Learning from Experience*

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Abstract

This paper examines whether individuals learn from experience by studying individual day traders’ decisions. It finds that the type of outcomes day traders observe determines their exit and trade size decisions: day traders take smaller gambles or quit after unsuccessful day trades. Wealth changes do not explain any of our results: day traders do not reduce trade sizes or quit simply because their wealth is smaller. These findings are consistent with a fully rational model where agents learn about the profitability of day trading: even an investor who currently believes that day traders lose money is often willing to day trade to learn because the potential gains are large. A commonly suggested alternative hypothesis, that day traders have stubborn beliefs about their own abilities, does not generate the dynamics we observe in the data. Our results suggest that individuals do learn from experience, casting doubt on behavioral models where investors exhibit permanent, welfare-reducing irrationality.
“Three years of falling markets have shaken out the inexperienced investors who signed up to online accounts without any real understanding of the markets.”

—Financial Times, “Day Trading: Those still trading online have kept their day jobs and learnt how to hedge” (February 22, 2003)

1 Introduction

Most economic models assume that investors are capable of very complicated computations. For example, Merton’s (1969) life-cycle model and its extensions assume that investors can solve Bellman equations while rational expectations equilibrium (REE) models assume that agents can solve fixed-point problems. In practice, individuals’ ability to carry out these computations may be limited (Hirshleifer 2001) but probably not zero. A broad literature in macroeconomics assumes that investors do not know everything about the economy but they learn about the parameters over time. A typical question posed in this adaptive learning literature is the following: suppose agents postulate a functional form for the prices in a rational expectations model; does the economy converge to equilibrium if the agents revise their estimates each period based on new observations?1 Similarly, Grossman and Stiglitz (1980) motivate their REE model as a statistical equilibrium where the economy converges because of learning.

Despite many theoretical arguments for why learning is crucial, empirical work on the issue is nonexistent. This is not surprising: it is difficult to examine how an investor changes her behavior in response to new information because (1) the investor’s prior beliefs are inherently unobservable and (2) we often do not even know what the investor observes. For example, if the investor actively trades in the stock market, she could eventually learn that her stock picking skills are not good enough to justify the high transaction costs. She might then stop and transfer her assets to an index fund. However, it would empirically be very difficult to detect such rational learning because the feedback from stock-picking is slow and noisy.

We address the important question of learning by studying the decisions of individual day traders. We argue that these investors are ideal for a study of learning for several reasons:

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1See Evans and Honkapohja (2001) for a textbook treatment.
1. **Dispersed Beliefs.** It is very difficult for an individual to learn about her own day trading skills without actually day trading. Many individuals may have a reason to entertain the possibility that day trading might be profitable: for example, a neighbor’s positive experiences may cast enough doubt so that the individual wants to find out for herself. Dispersed beliefs are also ideal for a study of learning: new data influence beliefs (and behavior) more when investors’ priors are uninformative.

2. **Immediate feedback.** A day trader tries to buy and sell stocks during a single trading day at profit. The profit or loss from each day trade provides the investor with immediate and precise feedback. This distinguishes day trading from general stock-picking where feedback is often slow and noisy.

3. **Sequential observations.** A day trader makes sequential decisions and each of these decisions is observable. For example, if the day trader learns from experience, outcomes drive changes in day trade sizes and determine when the day trader quits.

The first argument about dispersed priors is important. For example, the question why some individuals day trade is puzzling by itself because most of these day traders lose money (Barber, Lee, Liu, and Odean 2004; Linnaĩmäa 2005a).\(^2\) A casual explanation is that day traders have **stubborn beliefs** about their own abilities.\(^3\) This explanation argues that day traders day trade, lose money, and continue to day trade. However, an alternative possibility is that individuals may just be uncertain about the profitability of day trading.\(^4\) If this is true, investors may day trade just to learn: the potential gains are too high to be offset by the small costs of learning—time and transaction costs. (Section 2’s model makes this argument precise.)

\(^2\)Other studies that examine day traders and other short-term trades include Harris and Schultz (1998), Garvey and Murphy (2001), Jordan and Diltz (2002), and Seasholes and Wu (2004).

\(^3\)We use term “stubborn beliefs” to describe an investor whose subjective prior about the day trading return distribution is degenerate. Such an investor ignores all feedback and always acts as if the subjective prior is the truth. We avoid using term “overconfidence” because it only indicates that an agent’s subjective probability distribution is too tight (Alpert and Raiffa 1982; Kyle and Wang 1997). Many studies, however, do assume that overconfidence is a permanent or nearly permanent trait (see, e.g., Odean 1998). We use term “stubborn beliefs” instead of “overconfidence” to avoid ambiguity when discussing prior and posterior distributions.

\(^4\)The idea that some individuals do not perfectly understand financial markets is not novel. For example, Brennan (1995, pp. 61) contrasts an individual investor against the representative agent: “The representative investor is assumed to understand the economy and the process determining asset prices; the individual investor [is not].”
This paper tests whether individuals learn from experience by comparing two explanations for the existence of day traders. These competing hypotheses—rational learning versus stubborn beliefs—make very different predictions about day traders’ behavior. If individuals day trade because of stubborn beliefs, (1) they quit only after running out of money and (2) their trade sizes respond to prior outcomes only because of the outcomes’ wealth effects. In contrast, if individuals day trade because of uncertainty and learning, (1) they quit after updating their beliefs sufficiently downwards and (2) their trade sizes change as their beliefs evolve. For example, a positive experience leads an investor to revise her beliefs upwards, and hence, to increase her trade size. Moreover, individuals who believe that day trading is unprofitable—but entertain the possibility that it could be profitable—may place small exploratory bets to learn about the profitability.

We test the learning hypothesis using a unique data set from Finland. This data set contains all transactions and positions of all individual day traders in Finland. Many characteristics of these investors support the possibility that they are uncertain about the profitability of day trading. For example, many have surprisingly limited investment experience before starting to day trade: one of every ten day traders owned or traded stocks for the very first time less than three months before the first day trade. Furthermore, the majority of day traders day trade only once or twice. This may reflect learning: investors are pessimistic about the profitability of day trading to begin with and only need a few observations to confirm their beliefs.

Our results show that investors learn from experience. Day traders quit after experiencing losses and adjust their trade sizes in response to outcomes even after controlling for the outcomes’ wealth effects. Day traders also experiment with very small trades that are without speculative motives. We conclude that it is uncertainty and learning—not stubborn beliefs about one’s own abilities—that explains why (some) investors day trade. This intuitive result is important: a large body of behavioral finance literature assumes that behavioral biases persist over time. However, if investors learn from experience—as almost all non-behavioral studies assume either directly or by placing high demands on the agents’ cognitive abilities—an individual’s welfare-reducing behavioral biases will dissipate over the life-cycle.

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5We follow Lefèvre (1923, pp. 128): “It is simple arithmetic to prove that it is a wise thing to have the big bet down only when you win, and when you lose to lose only a small exploratory bet, as it were.”
The rest of the paper is organized as follows. Section 2 illustrates the effects of parameter uncertainty and learning using a three-period model. Section 3 discusses the data set and Section 4 presents the empirical results. Section 5 concludes.

2 Model

This section presents a tractable life-cycle model to demonstrate how parameter uncertainty and learning affect optimal investment. (We give a multiperiod extension of the model in Appendix A.2. This model extends the life-cycle model of Linnainmaa (2005b) by adding transaction costs and by dropping the assumption that the agent observes outcomes even without an investment.) We also compare the model's predictions against the predictions of the “stubborn beliefs” hypothesis.

2.1 Assumptions

We make the following assumptions:

- An agent lives for three dates, \( t = 0, 1, 2 \), and maximizes power utility over date 2 wealth,

\[
U(w_2) = \frac{w_2^{1-\gamma} - 1}{1 - \gamma}.
\]

- The agent can day trade an amount \( x_t \geq 0 \) at dates 0 and 1.

- The day trade's outcome has a binomial distribution:
  
  - The day trade is a success with probability \( p \) and pays off \( (1 + \delta)x \)
  
  - The day trade is a failure with probability \( 1 - p \) and pays off \( (1 - \delta)x \).

where \( 0 < \delta \leq 1 \).

- The agent must always pay a transaction cost \( c > 0 \) to day trade. The agent can pay the cost without day trading (i.e., by choosing \( x = 0 \)) if he only wants to observe the outcome.\(^6\)

\(^6\)The transaction cost may consist of items such as the spread and brokerage fee. An agent paying the cost and choosing \( x = 0 \) is effectively placing a very small trade just to learn from experience. For example, investors in actual markets may wish to try out a trading strategy with small trades. Our assumption about infinitesimal trades captures this intuition.
• There is a risk-free asset that pays no interest.

• The agent has initial wealth $w_0 \gg c$.

• The agent does not know parameter $p$ (the probability of a success) but has a prior about it:

  – The date 0 prior beliefs are Beta($\alpha_0$, $\beta_0$) distributed.
  – If the agent day trades—or pays the transaction cost and chooses $x = 0$—he observes the outcome and updates his beliefs as a Bayesian.
  – If he does not pay the cost, he does not get to observe the outcome.

The agent solves his problem in two steps: first, an inference problem in which the agent updates his estimate of $p$ and second, an optimization problem in which the agent chooses the optimal investment given the current wealth and the estimate of $p$ (Gennotte 1986; Brennan and Xia 2001). We consider first the inference problem.

### 2.2 The Agent’s Inference Problem

The agent has a conjugate prior distribution Beta($\alpha_0$, $\beta_0$) about $p$ at date 0. The assumption about a binomial stock price process and a Beta-distributed prior makes the inference problem particularly tractable. The date $t$ posterior after observing $N_t$ positive stock price movements is Beta($\alpha_0 + N_t$, $\beta_0 + t - N_t$)-distributed (see, e.g., DeGroot (1970, pp. 160)). The mean of the posterior distribution at date $t$ is

$$E_t(p) = \frac{\alpha_0 + N_t}{\alpha_0 + \beta_0 + t}. \quad (2)$$

We let $\alpha_t \equiv \alpha_0 + N_t$ and $\beta_t \equiv \beta_0 + t - N_t$ to denote the date $t$ belief parameters. The intuition for this updating is simple: the parameters of the Beta-distribution keep track of the number of good and bad observations. For example, if the agent starts with parameters (1, 2), the parameters are (2, 2) after a good outcome and (1, 3) after a bad outcome. Figure 1 illustrates the agent’s date 0 problem.
2.3 The Agent’s Optimization Problem

We first solve the agent’s date 1 problem and then move backwards and solve the date 0 problem. Because the agent has to pay \( c \) to learn, we must compare agent’s utilities under three different policies at each step:

- **Invest**: the agent pays the cost \( c \) and day trades a strictly positive amount, \( x_t > 0 \).
- **Pay the Cost**: the agent pays the cost \( c \) but only to observe the outcome, \( x_t = 0 \).
- **Do Nothing**: the agent does not pay the cost and does not observe the outcome.

2.3.1 The Date 1 Problem

We first compute the agent’s indirect utilities under different policies and then characterize the agent’s optimal behavior by comparing the utilities from following these policies. The agent’s date 1 Bellman equation solves the following problem under the *invest* policy:

\[
V_{1}^{\text{invest}}(w_1, (\alpha_1, \beta_1)) = \max_{x_1 \geq 0} \left\{ \frac{\alpha_1}{\alpha_1 + \beta_1} \left( w_1 + \delta x_1 - c \right)^{1-\gamma} + \frac{\beta_1}{\alpha_1 + \beta_1} \left( w_1 - \delta x_1 - c \right)^{1-\gamma} \right\}. \quad (3)
\]

The optimal investment from the first order condition is

\[
x_1^*(w_1, (\alpha_1, \beta_1)) = \frac{\frac{1}{\delta} \frac{1}{\alpha_1} - \frac{1}{\beta_1}}{\frac{1}{\alpha_1} + \frac{1}{\beta_1}} (w_1 - c). \quad (4)
\]
We rewrite the agent’s value function as

\[ V_{\text{invest}}^1(w_1, (\alpha_1, \beta_1)) = \begin{cases} (w_1 - c)^{1-\gamma} k_1(\alpha_1, \beta_1) & \text{if } x_1^*(w_1, (\alpha_1, \beta_1)) > 0 \\ -\infty & \text{otherwise} \end{cases} \]  

(5)

where \( k_1(\alpha_1, \beta_1) = \frac{\alpha_1^{1-\gamma}}{\alpha_1 + \beta_1} \left( \frac{1}{\alpha_1} + \beta_1 \right)^{\gamma} \). We make two observations about \( k_1(\alpha_1, \beta_1) \). First, \( k_1(\alpha_1, \beta_1) \geq 1 \) if \( \gamma < 1 \) and \( k_1(\alpha_1, \beta_1) \geq 1 \) if \( \gamma > 1 \). Second, \( \frac{\partial}{\partial \alpha_1} k_1(\alpha_1, \beta_1) > 0 \) if \( \gamma < 1 \) and \( \frac{\partial}{\partial \alpha_1} k_1(\alpha_1, \beta_1) > 0 \) if \( \gamma > 1 \).

If the agent chooses the do nothing policy, the indirect utility is \( V_{\text{do nothing}}^1(w_1, (\alpha_1, \beta_1)) = \frac{w_1^{1-\gamma}}{1-\gamma} \). Note that it is never optimal for the agent to just pay the cost at date 1 so we ignore this policy. The optimal date 1 rule is utility-maximizing:

\[ V_1(w_1, (\alpha_1, \beta_1)) = \arg \max_{\{\text{invest, do nothing}\}} \{ V_{\text{invest}}^1, V_{\text{do nothing}}^1 \}. \]  

(6)

Hence, the agent day trades if (i) \( \alpha_1 > \beta_1 \) and (ii) \( c \leq \left( 1 - k_1(\alpha_1, \beta_1)^{-\frac{1}{1-\gamma}} \right) w_1 \). (The second condition follows from Eq. 6 after substitution.) This condition says that the cost \( c \) cannot be too high or the agent will not invest. Let us denote the RHS of this condition by \( c_1(w_1, (\alpha_1, \beta_1)) \)—this is the upper bound for the cost—and note that \( \frac{\partial}{\partial w_1} c_1 > 0 \) and \( \frac{\partial}{\partial \alpha_1} c_1 > 0 \).

We can write the date 1 value function as

\[ V_1(w_1, (\alpha_1, \beta_1)) = \frac{(w_1 - b_1 * c)^{1-\gamma}}{1-\gamma} k_1(\alpha_1, \beta_1) \]  

(7)

where \( b_1 = 0 \) and \( k_1(\alpha_1, \beta_1) = 1 \) if the agent does not day trade at date 1.

### 2.3.2 The Date 0 Problem

The optimal behavior at date 0 can be solved in the same way as the date 1 behavior. We compute the indirect utilities of the three policies separately and then let the agent choose the one that yields the highest utility.

First, the agent can choose to day trade at date 0. Note that this can only happen if the agent may believe tomorrow that day trading is profitable: i.e., the necessary condition is that \( \alpha_0 + 1 > \beta_0 \). If this is not the case, the agent would never day trade tomorrow. Hence, we
temporarily assume that the agent would want to day trade tomorrow at least after a positive outcome today. The agent’s date 0 Bellman equation then solves

\[
V_{0}^{\text{invest}}(w_0, (\alpha_0, \beta_0)) = \max_{x_0 \geq 0} \left\{ \frac{\alpha_0}{\alpha_0 + \beta_0} V_1(w_0 + \delta x_0 - c, (\alpha_0 + 1, \beta_0)) + \frac{\beta_0}{\alpha_0 + \beta_0} V_1(w_0 - \delta x_0 - c, (\alpha_0, \beta_0 + 1)) \right\}.
\] (8)

The optimal investment from the first-order condition is

\[
x_0^*(w_0, (\alpha_0, \beta_0)) = \frac{1}{\delta} \left[ (\alpha k^+ + \beta k^-)^{\frac{1}{\gamma}} - (\beta k^- + \beta k^-)^{\frac{1}{\gamma}} w_0 - \frac{(\alpha k^+ + \beta k^-)^{\frac{1}{\gamma}} b^+ - (\beta k^- + \beta k^-)^{\frac{1}{\gamma}} b^-}{(\alpha k^+ + \beta k^-)^{\frac{1}{\gamma}} + (\beta k^- + \beta k^-)^{\frac{1}{\gamma}}} c \right].
\] (9)

where \(k^+\) and \(b^+\) are the coefficients of the date 1 value function (Eq. 7) after a positive outcome today and \(k^-\) and \(b^-\) are the coefficients after a negative outcome. (For example, if the agent invests in both states tomorrow, \(b^+ = b^- = 1\), but if the agent only invests in the upstate, \(b^+ = 1\) and \(b^- = 0\).) The value function can be rewritten as

\[
V_{0}^{\text{invest}}(w_0, (\alpha_0, \beta_0)) = \frac{(w_1 - b_0 * c)^{1-\gamma}}{1-\gamma} k_0(\alpha_0, \beta_0)
\] (10)

where \(b_0 = \begin{cases} 2 & \text{if invest in both date 1 states} \\ \frac{3}{2} & \text{if invest only after a good outcome today} \end{cases}\)

\[
k_0(\alpha_0, \beta_0) = \begin{cases} \frac{2^{1-\gamma}}{\alpha_0 + \beta_0} \left[ (\alpha_0 k_2(\alpha_0 + 1, \beta_0))^{\frac{1}{\gamma}} + (\beta_0 k_2(\alpha_0, \beta_0 + 1))^{\frac{1}{\gamma}} \right]^{\frac{1}{\gamma}} & \text{both states} \\ \frac{2^{1-\gamma}}{\alpha_0 + \beta_0} \left[ (\alpha_0 k_2(\alpha_0 + 1, \beta_0))^{\frac{1}{\gamma}} + (\beta_0 k_2(\alpha_0, \beta_0 + 1))^{\frac{1}{\gamma}} \right]^{\frac{1}{\gamma}} & \text{upstate only} \end{cases}
\]

Second, if the agent pays the cost, he derives indirect utility

\[
V_{0}^{\text{pay the cost}}(w_0, (\alpha_0, \beta_0)) = \frac{\alpha_0}{\alpha_0 + \beta_0} V_1(w_0 - c, (\alpha_0 + 1, \beta_0)) + \frac{\beta_0}{\alpha_0 + \beta_0} V_1(w_0 - c, (\alpha_0, \beta_0 + 1))
\] (11)

which is the tomorrow’s expected indirect utility after paying the cost today. Third, the indirect utility of doing nothing at date 0 is \(V_{0}^{\text{do nothing}}(w_0, (\alpha_0, \beta_0)) = \frac{w_0^{1-\gamma}}{1-\gamma}\). The optimal
date 0 rule is utility-maximizing:

\[ V_0(w_0, (\alpha_0, \beta_0)) = \arg \max \left\{ V_0^{\text{invest}}, V_0^{\text{pay the cost}}, V_0^{\text{do nothing}} \right\}. \] (12)

2.4 The Agent’s Optimal Behavior

We characterize the agent’s optimal behavior with three results before turning to examples. The proofs are in the appendix.

**Proposition 1.** The agent always pays at least the trading cost at date 0 if the prior is sufficiently dispersed and the trading cost \( c \) is small.

This result says that the agent always pays the cost if he is sufficiently unsure of himself and the cost is not “too high”. This an intuitive result: the agent is willing to pay a small cost because the potential gains—i.e., if day trading turns out to be profitable—are very large.

**Proposition 2.** The optimal investment is increasing in the number of positive outcomes, \( \alpha \).

This result says that, ceteris paribus, an agent with higher expectations about the profitability day trades more.

**Proposition 3.** The optimal investment is decreasing in the agent’s risk aversion, \( \gamma \).

This result shows that an agent with low risk-aversion is willing to day trade for a wider range of parameter values relative to an otherwise identical but more risk-averse agent.

2.5 Regions of Optimal Behavior

Figure 2 illustrates the agent’s optimal behavior in the model. The agent has an initial wealth of \( w_1 = \$10,000 \) and a risk-aversion of either \( \gamma = 2 \) or \( \gamma = 0.5 \), and has to pay \( c = \$20 \) to day trade. The three possible choices are drawn as functions of the parameters of the prior distribution. Note that the area below the diagonal (where \( \alpha_0 > \beta_0 \)) represents beliefs that day trading is profitable.

It is useful to consider where the agent gets his prior before discussing the results. A natural influence for the initial beliefs is the media and the word-of-mouth information sharing among neighbors (Hong, Kubik, and Stein 2004). For example, suppose the agent has three neighbors
Figure 2: **Optimal Behavior under Parameter Uncertainty.** This figure shows the optimal behavior of an agent who is uncertain about the profitability of day trading and maximizes power utility over terminal wealth. The agent has an initial wealth of $w_0 = 10,000$, a risk-aversion of $\gamma = 2$ (Panel A) or $\gamma = 0.5$ (Panel B) and can day trade amount $x \geq 0$ at dates 0 and 1 at a cost $c = 20$. The day trade is a success with a probability $p$ and pays off $(1 + \delta)x$; it is a failure with a probability $1 - p$ and pays off $(1 - \delta)x$. The investor has a Beta($\alpha_0$, $\beta_0$) distributed prior about $p$. If the agent pays the cost, he observes the outcome and updates his beliefs as a Bayesian. This figure plots the three possible actions that the agent can take at date 0: do nothing, pay the cost, or day trade (“invest”). The regions are plotted as functions of parameters of the prior distribution. An increase in $\alpha$ while keeping $\beta$ fixed means that the investor has a more positive view of the profitability. An increase in both $\alpha$ and $\beta$ for a fixed $\delta$ lowers the variance of the prior. An agent in the region above (below) the 45º line believes that day trading is unprofitable (profitable).

known for their active day trading. If two of them are now hiding from debtors while one is driving a new luxury car, the agent may pick Beta(1, 2) as a reasonable description of his beliefs (“one good, two bad observations”). However, the agent may be more uncertain than
this because his neighbors’ experiences may not be very informative of his own day trading
skills. If so, the agent beliefs might be better described by Beta(0.1, 0.2) distribution. The
agent has the same level of expectations as before but more uncertainty.\(^7\)

The figure reveals how the ability to learn affects behavior. We discuss three observations:
(1) agents are usually willing to pay the cost and day trade when they believe day trading
is even marginally profitable; (2) agents are often willing to pay the cost to learn even when
they believe day trading is probably unprofitable; and (3) the less risk-averse agent is willing
to day trade and pay the cost for a wider range of parameters. First, an agent who currently
believes that day trading is profitable usually pays the cost and day trades, \(x_0^* > 0\). The
only exceptions are the cases where the agent’s prior about \(p\) is very close to \(\frac{1}{2}\) and the agent
is very confident about his view. In this case, the cost of day trading makes the activity
unattractive. (These cases lie just below the diagonal outside the upper-right corner of Figure
2; not shown.) Put differently, even if the agent believes he could earn marginally positive
day trading returns before brokerage fees, brokerage fees reverse the situation.

Second, agents are often willing to place exploratory bets (i.e., they pay the cost to learn)
even when they believe that day trading is unprofitable. (This is the shaded area above the
diagonal in the figure.) The requirement is that the agent is sufficiently uncertain about his
beliefs; i.e., when \(\alpha_0\) and \(\beta_0\) tend closer to zero. Third, the level of risk-aversion significantly
affects optimal decisions. The Panel B’s agent day trades (or at least pays the cost) for a
wider range of parameters than the Panel A’s agent.

The model illustrates that parameter uncertainty can potentially explain why some in-
vestors day trade. We expect day traders to be a specific part of the population: those who
initially believe that day trading is profitable or those who are just sufficiently uncertain.
Moreover, ceteris paribus, an investor with lower risk-aversion is more likely to become a day
trader. This tractable Bayesian model leads to an important conclusion: even in a world
where everyone believes that day traders lose money, some investors become day traders.

\(^7\)The following two facts about Beta-distribution are useful. First, the ratio \(\frac{\alpha}{\beta}\) determines the mean. Second,
when this ratio is fixed and \(\alpha\) and \(\beta\) increase, the variance decreases. Hence, in our example, the agent’s belief
about the level of profitability is unchanged when he moves from parameters \((\alpha_0, \beta_0) = (1, 2)\)
to \((\alpha_0, \beta_0) = (0.1, 0.2)\) but the dispersion increases. (Pratt, Raiffa, and Schlaifer (1995) discuss at length how to parameterize
prior distributions.)
### Parameters:

<table>
<thead>
<tr>
<th>Initial wealth</th>
<th>$10,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$20</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>$\alpha, \beta$</td>
<td>(1,2)</td>
</tr>
</tbody>
</table>

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**Example 1**

Figure 3: **Day Trader’s Optimal Behavior under Parameter Uncertainty.** This figure illustrates optimal behavior of an agent who is uncertain about the profitability of day trading and maximizes power utility over terminal wealth. The agent has an initial wealth of $w_0 = $10,000, a risk-aversion of $\gamma = 2$ and can day trade amount $x \geq 0$ at dates 1, 2, 3, and 4 at a cost $c = $20. The day trade is a success with a probability $p$ and pays off $(1 + \delta)x$; it is a failure with a probability $1 - p$ and pays off $(1 - \delta)x$. There is also a risk-free asset than pays no interest. The investor has a Beta($\alpha_0, \beta_0$) distributed prior about $p$. If the agent pays the cost, he observes the outcome and updates his beliefs as a Bayesian. The agent’s prior in the first example is Beta(2, 1)-distributed and it is Beta(1, 2) in the second example. Each node in this figure shows the agent’s optimal decision, his beliefs and wealth, and how much he invests (the cost is denoted by “+$20”). Terminal nodes ($t = 5$) are not shown.

#### 2.5.1 A Numerical Example

How does the agent’s behavior change over time because of learning? Figure 3 addresses this question with two examples using a five-period version of the model (see Appendix A.2). The parameters of this example are the same as before: an initial wealth of $10,000, a cost of $c = $20, and a risk-aversion of $\gamma = 2$. The agent in the first example starts with a prior $(\alpha_0, \beta_0) = (1,2)$, i.e., with a belief that day trading is unprofitable while the agent in the second examples starts with a more optimistic prior, $(\alpha_0, \beta_0) = (2,1)$.

The first example shows how the agent’s behavior is affected by what may happen: the
agent is willing to pay the cost because day trading may turn out to be profitable. In fact, he requires a sequence of two positive outcomes before day trading more than the minimum amount (i.e., he then pays the cost and invests $930). If the outcome is again positive, the investment increases to $1,862 for the last period before the agent consumes his terminal wealth. However, a more probably scenario ex ante (given the agent’s prior) is that the agent observes a negative outcome already at date 0 or at date 1. If this happens, the agent quits. Note that the agent already rationally believes before date 0 that he will have quit after date 1 with probability $\frac{2}{3} + \frac{1}{3} \times \frac{1}{2} = \frac{5}{6}$ and after date 2 with probability $\frac{9}{10}$. Yet, the agent is willing to pay the small cost to learn because of the possibility that day trading may turn out to be profitable.
The second example illustrates how parameter uncertainty affects behavior even when the agent starts with a high prior. The agent’s trade size increases after positive outcomes and falls after negative outcomes. This is not surprising, but it is in strong contrast with a scenario where the agent does not learn. For example, suppose that the agent took the probability of a successful day trade (\( \hat{p} \)) as a fixed parameter. A quick computation shows that the agent would always invest a constant fraction (17.2%) of his after-trading costs wealth. Hence, the initial investment would be higher, $1,702 (versus $1,243 with learning). Second, his next period investment would be either $1,984 or $1400, depending on the first period’s outcome—the upstate investment is less than what our agent invests, and the downstate investment is more.

These two examples illustrate how the evolution of beliefs generates dynamics in trade sizes and exit decisions, and how the predictions of a stubborn agent’s behavior are very different from the behavior of agent who learns over time. We now make this comparison more precise.

2.6 Learning versus Stubborn Beliefs

We now show formally that the competing hypothesis of stubborn beliefs predicts that day traders’ behavior is “constant” over time. Investors with stubborn beliefs day trade because they believe they can make money by day trading. They do not revise their beliefs over time and, consequently, changes in wealth and horizon alone influence investment behavior. Suppose that an agent has a degenerate subjective distribution \( E_t(p) = \hat{p} \) —i.e., that the agent’s belief about the profitability of day trading is fixed. This type of an agent day trades only if \( \hat{p} > \frac{1}{2} \). If the agent does believe that day trading is profitable, he always day trades a constant amount of after-trading costs wealth:

\[
x_t^* = \frac{\hat{p}^\gamma}{\hat{p}^\gamma + (1 - \hat{p})^\gamma} (w_t - (T - t + 1) c)
\]

(13)

where \( T \) is the index of the last trading day. Note that the adjustment for trading costs takes into account all the costs the agent will pay over the life-cycle. This result shows that the ratio of two consecutive trade sizes is independent of the level of beliefs and that the investor exits only when \( w_t < (T - t + 1) c \).

This result is in strong contrast to the learning hypothesis: outcomes do not affect the
exit decision nor do they influence trade sizes after controlling for wealth changes. Moreover, only investors who learn over time place exploratory, minimum size day trades. Agents with stubborn beliefs either day trade strictly positive amount or they do not day trade at all. We now empirically test these predictions about trade sizes, exit decisions, and exploratory day trades.

3 Data and Sample

This section describes the institutional setting of the Finnish market and our data sets. We also construct the sample used in our tests, define day traders, and compare day traders to other investors in the same market.

3.1 Helsinki Exchanges

Trading on the Helsinki Exchanges (HEX) is divided into sessions. Each trading day starts at 10:10 am with an opening call. Orders that are not executed at the opening remain on the book and form the basis for the continuous trading session. This trading session takes place between 10:30 am and 5:30 pm in a fully automated limit order book, the automated trading and information system (HETI). After-hours trading (5:30 – 5:45 pm) takes place after the continuous trading session and again the next morning (9:30 – 10:00 am) before the next opening call. (Two changes to the trading schedule were made during the sample period. On August 31, 2000, the regular trading session was extended to 6:00 pm and the after-hours session was moved to match this change. On April 10, 2001, an evening session that extended trading hours to 9:00 pm was introduced.)

The HEX trading system displays the five best price levels of the book to the market participants. The public can view this book in a market-by-price form while financial institutions receive market-by-order feed. Simple rules govern trading on the limit order book. There are no designated market makers or specialists; the market is completely order-driven. An investor trades by submitting limit orders. The minimum tick size is €0.01. An investor who wants immediate execution must place the order at the best price level on the opposite side.

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8A market-by-price book displays the five levels on both sides of the market but only indicates the total number of shares outstanding at each price level. A market-by-order book shows each order separately and also shows which broker/dealer submitted each order.
of the book—consistent with the market convention, we call these orders market orders. An investor who wants to buy or sell more shares than what is currently outstanding at the best price level must “walk up or down the book” by submitting separate orders for each price level. If a limit order executes against a smaller order, the unfilled portion stays on the book as a new order. Time and price priority between limit orders is enforced. For example, if an investor submits a buy order at a price level that already has other buy orders outstanding, all the old orders must execute before the new order.

The total market value of the 158 companies in the Helsinki Exchanges was €383.14 billion in the middle of the sample period (May 2000). The average realized log-spread during the sample period was 0.43% for the 30 largest stocks and as a low as 0.13% for Nokia, the most actively traded company. The most popular online broker charged a fixed fee of €8.25 and 0.2% of the trade value towards the end of the sample period. There was also a flat 28% capital gains tax during the sample period.

3.2 Data

Our data are (1) the complete trading records and holdings information of all Finnish investors and (2) the transaction data for all the stocks listed on the HEX.

1. The Finnish Central Securities Depository registry (FCSD) provided us the investor holdings and trading records for the period from January 1995 to November 2002. Each record includes a date-stamp, price, volume, stock code, a code that identifies the investor type—a domestic institution, a domestic household, or a foreigner—and other demographic information. We classify investors as individuals or institutions for this study. Grinblatt and Keloharju (2000) give the full details of this data set.

2. The transactions data are derived from the supervisory files from the HEX for a period from January 1995 to October 23, 2001. Each entry in this data set is a single trade. Each entry contains a unique trade identifier, the price, volume, and brokerage firm identities for both the buyer and the seller.
3.3 Matching the Data Sets

We match the investor trading records against the transactions data to obtain information on at what time each trade is completed. We describe the matching procedure here. Each trade record in the transaction data contains all the same information as the investor trading records except the investor identity. We use common elements to link the data sets.

There is no ambiguity in matching two types of trades: trades with unique price-volume combinations and non-unique trades that must originate from the same investor. We call these trades uniquely matched trades. There is no one-to-one link between the data sets for the remaining trades. However, it is often possible to determine almost exactly when a trade took place. For example, even if there are two trades with a price/volume combination of 40.1 euros/200 shares, the trades may have occurred almost simultaneously. We follow this idea and compute the lower and upper bound for trade time (i.e., upper and lower time-stamps) for each non-unique entry in the investor trading records. Each investor trading record now contains either the exact time of the trade or at least bounds for the time of the trade.

3.4 Sample and Demographics

A day trade is a purchase and a sale of the same stock (in any order) on the same day. If the investor buys and sells the same number of shares, we say that the day trade is complete. If the amounts differ, we say that the day trade is partial. A day trader is an investor who day trades at least once. There are 1,055,505 individual investors in the FCSD registry with at least some holdings or trades during the sample period. A total of 22,529 investors day trade at least once. Most day traders (52.6%) day trade only once or twice. (There are very active day traders as well: 6.0% of individuals day trade at least 50 times, and the most active day trader in the sample day trades 1,715 times.) We now discuss the overall day trading phenomenon and then compare individual day traders against the rest of the individual investor population.

---

9 We say that a trade has a unique price-volume combination if, for example, there is only one trade (in one stock-day) with a price of €82 and a volume of 1,200 shares. A trade is non-unique if, say, three trades have the same price-volume combination. In this example, “all must originate from the same investor” would mean that a single investor in the investor data set is the buyer or the seller in all the three trades.

10 An important question is whether this ex post identification of day trades is valid. In particular, it is possible that investors always set target prices when they purchase shares and sometimes these targets are breached during the same-day, generating “day trades”. Linnanmaa (2005a) studies the same data as we do and tests—and strongly rejects—this hypothesis. For example, over 1/3 of the day trades result in losses but under the target price scenario there should be none.
Figure 4: Day Trading Activity in Finland, 1997–2002. This figure plots the level of the Helsinki Exchanges market index (the Finnish stock market; the value at the end of January 1997 is set to 100) and the number of Finnish individual investors that day trade each month. A day trade is a purchase and a sale of the same stock (in any order) on the same day. A total of 22,529 individuals completed at least one day trade between January 1995 and November 2002.

Figure 4 shows how day trading activity varied between January 1997 and November 2002. It plots the number of investors who completed at least one day trade each month. The figure shows that although the day trading activity reached its peak in 2000, it did not die off: over 1,500 investors day traded each month towards the end of the sample.\(^\text{11}\)

Table 1 compares demographics of day traders against all other Finnish investors. We highlight several regularities. First, day traders are mostly male (81% vs. 54%) and on average 10 years younger than other investors. Second, day traders trade more even after excluding trades directly related to day trading. The median day trade completes 56 trades whereas this number is only three for the rest of the investor population. Third, the demographics suggest that the average day trader is less risk-averse than what the other investors are. For example, the stocks that day traders purchase have an average beta of 1.58 compared to the 1.42 for the others (\(t\)-value for the difference is \(-46.1\)). Day traders’ activity in trading warrants (15% versus 2%) supports the same conclusion: the typical day trader is less risk-averse than what the typical investor in the rest of the population is. Fourth and finally, many day traders had

\(^{11}\)Although not shown in the figure, there is a constant in- and out-flow of day traders each month. For example, 357 of the investors who day traded in November 2002 day traded for the first time.
Table 1: Day Trader and Investor Population Demographics

A day trader is an individual investor who buys and sells the same stock (in any order) on the same day at least once during the sample period from January 1995 to November 2002. Panel A reports the demographics for all day traders and for the rest of the investor population. *Traded Warrants* is the fraction of investors that traded warrants during the sample period. *Average Beta* is the average beta of all purchases. (“Day trades” are ignored when computing *Number of Trades, Average Trade Size, Traded Warrants,* and *Average Beta.*) Day trade is *complete (partial)* if the sale and purchase volumes are equal (unequal). Panel B reports how much investment experience day traders had at the time of the first day trade. This experience is measured from the day the investor first acquired stocks. Investors who already owned stocks at the beginning of the sample are included into the “over a year” category.

Panel A: *Day Trader and Investor Population Demographics*

<table>
<thead>
<tr>
<th></th>
<th>Day Traders</th>
<th>Remaining Population</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>Birth Year</td>
<td>1958.3</td>
<td>1960</td>
</tr>
<tr>
<td>Proportion Male</td>
<td>81.3%</td>
<td></td>
</tr>
<tr>
<td>Number of Trades</td>
<td>99.9</td>
<td>56</td>
</tr>
<tr>
<td>Average Trade Size</td>
<td>€11,953</td>
<td>€5,037</td>
</tr>
<tr>
<td>Traded Warrants</td>
<td>15.1%</td>
<td></td>
</tr>
<tr>
<td>Average Beta</td>
<td>1.58</td>
<td>1.59</td>
</tr>
<tr>
<td>Number of Day Trades</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complete</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>Partial</td>
<td>9.3</td>
<td>2</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>22,529</td>
<td>1,032,976</td>
</tr>
</tbody>
</table>

Panel B: *Day Traders’ Investment Experience before the First Day Trade*

<table>
<thead>
<tr>
<th>Experience</th>
<th>Fraction of Day Traders</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>2.1%</td>
</tr>
<tr>
<td>Less than a Month</td>
<td>3.2%</td>
</tr>
<tr>
<td>One to Three Months</td>
<td>4.4%</td>
</tr>
<tr>
<td>Three to Six Months</td>
<td>5.0%</td>
</tr>
<tr>
<td>Six Months to a Year</td>
<td>7.8%</td>
</tr>
<tr>
<td>Over a Year</td>
<td>77.5%</td>
</tr>
</tbody>
</table>

very limited amount of investment experience before they began to day trade. Panel B in Table 1 shows that 483 (2.1%) day traders had never owned or traded stocks before their first day trade. Moreover, 3,311 (14.7%) day traders had less than six months of experience.

Many of these demographics are consistent with the learning hypothesis. First, most day traders day trade only a few times. This is not what one would expect if stubborn beliefs were the main reason for day trading. Second, the evidence about lower risk-aversion is consistent
with the model of Section 2: a mildly risk-averse investor day trades for a wide range of priors. Third, some day traders’ limited experience is also inconsistent with stubborn beliefs. If stubborn beliefs were the main reason for day trading, we would expect only heavy traders to turn into day traders. We argue that if there are any investors with “parameter uncertainty” about the profitability of day trading, they are most likely those with the least exposure to the market.\footnote{Many day traders are active traders as well, consistent with these investors’ being possibly overconfident about their stock-picking abilities (Barber and Odean 2001). However, this evidence against the learning hypothesis is not conclusive for several reasons. First, many day traders are very inactive day traders. Second, overconfidence in stock-picking skills does not imply that these active traders have perfectly stubborn beliefs about the profitability of day trading as well. Finally, many of the “other trades” may in fact be indirectly related to day trading: Linnainmaa (2005a) shows that individual day traders often abort their day trades when the stock price moves against them, leaving only an observation of a purchase in the data.}

4 Empirical Tests

We test the learning hypothesis by studying individual day traders’ two choices: the day trade size and the decision to quit day trading. We test following hypotheses:

**Hypothesis 1.** Day traders stop day trading after negative outcomes, controlling for the outcomes’ wealth effects.

**Hypothesis 2.** Individuals who day trade in small quantities relative to their wealth are more likely to quit day trading.

**Hypothesis 3.** Day traders increase their trade sizes after positive outcomes and decrease them after negative outcomes, controlling for the outcomes’ wealth effects.

These predictions follow the learning hypothesis, formalized in Section 2 as a Bayesian life-cycle model. For example, Hypotheses 1 and 3 test the most straightforward predictions of the learning model: investors revise their beliefs over time and decrease their trade sizes or quit after poor outcomes. Hypothesis 2 uses the idea that trade size is informative of the parameters of the prior distribution: the optimal trade size depends on the investor’s wealth and beliefs. Hence, an investor who commits only a small proportion of wealth to day trading is closer, ceteris paribus, to quitting than an investor who places a large day trade (see Proposition 2).
The stubborn beliefs hypothesis is the alternative hypothesis in our tests. This hypothesis predicts that only the outcomes’ wealth effects matter. A possible reaction to this choice is that this is a straw-man hypothesis: surely investors learn from their experience. However, this is precisely our point: numerous studies assume that investors are, e.g., permanently overconfident, or that their learning is biased in a way that ultimately leads to perfect overconfidence (Gervais and Odean 2001). Hence, our null hypothesis of perfectly stubborn beliefs is a natural starting point for studying whether investors learn from experience.\footnote{A potential direction for future research is consider possible biases in learning and the \textit{changes} in these biases over time: “do investors learn to learn over time?”}

4.1 Methodology

We use data on individual day traders’ \textit{sequential} day trades in our tests. Each entry contains information about a single day trade. We also link investors’ day trades so that we also have all the information for both the previous and following day trades (if any).\footnote{Each entry contains the following information: the number of shares purchased and sold; the average purchase, sale, and same-day closing prices; the intraday sequence of individuals trades (i.e., whether the investors first purchased shares and later sold them, or vice versa; this is blank when the upper and lower time-stamps in Section 3.3 overlap and create ambiguity about the sequence); the number of trading days from the same investor’s previous day trade; the gross profit or loss for each previous day trade; the portfolio value at yesterday’s close; the number of earlier trades and day trades; and the number of shares the investor owned yesterday along with the amount of unrealized capital gains or losses computed with the FIFO principle.}

4.1.1 Variable Definitions and Description of Controls

\textit{Definitions.} We compute day trading profits as

\begin{equation}
\Pi = \min(V_b, V_s) \times (p_s - p_b)
\end{equation}

where $V_b$ and $V_s$ are the number of shares purchased and sold, and $p_b$ and $p_s$ are the average purchase and sale prices, respectively. We define \textit{Loss} as a dummy variable that is set to one if the day trade loses money (i.e., $p_s > p_b$). We use the \textit{Loss} dummy to test the impact of outcomes; by ignoring the size of the gain or loss, we can control for the outcomes’ wealth effects. We define \textit{Proportion of Earlier Losses} as the the number of losing day trades divided by the total number of day trades. (For example, if the agent has taken losses in 60% of his
day trades before this one, we set the variable equal to 0.6.) We measure day trade size as

\[
\text{trade size} = \min(V_b, V_s) \times \text{yesterday's closing price.} \tag{15}
\]

We use this definition instead of the actual value of the trade (i.e., \( V_b \times p_b + V_s \times p_s \)) to avoid correlation between the day trade’s performance and the trade size.\(^{15}\)

**Controls.** We control for the day trading outcomes’ wealth effects to distinguish learning from stubborn beliefs. These controls are necessary because even a stubborn beliefs-agent would respond to wealth changes. We include two log-portfolio values: the value of the investor’s portfolio at yesterday’s close (“current portfolio”) and the value of the investor’s portfolio before the first day trade (“initial portfolio”). The trade size test—that examines changes in the sizes of consecutive day trades—replaces the initial portfolio value with the value of the investor’s portfolio before the previous day trade. These variables keep track of how much wealthier or poorer the investor is relative to an earlier date. We also include an additional wealth control into the exit regression: the fraction of money the investor has made or lost in day trading. We compute this as the sum of day trading profits (or losses; Eq. 14) divided by the value of the initial portfolio.

We control for the possibility that some day trades are tax motivated. The capital gains tax in Finland creates an incentive for investors to realize losses if they have already realized gains (Grinblatt and Keloharju 2004). Some day trades may not speculative because of this incentive; rather, an investor sells and purchases the shares back almost simultaneously to realize a loss. (This is illegal but the law is difficult to enforce.) We control for tax-loss trading by adding monthly dummies, an ownership dummy, and a *Capital Loss* dummy.\(^{16}\) We also include the following unreported control variables in the exit and trade size regressions:

---

\(^{15}\)To illustrate the problem that might otherwise arise, suppose that (1) an individual’s day trade outcomes are independently and identically distributed and that (2) the investor always buys shares worth the same amount of money and sells them later the same day. Then, a “good” day trade would have a higher trade size than a “bad” day trade (because \( p_s > p_b \) in the actual trade value computation, \( V \times (p_b + p_s) \)). Such an investor’s good performance would predict a lower trade size next period because of the i.i.d. assumption. Our method ensures that this does not affect our results. Moreover, as discussed in Section 4.5.4, the results are nearly identical even if we use the actual trade size. This shows that the mechanistic effect described here is empirically of only second-order importance.

\(^{16}\)The results are almost unchanged if these controls are omitted. Moreover, the results are very similar if we exclude all day trades where the investor previously owns shares in the stock that is day traded. This suggests that even though some day trades may be tax motivated, such day trades do not affect our analysis.
93 monthly dummies, 6 dummy variables to define the day trade type\(^{17}\), the number of stocks in the investor’s portfolio, “no portfolio” dummies for the current and the first portfolio, a dummy for the first day trade, a dummy variable for a missing FIFO price.

4.2 Regression Specifications

4.2.1 Specification of the Exit Regression

We estimate logistic regression to study whether individuals are more likely to quit day trading after observing poor outcomes. We set the dependent variable to one for each investor’s last day trade; otherwise, the variable is set to zero. Some day traders probably continued to day trade after the end of our sample. However, because it is impossible to distinguish a true exit from a pause in day trading at the end of the sample, we always use this definition. This treatment is conservative: we add noise to the data if some of our exits are only pauses. However, our monthly dummies help to control for the deterministic increase in the unconditional exit probability. (Section 4.5.4 shows that the results are robust to this specification.)

We include three main explanatory variables to test the predictions of the learning hypothesis specific to quitting: (1) the loss from the current day trade (a dummy variable), (2) the proportion of earlier losses, and (3) the log-trade size. We also include the control variables described in Section 4.1.1. We estimate the exit regression for two samples. First, we estimate the regression by including all 22,529 day traders. However, there is a concern that some very active day traders may drive the results. We address this possibility by estimating a second regression, this time only with those individuals who day trade at most 10 times. This second regression introduces a selection bias on purpose: the results are conditional on the investor quitting relatively soon after starting to day trade. For example, such investors may have more pessimistic expectations to begin with.

\(^{17}\)We use the same categorization as Linnainmaa (2005a): a day trade is classified based on whether it is complete (i.e., \(V_b = V_s\)) or partial (i.e., \(V_b \neq V_s\)) and whether the investor initiated the day trade with a purchase or a sale. An additional category represents day trades where the investor leaves open an overnight short position. We also include a dummy variable set to one if the day trade type is unknown (i.e., the upper and lower time stamps overlap for the investor’s trades).
4.2.2 Specification of the Trade Size Regression

We estimate an OLS regression to study whether investors adjust their trade sizes in response to earlier outcomes. The log-difference between the sizes of two consecutive day trades is the regression’s dependent variable. We use the trade size definition in Eq. 15 to avoid correlation between the trade sizes and performance.

We include similar explanatory variables into the trade size regression as we used in the exit regression. The distinction between these two regressions is that we now lag most of our observations by one day trade. Hence, the key learning variable is now the loss dummy from the previous day trade. We also include the proportion of earlier losses before the previous day trade. We also include the control variables described in Section 4.1.1 and estimate the regression again separately for all day traders and for those who day trade at most 10 times.

4.3 Results on Day Traders’ Decision to Quit Day Trading

The exit regression estimates in Table 2 show that negative outcomes induce day traders to quit day trading. This conclusion is supported by all three of our key learning variables. First, an investor is significantly more likely to quit after a negative outcome than after a positive outcome. The coefficient estimate of 0.38 (t-value is 21.5) for the entire sample shows that the odds-ratio for quitting is 1.47 times larger after a negative outcome. This strongly supports the hypothesis that day traders revise their beliefs depending on what type of outcomes they observe. Second, an investor with a higher proportion of bad outcomes in the past is more likely to exit today. For example, a switch from “no bad outcomes” to “all bad outcomes” increases the quitting odds-ratio by 1.21 times. This result is intuitive: ceteris paribus, an investor who has had more bad experiences has lower expectations about the profitability of day trading. This effect is—both economically and statistically—somewhat stronger in the second regression that only includes “less than ten times” day traders.

Third, investors with smaller day trades are more likely to quit. This finding is consistent with the hypothesis that a small day trade size implies “poor” beliefs. For example, ceteris paribus, if we double the trade size, the quitting odds-ratio is 1.17 times smaller. On the other hand, if one investor makes a day trade worth $10,000 while the other (otherwise identical) only day trades $1,000, our results indicate that the odds of the latter quitting are 1.7 times
Table 2: The Impact of Losses on the Decision to Quit Day Trading

This table estimates a logistic regression to examine when day traders quit day trading. Each observation is a single day trade, defined as a purchase and a sale of the same stock on the same day. The dependent variable is set to one for each investor’s last day trade. Current Loss is a dummy variable set to one if the average sale price for the day trade is less than the average purchase price. Proportion of Earlier Losses is the number of losses from the earlier day trades divided by the total number of earlier day trades. Trade Size is the minimum of purchase and sale volumes multiplied by the yesterday’s closing price. Current and First Portfolio Values are the values of the investor’s portfolio at yesterday’s close and the day before the first day trade. Cumulative Profits is the sum of day trading profits from all earlier day trades. Number of 3Mos Day Trades is the number of times the investor day traded during the previous three months. The number of trades is defined similarly. Ownership is a dummy variable set to one if the investor already owned shares in the stock that is being day traded. Unrealized Capital Loss is a dummy variable set to one if the FIFO price is higher than the yesterday’s closing price. The second column estimates the regression only for those individuals who day trade at most 10 times. (The regression also includes unreported control variables; see text.)

<table>
<thead>
<tr>
<th>Learning Variables</th>
<th>All Day Traders</th>
<th>Coeff.</th>
<th>t-value</th>
<th>Day Traders with $N_{dt} \leq 10$</th>
<th>Coeff.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Loss</td>
<td>0.38</td>
<td>21.5</td>
<td></td>
<td>0.32</td>
<td>14.0</td>
<td></td>
</tr>
<tr>
<td>Proportion of Earlier Losses</td>
<td>0.19</td>
<td>5.8</td>
<td></td>
<td>0.27</td>
<td>7.1</td>
<td></td>
</tr>
<tr>
<td>ln(Trade Size)</td>
<td>−0.23</td>
<td>−29.0</td>
<td></td>
<td>−0.15</td>
<td>−15.3</td>
<td></td>
</tr>
<tr>
<td>Wealth Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Current Portfolio Value)</td>
<td>0.07</td>
<td>7.2</td>
<td></td>
<td>0.02</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>ln(First Portfolio Value)</td>
<td>−0.02</td>
<td>−2.8</td>
<td></td>
<td>0.01</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Cumulative Profits / First Portfolio Value</td>
<td>0.12</td>
<td>5.3</td>
<td></td>
<td>0.22</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>Other Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Number of 3MOs Day Trades)</td>
<td>−0.60</td>
<td>−64.9</td>
<td></td>
<td>0.24</td>
<td>8.3</td>
<td></td>
</tr>
<tr>
<td>ln(Number of 3MOs Trades)</td>
<td>−0.72</td>
<td>−75.5</td>
<td></td>
<td>−0.48</td>
<td>−37.8</td>
<td></td>
</tr>
<tr>
<td>Ownership</td>
<td>0.35</td>
<td>7.0</td>
<td></td>
<td>0.30</td>
<td>4.9</td>
<td></td>
</tr>
<tr>
<td>Ownership * Unrealized Capital Loss</td>
<td>−0.06</td>
<td>−2.7</td>
<td></td>
<td>−0.02</td>
<td>−0.8</td>
<td></td>
</tr>
</tbody>
</table>

N = 281,350, 42,268
Nagelkerke $R^2$ = 41.1%, 18.5%

There is no support for the hypothesis that investors quit only after they have lost enough money. Note that the current portfolio log-value has a positive sign while the first portfolio log-value has a negative sign. This indicates that a positive change in the portfolio value is associated with a higher exit probability. Similarly, large cumulative day trading profits (this is the “cumulative profits / first portfolio value” variable in the regression) predicts higher
exit probability. However, both the economic and statistical significance of these results is weak.\textsuperscript{18} We also note that the regression estimates are fairly similar for both samples.\textsuperscript{19} This suggests that the presence of very active day traders is not a concern for our results. Hence, our exit regression results are consistent with the learning hypothesis: day trading seems to be often motivated simply by uncertainty about the profitability of day trading, not by stubborn beliefs.

4.4 Results on Day Traders’ Choice of Day Trade Sizes

The trade size regression results in Table 3 also strongly support the hypothesis that day traders revise their beliefs over time. A day trader decreases his day trade size by 5.9\% after a loss compared to the change after a gain. (The impact of negative outcomes is \(-3.0\%\) for the day traders in the second column.) Note that this figure is conditional on the investor continuing day trading: we are “missing” those observations where the investor drops the day trade size to zero.\textsuperscript{20}

A higher proportion of earlier losses predicts a modest increase in the trade size. Note that this variable’s value is negative correlated with the size of the previous day trade because of the way it is lagged: the previous day trade \((t - 1)\) is relatively small when there are more poor earlier observations \((\tau < t - 1)\). The estimate shows that, conditional on keeping on day trading after poor observations, day traders increase their trade sizes. However, the economic significance of this result is rather small: the increase in the relative trade size is predicted to be only 2.8\% if the investor switches from “no bad outcomes” to “all bad outcomes”.

The results in Table 3 show that the outcomes’ wealth effects also matter. Investors

\textsuperscript{18}Our results also debunk one alternative, behavioral explanation to why investors day trade. It is possible that investors derive gambling or horse race-type of entertainment from day trading and that they have made an initial allocation to a “day trading fund or mental account”. If so, investors day trade until they have exhausted their allocation. Our estimate for cumulative profits suggests that this is not the case, at least for most day traders. Note that it is likely that some investors derive entertainment from such activity (Grinblatt and Keloharju 2005); our conclusion is simply that this sensation-seeking explanation does not appear to describe the motivations of the majority of day traders.

\textsuperscript{19}The only significant differences are between “the number of earlier (day) trades”. This is not surprising. Because there is a small number of extremely active day traders but a large number of casual day traders, a high “past trades” value catches this skewness in the distribution.

\textsuperscript{20}We also examine day traders’ unconditional trade size changes. The average change in the trade size is \(-0.9\%\) for the whole sample and \(-1.6\%\) for the day traders in the second column. This indicates that there is a drift down in trade sizes, consistent with the idea that most day traders learn that they cannot make money by day trading. Collectively, our results show that investors revise their beliefs downwards, decrease their trade sizes, and eventually quit.
Table 3: The Impact of Losses on the Day Trade Size

This table estimates a regression where the dependent variable is the log-difference between the size of the current day trade and the size of the previous day trade. The size of the day trade is defined as the minimum of purchase and sale volumes multiplied by the stock price at the previous close. *Previous Loss* is a dummy variable set to one if the average sale price of the previous day trade is below the average purchase price and *Proportion of Earlier Losses* is the number of losses from the earlier day trades divided by the total number of day trades (this excludes also the previous day trade’s outcome). *Current and Previous Portfolio Values* are the values of the investor’s portfolio at yesterday’s close and the day before the previous day trade. *Number of 3Mos Day Trades* is the number of times the investor day traded during the previous three months. The number of trades is defined similarly. *Ownership* is a dummy variable set to one if the investor already owned shares in the stock that is being day traded. *Unrealized Capital Loss* is a dummy variable set to one if the FIFO price is higher than the yesterday’s closing price. *Days from the Previous Day Trade* is the number of trading days from the Previous Day Trade. The second column estimates the regression only for those individuals who day trade at most 10 times. (The regression also includes unreported control variables; see text.)

<table>
<thead>
<tr>
<th>Learning Variables</th>
<th>All Day Traders</th>
<th>Coeff.</th>
<th>t-value</th>
<th>Day Traders with $N_{dt} \leq 10$</th>
<th>Coeff.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous Loss</td>
<td>-0.059</td>
<td>-21.6</td>
<td></td>
<td>-0.030</td>
<td>-3.4</td>
<td></td>
</tr>
<tr>
<td>Proportion of Earlier Losses</td>
<td>0.028</td>
<td>4.4</td>
<td></td>
<td>0.035</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td>Wealth Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(\text{Current Portfolio Value})$</td>
<td>0.022</td>
<td>12.5</td>
<td></td>
<td>0.041</td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>$\ln(\text{Previous Portfolio Value})$</td>
<td>-0.022</td>
<td>-12.9</td>
<td></td>
<td>-0.042</td>
<td>-8.9</td>
<td></td>
</tr>
<tr>
<td>Other Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ln(\text{Number of 3MOs Day Trades})$</td>
<td>-0.015</td>
<td>-11.5</td>
<td></td>
<td>-0.050</td>
<td>-3.6</td>
<td></td>
</tr>
<tr>
<td>$\ln(\text{Number of 3MOs Trades})$</td>
<td>0.008</td>
<td>4.2</td>
<td></td>
<td>0.024</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>Ownership</td>
<td>0.023</td>
<td>2.2</td>
<td></td>
<td>0.041</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>Ownership * Unrealized Capital Loss</td>
<td>-0.062</td>
<td>-16.3</td>
<td></td>
<td>-0.077</td>
<td>-6.5</td>
<td></td>
</tr>
<tr>
<td>$\ln(\text{Days from the Previous Day Trade})$</td>
<td>-0.013</td>
<td>-12.7</td>
<td></td>
<td>-0.023</td>
<td>-9.9</td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>256,940</td>
<td></td>
<td></td>
<td>25,452</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>7.6%</td>
<td></td>
<td></td>
<td>7.8%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

increase their trade sizes if their investment wealth today is higher than what it was at the time of the previous day trade. For example, the point estimate in the first regression predicts that day traders increase their day trade sizes by 2.2% of the percentage change in the portfolio value. For example, if the portfolio value has increased by 20% from the previous day trade, the relative increase in the trade size is only 0.5%. Hence, this wealth effect is economically modest. These results support the hypothesis that individuals learn from experience: individuals revise their beliefs over time and change their trade sizes more than what they would if they ignored
feedback.

4.5  Robustness Tests

We now show that the conclusions drawn from our exit and trade size regressions are robust to alternative specifications and further tests. First, we argue that some of investors’ initial day trades appear to be exploratory (Section 4.5.1). Second, we show that investors avoid very risky strategies when day trading for the first time (Section 4.5.2). Third, we demonstrate that the link between an investor’s trade size and exit decisions leaves a very particular mark in the data: there is a positive drift in the average trade sizes of those investors who continue day trading (Section 4.5.3). Fourth, we describe additional robustness checks of model specifications and variable definitions (Section 4.5.4). These analyses support the learning hypothesis and are difficult to explain by the stubborn beliefs hypothesis.

4.5.1  Exploratory Day Trades

Our parameter uncertainty model predicts that an investor who does not believe that day trading is profitable may still place a small exploratory day trade to learn about this profitability. We now argue that some trades in the data do not appear to be speculative because of their small size.

Many investors day trading for the first time day trade in very small quantities. For example, 5.5% of the first day trades are less than €500 in size (Eq. 15) while this fraction is only 1.0% for all the other day trades. Day trades this small are suspicious because of the effect of trading costs. The cheapest online broker at the time of the sample charged a commission of €8.25 per trade and 0.2% of the trade value. Let us ignore the proportional commission and consider only the role of the fixed cost. If a trade of €500 is speculative, the investor must expect to predict correctly a price movement of approx. 3.3% (2 * 8.25/500) to break even! The median required price movement for these small initial day trades is 6.3%. This is an enormous daily price change and seems to contradict the assumption that these small day trades are speculative. Moreover, there is a second problem with the speculation argument. Even if we assume that these small day trades are speculative, their sizes are curious. If the investor expects to predict correctly a significant price change, he should be willing to trade more aggressively to exploit the opportunity. The only plausible explanation seems to be that
4.5.2 Initial Day Trades and Risk-Taking

The learning hypothesis suggests that many of the investors day trading for the first time are likely those who only day trade to learn more. Such investors want to limit the riskiness of their trades. We examine this possibility by studying whether investors avoid a particular type of a day trade: a day trade where the investor takes a short-position with the intent of covering the short later the same day at profit. (We call this the “short” strategy.) This strategy is riskier than an alternative sequence where an investor first buys shares with intent of selling them back later (the “long” strategy) for at least two reasons. First, the potential downside is unlimited because the stock price can make an arbitrarily large jump upwards. Second, the investor has to cover the short even if the liquidity in the stock dries up—i.e., if the spreads widen significantly or even disappear.

We test the hypothesis about the riskiness of initial day trades as follows. First, we drop day trades where the investor owns shares in the stock that is being day traded. Second, we classify investors into two groups: investors who are day trading for the first time and those who have already day traded at least once before. Third, we compute the number of “short” and “long” day trades for both groups for each day in the sample. We then drop days with none or only one day trade from either group. Next, we compute the proportion of “short” day trades for each day-group and take the difference between the two groups. The resulting daily time-series of differences has 752 observations. This time-series measures in a very controlled setting whether investors who are day trading for the first time prefer the safer “long” strategy.

The time-series average difference between the “first” and “other” groups is $-12.2\%$ (the s.e. is 0.7%). The investors who are day trading for the first time employ the riskier strategy in 13.2% of the cases; hence, more experienced day traders are approximately twice as likely to short. This result shows that investors day trading for the first time have a distaste for the riskier strategy, consistent with the hypothesis that investors learn from experience. Investors who have day traded before are less likely be day trading just to learn about the profitability of day trading.
4.5.3 Trade Sizes and the Decision to Exit

If investors day trading in smaller quantities are closer to quitting (Proposition 2), the cross-sectional average trade size will drift upwards. (“The cross-sectional average trade size” is the average day trade size of those investors who are day trading for the first time, for the second time, and so forth.) The intuition is that those who keep on day trading believe that day trading is profitable and day trade in larger quantities while those who quit drop out from the left tail of the trade size distribution. Hence, as more time passes, only those who believe that day trading is profitability remain. Note that this is a very strong prediction: we know from Section 4.4 (see footnote 20) that, unconditionally, the typical day trader decreases his trade size from one day trade to the next. This suggests that we should, if anything, find a negative drift in cross-sectional average day trade sizes.

The data strongly supports the link between trade sizes and the exit decision: the cross-sectional average trade size drifts up from €13,174 to €23,096 from the first day trade to the tenth day trade. All the intermediate changes (e.g., from the first day trade to the second day trade) are positive. Furthermore, outliers do not drive this dramatic increase: the median trade size increases from €4,250 to €10,750 with all intermediate changes positive. (The Mann-Whitney z-value for the difference between the tenth and the first day trade is 32.2.) These results show that investors who keep on day trading trade in larger quantities than those who drop out. Those who drop out have low expectations about the profitability and trade in small quantities because both the exit and trade size decisions are determined by beliefs.

4.5.4 Additional Robustness Results

We conclude our discussion of the empirical evidence by describing several robustness checks of our variable definitions and model specifications. First, the exit regression results are similar if the model is estimated as a linear probability model and not as a logistic model. Similarly, if we recode the dependent variable in the trade size regression as an increase / decrease dummy variable and estimate the model as a logistic regression, the results are qualitatively the same.

Second, we find that the results are nearly unchanged if the trading profit measure is changed to include a brokerage fee and the residual position (i.e., $V_b - V_s$) is taken into account
by marking it to the market at the closing price. Similarly, the results are nearly unchanged if we define trade size as the true value of the trade, or if we replace the \text{MIN}-function in Eq. 15 by the sum of purchase and sale volumes.

Third, we examine whether there are any problems in our methodology of identifying when day traders quit. (Our exit regression in Section 4.3 included all day trades from each investor and defined each investor’s last day trade as the exit. We argued that this is a conservative treatment and also included monthly dummies as control variables.) We address this concern by leaving out investors who complete any day trades during the last six months of our data set. This restriction guards against the possibility that some of the “late” day traders are just taking a brief pause from day trading and have not really quit. We find slightly stronger results for this subsample. (We report the whole sample results in Section 4.3 because our arbitrary six-month rule may create a selection bias: we are probably cutting out day traders who have discovered that they can make money by day trading.)

Fourth, we estimate the exit regression separately for several subsamples to show that the results are not created by, e.g., nonlinearities in the wealth controls or by heterogeneity across investors day trading for the \textit{i}th time versus those day trading for the first time. We partition all day trades into subsamples in two steps. In the first step, all day trades (with \(2 \leq i \leq 10\), where \(i\) is the number of the current day trade) are assigned into bins based on the number of the day trade, \(i\). In the second step, we assign each bin’s observations into deciles based on the investor’s cumulative day trading profits. By construction, investors in each of the resulting 90 subsamples have lost or gained an almost equal amount of money over the same number of day trades. We then estimate the exit regression for each of these subsamples.\(^{21}\) We use “the proportion of losses” to test the learning hypothesis. We define this variables as the number of losing day trades (including the current day trade) divided by the total number of day trades.\(^{22}\) The coefficient estimate for this variable is positive in 70% of the subsamples and has an average of 0.397 (the standard error for this average estimate is 0.094). This result shows that outcomes matter beyond their wealth effects, consistent with the learning hypothesis. It also suggests that omitted controls do not explain Table 2’s exit results.

\(^{21}\)We modify the regression of Table 2 to omit some variables that would cause multicollinearity and reduce the number of control variables because of smaller sample sizes. The details are available upon request.

\(^{22}\)For example, if the investor just day traded for the fifth time and has lost money in three day trades, we set this variable to 3/5.
5 Conclusions

This paper studies whether individual investors are capable of learning from experience by examining the complete trading records of all Finnish (individual) day traders. This is an important question because many economic models either assume hyperrational agents or, at the very least, assume that agents adjust their behavior over time to improve their well-being (i.e., adaptive learning). We circumvent the problem that beliefs (and the changes in beliefs) are inherently unobservable by studying individual day traders. These investors are ideal for studying learning for several reasons: first, they may have widely dispersed beliefs about the profitability; second, they receive immediate (and observable) feedback from their day trades; and finally, observations are sequential—it is easy to observe how the behavior changes from one day trade to the next. We compare two competing hypotheses:

- **The learning hypothesis.** Individuals do not know whether they can make money by day trading but have prior beliefs about their abilities. Some individuals—i.e., those with very dispersed priors, low risk-aversion, or optimistic priors—day trade to learn about the profitability, adjusting trade sizes or quitting depending on realized outcomes.

  An investor with pessimistic (but uncertain) beliefs about the profitability of day trading may want to day trade because the downside of being right is significantly smaller than the upside of being wrong. For example, the cost of a small round-trip trade was approx. €17 during the sample period. Hence, day trading is relatively cheap, and because of this, an investor wants to make sure that day trading really is unprofitable before ignoring it. Even in a world where everyone believes that day traders lose money, some investors become day traders.

- **The stubborn beliefs hypothesis.** Day traders are investors who believe in their abilities to make money by day trading. These investors have degenerate subjective prior distributions about their own abilities and adjust their behavior only because of the outcomes’ wealth effects. (This hypothesis is a common assumption in behavioral finance: investors are permanently overconfident about their own abilities.)
Many facts about day traders are consistent with the hypothesis that it is learning, not stubborn beliefs, that explains why investors day trade. For example, many individuals have very limited investment experience before starting to day trade; most day traders day trade only a few times; and many initial day trades do not seem to have a speculative motive. Our main evidence comes from the trade size and exit decisions: day traders are more likely to quit after negative outcomes and they respond to outcomes by increasing and decreasing their trade sizes. We control for the outcomes’ wealth effects in both cases: day traders do not reduce trade sizes or quit simply because their wealth is smaller. Collectively, our evidence suggests that only parameter uncertainty and learning, not stubborn beliefs, can explain the type of behavior observed in the data.\textsuperscript{23}

Our positive results about investors’ abilities to learn from experience are important. For example, many behavioral finance studies suggest that investors exhibit behavioral biases with costly consequences. However, as long as individuals learn from experience, these biases should not be too costly: it seems plausible that precisely those biases that cause significant welfare losses are those that the investor most easily observes. For example, an agent who reduces her performance by 10% each year is more likely to learn from experience than an agent who experiences a performance gap of only 1%. Even if individuals are not as hyperrational as some studies assume, it appears unlikely they could completely ignore feedback from any meaningful welfare losses.

\textsuperscript{23}We cannot rule out the possibility that some day traders day trade because they have stubborn beliefs about their abilities. However, given the promptness of the feedback from day trading, this does not seem plausible for explaining the motivations of the majority of day traders. It is possible that innately poor quality of feedback from stock picking prohibits regular investors from learning (Odean 1998), but such an argument does not apply to day trading. Our results suggest that in most cases, the reason for day trading is uncertainty, not stubborn beliefs.
A Appendix

A.1 Proofs

Proof of Proposition 1. Assume that the agent wants to day trade at date 1 if and only if the date 0 outcome is positive: i.e., suppose that $\beta_0 - 1 < \alpha_0 < \beta_0 + 1$. The indirect utility of the pay the cost policy is

$$V_{\text{pay the cost}}^0(w_0, (\alpha_0, \beta_0)) = \frac{\alpha_0}{\alpha_0 + \beta_0} \frac{(w_0 - 2 \cdot c)^{1-\gamma}}{1-\gamma} \cdot k_1(\alpha_0 + 1, \beta_0) + \frac{\beta_0}{\alpha_0 + \beta_0} \frac{(w_0 - c)^{1-\gamma}}{1-\gamma}. \quad (16)$$

Let $r_0 \equiv \frac{\beta_0}{\alpha_0}$, substitute $\beta_0 \equiv \alpha_0 r_0$ into the value function, take the limit $\alpha_0 \to 0$ (the agent’s prior is completely uninformative at the limit), and evaluate the expression at $c = 0$ to get

$$\lim_{\alpha_0 \to 0} V_{\text{pay the cost}}^0(w_0, (\alpha_0, r_0)) \bigg|_{c=0} = \frac{w_0^{1-\gamma}}{1-\gamma} \left[ \frac{2^{1-\gamma} + r_0}{1 + r_0} \right].$$

Suppose that $\gamma > 1$ (an identical argument applies for $\gamma < 1$). Then,

$$V_{\text{pay the cost}}^1 > V_{\text{nothing}} \iff 2^{1-\gamma} < 1.$$ 

This inequality holds because $\gamma > 1$. The continuity of $V_{\text{pay the cost}}^1(w_1, (\alpha_0, r_0))$ with respect to $\alpha_0$ and $c$ together with the strict inequality implies that “pay the cost” yields higher indirect utility than the “do nothing” policy even when $c$ is strictly positive and the variance of the prior is not completely uninformative.

Proof of Propositions 2 and 3. These propositions follow directly from differentiating the optimal investment (Eq. 9) with respect to $\alpha$ and $\gamma$, respectively. We only need to use the earlier result that $\frac{\partial}{\partial \alpha_1} k_1(\alpha_1, \beta_1) > 0$ if $\gamma < 1$ and $\frac{\partial}{\partial \alpha_1} k_1(\alpha_1, \beta_1) > 0$ if $\gamma > 1$. 

A.2 Extended Model

This section derives an approximate solution to a $T$-period version of the model of Section 2. The market incompleteness (i.e., short-sale restrictions) means that we can only express

\[24\text{If the lower bound is violated, it is always optimal to do nothing. If the upper bound is violated, it is always be optimal to invest.}\]
the optimal behavior recursively. We approximate the indirect utility of the “pay the cost”
policy to obtain simple recursion formulas. Assume that the value functions tomorrow can
be written as
\[ V_{t+1}^+ = \frac{(w_{t+1} - b^+ c)^{1-\gamma}}{1-\gamma} k^+ \]
and
\[ V_{t+1}^- = \frac{(w_{t+1} - b^- c)^{1-\gamma}}{1-\gamma} k^- \]
where superscripts + and - indicate the sign of the current outcome. (We later show that it satisfies this form.) The
actual value of the pay the cost policy is then
\[ V_{t}^{\text{pay the cost}}(w_t, (\alpha, \beta)) = \frac{\alpha}{\alpha + \beta} \frac{(w_t - c - b^+ c)^{1-\gamma}}{1-\gamma} k^+ + \frac{\beta}{\alpha + \beta} \frac{(w_t - c - b^- c)^{1-\gamma}}{1-\gamma} k^- \]  \(17\)

However, if \( c \) is small relative to \( w_t \), the following approximation is good:25
\[ V_{t}^{\text{pay}}(w_t, (\alpha, \beta)) \approx \frac{(w_t - \frac{b^+ + b^- + 2}{2} c)^{1-\gamma}}{1-\gamma} \frac{\alpha k_1 + \beta k_2}{\alpha + \beta}. \]  \(18\)

Then, the value functions conditional of three possible policies have the following recursive
forms:
\[ V_{t}^{\text{invest}}(w_t, (\alpha, \beta)) = \frac{(w_t - \frac{b^+ + b^- + 2}{2} c)^{1-\gamma}}{1-\gamma} \frac{\alpha k_1 + \beta k_2}{\alpha + \beta}, \]
\[ V_{t}^{\text{pay}}(w_t, (\alpha, \beta)) \approx \frac{(w_t - \frac{b^+ + b^- + 2}{2} c)^{1-\gamma}}{1-\gamma} \frac{\alpha k_1 + \beta k_2}{\alpha + \beta} k_{\text{invest}}, \]
\[ V_{t}^{\text{nothing}}(w_t, (\alpha, \beta)) = \frac{w_t^{1-\gamma}}{1-\gamma}. \]

We set \( V_{t}^{\text{invest}} \) equal to \(-\infty\) if the optimal investment \( x_t^* < 0 \).26 Note that all the value
functions corresponding to different policies are now of the form
\[ V = \frac{(w - b c)^{1-\gamma}}{1-\gamma} k \] for some \( b \) and \( k \), satisfying our assumption above. The strategy for solving the problem is as follows.

Starting at date \( T - 1 \), compute the indirect utilities of all policies and choose the utility-
maximizing. Next, write the indirect utility in each “node” as \( \frac{(w-bc)^{1-\gamma}}{1-\gamma} k \) and record \( b \) and \( k \). Repeat these steps for dates \( T - 2, T - 3, \ldots, 1 \) to get the complete optimal policy.\(^{27}\)

### A.3 Simulated Regressions

We use the extended model of Appendix A.2 to generate simulated day trader data. We show that the exit and trade size results from this data are similar to our empirical results in Tables 2 and 3. We use the following parameters for our simulations: \( T = 15, c = 20, \gamma = 2, \) and \( p = 0.45 \). We compute optimal day trading behavior for 2,500 day traders and record their decisions.\(^{28}\) Each day trader’s characteristics are generated as follows: (1) the initial wealth, \( w_i \), is uniformly distributed between $5,000 and $50,000; (2) the mean of the prior distribution, \( \mu_i \), is Beta distributed with parameters \( (3, 3, \frac{1-p}{p}) \); and (3) the variance of the prior distribution is distributed \( \sigma_i^2 \sim U\left(\frac{1}{3}(1-\mu_i)\mu_i, (1-\mu_i)\mu_i\right) \). The term \( \frac{1-p}{p} \) in the mean ensures that \( E[\mu_i] = p \); i.e., the average investor in the population has no misconceptions about how profitable day trading is. The distributional assumption for the variance determines that each individual’s variance is between 33% and 100% of the maximum possible variance. (We avoid creating investors who have almost degenerate beliefs.)

Table 4 summarizes the characteristics of our simulated data and shows the results from the exit and trade size regressions. We include the corresponding results from Tables 2 and 3) for comparison. The simulated results are similar to the empirical results: the key learning variables have the same signs in both data sets. (The magnitudes are different because the model is not calibrated to the data.) These simulations show that a fully-rational model can generate data that has same features as our real life day trader data. The simulated data has an attractive feature: 60% of the day traders have a prior \( E_0[p] < 0.5; \) i.e., they already initially believe that that day traders lose money.

\(^{27}\)A problem with this approach is that—because of the lump sum trading cost \( c \)—wealth is a state-variable that affects optimal decisions. A solution to this problem is as follows: (1) obtain a candidate for the complete optimal policy by making all the required evaluations using \( w_t = w^* \) for some reasonable choice of \( w^* \) and (2) verify that the candidate solution is optimal by first calculating all portfolio values forward using the candidate policy.

\(^{28}\)If the investor’s optimal decision at date \( t = 0 \) is to do nothing, we create a new investor. Similar to our treatment of the actual data, we also include investors who do not quit before the end of sample (i.e., we retain investors who still day trade at date \( t = T \)). We use the approximation described in Appendix A.2 to compute the solution.
Table 4: Exit and Trade Size Regressions with Simulated Data

This table estimates the exit and trade size regressions with simulated data and compares the results to the estimates from the actual data (from Tables 2 and 2). We create the simulated data from the extended portfolio choice model of Appendix A.2 for 2,500 day traders. The following parameters are fixed: \( T = 15, \ c = 20, \ \gamma = 2, \) and \( p = 0.45. \) Each investor’s initial beliefs and wealth are randomized: wealth is uniformly distributed between $5,000 and $50,000, the mean of the prior distribution drawn from a Beta\( (3,3) - \)distribution, and the variance of the prior is drawn from \( U[\frac{1}{4}(1-\mu_i)\mu_i, (1-\mu_i)\mu_i] \) (see text). We first simulate outcomes using the actual probability \( p \) and then solve for each day trader’s optimal behavior. The simulated sample has the following characteristics: the average day trader in the sample day trades 8.01 times (23.5\% of them day trade once or twice); the average mean of the prior distribution is 0.453; and the average cumulative profit is \(-$1,656 \) (max = $324,501, min = \(-$17,123\)).

Panel A: Exit Regression

<table>
<thead>
<tr>
<th>Learning Variables</th>
<th>Simulated Coeff.</th>
<th>t-value</th>
<th>Real Data Coeff.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Loss</td>
<td>3.53</td>
<td>38.2</td>
<td>0.38</td>
<td>21.5</td>
</tr>
<tr>
<td>Proportion of Earlier Losses</td>
<td>25.76</td>
<td>40.2</td>
<td>0.19</td>
<td>5.8</td>
</tr>
<tr>
<td>( \ln(\text{Trade Size}) )</td>
<td>-0.20</td>
<td>-9.3</td>
<td>-0.23</td>
<td>-29.0</td>
</tr>
<tr>
<td>Wealth Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln(\text{Current Portfolio Value}) )</td>
<td>5.31</td>
<td>22.7</td>
<td>0.07</td>
<td>7.2</td>
</tr>
<tr>
<td>( \ln(\text{First Portfolio Value}) )</td>
<td>-6.53</td>
<td>-26.0</td>
<td>-0.02</td>
<td>-2.8</td>
</tr>
<tr>
<td>Other Controls</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>( \ln(\text{Number of Earlier Day Trades}) )</td>
<td>8.91</td>
<td>40.8</td>
<td>-0.60</td>
<td>-64.9</td>
</tr>
</tbody>
</table>

\( N = 20,015 \)  
Nagelkerke \( R^2 \) 61.4\% 41.1\%

Panel B: Trade Size Regression

<table>
<thead>
<tr>
<th>Learning Variables</th>
<th>Simulated Coeff.</th>
<th>t-value</th>
<th>Real Data Coeff.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous Loss</td>
<td>-2.954</td>
<td>-104.1</td>
<td>-0.059</td>
<td>-21.6</td>
</tr>
<tr>
<td>Proportion of Earlier Losses</td>
<td>0.255</td>
<td>6.0</td>
<td>0.028</td>
<td>4.4</td>
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<td>Wealth Controls</td>
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<td></td>
</tr>
<tr>
<td>( \ln(\text{Current Portfolio Value}) )</td>
<td>-3.243</td>
<td>-23.4</td>
<td>0.022</td>
<td>12.5</td>
</tr>
<tr>
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<td>3.314</td>
<td>23.7</td>
<td>-0.022</td>
<td>-12.9</td>
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<tr>
<td>Other Controls</td>
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<td></td>
</tr>
<tr>
<td>( \ln(\text{Number of Prior Day Trades}) )</td>
<td>-0.021</td>
<td>-0.8</td>
<td>-0.015</td>
<td>-11.5</td>
</tr>
</tbody>
</table>

\( N = 17,514 \)  
Adjusted \( R^2 \) 48.6\% 7.6\%

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References


