Bond Pricing with Default Risk *

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Abstract

We price corporate debt from a structural model of firm default. We assume that the capital market brings about efficient firm default when the continuation value of the firm falls below the value it would have after bankruptcy restructuring. This characterization of default makes the model more tractable and parsimonious than the existing structural models. The model can be applied in conjunction with a broad range of default-free interest rate models to price corporate bonds. Closed-form corporate bond prices are derived for various parametric examples. The term structures of yield spreads and durations predicted by our model are consistent with the empirical literature. We illustrate the empirical performance of the model by pricing selected corporate bonds with varied credit ratings.

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Traditionally the credit risk literature has taken two approaches to the valuation of corporate debt. The 'structural' approach models the bankruptcy process explicitly. It defines both the event that triggers default and the payoffs to the bond holders at default in terms of the assets and liabilities of the firm. Substantial abstraction of the bankruptcy game is required to retain tractability. The structural approach to firm default has only been able to produce closed-form prices under extremely simplistic capital structure assumptions.

The 'reduced-form', or 'statistical' approach treats default as an event governed by an exogenously specified jump process.[1] The statistical approach is very tractable. Duffie and Singleton (1999) show that any default-free term structure model can be used to price bonds with default risk. One simply models the spot interest rate to include an instantaneous default spread. Affine term structure models can then be tweaked to produce closed-form corporate bond prices.

The tradeoff of realism for tractability in structural models has, so far, generated less than satisfactory empirical pricing performance. As a consequence, the applied literature has favored statistical models over structural models. However, beyond good in-sample fit, we are ultimately interested in linking the determinants of default to firm characteristics. For this purpose, the reduced-form approach is less suitable; hence, the appeal of structural models.

Our paper offers a highly tractable structural model of default which performs well

[1] Since this paper is structural in nature, we refer readers interested in the statistical models to Litterman and Iben (1991), Madan and Unal (1993), Fons (1994), Das and Tufano (1995), Jarrow and Turnbull (1995), Jarrow, Lando and Turnbull (1997), Lando (1997, 1998), and Duffie and Singleton (1999), and Duffie and Lando (2001) amongst other excellent treatments on the topic.

empirically. Like other papers in the structural literature, we characterize default as the first time the firm value V crosses a default boundary K. This approach begins with Black and Scholes (1973), Merton (1974) and Black and Cox (1976), continues with Longstaff and Schwartz (1995) and, more recently, with Briys and Varenne (1997), Taurén (1999) and Collin-Dufresne and Goldstein (2001).

In Black and Scholes (1973) and Merton (1974), all debts mature on the same day, and the firm defaults when its value is lower than the payment due. Hence, the default boundary K consists of a single point, equal to the face value of the maturing debt. If default occurs, the claimants receive the liquidation value of the firm in order of priority. Unfortunately, the model becomes intractable when debt obligations mature at various points in time.

Black and Cox (1976) and Longstaff and Schwartz (1995) assume that the firm is forced into default by its debt covenants the first time its value falls below a constant threshold K.[2] In this case, K can be viewed as the face value of the liabilities of a firm that has a constant dollar amount of debt outstanding at all times. Here, default may occur at any point in time, even when no payment is due. Black and Cox (1976) model the default payoffs like Black and Scholes (1973), which again makes the model intractable for realistic capital structures.

Longstaff and Schwartz (1995) introduce an innovative way of dealing with default payoffs. At default, the corporate bond is exchanged for a fraction $(1 - W)$ of a default-free bond, where W may depend on the priority and the maturity of the original corporate bond.[3]

[2]Black and Cox (1976) actually consider a default boundary of the form $K(t) = ke^{-c(T-t)}$, where T is the time to maturity and k is the face value of the maturing debt.

[3]This assumption is consistent with empirical studies (Franks and Torous (1989), Eberhart, Moore, and Roenfeldt (1990), Weiss (1990), and Betket (1995)), which suggest that priority rules are almost always

This allows the treatment of corporate coupon bonds as a portfolio of corporate zeros. More importantly, it allows the pricing of a corporate bond without a detailed specification of the rest of the firm's capital structure. Along a similar vein, Briys and Varenne (1997) respecify the default boundary of Black and Cox (1976) to allow for stochastic default-free interest rates, while adopting the default write-down treatment of Longstaff and Schwartz.[4] .

In both Black and Cox (1976), Longstaff and Schwartz (1995) and Briys and Varenne (1997), the debt issued by the firm is assumed to remain constant irrespective of the firm value. This suggests an unreasonable waste of the firm's debt capacity as the firm grows in value. Taurén (1999) and Collin-Dufresne and Goldstein (2001) realize that in reality the dollar amount of the firm's liabilities does not remain constant. They propose alternative models to reflect the firm's tendency to maintain a stationary leverage ratio. Following the interpretation that K is the face value of the firm's liabilities, the ratio V/K can be interpreted as the inverse debt ratio. V/K is then modeled as mean-reverting, and the firm is assumed to enter into default when V/K falls 'dangerously low'. However, Taurén (1999) concedes that empirical estimation with his model is almost impossible due to the computational intensity.[5,6] Further, the empirical performance of the model is poor, with pricing errors in yields in excess of 100 basis points.

Our model, by comparison, is both more realistic and more parsimonious than previous

violated, with junior claimants receiving payments even when senior claimants are not paid in full.

[4]Briys and Varenne (1997) model the boundary as $K(t) = k\alpha \exp(\int_t^T r(u)du)$, where k is the face value of the debt maturing at time T

[5]Collin-Dufresne and Goldstein (2001) do not address the empirical performance of their model.

[6]Many parameters in Taurén (1999) could not be estimated directly from corporate bond prices but must be separately assumed or calibrated. The high dimensionality of the estimation also makes it infeasible to estimate more than one zero coupon corporate bond.

approaches. The innovation in our model comes from the characterization of firm default. In an efficient capital market, default occurs when the continuation value of the firm under the current management and capital structure is less than the value that the firm would have after bankruptcy. Specifically, bankruptcy can result in a Chapter 7 liquidation, a Chapter 11 reorganization or liquidation, or a private debt restructuring. We assume that the bankruptcy code and corporate governance mechanism, coupled with the market for corporate control, ensure that bankruptcy occurs efficiently.

We concede that the ex ante efficiency of firm bankruptcies is a controversial subject. We briefly present objections to the efficiency hypothesis here and argue our case. Critics of the efficient bankruptcy theory argue that the cost of financial distress (estimated at 3% of firm value by Weiss (1990) and $10 - 20\%$ by Andrade and Kaplan (1998)) is too large for economic efficiency.[7] However, Easterbrook (1990) argues that the loss from distress is small in comparison to the loss that would occur if alternative resolutions are pursued. Critics of efficient bankruptcy also point out that distress can occur for firms with healthy operating incomes and margins; in which case, distress seems to arise from high leverage rather than bad firm performance. However, Andrade and Kaplan (1998) find that for firms which appear to suffer purely from financial distress, distress has lead in many cases to cost cutting initiatives and changes in the management. Chen and Wei (1993) also find that creditors are willing to waive violations of debt covenants (such as low book asset value or late interest payments) for firms with healthy operating ratios. These evidence suggest that economic efficiency does play an important role in effecting distress.

[7] In general, estimating the cost of financial distress is an extremely noisy exercise due to the difficulty of differentiating the impact in firm performance from economic distress and financial distress.

We define the default boundary K as the 'bankrupt' firm value, which can be either the value of the firm's liquidated assets or a recapitalized going concern. Market efficiency then predicts default to occur when the firm's continuation value V falls below K. This characterization of the default boundary necessarily makes K stochastic and correlated with V. Additionally, since K represents the value of an asset, its risk-adjusted return can be modeled to equal the default-free interest rate.

Other structural models focus on the physical events leading to default, rather than the equilibrium characteristics of default. As a result, K must reflect the firm's outstanding liabilities and simultaneously account for the bargaining game between the shareholders and the equity holders.[8] Therefore, the modeling of K becomes inevitably complex.

We follow Longstaff and Schwartz (LS, 1995) in assuming that at default a corporate bond is exchanged for an equivalent default-free bond at a write-down W, which depends on the bond's priority and maturity. As we mentioned, this feature allows the valuation of each debt issue independent of the rest of the firm's capital structure.

Our model of default can be coupled with virtually any model of the default-free term structure to price corporate bonds. In particular, when the default-free term structure has non-stochastic volatility, we are able to derive approximate analytical bond prices. The approximate analytic pricing solution is rapidly convergent and achieves high accuracy with only second order expansion terms. This reduces the computational intensity substantially. For other default-free term structure assumptions, we provide simple numerical methods for computing bond prices.

[8]See Leland (1994), Leland and Toft (1996), and Mella-Barral and Perraudin (1997) for models on the strategic game between the equity holders and the debt holders.

We estimate our model with bond data from individual firms with different credit ratings. Until now, the estimation problem has been extremely difficult due to the high dimensionality of the structural models. We cast the estimation problem in a GMM framework. We specify moment restrictions and a weighting matrix in a way which substantially reduces the dimensionality of the estimation. This makes the estimation fast enough to study multiple bond issues. The pricing errors from the model average 45 basis points, representing a substantial improvement over previous structural models. The quality of the corporate bond data makes a better fit unlikely. More importantly, the estimated parameters have a natural interpretation.

Our model predicts a monotone increasing term structure of credit spreads for high quality corporate bonds and hump shaped credit spreads for low quality bonds, which is consistent with the empirical findings of Sarig and Warga (1989) and Bohn (1999). Durations for bonds of identical promised cash flows are predicted to be increasing in credit quality, which is consistent with the findings of Chance (1990). However, we do not necessarily predict negative durations for extremely risky bonds, which is a crucial prediction of Longstaff and Schwartz (1995). Our model does allow for the seemingly counter-intuitive result predicted and empirically observed by LS that credit spreads narrow as default-free interest rates increase. However, our reason for this prediction is very different from LS. In LS, since the default boundary K is constant and $V/K = 1$ defines default, the ratio V/K has a risk neutral drift of r; intuitively, the greater the r, the more V/K drifts away from 1. Therefore, it follows immediately that the probability of default and the credit spread decrease as r increases; intuitively, leverage mechanically decreases with r in LS. In our model, both V and K are asset values and therefore have risk neutral drifts equal to r. The risk neutral

drift of the ratio V/K therefore, does not depend on r. However, V/K can be positively correlated with r if the bankruptcy value K is more sensitive than the continuation value V to interest rate shocks.

The remainder of this article is organized as follows. Section I develops the modeling framework. Section II derives closed-form solutions for risky corporate bonds under two particular parameterizations of the model and analytical approximate solutions and simulation methods for more general model specifications. Section III discusses the term structure of yield spreads and durations predicted by our model. Section IV examines the empirical performance and illustrates applications of the valuation method. Section V concludes the article.

I. The Valuation Framework

In this section, we present the assumptions about the firm value dynamics and the default boundary dynamics. We assume trading occurs continuously in a frictionless market. We also make the necessary assumptions for the existence of a unique equivalent martingale measure (EMM) Q under which the instantaneous expected rate of return on all assets is the default-free short rate r. Finally, we assume that two sources of risk – shocks to the firm fundamentals and shocks to the default-free interest rate – drive the variations in the continuation and the bankruptcy firm values. The shocks to the firm fundamentals under Q are characterized by the Brownian motion Z_v, while the shocks to the default-free interest rates are characterized by the Brownian motion Z_r. The instantaneous correlations between dZ_v and dZ_r is $\rho_{rv}dt$.

A. The Continuation Value Dynamics

We define V as the *continuation value* of the firm. It refers to the value of the firm without entering into bankruptcy restructuring. This will be distinguished from the *bankruptcy value* K, which refers to the value that the firm would have after the bankruptcy proceeding. We introduce the dynamics of V first. Under the unique EMM assumption, the instantaneous total return of V under the Q measure must equal the default-free short rate r. We also make additional restrictions. In particular, the shocks to the firm value delivered by changes in the firm fundamentals and the interest rates have constant volatilities. Summarizing, the risk-neutral dynamics of V are governed by:

$$\frac{dV(t)}{V(t)} = [r(t) - \delta_e(t, V, K, r) - \delta_d(t, V, K, r)]dt + \gamma_v dZ_v(t) + \gamma_r dZ_r(t) \tag{1}$$

where $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$ are the pay out rates to the equity and the debt holders and can be arbitrary functions of t, V, K and r. γ_v and γ_r are loadings on the shocks to firm fundamentals (dZ_v) and shocks to the default-free short rate (dZ_r). Note, $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$ can reflect complicated dividend and debt servicing policies. In addition, $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$ may be negative to reflect additional equity or debt issuing. This freedom in the modeling of $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$ cannot be achieved in other structural models, except for the stationary leverage ratio model of Collin-Dufresne and Goldstein (2001). We will see later that $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$ do not enter into the computation of the default probability and thus do not need to be explicitly modeled.

B. The Bankruptcy Value Dynamics

We wish to characterize the default boundary to reflect efficient bankruptcies. We assume that the capital market is efficient, which suggests that firm default occurs when the continuation value of the firm falls below the value the firm would have if it enters bankruptcy restructuring. The institutional features and mechanisms of the financial market such as bankruptcy codes, debt covenants, and the market for corporate control are suppose to bring about efficient defaulting.

We define the default boundary $K(t)$ as the time t bankruptcy value of the firm. Efficiency then predicts that default occurs the first time $V(t)$ falls below $K(t)$. We now make exact the definition of $K(t)$. For simplicity of exposition, suppose one specific plan of bankruptcy restructuring is available for each firm.[9] For some firms, if default occurs, bankruptcy results in a Chapter 7 or Chapter 11 liquidation. Suppose that the firm faces certain liquidation in the event of default. $K(t)$ then represents the sum value of the physical assets of the firm. Prior to default, the continuation value of the firm is greater than the value of the assets in a piecemeal sell off. However, at default, when $V(t)$ hits $K(t)$ for the first time, the firm is liquidated to generate $K(t)$, which is then distributed to the claimants of the firm. For other firms, bankruptcy results in a Chapter 11 reorganization. Suppose the reorganization plan liquidates the non-cash generating long term investments and retains only the cash generating assets. $K(t)$ then represents the value of this stripped down company, plus the sales proceeds, at t. When $V(t)$ hits $K(t)$, the firm defaults and is reorganized, and the claimants receive securities of this new entity.

So defined, K represents an asset value. Therefore, under Q, K's instantaneous total

[9] Alternatively, one might imagine that one particular form of liquidation/restructuring dominates the other forms of liquidation/restructuring for all t.

return must be equal to r. Again, we make the additional restriction of constant volatility on the K process and write the dynamics of K under Q as:

$$\frac{dK(t)}{K(t)} = [r(t) - \delta_e(t, V, K, r) - \delta_d(t, V, K, r)]dt + \beta_v dZ_v(t) + \beta_r dZ_r(t) \tag{2}$$

where γ_v and γ_r are loadings on the shocks to firm fundamentals (dZ_v) and shocks to the default-free short rate (dZ_r). It should be clear that K's drift is also adjusted by $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$.

We argue before that $K(t)$ represents the value of the firm in a piecemeal liquidation if the firm has no economic value as a going concern. Under this scenario, $K(t)$, for estimation purposes, might be proxied by the book value of the firm's asset. The ratio V/K can then be crudely interpreted as Tobin's q. A low V/K would indicate low economic value added. Financial distress and economic distress therefore occur at the same time. However, when the firm is expected to continue as a going concern in the event of a bankruptcy, V/K does not have the interpretation of a Tobin's q anymore. Financial distress is then a corporate control mechanism which forces reorganization of resources to deliver higher economic value added and can occur without obvious signs of economic distress.

It is important to stress that bankruptcy occurs when the firm's asset is insufficient to cover its liabilities. Our model is consistent with that definition of bankruptcy though it does not explicitly model the firm's liabilities. Note that $K(t)$ is not modeled as the outstanding debt of the firm as is done in the traditional structural literature. However, $K(t)$ is nonetheless related to the firm's outstanding debt. Recall that the creditors receive a fraction of the bankruptcy value $K(\tau)$ when the firm defaults at time $\tau > t$. Naturally, the firm's ability to finance its operations with debt (or its debt capacity) must be closely

related to $K(t)$, which is the conditional expected discounted value of $K(\tau)$; this linkage is substantiated in Williamson (1988) and Shleifer and Vishny (1992). We expect a decrease in $K(t)$ to lead to a reduction in the firm's debt capacity, which forces the firm to substitute some debt financing with equity financing. Similarly, we expect an increase in $K(t)$ to increase debt capacity, which encourages the firm to substitute some equity financing with debt financing. Consequently, when $K(\tau)$ increases or decreases without a proportional movement in $V(t)$, we expect the debt ratio to also move in the same direction. To some degree then, it is convenient to think of the bankrupt firm value $K(t)$ as $(1 - W)$ fraction of the firm's current book liability. Under this interpretation, our model is similar to a generalized version of Briys and Varenne (1997), where the book liability of the firm follows a diffusion process. In addition, insofar that γ_v, γ_r, β_v and β_r are selected such that the process $K(t)$ tracks $V(t)$, our model also captures the notion of the firm trying to maintain a stationary leverage.

C. The Solvency Ratio

We now define a new variable, the log-solvency ratio:

$$X(t) \equiv \log \frac{V(t)}{K(t)}. \tag{3}$$

We note that when $X(t)$ hits 0 for the first time, the firm enters into default. Restating default this way avoids having to keep track of both $V(t)$ and $K(t)$. The evolution of $X(t)$ completely describes the default probability.

The dynamics of $X(t)$ are given by:

$$dX(t) = \mu_X dt + \sigma_X dZ_X(t) \tag{4}$$

where Z_X is a new Brownian motion defined by:

$$\sigma_X Z_X(t) = (\gamma_r - \beta_r)Z_r(t) + (\gamma_v - \beta_v)Z_v(t) \tag{5}$$

and

$$(\sigma_X dZ_X)^2 = [(\gamma_r - \beta_r)^2 + (\gamma_v - \beta_v)^2 + 2\rho_{rv}(\gamma_r - \beta_r)(\gamma_v - \beta_v)]dt \tag{6}$$

and where the drift coefficient is given by:

$$\mu_X = \frac{1}{2}\left(\sigma_K^2 - \sigma_V^2\right) \tag{7}$$

where σ_K is the volatility of the bankruptcy value process:

$$\sigma_K^2 = \beta_r^2 + \beta_v^2 + 2\rho_{rv}\beta_r\beta_v \tag{8}$$

and where σ_V is the volatility of the continuation firm value process:

$$\sigma_V^2 = \gamma_r^2 + \gamma_v^2 + 2\rho_{rv}\gamma_r\gamma_v \tag{9}$$

Note, that dZ_X^Q is correlated with dZ_r^Q. The instantaneous correlation coefficient is given by:

$$\rho_{Xr}dt = \frac{\rho_{rv}(\gamma_v - \beta_v) + (\gamma_r - \beta_r)}{\sigma_X}dt. \tag{10}$$

We can now define the condition of default in terms of X. Default is defined to occur at τ, where τ is the first time X hits zero. Note that default does not depend on $\delta_e(t, V, K, r)$ and $\delta_d(t, V, K, r)$. This arises because the firm's dividend and debt interest payments impact both the firm value V and the bankrupt value K equally. This feature is distinctively different from other structural models with the exception of stationary leverage models.

D. Debt Write Downs when Default Occurs

We assume that debt restructuring occurs simultaneously for all debt issues once the firm defaults. This is in accordance with cross default provisions that are widely adopted in practice. Next, following Longstaff and Schwartz (1995), we assume that, at default, the holder of a corporate coupon bond receives $1-W$ of an otherwise equivalent Treasury bond; where the write down W is sometimes referred to by ratings agencies as the loss severity (see Cantor and Fons (1999)), and where W is larger for bonds with lower priorities. For example, consider two corporate bonds – a senior secured and a junior note. The write down W_1 for the senior note would be less than the write down W_2 for the junior note. For modeling simplicity W is assumed deterministic; equivalently, we may assume that W is stochastic but uncorrelated with other stochastic processes in the model.

Since a corporate coupon bond, in default, is exchanged for an equivalent Treasury bond, which is a portfolio of Treasury zeros, it can be replicated by a portfolio of corporate zeros in an obvious way. The firm's outstanding debt issues can then be modeled as a collection of corporate zeros that are defaulted on at the same time, and each corporate zero coupon bond can be priced independent of the other zeros. This formulation allows each corporate bond to be priced independently of the firm's other liabilities, making the detailed description of the firm's capital structure unnecessary for the pricing of the corporate bonds! We write the time t value of a corporate zero coupon bond with maturity T as:

$$\begin{aligned} C(t,T) &= \mathrm{E}_t^Q[(1-W\mathbf{1}(\{\tau \le T\}))e^{-\int_t^T r(u)du}] \\ &= P(t,T) - W\mathrm{E}_t^Q[\mathbf{1}(\{\tau \le T\})e^{-\int_t^T r(u)du}] \end{aligned} \tag{11}$$

where $P(t,T)$ is the price of the T-maturity Treasury zero, τ is the first time X hits zero, $\mathbf{1}(\{\tau \le T\})$ is an indicator function which takes on the value 1 if $\tau \le T$, and the expectation

is taken over Q.

E. The Short Rate Dynamics

We now specify the dynamics of the default-free term structure. For now we specify a general process for the default-free short rate. We will study particular parameterizations of the process when we solve for analytical pricing formulas.

The dynamics of the instantaneous default-free interest rate r are governed by:

$$dr(t) = \mu_r(r,t)dt + \sigma_r(r,t)dZ_r(t) \tag{12}$$

where $\mu_r(r,t)$ and $\sigma_r(r,t)$ can be functions of r and t (and are left unspecified for now), and $Z_r(t)$ is the same Brownian motion that shocks V and K. The introduction of stochastic default-free interest rates is important for examining the impact of interest rate risk on the default probability and for explaining the observed differences in credit spreads for firms with similar credit ratings. Jones, Mason and Rosenfeld (1984) and Ogden (1987) conclude from their empirical studies that the nonstochastic default-free interest rate specification may contribute to bond overpricing of the Merton model. Empirically, Eom, Helwege and Huang (2003) find that adding interest rate volatility has significant pricing impacts. However, for the structural models studied in Eom et. al., the pricing errors do not seem to be attenuated by the addition of stochastic default-free rates–suggesting rooms for improving how default-free interest rates are incorporated into structural credit risk models.

II. Valuation of Corporate Debt Securities

Examining (11), we need to specify a model of the term structure of default-free interest rates to compute the pure discount bond price $P(t,T)$. We need to further characterize the first hitting time τ to evaluate the expectation $\mathrm{E}_t^Q[\mathbf{1}(\{\tau \leq T\}) \exp(-\int_t^T r(u)du)]$. We postpone specifying the interest rate process until later since our framework is compatible with most commonly used default-free term structure models.

A. Bond Pricing Formula Under the Forward Measure Q_T

As is often true, using the discount bond with price $P(t,T)$ as the numeraire simplifies the algebra. We re-write the formula for a corporate discount bond in (11) as:

$$\begin{aligned} C(t,T) &= P(t,T)\mathrm{E}_t^{Q_T}[(1 - W\mathbf{1}(\{\tau \leq T\})] \\ &= P(t,T)(1 - W\mathrm{E}_t^{Q_T}[\mathbf{1}(\{\tau \leq T\})]) \\ &= P(t,T)(1 - W\Pi(t,T)) \end{aligned} \tag{13}$$

where the expectation is taken under the forward measure Q_T and where we define $\Pi(t,T)$ (the forward default probability) as the probability that default occurs between t and T under Q_T.[10] This formulation of the bond prices is easier to work with.

To make use of equation (13) to price corporate bonds, we need to re-express the dynamics of X under Q_T:

$$dX(t) = (\mu_X + \rho_{Xr}\sigma_X s(t,T))\, dt + \sigma_X dZ_X^{Q_T}(t) \tag{14}$$

where $s(t,T)$ is the volatility of the T-maturity discount bond and $Z_X^{Q_T}$ is a standard Brownian motion under Q_T (See Appendix A for an elaboration on (14)). The drift of X under

[10] See Baxter and Rennie (1996) or Duffie (1996) for references to the forward probability.

Q_T has an additional term $\rho_{Xr}\sigma_X s(t,T)$ that serves to correct for the interest rate risk.

For $\rho_{Xr} > 0$, the shocks to the log-solvency ratio are positively correlated with the shocks to the short rate. So increases in the default probability (decreases in the log-solvency ratio), which reduce corporate bond prices, are likely associated with decreases in the interest rate, which increase corporate bond prices. Intuitively, the two sources of risk partially offset each other, resulting in lower credit spreads than if $\rho_{Xr} \leq 0$

The forward risk adjusted probability of default $\Pi(t,T)$ can, in general, be computed by simulation, although closed-form and analytic approximation solutions are available under more restrictive assumptions. We present these special cases in the next section.

C. Computing Bond Prices

1. Independent Default Risk and Interest Rate Risk

Under specific parameterizations of the firm value process V and the default boundary process K, ρ_{Xr} will be zero, indicating that the default probability is independent of r under Q^T.[11] An equivalent assumption is made in Jarrow and Turnbull (1995) and is implicit in models where r is non-stochastic. When $\rho_{Xr} = 0$, $\Pi(t,T)$ can be computed in closed-form for any prescribed term structure model; it is simply the probability of an arithmetic Brownian motion, starting from the initial value $X(t)$, with drift μ_X and volatility σ_X, hitting zero before time T.

The first-passage time density of X evaluated at $\tau > t$ is (See Karatzas and Shreve

[11] Recall from (10) that ρ_{Xr} is zero when $\rho_{rv}(\gamma_v - \beta_v) + (\gamma_r - \beta_r) = 0$

(1991)):

$$\phi(\tau) = \frac{X(t)}{\sigma_X (2\pi)^{1/2} (\tau - t)^{3/2}} \exp\left\{ -\frac{[X(t) + \mu_X(\tau - t)]^2}{2\sigma_X^2 (\tau - t)} \right\} \tag{15}$$

so that

$$\Pi(t,T) = 1 - \mathcal{N}\left(\frac{X(t) + \mu_X(T-t)}{\sigma_X \sqrt{T-t}} \right) + \exp\left\{ \frac{-2\mu_X X(t)}{\sigma_X^2} \right\} \mathcal{N}\left(\frac{X(t) - \mu_X(T-t)}{\sigma_X \sqrt{T-t}} \right) \tag{16}$$

where $\mathcal{N}$ denotes the standard normal cumulative distribution function.

We note that this model gives closed-form corporate bond prices when coupled with any model of the default-free term structure that produces closed-form Treasury bond prices. We will examine the validity of the assumption that $\rho_{Xr} = 0$ in the empirical portion of the paper.

2. Deterministic Bond Volatilities

We are also able to derive analytical approximate bond prices when the volatility of the T-maturity Treasury bond is a deterministic function of time. Term structure models like Vasicek (1977), Ho and Lee (1986), Hull and White (1990), or other models in the Heath, Jarrow and Morton (1992) framework produce bond prices with deterministic volatilities. With this assumption, the first-passage time problem can be restated as the first-passage time of a standard arithmetic Brownian motion through a deterministic boundary. The formula for the boundary is given by:

$$B(\tau) = \frac{X(t) + \mu_X(\tau - t)}{\sigma_X} + \frac{\rho_{Xr}}{\sigma_X} \int_t^\tau s(u,T)du \tag{17}$$

where, again, $s(t,T)$ is the T-maturity Treasury bond return volatility (see Appendix B for the derivation).

The default probability Π_t^T can be approximated very efficiently in closed form. Durbin (1992) shows that the first-passage time probability can be approximated to a high degree of accuracy by the following approximation[12]:

$$\begin{aligned}\Pi(t,T) \approx & \int_t^T \left(\frac{B(u)}{u-t} - B'(u)\right)\varphi(u)du \\ & - \int_t^T \int_t^u \left(\frac{B(v)}{v-t} - B'(v)\right)\left(\frac{B(u)-B(v)}{u-v} - B'(u)\right)\varphi(u,v)dvdu \end{aligned} \tag{18}$$

where $B'(u)$ denotes the slope of the boundary at u; $\varphi(u)$ is the density of the Brownian motion at time u, evaluated at $B(u)$; and $\varphi(u,v)$ is the joint density of the Brownian motion at times u and v, evaluated at $B(u)$ and $B(v)$:

$$\varphi(u) = (2\pi(u-t))^{-1/2} \exp\left\{\frac{-(B(u)-B(t))^2}{2(u-t)}\right\} \tag{19}$$

and

$$\varphi(u,v) = \varphi(u)(2\pi(u-v))^{-1/2} \exp\left\{\frac{-(B(u)-B(v))^2}{2(u-v)}\right\} \tag{20}$$

The first-passage time probability can be easily computed with simple numeric quadrature methods to evaluate the integrals in equation (18).

III. The Term Structure of Yield Spreads and Duration

A. Yield Spread

In this section we plot the term structure of yield spreads for varying values of the parameters. We consider our model in conjunction with the Vasicek and the CIR short rate

[12] Additional expansion terms may be added to further improve accuracy.

processes. For the Vasicek case, corporate bond prices can be approximated analytically.[13] For the CIR case, we compute bond prices numerically. The yield spread is computed as

$$\begin{aligned} y(t,T) &= \frac{1}{T-t}\log\frac{P(t,T)}{C(t,T)} \\ &= \frac{1}{T-t}\log\frac{1}{1-W\Pi(t,T)} \end{aligned} \tag{21}$$

We note that the yield spread depends positively on the write down rate W and the forward default probability $\Pi(t,T)$. Since we can interpret W as a proxy for priority and the forward probability as a proxy for default risk, the model predicts yield spreads to be increasing in default risk and decreasing in priority, which agrees with intuition.

In figure 1, we plot the yield spread for various values of the solvency ratio $X(0)$. We find that less solvent firms display a humped credit spread term structure, while the term structure is monotone increasing for firms with high solvency. This is consistent with the empirical results of Sarig and Warga (1989) and Bohn (1999).[14] In addition, we find that shorter term bonds are more sensitive to changes in the solvency of the issuing firms and the sensitivity is highest for the low solvency (rating) bonds. For a corporate bond of four year maturity, a fall in the solvency ratio from five to three increases the yield spread from 10 to 80 basis points. A fall in solvency ratio from three to two increases the yield spread from 80 to 330 basis points. For a corporate bond of twenty year maturity, the similar changes in the solvency ratio raise the credit spread from 60 to 115 to 180 basis points, respectively. Note that as maturity goes to zero, credit spread also goes to zero. This is standard in a

[13] See Appendix C for the application of Durbin's rapidly convergent approximation formula for the Vasicek specification

[14] Helwege and Turner (1998) find no empirical support for humped credit spread. However, Bohn (1999), using a larger sample of low quality issues, finds strong evidence for humped credit spread.

perfect information environment where investors observes the firm's asset level.[15]

The intuition for the humped credit spread term structure is clear. For very low grade bonds, the probability of default does not increase dramatically with maturity beyond the first few years. More specifically, we can write the T year forward default probability $\Pi(0,T)$ as $\Pi(0,\frac{T}{2}) + \Pi(\frac{T}{2},T)$, where $\Pi(\frac{T}{2},T)$ is the default probability conditional on no default in the first T/2 years. In our model, $\Pi(\frac{T}{2},T) < \Pi(0,\frac{T}{2})$. In particular, for bonds with $X(0)$ close to 0, we have $\Pi(\frac{T}{2},T) << \Pi(0,\frac{T}{2})$. The easiest way to understand this decreasing conditional default probability is through a simple binomial example. Assume that $X(t)$ can go up or down by 25 percent with equal probability each period. We see that the lower support of the conditional distribution (conditional on surviving up to time t) is curtailed at $X(t) = 0$. However, the upper support is increasing with t. The decreasing time t conditional default probability is then a trivial consequence. Since our modeling approach allows the firm to migrate between credit classes as its solvency changes, the decreasing conditional default probability reflects a higher conditional expected credit worthiness. The prediction of a decreasing conditional probability of default and the associated humped-shape yield spread term structure for low solvency firms are, however, absent in the statistical models. Traditional statistical models assume instead a constant conditional default probability, which is appropriate for the examination of the swap spread or generic corporate spread for

[15] However, in a noisy environment, such as the one described in Duffie and Lando (2001), a firm with asset level lower than its book liability may be able to raise additional debt capital. In which case a credit spread would exist even for a debt of zero maturity. Huang and Huang (2002) find empirical evidence that yield spread does not shrink to zero with decreasing maturity. This empirical observation cannot be reconciled with any of the known structural models and is a defect of this literature. We investigate in the empirical section the degree to which this issue impacts our model's pricing performance.

a portfolio of bonds belonging to a given credit class, but is less well-suited for studying individual firm default. Fons (2002) reports that annually, on average, 25% of all rated corporate bonds migrate to different credit categories, suggesting that for the average firm, $X(t)$ is rather volatile.

In figure 2, we see that the yield spread is decreasing with μ_X. This is obvious. While there is no ex-ante reason to expect a positive or a negative μ_X, we would expect μ_X to be small in magnitude relative to the level of X for a firm not near bankruptcy. A large negative μ_X would indicate a capital structure policy, which leads quickly to bankruptcy – a scenario that appears unlikely. In the empirical section, we see that the estimated μ_X is small relative to the level of X (usually 1/100th the value of X) and is typically insignificantly different from zero. Contrasting to models with constant default boundaries, where the log-solvency ratio has a positive drift due to the firm value's risk-adjusted drift r, our model suggests a higher yield spread for corporate bonds.

In figure 3, we plot the term structure of yield spreads for various values of ρ_{Xr}. We see that the yield spread is decreasing in ρ_{Xr}. This observation has important implications for the yield spreads paid by counter-cyclical firms ($\rho_{Xr} < 0$) and cyclical firms ($\rho_{Xr} > 0$) which have otherwise identical credit ratings. For $\rho_{Xr} > 0$, the firm's default probability increases (or X decreases) when interest rate decreases. The former effect decreases the corporate bond price while the latter increases it, creating offsetting effects. It is worth noting that ρ_{Xr} impacts the default probability substantially in our static comparison, and the effect is intensified by both maturity and solvency. The difference in yields between the following two sets of parameters $\{X(0) = \log(2), \rho_{Xr} = 0\}$ and $\{X(0) = \log(2), \rho_{Xr} = 0.15\}$ at 4 year maturity is around 35 bps, while the difference between $\{X(0) = \log(5), \rho_{Xr} = 0\}$ and

$\{X(0) = \log(5), \rho_{Xr} = 0.15\}$ at the same maturity is less than 5 bps. So a less solvent firm suffers a larger increase in yield spread when ρ_{Xr} decreases. However, the yield difference between $\{X(0) = \log(5), \rho_{Xr} = 0\}$ and $\{X(0) = \log(5), \rho_{Xr} = 0.15\}$ at 20 year maturity increases to around 40 bps. So a longer maturity corporate debt suffers a larger increase in yield spread when ρ_{Xr} decreases.

Recall the formula for ρ_{Xr}:

$$\rho_{Xr} dt = \frac{\rho_{rv}(\gamma_v - \beta_v) + (\gamma_r - \beta_r)}{\sigma_X} dt, \tag{22}$$

where γ_r and β_r are, respectively, the sensitivity of the continuation (V) and the bankruptcy (K) values of the firm to interest rate shocks, and γ_v and β_v are, respectively, the sensitivity to firm fundamental shocks.

Positive shocks to the firm's fundamentals should increase both the firm's continuation value and bankruptcy value. Therefore, γ_v and β_v should be positive. In addition, we expect the continuation value of the firm to have a higher loading on firm fundamental shocks, resulting in $\gamma_v - \beta_v > 0$. We also expect the correlation between the shocks to the firm fundamentals and the interest rates (ρ_{rv}) to be negative. Therefore, for ρ_{Xr} to be positive (which is observed in seven out of the nine firms in our empirical study), β_r must be more negative than γ_r, suggesting that the bankruptcy value of the firm is more sensitive to interest rate shocks than the continuation value of the firm. This scenario is unlikely if the bankruptcy value represents the value of a portfolio of liquidated plants and equipments. However, if the bankruptcy value represents, instead, the value of a restructured new going-concern, then it is possible that the shadow value of the new going-concern (which is K) would have a higher sensitivity to interest rate shocks.

Kahl (2002) reports that a significant fraction of the firms in bankruptcy emerge out of Chapter 11 as restructured going-concerns rather than being liquidated. The process of restructuring often recapitalizes the new concerns (almost entirely) with short-term debt financing and then rolls the bridge financing into longer term loans as the firm's credit quality improves. Equity financing usually is not an option for these firms fresh out of Chapter 11. From the discussion above, we can extract the market's expectation of the bankruptcy outcome of the firm conditional on default from the estimated ρ_{Xr}. A positive ρ_{Xr} would suggest that the firm would likely restructure to become a new going concern upon entering bankruptcy, while a negative ρ_{Xr} would suggest that the firm would likely be liquidated.

In figure 4, we see that the yield spread is increasing with σ_X. The more volatile is the firm's log-solvency ratio, the more likely default occurs. From equation (6) we know that:

$$\sigma_X^2 = (\gamma_r - \beta_r)^2 + (\gamma_v - \beta_v)^2 + 2\rho_{rv}(\gamma_r - \beta_r)(\gamma_v - \beta_v). \tag{23}$$

Therefore, the volatility of the solvency ratio depends most importantly on the relative responses of V and K to the firm fundamental shocks and the interest rate shocks. If V and K respond in near tandem to the two shocks, then σ_X is small and vice versa. We note that for a growth firm, V is likely to be substantially more volatile than K, suggesting a high cost of debt financing even if X is high.

In figure 5, we plot the yield spread for various values of the write down ratio W. The yield spread increases with W as expected. The greater the loss of value to the principal during the debt renegotiation, the greater the risk premium demanded up front. We note, however, that since W does not impact the default probability; it also does not impact

the curvature of the yield spread. Thus, insofar as yield spreads and credit ratings issued by rating agencies reflect both the default probabilities and debt write downs, our model can disjoin the two effects easily by examining the shape and the level of the yield spread. For statistical models, disjoining the effects of the write down ratio W from the default intensity is impossible. The ability to estimate the write down ratio provides some real advantages. Traditionally, the recovery rates $(1 - W)$ for different debt priority classes can only be estimated from bond issues that have been defaulted on (see Altman (1992), Frank and Torous (1994), and Fons (2002) for studies on recovery rates for different priority and rating classes). This severely limits the size of the sample. With our model, we can estimate the recovery rates using all traded debt issues, which significantly increase the size of the sample.

B. Duration

We now turn our attention to the duration of corporate bonds. We can write the duration of the corporate discount bond as:

$$\frac{\partial C(t,T)/\partial r}{C(t,T)} = -\frac{\partial P(t,T)/\partial r}{P(t,T)} - \frac{\partial(1 - W\Pi_t^T)/\partial r}{(1 - W\Pi_t^T)} \tag{24}$$

where it is convenient to interpret $1-W\Pi(t,T)$ as the *risk and interest rate adjusted* recovery rate on the loan. Bond prices respond to changes in the short rate through two channels. First, the discounted value of the bond's promised cash flows depends critically on r; this is the Treasury component of the corporate bond. Second, the adjusted recovery rate may also depend on r since the probability of default may depend on r. For the Vasicek model, $\Pi(t,T)$ does not depend on r, so the duration is always equal to the duration of a Treasury

bond. For the CIR model, $\Pi(t,T)$ does depend on r. From equation (**??**), we see that $X(t)$ is increasing in $r(t)$ (or $\Pi(t,T)$ is decreasing in $r(t)$) when $\rho_{Xr} > 0$, and vice versa. Therefore, when $\rho_{Xr} > 0$, the risk and interest rate adjusted recovery rate is increasing in $r(t)$, suggesting a duration for the corporate bond that is lower than the Treasury discount bond, which is consistent with the observation of Chance (1990). However, numerically, the impact of an increase in the instantaneous short rate is non-existent on the forward default probability $\Pi(t,T)$. Numerically, we are unable to produce negative durations within reasonable or even extreme ranges of parameters. So we do not predict negative durations. This prediction distinguishes our model from credit models with constant default boundaries, where negative duration is easily produced for low grade bonds. We consider the prediction of non-negative durations intuitive and desirable; there is no empirical evidence substantiating the existence of negative duration bonds.

IV. Empirical Analysis

The tractability issue has limited the empirical analysis of structural models. To reduce the computational complexity of the estimation problem, model parameters are often calibrated rather than estimated from actual price data. The calibrated model is then used to fit the price data to determine model performance. Using this approach, Jones, Mason and Rosenfeld (1984), Ogden (1987) and Lyden and Saraniti (2000) have found Merton type models to overprice corporate bonds. In an expanded study, Eom, Helwege and Huang (2002) examine five structural models (including also Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001)) and find that

Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001) models is able to deliver higher yield spreads because of the inclusion of stochastic default-free rates; however, the pricing error is large (the average error for the models is more than 100% of the predicted yield spreads), and extremely large or small spreads are common. The poor performance may be attributed partly to the accuracy of the parameter calibration exercise. However, more interesting is the evidence that all five models are found to have particular difficulties pricing bond issues from firms with low leverage ratios and low firm value volatilities; bonds with short durations are also priced with greater errors. In this section we explicitly address the calibration concern by estimating all model parameters with price data. We find that our model produces pricing errors that are the same size as the bid-ask spreads of the bonds. In addition we do not find difficulties pricing bonds from firms with low leverage ratios and asset volatilities or with low durations.

For our empirical study, we use the parametric model with the Vasicek default-free term structure specification. We note that no essential benefits are gained by adopting the CIR specification, which is computationally much more intensive. Since our focus is on the term structure of the yield spread rather than on the Treasury yield curve, we adopt the more convenient Vasicek specification here.[16] We derive the approximation to the forward default probability for the Vasicek specification in Appendix C.

A. The Data

[16] In a numerical exercise, we find that term structures of yield spreads produced under the CIR specification can be reproduced almost exactly using the Vasicek specification. The average root mean square error between the CIR and the Vasicek term structures of yield spreads is 0.8 basis points.

We estimate the model for corporate bonds issued by nine different issuers with S&P bond ratings ranging from AAA to BB. The empirical exercise is not exhaustive due to the computational intensity of the estimation. We seek instead to illustrate the applicability of the model for various risk classes. Firms represented in this sample are selected to satisfy the following screening criteria.

1. NYSE listed and traded – A numbers of corporate bonds are now listed and traded on the NYSE Automated Bond System. Daily closing prices on these bonds are available from Datastream. In addition, the traded volume as well as the bid-ask spreads are available from the NYSE quote reporting system. This requirement ensures that the reported prices are traded prices rather than soft quotes or matrix inferred prices.

2. Liquid issues – as determined by examining the trading volume and daily price movements.

3. Dollar denominated.

4. Non-callable.

5. No sinking fund requirement.

6. Have at least 5 years of daily data ending in December 1999.

7. Have more than two bond issues in the same claimant class satisfying 1-6.

Bond issuer and issuance information are collected from SDC and cross-referenced against information in Datastream and the S&P Bond Guide. The summary statistics of the selected bond issuers and issues are listed in Table 1.

It is worth noting that the average bid-ask spread (quoted in yields) for the bond issuers in our sample is about 40 basis points. The bid-ask spread data are collected from the NYSE Automated Bond System for all currently traded bonds issued by the 9 firms in our sample. The large bid-ask spread alerts us to micro-structure problems such as return auto-correlation arising from bid-ask bounce. The concern, however, is moderated by using monthly frequency data.

The price data for the selected bonds are obtained from Datastream. To avoid the problem with stale data resulting from non-trading, we create monthly price data from daily data. Specifically, we mark consecutive days of identical prices as stale and then create monthly price data using the first trading day of the month. If the first trading day price is stale, a missing data flag is inserted and the particular data point is discarded in our estimation. Finally, prices are adjusted for accrued interest. For the Treasury rates, we construct the appropriate default-free term structure for each first trading day in the sample using the Vasicek model and the yield information from the corresponding Treasury strips. The fitted error of the Vasicek model in yield is 11 basis points, which is consistent with what is reported in Duffee (1999), who also calibrates a Vasicek model for computing credit spreads.

B. The Method

The time t price of corporate bond j with face value \$1, coupon rate D, and M remaining coupon payments is computed as:

$$V_{jt} = \sum_{m=1} D \cdot C(t, T_m; \theta, W_j, X(t)) + C(t, T_M; \theta, W_j, X(t)) \tag{25}$$

where $C(t, T_m; \theta, W_j, X(t))$ is computed by (11), and we make explicit the dependence on the parameter vector of firm characteristics $\theta = (\mu_X, \sigma_X, \rho_X)$, the write down W_j, and the state variable $X(t)$.

Under the null, at the true vector of firm characteristics θ^*, true write down W_j^*, and the true realized state variable $X^*(t)$, the observed bond yield $\tilde{Y}_{jt}$ for bond j at time t is:

$$\tilde{Y}_{jt} = Y_j(\theta^*, W_j^*, X^*(t)) + \tilde{\varepsilon}_{jt}. \tag{26}$$

where $\tilde{\varepsilon}_{jt}$ is measurement error.

Since we do not observe the true log-solvency ratio $X^*(t)$ directly, we must impose moment restrictions to estimate $\hat{X}_t$ for each time t.[17] This leads naturally to the following set of conditional orthogonality conditions (one restriction for each time t observation for each corporate bond j):

$$\mathrm{E}\left[\tilde{Y}_{jt}\right] = Y_{jt}(\theta^*, W_j^*, X^*(t)) \quad \text{for } j = 1 \cdots J \text{ and } \quad t = 1 \cdots T \tag{27}$$

where j indexes over the J bonds issued by the firm, and t indexes over the T price observations.

To further restrict the filtered time series of the state variable $\hat{X}$, the following orthogonality conditions on the conditional mean, variance, and covariance are needed:

$$\mathrm{E}\left[\Delta X^*(t)\right] = \mu_p^* \quad \text{for } t = 1 \cdots T \tag{28}$$

and

$$\mathrm{E}\left[\left(\Delta X^*(t) - \mu_p^*\right)^2\right] = \sigma_X^{*2} \quad \text{for } t = 1 \cdots T \tag{29}$$

[17] We use * to indicate the true parameter vector or state variable. We use $\hat{}$ to indicate the sample estimates for the true parameter vector or state variable.

and

$$\mathrm{E}\left[\left(\Delta X^*(t) - \mu_p^*\right)\left(\Delta r(t) - a(r(t-1) - b)\right)\right] = \sigma_r \sigma_X^* \rho_{Xr}^* \quad \text{for } t = 1 \cdots T \tag{30}$$

where μ_p^* is the physical drift of the process X^* and a, b, and σ_r are parameters of the Vasicek short rate process.

Additionally, we impose the restriction that, for a given issuer, bonds which belong to the same priority class, have the same write down W_j. This restriction is imposed directly in the bond pricing formula and does not appear in the set of moment restrictions.

Summarizing, for each time t observation, we have $J + 3$ orthogonality conditions, where J is the number of bonds included in the sample for the given issuer. Suppose we have 120 months of observations for four bonds in two priority classes from the same issuer; we would have $120 \cdot (4+3) = 840$ moment restrictions, with which to estimate 126 unknowns (120 $\hat{X}_t$'s, 2 $\hat{W}_j$'s, and 1 $\hat{\mu}_p$, $\hat{\mu}_X$, $\hat{\sigma}_X$, and $\hat{\rho}_X$).

While there are other ways to impose the restrictions on the filtered state variable $\hat{X}$, the point restrictions defined above prove to be convenient for stating the estimation problem in the context of GMM. The dimension of the estimation problem appears to be extremely high at first glance. However, with the judicious selection of a special block diagonal matrix, we can reduce the dimension of our minimization problem. We now state the objective

function to be minimized:[18]

$$Q(\theta, W, X, \mu_p) = \begin{bmatrix} \tilde{u}_1(\theta, W, X_1, \mu_p) \\ \tilde{u}_2(\theta, W, X_2, \mu_p) \\ \vdots \\ \tilde{u}_T(\theta, W, X_T, \mu_p) \end{bmatrix}' \begin{bmatrix} \hat{\Omega} & 0 & \cdots & 0 \\ 0 & \hat{\Omega} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \hat{\Omega} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{u}_1(\theta, W, X_1, \mu_p) \\ \tilde{u}_2(\theta, W, X_2, \mu_p) \\ \vdots \\ \tilde{u}_T(\theta, W, X_T, \mu_p) \end{bmatrix} \tag{31}$$

where $\tilde{u}_t(\theta, W, X_t, \mu_p)$ is the time t vector of sample moments corresponding to the restrictions defined in (27) to (30). We postpone the discussion on how to estimate $\hat{\Omega}$ until a few paragraphs later.

We know that the GMM estimates $\{\hat{\theta}, \hat{W}, \hat{X}, \hat{\mu_p}\}$ associated with our particular choice of weighting matrix converges in probability to the true parameters $\{\theta^*, W^*, X^*, \mu_p^*\}$. In fact, the block diagonal weighting matrix improves efficiency over the standard unrestricted weighting matrix from a two stage procedure when the pricing errors are uncorrelated across time (See Cochrane (2001) for a detailed discussion on choosing the GMM weighting matrix).

The benefit of our block diagonal weighting matrix is most evident when we rewrite equation (31) as:

$$Q(\theta, W, X, \mu_p) = \sum_{t=1}^{T} \left[\tilde{u}_t(\theta, W, X_t, \mu_p)\right]' \hat{\Omega}_t^{-1} \left[\tilde{u}_t(\theta, W, X_t, \mu_p)\right] \tag{32}$$

Note, for a particular choice of $\{\theta, W, \mu_p\}$, the t-th term in the summation depends only on $X(t)$. Therefore, minimizing Q, given $\{\theta, W, \mu_p\}$, is the trivial task of minimizing the summation term by term – or performing one variable minimization T times. This, then, allows us to treat $Q(\theta, W, X(\theta, W, \mu_p), \mu_p)$ as a function of only θ, W, and μ_p in our minimization.

We make the additional restriction $\{\Omega_t = \Omega; t = 1 \ldots T\}$.

[18] Note, we estimate each issuing firm separately, since our model does not specify the interactions between firms.

We estimate $\hat{\Omega}$ using the following two stage procedure. In the first stage, the identity matrix is used as the weighting matrix for the GMM estimation. Corresponding to the consistent first-stage parameter estimates $\{\hat{\theta}^1, \hat{W}^1, \hat{X}^1, \hat{\mu}_p^1\}$ we have the following sample moments $\{\tilde{u}_1(\hat{\theta}^1, \hat{W}^1, \hat{X}_1^1, \hat{\mu}_p^1), \ldots, \tilde{u}_T(\hat{\theta}^1, \hat{W}^1, \hat{X}_T^1, \hat{\mu}_p^1)\}$. The inverse weighting matrix $\hat{\Omega}$ is then estimated as:

$$\hat{\Omega} = \hat{\Gamma}_0 + \sum_{n=1}^{2} \left(1 - \frac{n}{3}\right) \left(\hat{\Gamma}_n + \hat{\Gamma}_n'\right) \tag{33}$$

where

$$\hat{\Gamma}_n = \frac{1}{T} \sum_{t=n+1}^{T} \left[\tilde{u}_t(\hat{\theta}^1, \hat{W}^1, \hat{X}_t^1, \hat{\mu}_p^1)\right] \left[\tilde{u}_t(\hat{\theta}^1, \hat{W}^1, \hat{X}_t^1, \hat{\mu}_p^1)\right]^T. \tag{34}$$

Note that the weighting matrix we estimate is adjusted for serial correlation between observations (see Hamilton (1994)).

Finally, it is important to realize that given the orthogonality conditions that we have defined and the pricing formula associated with the Vasicek specification of the default-free interest rate, some parameters can be reliably estimated while others cannot. From the formula for the default probability $Pi(t,T)$ presented in Appendix C, we see that we cannot separately identify X_t^*, μ_X^*, ρ_{Xr}^* and σ_X^* using only the moment conditions specified in (27), (28), and (29). Using these restrictions, only the standardized parameters $\frac{X_t^*}{\sigma_X^*}$, $\frac{\mu_X^*}{\sigma_X^*}$ and $\frac{\rho_{Xr}^*}{\sigma_X^*}$ can be estimated. The moment condition defined in (30) is required to disjoin these parameters. Performing the GMM estimation (to estimate only the standardized parameters) without the orthogonality condition defined in (30), we find that $\frac{\mu_X^*}{\sigma_X^*}$ and $\frac{\rho_{Xr}^*}{\sigma_X^*}$ can be estimated very accurately, while μ_X^*, ρ_{Xr}^* and σ_X^* are estimated with much lower degrees of confidence.

C. The Results

Table 2 presents the parameter estimates and their standard errors for each of the 9 corporate bonds. The row $RMSE$ reports the average bond pricing errors in yields for each firm. The average pricing error is 39 basis points. Recall that the average bid-ask spread for the firms in our sample is 41 basis points, so the $RMSE$ is quite low compared to the precision of the data.

We report the average solvency ratio V/K and log-solvency ratio $\hat{X}_t$ also. Observe that both the estimated $\hat{\mu_p}$ and $\hat{\mu_X}$ are between 1/10th to 1/100th the size of $\hat{X}_t$. In fact, we cannot conclude that $\hat{\mu_p}$ and $\hat{\mu_X}$ are different from zero! We expect this to be the case. If the μ's were large compared to X, then the firm drifts deterministically toward 100 percent equity financing or 100 percent debt financing. This cannot represent a stationary (or even reasonable) capital structure policy.

The estimated $\hat{\rho}_{Xr}$ is significantly different from zero for only 3 out the 9 firms; we have alerted the reader to the difficulty of estimating the non-standardized parameters before. The standardized estimates $\frac{\hat{\rho_{Xr}}}{\sigma_X}$, which are not separately reported here, are all significantly non-zero, suggesting that the interaction between the default-free interest rates and the default probability is important in determining the corporate yield spreads. (though the resulting pricing impact may be small).

In addition, for 7 out of the 9 firms, the estimated $\hat{\rho}_{Xr}$ is positive. A positive ρ_{Xr} indicates that the forward probability of default is negatively correlated with the interest rate, or that changes in the yield spread are negatively correlated with the changes in the interest rate. This is observed in Longstaff and Schwartz (1995) and confirmed in Taurén (1999) and Duffee (1999). However, in contrast to LS, increases in r do not necessarily reduce the default probability and the yield spread (and in the Vasicek case, interest rate shocks

do not 'cause' any change to the default probability and the spread). We emphasize that there is no causal relationship between the interest rate and the firm default probability (in the model we estimated). The *observed* negative correlation between yield spread changes and interest rate changes indicates that the firm's bankruptcy value K is more sensitive to interest rate shocks than the firm's market capitalization value V. Recall that K is more sensitive to the interest rate if, at default, the firm is expected to emerge from the debt renegotiation with high leverage. The 'shadow' value of this new going-concern (which is K) would then have a higher sensitivity to interest rate shocks than would the current firm value V.

As we mentioned before, our model allows W and the forward default probability to be estimated separately. Of the 9 issuers in our sample, 7 issuers had junior debts outstanding only. Franks and Torous (1994) and Altman (1992) report that the average write downs for junior debt issues are 0.693 and 0.720 respectively. In our sample, the average write down for junior debt issues is 0.7582. In addition, Franks and Torous (1994) also finds that the average write down for guaranteed issues is 0.395. We have only one issuer with 4 guaranteed notes outstanding, and we estimate a write-down of 0.3852 for those issues. For senior issues, Franks and Torous (1994) find an average write down of 0.530. We have one issuer with 2 senior bonds, and we estimate a write down of 0.4982 for those issues. One useful empirical feature of the model is the estimation of default write downs using bond data from non-defaulting firms. In Altman (1992) and Franks and Torous (1994), the samples include only firms that have filed for bankruptcy protections. The current method, by comparison, can estimate write downs using bond data on solvent issuers, which provides researchers with more cross-sectional observations.

We now present evidence that the model does not show systematic pricing errors with respect to the time to maturity of the corporate bonds or the leverage ratio or the volatility of the firm value. To examine the relationship between the model pricing errors and the time to maturity of the bond, we regress the pricing errors on time to maturity for each bond (33 regressions in total). The average absolute value of the t-statistic for the coefficient is 1.18, suggesting no statistical significant relationship. In our data sample, the shortest time to maturity was more than two years. Therefore, it is possible that mispricing at the very short maturities suggested by Duffie and Lando (2001) and Huang and Huang (2002) cannot be observed clearly. To address the pricing error bias reported by Eom, Helwege and Huang (2003), we examine the RMSE for each of the nine firm with respect to the volatility of the equity value and the book debt ratio as well as with respect to the volatility and the level of the log-solvency ratio. No discernable relationships are observed.

These results combined with the smaller pricing errors, relative to what have been empirically measured using other structural models, give us confidence that our model can be applied fruitfully in practice when combined with our proposed estimation technique. Specifically, our model can be used to determine the price of a firm's new or infrequently traded bond issues. We can estimate the bond pricing parameters for the issuer by applying the model to its liquidly traded bonds. The estimated parameters can then be used to compute the bond prices of the non-traded or less liquid bond issues that the issuer has outstanding.

Finally, we document the relationship between the extracted log-solvency and other firm characteristics. The extracted time series of the log-solvency ratio X_t is, of course, highly correlated with the time series of the firm's yield spreads. In Table 3, we report the

regression of $\frac{\Delta X_t}{X_t}$ on the firm's excess equity returns $(R_t - R_{Tbill,t})$ and changes in the 10-year yields $(\Delta Y_{10yr,t})$. This is similar to the analysis that is performed by Longstaff and Schwartz (1995) for average corporate yield changes (which reports R^2 of 0.65)–but performed on individual firms. We note that the X_t's are increasing with the equity returns in 6 out of 8 issuers.[19] The measured relationship between changes in the firm's log-solvency and its equity value is likely capturing the relationship between log-solvency and the firm's market debt ratio, since changes in firm's market debt ratio and equity value are highly correlated. The positive coefficient is therefore natural and should be expected. However, the regressions are extremely noisy and we are unable to produce small enough standard errors to conclude significance.[20] Firms which report positive ρ_{Xr} also have positive coefficients on the changes in the 10-year yields (with the exception of ARCO); this, of course, is tautological and serves as a check on our empirical procedure. However, only 2 of the 8 issuers have significantly positive coefficient estimates. The results from the regressions are unfortunately mostly insignificant. The low average R^2 further suggests that we are far from understanding the factors which drive the solvency of the firm.

From the empirical results, we believe that future structural models should attempt to identify the factors which drive the firm value V and the default boundary value K. In doing so we can place additional restriction on the solvency process X. This will allow us to examine more carefully the relationships between the firm's capital structure evolution and the firm's solvency as well as the costs of the firm's financing.

[19] Pacific Bell is not a publicly traded entity.

[20] The average number of the time series observations is 80.

V. Conclusion

This article presents a structural model of default risk that allows for tractable pricing of corporate fixed rate debts. The model can be specified in conjunction with a wide range of default-free term structure specifications. The theoretical properties of the model are attractive and consistent with the empirical literature on default yield spread and risky bond duration. In addition, the model compares favorablely with and offers improvement over the existing structural models. We also estimate the model using panel data of bond prices from 9 firms, and illustrate a relatively fast estimation technique as well as some useful applications of our model. Our estimation technique offers obvious advantages over the calibration techniques that have been applied to study structural models. We find that our model, combined with our GMM estimation, produces low pricing errors and do not suffer from the pricing biases observed by recent empirical studies on existing structural models.

Appendix

A. Proof of Equation (14)

Under the spot risk adjusted probability measure Q, we can decompose Z_x into $\rho_{xr}Z_r$ and $\sqrt{1-\rho_{xr}^2}Z_o$, with Z_o a standard Brownian motion orthogonal to Z_r. Shifting the processes to the forward risk adjusted measure Q_T the appropriate drift adjustments are:

$$d\begin{pmatrix} Z_r^{Q_T}(t) \\ Z_o^{Q_T}(t) \end{pmatrix} = d\begin{pmatrix} Z_r(t) \\ Z_o(t) \end{pmatrix} - \begin{pmatrix} s(t,T) \\ 0 \end{pmatrix} dt \tag{A1}$$

The two Brownian motions $Z_r^{Q_T}$ and $Z_o^{Q_T}$ are still orthogonal and therefore constitute a two-dimensional standard Brownian motion under Q_T. The dynamics of X^{Q_T} can be written with respect to $Z_r^{Q_T}$ and $Z_o^{Q_T}$ as:

$$dX^{Q_T}(t) = (\mu_X + \rho_{Xr}\sigma_X s(t,T))\,dt + \rho_{Xr}\sigma_X dZ_r^{Q_T}(t) + \sqrt{1-\rho_{Xr}^2}\sigma_X dZ_o^{Q_T}(t) \tag{A2}$$

To simplify, we can define a standard Brownian motion $Z_X^{Q_T}$:

$$Z_X^{Q_T}(t) = \rho_{Xr} Z_r^{Q_T}(t) + \sqrt{1-\rho_{Xr}^2} Z_o^{Q_T}(t) \tag{A3}$$

and (14) follows.

B. Proof of Equation (17)

Recall equation (14),

$$dX^{Q_T} = [\mu_X + \rho_{Xr}\sigma_X s(t,T)]dt + \sigma_X dW_X^{Q_T} \tag{B1}$$

rewriting, we have

$$dW_X^{Q_T} = \frac{1}{\sigma_X}\Big(dX^{Q_T}(t) - \mu_X dt - \rho_{Xr} s(t,T)dt\Big) \tag{B2}$$

integrating (and noting that $X^{Q_T}(y) = 0$), we have:

$$W_y^{Q_T} - W_t^{Q_T} = \frac{X^{Q_T}(t) + \mu_X(y-t)}{\sigma_X} + \frac{\rho_{Xr}}{\sigma_X}\int_t^y s(u,T)du. \tag{B3}$$

C. Durbin's Rapidly Convergent Approximation for the Forward Default Probability

We apply Durbin's rapidly convergent approximation method to compute the default probability for the Vasicek specification of our model. The Vasicek short rate process is

$$dr(t) = a(b - r(t))dt + \lambda dZ_r^Q(t) \tag{B4}$$

Applying equation (17) and realizing autonomous nature of our problem, the boundary formula is:

$$\begin{aligned} B(u) &= \frac{X(0) + \mu_X(u)}{\sigma_X} + \frac{\rho_{Xr}}{\sigma_X}\int_0^u \lambda \frac{1 - e^{-a(T-s)}}{a} ds & \text{(B5)} \\ &= \frac{X(0)}{\sigma_X} + \frac{\rho_{Xr}}{\sigma_X}\frac{\lambda e^{-aT}}{a^2}\left(1 - e^{au}\right) + \left(\frac{\mu_X}{\sigma_X} + \frac{\rho_{Xr}}{\sigma_X}\frac{\lambda}{a}\right) u & \text{(B6)} \end{aligned}$$

Applying Durbin's approximation (18), the second order approximation is given by:

$$\Pi(0,T)(\{\tau < T\}) \approx \int_0^T \left(\frac{X(0)}{\sigma_X}\frac{1}{u} + C_1\left(\frac{1}{au} - \frac{e^{au}}{au} + e^{au}\right)\right)\varphi(u)du \tag{B7}$$

$$-C_1^2\int_0^T\int_0^u \left(\frac{X(0)}{\sigma_X}\frac{1}{vC_1} + \left(\frac{1}{av} - \frac{e^{av}}{av} + e^{av}\right)\right)\left(e^{au} - \frac{e^{au} - e^{av}}{a(u-v)}\right)\varphi(u,v)dvdu \tag{B8}$$

where

$$\begin{aligned} \varphi(u) &= \frac{1}{\sqrt{2\pi u}}\exp\left\{-\frac{1}{2}\left(\frac{x(0)}{\sigma_X}\frac{1}{u} + C_2 + C_1\frac{1 - e^{au}}{au}\right)^2\right\} & \text{(B9)} \\ \varphi(u,v) &= \varphi(v)\frac{1}{\sqrt{2\pi(u-v)}}\exp\left\{-\frac{1}{2}\left(C_2 - C_1\frac{e^{au} - e^{av}}{a(u-v)}\right)^2\right\} & \text{(B10)} \\ C_1 &= \frac{\rho_{Xr}}{\sigma_X}\frac{\lambda}{a}e^{aT} & \text{(B11)} \\ C_2 &= \frac{\mu_X}{\sigma_X} + \frac{\rho_{Xr}}{\sigma_X}\frac{\lambda}{a} & \text{(B12)} \end{aligned}$$

We note from the formula above, it is not possible to simultaneously determine $X(0)$, μ_X, ρ_{Xr}, and σ_X. Only the standardized parameters $\frac{X(0)}{\sigma_X}, \frac{\mu_X}{\sigma_X}, and \frac{\rho_{Xr}}{\sigma_X}$ can be determined.

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Table 1: Summary Statistics for the Bond Issuers

	Exxon	ARCO	Eli Lilly	Pacific Bell	IBM	Ford	RJR Nabisco	Safeway	United Airline
Data Range	11/94–12/99	02/90–12/99	02/95–12/99	10/93–12/99	07/93–12/99	11/95–12/99	10/93–12/98	03/95–12/99	08/94–12/99
Bond Issues Selected	4	3	5	2	3	4	5	5	2
Claimant Class	(Guaranteed Notes)	(Debenture)	(Notes)	(Notes)	(Notes)	(Notes)	(Notes)	(Sr. Secured, Sr. Sub.)	(Debenture)
Bid-Ask Spread (% of bond price)	1.66%	1.27%	0.23%	0.38%	2.28%	0.94%	1.25%	1.74%	1.60%
Bid-Ask Spread (yield form [bps])	41.46	31.84	45.01	42.89	34.01	13.45	44.18	52.11	62.70
Average Issue Size ($mil)	638	417	260	312	850	438	695	185	370
S&P Rating on 12/99	AAA	AA+	AA	AA−	A+	A+	BBB−	BBB,BBB−	BB+
S&P Rating on 12/97	AAA	A	AA	AA−	AA	A	BBB−	BBB,BBB−	BB+
S&P Rating on 12/95	AAA	A	AA	AA−	AA	A+	BBB−	BBB,BB+	BB
S&P Rating on 12/93	−−	A+	−−	AA−	AA	−−	BBB−	−−	−−
S&P Rating on 12/91	−−	A+	−−	−−	−−	−−	−−	−−	−−

This table shows the summary statistics for the nine issuers in our sample. Each issuer has at least 60 months of price observations for each of its bonds included in the study. The claimant classes of the bonds are collected from the SDC database. Note that Safeway is the only issuer with bonds from two different claimant classes; 3 of the 5 Safeway bonds are rated senior subordinate, while 2 are rated senior secured. We also report the average bid-ask spread for the issuers in the yield form. This gives us an idea of the quality of the bond price data as well as a reasonable magnitude for the pricing errors. Finally, we report the ratings of the bonds from 1991 through to 1999. In general bonds in the same claimant class from the same issuer have the same rating. For 50% of the cases, we observe rating migration over the life of the bond.

Table 2: Parameter Estimates from GMM Estimation

	Exxon	ARCO	Eli Lilly	Pacific Bell	IBM	Ford	RJR Nabisco	Safeway	United Airline
μ_X	0.011*	−0.017	0.186*	0.0247	−0.0246*	−0.0013*	−0.0576*	0.1134	0.0642*
	(0.0095)	(0.047)	(0.189)	(0.0355)	(0.0178)	(0.0008)	(0.0266)	(0.1364)	(0.0355)
ρ_{Xr}	0.1568*	0.0399*	0.1518*	0.0839*	0.0624*	−0.291*	0.2482*	0.1177*	−0.1631*
	(0.1357)	(0.1076)	(0.1541)	(0.1209)	(0.1028)	(0.1439)	(0.1145)	(0.1415)	(0.0886)
σ_X	0.3196	0.057	0.5998	0.1288	0.1811	0.4585	0.4710	0.7449	0.3171
	(0.2765)	(0.1536)	(0.6088)	(0.185)	(0.32)	(0.2267)	(0.2174)	(0.8959)	(0.1737)
W	0.3852	0.6784	0.7968	0.7376	0.6821	0.7079	0.7458	0.498[1],0.328[2]	0.8827
	(0.098)	(0.020)	(0.026)	(0.0253)	(0.0909)	(0.029)	(0.120)	(0.078),(0.040)	(0.0321)
μ_P	-0.0797	-0.0039	-0.0579	-0.0024	0.03565	-0.142	0.0146	0.0127	0.0513
	(0.2228)	(0.0429)	(0.4301)	(0.0801)	(0.04273)	(0.3362)	(0.2887)	(0.4977)	(0.208)
RMSE (in yields bps)	26.84	43.99	49.05	29.76	47.56	37.64	41.61	43.53	27.08
Average(log(V/K))	1.24	0.37	1.92	0.40	0.23	1.71	1.61	2.02	1.08
Average(V/K)	3.46	1.44	6.85	1.49	1.26	5.54	5.02	7.55	2.93

This table shows the GMM estimates for our model. The reported estimates for $\mu_X, \mu_p, \rho_{Xr}, and \sigma_X$ are annualized. The standard errors are reported in the parentheses. The * signifies that the standardize estimate is significant at the 5% level. The pricing error for the bonds from each issuer is reported in the row RMSE. Note that these numbers are of the same magnitude as the bid-ask spread reported in Table 1.

Table 3: Regressing log-solvency Ratio (X) on Stock Returns and Changes in the 10-yr Treasury Yield

$$\frac{X_t - X_{t-1}}{X_{t-1}} = a + b1\ (R_t - R_f) + b2\ \Delta Y_t + \epsilon$$

	a	b1	b2	R^2
IBM	0.0016	0.5903	0.0564	0.0280
	(0.0177)	(0.4239)	(0.0938)	
Ford	-0.0069	0.0743	-0.0966	0.0510
	(0.0128)	(0.1581)	(0.1541)	(0.0740)
RJR	-0.0032	0.1888	0.1715	0.1611
	(0.0094)	(0.0996)	(0.0528)	
Eli Lilly	0.0040	-0.0315	0.0521	0.0081
	(0.0147)	(0.1652)	(0.0878)	
ARCO	-0.0004	0.1308	-0.0114	0.0216
	(0.0070)	(0.1188)	(0.0393)	
UAL	0.0066	0.0885	-0.0590	0.0277
	(0.0108)	(0.1070)	(0.0625)	
Exxon	0.0036	-0.2145	0.0301	0.0302
	(0.0096)	(0.1981)	(0.0575)	
Safeway	0.0056	0.0714	0.2294	0.1639
	(0.0121)	(0.1249)	(0.0726)	

This table reports the coefficients and R^2's from regressing changes in the log-solvency ratio on the excess firm stock returns and the changes in the 10-year Treasury yield. Standard errors are reported in the parentheses. Note that the regression is performed for only 8 out of the 9 firms in our sample, because Pac Bell is not a publicly traded entity.

Yield Spread for Different Values of X

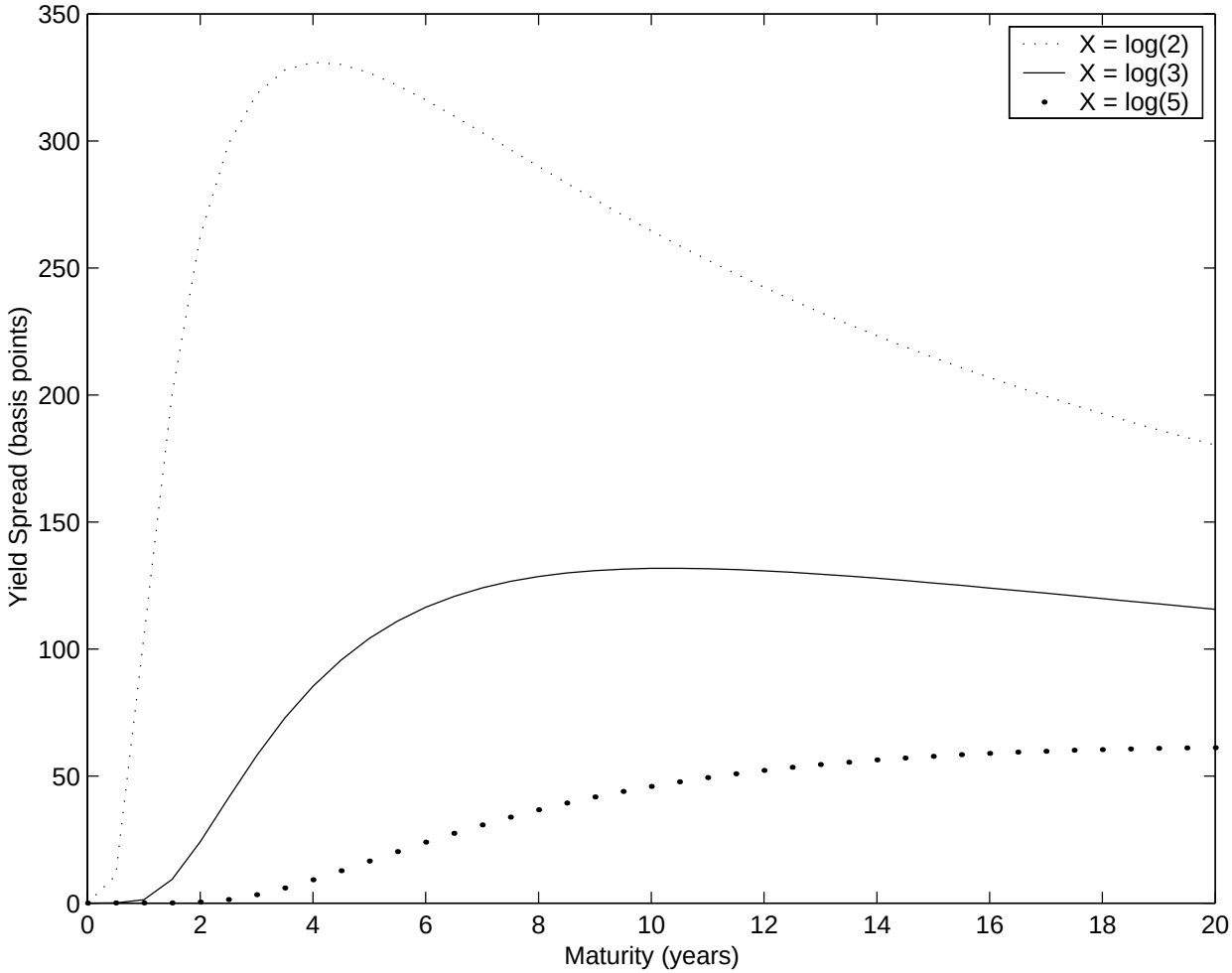

Figure 1. This figure plots the term structure of yield spread for different values of the log-solvency ratio X. The other parameters of the model are held fixed at $\mu_X = 0, \rho_{Xr} = 0, \sigma_X = 0.3$, and $W = 0.5$.

Yield Spread for Different Values of μ_X

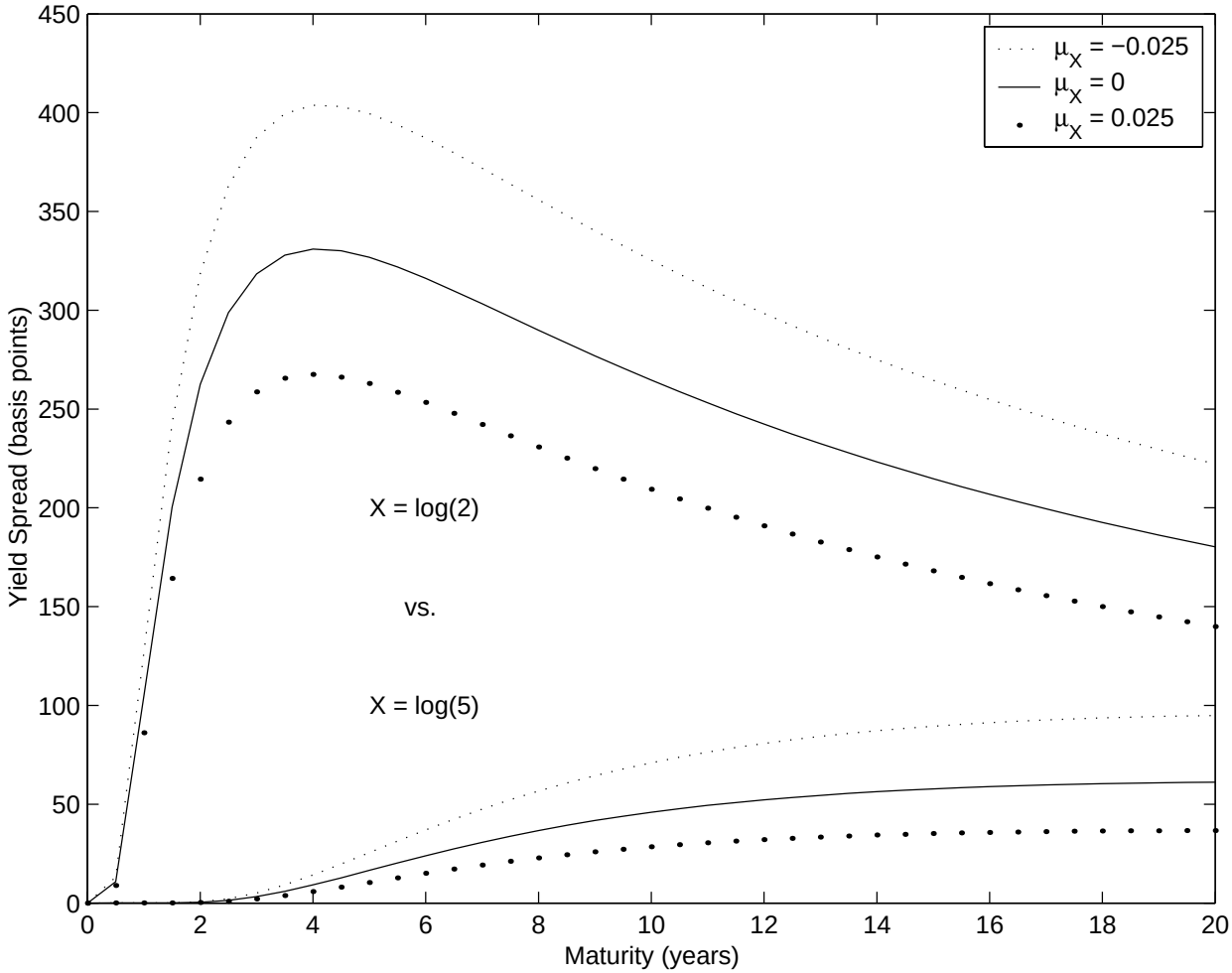

Figure 2. This figure plots the term structure of yield spread for different values of the risk neutral drift μ_X of the log-solvency ratio process at $X = \log(2)$ and $X = \log(5)$. The other parameters of the model are held fixed at $\rho_{Xr} = 0, \sigma_X = 0.3$, and $W = 0.5$.

Yield Spread for Different Values of ρ_{Xr}

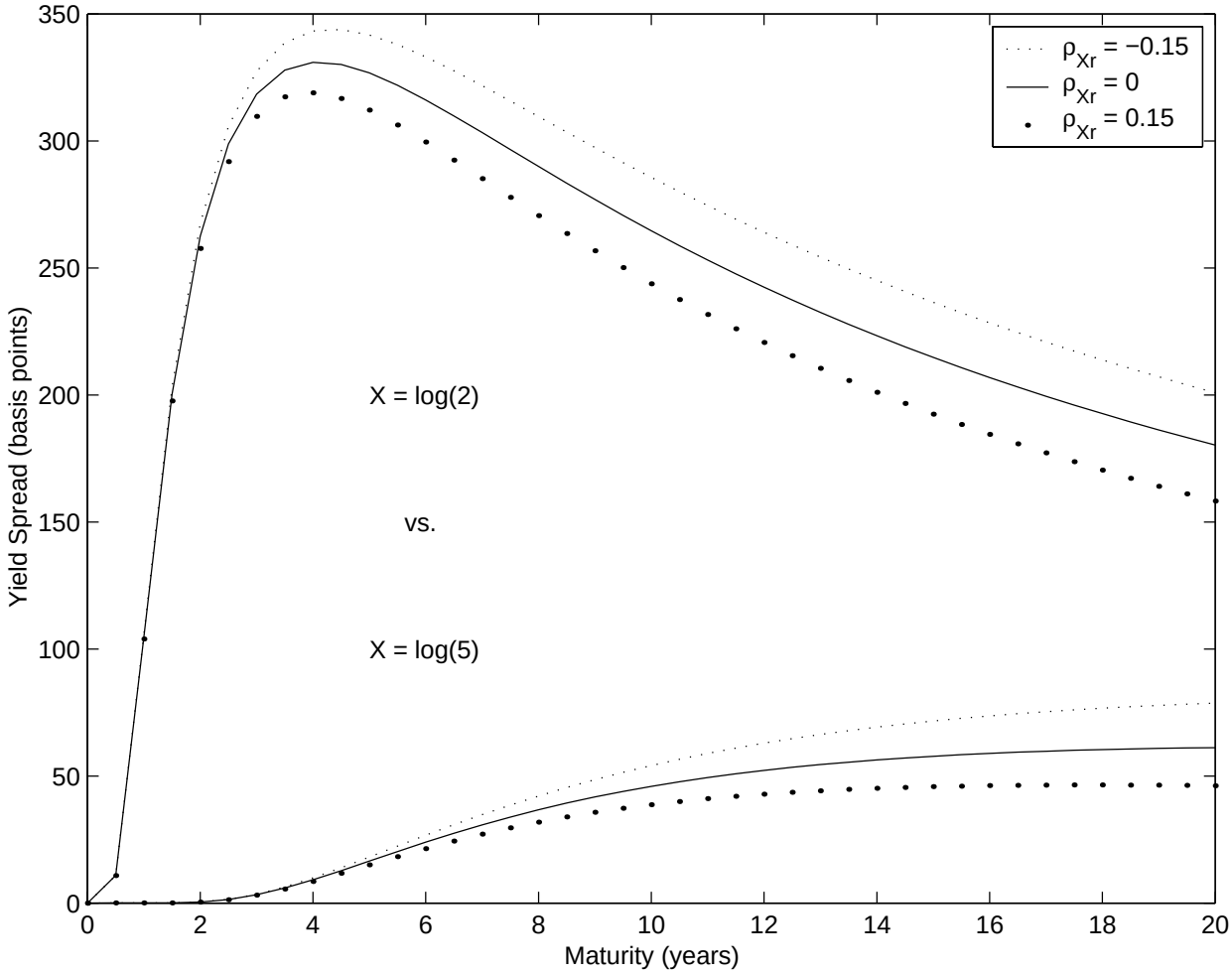

Figure 3. This figure plots the term structure of yield spread for different values of ρ_{Xr} at $X = \log(2)$ and $X = \log(5)$. The other parameters of the model are held fixed at $\mu_X = 0, \sigma_X = 0.3$, and $W = 0.5$.

Yield Spread for Different Values of σ_X

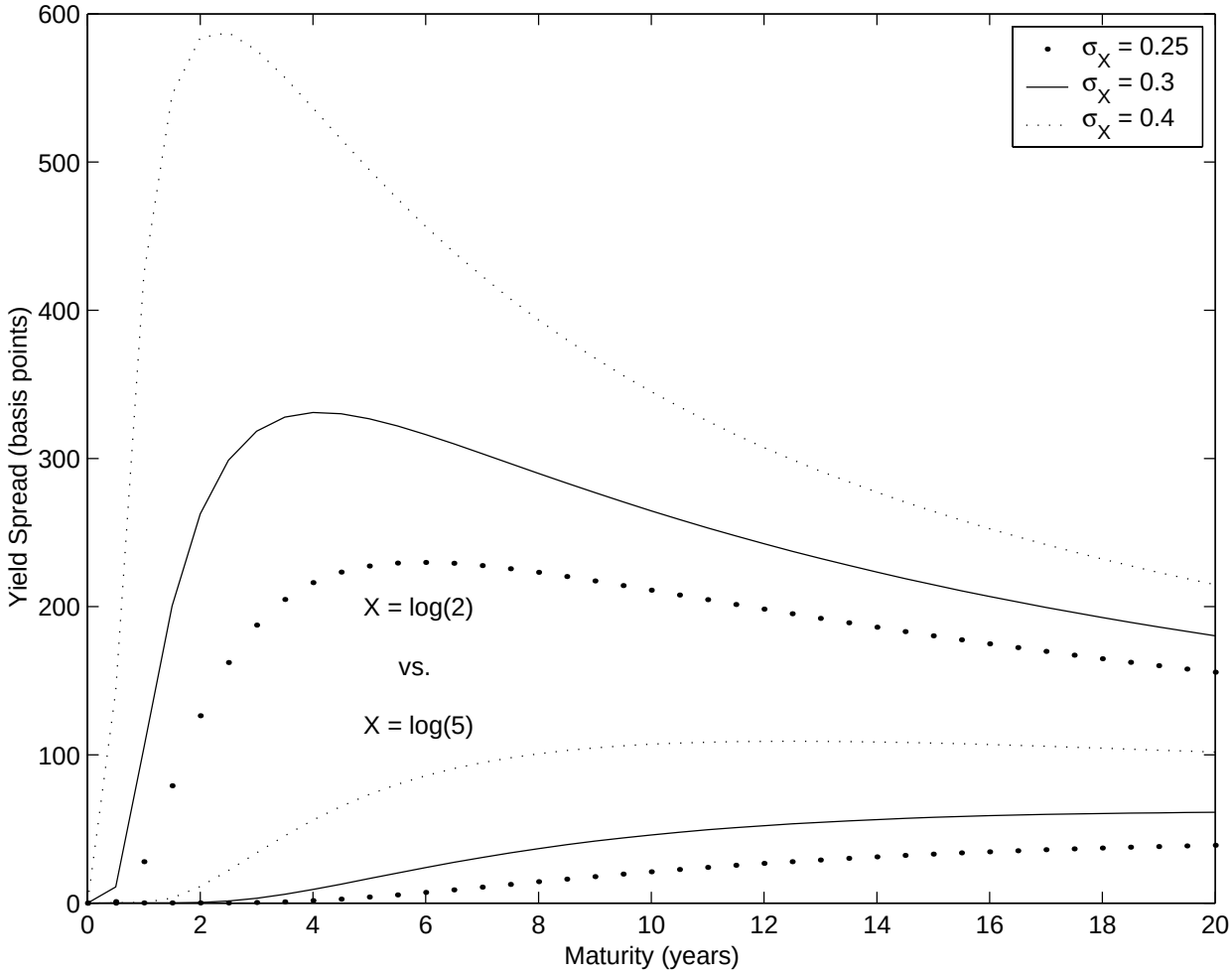

Figure 4. This figure plots the term structure of yield spread for different values of σ_X at $X = \log(2)$ and $X = \log(5)$. The other parameters of the model are held fixed at $\mu_X = 0, \rho_{Xr} = 0$, and $W = 0.5$.

Yield Spread for Different Values of W