The Effects of Leverage

On

The Pricing S&P 500 Index Call Options

By

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We thank Richard Roll for comments on an early draft of this paper. As this research is preliminary please do not quote or distribute without the author's permission. *Contact Geske by mail at The Anderson School at ***UCLA***, 110 Westwood Plaza, Los Angeles, California 90095, USA, by telephone at 310-825-3670, or by e-mail. rgeske@anderson.ucla.edu.

Abstract

The purpose of this paper is to examine whether leverage has a significant statistical and economic effect on the pricing of S&P 500 index options. This is the first paper to directly test for leverage effects in stock index options. To analyze these effects we use the Geske (1979) compound option model. The Geske model is closed form, implies stochastic equity volatility, is consistent with Modigliani and Miller, incorporates debt refinancing, and includes possibly differential default and bankruptcy. Black-Scholes (1973) is a special case of the Geske model. In this paper we show that during the years 1996-2004 the aggregate *market* based debt to equity (D/E) ratio of the firms comprising the S&P 500 equity index varies from about 40-120 percent. We believe this is the first presentation of a *market* D/E ratio derived from option theory. Next and more importantly we are the first to report the details of the statistically significant economic effects that market leverage has on pricing S&P 500 index call options. We measure that the Geske model improves the net option valuation of listed in the money (or out of the money) S&P 500 index call options on average by about 35% (28%) compared to Black-Scholes values. We demonstrate that the improvement is directly (and monotonically) related to both the time to expiration of the option and the amount of leverage in this market index. For options with longer expirations and/or periods of higher market leverage the improvement is greater, ranging from about 40% to 80%. We also demonstrate economic significance in basis points by showing that dealers making a book in index options can expect benefits of at least several 100 basis points using Geske instead of Black-Scholes.

1. Introduction

S&P 500 equity index options are the world's most widely traded index option. In the year 2004 when our data ends, the S&P 500 index option (SPX) volume was more than 49 million contracts compared to about 16 million contracts for the next most active equity index option, the S&P 100 (OEX), respectively.[1] The SPX options are European and the OEX options are American. The fact that the SPX options cannot be exercised early makes them good candidates for examining if there are any leverage effects in option prices, independent of the American early exercise premium.

When S&P 500 index option trading began in 1983, it was initially thought that the Black-Scholes model (1973) should do better pricing these options on a portfolio of stocks because they are European and because the central limit theorem suggests that the sum of returns on a large number of random variables is likely to be normally distributed, as Black-Scholes assumed. Since that time much research has shown that the Black-Scholes model does not do as well on these European index options as initially hoped.

Rubinstein (1978) demonstrated for individual stock options that Black-Scholes model exhibited both time and strike price biases with reversals which appeared to be "period dependent" and argued that alternative models may not be able to explain these periodic reversals. It is often thought that the biases in Black-Scholes arise because of stochastic volatility. Heston (1993) (Hull White (1987) and others) develops a closed-form stochastic volatility model with arbitrary correlation between volatility and asset returns and demonstrates that this model has the ability to improve on the Black-Scholes biases when the correlation is negative. Rubinstein (1994) (and others) develops a lattice approach to

[1] See CBOE Market Stats 2004.

best fit the cross-sectional structure of option prices wherein the volatility can depend on the asset price and time. Dumas, Fleming and Whaley (1998) describe the approach of Rubinstein and others as a deterministic volatility function (DTV) and find that these implied tree approaches work no better than an *ad* hoc version of Black-Scholes where the implied volatility is modified for strike price and time. Bakshi, Cao, and Chen (1997) also test stochastic volatility jump models compared to Black-Scholes with a focus on hedging errors, and demonstrate improvements. However, similar to Dumas, et al, when they allow Black-Scholes *ad hoc* modifications to be both delta and vega neutral, the advantage of stochastic volatility is diminished. Bates (2000) describes the post 1987 crash period in which he and others state the volatility smile became a negative skew. He documents Black-Scholes biases pricing S&P 500 equity index options, and he tests a model with stochastic volatility and jumps that shows some improvement. Heston and Nandi (2000) develop a closed-form GARCH option valuation model which exhibits the required negative skew and contains Heston's (1993) stochastic volatility model as a continuous time limit. They demonstrate that their out of sample valuation errors are lower than the *ad hoc* modified version of Black-Scholes which Dumas, Fleming, and Whaley developed.

Much of the literature described above which follows Black-Scholes (1973) is based on the idea that their distributional assumption of stock returns being instantaneously normally distributed with a constant instantaneous conditional volatility is not realistic. For example, the observed frequency of stock market crashes (1929 and 1987) cannot occur under this assumption. Stochastic volatility arising from crash fears is often modeled as variants of Merton's (1976) jump diffusion model. Alternatively, there is extensive evidence of an inverse relation between the level of equity prices and instantaneous conditional volatility observed for individual firms (Black, 1976) and for indexes (Nelson, 1991). This inverse relation has long been thought to be leverage related, and more recently has been described as "volatility feedback" effects (Campbell and Hentschel, 1992) where changes in volatility change the discount rates for future cash flows and dividends.

This paper is the first to directly test the extent of the leverage effect in S&P 500 index options by measuring and using the actual daily leverage of the index. We test the leverage effect with the closed-form model of Geske (1979). As originally derived Geske's compound option model requires the current total (total herein means debt + equity) market value of the 500 firms in the index, and the instantaneous volatility of the rate of growth of this total market value, and both are not directly observable. This problem is circumvented by observing the equity index price and the price of a call option on the index, and then solving three simultaneous equations for the total market value, V, market return volatility, σ_V , and the critical total market value, V*, for the option exercise boundary. The rest of the paper proceeds as follows. Section 2 describes the models and how they are implemented. Section 3 describes the data and explains in more detail how the necessary data inputs are calculated. Section 4 describes our results and reports both statistical and economic significance. Section 5 concludes the paper.

2. Models

The assumptions underlying the Black-Scholes model and their resulting equation are better known than the same for Geske's compound option model. Thus, to review, recall that Geske's option model applied to listed individual equity or index options transforms the state variable underlying the option from the stock to the total market value of the 500 firms comprising the index. In this case the volatility of the equity index will be random and inversely related to the value of the market equity. This model is consistent with Modigliani and Miller, and allows for default on the debt and bankruptcy. The Black-Scholes model is a special case of Geske's model which will reduce to their equation when either the dollar amount of leverage is zero or when the leverage is perpetuity. The boundary condition for the exercise of an option is also transformed from depending on the strike price and stock price to depending on a critical total market value, V*. If the conditions Black and Scholes assume for the stock price are assumed for the total market, then this results in the following equation for pricing S&P 500 index call options:[2]

$$C = VN_2(h_1 + \sigma_v\sqrt{T_1 - t}, h_2 + \sigma_v\sqrt{T_2 - t}; \rho) - Me^{-r_{F2}(T_2 - t)}N_2(h_1, h_2; \rho) - Ke^{-r_{F1}(T_1 - t)}N_1(h_1) \quad (1)$$

where

$$h_1 = \frac{\ln(V/V^*) + (r_{F1} - 1/2\sigma_v^2)(T_1 - t)}{\sigma_v\sqrt{T_1 - t}}$$

$$h_2 = \frac{\ln(V/M) + (r_{F2} - 1/2\sigma_v^2)(T_2 - t)}{\sigma_v\sqrt{T_2 - t}}$$

$$\rho = \sqrt{(T_1 - t)/(T_2 - t)}\,.$$

[2] See Geske (1979) for more detail, and to repeat, think of stock value as market equity index level and firm value as total market value of stock and debt outstanding for 500 firms each day.

Here V* at option expiration date t=T_1 is the critical total market value at which the equity index level, S_{T1} = K, and S_{T1} is deduced from Merton's application of the Black-Scholes equation which treats stock as an option:

$$S = V N_1(h_2 + \sigma_v \sqrt{T_2 - t}) - M e^{-r_{F2}(T_2 - t)} N_1(h_2) \qquad (2)$$

and thus at t=T_1 where S_{T1} = K,

$$S_{T1} = V_{T1}^{*} N_1(h_2 + \sigma_v \sqrt{T_2 - T_1}) - M e^{-r_{F2}(T_2 - T_1)} N_1(h_2) = K \qquad (3)$$

where h_2 is given above. The face value of all 500 firm's debt outstanding is M and T_2 is the duration of this debt. For Geske's compound option there are two correlated exercise opportunities at T_1 for the call option and at T_2 for the debt duration. The correlation is measured by $\rho = \sqrt{(T_1 - t)/(T_2 - t)}$ where index option expiration T_1 is less than or equal to market debt duration, T_2. When the firm has no debt or when the debt is perpetuity, V = S and σ_v = σ_s, and equation (1) reduces to the well known Black-Scholes equation:

$$C = S N_1(h_1 + \sigma_v \sqrt{T_1 - t}) - K e^{-r_{F1}(T_1 - t)} N_1(h_1) \qquad (4)$$

The notation for these models can be summarized as follows:

C = current market value of an index call option

S = current market value of the S&P equity index net of dividends D,

V = current total (debt + equity) market value of 500 firms in the S&P,

V* = critical total market value where V $\geq$ V* implies S$\geq$K,

M = face value of market debt (debt outstanding for S&P 500 firms),

K = strike price of the option,

r_{Ft} = the risk-free rate of interest to date t,

σ_v = the instantaneous volatility of the total market return,

σ_s = the instantaneous volatility of the equity index return,

t = current time,

T_1 = expiration date of the option,

T_2 = duration of the market debt,

$N_1(.)$ = univariate cumulative normal distribution function,

$N_2(\ldots)$ = bivariate cumulative normal distribution function,

ρ = correlation between the two option exercise opportunities at T_1 and T_2.

D = dividends

Because of leverage the volatility of an option is always greater than or equal to the volatility of the underlying state variable, and given these assumptions the exact relation between the volatility of the equity index and the volatility of the total market is expressed as follows:[3]

$$\sigma_s = \frac{\partial S}{\partial V} \frac{V}{S} \sigma_v \qquad (5)$$

Thus, while Black-Scholes assume the equity's return volatility is not dependent on the equity level, Geske's model implies that the volatility of the equity's return is

[3] See Geske (1979) for details.

inversely related to the equity index level. When the equity index level drops (rises), assuming the market does not react, the market leverage rises (falls), and the equity index volatility also rises (falls). In both models we adjust the equity index level for dividends. In the next section we describe the data necessary to test for the presence of any leverage effects in index call option prices.

3. Data Collected and Constructed

In order to test for the effects of leverage on S&P 500 index call options we need option price data, stock price data, stock dividend data, interest rate data, and balance sheet information. We also require the composition of the S&P 500 firms on a daily basis. We collect daily closing stock prices, daily shares outstanding, and the daily composition of the S&P 500 index from CRSP. The interest rate data are daily from the Federal Reserve for government securities with maturities ranging from 1 month to 10 years, which we adjust to use as discount factors for the market debt, option strike prices, and dividends. The option prices we use are from Option Metrics from January 4, 1996 through April 30, 2004. This 100 month sample period covering 8 1/3 years has about 2080 observation days. The data are the daily closing prices, if there was a trade at the close, or the closing *best* bid and *best* ask as a spread, which we average for the closing option price.[4] We also collect the option volume and open interest data and dividend data for the S&P 500 from Option Metrics. The at the money S&P 500 index options have the highest daily volume of all traded equity options and thus they should not exhibit much non-synchronicity.

[4] Option Metrics takes the *best* bid and ask from the exchange (CBOE, Phlx, Amex, Ise) that has the trade closest to the closing stock price which for the active at the money S&P 500 index options will almost always be synchronous. The in and out of the money options which trade less frequently may be non-synchronous.

However, in order to further minimize non-synchronous problems, first we check to see if there was an option trade on that day. Next we check to see if arbitrage bounds are violated ($C \leq S - K\,e^{-r_T T}$) and eliminate these option prices. If non-synchronicity occurred because the stock price moved up after the less liquid in or out of the money option last traded, then option under-pricing would be observed, and some of these options would be removed by the above arbitrage check. If non-synchronicity occurred because the stock price moved down after the less liquid in or out of the money option last traded, then option over-pricing would be observed. Because we cannot perfectly eliminate non-synchronous pricing for the in and out of the money options with this data base we keep track of the amount of under and over-pricing in order to relate this miss-pricing to the resultant under (over) pricing of in (out of) the money index call options.

The balance sheet information we collect from S&P's annual and quarterly Compustat. This debt data is categorized as due in years 1 through 5 (Data 44, 91,92,93,94), and greater than 5 (Data 9 – (91-94)), which we place at 7 years. To these categories we add current liabilities (Data 5), deferred charges (Data 152), accrued expenses (Data 153), deferred federal, foreign, and state taxes (Data 269,270,271), all in year 1, and debentures (Data 82), which we place in year 7, respectively. The debt due on each day in each quarter of each year for the S&P 500 firms is the sum of the debt due for all 500 firms for that day in that quarter of that year. This structure of the S&P 500 debt outstanding permits the computation of the daily duration of the market debt and the daily amount due at the duration date.

Next we calculate the daily market value (cap) of the S&P 500 (stock price times shares outstanding), and we find the factor *f* which was used to normalize the index, and we confirm that we match the reported index level each day during our sample. We use this same normalization factor for the daily S&P 500 debt outstanding. This procedure produces daily for the S&P 500 the market value of the equity and the face value and duration of the debt outstanding.

Now we have the data defined on page 6 as C, S, M, K, r_{FT}, t, T_1, T_2, D, and ρ, and are prepared to compute V, V*, and σ_v. In order to compute V, V *, and σ_v, we solve simultaneously equations (1), (2), and (3), given market values for C, S, and the contracted strike price K. In this paper we choose to test these models using the methodology of most professionals. Thus, we allow a term structure of volatility, possibly different for different option expirations but the same for all strikes of the same expiration, and compute this term structure of volatility daily. All the tests use out of sample data and forward looking implied volatilities for both models. Since we know from the open interest and volume data that the *most at-the-money* options are the deepest and most liquid we base the volatility term structure on the most at-the-money options. Thus, daily we compute the implied volatilities for the equity index from Black-Scholes and for the total market value from Geske for each time to expiration for the *most at the money* option, given the stock price, option price, and strike price. For different times to expiration we hold the equity index level, S, and total market value, V, constant and allow the implied volatility for the *most at the money* option to produce this option's market price. This is the methodology which we understand most professionals using Black-Scholes follow. Given the observed market prices of index call options this methodology produces the well documented Black-Scholes pricing biases observed for S&P 500 index call options.

The Black-Scholes model under prices the vast majority of in the money call options and over prices the vast majority of out of the money call options. We can now examine what improvement, if any, Geske's leverage based option model can provide. In the next section we discuss the results of this analysis and compare the two models.

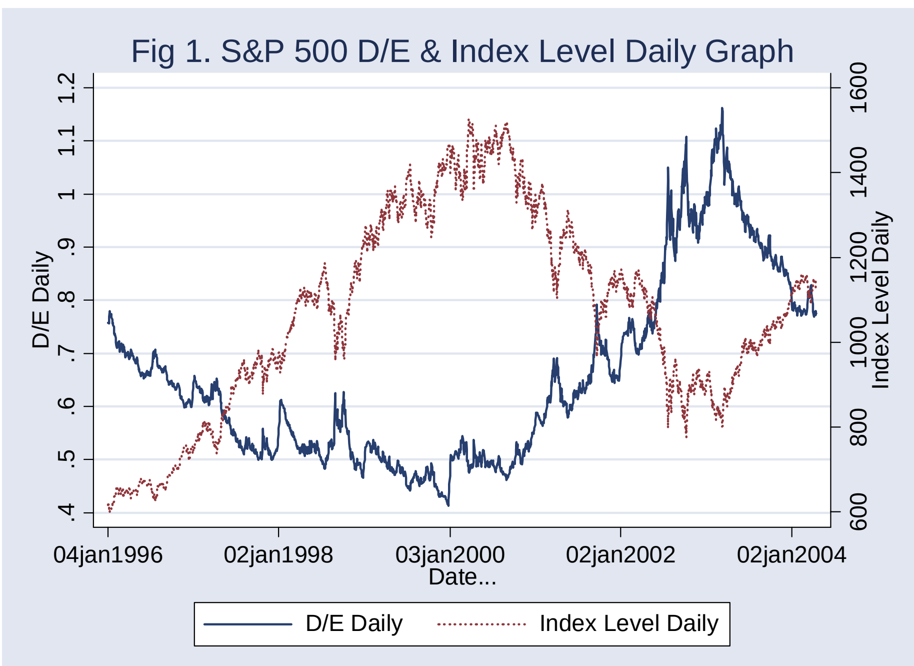
4. The Results

In this section we present evidence about the size of the *market* value of leverage in the S&P 500 firms deduced with option theory, more details about the model comparison methodology, graphs of the model errors with respect to the option's time to expiration and moneyness, and more detailed tables illustrating both the statistical and economic significance of the Black-Scholes errors and Geske's improvements with respect to moneyness and time to expiration by calendar year and by leverage.

4a. Market Leverage

The analysis described above permits the computation of the daily debt to equity ratio of the aggregate S&P 500 companies, where the equity is the actual market value of all stocks and the debt is the implied *market* debt value from the option pricing structure using market prices of the equity index and options on this index. We believe that Figure 1 is the first presentation of *market* equity values and *market* debt values implied from total market value producing *market* D/E ratios for the S&P 500 over time, and we plot D/E along with the level of the S&P 500 equity index. Figure 1 is presented for our sample period of option prices, ranging from January 1, 1996 through April 30, 2004.

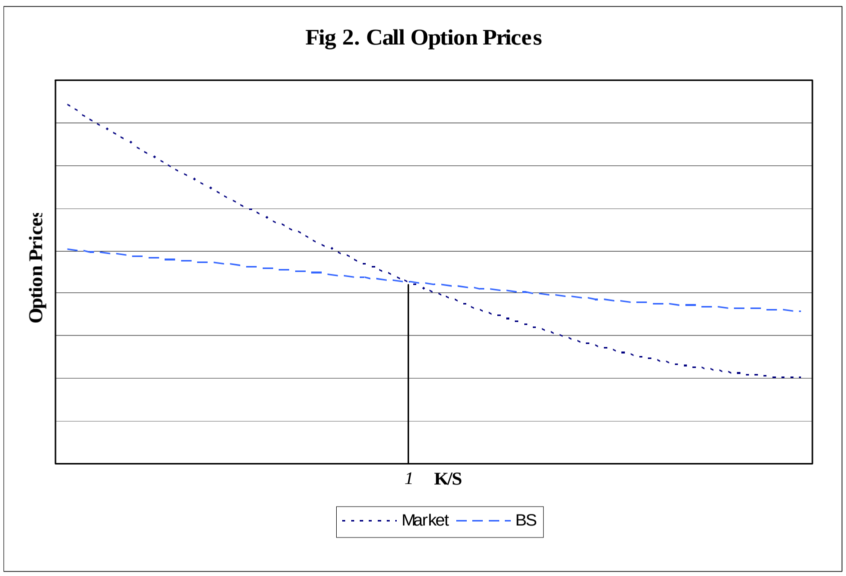
Figure 1 shows that during our sample period the market debt/equity ratio for the total of all firms in the S&P 500 ranges from a minimum of about 40% in January, 2000, to a maximum of 120 % in April, 2003.[5] As expected the debt/equity ratio and S&P 500 stock index level are inversely related. The graph shows the S&P 500 equity market level ranges between 600 and 1600 during this period. The S&P 500 index is based on a portfolio of 500 different stocks constructed as follows: 400 industrials, 40 utilities, 20 transportation companies, and 40 financial institutions. The weights of the stocks in this index portfolio at any given time are proportional to their market capitalization. The S&P 500 stock index represents about 80% of the market capitalization of all stocks listed on the New York Stock Exchange.



[5] Recall that with daily *market* prices for the very liquid equity index and equally liquid at the money options on the index, given Modigliani-Miller, we can solve directly for the daily implied *market* value of the market debt.

4b. Model Pricing Error Comparison

Figure 2 presents a graph of call index option market prices, Black-Scholes model values, and moneyness, K/S, which is representative of most research findings for the S&P 500 stock index call options.



Since the index level, S, is the same for all K at any point in time during or at the end of any day, as K varies in Figure 2, the in the money (ITM) stock index calls (low K) are shown to be under valued and the out of the money (OTM) index calls (high K) are shown to be over valued by the Black-Scholes model relative to the market prices.[6]

We show in Figure 3 that Geske's compound option model has the potential to improve or even eliminate these Black-Scholes valuation errors because of the leverage effect. The

[6] Figure 2 presents the most ubiquitous result from our data. However, for a very small number of matched pairs there are (15) different model distance comparisons that are made: both over market, both under, one over while the other is under, one equal to the market while the other is either over or under, both equal to each other but either over or under, both equal to each other and equal to the market, and furthermore, there are multiple cases for each situation when the models are not equal to each other.

reason for this, once again, is leverage creates a negative correlation between the index level and the index volatility. This interaction between the index level and index volatility implies that the index volatility is both stochastic and inversely related to the level of the index, and that the resultant implied index return distribution will have a fatter left tail and a thinner right tail than the Black-Scholes assumption of a normal return distribution. Thus, Geske's compound option model produces option values that are greater (less) than the Black-Schole's values for in (out of) the money European index call options, and could potentially eliminate the well known Black-Scholes bias.

Fig 3. Call Option Prices

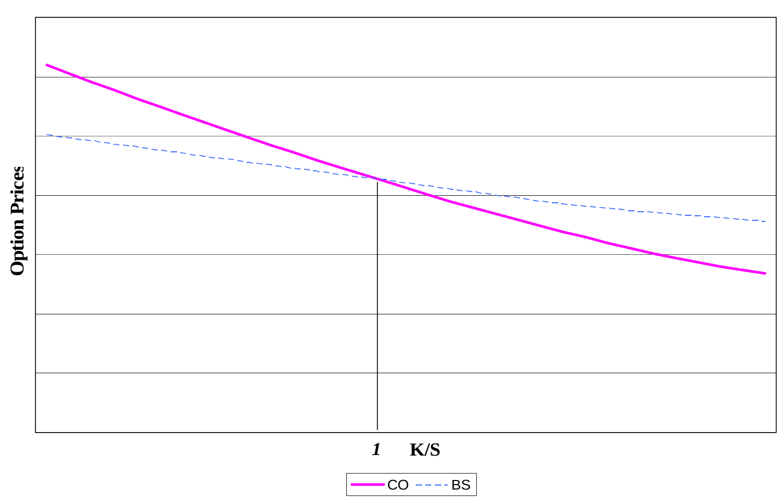
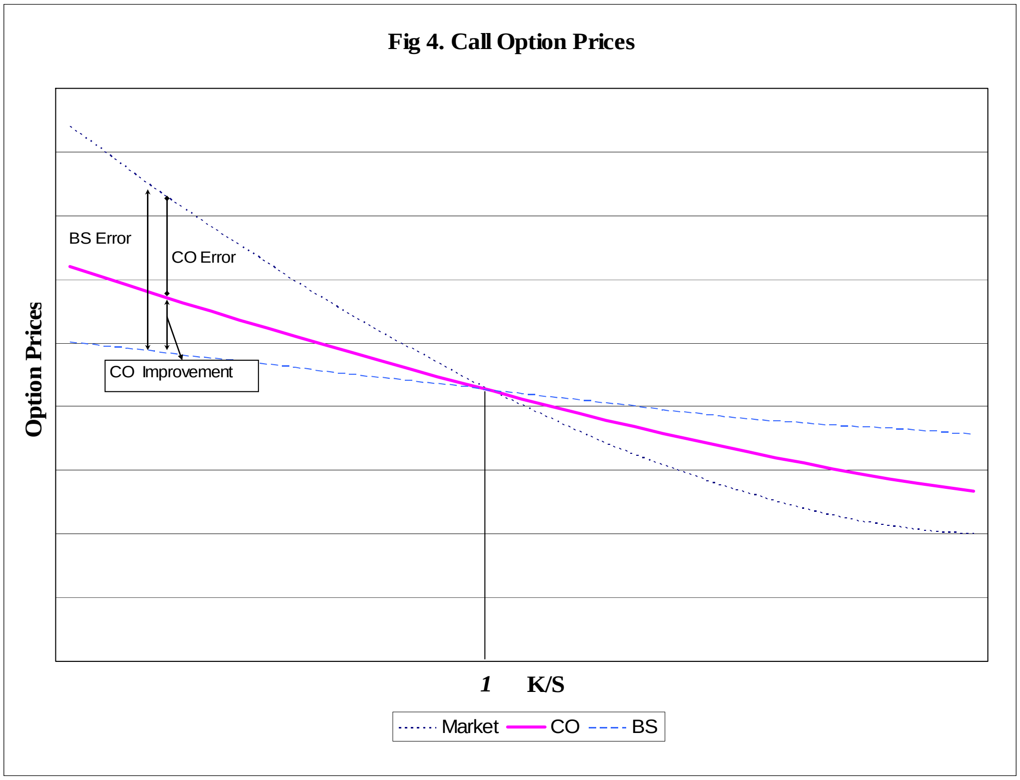


Figure 4 presents how we measure the amount of improvement Geske's model provides for S&P 500 stock index call options during this sample period. We create tens of thousands of matched pairs of all options for each expiration date and each strike price, and we measure the distance between each model's value and the market price. We compare the distance that each model value is from the market price for each matched pair, find the model that produces the closest distance to the market, and we compute the improvement of one model to the other for that pair. We then net these distances for all matched pairs in order to find which model is closest to the market for all matched pairs on average and how much net improvement, if any, is present. We present this analysis for all matched pairs of options for a variety of categories with different times to expiration, different moneyness, and for the different market leverage exhibited during our sample time period.



The improvement of Geske's compound option model compared to the Black-Scholes is calculated with the following formula:[7]

$$\frac{\text{BS error} - \text{CO error}}{\text{BS error}} = \frac{(\text{Market - BS}) - (\text{Market - CO})}{(\text{Market - BS})} \tag{6}$$

[7] Care must be taken with the sign of the variety of matched pair errors explained in footnote 4, especially if one model value distance is above and the other distance is below the market, when computing the average error across all matched pairs. However, the result depicted in Figures 2 and 3 is found for the vast number of all options (more than 98%).

To repeat, it is well documented that the Black-Scholes model under prices most in the money call options (low K) and over prices most out of the money call options (high K) on the S&P 500 index. This is the first paper to report on Geske's compound option model and its potential to correct these errors when used to price S&P 500 index call options.

4c. Graphs of Errors with respect to Time to Expiration

In the next two figures, 5A for in the money (ITM) and 5B for out of the money (OTM) S&P 500 index call options, we present the dollar pricing errors and net improvement of the Geske's compound option model compared to the Black-Scholes model as a function of time to option expiration in days. Here we consider at the money calls (ATM) to be within 5% of the index level, which is what most research reports as ATM.

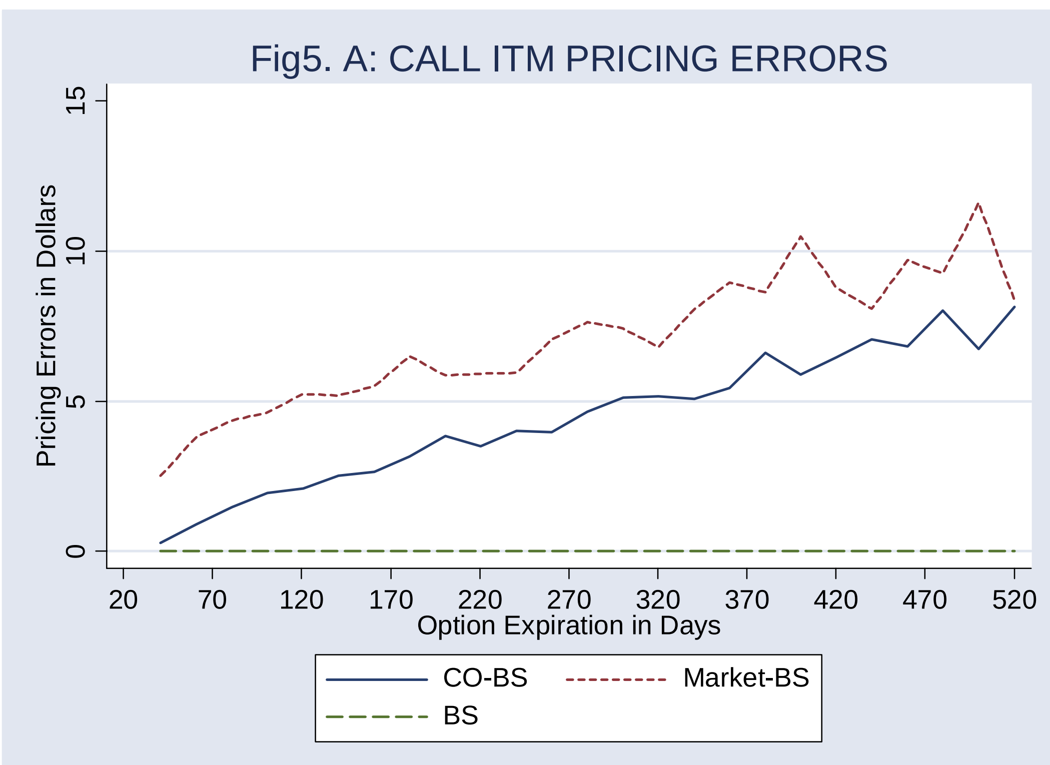
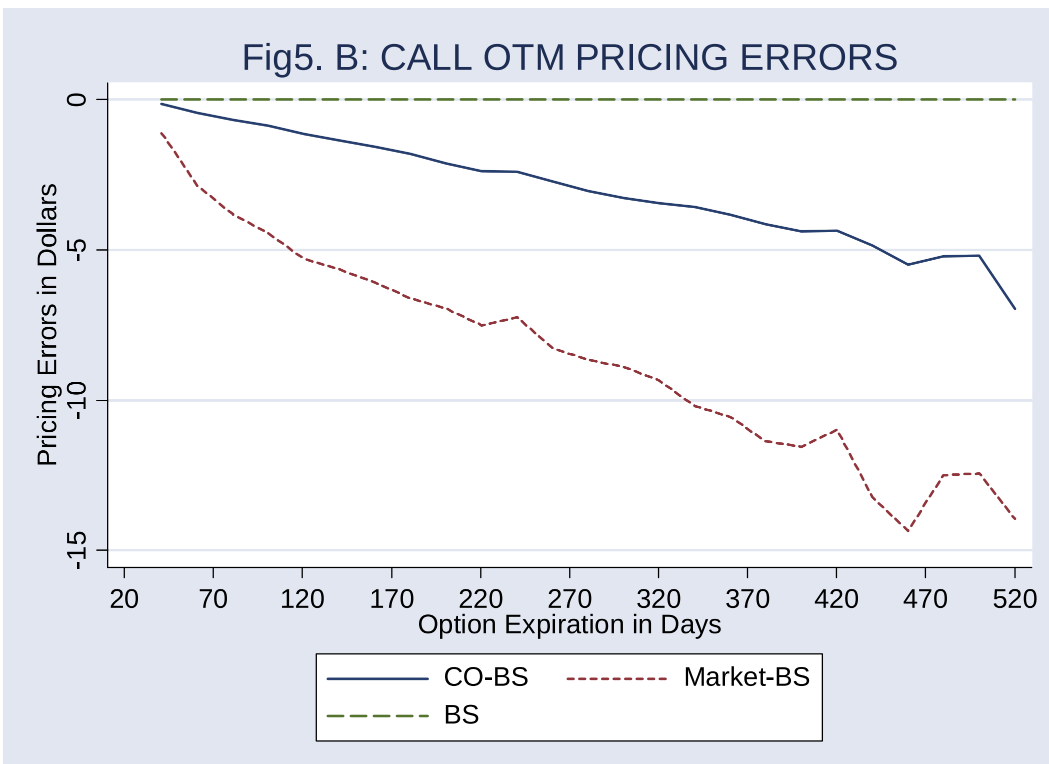


Figure 5A shows that the Black-Scholes (BS) under prices ITM index call options and this BS pricing error monotonically increases with time to expiration. The dollar range of this pricing error is from $3 to $12 as a function of time to expiration. Geske's compound option model shows considerable improvement (up to 80%) relative to Black-Scholes, and does not exhibit errors that are increasing with respect to time to expiration for ITM index call options. Table 2 presents a more detailed analysis of these errors and improvements with respect to time to expiration.

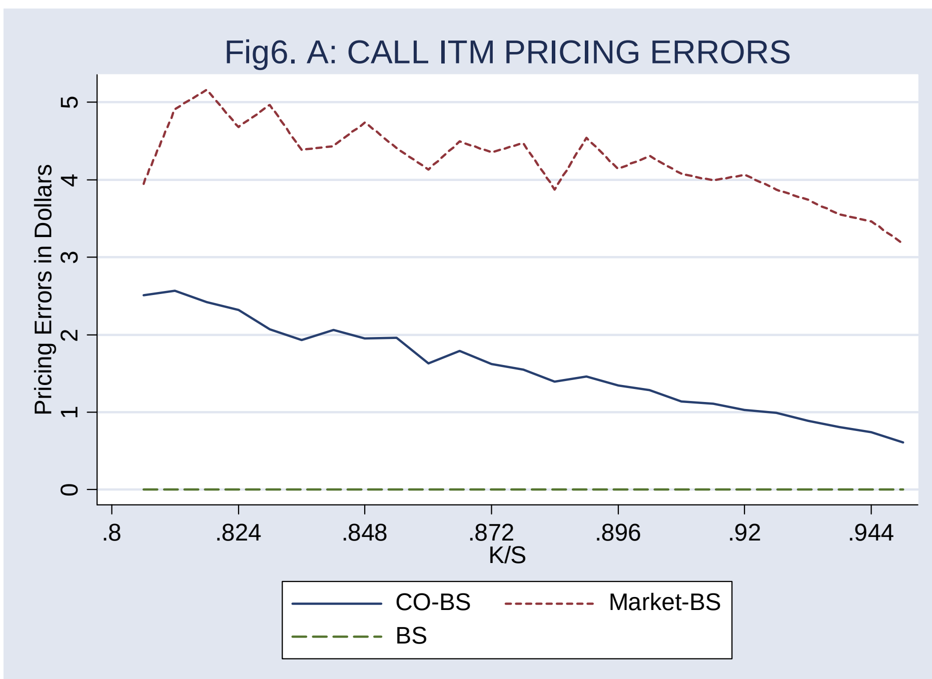
Figure 5B shows that the Black-Scholes over prices OTM index call options and again this pricing error monotonically increases with time to expiration. The dollar range of the pricing error is from $2 to $14 as a function of time to expiration. Geske's compound option model again improves upon Black-Scholes considerably. Tables 4 to 8 will present more details regarding these B-S errors and Geske's improvements.



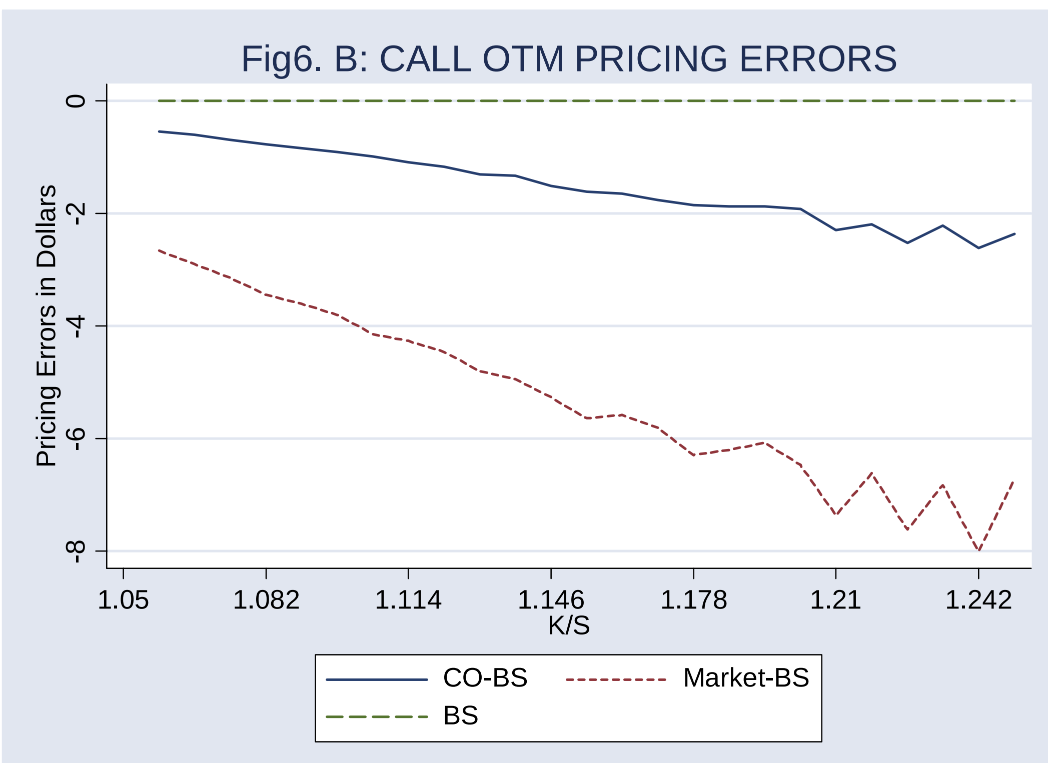
4d. Graphs of Errors with respect to Moneyness

In the next two figures, 6A and 6B, for ITM and OTM index call options respectively, we present the pricing errors in dollars and net improvement of Geske's compound option model compared to the Black-Scholes model as a function of option moneyness, K/S.

Figure 6A presents for ITM S&P 500 index call options the pricing errors in dollars as a function of the moneyness, K/S. The Black-Scholes model net under prices ITM call index options (K/S < 1) during this 8 1/3 year sample period.



For example, if the index level was 1000, 5% ATM calls would be with strikes between 950 and 1050. Thus, Figure 6A shows, for all ITM strike prices such that K/S ranges from 0.95 down to 0.8, the Black-Scholes model pricing error ranges just over \$3 to just over \$5 dollars, and the errors are increasing with in the moneyness as depicted in Figures 2 and 3. Figure 6A shows Geske's compound option model net improvement over Black-Scholes ranges from about $0.80 to $2.75 over this same in the money range for this sample period. Figure 6B presents for the out of the money (OTM) index call options the pricing errors in dollars as a function of the moneyness, K/S. The Black-Scholes model net over prices all OTM call index options (K/S > 1). Again, we consider at the money calls (ATM) to be within 5% of the index level, where as before, if the index level was 1000, ATM calls would be with strikes between 950 and 1050. Then for all OTM strikes such that K/S ranges from 1.05 up to 1.25, Figure 6B shows that the Black-Scholes model's over pricing error ranges from about $2.50 to $8 dollars. Figure 6B demonstrates that Geske's compound option models improvement over Black-Scholes ranges from about $0.70 to $2.70 over this same out of the money range.



4e. Tables of Errors Significance by Year, Expiration, Moneyness, and Leverage

In the following tables we present a more detailed analysis of the above results relating these ITM and OTM Black-Scholes pricing errors and Geske's improvements to the option's time to expiration by calendar year and by market leverage. We also present the number of options available in these categories during this time period, and examine both the statistical and economic significance of Geske's model relative to Black-Scholes.

First, we show that when the ATM option region is considered to be within 5% of the index level a large number of better priced but still miss-priced options are eliminated. If we consider only one option per day per time to expiration as the *most at the money option*, to be defined as *MATM*, then all but one previously eliminated 5% ATM's will now be either in or out of the money and priced with some error. This change in the definition of ATM option will increase the sample size of miss-priced options. We also would expect this ATM definitional change to reduce the average net pricing error because the ATM options exhibit smaller pricing errors.[8]

Consider the number of matched pairs of traded ITM call options presented in Table 1. Panel A illustrates that if we consider only one option to be ATM each day (the *most at the money* option), the sample of ITM call index options more than doubles from 18,047 to 45,664 matched pairs. The most active trading years for ITM call index options during our

[8] Given this methodology, the near or at the money options will always be better priced. However, independent of this methodology, the fact that most researchers report that the ATM volatility is a better predictor of actual realized volatility than the ITM or OTM volatilities is often attributed to the fact that the ATM's are the most widely traded options. This fact is probably why many professionals use this methodology.

sample period remain 1997, 1998, 1999, and 2003, containing 10,540 of the 18,047 (25,875 of the 45,664) total options. As expected, this table shows that the ITM near expiration calls are traded more heavily than the far expiration calls in every year. The nearest two expiration categories (6-20 & 21-70days) contain 12,096 of the 18,047 (32,699 of the 45,664) total options. Here the days to expiration range from 6 to over 365.[9]

In Table 1, Panel B, we present these same ITM index call options by time to expiration and now by debt/equity (D/E) ratio. The D/E ratio during this time period ranges between 40% and 120%, as depicted in Figure 1. Panel B shows that about 25% of this 5% ATM sample of ITM options traded when the market D/E ratio ranged from 80% to 120%.

TABLE 1

CALL ITM

TOTAL NUMBER OF OPTIONS

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	273	670	239	229	8	**1,419**
1997	542	1,056	331	444	36	**2,409**
1998	566	1,272	400	472	103	**2,813**
1999	455	1,085	329	429	80	**2,378**
2000	310	816	273	325	31	**1,755**
2001	381	852	232	327	67	**1,859**
2002	411	968	197	373	75	**2,024**
2003	598	1,417	455	470	0	**2,940**
2004	118	306	26	0	0	**450**
TOTAL	**3,654**	**8,442**	**2,482**	**3,069**	**400**	**18,047**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	1,039	2,210	596	557	37	**4,439**
1997	1,667	3,014	758	884	99	**6,422**
1998	1,672	3,136	788	936	204	**6,736**
1999	1,547	2,624	647	926	161	**5,905**
2000	1,144	1,997	616	696	107	**4,560**
2001	1,088	2,007	554	766	164	**4,579**
2002	1,135	2,215	534	849	160	**4,893**
2003	1,747	3,197	904	964	0	**6,812**
2004	515	745	58	0	0	**1,318**
TOTAL	**11,554**	**21,145**	**5,455**	**6,578**	**932**	**45,664**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	653	1,540	552	569	108	**3,422**
0.5-0.6	1,169	2,564	754	1,033	146	**5,666**
0.6-0.7	554	1,381	417	518	51	**2,921**
0.7-0.8	360	821	162	196	39	**1,578**
0.8-0.9	257	627	210	120	12	**1,226**
0.9-1.0	447	1,008	265	450	38	**2,208**
1.0-1.2	214	501	122	183	6	**1,026**
TOTAL	**3,654**	**8,442**	**2,482**	**3,069**	**400**	**18,047**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	2,220	3,720	1,106	1,257	242	**8,545**
0.5-0.6	3,552	6,518	1,619	2,069	331	**14,089**
0.6-0.7	1,951	3,972	1,048	1,182	160	**8,313**
0.7-0.8	1,295	2,275	423	545	99	**4,637**
0.8-0.9	828	1,570	457	270	19	**3,144**
0.9-1.0	1,210	2,124	549	857	64	**4,804**
1.0-1.2	498	966	253	398	17	**2,132**
TOTAL	**11,554**	**21,145**	**5,455**	**6,578**	**932**	**45,664**

[9] About 2% of this ITM sample contains long dated calls ranging form 365 to 600 days to expiration.

Table 2 presents the net pricing error improvement of Geske's model relative to Black-Scholes by calendar year and by D/E ratio for the various times to expiration and for the two definitions of ATM for all ITM call index option matched pairs during this sample period. Note in Panel A that the improvement of Geske's model with respect to time to expiration varies from 2% for shortest expirations to 77% for longest expirations, and is strictly monotonic across all years and ranges of leverage when the sample size is sufficient. Table 2 also illustrates, as expected, that when ATM is defined as a single *most at the money* option, the previously excluded but now ITM options which have smaller pricing errors reduce the net pricing improvement in all years and across all times to expiration except the nearest to expiration.

TABLE 2

CALL ITM

PRICING ERROR IMPROVEMENT

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0%	49%	91%	167%	286%	**84%**
1997	1%	34%	69%	108%	207%	**58%**
1998	1%	16%	28%	48%	64%	**29%**
1999	1%	15%	29%	38%	50%	**26%**
2000	1%	15%	30%	40%	64%	**25%**
2001	2%	17%	30%	48%	83%	**30%**
2002	4%	17%	31%	55%	131%	**34%**
2003	2%	36%	55%	81%	N/A	**49%**
2004	0%	43%	61%	N/A	N/A	**36%**
TOTAL	**2%**	**21%**	**39%**	**57%**	**77%**	**35%**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	2%	37%	75%	142%	256%	**59%**
1997	2%	27%	60%	99%	180%	**43%**
1998	2%	14%	26%	45%	60%	**23%**
1999	2%	13%	26%	35%	50%	**21%**
2000	2%	13%	26%	37%	59%	**21%**
2001	3%	16%	27%	45%	81%	**26%**
2002	5%	16%	28%	51%	123%	**29%**
2003	3%	30%	49%	74%	N/A	**39%**
2004	1%	32%	49%	N/A	N/A	**25%**
TOTAL	**2%**	**19%**	**35%**	**53%**	**74%**	**29%**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	1%	15%	27%	39%	52%	**25%**
0.5-0.6	1%	19%	41%	54%	81%	**34%**
0.6-0.7	2%	27%	50%	82%	89%	**45%**
0.7-0.8	2%	32%	49%	86%	128%	**45%**
0.8-0.9	1%	32%	58%	71%	128%	**44%**
0.9-1.0	4%	25%	46%	66%	128%	**41%**
1.0-1.2	5%	25%	32%	69%	59%	**36%**
TOTAL	**2%**	**21%**	**39%**	**57%**	**77%**	**35%**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	2%	13%	25%	36%	51%	**21%**
0.5-0.6	2%	16%	36%	51%	76%	**27%**
0.6-0.7	2%	23%	44%	75%	88%	**36%**
0.7-0.8	2%	27%	42%	77%	120%	**35%**
0.8-0.9	2%	26%	49%	63%	124%	**33%**
0.9-1.0	5%	23%	41%	62%	125%	**35%**
1.0-1.2	5%	23%	30%	64%	59%	**32%**
TOTAL	**2%**	**19%**	**35%**	**53%**	**74%**	**29%**

Table 2, Panel B, focuses on categories of leverage. Here it is shown that relative to Black-Scholes the improvement of Geske's model's increases with the D/E ratio almost monotonically for every time to expiration, especially when the sample number of options in each category produces a sufficiently large sample. [10]

Table 3 presents similar data to Table 1 for out of the money (OTM) index call options. First consider the number of traded index calls presented in Table 3 for OTM options. Panel A for ATM defined as 5% shows that the near expiration index calls again are traded more heavily than the far expiration calls in every year. Here the days to expiration again range from 6 to over 365.

TABLE 3

CALL OTM

TOTAL NUMBER OF OPTIONS

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	277	1,334	454	638	60	**2,763**
1997	467	1,786	659	1,269	127	**4,308**
1998	765	1,832	740	1,614	308	**5,259**
1999	606	2,157	1,060	1,519	294	**5,636**
2000	895	2,972	1,212	1,786	275	**7,140**
2001	894	2,631	1,206	2,250	609	**7,590**
2002	1,135	3,041	1,255	2,877	567	**8,875**
2003	788	2,193	970	1,887	0	**5,838**
2004	163	358	33	0	0	**554**
TOTAL	**5,990**	**18,304**	**7,589**	**13,840**	**2,240**	**47,963**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	1,230	3,789	1,197	1,275	125	**7,616**
1997	1,676	4,414	1,430	2,087	276	**9,883**
1998	1,964	4,046	1,346	2,448	562	**10,366**
1999	2,085	4,313	1,718	2,324	489	**10,929**
2000	2,424	5,044	1,856	2,456	420	**12,200**
2001	2,011	4,161	1,706	2,969	833	**11,680**
2002	2,151	4,720	1,849	3,755	790	**13,265**
2003	1,938	4,157	1,547	2,580	0	**10,222**
2004	658	860	70	0	0	**1,588**
TOTAL	**16,137**	**35,504**	**12,719**	**19,894**	**3,495**	**87,749**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	827	3,087	1,583	2,170	435	**8,102**
0.5-0.6	1,794	5,312	1,944	3,854	600	**13,504**
0.6-0.7	1,053	3,603	1,533	2,534	482	**9,205**
0.7-0.8	618	1,948	648	1,461	392	**5,067**
0.8-0.9	384	935	476	521	70	**2,386**
0.9-1.0	773	2,033	819	1,961	195	**5,781**
1.0-1.2	541	1,386	586	1,339	66	**3,918**
TOTAL	**5,990**	**18,304**	**7,589**	**13,840**	**2,240**	**47,963**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	2,926	6,089	2,577	3,296	719	**15,607**
0.5-0.6	4,949	10,796	3,467	5,708	1,060	**25,980**
0.6-0.7	2,946	7,446	2,711	3,789	716	**17,608**
0.7-0.8	1,859	3,969	1,154	2,133	566	**9,681**
0.8-0.9	1,041	2,027	813	708	89	**4,678**
0.9-1.0	1,499	3,202	1,214	2,583	266	**8,764**
1.0-1.2	917	1,975	783	1,677	79	**5,431**
TOTAL	**16,137**	**35,504**	**12,719**	**19,894**	**3,495**	**87,749**

[10] Note that in a few instances the pricing error correction is greater than 100%. This can happen when the two models errors are on opposite sides of the market price.

Panel A also illustrates that if we consider only one option to be ATM each day (the *most at the money*), the sample of matched pairs of OTM call index options increases from 47,963 to 87,749. The most active trading years for OTM call index options during our sample period are 2000, 2001, and 2002. Recall Figure 1 which shows this is the interval of our sample period when the S&P 500 index level was declining.

In Table 3, Panel B, we present these same OTM index call options by time to expiration and by debt/equity (D/E) ratio for the same ranges of time to expiration and leverage. The longer expiration options (121 to over 365 days) comprise about 34% of the sample, and the higher leverage categories (80% to 120%) again comprise about 25% of the sample.

Table 4 presents the net pricing error improvement by calendar year and by D/E ratio for the various times to expiration and for the two definitions of ATM for all OTM call index options during this sample period. In Panel A, when ATM is defined as 5%, the year 1996 again exhibits the greatest pricing improvement of 49% across all times to expiration, and the year 1999 exhibits the lowest pricing error improvement of 19%. Table 4 also illustrates that when ATM is defined as a single *most at the money* option, the previously excluded but now OTM options which have smaller pricing errors reduce the net pricing improvement in all years but does not have much effect across all times to expiration except the nearest to expiration. The greatest improvement is still year 1996 and the lowest improvement is still in year 1999. Table 4, Panel A, also illustrates that Geske's improvement increases monotonically with time to expiration.

Table 4, Panel B, demonstrates that Geske's compound option model's improvement also increases with the D/E ratio, almost monotonically for every time to expiration, especially when the sample number of options is sufficiently large.

TABLE 4

CALL OTM

PRICING ERROR IMPROVEMENT

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	35%	26%	32%	62%	124%	**49%**
1997	9%	16%	27%	55%	86%	**40%**
1998	9%	12%	15%	25%	39%	**22%**
1999	7%	11%	14%	22%	29%	**19%**
2000	11%	13%	17%	23%	33%	**21%**
2001	18%	19%	22%	33%	49%	**32%**
2002	19%	19%	24%	37%	59%	**35%**
2003	55%	30%	32%	49%	N/A	**42%**
2004	-9%	40%	33%	N/A	N/A	**41%**
TOTAL	**13%**	**16%**	**20%**	**31%**	**43%**	**28%**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	7%	23%	33%	63%	124%	**41%**
1997	6%	17%	27%	55%	86%	**35%**
1998	6%	11%	15%	25%	40%	**21%**
1999	5%	10%	14%	22%	30%	**18%**
2000	7%	12%	17%	23%	33%	**20%**
2001	11%	18%	22%	32%	49%	**31%**
2002	13%	18%	24%	37%	59%	**33%**
2003	19%	26%	31%	49%	N/A	**39%**
2004	9%	25%	30%	N/A	N/A	**23%**
TOTAL	**8%**	**15%**	**20%**	**31%**	**44%**	**26%**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	8%	12%	15%	22%	30%	**20%**
0.5-0.6	10%	13%	18%	28%	41%	**24%**
0.6-0.7	17%	20%	24%	39%	54%	**35%**
0.7-0.8	29%	23%	25%	40%	55%	**38%**
0.8-0.9	33%	24%	30%	44%	62%	**39%**
0.9-1.0	23%	21%	29%	42%	63%	**37%**
1.0-1.2	23%	24%	24%	42%	46%	**35%**
TOTAL	**13%**	**16%**	**20%**	**31%**	**43%**	**28%**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	5%	11%	15%	22%	30%	**18%**
0.5-0.6	7%	12%	18%	28%	41%	**22%**
0.6-0.7	10%	19%	25%	39%	55%	**33%**
0.7-0.8	10%	21%	25%	40%	55%	**36%**
0.8-0.9	12%	21%	29%	44%	62%	**34%**
0.9-1.0	16%	21%	28%	42%	63%	**35%**
1.0-1.2	19%	23%	24%	42%	46%	**34%**
TOTAL	**8%**	**15%**	**20%**	**31%**	**44%**	**26%**

In this research we have also tried a different volatility methodology of basing the aggregate net pricing errors and improvement of Geske's model compared to Black-Scholes on the volatility that minimizes the sum of squared errors. We find that this does not change the characteristics of our results, and this is evident regardless of whether we allow or do not allow a term structure of volatility. This result is not surprising because moving the pricing volatility that minimizes the sum of squared errors away from ATM toward either the ITM or OTM will exhibit a more than off-setting effect on the larger errors in the other direction. This greater off-setting effect will be present independent of the definition of ATM (5% or 0% for *most at the money*) and will generally require moving the minimizing volatility toward the ATM.

Statistical Significance

Here we use non-parametric statistics to test the significance of the differences between Black-Scholes and Geske's model, which is the same as the significance of the reported improvements using both the 5% ATM and the 0% *most at the money* ATM. As can be seen in Table 5 for ITM options and Table 6 for OTM options, we find Geske's model

TABLE 5

CALL ITM

Rank Sum Test *p* Value

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0.9453	0.0000	0.0000	0.0000	0.0002	**0.0000**
1997	0.8355	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.8298	0.0000	0.0000	0.0000	0.0000	**0.0000**
1999	0.8461	0.0000	0.0000	0.0000	0.0000	**0.0000**
2000	0.7896	0.0000	0.0000	0.0000	0.0000	**0.0000**
2001	0.6821	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.3886	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.5950	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.9293	0.0000	0.0003	0.0000	0.0000	**0.0000**
TOTAL	**0.4389**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0.7105	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.5659	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.5899	0.0000	0.0000	0.0000	0.0000	**0.0000**
1999	0.5820	0.0000	0.0000	0.0000	0.0001	**0.0000**
2000	0.5417	0.0000	0.0000	0.0000	0.0085	**0.0000**
2001	0.4379	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.2823	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.2426	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.7623	0.0000	0.0002	0.0000	0.0000	**0.0000**
TOTAL	**0.0641**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	0.8326	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.5-0.6	0.7358	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.6-0.7	0.8230	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.7-0.8	0.8100	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.8-0.9	0.8330	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.9-1.0	0.4823	0.0000	0.0000	0.0000	0.0000	**0.0000**
1.0-1.2	0.5681	0.0000	0.0000	0.0000	0.0022	**0.0000**
TOTAL	**0.4389**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	0.5328	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.5-0.6	0.3678	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.6-0.7	0.4919	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.7-0.8	0.5620	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.8-0.9	0.5441	0.0000	0.0000	0.0000	0.0001	**0.0000**
0.9-1.0	0.2763	0.0000	0.0000	0.0000	0.0000	**0.0000**
1.0-1.2	0.3491	0.0000	0.0000	0.0000	0.0612	**0.0000**
TOTAL	**0.0641**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

improvements are all significant at greater than the 99.99% level except for the very near maturity options.[11]

Near maturity when market option prices are converging to the in and out of the money boundaries there is much more noise in the pricing errors, especially for the out of the money options that are approaching zero.

TABLE 6

CALL OTM

Rank Sum Test *p* Value

PANEL A

5 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0.3937	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.3959	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.1678	0.0000	0.0000	0.0000	0.0000	**0.0000**
1999	0.3744	0.0000	0.0000	0.0000	0.0000	**0.0000**
2000	0.1937	0.0000	0.0000	0.0000	0.0000	**0.0000**
2001	0.1626	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.0167	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.0079	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.5638	0.0009	0.0010	0.0000	0.0000	**0.0035**
TOTAL	**0.0001**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

0 PERCENT

Option Expiration (in Days)

YEAR	Min-20	21-72	73-120	121-364	365-Max	TOTAL
1996	0.2748	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.2244	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.1457	0.0000	0.0000	0.0000	0.0000	**0.0000**
1999	0.2353	0.0000	0.0000	0.0000	0.0000	**0.0000**
2000	0.0829	0.0000	0.0000	0.0000	0.0000	**0.0000**
2001	0.0514	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.0056	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.0017	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.4392	0.0000	0.0003	0.0000	0.0000	**0.0001**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

PANEL B

5 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	0.2555	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.5-0.6	0.0817	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.6-0.7	0.0960	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.7-0.8	0.2271	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.8-0.9	0.2507	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.9-1.0	0.0320	0.0000	0.0000	0.0000	0.0000	**0.0000**
1.0-1.2	0.0211	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.0001**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

0 PERCENT

Option Expiration (in Days)

D/E	Min-20	21-72	73-120	121-364	365-Max	TOTAL
0.4-0.5	0.1291	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.5-0.6	0.0235	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.6-0.7	0.0390	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.7-0.8	0.1187	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.8-0.9	0.1776	0.0000	0.0000	0.0000	0.0000	**0.0000**
0.9-1.0	0.0108	0.0000	0.0000	0.0000	0.0000	**0.0000**
1.0-1.2	0.0064	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

[11] Furthermore, when using a volatility that minimizes the sum of squared errors the significance of our results also hold, and the near maturity options remain significantly different as others have reported (see Heston and Nandi (2000).

Economic Significance

In addition to the above tables, here we report the economic significance of these improvements for ITM options in Tables 7 and OTM options in Table 8. More specifically, Tables 7 and 8 show results when Geske's model is compared by the *number* of matched pairs that it is a closer absolute distance to the market price than Black-Scholes, by the *dollar improvement* of this distance advantage, and by the *basis points* (bp) that this improvement implies for a portfolio. These comparisons are categorized by both calendar year and by leverage

TABLE 7

CALL ITM

BASIS POINT IMPROVEMENTS

PANEL A

5 PERCENT

YEAR	PV	NUMBER TOTAL	NUMBER BS	NUMBER CO	BS	CO	BP
1996	88,768.92	1,131	213	918	295.30	931.81	**72**
1997	254,623.61	2,136	260	1,876	463.08	2,178.65	**67**
1998	413,062.26	2,493	101	2,392	191.67	3,616.70	**83**
1999	438,917.65	2,137	82	2,055	92.87	3,475.35	**77**
2000	329,356.39	1,626	73	1,553	58.43	2,001.27	**59**
2001	257,111.45	1,721	80	1,641	55.32	2,179.31	**83**
2002	238,084.70	1,934	188	1,746	278.13	1,954.16	**70**
2003	300,993.35	2,667	175	2,492	153.03	3,126.61	**99**
2004	40,772.25	355	28	327	16.63	275.72	**64**
TOTAL	**2,361,690.58**	**16,200**	**1,200**	**15,000**	**1,604.46**	**19,739.58**	77

0 PERCENT

YEAR	PV	NUMBER TOTAL	NUMBER BS	NUMBER CO	BS	CO	BP
1996	152,391.24	3,464	351	3,113	320.96	1,587.14	**83**
1997	390,648.40	5,434	452	4,982	489.62	3,038.84	**65**
1998	583,499.48	5,646	189	5,457	202.78	4,501.21	**74**
1999	620,498.43	4,913	168	4,745	95.60	4,279.23	**67**
2000	481,940.79	3,778	179	3,599	67.81	2,628.03	**53**
2001	377,854.73	3,735	160	3,575	61.64	2,845.65	**74**
2002	348,673.00	4,105	298	3,807	297.14	2,632.06	**67**
2003	433,256.75	5,885	297	5,588	158.35	4,119.52	**91**
2004	66,754.55	1,086	69	1,017	17.54	419.51	**60**
TOTAL	**3,455,517.37**	**38,046**	**2,163**	**35,883**	**1,711.44**	**26,051.19**	70

PANEL B

5 PERCENT

D/E	PV	NUMBER TOTAL	NUMBER BS	NUMBER CO	BS	CO	BP
0.4-0.5	615,901.06	3,061	118	2,943	130.91	4,440.51	**70**
0.5-0.6	812,091.56	5,086	339	4,747	567.33	6,691.75	**75**
0.6-0.7	298,886.74	2,562	243	2,319	316.65	2,790.10	**83**
0.7-0.8	162,054.77	1,359	188	1,171	250.10	1,283.80	**64**
0.8-0.9	123,936.65	1,065	67	998	63.45	1,183.78	**90**
0.9-1.0	235,953.40	2,076	162	1,914	192.90	2,380.12	**93**
1.0-1.2	112,866.40	991	83	908	83.12	969.52	**79**
TOTAL	**2,361,690.58**	**16,200**	**1,200**	**15,000**	**1,604.46**	**19,739.58**	77

0 PERCENT

D/E	PV	NUMBER TOTAL	NUMBER BS	NUMBER CO	BS	CO	BP
0.4-0.5	880,344.36	7,044	247	6,797	138.33	5,618.24	**62**
0.5-0.6	1,176,974.54	11,865	629	11,236	601.17	8,561.04	**68**
0.6-0.7	465,821.05	6,709	429	6,280	332.99	4,020.60	**79**
0.7-0.8	258,508.57	3,737	335	3,402	276.05	1,919.43	**64**
0.8-0.9	190,729.05	2,694	110	2,584	65.14	1,661.16	**84**
0.9-1.0	329,988.70	4,157	293	3,864	209.12	3,031.51	**86**
1.0-1.2	153,151.10	1,840	120	1,720	88.64	1,239.21	**75**
TOTAL	**3,455,517.37**	**38,046**	**2,163**	**35,883**	**1,711.44**	**26,051.19**	70

First, consider Table 7 for ITM options when ATM is defined as 5% (0%). In Panel A the columns left to right represent the year, the present value of all ITM matched pairs for that year, the total number of matched pairs, the number of matched pairs where Black-Scholes (BS) is closer to the market price in absolute distance, the number of matched pairs where

Geske's compound option (CO) model is closer to the market price, the dollar value of the BS improvement, the dollar value of the CO improvement, and the net basis point advantage of Geske's model for that year. Panel B of Table 7 presents the same information categorized by the D/E ratio in the first column which ranges from 40-120 %. The totals for each column and each row are also presented.

For the *number* of ITM matched pairs of options compared in Table 7, Panels A and B for the two ATM definitions, 16,200 (38,046), Geske's model is closer to the market price than the Black-Scholes model for 15,000 (35,663) of these matched pairs (about 95%) and Black-Scholes is closer on 1,200 (2163) pairs.

The *basis points* net improvement from using the Geske's model and being closer to the market price are on average 77 bp (70 bp) for ITM options (Table 7) and 901 bp (491 bp) for OTM options (Table 8) in a *one of each option* portfolio of options when ATM is defined as 5% (0%). These numbers are calculated by constructing a *one of each option* portfolio containing one option for each strike price and time to expiration for each day and finding the market value of that *one of each option* portfolio each day for all days in a year. The basis point and dollar value improvements would generally be much larger for professionals who do not hold a *one of each* option portfolio, but instead hold all options in multiple amounts. Each option at a specific strike price and time to expiration generally has a much larger volume of trading which professionals will capture.[12]

[12] The ATM options have the greatest volume and eliminating the 5% defined as ATM in order to find the pricing errors of the ITM's and OTM's excludes a large number of options as illustrated in these Tables. Furthermore, as mentioned in footnote 4, there are also some options where Black-Scholes and compound option models produce the same values and errors and these are excluded from this improvement analysis.

More specifically, *dollar improvement* for each model is measured by considering all those matched pairs where a specific model is closer to the market price than the alternative model in absolute distance measured in dollars. The basis point advantage of Geske's model is then computed by dividing the net dollar improvement for that year or leverage category by the total value of options in that category. For example, in Panel A, across the sample years 1996-2004 the Geske's compound option model has a total *dollar improvement* of $19,739.58 and Black-Scholes has a *dollar improvement* of $1,604.46. Thus, the net *dollar improvement* of Geske's compound option model is $18,135.12, and that divided by the total value of each option in this ITM portfolio, $2,361,690.58, produces the 77 net *basis point* improvement. When the 0% ATM definition of a single *most at the money* option is used this ITM portfolio value increases to $3,455,517.37 because of the inclusion of previously excluded 5% ATM options. This basis point improvement is generally monotonically increasing with leverage, especially when sample size is considered.

Table 8 presents the same *number*, *dollar improvement*, and *basis point* improvement analysis for OTM call index options during this sample period when ATM is again defined either as 5% or 0% and is again depicted both yearly and by D/E ratio. Table 8 also shows the total dollar value of options in a year or D/E category, the total number of options in that category, the number where Black-Scholes is closer to the market and the number where the Geske's model is closer to the market, the dollar value of both Black-Scholes and Geske's compound option improvements, and the basis point (bp) advantage of Geske's option model relative to Black-Scholes.

For the *number* of OTM matched pairs of options compared in Table 8, Panels A and B for the two ATM definitions, 45,614 (77,227), Geske's model is closer to the market price than the Black-Scholes model for 43,012 (72,999) of these matched pairs (about 95%).

Table 8, in Panel A, across the sample years 1996-2004 that Geske's compound option model now has a larger *dollar improvement* for OTM options of $58,141.01 and Black-Scholes has a smaller *dollar improvement* for OTM options of $326.23. Thus, the net *dollar improvement* of Geske's compound option model is $57,814.78, and that divided by

TABLE 8

CALL OTM

BASIS POINT IMPROVEMENTS

PANEL A

5 PERCENT

YEAR	PV	NUMBER TOTAL	NUMBER BS	NUMBER CO	BS	CO	BP
1996	13,384.02	2,627	147	2,480	48.94	2,018.51	**1472**
1997	52,082.65	4,184	231	3,953	83.71	4,452.16	**839**
1998	93,188.14	5,114	150	4,964	31.19	7,731.91	**826**
1999	113,171.21	5,487	106	5,381	4.75	8,313.78	**734**
2000	117,226.15	6,865	343	6,522	16.70	8,816.99	**751**
2001	107,449.75	7,173	418	6,755	17.33	11,013.66	**1023**
2002	99,232.93	8,524	695	7,829	100.06	10,218.27	**1020**
2003	45,132.03	5,425	559	4,866	19.63	5,503.81	**1215**
2004	618.21	415	153	262	5.92	71.92	**1068**
TOTAL	**641,485.09**	**45,814**	**2,802**	**43,012**	**328.23**	**58,141.01**	**901**

0 PERCENT

YEAR	PV	NUMBER TOTAL	NUMBER BS	NUMBER CO	BS	CO	BP
1996	48,210.65	6,479	319	6,160	68.27	3,041.86	**617**
1997	139,504.63	8,684	493	8,191	113.91	5,657.51	**397**
1998	202,889.30	9,123	262	8,861	49.88	8,903.31	**436**
1999	244,869.70	9,632	193	9,439	10.63	9,495.05	**387**
2000	238,048.21	10,906	511	10,395	28.52	9,877.95	**414**
2001	191,610.20	10,252	569	9,683	22.77	12,009.18	**626**
2002	179,226.32	11,860	873	10,987	115.73	11,258.25	**622**
2003	104,321.61	8,995	734	8,261	26.71	6,531.43	**624**
2004	6,607.19	1,293	271	1,022	8.63	175.66	**253**
TOTAL	**1,355,287.81**	**77,224**	**4,225**	**72,999**	**445.05**	**66,950.20**	**491**

PANEL B

5 PERCENT

D/E	PV	NUMBER TOTAL	NUMBER BS	NUMBER CO	BS	CO	BP
0.4-0.5	152,592.16	7,882	216	7,666	7.26	11,191.43	**733**
0.5-0.6	223,258.23	13,024	543	12,481	78.25	17,786.20	**793**
0.6-0.7	101,513.50	8,781	503	8,278	94.22	11,036.25	**1078**
0.7-0.8	52,933.18	4,720	425	4,295	32.48	6,008.80	**1129**
0.8-0.9	16,624.42	2,160	229	1,931	6.99	1,906.67	**1143**
0.9-1.0	54,661.15	5,480	529	4,951	69.00	6,050.04	**1094**
1.0-1.2	39,902.45	3,767	357	3,410	40.03	4,161.62	**1033**
TOTAL	**641,485.09**	**45,814**	**2,802**	**43,012**	**328.23**	**58,141.01**	**901**

0 PERCENT

D/E	PV	NUMBER TOTAL	NUMBER BS	NUMBER CO	BS	CO	BP
0.4-0.5	335,032.62	13,806	369	13,437	13.42	12,820.48	**382**
0.5-0.6	483,497.41	22,933	982	21,951	128.76	20,553.50	**422**
0.6-0.7	209,593.76	15,326	745	14,581	114.37	12,915.29	**611**
0.7-0.8	111,780.79	8,366	712	7,654	51.33	7,004.92	**622**
0.8-0.9	43,244.37	4,069	301	3,768	8.36	2,350.44	**542**
0.9-1.0	104,165.34	7,792	685	7,107	81.92	6,784.36	**643**
1.0-1.2	67,973.52	4,932	431	4,501	46.89	4,521.21	**658**
TOTAL	**1,355,287.81**	**77,224**	**4,225**	**72,999**	**445.05**	**66,950.20**	**491**

the total value of each option in this smaller valued OTM portfolio, $641,485.09, produces the 901 *basis point* improvement. When the ATM definition is a single *most at the money* option this OTM portfolio value increases to $1,355,267.61 because of the inclusion of previously excluded options, and Geske's net *basis point* improvement is now 491 bp. Again, this basis point improvement is generally monotonically increasing with leverage.

Table 9 presents a comparison of the per cent pricing errors and the root mean squared errors (RMSE) of Geske relative to the Black-Scholes model for these index call options. These results are presented for moneyness ranges of K/F (F is the forward index level) between 0.9 and 1.1, and for times to expiration ranging from six days to 100 days.[13]

Table 9

		CALL Days to Expiration								
			<40			[40-70]			>70	
Model	Moneyness	RMSE	%Error	Number	RMSE	%Error	Number	RMSE	%Error	Number
BS	[0.90/0.95)	2.73	3.50	4,493	3.57	4.25	2,016	3.88	4.42	879
	[0.95/0.99)	1.88	4.74	9,613	2.00	3.94	4,505	1.87	3.15	2,316
	[0.99/1.01)	0.60	2.95	6,851	0.92	2.79	3,812	1.18	2.78	2,286
	[1.01/1.05]	1.73	21.12	12,014	2.54	14.12	5,787	2.81	10.71	3,006
	(1.05/1.10]	1.87	87.44	7,947	3.13	48.20	4,020	3.58	31.48	1,909
CO	[0.90/0.95)	2.49	3.19	4,493	2.83	3.37	2,016	2.78	3.18	879
	[0.95/0.99)	1.76	4.44	9,613	1.70	3.34	4,505	1.50	2.53	2,316
	[0.99/1.01)	0.57	2.79	6,851	0.82	2.48	3,812	1.00	2.37	2,286
	[1.01/1.05]	1.60	19.53	12,014	2.21	12.27	5,787	2.32	8.85	3,006
	(1.05/1.10]	1.67	78.30	7,947	2.66	40.98	4,020	2.84	24.99	1,909

As Table 9 shows Geske's compound option model has lower RMSE and % errors for all times to expiration and for all moneyness categories. It does appear that Geske's model has more improvement for the longer times to expiration. The % errors are larger for both models for the out of the money index calls (K/F > 1.01) because the option prices are smaller, especially near maturity.

[13] The restricted range on moneyness and time to expiration exactly matches the restrictions in Heston Nandi (2000), Table 7, page 610. While our sample period is different and larger (1996-2004 versus 1993-1995), when RMSE's are adjusted for the different average index level, our results compare very favorably with Heston Nandi.

The improvement of Geske's compound option model relative to the Black-Scholes model for pricing the world's most widely traded equity index options on the S&P 500 is notable and probably should be considered by any professional managing a large portfolio of options. The data necessary to implement Geske's model is readily available. Also, once professionals are set up to price options based on the more fundamental firm value, they are also set up to produce output useful for dealing with credit derivatives, such as credit spreads and risk neutral default probabilities. Also, Geske's model has been extended in a variety of ways which would be appropriate for options which are different than the S&P 500 index options.

However, recall that Figures 5 and 6, A and B, demonstrate that leverage is not explaining all of the difference between Black-Scholes values and market prices for these S&P 500 call index options during this time period January, 1996 to April 30, 2004. The next section summarizes and presents our conclusions.

5. Conclusions

In this paper we have demonstrated that the Geske compound option model can be used to price the world's most widely traded equity index options on the S&P 500. This model takes the theory of option pricing deeper into the theory of the firm and market by incorporating the effects of leverage consistent with Modigliani and Miller. Herein, the Geske option model characterizes how leverage causes the market equity index risk to change stochastically and inversely with the equity price level, and incorporates default risk and bankruptcy. We thought that because the Geske model incorporates these additional factors not included in the Black-Scholes model that it might provide a better understanding of option prices. However, the Geske model requires inputs that, while readily available, are not necessarily as accurately observable as the inputs for the Black-Scholes model, and measurement error on the inputs can discredit the output. Herein, we demonstrate that with readily obtainable data and the use of market prices for the equity index and options on the index, we can imply the value and volatility of the total market debt and equity for the 500 S&P firms and improve relative to Black-Scholes on the pricing of index call options on the S&P 500. We demonstrate that this improvement is both statistically and economically significant for all strikes and all times to expiration. We also show that this improvement is greater the longer the time to expiration of the option and/or the greater the market leverage in the sample period. Finally, we show that although incorporating leverage effects can dramatically improve the pricing of S&P 500 equity index call options relative to the Black-Scholes model, the Geske model prices for ITM and OTM options are not eliminating all market pricing errors.

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