give futures prices F(Z,T) which are very close to F(S,δ,T) when the time to maturity of the contract is greater than three years.

Then, define the stochastic process for the shadow spot price as:

$$\frac{dZ}{Z} = (r-c)dt + \sigma_F(t)dz \tag{18}$$

Note that now the volatility is a function of time and is defined as in (10).

In the long term model, the futures price F with maturity T for a shadow spot price Z is then:

$$F(Z,T) = Z e^{(r-c)T} \tag{19}$$

Applying Ito's Lemma to (19) we see that the volatility of futures returns is given by $\sigma_F(T)$. In this model, then, the volatilities of futures returns are exactly the same as those in the two factor model and futures prices are very close to those in the two factor model for maturities greater than three years.

In the long term model the value of any claim on the commodity, V(Z,T), must satisfy the partial differential equation

$$\frac{1}{2}\sigma_F^2(T)Z^2V_{ZZ}+(r-c)ZV_Z-V_T-rV = 0 \tag{20}$$

By using the two factor model to redefine a single state variable, the shadow spot price of the commodity, with a volatility that depends on the maturity of the contract we want to value, and a constant long term convenience yield, we obtain a model very similar to the basic model. The only important difference is that the volatility is time dependant. This is not a critical issue in the valuation of European claims since the accumulated volatility to the maturity of the claim

$$v(T) = \int_0^T \sigma_F^2(t)\,dt \tag{21}$$

has a closed form solution

$$\begin{aligned} v(T) = {} & (\sigma_1^2+\frac{\sigma_2^2}{\kappa^2}-\frac{2\rho\sigma_1\sigma_2}{\kappa})T+\frac{\sigma_2^2(1-e^{-2\kappa T})}{2\kappa^3} \\ & +2\sigma_2(\sigma_1\rho-\frac{\sigma_2}{\kappa})\frac{(1-e^{-\kappa T})}{\kappa^2} \end{aligned} \tag{22}$$

The risk neutral distribution of the shadow spot price is log-normal with mean equal

to the forward price given in (19) and a variance (of the log price) given in (22). Note that this distribution has the same variance as the distribution of the spot price in the two factor model. The mean of the distribution approximates the mean of the distribution in the two factor model for longer maturities since (19) converges to (7).

For American claims, the fact that the volatility in (20) is time dependant does not complicate the numerical solution of the differential equation.

2.4 Valuing European Options

Since the two factor model and the long term model imply very similar means and the same variances for the risk neutral distributions of spot prices, the valuation of long term options will give very similar results in both models. Consider the valuation of an European call option on the commodity with an exercise price of K. In the forward price space the value of the call options in both model can be expressed as:

$$C(\cdot,T) = e^{-rT}c(F(\cdot,T),T) \tag{23}$$

where

$$c(F,T) = FN(d) - KN(d - v(T)) \tag{24}$$

where N(d) is the standard normal distribution function and

$$d = \frac{\ln\frac{F}{K}}{v(T)} + \frac{1}{2}v(T)$$

For each model the call price and the futures price are functions of the appropriate

state variables of the respective model: the spot price and the instantaneous convenience yield in the two factor model, and the shadow spot price in the long term model. The variance for both models is the same and it is given in (22). Naturally, at the maturity of the option the relevant variable for determining the exercise strategy is the actual spot price of the commodity and not the shadow price.

3. Implementation

In this section I will demonstrate how to implement the long term model using the parameters estimated for the two factor model in Schwartz (1997). Table 1 shows these parameters for oil and copper. The parameters for copper were estimated using publicly available futures prices for contracts for the period July 88 to June 95. All the contracts had maturities of less than two years. The parameters for oil were estimated using proprietary oil forwards curves which were made available by Enron for the period January 93 to May 96. For this data the maximum maturity used was 9 years. Note that μ, the true total expected return on the spot commodity, does not enter into our calculations and is only reported in Table 1 for completeness.

	Copper	Oil
Period	7/29/88 to 6/13/95	1/15/93 - 5/16/96
Contracts	F1, F3, F5, F7, F9	Enron Data
NOBS	347	163
μ	0.326 (0.110)	0.082 (0.120)
κ	1.156 (0.041)	1.187 (0.026)
α	0.248 (0.098)	0.090 (0.086)
$\sigma 1$	0.274 (0.012)	0.212 (0.011)
$\sigma 2$	0.280 (0.017)	0.187 (0.012)
ρ	0.818 (0.020)	0.845 (0.024)
λ	0.256 (0.114)	0.093 (0.101)
r	0.06	0.05

Table 1

(standard errors are in parenthesis)

Given the parameters of the two factor model and all the futures prices with their respective maturities on any given day, I use the futures pricing equation (7) in a double grid search routine to estimate the state variables, S and δ, which minimize the square deviations between model and market prices. With the estimated state variables S and δ, equation (7) allows us to construct the term structure of futures prices implied by the two factor model, and equation (17) allows us to compute the shadow spot price Z. Finally, equation (19) allows us to construct the term structure of futures prices implied by the long term model.

Figure 1
Market and Models Copper Futures Prices
3/31/97

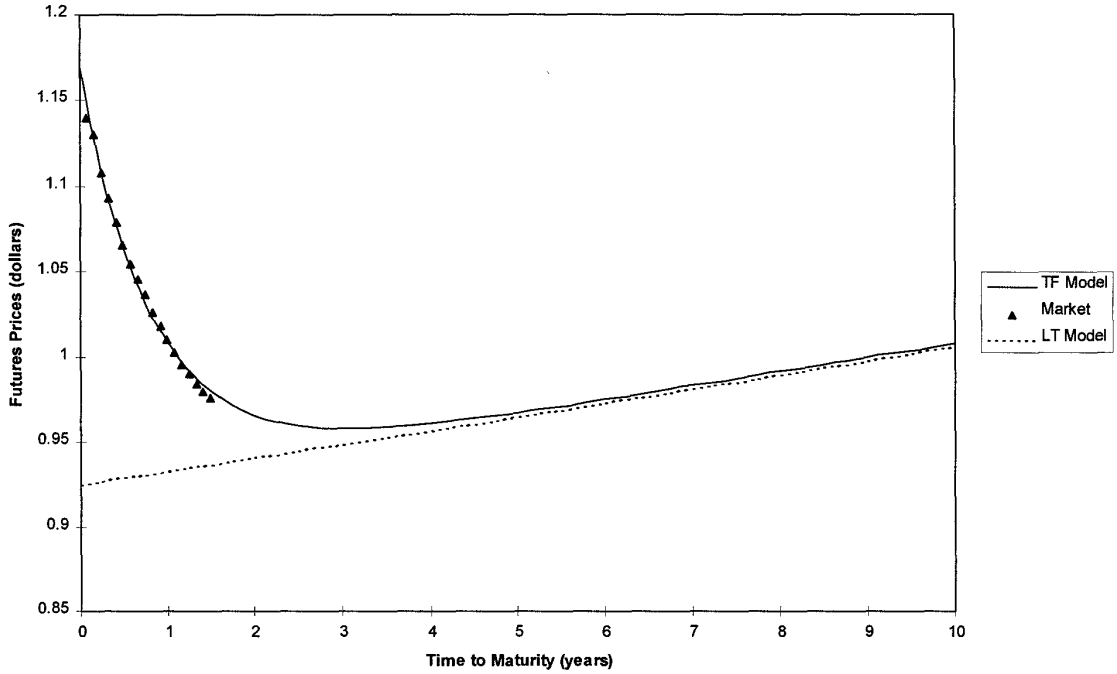


Figure 1 graphs the market prices of all the copper futures contracts reported on the Wall Street Journal for 3/31/97. The figure also shows the term structure of copper futures prices implied by both the two factor model and the long term model, for that day using the parameters for copper in Table 1. Note that the maximum maturity of existing contracts is less than two years. The estimated spot price is $1.169 and the estimated instantaneous convenience yield is 0.305. The corresponding shadow spot price is $0.9252 and the long term convenience yield is 0.0516. The figure shows that the long term model approximates the two factor model for maturities greater than 3 years.

Figure 2
Market and Models Oil Futures Prices
3/31/97

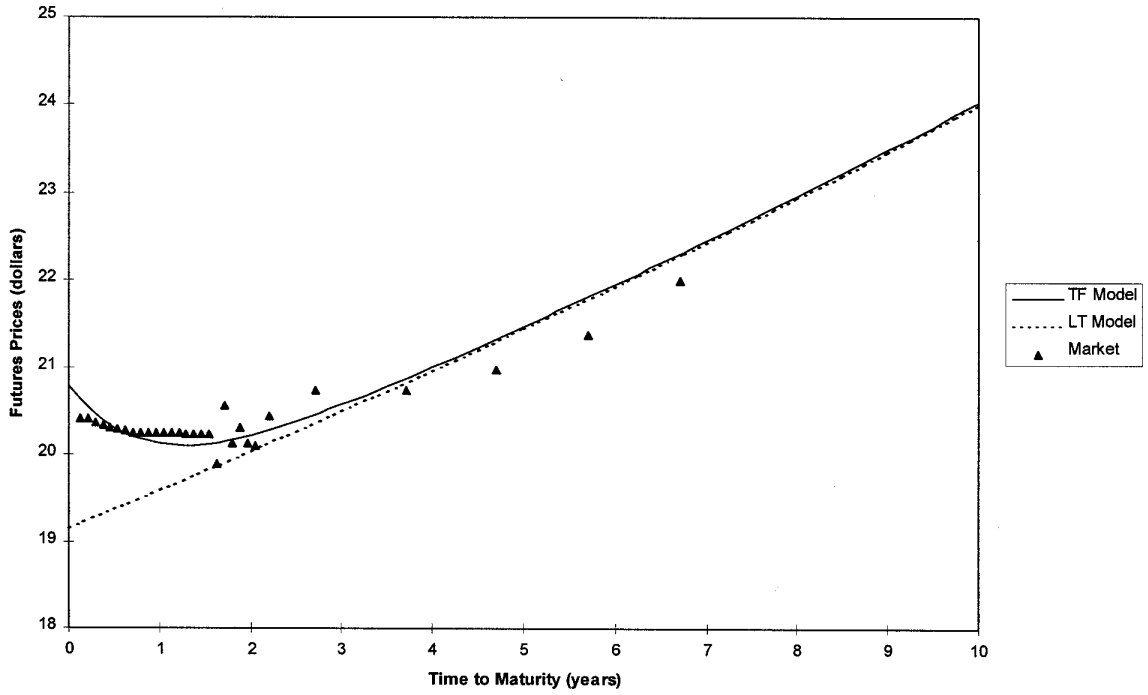
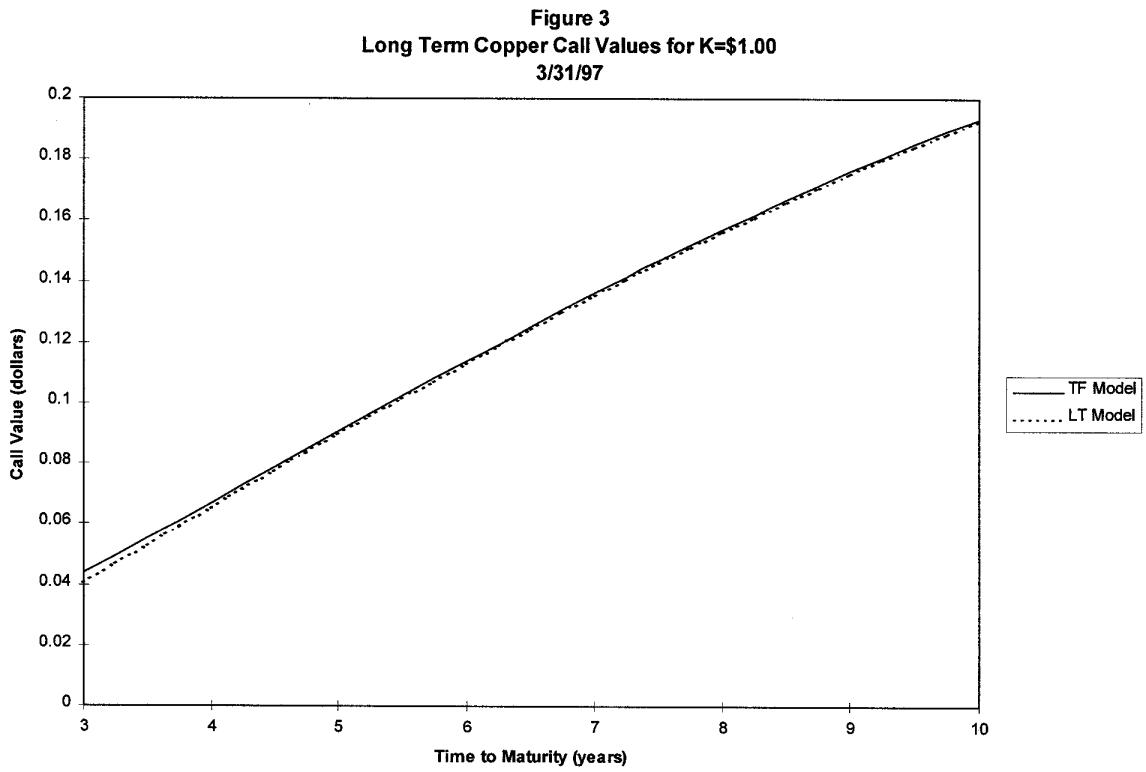


Figure 2 displays the market prices for all oil futures contracts reported in the Wall Street Journal for 3/31/97 and the term structure of futures prices implied by the two models for the oil parameters in Table 1. Note that in this case there are prices reported for contracts up to almost 7 years to maturity. The estimated spot price is $20.79 and the estimated instantaneous convenience yield is 0.117. The corresponding value for the shadow spot price is $19.16 and for the long term convenience yield is 0.0275. Note once again that both models give very close prices for contracts with maturities greater than 3 years. The models, however, do not fit the oil futures prices so well, though some of these prices do not actually represent trades.

Figure 3 graphs the values of European copper call values obtained from the two factor model and the long term model for 3/31/97 for an exercise price of $1.00. The figure shows



that for maturities greater than three years the two models give very similar results.

4. Optimal Exercise Criteria

I have shown that the simple long term model has very similar valuation implications to the more complex two factor model when valuing futures and options with maturities greater than three years. An issue that still remains to be discussed is how the models compare with respect to the optimal time to undertake a project or more generally the optimal exercise of American contingent claims. This is important since in the two factor model we have that the critical spot price

above which it is optimal to invest (or exercise the American option) depends on the current instantaneous convenience yield, whereas in the one factor long term model there is only one critical shadow spot price above which it is optimal to invest. In this section we investigate this issue in the context of a simple imaginary copper mine.

Consider a simple copper mine that can produce one ounce of copper at the end of each year for ten years. Assume that production starts at the end of the fourth year since it takes three years to realize the necessary investments in the project. Suppose that the present value of the initial investment required is K=\$2 and that the constant unit cost of production is C=\$0.40. Assume that once the initial investment is decided further investment will proceed for three years and production will go ahead for the following ten years; that is, we neglect in this analysis the options to close and open the mine and the option to abandon it[4], and concentrate entirely in the option to invest. The first step in the procedure consists in the determination of the net present value of the project once it has been decided to go ahead with the investment (this is the 'boundary condition' of the second step), and the second step consists in the evaluation of the option to invest. The net present value of the project once the investment has been decided is:

$$NPV(\cdot) = \sum_{T=4}^{13} e^{-rT} F(\cdot, T) - C \sum_{T=4}^{13} e^{-rT} - K \qquad \textbf{(26)}$$

For each model the futures prices and the net present value are functions of the appropriate state variables of the respective model: the spot price and the instantaneous convenience yield in the two

[4] For a detailed discussion of these options see Brennan and Schwartz (1985). Since the procedures to evaluate the mine are numerical it would be easy to incorporate them in the analysis.

factor model, and the shadow spot price in the long term model. Note that the summation starts at 4 since it is assumed that production starts at the end of year four.

To value the option to invest and determine the price of copper above which it is optimal to invest in the project we need to solve numerically partial differential equations (12) for the two factor model and (20) for the long term model, both with boundary condition (26).[5]

In the two factor model the value of the mine is a function of both the spot price of copper and the instantaneous convenience yield. The optimal spot price above which it is optimal to invest is a function of the instantaneous convenience yield. Figure 4 shows this optimal price as a function of the convenience yield[6]. Notice that for a convenience yield of zero the optimal price is $1.10, whereas for a convenience yield of 0.40 the optimal price is $1.52. Due to the strong mean reversion in the convenience yield and the high correlation between the convenience yield and the spot price, a high value of the convenience yield implies that spot prices will tend to come down, therefore requiring a higher trigger value for investment. The value of the mine on 3/31/97, when the spot price was $1.169 and the instantaneous convenience yield was 0.305, would have been $1.75 and the optimal trigger price for investment would have been $1.41. At the current copper spot price it would not have been optimal to start investing.

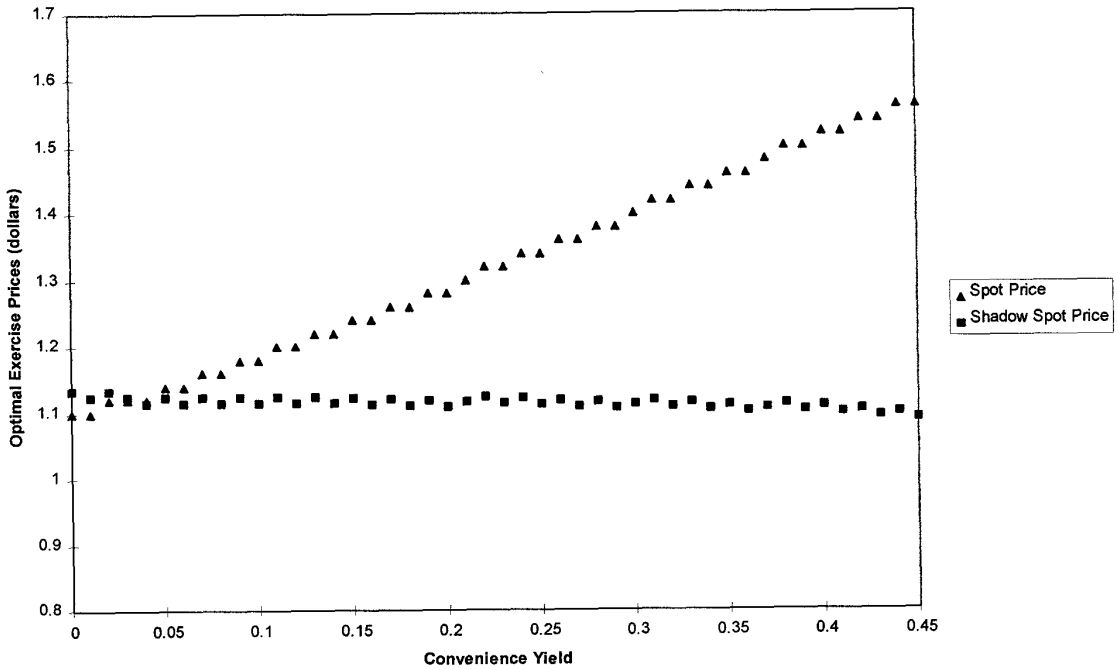
Figure 4 also shows the corresponding optimal shadow spot prices computed using equation (17) and implied by the combinations of optimal spot prices and convenience yields obtained in the two factor model. Notice that there is surprisingly little variation in these optimal

---

[5] In both cases I assume that the investment option has a maturity of ten years.

[6] The discreetness in the figure is due to the discreetness in the numerical solution.

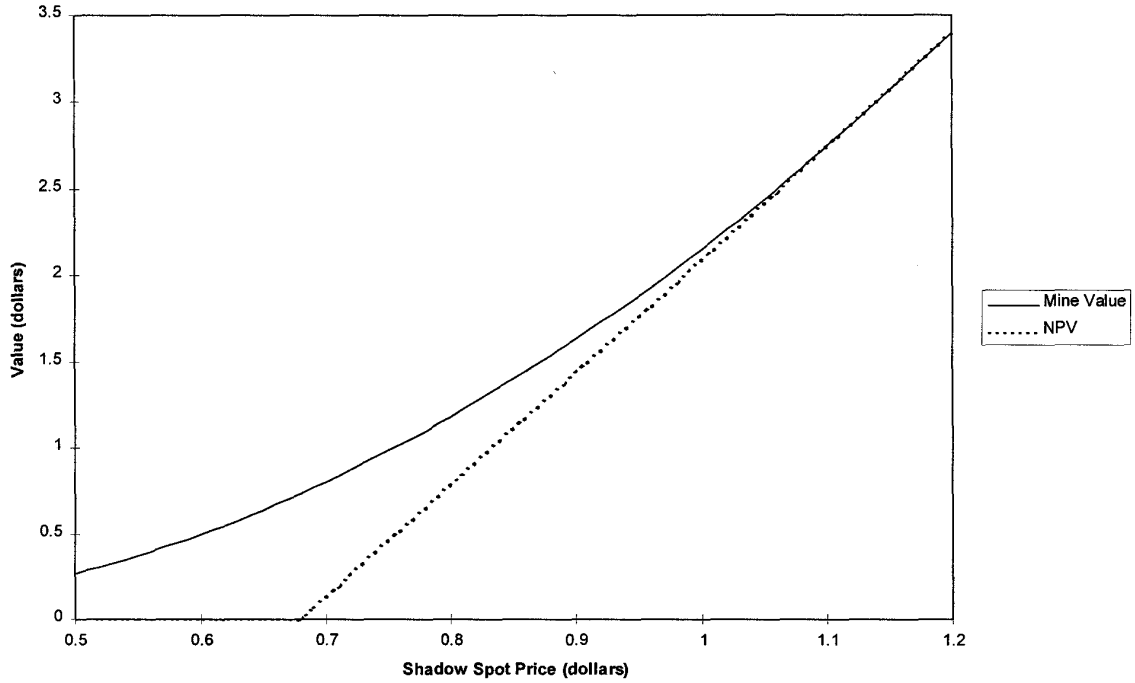
Figure 4
Simple Copper Mine
Optimal Exercise Prices



shadow spot prices: every combination of spot prices and convenience yield implies practically the same shadow spot price (values vary between $1.10 and $1.13). We see from the figure that only for very low levels of the convenience yield the optimal spot price is below the corresponding optimal shadow spot price.

When solving the partial differential equation for the one factor long term model we obtain a optimal shadow spot price for investment of $1.12, which is indistinguishable from the one obtained using the two factor model. Figure 5 graphs the value of the simple copper mine as a function of the shadow spot price. It also shows the net present value of the project, which is positive when the shadow spot price is greater that $0.69. The value of the investment option is larger than the net present value up to a price of $1.12 where both lines converge since at that point

Figure 5
Simple Copper Mine Value



it is optimal to invest in the project. The value of the mine on 3/31/97, when the shadow spot price was $0.9252, would have been $1.75, which to two significant figures is the same one obtained using the two factor model. Since the shadow spot price was bellow the trigger price for investment it would not have been optimal to start investing.

The previous discussion reveals that when valuing commodity projects where cash flows start a few years into the future a simple one factor model can give practically the same results as the two factor model. When using the long term model, the current convenience yield together with the optimal shadow spot price obtained from the model can be used in (17) to determine the actual spot price at which investment should proceed. If there are relevant cash flows in the first two

years, the value of the project can be adjusted after the valuation is made to take into account for possibly larger differences in futures prices between the two models in these years.

5. Conclusion

In my presidential address to the American Finance Association I showed that a two factor model for the stochastic behavior of commodity prices fitted quite well the term structure of futures prices and the term structure of futures return volatility for two key commercial commodities: copper and oil. The two factor model, however, is relatively difficult to apply when valuing complex investment projects with multiple options such as the investment option, the option to close the operation temporarily and then re-open it, and the abandonment option.

In this article I develop a simple one factor model which has practically the same implications as the two factor model when it is applied to value long term commodity assets. It derives directly from the two factor model: the single factor is a function of the two factors, the constant convenience yield in the model is a function of the parameters of the two factor model and the time dependant volatility is the same as in the two factor model. The inputs to the model are the prices of all current futures prices and the estimated parameters for the two factor model.

This long term model can then applied to value projects with complex options without sacrificing any of the advantages of the two factor model. Existing computer programs for one factor models can be easily modified to take into account most of the properties of the two factor model.

References

Bessembinder, H., J.F. Coughenour, P.J. Seguin, and M.M. Smoller, 1995, Mean reversion in equilibrium asset prices: evidence from the futures term structure, *Journal of Finance* 50:1, 361-375.

Brennan, M.J., 1991, The price of convenience and the valuation of commodity contingent claims, in D. Lund and B. Oksendal, eds., *Stochastic Models and Option Values,* North Holland.

Brennan, M.J., and E.S. Schwartz, 1985, Evaluating natural resource investments, *Journal of Business* 58, 133-155.

Cortazar, G., and E.S. Schwartz, 1994, The evaluation of commodity contingent claims, *Journal of Derivatives* 1, 27-39.

Fama, E.F., and K.R. French, 1997, Industry cost of equity, *Journal of Financial Economics* 43:2, 153-193.

Gibson, R., and E.S. Schwartz, 1990, Stochastic convenience yield and the pricing of oil contingent claims, *Journal of Finance* 45, 959-976.

Gibson, R., and E.S. Schwartz, 1991, Valuation of long term oil-linked assets, in D. Lund and B. Oksendal, eds., *Stochastic Models and Option Values,* North Holland.

Ross, S.A., 1995, Hedging long run commitments: exercises in incomplete market pricing, preliminary draft.

Schwartz, E.S., 1997, The stochastic behavior of commodity prices: Implications for valuation and hedging, *Journal of Finance* forthcoming.