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The Term Structure, the CAPM and the Market Risk Premium: An Interesting Puzzle

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1. Introduction

This note highlights an interesting puzzle involving the relation between the CAPM, the term structure and the market risk premium. The starting point for the analysis is the research on the decomposition of beta. Using the return decomposition technique developed by Campbell and Shiller (1988), Campbell and Mei (1993) investigate the extent to which systematic risk arises because of common variation in future required returns compared to common variation in expected cash flows. They conclude that "The betas of industry and size portfolios with the market are largely attributed to changing expected returns."

If Campbell and Mei are correct, then long-term Treasury bonds, whose prices also are affected by variation in expected returns, should have significant betas. This prediction turns out to be correct. Given Treasury bond beta estimates, the CAPM can be applied to analyze the term spread between long-term bonds and short-term bills. The puzzle that arises involves duration. Consistent with the Campbell-Mei analysis, Treasury bond betas are closely related to duration. However, there is no corresponding variation in yield spreads.

2. Empirical results

Figure 1 presents moving averages of beta for long-term Treasury bonds computed against the S&P 500. (Betas computed with respect to the CRSP value weighted index are marginally higher). The long-term Treasury bond returns are taken

from the data compiled by Ibbotson and Associates. The average maturity of the bonds in the Ibbotson sample is 20 years. Betas are calculated using a standard OLS regression with data from the 48 months preceding the point on the chart.

The sample period runs from January 1988 through December 1997. The starting point is selected to eliminate the impact of the crash. During the crash, there was a "flight to quality" and Treasury bond prices rose sharply. As a result, there was a very large negative covariance between returns on Treasury securities and returns on the market during the few months surrounding the crash. Analysis of the data back to 1974 reveals that the crash period is highly idiosyncratic. Rather than attempting to make any adjustments, the data used in this short note start after the crash.

Figure 1 provides striking confirmations for the Campbell-Mei analysis. The beta Ibbotson 20-year Treasury bond series is between 0.30 and 0.50 throughout the period from January 1992 to December 1997. From January of 1995 to December of 1997, the average beta is 0.45.[1] Furthermore, the estimated betas are highly significant because the standard error is less than that associated with beta estimates for common stock. For instance, for the final four-year period ending in December 1997, the estimated beta is 0.42 and the standard error is .107, so that the t-statistic is 3.93.

Using the 3-month bill rate as the risk-free rate, the CAPM can be applied to the 20-year bond to arrive at a "CAPM interpretation" of the yield curve. Specifically, the CAPM implies that

$$\text{Expected bond return} = \text{Bill yield} + \text{Bond beta} * (\text{Market risk premium}) . \qquad (1)$$

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[1] Because the estimates are overlapping, the data are highly autocorrelated and the "average" is not much different than the individual observations.

Assuming that the expected returns on Treasury bonds are approximately equal to the yield, equation (1) states that the yield spread should equal the bond beta times the market risk premium.[2]

Figure 2 plots the spread between the constant maturity 20-year Treasury security yield and the corresponding three-month bill yield as reported by the Federal Reserve. The plot begins in October 1993, rather than January 1992 because there is a gap in the Fed data for the 20-year security. The figure shows that the spread begins at about 300 hundred basis points. By the end of 1994, the spread falls to 200 basis. It then varies from approximately 100 to 200 basis points throughout the remainder of the sample period.

Putting Figures 1 and 2 together, a rough calculation can be performed to estimate the market risk premium. For instance, assume that representative estimates of beta and the yield spread, at least for the last several years of the sample period, are 0.45 for beta and 200 basis points for the spread. For the CAPM to work, the market risk premium over Treasury bills must be on the order of 4.4 percent. This is higher than the historical risk premium calculated over the 1926 to 1997 period, but on the order of the ex-ante risk premiums estimated by Blanchard (1993).

The Duration Puzzle

Although the foregoing CAPM interpretation of the yield spread seems reasonable, a puzzle emerges as soon as securities of different durations are considered. To illustrate, assume that the movements in the discount rate that account for the positive bond betas are permanent parallel shifts in the yield curve. Using the definition of beta,

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[2] This assumption is reasonable given the unpredictable nature of changes in long-term yields documented by Campbell and Shiller (1984), among others.

$$\text{Beta} = \text{cov}(r_b, r_m) / \text{var}(r_m), \tag{2}$$

these assumptions imply that,

$$\text{Beta of a bond} = -D\ \text{cov}(dr, r_m) / \text{var}(r_m) \tag{3}$$

where,

D = the modified duration,

dr = the change in the level of interest rates,

r_b = the return on the bond,

r_m = the return on the market portfolio.

Based on equation (3), the simple duration calculation implies that the ratio of the betas for two bonds should equal the ratio of the bonds' modified durations.

While equation (3) is too simple to capture the full variation in bond betas, it points to the fact that bonds with significantly different durations should have significantly different betas. If the CAPM interpretation is correct, then such the yield spreads over bills for such bonds also should be significantly different. Unfortunately, this prediction fails to hold even remotely.

To illustrate, consider a comparison of the 20-year Treasury bond with a 25-year Treasury strip. As of December 1997, the modified duration of a 20-year Treasury bond selling at par was approximately 11.5 years compared to a modified duration for the 25-year strip of 24.3 years. Using the Merrill Lynch 25-year strip index and 4 years of monthly data, the strip beta as of December 1997 is estimated to be 1.06, which is 2.5 times greater than the beta for the 20-year Treasury. The fact that this is reasonably close to the duration ratio of 2.1 confirms that equation (3) is a reasonable approximation.

The problem with the CAPM interpretation arises when duration is duration and beta are compared to the yield spread for different bonds. The CAPM interpretation implies that the yield spread between 20-year bonds and 25-year strips should be on the

order of the beta difference (0.64) times the market risk premium. Assuming that the market risk premium is 440 basis points, this comes to 288 basis points. In fact, the spread between the 20-year bond and the 25-year strip at the end of 1997 was only ***5*** basis points. These narrow spreads were characteristic of the sample period. As an illustration, the 30-year constant maturity yield is also plotted on Figure 2.[3] The figure shows that the yield curve is virtually flat beyond twenty-years during the entire sample period.[4] This is in direct contradiction to the CAPM interpretation because the duration of the 30-year bond is 20 percent greater than that of a 20-year bond. Furthermore, the beta for the 25-year strip seems unreasonable on its face. It implies that the expected return on the market is slightly less than the yield on the strip.

The puzzle is not that the CAPM interpretation fails to adequately characterize the term structure. A basic two-period model that ignores variation in the investment opportunity set could not be expected to be so robust. The puzzle is that it fails so badly. Consistent with the Campbell-Mei analysis, long-term Treasury securities have significant betas that are closely related to duration. The yield spreads between those securities,

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[3] The Fed does not report monthly data on the 25-year strip.

[4] In fact, the figure shows that the 30-year constant maturity yield is a sliver less than the 20-year yield. This is due to the inclusion of the more liquid "on-the-run" long bond in the Fed's 30-year calculation. But for that, the yield curve is flat beyond 20 years.

however, are wildly inconsistent with the CAPM. The puzzle is explaining why the model fails so completely. Hopefully, the search for an answer will help tie together what have heretofore been separate lines of research.

References

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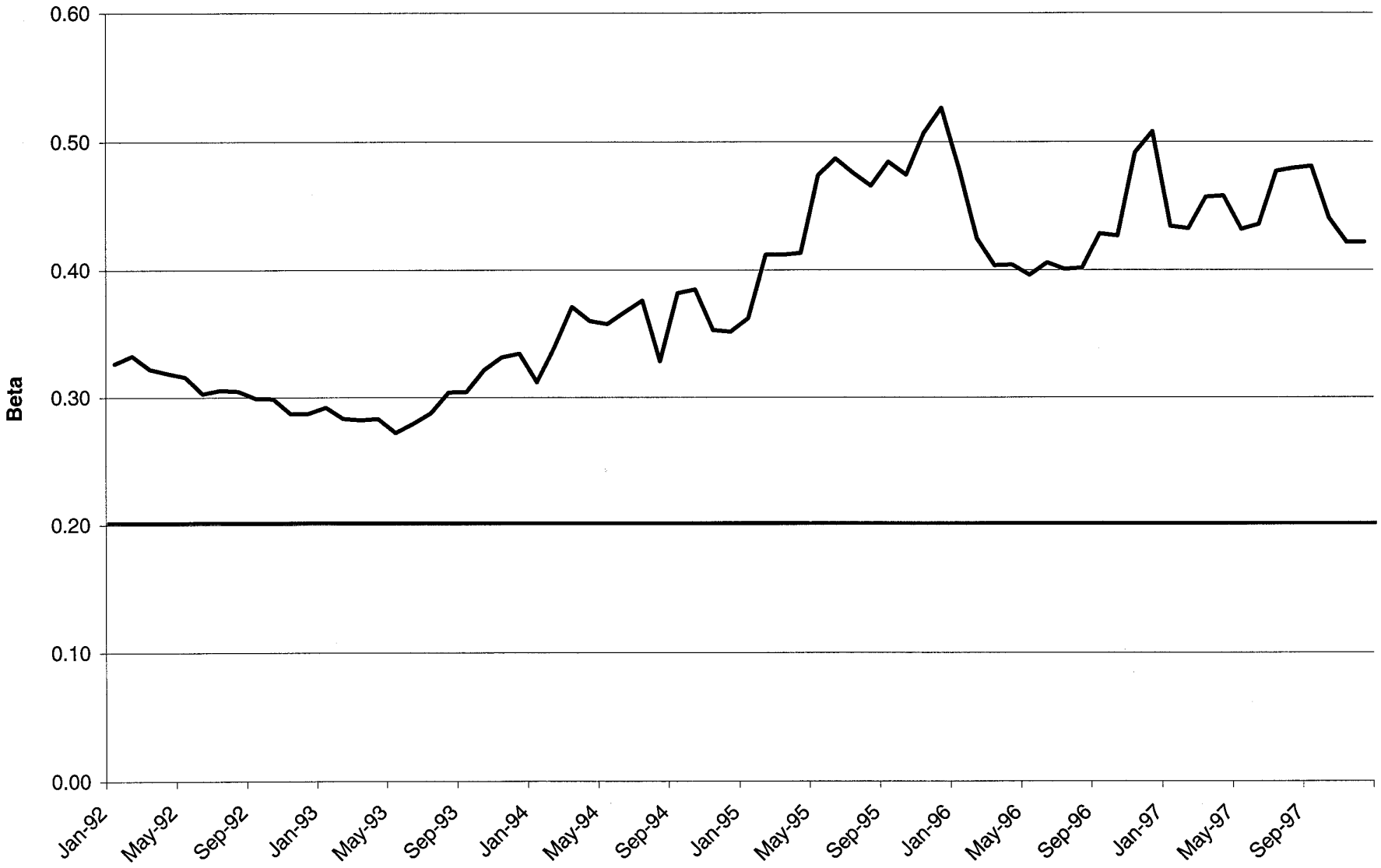
Figure 1

Betas for Long-term Treasury Bonds: 48 month moving average from January 1992 to December 1997



The Standard and Poor's 500 is used as the market index. Both the long-term Treasury returns and the market returns are taken from Ibbotoson and Associates.

Figure 2
Yield Spreads on Constant Maturity Treasury Securities