The American put option is continuously compounded.

The value of the American put option is given by the formula:

\[ P = e^{-rT} \text{max}(K - S, 0) \]

where:
- \( P \) is the price of the American put option.
- \( S \) is the current stock price.
- \( K \) is the exercise price.
- \( r \) is the risk-free interest rate.
- \( T \) is the time to expiration.

This formula is derived from the Black-Scholes model, with adjustments for the early exercise feature of American options.
The option price is derived from the Put-Call parity theorem, which states that:

\[ X - e^{-rT} S_0 \geq C \geq X - e^{-rT} P \]

where:
- \( X \) is the strike price of the option,
- \( S_0 \) is the current stock price,
- \( r \) is the risk-free interest rate,
- \( T \) is the time to expiration,
- \( C \) is the call option price,
- \( P \) is the put option price.

This inequality shows that the option price is bounded by the intrinsic value of the option and its time value. The upper bound is the intrinsic value plus the time value, and the lower bound is the intrinsic value minus the present value of the exercise fee.

**Proposition**

The price of a European call option is greater than or equal to the price of an equivalent American call option under certain conditions. The conditions are:

1. The stock price \( S_0 \) is above the exercise price \( X \).
2. The risk-free interest rate \( r \) is greater than zero.
3. The time to expiration \( T \) is positive.

In this case, the American call option will be exercised immediately if the stock price is above the exercise price, and the European call option will be exercised only at expiration. Therefore, the American call option will be more valuable than the European call option, leading to a higher option price for the American call option.

**Conclusion**

The option price is derived from the fundamental theorem of financial economics, where the option price is a function of the underlying asset's price, the risk-free interest rate, the time to expiration, and the strike price. The Put-Call parity theorem provides a relationship between call and put option prices, and the American call option price is bounded by the intrinsic value and the present value of the exercise fee.
\[ x_t = (1 - \lambda)^{\theta} (x_{t-1} - l^t \Delta x) + \lambda x_t \]

The values of components (a) and (b) in Proposition II are then given by the

\[ x_t = x + \Delta x \]

\[ x_t = [l^t \Delta x - (x_{t-1} - l^t \Delta x)] N(\Delta x, \Delta x^2) - [(d^t + l^t \Delta x) N(\Delta x, \Delta x^2)] N(\Delta x, \Delta x^2) \]

and

\[ x_t = [(l^t \Delta x - (x_{t-1} - l^t \Delta x)] N(\Delta x, \Delta x^2) - [(x_{t-1} - l^t \Delta x)] N(\Delta x, \Delta x^2) \]

The Black-Scholes formula

\[ \text{Black-Scholes formula} \]

is used for pricing the call option, which is the derivative of the underlying asset. The option is exercised if and only if the payoff is positive. The formula for the price of a call option on an underlying asset is given by:

\[ C = \text{Max} \left( 0, (S - K)^+ \right) \]

where:

- \( S \) is the current price of the underlying asset
- \( K \) is the strike price
- \( (S - K)^+ \) is the excess of the current price over the strike price

The Black-Scholes model assumes that the underlying asset follows a geometric Brownian motion. The model is used to price options and to determine the risk-neutral probability distribution of the underlying asset. The model is characterized by the following stochastic differential equation:

\[ dS_t = \mu S_t dt + \sigma S_t dW_t \]

where:

- \( S_t \) is the price of the underlying asset at time \( t \)
- \( \mu \) is the drift or expected return
- \( \sigma \) is the volatility
- \( dW_t \) is the increment of a standard Brownian motion