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A THEORY OF RANDOMIZED LAW ENFORCEMENT

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Abstract

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A much-neglected topic in regulatory economics is the relative advantage of rules-based vis-a-vis discretionary law enforcement. We explore this issue in a general setting as well as in an application to insider trading. Our primary premise is that unbiased discretion can be interpreted as randomizing the trigger above which individuals are caught and penalized for violating rules and regulations. We consider enforcement of regulation in an economy where agents are risk averse and have heterogeneous preferences. If violating laws carries risk other than the risk of being caught by law enforcers (e.g., in the case of speeding or fleeing after a theft, there is some chance that the violator will have an accident), the most risk tolerant agents are the ones which are most likely to violate society’s laws. Since randomization creates risk, only the most risk-tolerant agents violate such a rule. Thus, randomization of law enforcement has a primary advantage in that it can help screen out those individuals who are likely to be repeat violators of society’s laws. The degree of randomization depends on the resources available for law enforcement as well as the threshold risk tolerance above which rule violation takes place. Our work thus forms a first attempt to evaluate the desirability of different types of regulatory enforcement policies.
Introduction

Regulation is pervasive in several walks of life; e.g., consider three common areas: enforcement of speed limits, tax laws, and, in the context of financial markets, insider trading regulation. One feature of enforcement and regulation in these cases is that the enforcement policies of regulatory agencies are cloaked in a veil of unpredictability. Thus, while many of us believe that there is a cushion of speeds above the legally posted limit in which it is unlikely that we will be cited for speeding, we generally are unable to place a precise upper bound on this cushion. Similarly, part of the IRS audit procedure, specifically, the taxpayer compliance measurement program (TCMP), at least partially appears to be randomized (see Wiersema (1997), Weiss (1982), and Webster (1988)) in that under this program, a certain portion of tax returns are selected to be audited at random. Further, Kleinig (1996) suggests that given limited resources, police officers target suspected violators at random. With regard to the case of insider trading, Meulbroek (1992) suggests that while the majority of potential inside traders are prosecuted due to “unusual” price movements on the days they trade, the exact criteria for prosecution are not made public by the Securities and Exchange Commission.

It would seem that in the age of supercomputers and laser guns, deterministic rules in at least some areas of law enforcement could be framed and enforced, but this does not appear to be the case. While there has been much analysis of the economic costs and benefits of regulation (e.g., Chipty (1995), Gray (1987), Henderson (1996), and Pargal and Wheeler (1996)), the issue of how to set efficacious rules has received comparatively little attention. Notable exceptions to this are papers by Kaplow and Shavell (1994),
who analyze the optimal degree of penalty reduction when the violator confesses and Graves, Lee, and Sexton (1989), who make a distinction between enforcement and the setting of regulatory standards. These papers however, do not shed light on the following questions: Should society prosecute every offender? Or should it book only certain pre-specified categories of offenders? Or should it follow a randomized strategy, making “catching” somewhat random from the perspective of agents?

In this paper we analyze these issues. We argue that an important issue faced by society is that of future deterrence, in the sense that in a pool of agents who contemplate violating society’s rules and incurring a heavy penalty, exactly who should the enforcement policy be designed to catch, incarcerate, and thereby prevent from committing a repeat offence? It seems to us that the rules should be designed to catch those individuals, who once they violate and get off scot-free, are the ones most likely to violate the rules again. We thus consider the type of policies that are most likely to achieve this objective.

We consider a model with a pool of agents with differing degrees of risk aversion. Within this pool of agents we consider who would consider violating a randomized rule, i.e., a rule under which it is uncertain whether a particular violator will be caught. We find that the pool of people most likely to violate the rule are the risk-tolerant agents. This implies that randomization helps encourage violation by the less risk averse individuals and deter violation by the most risk averse individuals. We then show that the least risk averse agents are the ones most likely to be repeat offenders. This suggests that randomization helps ensnare those agents who are most likely to be repeat offenders. A sufficiently high penalty on these agents can then deter future violations by these agents.
Overall, we document a specific role for randomization, i.e., for making the enforcement of the rule less than predictable for the pool of agents, and thus creating randomness for the violators. The choice of the degree of randomness depends on the resources of the regulatory authority. If the resource limitation is sufficiently stringent, this constraint determines the degree of randomness. Otherwise, the degree of randomness is set to be that number which sets the violation cutoff as that level of risk tolerance above which offenders commit repeat offences.

We also apply the analysis to insider trading prosecution in a standard rational expectations model. We again consider enforcement rules with randomized triggers. Specifically, we consider insider prosecution rules that are triggered on extreme price moves and the insider having traded. We show that the greater the noise in the price trigger, the smaller is risk aversion cutoff below which insiders do not trade. Thus randomizing the trigger point can help deter the most risk averse insiders from trading and encourage the risk-takers to trade and be prosecuted. It is in society's interest to prosecute only the most risk tolerant insiders because it is these agents which cause the greatest losses for uninformed liquidity traders.

The analysis of optimal enforcement policy goes back to Becker (1968) who addresses the issue of what fine to levy on individuals when society has insufficient resources to prosecute every person who violates society's laws. Becker also addresses the notion that randomization may be necessary when society has limited resources. Our work is closely related to Polinsky and Shavell (1979) who analyze a variant of Becker's approach when the violator is risk averse. Another closely related line of research is that of Reinganum (1988, 1993), who analyzes the decision on whether to violate as well as the game between
the violator and the prosecution in terms of the type of sentence asked for while allowing for the possibility of plea bargaining. While the focus of Becker, Polinsky and Shavell, and Reinganum is on deriving the optimal combination of the proportion of individuals to be prosecuted and the fine to be levied on individuals, our focus is on the issue of whether deliberate randomization of law enforcement rules can serve a useful purpose in screening out certain categories of heterogeneous violators.

Related literature also includes the vast macroeconomic literature on rules-based versus discretionary monetary policy starting with Kydland and Prescott (1977). More recently Subrahmanyam (1995) has analyzed rules versus discretion in the context of market closure rules. Kleinig (1996) includes several analyses of the police use of discretion in law enforcement (see, for example, Heffernan (1996)). Our paper is related to these papers because deliberately injecting randomness into law enforcement can be interpreted as discretion, because in both cases, agents do not know whether they will be caught for sure when they violate a rule. The issue of deterministic rules versus randomness also arises in the case of insider trading regulation; the analysis of Meulbroek (1992) suggests that insider trading prosecutions have a random component to them. (See DeMarzo, Fishman, and Hagerty (1998) and Leland (1992) for other analyses of insider trading).

This paper is organized as follows. Section 1 discusses the general setting. Section 2 analyzes the complete model. Section 3 discusses the application to insider trading. Section 4 concludes.
1 The Basic Results

In this section we derive our basic results in the context of a simple model. We first consider a situation without law enforcement and ask who among a set of individuals with varying degrees of risk aversion would consider taking an extreme action which inflicts an externality on society. We then consider the type of law enforcement policy which would ensnare such individuals.

1.1 Who Violates?

In this section, we consider a set of agents with heterogeneous degrees of risk aversion, who are confronted with whether to take an extreme action $A_E$, or a moderate action $A_M$. When they take an extreme action $A_E$, they inflict an externality $E$ on society and, with a probability $q$, incur personal costs of $C$. This cost $C$ represents some exogenous effect of the extreme action and could, for example, correspond to an accident (in the case of speeding), being caught and castigated by peers (in the case of other types of lawbreaking such as stealing), and so on. With the complementary probability, $1 - q$, the agent who takes the extreme action obtain a private benefit of $\pi$. This represents the thrill of speeding, or the monetary benefits of stealing or trading on inside information. We assume that the benefit obtains only when the adverse effects of the action do not obtain; however, this aspect is not crucial to the analysis.

Now, suppose that each agent $l$ has mean-variance utility with a coefficient $R_l$ so that his utility is given by

$$E(W) - \frac{R_l}{2} \text{var}(W)$$
where \( W \) denotes wealth. We assume that only one agent takes a decision at any given time. For later convenience, we will bound the possible values of risk aversion and assume that the risk aversion of an agent can take on any value between \( R_s \) and \( R_B \), with \( R_s > 0 \) and \( R_B > 0 \). Now, if the agent takes the action \( A_E \), we have

\[
E(W) = (1 - q)\pi - qC
\]

and

\[
\text{var}(W) = (\pi + C)^2q(1 - q)
\]

If the agent takes the action \( A_M \), the expected utility is zero. This implies that the agent takes the action \( A_E \) if and only if

\[
E(W) - \frac{R_l}{2}\text{var}(W) > 0
\]

This leads us to the following proposition.

**Proposition 1** An agent takes the extreme action \( A_E \) if and only if his risk aversion

\[
R_l < \frac{2[(1 - q)\pi - qC]}{(\pi + C)^2q(1 - q)}.
\]

Thus the externality \( E \) is inflicted on society by the least risk averse agents. This result suggests that society should attempt to prosecute and preclude offences by the more risk-tolerant ("thrill-seeking") agents in the economy.

### 1.2 Enforcement

Consider a different scenario in which law enforcers are present and can prosecute and impose penalties on agents who take the extreme action. In this scenario, the agent
can take two actions, $A_{E1}$ and $A_{M1}$. If he takes the action $A_{M1}$ his expected utility is zero. On the other hand, if he takes the action $A_{E1}$ he violates a rule, and this violation carries a penalty $C_1$. However, upon violation, the probability of being caught and levied the penalty is not unity, but a number $p$, with $0 < p < 1$. If the individual takes the action $A_{E1}$ and is caught, then he is levied the penalty of $C_1$; with the complementary probability, he derives a private benefit of $\pi$.

Again, we assume a mean-variance utility function. In this case, the individual will violate if and only if

$$E(X) - \frac{R_t}{2} \var(X) > 0$$

where $X$ is the random variable which takes on the value $-C_1$ with probability $p$ and $\pi$ with probability $1 - p$. Substituting for the moments of $X$ into (1), the condition for violation reduces to

$$R_t < \frac{2[\pi - p(\pi + C_1)]}{(\pi + C_1)^2 p(1 - p)}$$

This implies that the least risk averse individuals are the ones most likely to violate the rule. Obviously the above condition is a meaningful restriction on the degree of risk aversion if and only if

$$p < \frac{\pi}{\pi + C_1}$$

Further, if $R_c$ denotes that value of risk aversion for which (2) holds as an equality, then it is easy to show that

$$\text{sgn}\frac{dR_c}{dp} = \text{sgn}[ - p^2 C_1 - \pi(1 - p)^2 ] < 0$$

This implies the following proposition.
Proposition 2 An agent violates the rule if and only if $R_i$ is less than the right-hand side of (2). Further, the threshold level of risk aversion below which the agent violates is decreasing in $p$, the probability of being caught.

The above proposition implies that the greater the probability of being caught, the lower is the risk aversion threshold below which the agent violates. This implies that subject to the constraint (3), the regulatory authority can manipulate $R_c$ by an appropriate choice of $p$.

In sum, the randomized enforcement rule has the property that only the most risk-tolerant agents take the extreme action $A_{E1}$. As these are the agents who are most likely to be repeat offenders (from Section 1.1), it can be seen that randomized enforcement potentially has the property of ensnaring repeat offenders. In the next section, we present an integrated analysis of the results of the two aspects of the problem considered here.

Before doing this, however, we consider the case where the agent is allowed to take a continuous action, rather than a discrete one, and show that the result of this section continues to hold in such a setting.

1.3 When the Agent Takes a Continuous Action

Suppose that instead of a discrete action (violation of a rule or not), the agent takes a continuous action $a$. The regulatory authority catches and prosecutes if and only if $a > \bar{a} + \delta$, where $\delta$ is a random variable. The agent receives the private benefit $\pi$ if and only if $a > \bar{a}$. We now show that even if the agent can choose the optimal action $a$, the randomized rule will cause the least risk averse individuals to violate the rule.
Suppose that \( \delta \) is uniformly distributed on \([k_1, k_2]\). Then

\[
Pr(caught) \equiv q = \frac{a - \bar{a} - k_1}{k_2 - k_1}
\]

(4)

Now, if the agent chooses \( a < \bar{a} \) then he receives a utility of zero. Thus provided the utility from choosing \( a > \bar{a} \) is positive, the agent maximizes

\[
q\pi - (1 - q)C_1 - \frac{R_i}{2}(\pi + C_1)^2q(1 - q)
\]

(5)

with respect to \( q \) (since \( q \) is a linear function of \( a \)). Differentiating with respect to \( q \), and setting the derivative to zero yields a linear equation for the optimal \( q \), and the solution to this equation is

\[
q = \frac{1}{2} + \frac{1}{R_i(\pi + C_1)}
\]

(6)

The agent will take an action \( a > \bar{a} \) if and only if the quantity in (5) is positive. Substituting for \( q \) from (6) above into (5) and calculating the condition for the resulting expression being positive, the following proposition obtains.

**Proposition 3** The agent takes an action \( a > \bar{a} \) if and only if

\[
\frac{4 + R_i^2(\pi + C_1)^2}{R_i} < \pi - C_1
\]

(7)

The left-hand side of this inequality is increasing in \( R \) if and only if

\[
R_i > \frac{2}{\pi + C_1}
\]

Thus, so long as the lower bound on agents' risk aversion \( R_S \) is sufficiently high, if the agent is allowed to take a continuous action and the probability of being caught is allowed to depend on the action, the least risk-averse agents violate and get ensnared. That the
least risk averse agents also inflict the externality can easily be shown by using arguments similar to those in Section 1.1; this exercise is omitted for brevity. The above analysis thus is consistent with the analysis using a binary action space in Sections 1.1 and 1.2.

2 An Integrated Analysis

Having demonstrated the basic results in the previous section, we now present a fuller analysis in this section. We assume for simplicity that the agent's action space is binary (as in Sections 1.1 and 1.2). There are a continuum of agents with differing risk aversion, and two dates, 1 and 2. An agent can choose whether to take an extreme action $A_E$ or a moderate action $A_M$ at each of the two dates.

We assume that because of limited resources it is impossible to enforce laws at all places and at all times. Thus, we stipulate that if the agent takes the extreme action at date 1, he may be caught but the regulation cannot be enforced at date 2, so that if the agent takes the extreme action at date 2 he is not caught. Date 2 of the model can be interpreted as the opportunity to speed or commit other crimes in remote areas where patrolling is not possible, or trading on inside information in very active markets where laws cannot be enforced. We also assume that if the agent is caught at date 1, he is precluded from the opportunity of taking the extreme action at date 2. This corresponds to incarceration or being placed on probation, for example. We also assume that only one agent plays the game at any point in time, and that his risk aversion is unknown to the regulatory authority.

The issue is to solve for the dynamic problem of the agent given the regulators'
randomized enforcement policy $p$, which represents the probability of being caught, and then to solve for the regulator’s choice of $p$. The basic payoff structure is as follows. At date 2, if the agent takes an extreme action $A_E$, with probability $q$ he suffers a cost of $C$ and inflicts an externality $E$ on society. Again, $C$ represents an exogenous effect of the extreme action such as an accident (in the case of speeding), or the negative impact inflicted by peers and society upon being found out as a lawbreaker. However, $C$ is not crucial to the model and can be set to zero without loss of generality. With probability $1 - q$ the agent who takes the extreme action obtains private benefits of $\pi$. We assume that the regulatory penalty at date 1 upon being caught taking the extreme action is $C_1$ and the agent again obtains private benefits of $\pi$ if he takes the extreme action but does not get caught at this date. The moderate action yields a zero payoff at both dates 1 and 2.

Let $\mu \equiv -qC + (1 - q)\pi$ and $V \equiv (\pi + C)^2 q(1 - q)$. Consider date 2 first. There are two situations under which an agent has to choose an action at date 2: he takes an extreme action at date 1 and does not get caught, or he takes the moderate action at date 1. Regardless of which situation he is in, it is easy to see that agent $l$ takes an extreme action at date 2 if and only if

$$R_l < \frac{2\mu}{V} = \frac{2[q\pi - (1 - q)C]}{(\pi + C)^2 q(1 - q)}$$

We will again denote the critical value of $R_l$ as $R_c \equiv \frac{2[q\pi - (1 - q)C]}{(\pi + C)^2 q(1 - q)}$. Now consider date 1. An agent with $R_l > R_c$ does not take an extreme action at date 2. He will take an extreme action at date 1 if and only if his risk aversion

$$R_{c1} < \frac{2[\pi - p(\pi + C_1)]}{(\pi + C_1)^2 p(1 - p)}$$
(from Section 1). But agents with \( R_i > R_c \) don’t inflict any externalities on society at date 2. Thus, there is no point to catching these agents at date 1.

Now let us consider an agent with \( R_i < R_c \). Such an agent takes the extreme action at date 2. His payoff if he takes the extreme action at date 1, which we denote by the random variable \( Y \), is described as follows: with probability \( p \) the agent gets \(-C_1\), with probability \((1 - p)q\) he gets \( \pi - C \), and with probability \((1 - p)(1 - q)\) he gets \(2\pi\). We can again calculate the expected utility as

\[
E(Y) = \frac{R_i}{2} \text{var}(Y)
\]

where

\[
E(Y) = -pC_1 + (1 - p)q(\pi - C) + 2(1 - p)(1 - q)\pi
\]

and

\[
\text{var}(Y) = p[C_1(1 - p) - (1 - p)q(\pi - C) - 2(1 - p)(1 - q)\pi]^2
\]

\[
+ (1 - p)q[pC_1 + (\pi - C)(1 - q + pq) - 2(1 - p)(1 - q)\pi]^2
\]

\[
+ (1 - p)(1 - q)[pC_1 - (1 - p)q(\pi - C) + 2\pi \{1 - (1 - p)(1 - q)\}]^2
\]

The expected utility if the agent takes a moderate action at date 1 is given simply by

\[
-qC + (1 - q)\pi - \frac{R_i}{2}(\pi + C)^2q(1 - q)
\]

Given the expected utility, it is again possible to calculate the threshold risk aversion (below which the agent takes the extreme action at date 1) as a function of \( p \). This function is described by that value of \( R_i \) which makes the expected utility of the extreme action equal to the expected utility of the moderate action for a given value of \( p \). The
agent thus takes the extreme action at date 1 if and only if by

\[
R_t < \frac{2[E(Y) - \{(1 - q)\pi - qC\}]}{\text{var}(Y) - (\pi + C)^2 q(1 - q)}
\]  

(8)

We denote the threshold level of risk aversion for which the above restriction holds as an equality as \( R_{cc} \).

The basic point is that \( R_{cc} \) can be manipulated by changing \( p \). While general results on this are difficult to derive, the appendix proves the following proposition.

**Proposition 4** Locally around \( p = 0 \) and \( C = 0 \), the derivative of the right-hand side of (8) with respect to \( p \) is negative so that for small \( p \), the threshold level of risk aversion below which the agent violates is decreasing in \( p \).

In Figure 1 we provide the expected utility as a function of \( p \), for \( R_t = R_c \). The parameter values used for this figure and the next are \( C = 0.1, \pi = 0.3, C_1 = 0.1, \) and \( q = 0.5 \), corresponding to an \( R_c = 5 \).

Please insert Figures 1 and 2 here.

For \( p > 0.38 \), the agent does not take the extreme action. Figure 2 plots the expected utility as a function of the risk aversion coefficient for \( p = 0.38 \) and shows that agents with \( R_t = R_c < 5 \) are the only ones who violate. Thus only agents who take the extreme action at date 2 are ensnared at date 1. Of course, the randomized enforcement implies that only a portion of these agents are caught. We now address this issue.

Let \( p_c \) denote that value of \( p \) such that agents with \( R_t < R_c \) violate and agents with \( R_t > R_c \) do not. Suppose the law enforcement authorities have limited resources of \( S \).
Suppose that there is a variable cost of enforcing the law per capita, which is \( s \). Thus, the maximum mass of agents that can be caught is \( S/s \) (rounded to the nearest integer). If the total mass of agents is \( \rho \), \( S/(\rho s) \) represents the maximum proportion of agents that can be caught. Let us call this proportion \( P^* \). Thus, if \( p_c < P^* \), then \( p \) is set to equal \( p_c \) else it is set to be the maximum allowable \( p \), i.e., \( P^* \). The basic premise here is that when resources are limited, society will not be able to prosecute all violators. In such cases, applying a randomized enforcement policy causes the likely repeat offenders to violate the rule, and prosecuting a portion of these agents represent a better deployment of the limited resources of the law enforcement authority.

3 Application to Insider Trading

In this section, we apply our ideas to the case of insider trading. We consider a single strategic insider, who can be caught and levied a penalty if he trades. But the probability of his being caught given that he trades is not unity. We show that in this case, for given information realizations, again the least risk-tolerant insiders are the individuals who indulge in insider trading.

3.1 Liquidity and Risk Aversion

Consider a risky security that pays off \( \theta + \epsilon \). There is a single risk averse insider who is strategic and has negative exponential utility with a risk aversion coefficient of \( R_i \). The noise trade is \( z \) and prices are set by risk-neutral market makers who see only the combined order flow, as in Admati and Pfleiderer (1988), Subrahmanyam (1991), or Kyle (1985). The informed trader's order is denoted by \( y \) and we also assume that the insider
knows the realization of the noise trade \( z \), in order to avoid the problem of the risk averse insider facing execution risk, which is likely not an important factor in reality. Finally, we stipulate a linear pricing rule \( P = \xi(y + z) \). The insider maximizes

\[
E(X) - \frac{R_t}{2} \text{var}(X)
\]

where

\[
X = y[\theta + \epsilon - \xi(y + z)]
\]

which implies that

\[
y = \frac{\theta - \xi z}{2\xi + Av_e}
\]

The appendix shows that \( \xi \) solves the equation

\[
\xi v_z(\xi + Av_e) - v_\theta = 0
\]

(9)

Taking the positive root of the above quadratic equation we have

\[
\xi = \frac{-R_tv_zv_e + \sqrt{R_t^2v_z^2v_e^2 + 4v_\theta v_z}}{2v_z}
\]

(10)

It is easy to show that \( d\xi/dR_t < 0 \) so that market liquidity is decreasing in the risk tolerance of the insider. Since the losses of liquidity traders are \( E[z(P - F)] = \xi v_z \), this leads to the following proposition.

**Proposition 5** The adverse selection costs of the liquidity traders are increasing in the risk tolerance of the insider.

Thus, one reasonable goal of policy-makers would be to lower \( \xi \), which would imply a rule designed to catch the least risk averse insiders.
3.2 Ex Ante Information Acquisition

We now present an ex ante analysis in which the insider decides whether to acquire the information or not, based on the randomization policy chosen by the regulator. We assume the regulator can accurately verify if the insider has the information and preclude the insider from trading. The Appendix shows that the ex ante utility from acquiring and trading on the information, and not being caught, is given by the expression

\[-1/\sqrt{\text{Det}}\]

where

\[
\text{Det} = \frac{R_i v_e + 2\xi}{R_i (\xi^2 v_z + v_e + v_\theta) + 2\xi}
\]

(11)

Substituting for \(\xi\) from (10), we have

\[
\text{Det} = \frac{2}{R_i v_z \left[ R_i^2 v_z^2 v_x + 4v_\theta \right] - R_i^2 v_x v_z + 2}
\]

It can easily be shown that

\[
\text{sgn} \frac{d \text{Det}}{d R_i} = -\text{sgn}[R_i^2 v_z^2 v_x + 2v_\theta - R_i v_x v_z^{\frac{1}{2}} \{R_i^2 v_z^2 v_x + 4v_\theta\}^{\frac{1}{2}}]
\]

Since the term in square brackets is of the form \(a + b - \sqrt{a^2 + 2ab}\), where \(a\) and \(b\) are positive, we have \(d \text{Det}/d R_i < 0\). This leads us to the following proposition.

**Proposition 6** The ex ante expected utility of the insider from trading on information is decreasing in the degree of risk aversion.

Now the insider will acquire information if

\[-\left[ (1 - p) \frac{1}{\sqrt{\text{Det}}} + p \exp(R_i C) \right] > -1\]
i.e., if

\[
(1 - p) \frac{1}{\sqrt{\text{Det}}} + p \exp(R_t C) < 1
\]

Since Det is increasing in \(R_t\) the left-hand side of the second inequality above is increasing in \(R_t\). This implies that insiders with lower values of risk aversion would be more likely to trade on inside information. Further, the randomization causes the least risk averse agents to acquire the information. For example, consider the parameter values \(v_c = 0.4\), \(C = 0.8\), \(v_\theta = v_\xi = 1\), and \(p = 0.3\). In this case, insiders with \(R_t < 0.36\) acquire information, whereas insiders with \(R_t > 0.36\) do not. Again, the randomized rule induces the least risk averse agents to violate the rule.

### 3.3 The Analysis Conditional on Information

Suppose that the insider is not detected unless he trades. Now, conditional on not being caught, the insider’s expected utility is given by

\[-\exp(-R_t U)\]

where

\[
U = \frac{\theta(\theta - \xi z)}{2\xi + R_t v_c} - \frac{\xi(\theta - \xi z)^2}{(2\xi + R_t v_c)^2} - \frac{\xi(\theta - \xi z) z}{2\xi + R_t v_c} - \frac{R_t(\theta - \xi z)^2 v_c}{2(2\xi + R_t v_c)^2}
\]

Now, if the insider is caught he is levied a penalty of \(C\) and gives up his trading profits. Let the probability of being caught be \(p\). The ex ante expected utility of the insider is therefore

\[-\exp(-R_t U)\]

with probability \(1 - p\) and

\[-\exp(R_t C)\]
with probability $p$. Assuming the wealth from not trading is zero, the insider trades if and only if

$$-p \exp(R_tC) - (1 - p) \exp(-R_tU) > -1$$

While the model cannot be solved in closed-form, numerical simulations again indicate that for any given $p$, insiders with risk aversion less than a cutoff trade on information and relatively less risk-tolerant insiders do not do so. For example, consider the parameter values $v_g = v_s = 1$, $v_c = 0.4$, $\theta = 3.2$, $z = 1$, and $C = 1$. Then, for $p = 0.47$, insiders with $R_t < 0.3$ trade, and insiders with $R_t > 0.3$ do not trade.

Thus, if the policy goal is to reduce the losses of the liquidity traders, the most risk tolerant insiders should be induced to trade in inside information, be prosecuted, and forced to give up their trading profits. Of course, in the numerical example above, because of the randomized enforcement, it is not possible to catch all insiders with risk aversion below the 0.3 cutoff. So, while on one hand, the rule allows violation by those insiders which cause the greatest harm to the liquidity traders, on the other, it does not permit prosecution of all these traders. Again, if resources are limited, it is possible to establish a threshold for the losses of liquidity traders that society is willing to tolerate and select $p$ such that the mass of insiders ensnared is such that the aggregate expected losses of liquidity traders equal this threshold.

4 Conclusion

It is often argued that discretion in law enforcement should be restricted because doing so will reduce unpredictability and unfair tactics (see, for example, Fisher (1996), Livingston
(1997), and Hall (1996)). Unbiased discretion, however, may also be interpreted as randomization, i.e., making enforcement imperfectly predictable. As Becker (1968) points out, randomization will clearly be necessary when resources are limited so that only a proportion of lawbreakers can be prosecuted. However, resource constraints may not be binding in all settings, so that society may be able to catch all violators in certain specific situations. Catching all violators in all possible settings is likely to be impossible (e.g., it may be difficult to enforce speeding laws in remote areas, or insider trading laws in active markets where insiders may have considerable anonymity). We argue that in such settings, randomization of law enforcement can substitute for unbiased discretion and play a useful role in screening out potential repeat violators. We argue that such randomization causes only the most risk-tolerant agents to violate the rule, so that it serves to catch and penalize the individuals most likely to violate in unenforceable settings.

An implication of our analysis is that such randomization is most likely to be observed when repeat offenses are relatively costly to society. Note also that in our model, randomization deters the most risk averse agents and causes the least risk averse agents to violate. It may argued that there is a case for deterring the least risk averse agents because they may lead to the most social damage in their crimes. While this argument has merit, our point is that laws cannot be enforced in all settings. Thus, deterring the least risk averse agents in a setting where laws can be enforced does not preclude the agent from committing the violating in an unenforceable setting (such as robbing someone in a dark alley or speeding in a remote section of a highway). Allowing the least risk averse agents to violate and be penalized, on the other hand, can preclude these agents from
committing a repeat violation in a setting where laws cannot be enforced. We argue that this is desirable because it is the least risk averse agents who are most likely to violate laws in unenforceable settings.

In the above context, it is important to note that our rationale for randomization works only if the penalty levied on individuals is sufficiently high that it precludes agents from repeat violations. Thus, if speeding laws are randomized but being caught for speeding entails only a small fine, randomization will likely not deter individuals from speeding again. However, if speeding carries a sufficiently high penalty (e.g., revoking driving privileges after two or three violations) then such laws would indeed be useful in deterring the so-called “thrill-seekers” (a metaphor for relatively risk-tolerant agents) from repeated speeding violations. It also is worth noting that our rationale for randomization obviously does not work in serious offenses such as homicides and rapes. In such situations, obviously, the cost to society of the single offense is so high that one cannot use randomized law enforcement as a tool for future deterrence.

Finally, it is not the goal of this paper to argue that our arguments for randomization are the only relevant ones. For example, other issues such as the severity of the offence (e.g., how far above the speed limit was the violator driving), the type of place where the offence was committed (the consequences of speeding on a deserted highway are much less severe than those of speeding on crowded freeways criss-crossing major cities) may also dictate the degree of randomization. Similarly, in the case of insider trading, discretion may have to be used because there may be a lack of hard evidence against the insider, and there may be difficulties in proving scienter (that the defendant knew the information was material and thus traded with intent to defraud). Nevertheless, we believe our model
takes first things first and provides useful arguments for randomization. We hope that our work serves as a starting point for more research on the subject.
Appendix

Proof of Proposition 4: The derivative of the reciprocal of the right-hand side of (8) can be written as

\[
\frac{A}{B}
\]

where

\[
A \equiv p^2(\pi^3(q - 2)^3 + 3\pi^2(q - 2)^2(Cq - C_1) + (Cq - C_1)^3 \\
+ 2\pi p(\pi^2(q - 2)^2 + 2\pi(q - 2)(Cq - C_1) + (Cq - C_1)^2 \\
- \pi(\pi^2(2q^2 - 5q + 4) + 2\pi(Cq(2q - 3) + C_1(2 - q)) + C_1(2 - q)) \\
+ C^2q(2q - 1) - 2CC_1q + C_1^2
\]

and

\[
B \equiv 2[\pi(p(q - 2) + 1) + p(Cq - C_1)]^2
\]

For \( p = 0 \) and \( C = 0 \), \( A/B \) reduces to

\[
\frac{\pi^2(2q^2 - 5q + 4) + 2\pi C_1(2 - q) + C_1^2}{2\pi}
\]

which is positive (for \( 0 < q < 1 \)). Of course the restriction in the inequality (8) makes sense only if the right-hand side of this inequality is positive. However, it is easy to find parameter values such (8) is positive for sufficiently small values of \( p \) and \( C \).

Derivation of the Equation for \( \xi \): Now, note that

\[
y + z = \frac{\theta}{2\xi + R_i v_e} + z \frac{\xi + R_i v_e}{2\xi + R_i v_e}
\]

Thus,

\[
\xi = \frac{\text{cov}(\theta, x + z)}{\text{var}(x + z)}
\]

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which yields
\[ \xi v_\theta + R_i v_\epsilon v_\theta = \xi v_\epsilon (\xi + R_i v_\epsilon)^2 \]

and is equivalent to (9).

**Derivation of Equation (11):** Define a vector of random variables \( T = [\theta, \epsilon, z] \). Then the payoff of the informed agent can be written as \( x(\beta \gamma) x' \), where
\[
\beta = \begin{bmatrix} \frac{1}{2 \xi + R_i v_\epsilon} & 0 & -\frac{\xi}{2 \xi + R_i v_\epsilon} \end{bmatrix}
\]
and
\[
\xi = \begin{bmatrix} \frac{\xi + R_i v_\epsilon}{2 \xi + R_i v_\epsilon} & 1 & -\frac{\xi(\xi + R_i v_\epsilon)}{2 \xi + R_i v_\epsilon} \end{bmatrix}
\]

We now make use of the following lemma, which is a standard result on normally distributed random variables.

**Lemma 1** Let \( Q(v) \) be a quadratic function of the random vector \( v \): \( Q(v) = C + B'v - v'Av \), where \( v \sim N(\mu, \Sigma) \), and \( A \) is a square, symmetric matrix. We then have
\[
E[\exp(Q(v))] = |\Sigma|^{-\frac{1}{2}} |2A + \Sigma^{-1}|^{-\frac{1}{2}} \exp \left( C + B'\mu + \mu' A \mu + \frac{1}{2} (B' - 2 \mu'A') (2A + \Sigma^{-1})^{-1} (B - 2 \mu) \right).
\]

To construct the matrix \( A \), we construct the \( 3 \times 3 \) square, symmetric matrix with elements \( a_{ij} = R_i (b_i g_j + b_j g_i) \), \( i = 1, \ldots, 3, j = 1, \ldots, 3 \), where \( b_i \) and \( g_i \) are the elements of the \( \beta \) and \( \gamma \) vectors. The matrix \( \Sigma \) in our case is simply \( \text{Diag}(v_\theta, v_\epsilon, v_z) \). Applying the above lemma, we find that the determinant of \( R_i \Sigma + I \) is given by
\[
\frac{R_i (\xi^2 v_\epsilon + v_\epsilon + v_\theta) + 2\xi}{R_i v_\epsilon + 2\xi}
\]

The expression in (11) thus follows. \( \Box \)
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Figure 1

Expected utility vs. the probability of being caught

Expected utility

Prob(caught)
Figure 2

Expected utility vs. risk aversion, for the value of $p$ such that $R_{cc} = R_c$