Convergence within the European Union: Evidence from Interest Rates

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Abstract

The economic and political changes which are taking place in Europe affect interest rates. This paper develops a two factor model for the term structure of interest rates specially designed to apply to the European Monetary Union countries. In addition to the participant country's short-term interest rate, we include as a second factor a "European" short-term interest rate. We assume that the "European" rate follows a mean reverting process as in Vasicek. The domestic interest rate also follows a mean reverting process, but the convergence is to a stochastic mean which is identified with the "European" rate. Closed-form solutions for prices of zero coupon discount bonds and options on these bonds are provided. A special feature of the model is that both the domestic and the European interest rate risks are priced. This paper also provides an empirical estimation focusing on the Spanish short-term interest rate. The "European" rate is proxied by the Ecu's interest rate. Through a comparison of the performance of our convergence model with a Vasicek model for the Spanish bond market, we show that our model provides a better fit both in-sample and out-of sample and that the difference in performance between the models is greater the longer the maturity of the bonds.

1 INTRODUCTION

The economic and political changes that are taking place in Europe are affecting financial markets, and interest rates are no exception. Changes in the European level of interest rates affect the domestic interest rates and we can no longer study these in isolation. In this article we develop a *convergence model* which takes into account the influence of the European rate on the behavior of interest rates of European Monetary Union (EMU) countries.

Since the seminal papers by Merton (1973) and Vasicek (1977), many interest rate models have been developed. In the simplest form, interest rates have been modelled as one-factor Markovian processes where the term structure is dependent on the short-term rate which itself is normally distributed. Empirical research, however, has suggested that multi-factor models do significantly better than single-factor models in describing the behavior of the term structure in the real world.

Although single factor models represent the short end of the term structure fairly well, they are inadequate to describe the behavior of long-term rates. This has led to two factor models, which include either the volatility (Longstaff and Schwartz (1992)) or the mean rate (e.g. Balduzzi, Das, Foresi and Sundaram (BDFS 1997), Balduzzi, Das and Foresi (1996)) as additional factors. Recently, some studies consider three or four factors such as BDFS (1996) and Chacko (1997). Both papers use, besides the short rate, stochastic mean and volatility as additional factors. Chacko (1997) in addition includes a fourth factor which is independent of the drift and diffusion of the interest rate process. While BDFS (1996) conclude that volatility influences mainly short to medium-term yields and the mean rate affects long yields more strongly, Chacko (1997) states that the long-run mean is found to be the most important factor for the middle of the yield curve, the stochastic volatility having a minor impact on the yield curve.

In this paper we develop and estimate a two factor model of the term structure of interest rates. Following common practice the first factor is identified with the level of the short term rate. The second factor is identified with the central tendency of the short rate, which itself changes stochastically over time. We refer to this model as a Stochastic Mean Reverting Model or *Convergence Model.*

[0]Some studies that test these models are Stambaugh (1988), Longstaff and Schwartz (1992), Litterman and Scheinkman (1991).

Stochastic Mean Reverting Models have been successful in describing the process followed by short term interest rates. Among the reasons for the use of stochastic mean models BDFS (1997) note that there is considerable evidence of leptokurtosis ("fat tails") in the distribution of changes of interest rates, which can be a consequence of time variation in the mean level. This issue is specially important for the pricing of options where volatility plays a crucial role. Moreover, a model which imposes a constant mean level may overstate the volatility of the short interest rate because changes in the mean get lumped into the volatility parameter. Finally, there are macroeconomic based reasons, for example changes in the level of inflation or exchange rates, which are likely to be reflected in mean shifts in interest rates.

These reasons make stochastic mean models a convenient framework to study the term structure of interest rates of European countries, given that for some years after forming the European Monetary Union, in January 1999, participant countries will be allowed to issue debt in their home currency while simultaneously European debt will be issued in Euros.[1]

In May 1998 a Special European Council Summit decided on EMU participants.[2] Participant countries will fix their bilateral exchange rate against the Euro in January 1999, but the level of this exchange rate was determined, almost exactly, in the May's Summit.[3] With fixed exchange rates, risk free interest rates across countries should be the same. Even before the formation of the EMU block we could observe how interest rates in the participating European countries were converging. Nevertheless, even with a single currency risk free interest rates across countries may be slightly different reflecting different sovereign risks.

Because participant countries will be allowed to issue domestic debt contemporaneously with the debt issued by the European Central Bank, it is very likely that small differences may still exist even when the single currency is implemented. Since the level of interest rates will affect bond prices

[1] By 2002 the Euro will be the only currency but still participant countries will be allowed to issue domestic debt contemporaneously to the debt issued by the European Central Bank.

[2] EMU will start with the participation of eleven countries of the fifteen countries member of EU: Germany, France, Spain, Luxemburg, Italy, Ireland, Belgium, Portugal, Finland, Holland and Austria. The four absent countries are Denmark, Greece, Sweden and the UK.

[3] The conversion rate, for each country, at which exchange rates will be fixed is an interesting and polemic topic (see i.e. Obstfeld (1998)), but it lies beyond the scope of this paper.

and prices of interest rates derivatives, developing a model that incorporates both the domestic rate and the central rate will improve the valuation and hedging of these instruments.

Our term structure model is exponentially affine and we obtain closed-form solutions for the prices of bonds and European options on discount bonds. American type options, options on coupon bonds and other exotic derivatives can be easily solved using numerical procedures.

It should be noted that in the model presented here, the market price of risk for both stochastic processes is priced and can be estimated directly from the data, without any need for further assumptions.

The characterization of the term structure dynamics developed in this paper not only applies to a fixed exchange or single currency situation, but also to the situation in which markets anticipate the formation of the EMU block, when interest rates were induced to converge.[4] In this context, we test the model for the Spanish term structure. As a proxy for the central tendency we use the ECU's interest rate. The short term interest rate of the ECU's deposits serves as a good proxy since from January 4th, 1998 all debt in ECU's will become debt in Euros, and the interest rate of this debt will become the reference for all economies joining EMU.[5]

Following Chan, Karoly, Longstaff and Sanders (CKLS, 1992) the method used for estimation purposes is the Generalized Method of Moments. This method provides a simple but flexible framework that is robust to misspecifications in the behavior of the residuals, and is very suitable to estimate the system of equations we obtain, making use only of the certain moment conditions and avoiding oversimplifying assumptions.

For a cross-section of Spanish discount bonds during the period June 1990 to December 1997, we compare the fit of our convergence model with that of a Vasicek model which assumes a constant mean rate. The average in-sample root-mean-square-error for the convergence model is 3.9%, whereas for the Vasicek model is 4.3%. The average out-of-sample errors are also smaller for the convergence model than for the Vasicek model. The differences in favor of the convergence model are greater the longer the maturity of the bonds for both in-sample and out-of-sample fit.

[4] The Maastricht Treaty establishes among the three criteria that a country should meet for converging, that the interest rate on long-term government bonds shouldn't exceed those of the 3 lowest-inflation members by more than 2 percentage points.

[5] We are aware that the Ecu is not an exact proxy for the Euro because it includes currencies like drachma and sterling that will not be included in the Euro.

When the sample period is divided into two subperiods (9/90-4/94 and 5/94-12/97) we show that the significance of the parameter's estimation increases during the second subsample, indicating that convergence had more influence on interest rates movements during the recent past.

The results presented here should be interpreted with caution since the distribution of interest rates in Spain has been changing in the recent past given that economic and political factors are forcing interest rates to converge. The performance of the model can be expected to improve once EMU becomes a reality, since then the reference rate will be clearly defined and the distribution of interest rates more stable.

The paper is organized as follows. In section 2 we introduce the stochastic process followed by the European and the domestic interest rate. In section 3 we derive the closed-form solutions for zero-coupon discount bond prices, and in section 4 we derive the closed-form solutions for European options on the domestic bond. In section 5 we present the data and discuss the empirical implementation of the model. Finally, we conclude in section 6.

2 A CONVERGENCE MODEL FOR INTEREST RATES

The basic element in the pricing of bonds and interest rates derivatives is the specification of the interest rate process.[6] In this section we define the process followed by the domestic short-term interest rate, modelled as a two factor process, and the process followed by the European rate which is taken as the benchmark process.

2.1 The Process Followed by the Domestic Short Rate of Interest

Definition 1 *The domestic short term interest rate follows a stochastic mean reverting process given by an Stochastic Differential Equation (SDE) of the form*

[6] General equilibrium models that are consistent with these specifications can be constructed. Some examples are Longstaff and Schwartz (1992) for two factor term structures, Goldstein and Zapatero (1996) for one factor term structure.

$$dr_d = [a + b(r_e - r_d)]dt + \sigma_d dz_d \tag{1}$$

where a, b and σ_d are constants, r_d is the domestic rate and r_e the European rate. The mean reverting level is the European interest rate which itself evolves stochastically over time.

The presence of a in the mean reflects the fact that the convergence of interest rates within the European Monetary Union will take place at the average interest level of the core countries of the Exchange Rate Mechanism, so the domestic interest rate does not need to replicate exactly the central interest level, and minor divergences may exist.

$(r_e - r_d)$ represents the reversion of r_d towards r_e. b is the speed of adjustment coefficient, and dz_d is an increment to a Wiener process.

This model provides a richer pattern of both term structure movements and volatility structures than the one factor models.[7]

2.2 The Process for the European Short Rate of Interest

Definition 2 *The SDE followed by the European interest rate is a mean reverting Ornstein-Uhlenbeck process,*

$$dr_e = c(d - r_e)dt + \sigma_e dz_e \tag{2}$$

where c is the speed of adjustment coefficient and d is the long run mean level of r_e and dz_e is an increment to a Wiener process.

The two processes are correlated with coefficient ρ

$dz_d dz_e = \rho dt$.

Given that the errors are normal, the specifications of both processes, domestic and European, allow interest rates to become negative. It is well known that if the current short term is well above zero, there is only a very small probability of reaching a negative level.[8]

One way to prevent the occurrence of negative interest rates is to assume that the short-term rate's diffusion coefficient is proportional to r^α when $\alpha > 0$.[9] Cox, Ingersoll and Ross (CIR, 1985) analyze the case where $\alpha = \frac{1}{2}$.

[7] See Hull and White (1994).

[8] For a detailed study of this topic, see Rogers (1995).

[9] In some cases additional technical conditions are required.

All the results of this paper can be extended in a straightforward manner to the CIR model.

3 BOND-PRICING EQUATIONS

In this section we derive the bond pricing formulas for the European and domestic bonds. The equation for the European bond is well known since it corresponds exactly to the Vasicek case, so we discuss it briefly. The European market price of risk obtained here is an input to the equation for the domestic bonds.

3.1 The European Bond

Proposition 1 Let $P(r_e, \tau)$ *be the price of a zero coupon discount bond with face value 1 ECU and* τ *years to maturity when the interest rate is* r_e *and following process (2). Its price is given by*

$$P(r_e, \tau) = \exp[F(\tau) - r_e G(\tau)] \tag{3}$$

where

$$G(\tau) = \frac{1 - \exp(-c\tau)}{c}$$

and

$$F(\tau) = \frac{(G(\tau) - \tau)(c(cd - \lambda_e \sigma_e) - \sigma_e^2/2)}{c^2} - \frac{\sigma_e^2 (G(\tau))^2}{4c}$$

Proof of Proposition 1 See Appendix B

3.2 The Domestic Bond

Proposition 2 *Let* $P(r_d, r_e, \tau)$ *be the price of a zero coupon discount bond with face value 1 domestic currency unit and* τ *years to maturity when the domestic interest rate is* r_d *and the central rate is* r_e*. Its price is given by*

$$P(r_d, r_e, \tau) = \exp[A(\tau) - r_d B(\tau) - r_e C(\tau)] \tag{4}$$

where $A(\cdot)$, $B(\cdot)$ *and* $C(\cdot)$ *are functions that depend on* τ *but not on* r_d *or* r_e*. The exact form of these functions is*

$$B(\tau) = \frac{1 - \exp(-b\tau)}{b}$$

$$C(\tau) = \frac{bB(\tau)(1 - \exp(-c\tau))}{c}$$

$$A(\tau) = \frac{1}{2}\tau[-2aB(\tau) + C(\tau)(-2cd + C(\tau)\sigma_e^2 + 2\sigma_e\lambda_e) +$$

$$+B(\tau)(2C(\tau)\rho\sigma_d\sigma_e + \sigma_d(2\lambda_d + B(\tau)\sigma_d))]$$

Proof of Proposition 2. See Appendix B.

4 INTEREST RATE DERIVATIVES

In this section we derive the closed-form solution to price *European options* on the domestic zero-coupon discount bond. Throughout this section the term European (vs American) is used to denote the exercise condition of the option. We do not discuss the valuation of derivatives of the central interest rate since they are well known in the literature.

4.1 Options on Domestic Discount Bonds

From (4) we can write the stochastic process followed by bond prices $P(r_d, r_e, \tau)$ as:

$$\frac{dP(r_d, r_e, \tau)}{P} = \mu(r_d, r_e, \tau)d\tau + \upsilon_d(\tau)dz_d + \upsilon_e(\tau)dz_e$$

where $\mu(r_d, r_e, \tau)$, $\upsilon_d(\tau)$, and $\upsilon_e(\tau)$ are known functions of r_d , r_e , τ and the parameters of the interest rate processes (υ_d, and υ_e are only functions of time).

Using Ito's lemma, the volatility of $P(r_d, r_e, \tau)$ is

$$\sigma^2(\tau) = \sigma_d^2 B(\tau)^2 + \sigma_e^2 C(\tau)^2 + 2\rho\sigma_d\sigma_e B(\tau)C(\tau)$$

where we have used the fact that $E[dz_d, dz_e] = \rho d\tau$.

Since this is independent of the level of r_d and r_e , the distribution of the bond price at any given time conditional on its price at an earlier time must be lognormal.

Consider a European call option $C(r_d, r_e, t, T_C)$ on a discount bond with exercise price K. Suppose that the current time is t, the option expires at T_C, and the bond expires at time T ($t \leq T_C \leq T$).

Given that C(r_d, r_e, t, T_C) depends on the same random variables r_d and r_e, it too must satisfy the equation (7, see Appendix B). The only difference is that the terminal value for the option is

$$C(r_d, r_e, t, T_C) = \max(P(r_d, r_e, T_C, T) - K, 0)$$

Merton (1973) extends the Black-Scholes option pricing model to accommodate for a stochastic term structure. His model applies to the process followed by $P(r_d, r_e, \tau)$ since the drift coefficient can be of general specification, while the diffusion coefficient must be equal to a deterministic function times the current bond price.

From the lognormal property, and the results in Merton (1973) and Langetieg (1980) it follows that the option price C is given by

$$C = P(r_d, r_e, t, T)N(h) - KP(r_d, r_e, t, T_C)N(h - \sigma_P)$$

where

$$h = \frac{1}{\sigma_P} \log \frac{P(r_d, r_e, t, T)}{P(r_d, r_e, t, T_C)K} + \frac{\sigma_P}{2}$$

and σ_P^2 is the variance of the logarithm of the price of the underlying bond at option expiration date

$$\sigma_P^2 = \int_t^{T_C} \sigma^2(\tau) d\tau$$

or

$$\sigma_P^2 = \int_t^{T_C} \{\sigma_d^2[B(\tau, T) - B(\tau, T_C)]^2 + \sigma_e^2[C(\tau, T) - C(\tau, T_C)]^2 +$$

$$+2\rho\sigma_d\sigma_e[B(\tau, T) - B(\tau, T_C)][C(\tau, T) - C(\tau, T_C)]\} d\tau$$

and finally

$$\sigma_P^2 = \nu_d(t,T_C)^2 B(T_C,T)^2 + \nu_e(t,T_C)^2 C(T_C,T)^2 + 2\rho\nu_d(t,T_C)\nu_e(t,T_C)B(T_C,T)C(T_C,T)$$

where $\nu_d^2(t,T_C)$, $\nu_e^2(t,T_C)$ are the variances of r_d and r_e respectively,

$$\nu_e^2(t,T_C) = \sigma_e^2 \frac{1-e^{-2c(T_C-t)}}{2c}$$

$$\nu_d^2(t,T_C) = \sigma_e^2 \frac{1-e^{-2c(T_C-t)}}{2c}(1-e^{-b(T_C-t)})^2 + \sigma_d^2 \frac{1-e^{-2b(T_C-t)}}{2b}$$

The derivation of $\nu_d^2(t,T_C)$, $\nu_e^2(t,T_C)$ can be found in Appendix C.

The price of a European Put can be easily found from call-put parity.

Unlike the case with a single stochastic factor, where we can decompose an option on a coupon-bearing bond into a portfolio of options on discount bonds,[10] it is not possible to do so in a two factor model. But we can still find a numerical solution.[11] These solutions can also be applied to value American options.

5 ESTIMATION

In this section we first present the econometric methodology employed in the estimation. Then we describe the data used in the empirical application of the convergence model. The empirical implementation takes the Spanish short term interest rate as the domestic rate and the ECU's short term interest rate as a proxy for the stochastic central tendency. Finally, we discuss the results obtained and compare them with those obtained using a one factor constant mean Vasicek model.

5.1 Methodology

The econometric approach used in estimating the parameters of the interest rate models is the Generalized Method of Moments. This technique is robust to misspecifications in the behavior of the residuals since it allows us to

[10] See Jamshidian (1989).

[11] Possible approaches are Hull and White (1994) or Broadie and Glasserman (1997).

use certain moment conditions without specifying the full density function.[12] Some studies that already use this technique are BDFS (1997), CKLS (1992).

Following the usual practice in these type of studies, we estimate the parameters of the continuous-time model using a discrete-time version of process (1)

$$r_{st+1} - r_{st} = [a + b(r_{et} - r_{st})]\triangle t + \sigma_s \xi_{st}\sqrt{\Delta t} \tag{5}$$

and (2)

$$r_{et+1} - r_{et} = c(d - r_{et})\triangle t + \sigma_e \xi_{et}\sqrt{\Delta t} \tag{6}$$

where the ξ_{st} and the ξ_{et} are correlated i.i.d. draws from a standard Normal distribution. The r_{st} are observations of the Spanish short term interest rates at each moment of time and r_{et} are observations of the European short term interest rates at each moment in time.

We define $\epsilon_{st} = r_{st+1} - r_{st} - [a + b(r_{et} - r_{st})]\triangle t$ from (5)
and $\epsilon_{et} = r_{et+1} - r_{et} - c(d - r_{et})\triangle t$ from (6)
Then the moment equations for our model are given by
$E[\epsilon_{st}] = 0$, $E[\epsilon_{st}^2] = \sigma_s^2 \triangle t$, $E[(r_{et} - r_{st})\ \epsilon_{st}] = 0$
$E[\epsilon_{et}] = 0$, $E[\epsilon_{et}^2] = \sigma_e^2 \triangle t$, $E[r_{et}\ \epsilon_{et}] = 0$
$E[\epsilon_{st}\epsilon_{et}] = \rho\sigma_s\sigma_e$

In addition, for the estimation we use also the information contained in the third moments,
$E[\epsilon_{et}^3] = 0$ and $E[\epsilon_{st}^3] = 0$

The value obtained for the quadratic form after convergence, that under the null hypothesis (that the third moments are zero, that is, that our variables are symmetric) follows a chi-squared distribution with two degrees of freedom, the number of overidentifying moments.

Estimations using also the fourth moments of the residuals were performed and the results turned out to be robust to the ones with just the third moments. However, the convergence process of the estimates involving fourth moments tended to be more unstable and sensible to starting values, due to the increasingly complex behavior of the objective function as higher order moments are included. For this reason we do not report those estimates.

[12] Additionally, it is useful in models where the diffusion varies with the level of interest rates. This fact would allow us to perform comparisons with models like CIR (1985) in a unified framework.

Were we to estimate the two regressions separately we could use maximum likelihood to obtain more efficient results, since the model implies that ϵ_{st} and ϵ_{et} are normally distributed. The estimate of the correlation coefficient, however, would not be efficient. GMM allows us to provide asymptotically efficient estimates of the seven parameters. Assuming that ϵ_{st} and ϵ_{et} follow a bivariate normal distribution (BVN), maximum likelihood would allow to efficiently estimate the seven parameters; but bivariate normality is not implied by the model.[13]

It is important to acknowledge that the discrete version of the process in (5) and (6) is only an approximation of the continuous-time specification. However, for short time sampling intervals such as the one we use in our study (one week) this approximation is almost exact.[14]

We also estimate a one-factor Vasicek model for the domestic term structure. The results of this estimation are used as a benchmark to compare them with the results of the convergence model. We also use the GMM method for consistency with the previous estimation.

In the Vasicek case the discrete-time version is

$r_{st+1} - r_{st} = c(d - r_{st})\triangle t + \sigma_s \xi_t \sqrt{\Delta t}$ where, as before, ξ_t are i.i.d. draws from a standard Normal distribution and the r_{st} are observations of the Spanish short term interest rates at each moment in time. Defining

$\epsilon_{st} = r_{st+1} - r_{st} - c(d - r_{st})\triangle t$

the moment equations are

$E[\epsilon_{st}] = 0$, $E[\epsilon_{st}^2] = \sigma_s^2 \triangle t$, $E[r_{st}\ \epsilon_{st}] = 0$

We again use the information in the third moment, $E[\epsilon_{st}^3] = 0$. In this case, under the null hypothesis (the third moment is zero), the quadratic form after convergence will follow a chi-squared distribution with one degree of freedom.

In order to use the pricing models developed, we still need to obtain

[13] Nevertheless, we estimated the system by a two-step maximum likelihood procedure (the joint likelihood of the seven parameters is not globally concave, and the joint estimation of the seven parameters is highly unstable). Thus, we first estimated the covariance matrix using the OLS residuals. Then we used this estimated covariance matrix in the bivariate normal likelihood to get estimates of the interceipts and slopes. Results (both after a first iteration, already efficient, and after several iterations of the two-step procedure with strong convergence criteria) were generally consistent with those of GMM, except for some subsets of the data. This discrepancy, and the fact that BVN is not implied by the model, led us to stick to the GMM results, more robust to nonnormal behavior of the errors.

[14] See for example Schwartz(1997).

parameter values for the markets price of risk (λ_s and λ_e). To do so, for each model, we search over different values of λ until we minimize the prediction error (RMSE) over a sample of Spanish and European bonds. The Root Mean Squared Error is computed by

$$RMSE = \sqrt{\frac{1}{N}\sum_{i-1}^{N}[P_i - A_i]^2}$$

where N is the number of observations, P_i is the model predicted price and A_i is the actual market price for the ith observation.

5.2 Data

We follow standard practise and treat the one month interest rate as a proxy for the instantaneous rate.

The short-term interest rates used are the one month interbank middle (between bid and ask) rate in the case of the Spanish market and the middle interest rate for one month deposits in the ECU one. We use weekly data from September 1990 to December 1997. Therefore we have 382 weekly observations of the interest rates. The period chosen was determined by the availability of data and the need to have enough observations to perform the estimations. The database used is Datastream.

Tables 1 and 2 provide the descriptive statistics of the interest rate data, and Figure 1 shows the evolution of both rates.

To obtain the weekly prices for European and Spanish zero coupon discount bonds, we use the estimation of the yield curve[15] with maturities 1, 2, 3, 5 and 10 also available in Datastream. Discount bond prices given in terms of the yields are

$$P(t,T) = \exp[-(T-t)Y(t,T)]$$

where $P(t,T)$ is the price at t of the discount bond maturing at T, and $Y(t,T)$ is its corresponding yield.

We have a total of 1910 discount bond prices (5 observations per week during 382 weeks) for each class of bonds (European and Spanish).

The first four months of 1998 are used for prediction purposes. Again we have 5 observations per week corresponding to bonds with 1, 2, 3, 5 and

[15] We use the curve to the power of three.

10 years to maturity which gives a total of 90 observations. Note that the prediction period is not used for the computations of the parameters of the interest rates processes nor of the market prices of interest rate risk.

5.3 Results

The results of the estimation for the parameters of the interest rates processes for the convergence model are presented in Table 3. The coefficients are more significant in the case of r_s than in the case of r_e. b, the coefficient that measures the speed of adjustment turns out to be the most significant parameter, thus confirming the reversion character of the Spanish interest rates towards the European ones. As shown in the table we cannot reject the null hypothesis that the third moments are zero.

The standard errors of the intercepts and slopes obtained with GMM are similar to the white-corrected errors (those corrected for heteroscedasticity). This doesn't happen for the standard errors of the variances and covariance: with the GMM estimation these standard errors estimation differ substantially from the maximum likelihood or least squares ones, and are only valid asymptotically. This is the reason why we do not report the t-ratio of the standard deviations and the correlation.

Figure 2 graphs the weekly changes in the Spanish interest rate and its weekly drift over the sample period. Excluding the period from September 92 to September 93 which corresponds to the European monetary crisis, changes in the Spanish interest rate followed closely the drift of the convergence model.

Using a cross-section of ECU' bond prices with 1, 2, 3, 5 and 10 years to maturity during the period September 1990 to December 1997, Table 4 shows that the best fit was achieved for a market price of European interest rate risk of $\lambda_e = -0.655$.

Using a cross-section of Spanish bond prices with 1, 2, 3, 5 and 10 years to maturity, the estimation of the Spanish price of risk and the errors incurred can be found in Tables 5 and 6. Using the previously estimated $\lambda_e = -0.655$, we obtain $\lambda_s = 3.315$. The in-sample RMSE is 3.9% and the mean error is 0.14%. RMSE vary from 0.95% for 1 year bonds to 6.1% for ten year bonds.

Tables 7 and 8 show the same set of parameter estimates as Table 3 when we divide the sample period into two subperiods. This allows us to see -by looking at the t-statistic of b,- that the convergence has been more significative during the last three years and a half , although it has also been

important during the first subperiod. The parameter a losses importance at the end reflecting that the rates are getting closer so we are not able to get with confidence the level of a, although we are able to do it for the whole period.

Table 9 reports the results of the estimation for the Vasicek model. The speed of adjustment coefficient, though positive, is not significant. Again we cannot reject that the third moment is zero. In Table 10 we report the estimation of the Spanish market price of interest rate risk and the errors. In this case $\lambda_s = -0.4316$, the in sample RMSE is 4.3% and the mean error 0.19%. The fit of the model is worse than the one obtained with the convergence model. It can also be seen that the errors get worse in the Vasicek model as the time to maturity of the bond grows. For one year bonds the RMSE is 0.93% for the convergence model and 0.95% for the Vasicek model, but for 10 years bonds the RMSE with Vasicek are 6.9% and with the convergence model are 6.1%. The convergence model seems to be more adequate to value long-term interest rates derivatives.

The average out-of-sample RMSE is 10.0% for the convergence model and 10.5% for the Vasicek one. Once more the differences between both models get greater as the time to maturity of the bonds gets longer.

6 CONCLUSIONS

We have developed a two factor term structure of interest rates model that applies to EMU countries, and provided closed-form solutions for the prices of bonds and European options on zero-coupon discount bonds.

The model is a stochastic mean reverting model. The first factor is identified with the level of the short-term interest rate of the country participating in EMU, and the second factor is identified with a "central rate" or a rate of the debt issued by a European Central Bank. The economic intuition underlying the model is provided by the fact that after EMU is established, each country will be able to issue national debt contemporaneously with the debt issued by the Central Bank. With a fixed Exchange Mechanism or with a single currency, economic theory and empirical evidence indicate that the interest rates of both debts will have to be very close, but not necessarily equal. The fact that domestic interest rates are converging to the European rate should be taken into account for valuation purposes, especially for the case of interest rates derivatives.

One important contribution of the model is that the prices of risk of both factors can be estimated from easily accessible data.

To asses the model's performance with some degree of confidence we need to wait until the EMU has been in place for some time. Nevertheless we do a preliminary estimation for the case of the Spanish term-structure taking the one month interbank interest rate as a proxy for the first factor and the one month interest rate on the ECU deposits as the stochastic central mean. We obtain a good in-sample and an out-of-sample fit. We also provide a comparison with the results of a Vasicek model estimation for the Spanish interest rate. It is showed that, when valuing zero coupon bonds the errors are smaller with our convergence model, and that this difference is specially remarkable for long-term bonds.

8 APPENDIX A

Table 1
Descriptive Statistics. Weekly Data from 9/6/90-31/12/97

	r_s	dr_s	r_e	dr_e	$r_e - r_s$
Mean	0.10	-0.00026	0.071357	-0.00014	-0.0292
Std Dev	0.0319	0.0046	0.0244	0.00226	0.0129
Skewness	0.197	-2.0087	0.2228	0.2556	-1.01
Kurtosis	-1.12	70.48	-1.4743	14.4	2.28
Max	0.1812	0.0431	0.12687	0.01437	-0.0055
Min	0.048	-0.0544	0.04	-0.015	-0.0929

Table 2
Correlations. Data from 9/6/90-31/12/97

	dr_e	$r_e - r_s$
dr_s	0.08	0.1948

	r_s
r_e	0.929

Table 3
Method of Moment Estimation for Convergence Model specified in equations 5 and 6. Weekly Data from September 1990-December 1997.(382 obs). Annualized coefficients (52 weeks per year).

	a	b	σ_s	c	d	σ_e	ρ	χ^2_2
Coeff	0.0938	3.67	0.032	0.2087	0.035	0.016	0.219	0.0003
t-stat	1.72	1.85		0.43	0.79			

Table 4

Estimation of the European market price of risk corresponding to the parameters in Table 3.

This table reports the price of risk using weekly data of a panel of European bonds. We report the In-Sample Pricing Errors (Mean Sum of Squared Errors (MSSE), Root Mean Squared Errors (RMSE), Mean Error (ME)). Estimation period: September 1990 to December 1997.

Years to mat	*No. Obs*	λ_e	*RMSE*	*ME*
1,2,3,5 and 10	1910	*-0.655*	0.0241	0.017

Table 5

Estimation of the Spanish market price of risk corresponding to the parameters in Table 3.

This table reports the prices of risk using weekly data. In-Sample Pricing Errors.

Period: September 1990 to December 1997.

		$\lambda_e =$	*-0.655*	
Years to mat	*No. Obs*	λ_s	*RMSE*	*ME*
1	382	*3.465*	0.0095	-0.000007
2	382	*3.020*	0.0215	0.000204
3	382	*3.105*	0.033	0.000464
5	382	*3.39*	0.048	0.000900
10	382	*3.391*	0.061	0.002860
1,2,3,5 and 10	1910	*3.315*	0.039	0.001446

Table 6

Out of Sample Test. Convergence Model.

The prediction period: 18 weeks, January 1998 - May 1998

(Estimation period: September1990-December 1997)

$\lambda_e = -0.655$	$\lambda_s = 3.315$		
Years to mat	*No. Obs*	*RMSE*	*ME*
1	18	0.0182	-0.01818
2	18	0.03967	-0.03963
3	18	0.0649	-0.0649
5	18	0.11357	-0.11348
10	18	0.1766	-0.1764
1,2,3,5 and 10	90	0.100	-0.0825

Table 7

Estimation of the Convergence Model. September 1990-April 1994.

191 Observations

Annualized Coefficients

	a	b	σ_s	c	d	σ_e	ρ	χ^2_2
Coeff	0.1877	6.0639	0.0457	0.1869	0.0346	0.0198	0.20	9.15e-06
t - stat	1.5	1.54		0.3	0.4			

Table 8

Estimation of the Convergence Model. May 1994- December 1997

191 Observations

Annualized Coefficients

	a	b	σ_s	c	d	σ_e	ρ	χ^2_2
Coeff	-0.0085	0.687	0.0087	1.01	0.045	0.0101	0.31	0.0058
t - stat	0.95	1.76		1.26	0.98			

Table 9

Estimation of a Vasicek Model for the Spanish Term Structure

$$dr_s = c(d - r_s)dt + \sigma_s dz_s$$

Weekly Data from September 1990-December 1997.(382 obs). Annualized coefficients (52 weeks per year).

	c	d	σ_s	χ^2_1
Coeff	0.524	0.0747	0.032	0.0003
t- stat	1.47	1.29		

Table 10

Estimation of the Spanish market price of risk corresponding to the above parameters.

This table reports the prices of risk using weekly data. In-Sample Pricing Errors.

Years to mat	*No. Obs*	λ_s	*RMSE*	*ME*
1	382	*-0.39*	0.0093	0.00012
2	382	*-0.395*	0.022	0.00039
3	382	*-0.395*	0.0348	0.00093
5	382	*-0.407*	0.0525	0.00172
10	382	*-0.449*	0.069	0.00297
1,2,3,5 and 10	1910	*-0.4316*	0.0433	0.00191

Table 11

Out of Sample Test. Vasicek Model

Out-of Sample Pricing Errors for Spanish bonds.

The prediction period: 18 weeks, 7/January/98 - 30/April/98, and estimation period: September1990-December 1997.

	λ_s =*-0.4316*		
Years to mat	*No. Obs*	*RMSE*	*ME*
1	18	0.0155	-0.0155
2	18	0.0415	-0.0415
3	18	0.0698	-0.0698
5	18	0.1193	-0.1192
10	18	0.1829	-0.1827
1,2,3,5 and 10	90	0.105	-0.0857

9 APPENDIX B

Proof of Proposition 1

This is quite straightforward since $P(r_e, \tau)$ must satisfy the SDE,

$$\frac{dP(r_e,\tau)}{P} = P_{r_e} dr_e + P_\tau d\tau + \frac{1}{2}\sigma_2^2 P_{r_e r_e} d\tau$$

and arbitrage arguments lead us to the PDE

$$E(DP) - \lambda_e \sigma_e P - r_e P = 0$$

Where D denotes the Dynkin differential operator, λ_e is the market price of European interest rate risk, and P_{r_e} and P_τ are partial derivatives with respect to r_e and τ respectively. The boundary condition is $P(r_e, 0) = 1$.

When we solve the PDE subject to its boundary condition we obtain the result (3)[16].

■

Proof of Proposition 2

If $P(r_d, r_e, \tau)$ is the price of a discount bond with face value 1 domestic currency and τ years to maturity dependent on $r_d(r_d, r_e, \tau)$ (1) by Ito's lemma, P must follow the SDE:

$$\frac{dP(r_d,r_e,\tau)}{P} = P_{r_d} dr_d + P_{r_e} dr_e + P_\tau d\tau + \frac{1}{2}\sigma_d^2 P_{r_d r_d} d\tau + \frac{1}{2}\sigma_e^2 P_{r_e r_e} d\tau + \rho\sigma_d\sigma_e P_{r_d r_e} d\tau$$

as before, P_{r_d}, P_{r_e}, P_τ, $P_{r_d r_d}$, $P_{r_e r_e}$, $P_{r_d r_e}$ denote partial derivatives.

Standard no-arbitrage conditions will lead us to obtain the following differential equation for the bond price

$$E(DP) - r_d P - \lambda_d \sigma_d P_{r_d} - \lambda_e \sigma_e P_{r_e} = 0$$

where D denotes the Dynkin differential operator, λ_d is the market price of the domestic interest rate risk and λ_e is the market price of the European interest rate risk. Alternatively we can write:

$$[a + b(r_e - r_d) - \lambda_d \sigma_d] P_{r_d} + [c(d - r_e) - \lambda_e \sigma_e] P_{r_e} + \frac{1}{2}\sigma_d^2 P_{dd} + \frac{1}{2}\sigma_e^2 P_{ee} + \rho\sigma_d\sigma_e P_{de} - P_t - r_d P = 0 \tag{7}$$

[16] Alternatively we can follow the method used to solve the PDE in the case of the domestic bond.

The boundary condition for this PDE is $P(r_d, r_e, 0) = 1$

We guess a solution of the form(4).[17] These class of solutions are often denoted as the 'exponential-affine' form, following the work by Duffie and Kan [1992].

Substituting the derivatives of the posited guess into equation (7), and then simplifying by separating terms as coefficients of r_s and r_g we arrive at the following transformation of the PDE:

$$r_d[B(\tau)b+B_t-1]+r_e[C(\tau)c-B(\tau)b+C_t]+[-aB(\tau)+B(\tau)\lambda_d\sigma_d-C(\tau)cd+C(\tau)\lambda_e\sigma_e+$$

$$+\frac{1}{2}\sigma_d^2B^2(\tau)+\frac{1}{2}\sigma_e^2C^2(\tau)+C(\tau)B(\tau)\rho\sigma_d\sigma_e-A_t]=0 \qquad (8)$$

for (8) to be uniformly satisfied over the support of r_d and r_e, all three of the terms in brackets must be equal to zero. The solutions for $A(\tau)$, $B(\tau)$, and $C(\tau)$ in 4 are found by integrating a system of ODEs:

$$B(\tau)b + B_t - 1 = 0$$

$$C(\tau)c - B(\tau)b + C_t = 0$$

$$-aB(\tau)+B(\tau)\lambda_d\sigma_d-C(\tau)cd+C(\tau)\lambda_e\sigma_e+\frac{1}{2}\sigma_d^2B^2(\tau)+\frac{1}{2}\sigma_e^2C^2(\tau)+C(\tau)B(\tau)\rho\sigma_d\sigma_e-A_t=0$$

subject to three boundary conditions: $B(0) = 0$, $C(0) = 0$ and $A(0) = 0$.

■

[17] This is quite standard, as we know that assuming affine functions for the drift and variance terms in the processes for our state variables assures an affine solution (see Duffie and Kan [1996])

10 APPENDIX C

Derivation of $\nu_d^2(t, T_C)$, $\nu_e^2(t, T_C)$.

Given the specifications

$$dr_e = c(d - r_e)dt + \sigma_e dz_e$$

and

$$dr_d = [a + b(r_e - r_d)]dt + \sigma_d dz_d$$

the processes followed by $r_e(\tau)$ and $r_d(\tau)$, with $\tau = Tc - t$, are respectively

$$r_e(\tau) = e^{-c\tau} r(t) + cd \int_0^\tau e^{-c(\tau-u)} du + \sigma_e \int_0^\tau e^{-c(\tau-u)} dz_e(u)$$

$$r_d(\tau) = e^{-b\tau}[r_d(t) + a \int_0^\tau e^{bs} ds + b \int_0^\tau e^{bs}(e^{-c\tau} r(t) + cd \int_0^\tau e^{-c(\tau-u)} du$$

$$+\sigma_e \int_0^\tau e^{-c(\tau-u)} dz_e(u))ds + \sigma_d \int_0^\tau e^{bs} dz_d(s)]$$

In the case of $r_e(\tau)$ it is well known (see, i.e., Jamshidiam 1989) that the variance of the process is

$$\nu_e^2(\tau) = \sigma_e^2 \frac{1 - e^{-2c(\tau)}}{2c}$$

In the case of $r_d(\tau)$ the random part can be written as

$$b\sigma_e \int_0^\tau e^{-b(\tau-s)} \int_0^\tau e^{-c(\tau-u)} dz_e(u) ds + \sigma_d \int_0^\tau e^{-b(\tau-s)} dz_d(s)$$

and by Fubini's Theorem

$$b\sigma_e \int_0^\tau e^{-b(\tau-s)} \int_0^\tau e^{-c(\tau-u)} dz_e(u) ds = b\sigma_e \int_0^\tau e^{-c(\tau-u)} \int_0^\tau e^{-b(\tau-s)} ds dz_e(u)$$

or,

$$\sigma_e(1 - e^{-b(\tau)}) \int_0^\tau e^{-c(\tau-u)} dz_e(u)$$

So, the variance of $r_d(\tau)$ is

$$\nu_d^2(\tau) = \sigma_e^2 \frac{1 - e^{-2c(\tau)}}{2c}(1 - e^{-b(\tau)})^2 + \sigma_d^2 \frac{1 - e^{-2b(\tau)}}{2b}$$

Figure 1

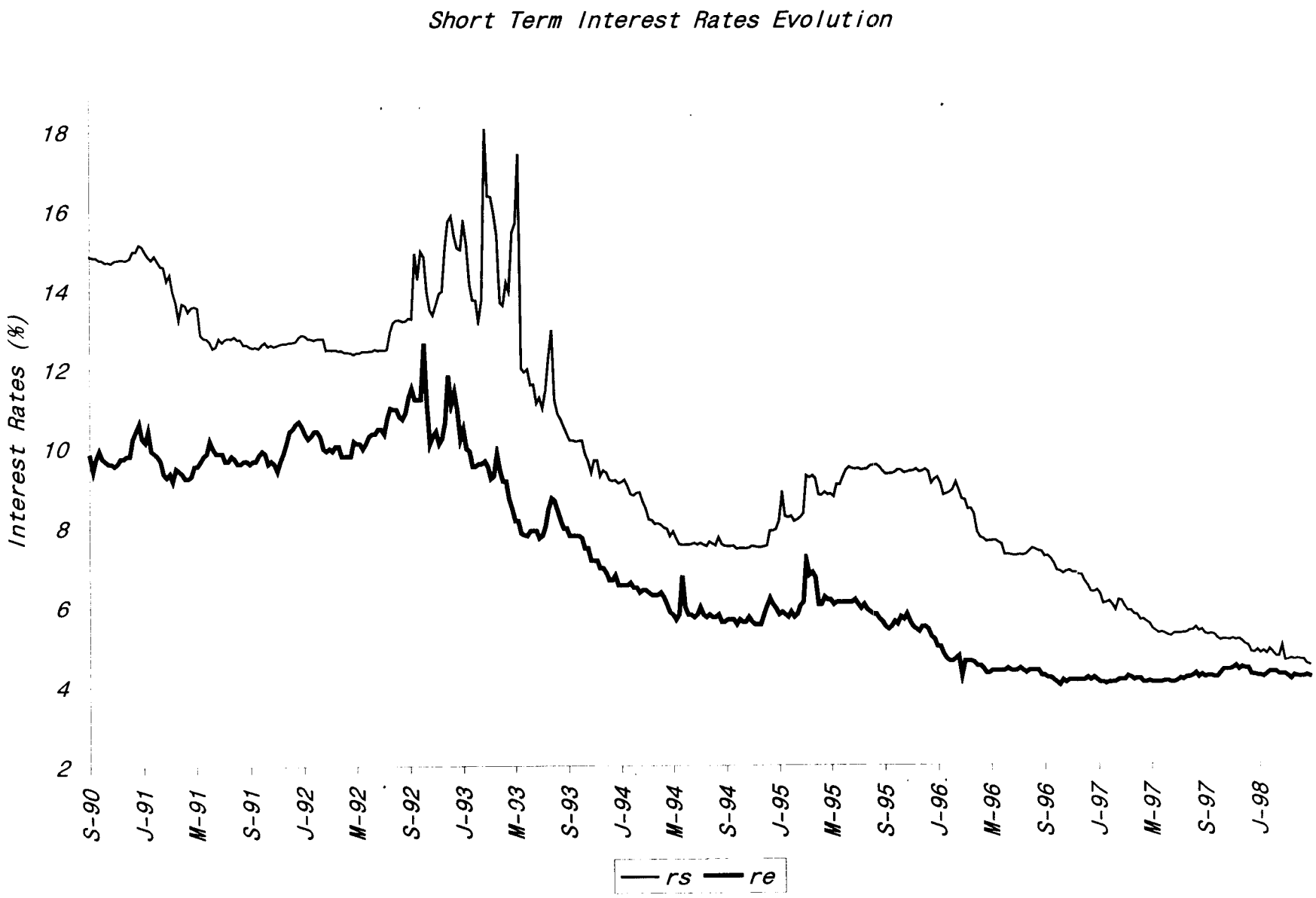
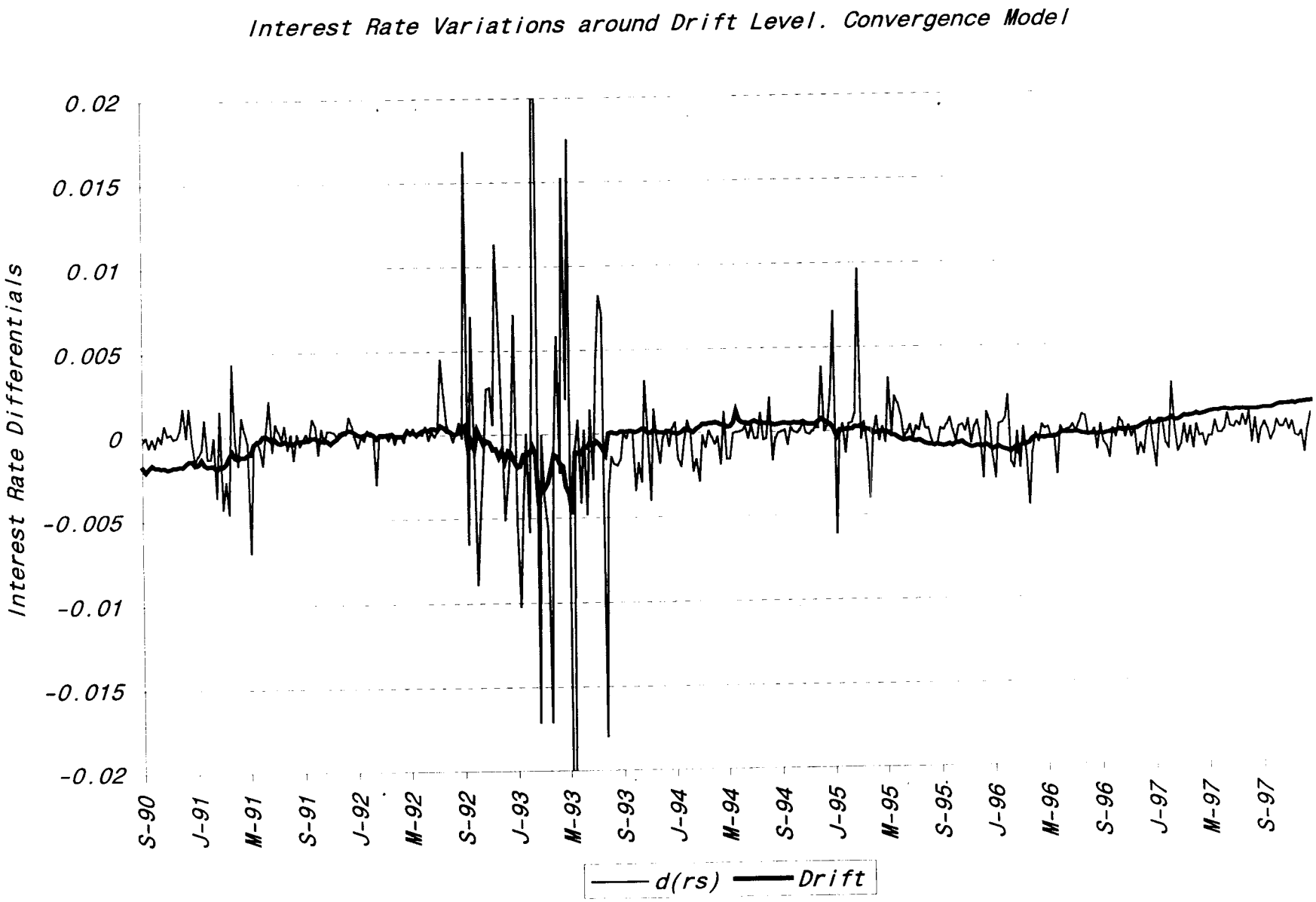

Short Term Interest Rates Evolution
Interest Rates (%)
18
16
14
12
10
8
6
4
2
S-90
J-91
M-91
S-91
J-92
M-92
S-92
J-93
M-93
S-93
J-94
M-94
S-94
J-95
M-95
S-95
J-96
M-96
S-96
J-97
M-97
S-97
J-98
rs
re

Figure 2


Interest Rate Variations around Drift Level. Convergence Model
Interest Rate Differentials
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
S-90
J-91
M-91
S-91
J-92
M-92
S-92
J-93
M-93
S-93
J-94
M-94
S-94
J-95
M-95
S-95
J-96
M-96
S-96
J-97
M-97
S-97
d(rs)
Drift