DYNAMIC ASSET ALLOCATION WITH EVENT RISK

Jun Liu
Francis A. Longstaff
Jun Pan*

Initial version: November 2000.
Current version: August 2001.

*Jun Liu is with the Anderson School at UCLA, jun.liu@anderson.ucla.edu. Francis Longstaff is with the Anderson School at UCLA, francis.longstaff@anderson.ucla.edu. Jun Pan is with the MIT Sloan School of Management, junpan@mit.edu. We are particularly grateful for helpful discussions with Tony Bernardo and Pedro Santa-Clara, for the comments of Jerome Detemple, Harrison Hong, Paul Pfleiderer, Raman Uppal, and participants at the 2001 Western Finance Association meetings, and for the many insightful comments and suggestions of the editor Richard Green and the referee. All errors are our responsibility. Copyright 2001.

ABSTRACT

An inherent risk facing investors in financial markets is that a major event may trigger a large abrupt change in stock prices and market volatility. This paper studies the implications of jumps in prices and volatility on investment strategies. Using the event-risk framework of Duffie, Pan, and Singleton, we provide an analytical solution to the optimal portfolio problem. We find that event risk dramatically affects the optimal strategy. An investor facing event risk is less willing to take leveraged or short positions. In addition, the investor acts as if some portion of his wealth may become illiquid and the optimal strategy blends elements of both dynamic and buy-and-hold portfolio strategies. Jumps in prices and volatility both have an important influence on the optimal strategy.

1. INTRODUCTION

One of the inherent hazards of investing in financial markets is the risk of a major event precipitating a sudden large shock to security prices and volatilities. There are many recent examples of this type of event risk such as the stock market crash of October 19, 1987 in which the Dow index fell by 508 points, the October 27, 1997 drop in the Dow index by more than 554 points, and the flight to quality in the aftermath of the Russian debt default where swap spreads increased on August 27, 1998 by more than twenty times their daily standard deviation, leading to the downfall of Long Term Capital Management and many other highly-leveraged hedge funds. Each of these events was accompanied by major increases in market volatility.[1]

The risk of event-related jumps in security prices and volatility changes the standard dynamic portfolio choice problem in several important ways. In the standard problem, security prices are continuous and instantaneous returns have infinitesimal standard deviations; an investor considers only small local changes in security prices in selecting a portfolio. With event-related jumps, however, the investor must also consider the effects of large security price and volatility changes when selecting a dynamic portfolio strategy. Since the portfolio that is optimal for large returns need not be the same as that for small returns, this creates a strong conflict that must be resolved by the investor in selecting a portfolio strategy.

This paper studies the implications of event-related jumps in security prices and volatility on optimal dynamic portfolio strategies. In modeling event-related jumps, we use the double-jump framework of Duffie, Pan, and Singleton (2000). This framework is motivated by evidence by Bates (2000) and others of the existence of volatility jumps, and has received strong empirical support from the data.[2] In this model, both the security price and the volatility of its returns follow jump-diffusion processes. Jumps are triggered by a Poisson event which has an intensity proportional to the level of volatility. This intuitive framework closely parallels the behavior of actual financial markets and allows us to study directly the effects of event risk on portfolio choice.

To make the intuition behind the results as clear as possible, we focus on the simplest case where an investor with power utility over end-of-period wealth allocates his portfolio between a riskless asset and a risky asset that follows the double-jump process. Because of the tractability provided by the affine structure of the model, we are able to reduce the Hamilton-Jacobi-Bellman partial differential equation for the

[1]For example, the VIX index of S&P 500 stock index option implied volatilities increased 313 percent on October 19, 1987, 53 percent on October 27, 1997, and 28 percent on August 27, 1998.

[2]For example, see the extensive recent study by Eraker, Johannes, and Polson (2000) of the double-jump model.

indirect utility function to a set of ordinary differential equations. This allows us to obtain an analytical solution for the optimal portfolio weight. In the general case, the optimal portfolio weight is given by solving a simple pair of non-linear equations. In a number of special cases, however, closed-form solutions for the optimal portfolio weight are readily obtained.

The optimal portfolio strategy in the presence of event risk has many interesting features. One immediate effect of introducing jumps into the portfolio problem is that return distributions may display more skewness and kurtosis. While this has an important influence on the portfolio chosen, the full implications of event risk for dynamic asset allocation run much deeper. We show that the threat of event-related jumps makes an investor behave as if he faced short-selling and borrowing constraints even though none are imposed. This result parallels Longstaff (2001) where investors facing illiquid or nonmarketable assets endogenously restrict their portfolio leverage. Interestingly, we find that the optimal portfolio is a blend of the optimal portfolio for a continuous-time problem and the optimal portfolio for a static buy-and-hold problem. Intuitively, this is because when an event-related jump occurs, the portfolio return is on the same order of magnitude as the return that would be obtained from a buy-and-hold portfolio over some finite horizon. Since these two returns have the same effect on terminal wealth, their implications for portfolio choice are indistinguishable, and event risk can be interpreted or viewed as a form of liquidity risk. This perspective provides new insights into the effects of event risk on financial markets.

To illustrate our results, we provide two examples. In the first, we consider a model where the risky asset follows a jump-diffusion process with deterministic jump sizes, but where return volatility is constant. This special case parallels Merton (1971) who solves for the optimal portfolio weight when the riskless rate follows a jump-diffusion process. We find that an investor facing jumps may choose a portfolio very different from the portfolio that would be optimal if jumps did not occur. In general, the investor holds less of the risky asset when event-related price jumps can occur. This is true even when only upward price jumps can occur. Intuitively, this is because the effect of jumps on return volatility dominates the effect of the resulting positive skewness. Because event risk is constant over time in this example, the optimal portfolio does not depend on the investor's horizon.

In the second example, we consider a model where both the risky asset and its return volatility follow jump-diffusion processes with deterministic jump sizes. The stochastic volatility model studied by Liu (1999) can be viewed as a special case of this model. As in Liu, the optimal portfolio weight does not depend on the level of volatility. The optimal portfolio weight, however, does depend on the investor's horizon since the probability of an event is time varying through its dependence on the level of volatility. We find that volatility jumps can have a significant effect on the optimal portfolio above and beyond the effect of price jumps. Surprisingly, investors may even choose to hold more of the risky asset when there are volatility

jumps than otherwise. Intuitively, this means that the investor can partially hedge the effects of volatility jumps on his indirect utility through the offsetting effects of price jumps. Note that this hedging behavior arises because of the static buy-and-hold component of the investor's portfolio problem; this static jump-hedging behavior differs fundamentally from the usual dynamic hedging of state variables that occurs in the standard pure-diffusion portfolio choice problem.

We provide an application of the model by calibrating it to historical U.S. data and examining its implications for optimal portfolio weights. The results show that even when large jumps are very infrequent, an investor still finds it optimal to reduce his exposure to the stock market significantly. These results suggest a possible reason why historical levels of stock market participation have tended to be lower than would be optimal in many classical portfolio choice models.

Since the original work by Merton (1971), the problem of portfolio choice in the presence of richer stochastic environments has become a topic of increasing interest. Recent examples of this literature include Brennan, Schwartz, and Lagnado (1997) on asset allocation with stochastic interest rates and predictability in stock returns, Kim and Omberg (1996), Campbell and Viceira (1999), Barberis (2000), and Xia (2000) on predictability in stock returns (with or without learning), Lynch (2000) on portfolio choice and equity characteristics, Schroder and Skiadas (1999) on a class of affine diffusion models with stochastic differential utility, Balduzzi and Lynch (1999) on transaction costs and stock return predictability, and Brennan and Xia (1998), Liu (1999), Wachter (1999), and Campbell and Viceira (2001) on stochastic interest rates. Although Merton (1971), Common (2000), and Das and Uppal (2001) study the effects of price jumps and Liu (1999), Chacko and Viceira (2000), and Longstaff (2001) study the effects of stochastic volatility, this paper contributes to the literature by being the first to study the effects of event-related jumps in both stock prices and volatility.[3]

The remainder of this paper is organized as follows. Section 2 presents the event-risk model. Section 3 provides analytical solutions to the optimal portfolio allocation problem. Section 4 presents the examples and provides numerical results. Section 5 calibrates the model and examines the implications for optimal portfolio choice. Section 6 summarizes the results and make concluding remarks.

2. THE EVENT-RISK MODEL

We assume that there are two assets in the economy. The first is a riskless asset paying a constant rate of interest r. The second is a risky asset whose price S_t is

[3]Wu (2000) studies the portfolio choice problem in a model where there are jumps in stock prices but not volatility, but does not provide a verifiable analytical solution for the optimal portfolio strategy.

subject to event-related jumps. Specifically, the price of the risky asset follows the process

$$dS_t = (r + \eta V_t - \mu\lambda V_t)\ S_t\ dt\ +\ \sqrt{V_t}\ S_t\ dZ_{1t}\ +\ X_t\ S_{t-}\ dN_t, \tag{1}$$

$$dV_t = (\alpha - \beta V_t - \kappa\lambda V_t)\ dt\ +\ \sigma\sqrt{V_t}\ dZ_{2t}\ +\ Y_t\ dN_t, \tag{2}$$

where Z_1 and Z_2 are standard Brownian motions with correlation ρ, V is the instantaneous variance of diffusive returns, and N is a Poisson process with stochastic arrival intensity λV. The parameters α, β, κ, λ, and σ are all assumed to be non-negative. X is a random price-jump size with mean μ and is assumed to have support on $(-1, \infty)$ which guarantees the positivity (limited liability) of S. Similarly, Y is a random volatility-jump size with mean κ and is assumed to have support on $[0, \infty)$ to guarantee that V remains positive. In general, the jump sizes X and Y can be jointly distributed with non-zero correlation. The jump sizes X and Y are also assumed to be independent across jump times and independent of Z_1, Z_2 and N.

Given these dynamics, the price of the risky asset follows a stochastic-volatility jump-diffusion process and is driven by three sources of uncertainty: diffusive price shocks from Z_1, diffusive volatility shocks from Z_2, and realizations of the Poisson process N. Since a realization of N triggers jumps in both S and V, a realization of N has the natural interpretation of a financial event affecting both prices and market volatilities. In this sense, this model is ideal for studying the effects of event risk on portfolio choice. Because the jump sizes X and Y are random, however, it is possible for the arrival of an event to result in a large jump in S and only a small jump in V, or a small jump in S and a large jump in V. This feature is consistent with observed market behavior; although financial market events are generally associated with large movements in both prices and volatility, jumps in only prices or only volatility can occur. Since μ is the mean of the price-jump size X, the term $\mu\lambda VS$ in (1) compensates for the instantaneous expected return introduced by the jump component of the price dynamics. As a result, the instantaneous expected rate of return equals the riskless rate r plus a risk premium ηV. This form of the risk premium follows from Merton (1980) and is also used by Liu (1999), Pan (2001), and many others. Note that the risk premium compensates the investor for both the risk of diffusive shocks and the risk of jumps.[4]

These dynamics also imply that the instantaneous variance V follows a mean-reverting square-root jump-diffusion process. The Heston (1993) stochastic-volatility

[4]Although the risk premium could be separated into the two types of risk premia, the portfolio allocation between the riskless asset and the risky asset in our model is independent of this breakdown. If options were introduced into the market as a second risky asset, however, this would no longer be true. See Pan (2001).

model can be obtained as a special case of this model by imposing the condition that $\lambda = 0$, which implies that jumps do not occur. Liu (1999) provides closed-form solutions to the portfolio problem for this special case.[5] Also nested as special cases are the stochastic-volatility jump-diffusion models of Bates (1996) and Bakshi, Cao, and Chen (1997). Again, since κ is the mean of the volatility jump size Y, $\kappa\lambda V$ in the drift of the process for V compensates for the jump component in volatility.

This bivariate jump-diffusion model is an extended version of the double-jump model introduced by Duffie, Pan, and Singleton (2000). Note that this model falls within the affine class because of the linearity of the drift vector, diffusion matrix, and intensity process in the state variable V. The double-jump framework has received a significant amount of empirical support because of the tendency for both stock prices and volatility to exhibit jumps. For example, a recent paper by Eraker, Johannes, and Polson (2000) finds strong evidence of jumps in volatility even after accounting for jumps in stock returns.[6] Duffie, Pan, and Singleton also show that the double-jump model implies volatility 'smiles' or skews for stock options that closely match the volatility skews observed in options markets.[7]

3. OPTIMAL DYNAMIC ASSET ALLOCATION

In this section, we focus on the asset allocation problem of an investor with power utility

$$U(x) = \begin{cases} \frac{1}{1-\gamma}\, x^{1-\gamma}, & \text{if } x > 0, \\ -\infty, & \text{if } x \leq 0, \end{cases} \tag{3}$$

where $\gamma > 0$, and the second part of the utility specification effectively imposes a non-negative wealth constraint. This constraint is consistent with Dybvig and Huang (1988) who show that requiring wealth to be non-negative rules out arbitrages of the type described by Harrison and Kreps (1979). As demonstrated by Kraus and Litzenberger (1976), an investor with this utility function has a preference for positive skewness.

Given the opportunity to invest in the riskless and risky assets, the investor starts with a positive initial wealth W_0 and chooses, at each time t, $0 \leq t \leq T$, to

[5]See Chacko and Viceira (2000) and Longstaff (2001) for solutions to the dynamic portfolio problem for alternative stochastic volatility models.

[6]Similar evidence is also presented in Bates (2000), Pan (2001), and others.

[7]See also Bakshi, Cao, and Chen (1997) and Bates (2000) for empirical evidence about the importance of jumps in option pricing.

invest a fraction ϕ_t of his wealth in the risky asset so as to maximize the expected utility of his terminal wealth W_T,

$$\max_{\{\phi_t,\ 0\le t\le T\}} E_0\left[\ U(W_T)\ \right], \tag{4}$$

where the wealth process satisfies the self-financing condition

$$dW_t = (r + \phi_t\ (\eta - \mu\lambda)\ V_t)\ \ W_t\ dt + \phi_t\sqrt{V_t}\ W_t\ dZ_{1t} + X_t\ \phi_{t-}\ W_{t-}\ dN_t. \tag{5}$$

Although the model could be extended to allow for intermediate consumption, we use this simpler specification to focus more directly on the intuition behind the results.

Before solving for the optimal portfolio strategy, let us first consider how jumps affect the nature of the returns available to an investor who invests in the risky asset. When a risky asset follows a pure diffusion process without jumps, the variance of returns over an infinitesimal time period Δt is proportional to Δt. This implies that as Δt goes to zero, the uncertainty associated with the investor's change in wealth ΔW also goes to zero. Thus, the investor can rebalance his portfolio after every infinitesimal change in his wealth. Because of this, the investor retains complete control over his portfolio composition; his actual portfolio weight is continuously equal to the optimal portfolio weight. An important implication of this is that an investor with leveraged or short positions in a market with continuous prices can always rebalance his portfolio quickly enough to avoid negative wealth if the market turns against him.

The situation is very different, however, when asset price paths are discontinuous because of event-related jumps. For example, given the arrival of a jump event at time t, the uncertainty associated with the investor's change in wealth $\Delta W_t = W_t - W_{t-}$ does not go to zero. Thus, when a jump occurs, the investor's wealth can change significantly from its current value before the investor has a chance to rebalance his portfolio. An immediate implication of this is that the investor's portfolio weight is not completely under his control at all times. For example, the actual portfolio weight will typically differ from the optimal portfolio weight immediately after a jump occurs. This implies that significant amounts of portfolio rebalancing may be observed in markets after an event-related jump occurs. Without complete control over his portfolio weight, however, an investor with large leveraged or short positions may not be able to rebalance his portfolio quickly enough to avoid negative wealth.

Because of this, the investor not only faces the usual local-return risk that appears in the standard pure diffusion portfolio selection problem, but also the risk that large changes in his wealth may occur before having the opportunity to adjust his portfolio. This latter risk is essentially the same risk faced by an investor who holds illiquid assets in his portfolio; an investor holding illiquid assets may also

experience large changes before having the opportunity to rebalance his portfolio. Because of this event-related 'illiquidity' risk, the only way that the investor can guarantee that his wealth remains positive is by avoiding portfolio positions that are one jump away from ruin. This intuition is summarized in the following proposition which places bounds on admissible portfolio weights.[8]

Proposition 1. Bounds on Portfolio Weights. *Suppose that for any t, $0 < t \leq T$, we have*

$$0 < E_t \left[\exp\left(- \int_t^T \lambda V_s \, ds \right) \right] < 1, \tag{6}$$

where λV_t is the jump arrival intensity. Then, at any time t, the optimal portfolio weight ϕ_t^* for the asset allocation problem must satisfy

$$1 + \phi_t^* \, X_{\text{Inf}} > 0 \qquad \text{and} \qquad 1 + \phi_t^* \, X_{\text{Sup}} > 0, \tag{7}$$

where X_{Inf} and X_{Sup} are the lower and upper bounds of the support of X_t (the random price jump size). In particular, if $X_{\text{Inf}} < 0$ and $X_{\text{Sup}} > 0$,

$$-\frac{1}{X_{\text{Sup}}} < \phi_t^* < -\frac{1}{X_{\text{Inf}}}. \tag{8}$$

Proof. *See Appendix.*

Thus, the investor endogenously restricts the amount of leverage or short selling in his portfolio as a hedge against his inability to continuously control his portfolio weight. If the random price jump size X can take any value on $(-1, \infty)$, then this proposition implies that the investor will never take a leveraged or short position in the risky asset.

These results parallel Longstaff (2001) who studies dynamic asset allocation in a market where the investor is restricted to trading strategies that are of bounded variation. In his model, the investor protects himself against the risk of not being able to trade his way out of a leveraged position quickly enough to avoid negative wealth by endogenously restricting his portfolio weight to be between zero and one. Intuitively, the reason for this is the same as in our model. Having to hold a portfolio over a jump event has essentially the same effect on terminal wealth as having a buy-and-hold portfolio over some discrete horizon. In this sense, the problem of illiquidity parallels that of event-related jumps. Interestingly, discussions of major financial market events in the financial press often link the two problems together.

[8]This result is also closely related to the classical ruin problem which has been extensively studied in the stochastic process literature. Examples include Aase (1984, 1986), Aase and Øksendal (1988), Jeanblanc-Picqué and Pontier (1990), and Bardhan and Chao (1995). We are grateful to the referee for this insight.

One issue that is not formally investigated in this paper is the role of options in alleviating the cost associated with the jump risk. Intuitively, put options could be used to hedge against the negative jump risk, allowing investors to break the jump-induced constraint and hold leveraged positions in the underlying risky asset.[9] In practice, the benefit of such option strategies depends largely on the cost of such insurance against the jump risk. Moreover, in a dynamic setting with jump risk, it might be hard to perfectly hedge the jump risk with finitely many options. Putting these complications aside, it is potentially fruitful to introduce options to the portfolio problem, particularly in light of our results on the jump-induced constraints.[10] A formal treatment, however, is beyond the scope of this paper.

We turn now to the asset allocation problem in (4) and (5). In solving for the optimal portfolio strategy, we adopt the standard stochastic control approach. Following Merton (1971), we define the indirect utility function by

$$J(W,V,t) = \max_{\{\phi_s,\ t\le s\le T\}} E_t[\ U(W_T)\ ]. \tag{9}$$

The principle of optimal stochastic control leads to the following Hamilton-Jacobi-Bellman (HJB) equation for the indirect utility function J,

$$\begin{aligned}\max_{\phi}\Bigg(&\frac{\phi^2W^2V}{2}J_{WW}+\phi\rho\sigma WVJ_{WV}+\frac{\sigma^2V}{2}J_{VV}\\&+(r+\phi(\eta-\mu\lambda)V)WJ_W+(\alpha-\beta V-\kappa\lambda V)J_V\\&+\lambda V\big(E[J(W(1+\phi X),V+Y,t)]-J\big)+J_t\Bigg)=0,\end{aligned} \tag{10}$$

where J_W, J_V, and J_t denote the derivatives of $J(W,V,t)$ with respect to W, V, and t, and similarly for the higher derivatives, and the expectation is taken with respect to the joint distribution of X and Y.

We solve for the optimal portfolio strategy ϕ^* by first conjecturing (which we later verify) that the indirect utility function is of the form

[9]Imposing buy-and-hold constraints on an otherwise dynamic trading strategy parallels our jump-induced constraint. Haugh and Lo (2001) show that options can alleviate some of the cost associated with the buy-and-hold constraint.

[10]We thank the referee for pointing out the role that options might play in mitigating the effects of event risk.

$$J(W,V,t) = \frac{1}{1-\gamma} W^{1-\gamma} \exp(A(t) + B(t)V), \tag{11}$$

where $A(t)$ and $B(t)$ are functions of time but not of the state variables W and V. Given this functional form, we take derivatives of $J(W,V,t)$ with respect to its arguments, substitute into the HJB equation in (10), and differentiate with respect to the portfolio weight ϕ to obtain the following first-order condition,

$$(\eta - \mu\lambda)V + \rho\sigma BV - \gamma\phi^* V + \lambda V E[(1+\phi^* X)^{-\gamma} X e^{BY}] = 0. \tag{12}$$

Before solving this first-order condition for ϕ^*, it is useful to first make several observations about its structure. In particular, note that if λ is set equal to zero, the risky asset follows a pure diffusion process. In this case, the investor faces a standard dynamic portfolio choice problem in which the first-order condition for ϕ^* becomes

$$\eta V + \rho\sigma BV - \gamma\phi^* V = 0. \tag{13}$$

Alternatively, consider the case where the investor faces a static single-period portfolio problem where the return on his portfolio during this period equals $(1 + \phi X)$. In this case, the investor maximizes his expected utility over terminal wealth by selecting a portfolio to satisfy the first-order condition,

$$E[(1+\phi^* X)^{-\gamma} X] = 0. \tag{14}$$

Now compare the first-order conditions for the standard dynamic problem and the static buy-and-hold problem to the first-order condition for the event-risk portfolio problem given in (12). It is easily seen that the left-hand-side of (12) essentially incorporates the first-order conditions in (13) and (14). In the special case where μ and Y equal zero, the left-hand-side of (12) is actually a simple linear combination of the first-order conditions in (13) and (14) in which the coefficients for the dynamic and static first-order conditions are one and λV respectively. This provides some economic intuition for how the investor views his portfolio choice problem in the event-risk model. At each instant, the investor faces a small continuous return, and with probability λV, may also face a large return similar to that earned on a buy-and-hold portfolio over some discrete period. Thus, the first-order condition for the event-risk problem can be viewed as a blend of the first-order conditions for a standard dynamic portfolio problem and a static buy-and-hold portfolio problem.

So far, we have placed little structure of the joint distribution of the jump sizes X and Y. To guarantee the existence of an optimal policy, however, we require that the following mild regularity conditions hold for all ϕ that satisfy the conditions of Proposition 1,

$$M_1 \equiv E[(1+\phi^* X)^{-\gamma} X e^{BY}] < \infty, \tag{15}$$

$$M_2 \equiv E[(1+\phi^* X)^{1-\gamma} e^{BY}] < \infty. \tag{16}$$

The following proposition provides an analytical solution for the optimal portfolio strategy.

Proposition 2. Optimal Portfolio Weights. *Assume that the regularity conditions in (15) and (16) are satisfied. Then the asset allocation problem in (4) and (5) has a solution ϕ^*. The optimal portfolio weight is given by solving the following non-linear equation for ϕ^*,*

$$\phi^* = \frac{\eta - \mu\lambda}{\gamma} + \frac{\rho\sigma B}{\gamma} + \frac{\lambda M_1}{\gamma}, \tag{17}$$

subject to the constraints in (7), and where B is defined by the ordinary differential equation

$$B' + \sigma^2 B^2/2 + \left(\phi^* \rho\sigma(1-\gamma) - \beta - \kappa\lambda\right) B + \left(\frac{\gamma(\gamma-1)\phi^{*2}}{2} + (\eta - \mu\lambda)(1-\gamma)\phi^* + \lambda M_2 - \lambda \right) = 0. \tag{18}$$

Proof. See Appendix.

From this proposition, ϕ^* can be determined under very general assumptions about the joint distribution of the jump sizes X and Y by solving a simple pair of equations. Given a specification for the joint distribution of X and Y, equation (17) is just a non-linear expression in ϕ^* and B. Equation (18) is an ordinary differential equation for B with coefficients that depend on ϕ^*. These two equations are easily solved numerically using standard finite difference techniques. Starting with the terminal condition $B(T) = 0$, the values of ϕ^* and B at all earlier dates are obtained by solving pairs of non-linear equations recursively back to time zero. Given the simple forms of (17) and (18), the recursive solution technique is virtually instantaneous. Observe that solving this pair of equations for ϕ^* and B is far easier than solving the two-dimensional HJB equation in (10) directly. For many special cases, the optimal portfolio weight can actually be solved in closed-form, or can be obtained directly by solving a single non-linear equation in ϕ^*. Several examples are presented in the next section.

We first note that the optimal portfolio weight is independent of the state variables W and V. In other words, there is no 'market timing' in either wealth or stochastic volatility. The reason why the portfolio weight is independent of wealth stems from the homogeneity of the portfolio problem in W. The reason the optimal

portfolio does not depend on V is formally due to the fact that we have assumed that the risk premium is proportional to V. Intuitively, however, this risk premium seems sensible since both the instantaneous variance of returns and the instantaneous risk of a jump are proportional to V; by requiring the risk premium to be proportional to V, we guarantee that all of the key instantaneous moments of the investment opportunity set are of the same order of magnitude.

Recall from the earlier discussion that the event-risk portfolio problem blends a standard dynamic problem with a static buy-and-hold problem. Intuitively, this can be seen from the expression for the optimal portfolio weight given in (17). As shown, the right-hand-side of this expression has three components. The first consists of the instantaneous risk premium $\eta - \mu\lambda$ divided by the risk aversion parameter γ. It is easily shown that when $\lambda = 0$ and V is not stochastic, the instantaneous risk premium become η and the optimal portfolio policy is η/γ. Thus, the first term in (17) is just the generalization of the usual myopic component of the portfolio demand. The second component is directly related to the correlation coefficient ρ between instantaneous returns on the risky asset and changes in the volatility V. When this correlation is non-zero, the investor can hedge his expected utility against shifts in V by taking a position in the risky asset. Thus, this second term can be interpreted as the volatility hedging demand for the risky asset. A similar volatility hedging demand for the risky asset also appears in stochastic-volatility models such as Liu (1999). Note that in this model, the hedging demand arises not only because the state variable V impacts the volatility of returns, but also because it drives the variation in the probability of an event occurring. Thus, investors have a double incentive to hedge against variation in V through their portfolio holdings of the risky asset. Finally, the third term in (17) is directly related to the first-order condition for the static buy-and-hold problem from (14). Thus, this term can be interpreted as the event-risk or 'illiquidity' hedging term; this term does not appear in portfolio problems where prices follow continuous sample paths.

4. EXAMPLES

In this section, we illustrate the implications of event-related jumps for portfolio choice through several simple examples.

4.1 Constant Volatility and Deterministic Jump Size.

In the first example, V is assumed to be constant over time. This implies that $\alpha = \beta = \kappa = \sigma = Y = 0$. In addition, we assume that price jumps are deterministic in size, implying $X = \mu$. In this case, the risky asset follows a simple jump-diffusion process. This complements Merton (1971) who studies asset allocation when the riskless asset follows a jump-to-ruin process.

In this example, the model dynamics reduce to

$$dS_t = (r + \eta V_0 - \mu\lambda V_0)\ S_t\ dt + \sqrt{V_0}\ S_t\ dZ_{1t} + \mu\ S_{t-}\ dN_t, \tag{19}$$

$$dV_t = 0. \tag{20}$$

Substituting in the parameter restrictions and solving gives the following simple expression for the optimal portfolio weight,

$$\phi^* = \frac{\eta - \mu\lambda}{\gamma} + \frac{\mu\lambda}{\gamma}(1 + \mu\phi^*)^{-\gamma}, \tag{21}$$

which is easily solved for ϕ^*. Assuming that $\eta > 0$, it is readily shown that $\phi^* > 0$. Note that the optimal portfolio strategy does not depend on time or the investor's horizon. This occurs since the instantaneous distribution of returns does not vary over time; the instantaneous expected return, return variance, and probability of a jump are constant through time.

There are several interesting subcases for this example which are worth examining. For example, consider the subcase where $\lambda = 0$, implying that the price follows a pure diffusion. In this situation, the optimal portfolio weight is simply

$$\phi^* = \frac{\eta}{\gamma}. \tag{22}$$

Alternatively, consider the related (but non-nested) case where the price of the risky asset follows a pure jump process; where the diffusion component of the price dynamics is set equal to zero. In this situation, the optimal portfolio weight becomes

$$\phi^* = \frac{1}{\mu}\left[\left(1 - \frac{\eta}{\mu\lambda}\right)^{-\frac{1}{\gamma}} - 1\right]. \tag{23}$$

These cases make clear that the portfolio that is optimal when the price process follows a pure diffusion is very different from the optimal portfolio when the price process follows a pure jump process. When the price process follows a jump diffusion, the investor has to choose a portfolio that captures aspects of both of these special cases. Because of the non-linearity inherent in the expression for the portfolio weight in (21), however, the optimal portfolio cannot be expressed as a simple linear combination or portfolio of the optimal portfolios for the two special cases given in (22) and (23).

Differentiating ϕ^* with respect to the parameters implies the following comparative static results,

$$\frac{\partial \phi^*}{\partial \eta} > 0, \qquad \frac{\partial \phi^*}{\partial \lambda} < 0, \qquad \frac{\partial \phi^*}{\partial \gamma} < 0, \tag{24}$$

provided $\eta > 0$. Interestingly,

$$\frac{\partial \phi^*}{\partial \mu} > 0, \quad \text{if} \quad \mu < 0, \qquad \frac{\partial \phi^*}{\partial \mu} \leq 0, \quad \text{if} \quad \mu \geq 0. \tag{25}$$

To illustrate this result, the top graph in Figure 1 plots the optimal portfolio weight as a function of the value of the jump size μ. As shown, the optimal portfolio weight is highly sensitive to the size of the jump μ. When the jump is in the downward direction, the investor takes a smaller position in the risky asset than he would if jumps did not occur. Surprisingly, however, the investor also takes a smaller position when the jump is in the upward direction. The rationale for this is related to the effects of jumps on the variance and skewness of the distribution of terminal wealth. Holding fixed the risk premium, jumps in either direction increase the variance of the distribution. On the other hand, jumps also affect the skewness (and other higher moments) of the return distribution and the investor benefits from positive skewness. Despite this, the variance effect dominates and the investor takes a smaller position in the risky asset for non-zero values of μ. The skewness effect, however, explains why the graph of ϕ^* against μ is asymmetric.

To illustrate just how different portfolio choice can be in the presence of event risk, the second graph in Figure 1 plots the optimal portfolio as a function of the risk aversion parameter γ for various jump sizes μ. When $\mu = 0$ and no jumps occur, the investor takes an unboundedly large position in the risky asset as $\gamma \to 0$. In contrast, when there is any risk of a downward jump, the optimal portfolio weight is bounded above as $\gamma \to 0$. This feature is a simple implication of Proposition 1, but serves to illustrate that the optimal portfolio in the presence of event risk is qualitatively different from the optimal portfolio when event risk is not present.

This also makes clear that the optimal strategy is not driven purely by the effects of jumps on return skewness and kurtosis. For example, skewness and kurtosis effects are also present in models where volatility is stochastic and correlated with risky asset returns, but jumps do not occur. In these models, however, investors do not place endogenous bounds on their portfolio weights of the type described in Proposition 1. Furthermore, the optimal portfolio in these models does not involve any static buy-and-hold component. This underscores the point that many of the features of the optimal portfolio strategy in our framework are uniquely related to the event risk faced by the investor.

To provide some specific numerical examples, Table 1 reports the value of ϕ^* for different values of the parameters. In this table, the risk premium for the risky asset is held fixed at 7 percent and the standard deviation of the diffusive portion of risky asset returns is held fixed at 15 percent. As shown, relative to the benchmark

where $\mu = 0$, the optimal portfolio weight can be significantly less even when the probability of an event occurring is extremely low. For example, even when a negative 90 percent jump occurs at a 100-year frequency, the portfolio weight is typically much less than 50 percent of what it would be without jumps. Note that this effect is not symmetric; a positive 90 percent jump at a 100-year frequency has a much smaller effect on the portfolio weight. Also observe that the effects of jumps on portfolio weights are much more pronounced for investors with lower levels of risk aversion. This counterintuitive effect occurs because less-risk-averse investors would prefer to hold more leveraged positions, but cannot because they do not have full control over their portfolio. Thus, the effects of event risk fall much more heavily on investors with low levels of risk aversion who would otherwise be more aggressive.

4.2 Stochastic Volatility and Deterministic Jump Sizes.

In the second example, V is also allowed to follow a jump-diffusion process. The two jump sizes X and Y, however, are assumed to be constants with values μ and κ respectively. The jump size μ can be positive or negative. The jump size κ can only be positive.

In this example, the model dynamics become

$$dS_t = (r + \eta V_t - \mu\lambda V_t)\ S_t\ dt + \sqrt{V_t}\ S_t\ dZ_{1t} + \mu\ S_{t-}\ dN_t, \tag{26}$$

$$dV_t = (\alpha - \beta V_t - \kappa\lambda V_t)\ dt + \sigma\sqrt{V_t}\ dZ_{2t} + \kappa\ dN_t. \tag{27}$$

Applying the results in Proposition 2 to this model gives the following expression for the optimal portfolio weight

$$\phi^* = \frac{\eta - \mu\lambda}{\gamma} + \frac{\rho\sigma B}{\gamma} + \frac{\lambda\mu}{\gamma}(1 + \mu\phi^*)^{-\gamma} e^{\kappa B}, \tag{28}$$

which can be solved for ϕ^* jointly with the equation for B given in equation (18).

Because of the dependence on B, the optimal portfolio weight is now explicitly a function of the investor's investment horizon. Examining (28) indicates that there are several ways in which the investment horizon affects the optimal portfolio weight. Specifically, B appears in conjunction with the correlation coefficient ρ reflecting that there is a dynamic hedging component to the investor's demand for the risky asset. Since V is mean reverting, the horizon over which investment decisions are made is important. However, dynamically hedging shifts in V is not the only reason why there is time dependence in the optimal portfolio weight. For example, when $\rho = 0$, the risky asset cannot be used to hedge against shifts in the investment opportunity set arising from variation in V. Despite this, the optimal portfolio weight still depends on the investor's horizon through the $e^{\kappa B}$ term in (28). Thus,

time dependence enters the problem both through the dynamic hedging component as well as through the static hedging component.

The top graph in Figure 2 plots the optimal portfolio weight as a function of the investor's horizon for various values of the dynamic hedging parameter ρ. In this case, ϕ^* is an increasing function of the horizon for each of the values of ρ plotted. We note, however, that ϕ^* can be a decreasing function of the investor's horizon when $\gamma < 1$. This graph also illustrates that the optimal portfolio weight converges to a constant as $T \to \infty$. Furthermore, the dependence of the optimal portfolio weight on ρ indicates that an important part of the demand for the risky asset comes from its ability to dynamically hedge the continuous portion of changes in V.

An important feature of this event-risk model is that both prices and volatility are allowed to jump. The previous section illustrated that the presence of price jumps in either direction induces investors to take smaller positions in the risky asset. Intuitively, one might suspect that introducing jumps in volatility would have a similar effect on the optimal portfolio weight. Surprisingly, this is not true in general. This can be seen from the second graph in Figure 2 which plots the optimal portfolio weight as a function of the size of the volatility jump κ for different values of μ. As shown, the optimal portfolio weight can be an increasing function of κ for some values of μ.

This result illustrates the important point that in addition to its ability to dynamically hedge against continuous changes in V, the risky asset can also be used as a static hedge against the effects of jumps in V. This second hedging role is one that does not occur in traditional portfolio choice models where state variables have continuous sample paths. The fact that the risky asset can be used to hedge in two different ways, however, makes it evident that the investor faces a dilemma in choosing a portfolio strategy. In particular, the portfolio that hedges against small local diffusion-induced changes in the state variables is not the same as the portfolio that hedges against large jumps in the state variables. This problem is inherent in the fact that when there is event risk, the portfolio problem has features of both a dynamic portfolio problem and an illiquid buy-and-hold problem.

Finally, if we impose the parameter restrictions $\rho = 0$ and $\kappa = 0$, volatility is still stochastic but the optimal portfolio weight becomes the same as in Section 4.1 where volatility is not stochastic. Thus, continuous stochastic variation in V only affects the optimal portfolio weight if it is hedgable through a non-zero value of ρ.

5. IMPLICATIONS FOR PORTFOLIO CHOICE

Moving beyond the numerical examples presented in the previous section, it is useful to explore how event risk might affect the optimal portfolio of an investor in a specific market. To this end, we calibrate the model to be roughly consistent with historical

stock index returns and stock index return volatility in the U.S. To make this process as straightforward as possible, we focus on the simple stochastic volatility model with deterministic jump sizes described in the previous section. Once calibrated to U.S. data, we explore the key implications of the model for investors.

In parameterizing this model of event risk, it is important to recognize that the major financial events addressed by our model are infrequent by their nature. Ideally, we would like to use a calibration approach that minimizes the effects of the inherent 'Peso problem' on the results. Although there are many ways to do this, we use the following informal (but hopefully intuitive) approach to allow us to estimate the size and frequency of events from the longest time series available.[11]

We first obtain the monthly return series for U.S. stocks during the 1802-1925 period created and described in Schwert (1990). We then append the CRSP monthly value-weighted index returns for the 1926-2000 period to give a time series of returns spanning nearly 200 years. A review of the data shows that there are eight observations where the stock index dropped by 20 or more percent. These observations include the beginning of the Civil War in May 1861, the black Friday crash of October 1929, and the October 1987 stock market crash. Interestingly, four of the eight observations are clustered in the high-volatility decade of the 1930s, consistent with the double-jump model. Since these observations are roughly five standard deviations below the mean, it is not unreasonable to view these negative returns as being due to a jump event. A back-of-the-envelope calculation suggests calibrating the model to allow a -25 percent jump (the average of the eight observations) at an average frequency of about 25 years. To provide an rough estimate of the size of the volatility jump, we compute the standard deviation of returns for the five-month window centered at the event month. The average of these standard deviation estimates is just under 50 percent. Given this, we make the simplifying assumption that when a jump occurs, the volatility of the stock return jumps by an amount equal to the difference between 50 percent and its mean value.

The remaining parameter estimates are obtained from Table 1 of Pan (2001). Using S&P 500 stock index returns and stock index option prices, Pan estimates the parameters of several versions of a jump-diffusion model. For simplicity, we use the parameter values Pan estimates for her SV0 model, and adjust them slightly to be

[11] As a diagnostic check, we also explored a number of other calibration approaches. The basic results of our analysis, however, are robust to the specific choice of parameter estimates and little additional insight was provided by using more complex estimation techniques. Although beyond the scope of this paper, we note that the general double-jump model could be formally estimated using either the efficient method of moments (EMM) approach applied by Andersen, Benzoni, and Lund (2001) or the Monte Carlo Markov chain (MCMC) technique used by Eraker, Johannes, and Polson (2000).

consistent with our estimates of jump sizes and frequencies.[12] Specifically, we use Pan's estimates of $\beta = 5.3$ and $\rho = -.57$. To obtain estimates of η, α and σ, we note that in our model, the expected instantaneous equity premium is $\alpha\eta/\beta$, the expected instantaneous variance of returns is $\alpha(1+\mu^2\lambda)/\beta$, and the the expected instantaneous variance of changes in V is $\alpha(\sigma^2 + \kappa^2\lambda)/\beta$. Setting these three moments equal to the corresponding estimates of .1055, .0242, and .3800 from Table 1 of Pan provides us with three equations which can be solved for the values of η, α, and σ. By doing this, we guarantee that the calibrated model matches the moments of returns and volatility estimated by Pan. This approach leads to the following parameter values for the baseline case where jumps occur with an average frequency of 25 years: $\alpha = .11512$, $\beta = 5.3000$, $\sigma = .22478$, $\eta = 4.90224$, $\rho = -.57000$, $\mu = -.25000$, $\kappa = .22578$, and $\lambda = 1.84156$.

To illustrate the effects that event risk has on the optimal portfolio choice for an investor where the model is calibrated to historical U.S. returns in this manner, Table 2 reports the portfolio weights for various levels of investor risk aversion. To facilitate comparison, we report the portfolio weights for the case where there are no jumps, where there are only jumps in the stock index, and the baseline case where there are jumps in both the stock index and volatility. Note that for the non-benchmark cases, we recalibrate the model so that we match the expected instantaneous moments estimated by Pan (2001) using the procedure described in the previous paragraph. In each case, the investor has a five-year investment horizon.

Table 2 shows that the possibility of a 25 percent downward jump in stock prices has an important effect on the investor's optimal portfolio, even though this type of event happens only every 25 years on average. For example, the optimal portfolio weight for an investor with a risk aversion parameter of two is 2.305 if no jumps can occur, is 1.929 if only jumps in the stock price can occur, and is 2.010 if both jumps in stock prices and volatility can occur. Observe that from Proposition 1, the investor never takes a position in the risky asset greater than 4 since jumps of -25 percent can occur. Table 2 shows that the risk of a downward jump always induces the investor to take a smaller position in the stock market than he would otherwise.

Table 2 also makes clear that while jumps in volatility do not have as much of an effect as jumps in the stock price, they do have a significant influence on the optimal portfolio. Interestingly, jumps in volatility decrease the optimal portfolio weight when $\gamma < 1$, and increase the optimal portfolio weight when $\gamma > 1$. Thus, for higher levels of risk aversion, the effect of a jump in volatility on the optimal

[12]The advantage of using the parameter estimates for Pan's SV0 model is that they represent parameter estimates for the stochastic volatility model in the absence of jumps. This then allows us to calibrate the model for different jump sizes using a particularly simple algorithm. As pointed out by Pan, allowing for jumps significantly enhances the ability of the stochastic volatility model to capture the properties of the data.

portfolio weight is to partially offset the effect of a jump in the value of the asset on the portfolio weight. Intuitively, the reason for this is that a jump in volatility not only increases the risk of portfolio returns, but also increases the expected risk premium on the stock. For $\gamma < 1$, the positive effect of a persistent volatility shock on expected returns offsets the effect on portfolio return riskiness. Thus, an investor with $\gamma < 1$ invests more in the risky asset than he would if there were no volatility shocks. This effect can also be viewed as a consequence of the investor's preference for positive serial correlation in his intertemporal returns.

Although we have calibrated the model to historical U.S. returns, it is important to recognize that U.S. returns may not fully reflect the size of potential jump events. The reason for this is the possibility of a survivorship bias since the U.S. has experienced historically high returns. This point is also consistent with Jorion and Goetzmann (1999) who show that many countries have experienced huge market declines during relatively short periods of time during the past century. In many cases, major events such as wars or political crises have actually led to stock markets being closed for years (or even decades). These closures have often resulted in catastrophic losses for investors. To reflect this downside risk to financial markets, we also consider a scenario where stock market jumps of -50 percent and volatility jumps to 70 percent occur at an average frequency of 100 years. Following the same calibration approach as before implies parameter values for this scenario of $\alpha = .11512$, $\beta = 5.3000$, $\sigma = .21099$, $\eta = 4.90224$, $\rho = -.57000$, $\mu = -.50000$, $\kappa = .46578$, and $\lambda = .46029$.

Table 3 reports the optimal portfolio weights for this alternative scenario. Even though the frequency of an event is much less, it has an even larger effect on the optimal portfolio weight than in Table 2. For example, the optimal portfolio weight for an investor with a risk aversion parameter of two is still 2.305 if no jumps can occur. If only jumps in the stock price can occur, then the portfolio weight is now 1.395 rather than 1.929. If both jumps in the stock price and volatility can occur, the optimal portfolio weight is now 1.481 rather than the value of 2.010 given in Table 2. As before, jumps in volatility increase the optimal portfolio weight for $\gamma < 1$, and vice versa.

Admittedly, we have focused only on simple calibrations of one of the simplest versions of the model. Despite this, however, we believe that several important general insights about the role that event risk could play in real-world portfolio decisions emerge from this analysis. Foremost among these is that investors have strong incentives to significantly reduce their exposure to the stock market when they believe that there is event risk. This is true even when the probability of a major downward jump in stock prices is very small, as in the scenario of a -50 percent jump occurring every 100 years on average. Certainly, jumps of this magnitude and frequency cannot be ruled out; it is all too easy to think of extreme situations where a downward jump of this magnitude could occur during the next century even in the U.S. Our analysis suggests a possible reason why historical levels of participation in

the stock market have been much lower than standard portfolio choice models would view as optimal.[13]

6. CONCLUSION

In this paper, we study the effects of event-related jumps in prices and volatility on investment strategies. Using the double-jump framework of Duffie, Pan, and Singleton (2000), we take advantage of the affine structure of the model to provide analytical solutions to the optimal portfolio problem.

The presence of event risk changes the standard portfolio problem in several important ways. First, since the investor no longer has complete control over his wealth, the investor acts as if some part of his portfolio consists of illiquid assets and he is much less willing to take leveraged or short positions. The optimal portfolio strategy blends elements of both a standard dynamic hedging strategy and a buy-and-hold or 'illiquidity' hedging strategy. Furthermore, event risk affects investors with low levels of risk aversion more that it does highly risk-averse investors. These results illustrate that the implications of event risk for the optimal portfolio strategy are both subtle and complex. Our analysis suggests that jumps in both prices and volatility have important effects on optimal portfolios. Finally, our results suggest that if market participants believe that there is even a remote chance of a sudden market collapse, their portfolio behavior could be very different from that implied by classical portfolio choice models which abstract from event risk.

This paper is only a first attempt to systematically study the effect of event risk on optimal portfolio choice. Along with other studies in the field of asset allocation, we use a partial equilibrium approach by taking prices as given. Clearly, however, an equilibrium study would be necessary to provide a complete understanding of the interaction between price dynamics and investor's portfolio choices. Nevertheless, we hope that this partial equilibrium study provides some understanding of the complete picture.

[13]For example, see Mankiw and Zeldes (1991), Heaton and Lucas (1997), and Basak and Cuoco (1998).

APPENDIX

Proof of Proposition 1.

Let $\{W_t^*, 0 \leq t \leq T\}$ be the wealth process attained by an investor who follows the optimal portfolio process ϕ^*. We first remark that W_T^* must be positive almost surely. Otherwise, a non-zero probability of $W_T^* \leq 0$ will result in $E[U(W_T^*)] = -\infty$, which is inferior to investing all of the positive initial wealth in the riskless asset.

We next show that for W_T^* to be positive almost surely, W_t^* must be positive almost surely for any $t < T$. To see this, we first condition on the event that there is no jump between t and T. This implies

$$W_T = W_t \exp\left(\int_t^T \left(r + \phi_\tau(\eta - \mu\lambda)V_\tau - \frac{\phi_\tau^2 V_\tau}{2}\right) d\tau + \phi_\tau \sqrt{V_\tau}\, dZ_1\right),$$

for any portfolio policy ϕ. Such an event of no jump between t and T has a positive probability given the assumption that

$$0 < E_t\left[\exp\left(-\int_t^T \lambda V_\tau\, d\tau\right)\right] < 1.$$

So $W_T^* > 0$ almost surely implies $W_t^* > 0$ almost surely for any t.

Finally, we show that for $W_t^* > 0$ almost surely, the optimal portfolio weight ϕ^* must satisfy (7). Suppose (7) is not satisfied for some t. Then there is a positive probability of a jump event between t and $t + \Delta t$ for some $\Delta t > 0$. Conditioning on such a jump event, the time-t wealth is $W_t = W_{t-}(1 + \phi X)$, where W_{t-} is the wealth before the jump event, and where X is the jump size. By the definition of X_{Inf} and X_{Sup}, we have for an arbitrary $\epsilon > 0$, a positive probability of $X \in (X_{\text{Inf}}, X_{\text{Inf}} + \epsilon)$ and a positive probability of $X \in (X_{\text{Sup}} - \epsilon, X_{\text{Sup}})$. Thus, if (7) is not satisfied, there is a positive probability of $W_t^* \leq 0$, which contradicts the assumption that W_t^* is the wealth process generated by the optimal portfolio weight ϕ^*.

Proof of Proposition 2.

Suppose that the indirect utility function J is of the conjectured form in (11) with state-independent time-varying coefficients $A(t)$ and $B(t)$ to be determined shortly. Then the first-order condition of the HJB equation (10) implies

$$\begin{aligned}\phi_t^* &= -\frac{J_W}{W J_{WW}}\left((\eta - \mu\lambda) + \rho\sigma\frac{J_{WV}}{JW} + \lambda M_1 \frac{J}{W J_W}\right) \\ &= \frac{\eta - \mu\lambda}{\gamma} + \frac{\rho\sigma B}{\gamma} + \frac{\lambda M_1}{\gamma},\end{aligned}$$

which is the optimal portfolio weight given in (17). It should be noted that ϕ^* is state independent and a non-linear function of B.

We now proceed to derive the ordinary differential equations for the time-varying coefficients $A(t)$ and $B(t)$, under which the conjectured form (11) for the indirect utility function J indeed satisfies the HJB equation (10). For this, we substitute (11) and (12) into the HJB equation and obtain,

$$-\frac{\gamma\phi^{*2}V}{2}+\phi^*\rho\sigma BV+\frac{\sigma^2B^2V}{2(1-\gamma)}+(r+\phi^*(\eta-\mu\lambda)V)$$
$$+(\alpha-\beta V-\kappa V)\frac{B}{1-\gamma}+\frac{\lambda V}{1-\gamma}M_2-\frac{\lambda V}{1-\gamma}+\frac{1}{1-\gamma}(A'+B'V)=0.$$

The left-hand-side of this expression is an affine function in V. For this expression to hold for all V, the constant term and the linear coefficient of V on the left-hand-side must be set equal to zero separately, which leads to the ordinary differential equation for $B(t)$ given in (18) and the following ordinary differential equation for $A(t)$

$$A'+\alpha B+(1-\gamma)r=0.$$

REFERENCES

Aase, K., 1984, Optimum Portfolio Diversification in a General Continuous-Time Model, *Stochastic Processes and Their Applications* 18, 81-98.

Aase, K., 1986, Ruin Problems and Myopic Portfolio Optimization in Continuous Trading, *Stochastic Processes and Their Applications* 21, 213-227.

Aase, K., and B. Øksendal, 1988, Admissible Investment Strategies in Continuous Trading, *Stochastic Processes and Their Applications* 30, 291-301.

Andersen, T., L. Benzoni, and J. Lund, 2001, An Empirical Investigation of Continuous-Time Equity Return Models, *Journal of Finance*, forthcoming.

Ang, A. and G. Bekaert, 2000, International Asset Allocation with Time-Varying Correlations, Working paper, Columbia University.

Bakshi, G., C. Cao, and Z. Chen, 1997, Empirical Performance of Alernative Option Pricing Models, *Journal of Finance* 52, 2003-2049.

Balduzzi, P. and A. Lynch, 1999, Transaction Costs and Predictability: Some Utility Cost Calculations, *Journal of Financial Economics* 52, 47-78.

Barberis, N., 2000, Investing for the Long Run when Returns are Predictable, *Journal of Finance* 55, 225-264.

Bardhan, I., and X. Chao, 1995, Martingale Analysis for Assets with Discontinuous Returns, *Mathematics of Operations Research* 20, 243-256.

Basak, S., and D. Cuoco, 1998, An Equilibrium Model with Restricted Stock Participation, *Review of Financial Studies* 11, 309-341.

Bates, D., 1996, Jump and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options, *Review of Financial Studies* 9, 69-107.

Bates, D., 2000, Post-'87 Crash Fears in S&P 500 Futures Options, *Journal of Econometrics* 94, 181-238.

Brennan, M., E. Schwartz, and R. Lagnado, 1997, Strategic Asset Allocation, *Journal of Economic Dynamics and Control* 21, 1377-1403.

Brennan, M. and Y. Xia, 1998, Resolution of a Financial Puzzle, Working paper, UCLA.

Campbell, J. and L. Viceira, 1999, Consumption and Portfolio Decisions when Expected Returns are Time varying, *The Quarterly Journal of Economics* 114, 433-495.

Campbell, J. and L. Viceira, 2001, Who Should Buy Long-Term Bonds? *American Economic Review*, forthcoming.

Chacko, G. and L. Viceira, 2000, Dynamic Consumption and Portfolio Choice with Stochastic Volatility in Incomplete Markets, Working paper, Harvard University.

Common, E., 2000, Extreme Events and the Role of Learning in Financial Markets, Working paper, Harvard University.

Das, S. and R. Uppal, 2001, Systemic Risk and International Portfolio Choice, Working paper, Santa Clara University.

Duffie, D., J. Pan, and K. Singleton, 2000, Transform Analysis and Asset Pricing for Affine Jump Diffusion, *Econometrica* 68, 1343-1376.

Dybvig, P. and C.-F. Huang, 1988, Nonnegative Wealth, Absence of Arbitrage, and Feasible Consumption Plans, *Review of Financial Studies* 1, 377-401.

Eraker, B., M. S. Johannes, and N. G. Polson, 2000, The Impact of Jumps in Returns and Volatility. Working paper, The University of Chicago.

Harrison, J. M. and D. M. Kreps, 1979, Martingales and Arbitrage in Multiperiod Securities Markets, *Journal of Economic Theory* 20, 381-408.

Haugh, M., and A. Lo, 2001, Asset Allocation and Derivatives, *Quantitative Finance* 1, 45-72.

Heaton, J., and D. Lucas, 1997, Market Frictions, Savings Behavior, and Portfolio Choice, *Macroeconomic Dynamics* 1, 76-101.

Heston, S., 1993, A Closed-Form Solution of Options with Stochastic Volatility with Applications to Bond and Currency Options, *Review of Financial Studies* 6, 327-343.

Jeanblanc-Picqué, M., and M. Pontier, 1990, Optimal Portfolio for a Small Investor in a Market Model with Discontinuous Prices, *Applied Mathematics and Optimization* 22, 287-310.

Kim, T. S. and E. Omberg, 1996, Dynamic Nonmyopic Portfolio Behavior, *Review of*

Financial Studies 9, 141-161.

Kraus, A. and R. Litzenberger, 1976, Skewness Preference and the Valuation of Risky Assets, *Journal of Finance* 31, 1085-1100.

Liu, J., 1999, Portfolio Selection in Stochastic Environments, Working paper, UCLA.

Longstaff, F. A., 2001, Optimal Portfolio Choice and the Valuation of Illiquid Assets, *Review of Financial Studies* 14, 407-431.

Lynch, A., 2000, Portfolio Choice and Equity Characteristics: Characterizing the Hedging Demand Induced by Return Predictability, *Journal of Financial Economics* forthcoming.

Mankiw, G., and S. Zeldes, 1991, The Consumption of Stockholders and Nonstockholders, *Journal of Financial Economics* 29, 97-112.

Merton, R., 1971, Optimum Consumption and Portfolio Rules in a Continuous-Time Model, *Journal of Economic Theory* 3, 373-413.

Merton, R., 1980, On Estimating the Expected Return on the Market: An Exploratory Investigation, *Journal of Financial Economics* 8, 232-361.

Pan, J., 2001, The Jump-Risk Premia Implicit in Option Prices: Evidence from an Integrated Time-Series Study, *Journal of Financial Economics*, forthcoming.

Schroder, M. and C. Skiadis, 1999, Optimal Consumption and Portfolio Selection with Stochastic Differential Utility, *Journal of Economic Theory* 89, 68-126.

Schwert, W., 1990, Index of United States Stock Prices from 1802 to 1987, *Journal of Business* 63, 399-426.

Wachter, J., 1999, Risk Aversion and Allocations to Long-Term Bonds, Working paper, New York University.

Wu, L., 2000, Jumps and Dynamic Portfolio Decisions, Working paper, Fordham University.

Xia, Y., 2000, Learning about Predictability: The Effect of Parameter Uncertainty on Dynamic Asset Allocation. Working paper, The University of Pennsylvania.

Table 1

Portfolio Weights with Constant Volatility and Deterministic Price Jump Size. This table reports the portfolio weights for the risky asset in the case where the volatility of the asset's returns is constant and the percentage size of the jump in the asset's price is also constant. The risk premium for the risky asset is held fixed at 7 percent and the volatility of diffusive returns is held fixed at 15 percent throughout the table. The frequency of jumps is expressed in years and equals the reciprocal of the jump intensity.

Risk Aversion Parameter	Frequency of Jumps	Percentage Jump Size				
		−90	−20	0	20	90
.50	1	.151	1.795	6.222	2.736	.189
	2	.269	2.511	6.222	3.970	.411
	5	.508	3.431	6.222	5.161	1.234
	10	.721	4.008	6.222	5.662	2.600
	100	1.091	4.927	6.222	6.163	5.744
1.00	1	.078	.970	3.111	1.289	.091
	2	.144	1.394	3.111	1.891	.190
	5	.290	1.963	3.111	2.516	.529
	10	.444	2.333	3.111	2.793	1.111
	100	.938	2.980	3.111	3.077	2.824
2.00	1	.040	.504	1.556	.624	.045
	2	.074	.730	1.556	.919	.092
	5	.155	1.033	1.556	1.238	.244
	10	.247	1.222	1.556	1.384	.503
	100	.641	1.509	1.556	1.537	1.395
5.00	1	.016	.206	.622	.245	.018
	2	.030	.300	.622	.361	.036
	5	.065	.424	.622	.490	.093
	10	.105	.499	.622	.550	.188
	100	.305	.606	.622	.614	.553

Table 2

Portfolio Weight Comparisons for the Calibrated Model where Jumps Occur Every 25 Years on Average. This table reports portfolio weights for the stochastic volatility model with deterministic jumps in prices and volatility. The average frequency of an event is 25 years. The first column reports the portfolio weights when the jump sizes are both zero (no jumps). The second column reports the portfolio weights when the stock price jump is −25 percent and the volatility jump is zero (stock jumps only). The third column reports the baseline case where the stock price jump is −25 percent and the volatility jumps to 50 percent. Each scenario is calibrated to match the parameter estimates in Table 1 of Pan (2001).

Risk Aversion Parameter	No Jumps	Stock Jumps Only	Both Stock and Volatility Jumps
.50	8.106	3.914	3.865
1.00	4.396	3.163	3.163
2.00	2.305	1.929	2.010
3.00	1.564	1.356	1.432
4.00	1.183	1.042	1.107
5.00	.952	.845	.901

Table 3

Portfolio Weight Comparisons for the Calibrated Model where Jumps Occur Every 100 Years on Average. This table reports portfolio weights for the stochastic volatility model with deterministic jumps in prices and volatility. The average frequency of an event is 100 years. The first column reports the portfolio weights when the jump sizes are both zero (no jumps). The second column reports the portfolio weights when the stock price jump is −50 percent and the volatility jump is zero (stock jumps only). The third column reports the baseline case where the stock price jump is −50 percent and the volatility jumps to 70 percent. Each scenario is calibrated to match the parameter estimates in Table 1 of Pan (2001).

Risk Aversion Parameter	No Jumps	Stock Jumps Only	Both Stock and Volatility Jumps
.50	8.106	1.993	1.987
1.00	4.396	1.859	1.859
2.00	2.305	1.395	1.481
3.00	1.564	1.059	1.174
4.00	1.183	.844	.956
5.00	.952	.698	.801

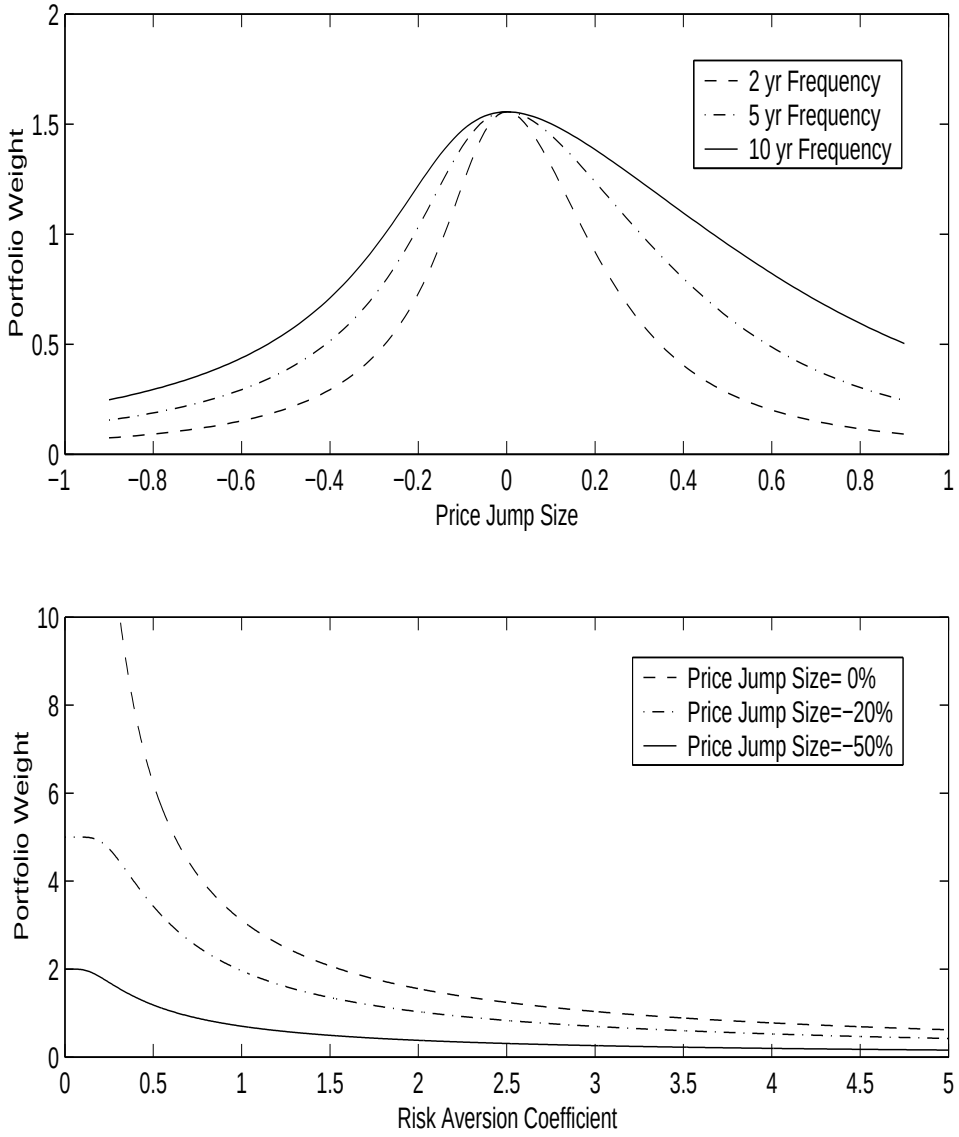

Figure 1. Optimal Portfolio Weights for the Constant-Volatility Case. The top panel graphs the optimal portfolio weight as a function of the size of the price jump for three different values of the jump frequency. The bottom panel graphs the optimal portfolio weight as a function of the risk aversion coefficient for three different values of the size of the price jump.

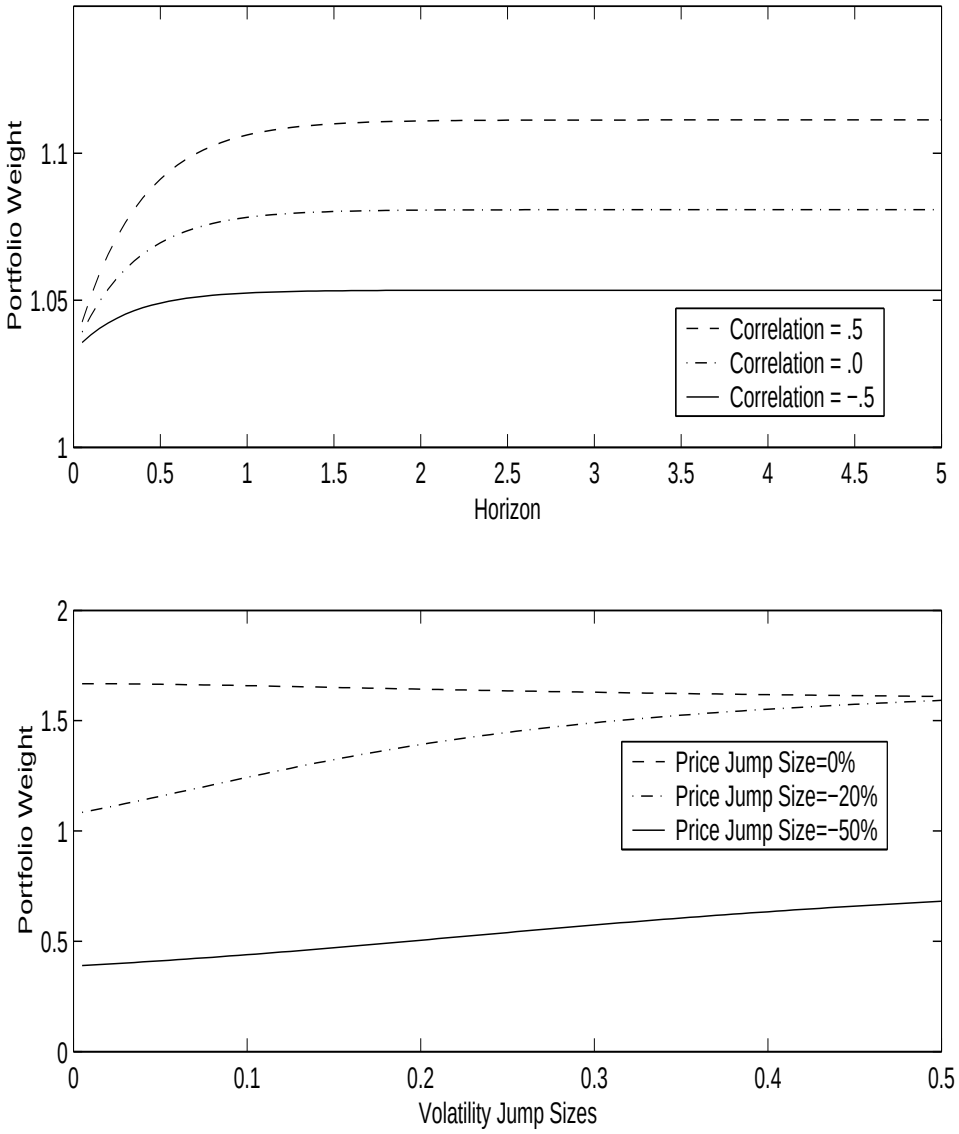

Figure 2. Optimal Portfolio Weights for the Stochastic-Volatility Case. The top panel graphs the optimal portfolio weight as a function of the investor's horizon measured in years for three different values of the correlation coefficient. The bottom panel graphs the optimal portfolio weight as a function of the size of the volatility jumps for three different values of the size of the price jump.