Capital budgeting in multi-division firms: Information, agency, and incentives*

Antonio E. Bernardo†

Hongbin Cai‡

Jiang Luo§

June 20, 2003

*We thank Naveen Daniel, Diego Garcia, Tano Santos, two anonymous referees, the editor (Mike Fishman), and seminar participants at Chicago, City University of Hong Kong, Georgia Tech, HKUST, MIT, Northwestern, Rochester and Yale for helpful comments and suggestions. Luo would also like to acknowledge financial support from the Wei Lun Fellowship. All errors are ours.

†Corresponding author: The Anderson School at UCLA, Los Angeles, CA 90095-1481, USA. ph: (310) 825-2198. e-mail: abernard@anderson.ucla.edu

‡Department of Economics, UCLA, Los Angeles, CA 90095-1477, USA

§Department of Finance, HKUST, Clear Water Bay, Hong Kong

Abstract

We examine optimal capital allocation and managerial compensation in a firm with two investment projects (divisions) each run by a risk-neutral manager who can provide (i) (unverifiable) information about project quality and (ii) (unverifiable) access to value-enhancing, but privately costly, resources. The optimal managerial compensation contract offers greater performance pay and a lower salary when managers report that their project is higher quality. The firm generally underinvests in capital and managers underutilize resources (relative to first-best). We also derive cross-sectional predictions about the sensitivity of investment in one division to the quality of investment opportunities in the other division, and the relative importance of division-level and firm-level performance-based pay in managerial compensation contracts.

1 Introduction

The long-term health of a firm is determined by the quality of its investments. In a typical firm, capital is allocated to investment projects based on reports by division managers who have access to private information about project quality. The eventual success of any project may also require the input and cooperation of other managers who control access to valuable resources within the firm.[1] The firm, however, may not be able to either independently verify the managers' reports or monitor the actions of managers to ensure that they are deploying and sharing their resources appropriately. Thus, to secure its long-term health, the firm must provide incentives to encourage truthful information and the appropriate utilization of managerial resources.

In this paper, we present a simple model to illustrate these information and incentive problems in a multi-division firm. Specifically, we consider a firm with unlimited access to capital and two investment projects. The optimal amount of capital to allocate to each project depends on its quality which is unknown to the firm's headquarters. For each project, however, the firm can hire a risk-neutral manager with private information which is useful for assessing the quality of both projects. Once hired, each manager reports her information (not necessarily truthfully) to headquarters which then allocates capital according to the reports. The veracity of the reports is assumed to be non-verifiable and non-contractible. Once capital is allocated, project cash flows can be enhanced by either or both managers by deploying resources under their control. The use of these resources is assumed to be privately costly, non-verifiable, and non-contractible.

[1] The fact that the success of investments in one division also depends on the resources available in other divisions may very well be the reason the divisions exist within the same firm. However, we do not explicitly analyze the boundaries of the firm in this paper; rather, we take as given the scope of the firm. We also assume the firm has access to capital but division managers do not, so the firm's headquarters is indispensable to the production process. Gertner, Scharfstein, and Stein (1994) and Stein (1997) explicitly model the productive role of headquarters to help understand the costs and benefits of internal and external capital markets.

To fix ideas, consider a firm with two divisions selling computer hardware and services. Each division has a manager with private information about the demand for its product obtained from extensive time in the field. The demand for hardware is related to the demand for services (and vice-versa) so each manager's private information is relevant for assessing the quality of investment projects in both divisions. Moreover, each manager controls resources that are valuable to both projects; for example, the hardware division manager may have relationships with buyers that can be exploited to sell services (and vice-versa). The use of this relationship capital with buyers is costly to the manager and likely to be non-verifiable by the firm.

In this paper, we demonstrate how capital budgeting and managerial compensation contracts can be jointly designed to help mitigate these information and incentive problems in a multi-division firm. In our model, the optimal managerial compensation contract is linear in both divisions' cash flows. Moreover, the firm provides more capital, greater performance-based pay, and a lower salary to division managers when they report (truthfully, in equilibrium) that their project is higher quality. The lower salary combined with higher performance-based pay effectively encourages truthtelling by making the managers "buy" shares in the project cash flows when they are very optimistic.

In general, the firm invests too little capital and managers allocate too few resources (relative to first-best) to both divisions and this underinvestment problem is more severe when both divisions have relatively poor investment opportunities, the asymmetric information between headquarters and division managers is greater, division managers have more firm-specific human capital, and division managers have less performance-based pay. We also show that investment in one division is positively related to the quality of investment opportunities in other divisions and this relation is stronger when managerial resource sharing is more important, division managers have more discretion, and division managers have more firm-specific human capital. These predictions suggest refinements to the empirical literature examining the sensitivity of investment in

one division to the cash flows and investment opportunities of other divisions in the firm (e.g., Lamont, 1997; Shin and Stulz, 1998). The excess sensitivity of investment to the cash flows of other divisions implies that multi-division firms will invest more (less) in a division than a single-division firm when other divisions in the firm are expected to perform well (poorly). Finally, we derive novel implications for the composition of division manager compensation contracts. For example, consistent with the empirical evidence in Bushman, et al. (1995) and Keating (1997), we predict that division managers will receive relatively more firm-level performance pay compared to division-level performance pay when they control resources that are more valuable in other divisions.

There is a large theoretical literature examining optimal capital budgeting mechanisms in single-division firms (e.g., Harris and Raviv, 1996; Holmstrom and Ricart i Costa, 1986; Zhang, 1997; Bernardo, Cai, and Luo, 2001; Garcia, 2001, 2002; Berkovitch and Israel, 2003). This paper extends the analysis in Bernardo, Cai, and Luo (2001) to multi-division firms with productive and information interactions among divisions. Harris, Kriebel, and Raviv (1982) and Antle and Eppen (1985) present models of multi-division firms in which the division manager has private information about the production technology and a preference for capital. These papers do not consider optimal managerial compensation mechanisms but rather focus on the role of transfer prices in allocating capital across divisions. Harris and Raviv (1998) present a model of a single-division firm with multiple investment projects in which the division manager has private information and a preference for capital. Headquarters can learn the information through a costly audit. The optimal mechanism trades off the distortion due to decentralized information and managerial preference for capital against the costs of (endogenously determined) probabilistic auditing. As in Harris, Kriebel, and Raviv (1982), they find regions of under- and over-investment, whereas we find only under-investment. These different predictions follow from their assumption of exogenous compensation contracts (see Bernardo, Cai, and Luo, 2001). Finally, Rajan, Servaes, and Zingales (2000) propose a theoretical model of internal capital markets in which division managers exhibit

rent-seeking behavior. In contrast, we consider agency costs due to asymmetric information and managerial moral hazard. Their model assumes managerial incentive schemes are exogenously determined whereas we derive jointly the optimal capital budgeting and managerial compensation mechanism.

The remainder of the paper is organized as follows. Section 2 presents our model. Section 3 derives the first-best capital and managerial resource allocations to be used as a benchmark for the optimal second-best mechanism derived in Section 4. Section 5 discusses the important features of the second-best mechanism and provides directions for future empirical work on capital budgeting and managerial compensation. Section 6 examines the robustness of our capital underinvestment result to alternative specifications for managerial preferences. Section 7 concludes and gives direction for future research.

2 The model

We consider a firm run by a headquarters acting in the interest of the firm's risk-neutral shareholders. Headquarters has unlimited access to capital and two investment projects. The optimal amount of capital to invest in each project depends on its quality, which is unknown to headquarters. Headquarters, however, can hire a risk-neutral manager for each project who can add value in two ways: first, they have information about their own project's quality (which may also be relevant for assessing the other project's quality) and second, they have access to resources which can enhance the cash flows of either or both projects. The deployment of such resources is assumed to be privately costly to the manager. Examples of managerial resources include key personnel, scarce and valuable assets (e.g., distribution network), or access to a relationship (e.g., suppliers and buyers). For what follows, we will use the standard terminology in the principal-agent literature and refer to such resources as the manager's own "effort". The key point is that the use of managerial resources ("effort") enhances cash flows but imposes costs on the manager

who controls them (e.g., restricts its use in an alternative pet project, depletes valuable political capital) and cannot be contracted upon.[2]

Once hired, each manager is asked to report (simultaneously) her private information to the headquarters, which then chooses the capital allocation to each project based on both reports. Specifically, we assume the project cash flows, denoted V_i for project $i = 1, 2$, are given by:

$$V_i = nk_i + (\alpha e_i + \beta e_{ji})k_i + (\delta t_i + \phi t_j)k_i - 0.5k_i^2 + \epsilon_i,$$

where the subscript $j = 2$ ($j = 1$) when $i = 1$ ($i = 2$).

In this specification, k_i denotes the amount of capital allocated to project i, t_i denotes the private information of manager i, e_i denotes the effort of manager i on her own project i, and e_{ij} denotes the effort of manager i on the other project j. The "quality" of project i depends on both t_i and t_j where the parameters $\delta \geq \phi \geq 0$ determine the relative importance of each manager's information. For example, if $\delta = 1$ and $\phi = 0$ only the manager of project i has relevant information about the quality of project i. The parameters $\alpha \geq \beta \geq 0$ reflect the impact of managerial effort on the project cash flows. The assumptions $\delta \geq \phi$ and $\alpha \geq \beta$ capture the idea that managers have more impact, via information and effort, on their own project.[3] The ϵ_i are independent noise terms with mean zero. Finally, $n \geq 0$ is a constant that ensures it is always worthwhile to invest some capital in each project in the socially efficient solution.

[2]Our model builds on Laffont and Tirole (1986, 1993) who also examine the fundamental tradeoff between moral hazard and asymmetric information. Laffont and Tirole consider the problem of regulating a single monopoly with unobserved efficiency and non-contractible effort. While many of our proof techniques follow their arguments closely, our model has many different features (e.g., multiple agents, capital budgeting) and we are concerned with a very different problem; namely, how information and resource spillovers across divisions affect capital allocations and incentives within a firm.

[3]In the special case $\phi = \beta = 0$ the two projects are completely independent and we can analyze each firm separately as in Bernardo, Cai, and Luo (2001).

The cash flow specification, V_i, has many standard and intuitive features. Capital and managerial effort are complementary, implying that the marginal products of each are increasing in the levels of the other. This assumption will be important for the main results of the paper although the specific functional form is not important. Furthermore, the marginal product of capital is increasing in the quality of the project, defined as $(\delta t_i + \phi t_j)$, which is intuitively appealing and implies that the headquarters will want to allocate more capital to higher quality projects. The noise terms ϵ_i are independently distributed and capture underlying uncertainty about or measurement errors of project cash flows; we show below that since all agents are risk neutral the additive, mean zero noise terms have no effect on our results.

Headquarters does not know the information, t_i, but only knows that t_i is drawn from the interval $[0, \bar{t}]$ according to a distribution $F_i(t)$ with density function $f_i(t)$, where $f_i(t) > 0, \ \forall t$. We assume that t_1 and t_2 are independently distributed and that the t_i and ϵ_i are independently distributed. As is standard in the mechanism design literature, we also assume that the inverse of the hazard rate of $F_i(\cdot)$, denoted $\mu_i(t) = (1 - F_i(t))/f_i(t)$, is decreasing in t. It is well known that many common distributions such as the uniform and (truncated) normal distribution have increasing hazard rates.

For tractability, we assume a specific functional form for the managers' (private) cost of effort; specifically, the division managers' expected utilities are given by:

$$EU_i = Ew_i - 0.5\gamma(e_i^2 + e_{ij}^2)$$

where w_i is compensation, Ew_i is expected compensation, and γ parameterizes the managers' effort cost.[4] In Section 6, we examine the effect of including managerial

[4]An alternative specification for the cost function is $0.5\gamma(e_i + e_{ij})^2$. However, this specification is problematic because it implies perfect substitution between e_i and e_{ij} and hence always gives corner solutions for effort allocations. In our cost specification, we assume no interactions between e_i and e_{ij}. We have also considered other specifications which allow for interactions. For example, a more general specification is $\gamma(e_i^2 + e_{ij}^2 + \tau e_i e_{ij})$, where $\tau < 0$ implies that e_i and e_{ij} are complementary and $\tau > 0$

preferences for capital. Finally, we assume that managers have outside employment opportunities offering the reservation utility $\bar{U} \geq 0$.

The headquarters' problem is to maximize the expected payoff to shareholders, the residual claimants of the cash flows from the two projects. We assume there are no conflicts of interest between headquarters and shareholders because these issues are not central to our thesis. Headquarters can use two instruments, capital and compensation, to provide incentives for the division managers to tell the truth about project quality and to provide effort. Specifically, headquarters designs an optimal mechanism consisting of (i) a capital allocation policy $k_i(\hat{t}_1, \hat{t}_2)$ depending on each of the division manager's reports about project quality, $\hat{t}_i$, and (ii) a compensation schedule $w_i(\hat{t}_1, \hat{t}_2, V_1, V_2)$ depending on both reports and both project outcomes. Importantly, we assume that the private information, t_i, and the managers' effort allocations, e_i and e_{ij}, are not directly observable or verifiable by the headquarters ex post, therefore, contracts cannot be written on these directly.

The sequence of moves of the game is as follows:

date 0: Headquarters offers managers a mechanism $\{w_i(\hat{t}_1, \hat{t}_2, V_1, V_2), k_i(\hat{t}_1, \hat{t}_2)\}$

date 1: Division managers simultaneously report $\hat{t}_i$.

date 2: Headquarters allocates capital of $k_i(\hat{t}_1, \hat{t}_2)$ to division i.

date 3: Division managers allocate effort to each project, e_i and e_{ij}.

implies that e_i and e_{ij} are substitutes. When e_i and e_{ij} are complementary, higher e_i reduces the marginal cost of e_{ij} and hence induces higher e_{ij}. Intuitively, one expects to see more "convergence" between divisions in this case. When e_i and e_{ij} are substitutes, higher e_i increases the marginal cost of e_{ij} and hence leads to lower e_{ij}. Intuitively, one expects to see more "divergence" between divisions. However, solving for the closed form solution in this general case is quite involved technically. In this paper, we focus on the simple case without cost interactions (i.e., $\tau = 0$).

date 4: The project cash flows are realized and distributed to shareholders less the compensation $w_i(\hat{t}_1, \hat{t}_2, V_1, V_2)$ paid to division managers.

We make the standard assumption in these types of models that headquarters can commit to the capital allocation scheme offered to the managers at date 0. Absent a commitment device, it would be optimal for headquarters to allocate a different level of capital at date 2 than the amount offered at date 0. If the managers knew this, however, they would not report truthfully. Headquarters' commitment could be the result of (unmodelled) reputational concerns if it intends to play such a game repeatedly (potentially with other managers) in the future. Finally, as is standard in all models with asymmetric information and risk-neutral agents, it is critical for what follows that each division manager observes her private information prior to date 0; otherwise, it would be optimal for the headquarters to sell the firm to the managers because there is no asymmetric information at the time of contracting and the risk-neutral managers are equally efficient at bearing the project quality risk as the risk-neutral headquarters. One plausible example of our timing assumption is that at the time each division manager is hired or promoted, she has knowledge of external factors (e.g., market demand, competitors' strategies, industry trends, etc.) relevant to the cash flows of projects the firm may consider in the future. Another possibility is that the firm recently acquired one of the divisions and kept its manager to run it in the future. The firm must then choose the appropriate capital budget for the division and the manager's compensation with the understanding that the manager already has private information about the quality of the division's existing and potential investment projects.

3 Benchmark case: First-best outcome

To provide a benchmark, we first determine the socially efficient (first-best) solution of the model. The first-best maximizes the expected total surplus (expectation over ϵ_i):

$$\max_{k_1,k_2,e_1,e_2,e_{12},e_{21}} E_{\{\epsilon_1\}}V_1 + E_{\{\epsilon_2\}}V_2 - 0.5\gamma(e_1^2 + e_{12}^2) - 0.5\gamma(e_2^2 + e_{21}^2).$$

Before we proceed, we make the following parameter assumptions throughout the paper:

(A1) $\gamma > \alpha^2 + \beta^2$;

(A2) $n\left[1 - \sqrt{1 - \frac{\beta^2}{2(\gamma - \alpha^2)}}\right] \geq (\delta + \phi)\left[\mu_0 + \bar{t}\sqrt{1 - \frac{\beta^2}{2(\gamma - \alpha^2)}}\right]$,

where $\mu_0 \equiv \max(\mu_1(0), \mu_2(0))$. Assumption (A1) requires that the marginal cost of providing effort is increasing relatively fast compared to the marginal productivity of effort, so that the objective function of the optimal mechanism design program is concave. Assumption (A2) requires that (i) the expected net cash flow from each project is sufficiently high (large n) or (ii) asymmetric information is not too severe (small δ and ϕ) or (iii) effort spillovers are relatively strong (large β relative to $\gamma - \alpha^2$), so that the solution to the mechanism design program yields interior capital allocations ($k_i > 0$) and effort allocations ($e_i > 0$ and $e_{ij} > 0$). For example, if n is large enough the firm will always want to allocate at least some capital and some profit-sharing to motivate managers to provide effort to each project. This assumption allows us to focus on the most interesting parameter region without having to consider cases where one or more choice variables is at a boundary. None of the qualitative results of the model are affected by this assumption.

Proposition 1. *The first-best capital allocations and managerial effort allocations are given by:*

$$\begin{aligned} k_i^{FB} &= (1 - \frac{\alpha^2}{\gamma} - \frac{\beta^2}{\gamma})^{-1}(n + \delta t_i + \phi t_j) \\ e_i^{FB} &= \frac{\alpha k_i^{FB}}{\gamma} \end{aligned}$$

$$e_{ji}^{FB} = \frac{\beta k_i^{FB}}{\gamma}$$

The proof is in the Appendix. It is intuitively clear and straightforward to show that, in the first-best outcome, the capital allocations and managers' effort allocations increase in α and β (importance of effort), increase in δ and ϕ (importance of project quality), increase in t_1 and t_2 (information about project quality), and decrease in γ (managers' private cost of effort). If headquarters could observe the information t_i, and the managers' effort allocations, e_i and e_{ij}, then it should write a complete contract with each division manager specifying the capital allocations and effort choices described in Proposition 1. The salary should be set to levels satisfying the division managers' participation constraint.

4 Second-best outcome

We now solve for the headquarters' optimal mechanism, under the assumption that it cannot contract on the managers' private information, t_i, or the managers' effort allocations, e_i and e_{ij}. By the Revelation Principle we can, without loss of generality, restrict our attention to direct revelation mechanisms in which both division managers report their information truthfully. Thus, the headquarters' mechanism design problem can be stated as:

$$\max_{w_i,k_i,e_i,e_{ij}} \int_0^{\bar{t}} \int_0^{\bar{t}} \int_{\epsilon_1} \int_{\epsilon_2} [V_1 + V_2 - w_1 - w_2] dG_2 dG_1 dF_1 dF_2$$

such that

(i) $\{e_i, e_{ij}\} \in \arg\max E_{\{\epsilon_1,\epsilon_2\}} w_i(t_1, t_2, V_1, V_2) - 0.5\gamma(e_i^2 + e_{ij}^2),$ (IC1)

(ii) $t_i \in \arg\max E_{\{t_j,\epsilon_1,\epsilon_2\}} U_i(t_i, \hat{t}_i),$ (IC2)

(iii) $\forall\{t_1, t_2\}, \quad E_{\{t_j,\epsilon_1,\epsilon_2\}} U_i(t_i, t_i) \geq \bar{U},$ (IR)

(iv) $\forall\{t_1, t_2\}, \quad k_i, e_i, e_{ij} \geq 0,$ (NN)

where G_i is the distribution of ϵ_i; $E_{\{\epsilon_1,\epsilon_2\}}$ denotes expectations over the random variables $\{\epsilon_1, \epsilon_2\}$; and $U_i(t_i, \hat{t}_i)$ is the utility of manager i who reports $\hat{t}_i$, has true type t_i, and assumes that the other manager is reporting her true type. Specifically, $U_i(t_i, \hat{t}_i) \equiv w_i(\hat{t}_i, t_j, V_1, V_2) - 0.5\gamma(e_i^2 + e_{ij}^2)$, where $V_i = V_i(t_1, t_2, k_i(\hat{t}_i, t_j), e_i, e_{ji}, \epsilon_i)$ and the effort allocations, e_i and e_{ji}, are ex post optimal for the division managers.

The first incentive compatibility constraint (IC1) requires that the division managers' effort allocations are ex post optimal. The second incentive compatibility constraint (IC2) requires that it is optimal for division managers to report truthfully given that the other division manager will also report truthfully. The constraint (IR) is the standard interim individual rationality constraint, requiring that for each division manager type t_i, her expected equilibrium payoff should be at least as large as her outside reservation utility, $\bar{U}$. The last constraint (NN) requires that capital allocations and effort allocations are non-negative.

Proposition 2. *The headquarters' maximum expected payoff is*

$$\begin{aligned} E\Pi &= 0.5E_{\{t_1,t_2\}}\left[\frac{(n+\delta t_1+\phi t_2-\delta\mu_1-\phi\mu_2)^2}{1-\alpha^2/\gamma-\beta^2/\gamma}+\frac{(n+\delta t_2+\phi t_1-\delta\mu_2-\phi\mu_1)^2}{1-\alpha^2/\gamma-\beta^2/\gamma}\right. \\ &\quad \left.+\gamma(\mu_1^2+\mu_2^2)(\frac{\delta^2}{\alpha^2}+\frac{\phi^2}{\beta^2})\right]-2\bar{U}. \end{aligned}$$

The optimal mechanism can be implemented in dominant strategies by the following capital allocation policy and linear compensation scheme:

$$\begin{aligned} k_i &= (1-\frac{\alpha^2}{\gamma}-\frac{\beta^2}{\gamma})^{-1}(n+\delta t_i+\phi t_j-\delta\mu_i-\phi\mu_j), \\ w_i &= a_i+b_iV_i+b_{ij}V_j \end{aligned}$$

where

$$\begin{aligned} b_i &= 1-\frac{\delta\gamma\mu_i}{\alpha^2k_i}, \qquad b_{ij}=1-\frac{\phi\gamma\mu_i}{\beta^2k_j}, \\ a_i &= \bar{U}+\int_0^{t_i}(\delta b_ik_i+\phi b_{ij}k_j)ds+0.5\gamma(e_i^2+e_{ij}^2) \\ &\quad -b_i[nk_i+(\alpha e_i+\beta e_{ji})k_i+(\delta t_i+\phi t_j)k_i-0.5k_i^2] \\ &\quad -b_{ij}[nk_j+(\alpha e_j+\beta e_{ij})k_j+(\delta t_j+\phi t_i)k_j-0.5k_j^2]. \end{aligned}$$

The managerial effort allocations implemented are:

$$e_i = \frac{\alpha k_i}{\gamma}-\frac{\delta\mu_i}{\alpha}, \qquad e_{ij}=\frac{\beta k_j}{\gamma}-\frac{\phi\mu_i}{\beta}.$$

The proof is in the Appendix.[5]

The optimal mechanism can be implemented by a linear managerial compensation contract consisting of salaries, a_i, shares of own-division cash flows, b_i, and shares of

[5]Our proof follows closely the arguments of Laffont and Tirole (1986, 1993), although our model is more general (includes multiple agents and capital budgeting). Laffont and Tirole (1993, pp. 68-73 and pp. 171-172) provides sufficient conditions for the optimality of linear contracts in a model of the optimal regulation of a monopoly. See also McAfee and MacMillan (1987) and Holmstrom and Milgrom (1987) for the description of other settings in which the linear contract is optimal.

other-division cash flows, b_{ij}; and capital allocation schedules, k_i. Moreover, since the optimal mechanism is implementable in dominant strategies, it is robust to each manager's beliefs about the other manager's actions. Finally, it is interesting to note that the optimal contract is unaffected by the noise terms, ϵ_i. The reason is that the contract trades off the benefit of providing incentives for managers to provide effort against the costs of eliciting truthful reporting. In our model, the additive noise term does not affect either (i) the fundamental tradeoff between moral hazard and asymmetric information (e.g., if the noise term $\epsilon_1 \equiv 0$ and e_{21} and t_2 are fixed then observing V_1 will allow the headquarters to infer $\alpha e_1 + \delta t_1$ but not e_1 or t_1 separately) or (ii) the cost of providing incentives when all agents are risk-neutral. In contrast, noise terms do affect the fundamental tradeoff between providing incentives and risk-sharing in models with moral hazard and risk-averse agents and thus affect the optimal contract in these settings (e.g., the informativeness principle). We include the noise term because it shows that our results hold more generally (deterministic cash flow is a special case) and it is natural to assume that cash flows are subject to measurement errors and other random shocks.[6]

The following corollary illustrates some important features of this mechanism.

Corollary 1. Monotonicity and comparative statics

(i) The mechanism $\{a_i(t_1,t_2), b_i(t_1,t_2), b_{ij}(t_1,t_2), k_i(t_1,t_2)\}$ *and managers' effort allocations,* $e_i(t_1,t_2)$ *and* $e_{ij}(t_1,t_2)$*, are continuous and monotonic over the whole domain* $[0,\bar{t}]X[0,\bar{t}]$*. The profit-sharing rules,* $b_i(t_1,t_2)$ *and* $b_{ij}(t_1,t_2)$*, the capital allocations,* $k_i(t_1,t_2)$*, and the managers' effort allocations,* $e_i(t_1,t_2)$ *and* $e_{ij}(t_1,t_2)$ *are non-decreasing in each argument. The salary* $a_i(t_1,t_2)$ *is non-increasing in* t_i*.*

(ii) The capital allocations, k_i*, increase with* α*,* β*,* t_1*,* t_2*, and* n*; decrease with* γ*; and increase with* δ *(*ϕ*) when* t_i *(*t_j*) is high and decrease when* t_i *(*t_j*) is low.*

[6] A more technical proof of the robustness of the linear contract to additive noise is given in Laffont and Tirole (1993, pp. 72-73) and the references therein.

(iii) The performance-pay b_i (b_{ij}) increases with α, β, t_1, t_2, and n and decreases with γ; b_i decreases with δ and increases (decreases) with ϕ when t_j is high (low); and b_{ij} decreases with ϕ and increases (decreases) with δ when t_j is high (low).

(iv) The managers' effort allocations e_i (e_{ij}) increase with α, β, t_1, t_2, and n, and decrease with γ; e_i increases in δ (ϕ) when t_i (t_j) is high and decreases in δ (ϕ) when t_i (t_j) is low; and e_{ij} increases in δ (ϕ) when t_j (t_i) is high and decreases in δ (ϕ) when t_j (t_i) is low.

The proof is in the Appendix. The optimal mechanism allocates more capital and greater profit-sharing to the managers when they report a higher project quality. More capital and profit-sharing are also offered to manager i when manager j reports a higher project quality. To induce truthtelling, the salary component of the compensation scheme is lower for higher reported project qualities.

The comparative statics for the optimal mechanism can be understood by considering the benefits and costs of providing capital and profit-sharing. On one hand, the marginal benefit of providing capital to project i increases in the project quality ($\delta t_i + \phi t_j$) and in the importance of managerial effort ($\alpha e_i + \beta e_{ji}$). Moreover, because managerial effort allocations to project i are given by $e_i = \frac{\alpha b_i k_i}{\gamma}$ and $e_{ji} = \frac{\beta b_{ji} k_i}{\gamma}$, the marginal benefit of providing profit-sharing increases in α, β, and k_i, and decreases in γ. On the other hand, the marginal cost of providing capital and managerial incentives depends on the cost of maintaining incentive compatibility. Specifically, we show in the proof of Proposition 2 in the Appendix (see Equation (8)) that incentive compatibility for manager i requires:

$$EU_i(t_i) = \bar{U} + \int_0^{t_i} \int_0^{\bar{t}} (\delta b_i k_i + \phi b_{ij} k_j) dF_j ds_i. \tag{1}$$

This states that to induce truthtelling, manager i of type t_i must receive expected utility of $EU_i(t_i)$ as in Equation (1). The term $\int_0^{t_i} \int_0^{\bar{t}} (\delta b_i k_i + \phi b_{ij} k_j) dF_j ds_i$ represents the type-t_i manager's *information rents*. These information rents are increasing in t_i. Truthtelling can be achieved with any contract yielding the manager $EU_i(t_i)$; however,

the marginal cost of providing managerial incentives, b_i and b_{ij}, and capital, k_i and k_j, spills over into the information rents that must be paid to all managers with higher quality projects. This increases the marginal cost of offering profit-sharing and capital allocations (especially for low-t_i managers). Moreover, these costs are increasing when asymmetric information is more important (high δ and ϕ).

Thus, an increase in quality, t_i, increases (decreases) the marginal benefit (cost) of capital and profit-sharing; therefore, capital, effort, and profit-sharing will increase. An increase in the impact of the managers' effort, α and β, increases the marginal benefit of capital and profit-sharing; therefore, capital, effort, and profit-sharing will increase. On the other hand, an increase in the cost of effort, γ, reduces the marginal benefit of profit-sharing and reduces the marginal product of capital (since effort and capital are complementary); therefore, capital, effort, and profit-sharing decrease. Finally, an increase in the importance of private information, δ and ϕ, increases the marginal benefit of capital; however, it also raises the marginal cost of providing capital and managerial incentives, especially for low-quality projects. Thus, for high-project qualities the former effect dominates and capital allocations increase whereas for low-project qualities the latter effect dominates and capital allocations decrease.

The information rents specification in Equation (1) highlights the importance of both asymmetric information and moral hazard in our model. If there is no asymmetric information ($\delta = \phi = 0$) headquarters can choose full profit-sharing and the first-best level of capital without increasing managerial information rents. Moreover, if there is no moral hazard ($\alpha = \beta = 0$) headquarters can implement first-best by choosing profit-shares equal to zero and setting capital to their first-best levels without increasing managerial information rents.

5 Interpretation and Empirical Implications

For what follows it will be useful to interpret the key parameters in our model: α, β, δ, ϕ, and γ. The parameter α represents both the importance of unverifiable managerial resources (heretofore labelled "effort") for the manager's own project cash flows and the degree of complementarity between these resources and capital, thus we expect α to be greater, for example, when managers have firm-specific human capital. The parameter β represents the importance of managerial resources for the cash flows of other projects in the firm, thus we expect β to be smaller in more diversified firms. The parameters δ (ϕ) represent the importance of the division managers' information for her own (other) project's cash flows. We expect δ to be greater when asymmetric information between headquarters and the managers is more severe (e.g., R&D on a new drug or technology), and ϕ to be greater when there are common factors affecting project cash flows (e.g., market demand in adjacent sales regions, industry trends). Finally, the parameter γ represents the managers' effort cost; however, an alternative and more empirically useful interpretation of $1/\gamma$ (more generally, the inverse of the second derivative of the cost function $1/C''(e)$) is the responsiveness of unverifiable actions to an increase in incentives (Milgrom and Roberts, 1992). In this interpretation, γ is higher when the managers have less discretion over job tasks.

5.1 Investment in Capital and Managerial Resources

The following implication demonstrates that in the second-best mechanism headquarters provide too little capital and managers provide too little effort relative to the first-best allocation.

Implication 1:

(i) The capital allocation is lower than the first-best solution. For each division, the underinvestment in capital is most severe for low-quality projects, becomes less

severe when either manager's report increases, and vanishes for the highest possible quality projects. For a given project quality, the underinvestment decreases in γ, and increases in α, β, δ and ϕ.

(ii) Managerial effort is lower than the first-best solution. For each division, the under-utilization of effort is most severe for low-quality projects, becomes less severe when either manager's report increases, and vanishes for the highest possible quality projects. For a given project quality, the under-utilization of effort in the managers' own project decreases in γ, increases in β, δ and ϕ, and is ambiguous in α; and the under-utilization of effort in the other project decreases in γ, increases in α, δ and ϕ, and is ambiguous in β.

The proof is in the Appendix. As we demonstrated above, incentive compatibility (in the second-best mechanism) requires that when the firm increases its capital allocation to any manager it must also increase the information rents to *all* higher-type managers. This makes the marginal cost of providing capital high, especially for low-quality projects. Consequently, there is underinvestment of capital in the optimal mechanism (relative to the first-best) and the underinvestment problem is more severe for lower quality projects. By allocating less capital and less profit-sharing to low-quality projects, headquarters reduces the incentive costs for high-quality projects where it matters most. From Equation (1) we see that the marginal cost of providing capital is more severe when δ and ϕ are larger, thus the underinvestment problem is more severe when there is more asymmetric information. Moreover, the marginal cost of providing capital is higher when profit-sharing is higher and the latter increases in α and β and decreases in γ. Thus, the underinvestment problem is more severe when managerial effort is more important and when the managers have more discretion. Finally, managerial effort allocations are increasing in the managers' profit share and the capital allocated to each project. However, since the second-best mechanism provides too little capital and less-than-full profit-sharing, managers also provide too little effort relative to the first-best.

Our underinvestment result contrasts with Harris and Raviv (1996, 1998) and Harris, Kreibel, and Raviv (1982), who found that overinvestment occurs for the lowest quality projects while underinvestment occurs for the highest quality projects.[7] Furthermore, our result is consistent with the evidence that firms adopt higher hurdle rates of return than predicted by standard finance theory (Poterba and Summers, 1992). Several other theories are consistent with this prediction including, for example, real options models in which there is value to waiting to invest in a project. However, our model makes further predictions which distinguish our theory from the others. For example, we predict that the underinvestment problem in a given division is more severe when (i) asymmetric information between headquarters and the division managers is greater (larger δ and ϕ), (ii) the division is more human-capital intensive (larger α), and (iii) the other divisions in the firm are performing poorly. For example, we predict that the difference between observed hurdle rates and predicted hurdle rates will be large for companies with high R&D expenses because project managers/scientists in these firms typically have considerable private information and firm-specific human capital.

Moreover, the underinvestment problem is associated with the division managers receiving less performance-based pay. Consistent with this prediction, Palia (2000) finds empirical evidence that underinvestment in relatively strong divisions of diversified firms is less severe when the division managers have a higher percentage of the firm's equity in options and shares. Interestingly, in our model division managers receive greater performance-based pay *because* they manage higher quality projects, not necessarily that greater performance-based pay *causes* firm value to increase. Thus, we argue that one must be careful interpreting results such as those found in Palia (2000) since the theory does not suggest an obvious causal link between performance-pay and eventual performance.

[7] We demonstrate in an earlier paper (Bernardo, Cai, and Luo, 2001) that this difference results from their assumption of exogenous compensation contracts.

Implication 2:

(i) The capital allocated to each project is more sensitive to the other manager's information in the second-best solution than in the first-best solution.

(ii) The difference in sensitivity to the other manager's information is stronger for higher α, β, and ϕ; and lower γ.

The proof is in the Appendix. The intuition for result (i) is that positive information about t_2 has two effects on the capital allocated to project 1: first, high t_2 also implies that project 1 has better prospects so more capital is allocated to it, and second, since manager 2 is a higher-type the firm optimally provides higher-powered incentives for her to provide effort to *both* projects. The extra effort given by manager 2 in project 1 increases the marginal product of capital in project 1. The first effect is present in both the first-best and second-best mechanisms whereas the second effect provides a motivation for increasing the capital allocation to project 1 not present in the first-best solution. Consequently, the optimal amount of capital to allocate to project 1 is more sensitive to the quality (cash flows) of project 2 in the second-best mechanism.

The sensitivity of investment in one division to the cash flows and investment opportunities in other divisions in the firm has been studied extensively (see, e.g, Lamont, 1997; Shin and Stulz, 1998; Rajan, Servaes, and Zingales, 2000; Chevalier, 2000; Whited, 2001; Billett and Mauer, 2003). For example, Shin and Stulz argue that we should expect to see investment in one division falling as the investment opportunities in other divisions improve if internal capital markets are redirecting capital to its best use. They find, however, that investment-to-asset ratios in divisions of conglomerate firms are relatively insensitive to empirical proxies of the investment opportunities of other ostensibly unrelated divisions and interpret this to mean that internal capital markets are inefficient.[8] A key element of the Shin-Stulz argument is that the firm is capital constrained.

[8] Chevalier (2000), Whited (2001), and Villalonga (2003) argue against this interpretation because of

By contrast, in our model the firm does not face a capital constraint and, as a result, we argue that we should expect investment in one division to be insensitive to the investment opportunities of other divisions if these other divisions' investment opportunities are independent (just as Shin and Stulz observed). Moreover, we predict that the sensitivity of investment in one division to the investment opportunities in another division should be greater when divisions are more related (larger β and ϕ), managers have more discretion (smaller γ), and managers have more firm-specific human capital (larger α).

5.2 Managerial Compensation

For what follows, we re-write each manager's wage contract:

$$w_i = a_i + b_i V_i + b_{ij} V_j \equiv a_i + b_{ij}(V_i + V_j) + (b_i - b_{ij}) V_i.$$

Thus, we can interpret the coefficient b_{ij} as the division managers' firm-level performance-based pay and $(b_i - b_{ij})$ as the division managers' division-level performance-based pay when $(b_i - b_{ij}) > 0$. The following implication follows immediately from Corollary 1:

Implication 3: Division managers receive greater firm-level performance pay when (i) project quality is higher (higher t_i), (ii) managerial effort is more valuable in their own division (larger α), (iii) managerial effort is more valuable in other divisions (larger β), and (iv) division managers have more discretion (smaller γ).

The following implication considers the use of division-level performance pay both in absolute terms and in relation to firm-level performance pay (i.e., $\frac{b_i - b_{ij}}{b_{ij}}$).

Implication 4: Holding division size constant ($k_1 = k_2$), division managers receive more firm-level performance pay in both absolute terms and relative to division-level

selection biases and measurement errors inherent in the Shin-Stulz empirical strategy which (i) defines a division to be unrelated if it is in a different two-digit SIC code and (ii) uses the average Tobin's q of (traded) stand-alone firms to measure the quality of investment opportunities in a (non-traded) division of a conglomerate firm.

performance pay when (i) managerial effort is more valuable in other divisions (larger β) and (ii) their private information is more important for her project's cash flow (larger δ).

The proof is in the Appendix. The intuition for part (i) is that when managerial effort is valuable in many divisions the firm wants to provide incentives for the managers to give effort in many divisions by emphasizing firm-level performance pay. The intuition for part (ii) is that when δ is large, information rents are high to induce truthful reports on the division managers' own division thus the firm again de-emphasizes division-level performance pay. Our implication is consistent with the empirical work of Bushman, et al. (1995) who find that the use of firm-level performance pay is increasing when divisions (groups) are more related whereas unrelated division manager pay is more closely related to division performance. Keating (1997) also finds that the use of firm accounting metrics in division manager compensation is positively related to the impact the manager being evaluated has on other divisions in the firm.

Implication 5: Firm-level performance-based pay is positively correlated across divisions.

The proof is in the Appendix. The intuition for this result is as follows. If, for example, manager 1 reports a high t_1 then both projects have higher quality and the firm optimally allocates more capital to both divisions. Because effort and capital are complementary this increases the marginal benefit of providing firm-level performance pay to encourage managers to allocate effort to all divisions. Consistent with this result, Palia (2000) finds that firm-level performance pay (measured by the share of firm equity) is positively correlated across divisions.

5.3 Comparison of multi-division firm to single-division firm

We now compare capital allocations in single-division and multi-division firms. To do so, we assume that division 1 of our multi-division firm has a single-division counterpart which obtains some market-average effort (i.e., managerial resource) spillover $\underline{e}_{21}$ and can only contract on its own net cash flows ($b_{12} = 0$). Moreover, it does not observe a precise report $\hat{t}_2$ but observes some market signal of t_2, $\underline{t}_2$. As an example of the resource spillover, $\underline{e}_{21}$, consider a firm consisting of an auto assembly division and an auto engine division that splits into two single-division firms. Even though the two separate firms may not be able to co-ordinate and utilize the resource spillovers as well as the headquarters of the two-division firm can, they probably maintain close business relationships through market transactions and some of the spillovers should still be realized.

Under these assumptions, it can be shown that the optimal capital allocation in the second-best mechanism for the single-division firm is given by:

$$k_1^1 = \frac{\gamma}{\gamma - \alpha^2}(n + \delta(t_1 - \mu_1) + \phi\underline{t}_2 + \underline{e}_{21}).$$

We are interested in comparing investment policies in a multi-division firm to a single-division firm *holding project quality constant.* We implement this strategy by holding t_1 fixed and assuming the multi-division firm receives a report $\hat{t}_2 = \underline{t}_2$. Comparing investment in the single-division firm, k_1^1, to the investment in division 1 of the multi-division firm, k_1 (in Proposition 2), we get the following implication.

Implication 6: The multi-division firm invests more (less) in a division than a single-division firm when other divisions in the multi-division firm are performing well (poorly). On average, the multi-division firm invests more than the single-division firm when managerial effort is more valuable in other divisions (larger β).

The proof is in the Appendix. The first statement follows closely from our Implication 2. The intuition for the second statement is that each division in the multi-division firm

derives benefits from the other division manager's effort (and these benefits increase in β) but also pays greater information rents in order to get the other division manager to report truthfully. For large β, the effort benefits exceed the information rent costs.

6 Alternative Managerial Preferences

We now check the robustness of our results to alternative specifications for managerial preferences. In our model, we chose to consider the importance of asymmetric information and managerial moral hazard. Another well-accepted and reasonable conflict that might emerge between shareholders and management is managerial preference for capital. This may reflect enhanced reputation from controlling bigger projects, a preference for "empire-building," or greater perquisite consumption that comes from running a larger business. One reasonable specification is to assume that the managers derive utility from monetary rewards and from controlling large (high k), high quality (high t) projects so that managerial preferences are given by:

$$EU_i = Ew_i - 0.5\gamma(e_i^2 + e_{ij}^2) + \eta(\delta t_i + \phi t_j)k_i,$$

where $\eta \geq 0$ measures the degree of managerial preferences for capital. Our original model corresponds to the special case of $\eta = 0$.

With these modified managerial preferences and everything else being identical to our earlier model, it is straightforward to show that the first-best capital allocation is:

$$k_i^{FB} = (1 - \frac{\alpha^2}{\gamma} - \frac{\beta^2}{\gamma})^{-1}[n + (1+\eta)\delta t_i + (1+\eta)\phi t_j].$$

The first-best capital allocation increases in the managerial preference for capital, η, because the headquarters can internalize the managers' preference perfectly by reducing her salary (negative compensation may be needed).

With asymmetric information and moral hazard, it can be shown that maintaining incentive compatibility in the second-best mechanism requires:

$$EU_i(t_i) = \bar{U} + \int_0^{t_i} \int_0^{\bar{t}} (\delta(b_i + \eta)k_i + \phi b_{ij} k_j) dF_j ds_i.$$

Comparing this to Equation (1) we see that the marginal cost of providing capital increases with η. Applying the same method of solving for the optimal mechanism, the second-best capital allocation is given by:

$$k_i = (1 - \frac{\alpha^2}{\gamma} - \frac{\beta^2}{\gamma})^{-1}[n + (1+\eta)\delta(t_i - \mu_i) + (1+\eta)\phi t_j - \phi\mu_j].$$

Thus, underinvestment in the second-best solution is

$$k_i^{FB} - k_i = (1 - \frac{\alpha^2}{\gamma} - \frac{\beta^2}{\gamma})^{-1}[(1+\eta)\delta\mu_i + \phi\mu_j].$$

Clearly, $k_i^{FB} - k_i$ is increasing in η. Moreover, since

$$\frac{\partial k_i}{\partial t_j} - \frac{\partial k_i^{FB}}{\partial t_j} = -(1 - \frac{\alpha^2}{\gamma} - \frac{\beta^2}{\gamma})^{-1}\phi\frac{d\mu_j}{dt_j} > 0,$$

from Implication 2 we can see that the (over-)sensitivity of investment in one division to the other manager's information (relative to the first best) is unchanged with preferences for capital. Summarizing we have the following result:

Proposition 3. *Managerial preference for capital exacerbates the underinvestment problem but has no effect on the cross-division (over-)sensitivity of investment to information (relative to the first- best solution).*

The discussion above indicates that managerial preference for capital has only secondary effects on capital budgeting, and its effects operate through its impact on the optimal incentive scheme to control for asymmetric information and moral hazard.[9] This

[9] A similar conclusion holds when managers have preferences for managing a *relatively* large and high-quality division.

point can be made clearer if we modify the managers' preferences for capital and ignore the project "quality":

$$EU_i = Ew_i - 0.5\gamma(e_i^2 + e_{ij}^2) + \eta k_i.$$

In this case, managers have no control over the "control benefits" (ηk_i), thus the costs of maintaining incentive compatibility are identical to our main model in which managers have no preferences for capital. With this specification, headquarters can internalize managerial preferences for capital perfectly even in the second-best solution and the underinvestment problem will not be exacerbated.

7 Conclusions

Firms make capital budgeting decisions based on the reports of project (division) managers. In many cases, these reports can neither be independently verified before significant capital investments are made nor contracted upon ex post. Moreover, the eventual success of these projects may also depend on the (unverifiable) energy and resources ("effort") devoted to it by managers throughout the firm. In such cases, the firm must put in place explicit incentives for managers to provide truthful reports and to utilize and share resources efficiently.

In this paper, we examined the extent to which capital budgeting and managerial compensation mechanisms can mitigate these information and incentive problems in a multi-division firm. In the optimal mechanism, firms underinvest in capital and managers underinvest resources in all divisions. Moreover, this underinvestment problem in one division is more severe if the division has poor prospects, the other divisions in the firm have poor prospects, asymmetric information between headquarters and all division managers is more severe, and managers have a preference for capital (i.e., managing larger divisions).

We believe our model is best suited to cases where division managers request capital

infrequently (e.g., R&D on a new drug). In such cases, compensation schemes such as shares in the firm (with restrictions on selling) or stock options (with a long vesting period) are likely to provide powerful incentives for managers to report truthfully their private information and allocate efficiently the resources under their control. However, our model is not well-suited to cases in which division managers request capital frequently. In such cases, reputation effects provide powerful incentives because the firm can always deny managers future capital requests if their earlier analyses were off the mark. In future research, we would like to determine the extent to which such reputation effects substitute for explicit managerial incentives in the capital budgeting process.

Appendix

Proof of Proposition 1. In the first-best, headquarters maximizes total expected surplus:

$$
\begin{aligned}
& nk_1 + (\alpha e_1 + \beta e_{21})k_1 + (\delta t_1 + \phi t_2)k_1 - 0.5k_1^2 \\
& + nk_2 + (\alpha e_2 + \beta e_{12})k_2 + (\delta t_2 + \phi t_1)k_2 - 0.5k_2^2 \\
& - 0.5\gamma(e_1^2 + e_{12}^2) - 0.5\gamma(e_2^2 + e_{21}^2).
\end{aligned}
$$

The first-order (necessary) conditions are (taking derivatives with respect to k_i, e_i, and e_{ij}, respectively):

$$
\begin{aligned}
0 &= n + \alpha e_i + \beta e_{ji} + \delta t_i + \phi t_j - k_i, \\
0 &= \alpha k_i - \gamma e_i, \\
0 &= \beta k_j - \gamma e_{ij},
\end{aligned}
$$

which yields the result. The second-order condition for the total surplus-maximization problem requires that the Hessian matrix of second derivatives of the objective function is negative semi-definite which is easy to verify under Assumption (A1). Q.E.D.

Proof of Proposition 2. Our proof strategy for finding the optimal Bayesian-Nash mechanism follows Laffont and Tirole (1986) and McAfee and McMillan (1987). The proof consists of two steps. In Step 1, we relax the IC constraints in such a way that it is possible to derive the optimal capital allocation and compensation mechanism and hence the headquarters' expected payoff. Since this relaxed program has fewer constraints than the original program, the headquarters' expected payoff in this relaxed program provides an upper bound of the value attainable in the original program. In Step 2, we consider

a narrower class of mechanisms than in the original program. Specifically, we focus on mechanisms with linear compensation schemes. Clearly the headquarters' expected payoff in this class provides a lower bound of the value attainable in the original program. We then demonstrate that this lower bound is identical to the upper bound derived in Step 1 and thus the headquarters' optimal mechanism can be implemented with the linear compensation scheme.

Step 1: Let $e_i^*(t_1, t_2)$ and $e_{ij}^*(t_1, t_2)$ represent the headquarters' desired effort allocations for truthful reports $\{t_1, t_2\}$. Suppose that headquarters could observe the actual values of $\alpha e_i + \delta t_i$ and $\beta e_{ij} + \phi t_i$ for both division managers and suppose manager j reports $\hat{t}_j$. If division manager i reports $\hat{t}_i$, headquarters wishes to see $\alpha e_i^*(\hat{t}_1, \hat{t}_2) + \delta \hat{t}_i$ and $\beta e_{ij}^*(\hat{t}_1, \hat{t}_2) + \phi \hat{t}_i$; otherwise, it knows that one of the division managers has lied about her information or has not followed the recommended effort allocation policies, and hence will punish (with arbitrary severity) the managers for their deviations. Thus, conditional on manager j reporting $\hat{t}_j$, if division manager i reports $\hat{t}_i$, she will have to choose $\hat{e}_i$ and $\hat{e}_{ij}$ to be consistent with her report; that is, $\alpha \hat{e}_i + \delta t_i = \alpha e_i^*(\hat{t}_1, \hat{t}_2) + \delta \hat{t}_i$ and $\beta \hat{e}_{ij} + \phi t_i = \beta e_{ij}^*(\hat{t}_1, \hat{t}_2) + \phi \hat{t}_i$, which yields:

$$\begin{aligned} \hat{e}_i &= e_i^*(\hat{t}_1, \hat{t}_2) + \delta(\hat{t}_i - t_i)/\alpha, \\ \hat{e}_{ij} &= e_{ij}^*(\hat{t}_1, \hat{t}_2) + \phi(\hat{t}_i - t_i)/\beta. \end{aligned} \tag{2}$$

Note that by Equation (2), if the division managers report truthfully $\{\hat{t}_1 = t_1, \hat{t}_2 = t_2\}$, they must follow headquarters' recommended effort allocation policies. Thus the (IC1) constraint is completely relaxed.

Denote the headquarters' original problem as (P1) and now consider the following problem (P2):

$$\max_{w_i, k_i, e_i, e_{ij}} \int_0^{\bar{t}} \int_0^{\bar{t}} \int_{\epsilon_1} \int_{\epsilon_2} [V_1 + V_2 - w_1 - w_2] dG_2 dG_1 dF_1 dF_2$$

such that

$$\text{(i)} \;\; t_i \in \arg\max E_{\{t_j,\epsilon_1,\epsilon_2\}} U_i(t_i, \hat{t}_i), \tag{IC2}$$

$$\text{(ii)} \;\; \forall\{t_1, t_2\}, \quad E_{\{t_j,\epsilon_1,\epsilon_2\}} U_i(t_i, t_i) \geq \bar{U} \tag{IR}$$

$$\text{(iii)} \;\; \forall\{t_1, t_2\}, \quad k_i, e_i, e_{ij} \geq 0 \tag{NN}$$

where $U_i(t_i, \hat{t}_i) = w_i(\hat{t}_i, t_j, V_1, V_2) - 0.5\gamma(e_i^2 + e_{ij}^2)$ is the utility to manager i who reports $\hat{t}_i$, has true type t_i, and assumes that the other manager is reporting her true type; $V_i = V_i(t_1, t_2, k_i(\hat{t}_i, t_j), e_i, e_{ji}, \epsilon_i)$; $e_i = \hat{e}_i$ and $e_{ji} = e_{ji}(\hat{t}_i, t_j)$.

Program (P2) replaces the (IC1) constraint of Program (P1) with Equation (2) and incorporates it in the (IC2) constraint, so (P2) is a relaxed program of (P1). Ignoring the (NN) constraint for the moment, we now solve (P2).

Given Equation (2), for any true types (t_1, t_2) and any reports $\{\hat{t}_1, \hat{t}_2\}$, the values of cash flows V_1 and V_2 can be expressed as:

$$\begin{aligned}
V_1(\hat{t}_1, \hat{t}_2, \epsilon_1) &= nk_1(\hat{t}_1, \hat{t}_2) + (\alpha e_1^*(\hat{t}_1, \hat{t}_2) + \beta e_{21}^*(\hat{t}_1, \hat{t}_2))k_1(\hat{t}_1, \hat{t}_2) \\
&\quad + (\delta \hat{t}_1 + \phi \hat{t}_2)k_1(\hat{t}_1, \hat{t}_2) - 0.5k_1(\hat{t}_1, \hat{t}_2)^2 + \epsilon_1 \\
V_2(\hat{t}_1, \hat{t}_2, \epsilon_2) &= nk_2(\hat{t}_1, \hat{t}_2) + (\alpha e_2^*(\hat{t}_1, \hat{t}_2) + \beta e_{12}^*(\hat{t}_1, \hat{t}_2))k_2(\hat{t}_1, \hat{t}_2) \\
&\quad + (\delta \hat{t}_2 + \phi \hat{t}_1)k_2(\hat{t}_1, \hat{t}_2) - 0.5k_2(\hat{t}_1, \hat{t}_2)^2 + \epsilon_2.
\end{aligned}$$

Thus the cash flows are independent of the true types (t_1, t_2), which is critical for what follows. Note also that the compensations may be written as $w_i(\hat{t}_1, \hat{t}_2, V_1(\hat{t}_1, \hat{t}_2, \epsilon_1), V_2(\hat{t}_1, \hat{t}_2, \epsilon_2))$ so we can write

$$E_{\{\epsilon_1,\epsilon_2\}}[w_i(\hat{t}_1, \hat{t}_2)] = E_{\{\epsilon_1,\epsilon_2\}}[w_i(\hat{t}_1, \hat{t}_2, V_1(\hat{t}_1, \hat{t}_2, \epsilon_1), V_2(\hat{t}_1, \hat{t}_2, \epsilon_2))].$$

The (IC2) constraint in (P2) now states that conditional on $\hat{t}_j = t_j$,

$$
\begin{aligned}
t_i &\in \arg\max EU_i(t_i, \hat{t}_i) \\
&= E_{t_j}[E_{\{\epsilon_1,\epsilon_2\}}[w_1(\hat{t}_i, t_j)]] - 0.5\gamma E_{t_j}\Big[e_i^*(\hat{t}_i, t_j) + \frac{\delta(\hat{t}_i - t_i)}{\alpha}\Big]^2 - 0.5\gamma E_{t_j}\Big[e_{ij}^*(\hat{t}_i, t_j) + \frac{\phi(\hat{t}_i - t_i)}{\beta}\Big]^2.
\end{aligned}
$$

By the Envelope Theorem, the (IC2) condition implies

$$
\begin{aligned}
\frac{dEU_i(t_i, t_i)}{dt_i} &= \frac{\partial EU_i(t_i, \hat{t}_i)}{\partial t_i}|_{\hat{t}_i = t_i} + \frac{\partial EU_i(t_i, \hat{t}_i)}{\partial \hat{t}_i}|_{\hat{t}_i = t_i} \\
&= \frac{\partial EU_i(t_i, \hat{t}_i)}{\partial t_i}|_{\hat{t}_i = t_i} = \frac{\delta\gamma E_{t_j}[e_i^*(t_i, t_j)]}{\alpha} + \frac{\phi\gamma E_{t_j}[e_{ij}^*(t_i, t_j)]}{\beta}. \qquad (3)
\end{aligned}
$$

Integration yields

$$
EU_i(t_i) = U_i(0) + \int_0^{\bar{t}} \int_0^{t_i} (\frac{\delta\gamma e_i^*(s, t_j)}{\alpha} + \frac{\phi\gamma e_{ij}^*(s, t_j)}{\beta}) ds dF_j.
$$

By the (IR) constraint, it must be that $U_i(0) = \bar{U}$. Taking the expectation with respect to t_i yields

$$
\begin{aligned}
EU_i &= \bar{U} + \int_0^{\bar{t}} \int_0^{\bar{t}} \int_0^{t_i} (\frac{\delta\gamma e_i^*(s, t_j)}{\alpha} + \frac{\phi\gamma e_{ij}^*(s, t_j)}{\beta}) ds dF_j dF_i \\
&= \bar{U} + \int_0^{\bar{t}} \int_0^{\bar{t}} (\frac{\delta\gamma e_i^*(t_i, t_j)}{\alpha} + \frac{\phi\gamma e_{ij}^*(t_i, t_j)}{\beta}) \mu_i dF_j dF_i
\end{aligned}
$$

where $\mu_i = (1 - F_i)/f_i$.

Substituting the expressions for wages, $Ew_i = EU_i + 0.5\gamma E[e_i^{*2} + e_{ij}^{*2}]$, into the expected payoff yields

$$
\begin{aligned}
E\Pi &= \int_0^{\bar{t}} \int_0^{\bar{t}} \Big\{ nk_1 + (\alpha e_1^* + \beta e_{21}^*)k_1 + (\delta t_1 + \phi t_2)k_1 - 0.5k_1^2 - U_1 - 0.5\gamma(e_1^{*2} + e_{12}^{*2}) \\
&\quad + nk_2 + (\alpha e_2^* + \beta e_{12}^*)k_2 + (\delta t_2 + \phi t_1)k_2 - 0.5k_2^2 - U_2 - 0.5\gamma(e_2^{*2} + e_{21}^{*2}) \Big\} dF_1 dF_2 \\
&= \int_0^{\bar{t}} \int_0^{\bar{t}} \Big\{ nk_1 + (\alpha e_1^* + \beta e_{21}^*)k_1 + (\delta t_1 + \phi t_2)k_1 - 0.5k_1^2 + nk_2 + (\alpha e_2^* + \beta e_{12}^*)k_2 \\
&\quad + (\delta t_2 + \phi t_1)k_2 - 0.5k_2^2 - 0.5\gamma(e_1^{*2} + e_{12}^{*2}) - 0.5\gamma(e_2^{*2} + e_{21}^{*2}) \\
&\quad - (\frac{\delta\gamma e_1^*}{\alpha} + \frac{\phi\gamma e_{12}^*}{\beta})\mu_1 - (\frac{\delta\gamma e_2^*}{\alpha} + \frac{\phi\gamma e_{21}^*}{\beta})\mu_2 \Big\} dF_1 dF_2 - 2\bar{U}. \qquad (4)
\end{aligned}
$$

Point-wise differentiation of the integrand gives the following first order conditions

$$\begin{aligned} 0 &= n + \alpha e_i^* + \beta e_{ji}^* + \delta t_i + \phi t_j - k_i, \\ 0 &= \alpha k_i - \gamma e_i^* - \frac{\delta\gamma}{\alpha}\mu_i, \\ 0 &= \beta k_i - \gamma e_{ji}^* - \frac{\phi\gamma}{\beta}\mu_j. \end{aligned} \tag{5}$$

Thus,

$$\begin{aligned} k_i &= (1 - \frac{\alpha^2}{\gamma} - \frac{\beta^2}{\gamma})^{-1}(n + \delta t_i + \phi t_j - \delta\mu_i - \phi\mu_j), \\ e_i^* &= \gamma^{-1}(\alpha k_i - \frac{\delta\gamma}{\alpha}\mu_i), \\ e_{ij}^* &= \gamma^{-1}(\beta k_j - \frac{\phi\gamma}{\beta}\mu_i). \end{aligned} \tag{6}$$

The s.o.c. holds under Assumption (A1). The maximized integrand in Equation (4) is given by

$$\begin{aligned} M_{11,11} &= 0.5\left[\frac{(n + \delta t_1 + \phi t_2 - \delta\mu_1 - \phi\mu_2)^2}{1 - \alpha^2/\gamma - \beta^2/\gamma} + \frac{(n + \delta t_2 + \phi t_1 - \delta\mu_2 - \phi\mu_1)^2}{1 - \alpha^2/\gamma - \beta^2/\gamma}\right. \\ &\quad \left. + \gamma(\mu_1^2 + \mu_2^2)(\frac{\delta^2}{\alpha^2} + \frac{\phi^2}{\beta^2})\right]. \end{aligned}$$

It is easy to check that $\{k_i, e_i^*, e_{ij}^*\}$ are all increasing in (t_i, t_j). By standard arguments (Mirrlees, 1971), the (IC2) constraint is satisfied. Plugging Equation (6) into $E\Pi$, the headquarters' expected payoff can now be expressed as

$$\begin{aligned} E\Pi^{P2} &= 0.5E_{\{t_1,t_2\}}\left[\frac{(n + \delta t_1 + \phi t_2 - \delta\mu_1 - \phi\mu_2)^2}{1 - \alpha^2/\gamma - \beta^2/\gamma}\right. \\ &\quad \left. + \frac{(n + \delta t_2 + \phi t_1 - \delta\mu_2 - \phi\mu_1)^2}{1 - \alpha^2/\gamma - \beta^2/\gamma} + \gamma(\mu_1^2 + \mu_2^2)(\frac{\delta^2}{\alpha^2} + \frac{\phi^2}{\beta^2})\right] - 2\bar{U}. \end{aligned} \tag{7}$$

Under Assumption (A2), it can be checked that the solution $\{k_i, e_i^*, e_{ij}^*\}$ given by Equation (6) is strictly positive for all (t_1, t_2). This means the (NN) constraint is satisfied and non-binding.

To prove that Equation (6) gives an optimal mechanism for Program (P2), we need to show that it dominates mechanisms under which the (NN) constraint is at least partly binding. This is necessary because a binding (NN) constraint may relax the (IC) constraints, as the set of possible deviations for the managers is reduced. We can immediately rule out the boundary case $k_i = 0$ since capital is critical to project cash flows.

Consider an arbitrary mechanism such that $e_1 = 0$ and all other e's are strictly positive. For this mechanism, let us completely relax the (IC1) and (IC2) constraints for manager 1. That is, he must report truthfully t_1 and follow the recommended e_{12}^*. Clearly no information rents are paid to manager 1. Manager 2's incentive constraints can be handled as before, which leads to maximizing

$$\begin{aligned} E\Pi &= \int_0^{\bar{t}}\int_0^{\bar{t}}\Big\{nk_1 + \beta e_{21}k_1 + (\delta t_1 + \phi t_2)k_1 - 0.5k_1^2 + nk_2 + (\alpha e_2 + \beta e_{12})k_2 \\ &\quad + (\delta t_2 + \phi t_1)k_2 - 0.5k_2^2 - 0.5\gamma e_{12}^2 - 0.5\gamma(e_2^2 + e_{21}^2) - (\frac{\delta\gamma e_2}{\alpha} + \frac{\phi\gamma e_{21}}{\beta})\mu_2\Big\}dF_1dF_2 - 2\bar{U}. \end{aligned}$$

It can be easily checked that the maximized integrand in this case is

$$M_{01,11} = 0.5\left[\frac{(n+\delta t_1+\phi t_2-\phi\mu_2)^2}{1-\beta^2/\gamma} + \frac{(n+\delta t_2+\phi t_1-\delta\mu_2)^2}{1-\alpha^2/\gamma-\beta^2/\gamma} + \gamma\mu_2^2(\frac{\delta^2}{\alpha^2}+\frac{\phi^2}{\beta^2})\right].$$

Similarly we can derive the maximized integrands in other cases involving binding (NN) constraints:

$$\begin{aligned} M_{11,10} &= 0.5\left[\frac{(n+\delta t_1+\phi t_2-\delta\mu_1)^2}{1-\alpha^2/\gamma} + \frac{(n+\delta t_2+\phi t_1-\phi\mu_1)^2}{1-\alpha^2/\gamma-\beta^2/\gamma} + \gamma\mu_1^2(\frac{\delta^2}{\alpha^2}+\frac{\phi^2}{\beta^2})\right], \\ M_{11,01} &= 0.5\left[\frac{(n+\delta t_1+\phi t_2-\delta\mu_1)^2}{1-\alpha^2/\gamma-\beta^2/\gamma} + \frac{(n+\delta t_2+\phi t_1-\phi\mu_1)^2}{1-\beta^2/\gamma} + \gamma\mu_1^2(\frac{\delta^2}{\alpha^2}+\frac{\phi^2}{\beta^2})\right], \\ M_{10,11} &= 0.5\left[\frac{(n+\delta t_1+\phi t_2-\phi\mu_2)^2}{1-\alpha^2/\gamma-\beta^2/\gamma} + \frac{(n+\delta t_2+\phi t_1-\delta\mu_2)^2}{1-\alpha^2/\gamma} + \gamma\mu_2^2(\frac{\delta^2}{\alpha^2}+\frac{\phi^2}{\beta^2})\right], \end{aligned}$$

$$
\begin{aligned}
M_{10,10} &= 0.5\left[\frac{(n+\delta t_1+\phi t_2)^2}{1-\alpha^2/\gamma}+\frac{(n+\delta t_2+\phi t_1)^2}{1-\alpha^2/\gamma}\right],\\
M_{10,01} &= 0.5\left[\frac{(n+\delta t_1+\phi t_2)^2}{1-\alpha^2/\gamma-\beta^2/\gamma}+(n+\delta t_2+\phi t_1)^2\right],\\
M_{01,10} &= 0.5\left[(n+\delta t_1+\phi t_2)^2+\frac{(n+\delta t_2+\phi t_1)^2}{1-\alpha^2/\gamma-\beta^2/\gamma}\right],\\
M_{01,01} &= 0.5\left[\frac{(n+\delta t_1+\phi t_2)^2}{1-\beta^2/\gamma}+\frac{(n+\delta t_2+\phi t_1)^2}{1-\beta^2/\gamma}\right],
\end{aligned}
$$

where a 0 in the subscript indicates a binding (NN) constraint, for example $\{11, 10\}$ means that only $e_{21} = 0$ and $\{10, 01\}$ means $e_{12} = 0$ and $e_2 = 0$.

Under Assumption (A2), it can be checked that $M_{11,11}$ is greater than all other M's listed above for all (t_1, t_2). Since $M_{11,11}$ is globally optimal, it is also greater than any combination of other M's mixed over different regions of (t_1, t_2). This implies that the mechanism given by Equation (6) gives greater expected payoff to the headquarters than any other mechanism that involves binding (NN) constraints. Therefore, Equation (6) gives an optimal mechanism for Program (P2).

Since (P2) is a relaxed program of (P1), we must have

$$E\Pi^{P2} \geq E\Pi^{P1}$$

where $E\Pi^{P2}$ is given by Equation (7) and $E\Pi^{P1}$ is the headquarters' maximum expected payoff in the original program (P1).

Step 2: Now we go back to the original program (P1), but restrict our attention to a class of mechanisms with linear compensation rules:

$$w_i = a_i + b_i V_i + b_{ij} V_j$$

where $\{a_i, b_i, b_{ij}\}$ are functions of the reported types $\{\hat{t}_1, \hat{t}_2\}$. Substituting $E_{\{\epsilon_1,\epsilon_2\}} w_i$ into manager i's utility function gives:

$$
\begin{aligned}
U_i &= a_i + b_i\Big[nk_i + (\alpha e_i + \beta e_{ji})k_i + (\delta t_i + \phi t_j)k_i - 0.5k_i^2\Big] \\
&\quad + b_{ij}\Big[nk_j + (\alpha e_j + \beta e_{ij})k_j + (\delta t_j + \phi t_i)k_j - 0.5k_j^2\Big] - 0.5\gamma(e_i^2 + e_{ij}^2).
\end{aligned}
$$

The first order conditions with respect to e_i and e_{ij} are:

$$
\begin{aligned}
\frac{\partial U_i}{\partial e_i} &= \alpha b_i k_i - \gamma e_i = 0, \\
\frac{\partial U_i}{\partial e_{ij}} &= \beta b_{ij} k_j - \gamma e_{ij} = 0.
\end{aligned}
$$

Ignoring the (NN) constraint for now, we have $e_i = \alpha b_i k_i/\gamma$ and $e_{ij} = \beta b_{ij} k_j$. This is the (IC1) constraint in the class of linear mechanisms.

The division managers' choices of $\{e_i, e_{ij}\}$ are again independent of the true types t_i. The (IC2) constraint can be rewritten as:

$$
\begin{aligned}
t_i &\in \arg\max EU_i(t_i, \hat{t}_i) \\
&= W_i(\hat{t}_i) + t_i E_{t_j}[\delta b_i(\hat{t}_i, t_j)k_i(\hat{t}_i, t_j) + \phi b_{ij}(\hat{t}_i, t_j)k_j(\hat{t}_i, t_j)].
\end{aligned}
$$

where $W_i(\hat{t}_i)$ represents the terms in manager i's expected utility that only depend on her own report, $\hat{t}_i$.

The Envelope Theorem implies

$$
\frac{dEU_i(t_i, t_i)}{dt_i} = E_{t_j}[\delta b_i(t_i, t_j)k_i(t_i, t_j) + \phi b_{ij}(t_i, t_j)k_j(t_i, t_j)].
$$

Integrating the above expression and imposing $EU_i(0) = \bar{U}$ from the (IR) constraint yields:

$$
EU_i(t_i) = \bar{U} + \int_0^{t_i}\int_0^{\bar{t}} (\delta b_i k_i + \phi b_{ij} k_j) dF_j ds_i. \tag{8}
$$

Integrating by parts over t_i yields:

$$EU_i = \bar{U} + \int_0^{\bar{t}} \int_0^{\bar{t}} (\delta b_i k_i + \phi b_{ij} k_j) \mu_i dF_i dF_j = \bar{U} + \int_0^{\bar{t}} \int_0^{\bar{t}} (\frac{\delta \gamma e_i}{\alpha} + \frac{\phi \gamma e_{ij}}{\beta}) \mu_i dF_i dF_j$$

where the last equality follows from optimal choices of e_i and e_{ij} by manager i.

Substituting $Ew_i = EU_i + 0.5\gamma E(e_i^2 + e_{ij}^2)$ into $E\Pi$ gives

$$\begin{aligned} E\Pi &= \int_0^{\bar{t}} \int_0^{\bar{t}} \Big\{ nk_1 + (\alpha e_1 + \beta e_{21})k_1 + (\delta t_1 + \phi t_2)k_1 - 0.5k_1^2 - U_1 - 0.5\gamma(e_1^2 + e_{12}^2) \\ &\quad + nk_2 + (\alpha e_2 + \beta e_{12})k_2 + (\delta t_2 + \phi t_1)k_2 - 0.5k_2^2 - U_2 - 0.5\gamma(e_2^2 + e_{21}^2) \Big\} dF_1 dF_2 \\ &= \int_0^{\bar{t}} \int_0^{\bar{t}} \Big\{ E_{\epsilon_1} V_1 + E_{\epsilon_2} V_2 - 0.5\gamma(e_1^2 + e_{12}^2) - 0.5\gamma(e_2^2 + e_{21}^2) \\ &\quad - (\frac{\delta \gamma e_1}{\alpha} + \frac{\phi \gamma e_{12}}{\beta})\mu_1 - (\frac{\delta \gamma e_2}{\alpha} + \frac{\phi \gamma e_{21}}{\beta})\mu_2 \Big\} dF_1 dF_2 - 2\bar{U}. \end{aligned}$$

Note that this is identical to the objective function in problem (P2). Therefore, pointwise differentiation of the integrand will give identical first-order conditions as Equation (5), leading to the same solution $\{k_i, e_i^*, e_{ij}^*\}$ as Equation (6). To derive other parts of the optimal linear mechanism, we have $b_i = \frac{\gamma e_i^*}{\alpha k_i} = 1 - \frac{\delta \gamma \mu_i}{\alpha^2 k_i}$ and $b_{ij} = \frac{\gamma e_{ij}^*}{\beta k_j} = 1 - \frac{\phi \gamma \mu_i}{\beta^2 k_j}$. The salary portion of the compensation can be recovered from

$$\begin{aligned} a_i &= \bar{U} + \int_0^{t_i} (\delta b_i k_i + \phi b_{ij} k_j) ds + 0.5\gamma(e_i^{*2} + e_{ij}^{*2}) \\ &\quad - b_i \Big[nk_i + (\alpha e_i^* + \beta e_{ji}^*)k_i + (\delta t_i + \phi t_j)k_i - 0.5k_i^2 \Big] \\ &\quad - b_{ij} \Big[nk_j + (\alpha e_j^* + \beta e_{ij}^*)k_j + (\delta t_j + \phi t_i)k_j - 0.5k_j^2 \Big]. \end{aligned}$$

Since this optimal linear mechanism has the same solution $\{k_i, e_i^*, e_{ij}^*\}$ as the optimal solution to problem (P2), we have

$$E\Pi^L = E\Pi^{P2},$$

where $E\Pi^L$ is the headquarters' expected payoff in the optimal linear mechanism.

On the other hand, note that the optimal linear mechanism satisfies all the constraints in (P1) hence it is a feasible mechanism of (P1) so it follows that

$$E\Pi^L \leq E\Pi^{P1}.$$

Combining these two, we have $E\Pi^{P2} \leq E\Pi^{P1}$. But Step 1 establishes that $E\Pi^{P2} \geq E\Pi^{P1}$. Therefore, $E\Pi^{P2} = E\Pi^{P1}$, so the optimal linear mechanism is an optimal mechanism in Program (P1).

So far we have studied the Bayesian Nash implementation of the headquarters' mechanism design problem. Since the optimal mechanism is monotonic in both t_1 and t_2, by the results of Mookherjee and Reichelstein (1992), it can also be implemented in dominant strategies. This completes the proof. Q.E.D.

Proof of Corollary 1: The monotonicity of the capital allocations, managerial effort allocations, and performance-pay are immediate after taking the derivative of each with respect to t_1 and t_2.

We now prove that salary $a_1(t_1, t_2)$ is decreasing in t_1 (a similar argument holds for $a_2(t_1, t_2)$ decreasing in t_2). Consider the two values, M_1 and M_2, given by:

$$\begin{aligned} M_1 &= \int_0^{t_1} (\delta b_1 k_1) ds_1 - b_1\Big[nk_1 + (\alpha e_1 + \beta e_{21})k_1 + (\delta t_1 + \phi t_2)k_1 - 0.5k_1^2\Big] + 0.5\gamma e_1^2, \\ M_2 &= \int_0^{t_1} (\phi b_{12} k_2) ds_1 - b_{12}\Big[nk_2 + (\alpha e_2 + \beta e_{12})k_2 + (\delta t_2 + \phi t_1)k_2 - 0.5k_2^2\Big] + 0.5\gamma e_{12}^2. \end{aligned}$$

Since the sum of M_1 and M_2 is the equal to the salary (less a constant $\bar{U}$), it suffices to show that M_1 and M_2 decrease with t_1. Note that M_1 can be written as

$$M_1 = \int_0^{t_1} (\delta b_1 k_1) ds_1 - \delta b_1 k_1 t_1 - 0.5 b_1 k_1 (n + \beta e_{21} + \phi t_2 - \delta t_1).$$

Thus,

$$\begin{aligned} \frac{\partial M_1}{\partial t_1} &= -0.5\frac{\partial (b_1 k_1)}{\partial t_1}(n + \beta e_{21} + \delta t_1 + \phi t_2) - 0.5\beta b_1 k_1 \frac{\partial e_{21}}{\partial t_1} + 0.5\delta b_1 k_1 \\ &< -0.5 b_1 \Big[\frac{\partial k_1}{\partial t_1}(n + \beta e_{21} + \delta t_1 + \phi t_2) - \delta k_1\Big] \end{aligned}$$

$$
\begin{aligned}
&= -0.5\delta b_1(1-\frac{\alpha^2}{\gamma}-\frac{\beta^2}{\gamma})^{-1}\Big[\beta e_{21}+\delta\mu_1+\phi\mu_2-(n+\beta e_{21}+\delta t_1+\phi t_2)\frac{d\mu_1}{dt_1}\Big] \\
&< 0
\end{aligned}
$$

where the first inequality follows from the monotonicity of b_1 and e_{21}, and the second inequality follows from the monotonicity of μ_1 and Assumptions (A1) and (A2). Similarly we can show that M_2 also decreases with t_1. Q.E.D.

Proof of Implication 1:

Using the solutions for the first-best capital allocation, k_i^{FB}, and the second-best capital allocation, k_i, from Propositions 1 and 2 we have:

$$k_1^{FB}-k_1=(1-\frac{\alpha^2}{\gamma}-\frac{\beta^2}{\gamma})^{-1}(\delta\mu_1+\phi\mu_2)>0 \tag{9}$$

implying capital underinvestment in the second-best mechanism. Since $\frac{d\mu_1}{dt_1}<0$ and $\frac{d\mu_2}{dt_2}<0$, $k_1^{FB}-k_1$ decreases with t_1 and t_2. The comparative statics are obvious after taking the partial derivatives with respect to the parameters. Similarly, we can prove the properties of $k_2^{FB}-k_2$.

Note also that:

$$e_1^{FB}-e_1=\frac{\alpha}{\gamma}(k_1^{FB}-k_1)+(\frac{\delta}{\alpha})\mu_1>0.$$

All the results for underinvestment in effort deployed in the manager's own project, e_1, follow immediately. Similar arguments apply to e_2, e_{12} and e_{21}. Q.E.D.

Proof of Implication 2:

Using the solutions for the first-best capital allocation, k_i^{FB}, and the second-best capital allocation, k_i, from Propositions 1 and 2 we have:

$$\frac{\partial(k_i-k_i^{FB})}{\partial t_j}=\frac{\phi(-\mu_j')}{1-\alpha^2/\gamma-\beta^2/\gamma}>0$$

where the inequality results from the monotonicity of μ_j and Assumption (A1). This proves part (i). Part (ii) follows from taking the derivative of the right-hand side with respect to each parameter. Q.E.D.

Proof of Implication 4:

If $k_1 = k_2 = k$ then the division-level performance pay is given by:

$$b_i - b_{ij} = \frac{\phi\gamma\mu_i}{\beta^2 k_j} - \frac{\delta\gamma\mu_i}{\alpha^2 k_i} = \frac{\gamma\mu_i}{k}(\frac{\phi}{\beta^2} - \frac{\delta}{\alpha^2}). \tag{10}$$

The desired result is clear when $(b_i - b_{ij}) > 0$ and you substitute the second-best capital allocation from Proposition 2 into the above.

If $k_1 = k_2 = k$ then the relative division-level performance pay compared to firm-level performance pay is given by:

$$\frac{b_i - b_{ij}}{b_{ij}} = \frac{b_i}{b_{ij}} - 1 = \frac{1 - \frac{\delta\gamma\mu_i}{\alpha^2 k}}{1 - \frac{\phi\gamma\mu_i}{\beta^2 k}} - 1$$

Taking logs of the first term on the right hand side yields:

$$\ln(\frac{1 - \frac{\delta\gamma\mu_i}{\alpha^2 k}}{1 - \frac{\phi\gamma\mu_i}{\beta^2 k}}) = \ln(1 - \frac{\delta\gamma\mu_i}{\alpha^2 k}) - \ln(1 - \frac{\phi\gamma\mu_i}{\beta^2 k}) \approx \frac{\gamma\mu_i}{k}(\frac{\phi}{\beta^2} - \frac{\delta}{\alpha^2})$$

where the approximation is valid if n is significantly large. Since this last expression is identical to the above, the same comparative statics results hold. Q.E.D.

Proof of Implication 5:

We need to show that b_{ij} and b_{ji} are positively correlated. For what follows, we use the fact: $\text{Cov}(p(x), q(x)) > 0$ if $p'(x),\ q'(x) > 0$. Denote $y_{ij} = \dfrac{\mu_i}{n + \delta(t_j - \mu_j) + \phi(t_i - \mu_i)}$. It suffices to show that y_{ij} and y_{ji} are positively correlated since $\text{Corr}(b_{ij}, b_{ji}) = \text{Corr}(y_{ij}, y_{ji})$. Given t_i, y_{ij} and y_{ji} increase with t_j. Using our fact above we have $E(y_{ij}y_{ji}|t_i) > E(y_{ij}|t_i)E(y_{ji}|t_i)$. Also, since $E(y_{ij}|t_i)$ and $E(y_{ji}|t_i)$ increase with t_i, $E\Big[E(y_{ij}|t_i)E(y_{ji}|t_i)\Big] > E\Big[E(y_{ij}|t_i)\Big]E\Big[E(y_{ji}|t_i)\Big]$. Therefore,

$$\begin{aligned} E(y_{ij}y_{ji}) &= E\Big[E(y_{ij}y_{ji}|t_i)\Big] > E\Big[E(y_{ij}|t_i)E(y_{ji}|t_i)\Big] \\ &> E\Big[E(y_{ij}|t_i)\Big]E\Big[E(y_{ji}|t_i)\Big] = E(y_{ij})E(y_{ji}), \end{aligned}$$

implying y_{ij} and y_{ji} are positively correlated. Q.E.D.

Proof of Implication 6: For the first statement, note that:

$$k_1 - k_1^1 \quad = \quad \frac{n + \delta(t_1 - \mu_1) + \phi(\underline{t}_2 - \mu_2(\underline{t}_2))}{1 - \frac{\alpha^2}{\gamma} - \frac{\beta^2}{\gamma}} - \frac{n + \delta(t_1 - \mu_1) + \phi\underline{t}_2 + \underline{e}_{21}}{1 - \frac{\alpha^2}{\gamma}}$$

which increases in $\underline{t}_2$. If there exists $\underline{t}_2^{crit} \in (0, \bar{t})$ such that $k_1 - k_1^1 = 0$, then clearly k_1 is larger (smaller) than k_1^1 when t_2 is larger (smaller) than $\underline{t}_2^{crit}$.

For the second statement, note that $E_{\{t_1,t_2\}}[k_1 - k_1^1]$ increases in β. If there exists $\beta^{crit} > 0$ such that $E_{\{t_1,t_2\}}[k_1 - k_1^1] = 0$ then $E_{\{t_1,t_2\}}[k_1]$ is larger (smaller) than $E_{\{t_1,t_2\}}[k_1^1]$ when β is larger (smaller) than β^{crit}. Q.E.D.

References

[1] Antle, R., and G. Eppen, 1985, "Capital Rationing and Organizational Slack in Capital Budgeting," *Management Science*, 31, 163-174.

[2] Berkovitch, E., and R. Israel, 2003, "Why the NPV Criterion does not Maximize NPV," Working paper. Tel Aviv University; forthcoming in *Review of Financial Studies.*

[3] Bernardo, A.E., H. Cai, and J. Luo, 2001, "Capital Budgeting and Compensation with Asymmetric Information and Moral Hazard," *Journal of Financial Economics*, 61, 311-344.

[4] Billett, M.T., and D.C. Mauer, 2003, "Cross-Subsidies, External Financing Constraints, and the Contribution of the Internal Capital Market to Firm Value," Working paper. University of Iowa; forthcoming in *Review of Financial Studies.*

[5] Bushman, R. M., R.J. Indjejikian, and A. Smith, 1995, "Aggregate Performance Measures in Business Unit Manager Compensation: The Role of Intrafirm Interdependencies," *Journal of Accounting Research*, 33 Supplement, 101-128.

[6] Chevalier, J., 2000, "What do we know about Cross-subsidization? Evidence from the Investment Policies of Merging Firms," working paper, Yale University.

[7] Garcia, D., 2001, "Retained Equity, Investment Decisions and Private Information," working paper, Dartmouth College.

[8] Garcia, D., 2002, "Optimal Contracts with Privately Informed Agents and Active Principals," working paper, Dartmouth College.

[9] Gertner, R., D.S. Scharfstein, and J.C. Stein, 1994, "Internal versus External Capital Markets," *Quarterly Journal of Economics*, 109, 1211-1230.

[10] Harris, M., and A. Raviv, 1996, "The Capital Budgeting Process: Incentives and Information," *Journal of Finance*, 51, 1139-1174.

[11] Harris, M., and A. Raviv, 1998, "Capital Budgeting and Delegation," *Journal of Financial Economics*, 50, 259-289.

[12] Harris, M., C.H. Kriebel, and A. Raviv, 1982, "Asymmetric Information, Incentives, and Intrafirm Resource Allocation," *Management Science*, 28, 604-620.

[13] Holmstrom, B., and P. Milgrom, 1987, "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 55, 597-619.

[14] Holmstrom, B., and J. Ricart i Costa, 1986, "Managerial Incentives and Capital Management," *Quarterly Journal of Economics*, 101, 835-860.

[15] Keating, A. S., 1997, "Determinants of Divisional Performance Evaluation Practices," *Journal of Accounting and Economics*, 24, 243-274.

[16] Laffont, J.-J., and J. Tirole, 1986, "Using Cost Observation to Regulate Firms," *Journal of Political Economy*, 94, 614-641.

[17] Laffont, J.-J., and J. Tirole, 1993, *A Theory of Incentives in Procurement and Regulation*, MIT Press, Cambridge.

[18] Lamont, O., 1997, "Cash Flow and Investment: Evidence from Internal Capital Markets," *Journal of Finance*, 52, 83-109.

[19] McAfee, R. P., and J. McMillan, 1987, "Competition for Agency Contracts," *RAND Journal of Economics*, 18, 296-307.

[20] Milgrom, P., and J. Roberts, 1992, *Economics, Organizations and Management*, Prentice Hall, Englewood Cliffs.

[21] Mirrlees, J.A., 1971, "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38, 135-208

[22] Mookherjee, D., and S. Reichelstein, 1992, "Dominant Strategy Implementation of Bayesian Incentive Compatible Allocation Rules," *Journal of Economic Theory*, 56, 378-99.

[23] Palia, D., 2000, "Corporate Governance and the Diversification Discount: Evidence from Panel Data," working paper, Rutgers University.

[24] Poterba, J., and L. Summers, 1992, "Time Horizons of American Firms: New Evidence from a Survey of CEOs," working paper, Harvard University.

[25] Rajan, R., H. Servaes, and L. Zingales, 2000, "The Cost of Diversity: The Diversification Discount and Inefficient Investment," *Journal of Finance*, 55, 35-80.

[26] Shin, H., and R. Stulz, 1998, "Are Internal Capital Markets Efficient?", *Quarterly Journal of Economics*, 113, 531-552.

[27] Stein, J.C., 1997, "Internal Capital Markets and the Competition for Corporate Resources," *Journal of Finance*, 52, 111-133.

[28] Villalonga, 2003, "Diversification Discount or Premium: New Evidence from BITS Establishment-Level Data," Working Paper. Harvard University; forthcoming in *Journal of Finance*.

[29] Whited, T., 2001, "Is it Inefficient Investment that Causes the Diversification Discount?," *Journal of Finance*, 56, 1667-1692.

[30] Zhang, G., 1997, "Moral Hazard in Corporate Investment and the Disciplinary Role of Voluntary Capital Rationing," *Management Science*, 43, 737-750.