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On Distinguishing Between Rationales for Short-Horizon Predictability of Stock Returns

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Abstract

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In this paper, we shed theoretical and empirical light on short-horizon return reversals. We provide a model of price formation that captures both behavioral and microstructure effects. This model allows us to obtain analytical implications for the conditions required to obtain short-horizon price reversals. Key to distinguishing between inventory and overreaction explanations for price reversal is the role of order flow. The inventory rationale requires a relation between returns and past order flow, whereas a reversion in beliefs can cause reversal in returns independent of order flow. The cross-sectional evidence indicates that belief reversal appears to more accurately capture return patterns at monthly horizons, but it takes more than a month and at least three months for order flow innovations to be reversed out in stock returns.

1 Introduction

The predictability of stock returns has received a lot of attention in the literature. Prominent among controversies regarding this issue is the role of short-run predictability, defined as reversals in stock returns that occur at horizons ranging from a week to a month. Evidence of this predictability appears in papers by Cootner (1964), Fama (1964), Jegadeesh (1990), Lehmann, (1990), and Kaul and Nimalendran (1990). Since this empirical finding contradicts the notion that stock prices follow a random walk, and thus is *prima facie* reason to suspect a violation of weak-form market efficiency, as defined by Fama (1970), it deserves a deep understanding by finance scholars.

Extant research suggests that the source of this short-horizon predictability remains an unresolved debate. Some authors, e.g., Conrad, Kaul, and Nimalendran (1990) and Jegadeesh and Titman (1995) take the position that market microstructure phenomena such as inventory control effects[1] and bid-ask bounce are the cause of short-horizon return reversals. Others, such as Mase (1999) are of the opinion that it is market overreaction and correction. Those leaning towards the former cause generally relate return reversals to a measure of the bid-ask bounce such as that developed by Roll (1984), whereas Mase (1999) points to a relation between unsigned trading activity and the strength of the reversal to support his position.

We take the view that more progress can be made on resolving the above debate in the presence of a specific equilibrium model that incorporates both microstructure and behavioral effects. The sharp empirical implications from such a model could

[1] The inventory theory of price formation has been elucidated by Stoll (1978), Ho and Stoll (1983), O'Hara and Oldfield (1986), and Spiegel and Subrahmanyam (1995).

potentially go a long way in helping us develop a better understanding of what drives short-horizon predictability. Motivated by this observation, in this paper we develop a model where risk-averse, overconfident agents absorb liquidity shocks. The implications from such a model indicate that the key to distinguishing between the two explanations (microstructure and overreaction) is the role of order flow.

Note that since market making agents absorb liquidity shocks at a premium, any microstructural explanation for short horizon reversals must necessarily involve a negative relation between returns and lagged order flow. On the other hand, we show that overreaction due to belief misperceptions relates current returns negatively to lagged returns without requiring any relation between order flow and past returns. We also show that in a multiple regression of returns on lagged order flow and lagged returns, order flow dominates return completely if it contains exogenous liquidity orders as well as the trades of biased agents. However, when the order flow consists solely of exogenous liquidity orders, and biased agents take the other side of the order flow, the coefficient on order flow captures reversal due to inventory, whereas the coefficient on lagged return captures reversals due to overreaction. Thus, overall, the theoretical analysis delineates the crucial role for lagged order flow in predicting returns.

The above implications can potentially be explored in depth if one has order flow data in addition to CRSP data. We use order imbalance data for NYSE/AMEX stocks in conjunction with return data based on quote mid-point returns to shed light on the sources of short-horizon return reversals. The use of mid-point returns allays concerns about bid-ask phenomena causing the reversal. Further, due to the concerns about drawing conclusions based on portfolio returns (Lo and Mackinlay, 1990; Brennan,

Chordia, and Subrahmanyam, 1996), we focus on individual security returns in our analysis.

Our conclusions are as follows. We show that at a monthly horizon, there is no relation between returns and past order flow, suggesting that reversals at this horizon are not likely to be caused by inventory effects. This result obtains both for time-series and cross-sectional analyses. However, we uncover an intriguing result: that in the cross-section, order flow innovations take up to three months to be completely absorbed by the price formation process. This indicates that microstructure effects can be rather long-lived, supporting the analyses of Madhavan and Smidt (1993), and Hasbrouck and Sofianos (1993).

This paper is organized as follows. In the next section we present our model. Section 3 describes the data. Section 4 presents the empirical analysis. Section 5 concludes.

2 The Theory

2.1 The Basic Model

Consider a risky security which trades at dates 1 and 2, and pays off a random amount θ at date 3, where θ is normally distributed with mean zero. There is a mass unity of overconfident agents which absorbs liquidity shocks that appear in the market. Each such agent has CARA utility with coefficient R. The overconfident agents receive a signal $\theta+\epsilon$ at date 1, where ϵ is normally distributed, independent of θ and has variance v_ϵ. Overconfidence causes this variance to be misassessed at a level $v_c < v_\epsilon$. A demand shock of z arrives at the market on date 2; z is normally distributed with mean 0

and is independent of all other random variables in the model. We assume that no information is possessed by agents at date 1, and there are no liquidity shocks to be absorbed at this date.

Throughout the paper, v_X is assumed to denote $\mathrm{var}(X)$. Then, in this model the following Lemma (proved in Appendix A) holds.

Lemma 1 *The equilibrium value of the price at date 2, P_2, is given by*

$$P_2 = Rkv_\theta z + k(\theta + \epsilon) \tag{1}$$

where $k \equiv v_\theta/(v_\theta + v_c)$. The date 1 price is nonstochastic, i.e., it does not depend on any of the random variables, θ, ϵ, or z.

As can be seen, the price has two components. The first one is the premium demanded for absorbing the liquidity shock, and the second one is the (biased) conditional expectation of the asset's value. Both components cause reversals in price changes, as indicated in the following proposition, the proof of which appears in Appendix A:

Proposition 1 *The equilibrium value of the serial covariance* $\mathrm{cov}(\theta - P_2, P_2 - P_1)$ *is given by*

$$\mathrm{cov}(\theta - P_2, P_2 - P_1) = -\frac{v_\theta^2}{v_\theta + v_c}(v_\epsilon - v_c) - \frac{R^2 v_\theta^2 v_c^2 v_z}{(v_\theta + v_c)^2} \tag{2}$$

Thus, the two terms on the right-hand side of Eq. (2) respectively capture the two reasons for which reversals occur, namely, overreaction-correction and the natural inventory effect. Note that when agents are rational, i.e., $v_c = v_\epsilon$, the first component

goes to zero, and as the risk aversion coefficient becomes vanishingly small, so does the second component.

It is important to note that the inventory effect necessarily involves a relation between lagged order flow and returns. Specifically, the reversal occurs because a conditional risk premium is reversed out of asset returns. This conditional risk premium causes a negative relation between price changes and lagged imbalance. In particular, $\text{cov}(\theta - P_2, x_2) = -Rkv_\theta v_z < 0$, where $x_2 = z$ is the date 2 order flow absorbed by market makers. Belief reversion, however does not require a relation between lagged imbalance and returns. In particular, under risk-neutrality, there are no inventory effects, and only the first term in Eq. (2) contributes to reversal. In this situation, the date 2 price is simply the biased expectation $P_2 = [v_\theta/(v_\theta + v_c)][\theta + \epsilon]$, and there also is no relation between returns and lagged order flow because there are no risk premia required to absorb the flow. The proposition below follows directly from the above discussion:

Proposition 2 *1. If market makers are risk-averse (i.e., there are inventory effects), there is return reversal and also a relationship between lagged order flow and returns.*

2. If market makers are risk-neutral and rational, there is no return reversal and no relationship between order flow and returns.

3. If market makers are risk-neutral but overreact to information, there is return reversal but no relationship between order flow and returns.

Note that in our specification, the order flow emanates only from liquidity traders and

not from biased agents (who are assumed to take the other side of the order flow as market makers). Because of this, if one regresses price changes on lagged price changes as well as order flow, the lagged price change picks up the reversal in beliefs, while the lagged order flow captures the inventory effect. More specifically, we can write

$$E(\theta - P_2 | Q_1, P_2 - P_1) = \alpha Q_1 + \beta(P_2 - P_1) \tag{3}$$

where

$$\alpha = -\frac{R v_\theta^2}{v_\epsilon + v_\theta} \tag{4}$$

and

$$\beta = -\frac{v_\epsilon - v_c}{v_\epsilon + v_\theta} \tag{5}$$

The above expressions directly lead to the following proposition.

Proposition 3 *Consider a multivariate regression of price changes on lagged price changes and lagged order flow, where the order flow does not contain the trades of biased agents. This regression is linear, and both coefficients are nonpositive. The coefficient on order flow is zero if there are no inventory effects (market makers are risk-neutral) while the coefficient on price change is zero if agents are rational.*

Thus, a regression of the type indicated in the above proposition can potentially shed light on inventory and overreaction effects. However the result in the proposition is sensitive to model specification, as the next subsection illustrates.

2.2 Rational Market Makers

In the previous model, overconfident marketmaking agents interact with price-inelastic liquidity traders. In case the construct of just these two types of agents is not appealing

to the reader, let us now consider a slightly more complicated model where rational market makers trade with utility-maximizing overconfident agents, and also absorb liquidity shocks. Thus, suppose that there are two equal (unit) masses of agents, each with CARA utility and a common risk aversion coefficient R, the first class assesses the variance of ϵ correctly, whereas the second class, as before assesses it at $v_c < v_\epsilon$. The first class is assumed to absorb the order flow of the other investors. Standard mean variance analysis indicates that demands of the rational and overconfident agents are respectively given by $\frac{k_r(\theta+\epsilon)-P_2}{Rv_r}$ and $\frac{k_c(\theta+\epsilon)-P_2}{Rv_o}$ where $k_r \equiv v_\theta/[v_\theta+v_\epsilon]$, $k_c \equiv v_\theta/[v_\theta+v_c]$, $v_r \equiv v_\theta v_\epsilon/[v_\theta+v_\epsilon]$, and $v_o \equiv v_\theta v_c/[v_\theta+v_c]$. In this case, we can write the market clearing condition as

$$\frac{k_r(\theta+\epsilon)-P_2}{Rv_r} + \frac{k_c(\theta+\epsilon)-P_2}{Rv_o} + z = 0$$

This implies that

$$P_2 = \frac{\theta+\epsilon}{1/v_r+1/v_o}\left[\frac{k_r}{v_r}+\frac{k_c}{v_o}\right] + \frac{Rz}{1/v_r+1/v_o} \tag{6}$$

From the above expression for the price, we obtain the following proposition:

Proposition 4 *In the version of the model where rational market makers absorb the trades of overconfident agents and price-inelastic liquidity traders, the equilibrium value of the serial covariance is given by*

$$\mathrm{cov}(\theta-P_2, P_2-P_1) = -\frac{v_\theta^2 v_\epsilon(v_\epsilon-v_c)(v_\epsilon+v_c)}{[v_c(2v_\epsilon+v_\theta+v_\epsilon v_\theta)]^2} - \frac{R^2 v_\epsilon^2 v_c^2 v_\theta^2 v_z}{[v_c(2v_\epsilon+v_\theta+v_\epsilon v_\theta)]^2} \tag{7}$$

One learns from the above expression that similar intuitions apply in this case as well. Since the rational agents are risk averse, they are not able to completely arbitrage the

mispricing component illustrated in the first term. At the same time the standard inventory effect also contributes to reversal (illustrated in the second term).

Note that the order flow at date 2, denoted by Q_2 is given by $Q_2 = \frac{k_c(\theta+\epsilon-P_2)}{Rv_o} + z$. Thus, in this case the covariance between price changes and lagged order flow is given by

$$\text{cov}\left[\theta - P_2, \frac{k_c(\theta+\epsilon-P_2)}{Rv_o} + z\right]$$

which reduces to

$$-\frac{Rv_c^2 v_\epsilon v_\theta v_z(v_\epsilon + v_\theta)}{[v_c(2v_\epsilon + v_\theta + v_\epsilon v_\theta)]^2} - \frac{v_\epsilon v_\theta (v_\epsilon - v_c)^2}{R[v_c(2v_\epsilon + v_\theta + v_\epsilon v_\theta)]^2}$$

Thus, even if agents are rational ($v_c = v_\epsilon$), there is a relation between returns and lagged imbalance. The basic point we wish to reiterate is the following. For return reversals to be caused by inventory effects, there should a relation between returns and lagged imbalances. The overreaction-correction phenomenon also causes reversals, but this reversal in beliefs does not require an imbalance-return relation.

We now turn to the multivariate regression of price moves on lagged price changes and order flow. In this case, the following proposition obtains.

Proposition 5 *The regression of $\theta - P_2$ on Q_2 and $P_2 - P_1$ is linear. The conditional expectation may be expressed as*

$$E(\theta - P_2 | Q_1, P_2 - P_1) = \alpha' Q_1 + \beta'(P_2 - P_1) \tag{8}$$

where

$$\alpha' = -\frac{Rv_\epsilon v_\theta}{v_\epsilon + v_\theta} \tag{9}$$

and

$$\beta' = 0 \tag{10}$$

Thus, in this case, where the order flow contains overreaction as well as exogenous liquidity trades, the conditional expectation has zero loading on lagged price changes and a negative loading on order flow. This is because the order flow completely captures both sources of reversal, belief reversal as well as the inventory effect.

As another extension, we also provide expressions for the covariances when irrational agents and market makers have differing degrees of risk aversion (denoted by R_c and R_m, respectively). The analysis for this case is a simple extension of the preceding derivations, and is therefore omitted. In this case, we have

$$\begin{aligned}\mathrm{cov}(\theta - P_2, P_2 - P_1) = &-\frac{R_m^2 R_c^2 v_\epsilon^2 v_\theta^2 v_c^2 v_z}{[R_c v_c(v_\epsilon + v_\theta) + R_m v_\epsilon(v_c + v_\theta)]^2} \\ &-\frac{(v_\epsilon - v_c)(R_c v_c + R_m v_\epsilon)}{[R_c v_c(v_\epsilon + v_\theta) + R_m v_\epsilon(v_c + v_\theta)]^2}\end{aligned} \tag{11}$$

and the covariance between imbalance at date 2 and the price move across dates 2 and 3 reduces to

$$-\frac{R_m v_\epsilon v_\theta [R_c^2 v_c^2 v_z(v_\epsilon + v_\theta) + (v_\epsilon - v_c)^2]}{[R_c v_c(v_\epsilon + v_\theta) + R_m v_\epsilon(v_c + v_\theta)]^2}$$

It can be seen from the above expression that as inventory effects become vanishingly small ($R_m \to 0$), the covariance between lagged order flow and returns approaches zero. On the other hand, as the risk aversion of the irrational agents approaches zero ($R_c \to 0$), the covariance does *not* approach zero, because while the risk-neutral but biased agents absorb the order flow, the risk-neutral price in that case approaches the biased price set by irrational agents, and the imbalance is negatively related to the

subsequent reversal. The expression for the coefficients of the conditional expectation in Eq. (8) remain the same, except that R in Eq. (9) is replaced by R_m.

The basic point still obtains: inventory effects necessitate a relation between lagged imbalances and returns. However, it is worth noting that the causality does not obtain in that same manner for behavioral effects: reversals due to overreaction and correction neither necessitate nor rule out the possibility of a relation between returns and lagged order flow.

2.3 A Model with Two Rounds of Trade

The preceding models have involved only one round of trade, but they suffice to explain the basic intuition. In order to make the analysis more convincing, we now consider a dynamic model where liquidity shocks arrive in each of two periods, with liquidation at date 3. In this model, liquidity shocks of z_1 and z_2 arrive at each of periods 1 and 2. The initial date is now date 0, where no liquidity shocks arrive and no information signals are available. The functional form of the date 2 price remains unchanged from Eq. (1), and obtains by replacing z in this equation with $z_1 + z_2$:

$$P_2 = Rkv_\theta(z_1 + z_2) + k(\theta + \epsilon)$$

This immediately implies that the covariances and the coefficients of the conditional expectation at date 2 are unchanged from Eqs. (2), (4), and (5). Our goal is to look for relations between price moves and longer lags of price moves and order flows.

The calculation of the date 1 price is more complex. First, cumbersome algebra (which appears in Appendix A) demonstrates that the date 1 demand of the overcon-

fident agents is given by

$$x_1 = \frac{E(P_2|z_1, \theta+\epsilon) - P_1}{R}\left[\frac{1}{\text{var}(P_2|z_1, \theta+\epsilon)} + \frac{1}{v}\right] + E(x_2|\theta+\epsilon, z_1) \tag{12}$$

Let $\text{var}(P_2|z_1, \theta+\epsilon)$ and $\text{var}(\theta|\theta+\epsilon)$ (as calculated by the overconfident agent) be denoted by v_{p2} and v, respectively. Then the above expression simplifies to

$$\frac{k(\theta+\epsilon) + Rkv_\theta z_1 - P_1}{Rv_{p2}} + \frac{k(\theta+\epsilon-P_1)}{Rv}$$

and the market clearing condition at date 1 therefore becomes

$$\frac{k(\theta+\epsilon) + Rkv_\theta z_1 - P_1}{Rv_{p2}} + \frac{k(\theta+\epsilon-P_1)}{Rv} + z_1 = 0$$

This implies that

$$P_1 = k(\theta+\epsilon) + k' z_1$$

where

$$k' = \frac{R}{D}\left(1 + \frac{kv_\theta}{v_{p2}}\right)$$

with $D \equiv \frac{1}{v} + \frac{1}{v_{p2}}$.

This implies that

$$\text{cov}(\theta - P_2, P_1 - P_0) = -\frac{kv_\theta(v_\epsilon - v_c)}{v_\theta + v_c} - Rkk'v_\theta v_z$$

The two terms in the above expression again have an interpretation similar to that in the static models; the first captures overreaction due to overconfidence, while the second captures the inventory effect. The additional result is that the dynamic model permits reversals lasting over longer lags. Note also that $\text{cov}(\theta - P_2, Q_1) = -Rkv_\theta v v_z$, and is also negative.

The multiple regression of $\theta - P_2$ on $P_1 - P_0$ and the date 1 order flow z_1 is linear in these variables. With the coefficients being denoted by $\alpha^\dagger$ and $\beta^\dagger$, we have (see the Appendix)

$$\alpha^\dagger = -\frac{v_\epsilon - v_c}{v_\epsilon + v_\theta} \tag{13}$$

and

$$\beta^\dagger = -\frac{Rv_\theta^2\left[R^2v_\theta^2v_z\left\{v_c(v_c - v_\epsilon) + v_\theta(v_\epsilon + v_\theta)\right\} + v_c(v_c + v_\theta)^2\right]}{\left[R^2v_\theta^3v_z + v_c(v_c + v_\theta)\right]\left[v_c + v_\theta\right]\left[v_\epsilon + v_z\right]} \tag{14}$$

While $\alpha^\dagger$ is negative, $\beta^\dagger$ can be negative or positive. The reason is that the risk averse, overconfident agents have a tendency to reverse their positions at date 2 to reduce their risk exposure. Since their trades can be negatively correlated with the price move across dates 2 and 3, and they take the opposite side of z_1, the date 1 order flow can be positively correlated with the price move across dates 2 and 3. The greater the risk aversion the greater the tendency for this positive autocorrelation. As can be seen from (14), however, the coefficient is negative under a wide range of parameter values since only one of the terms, $v_c - v_\epsilon$, attenuates the negative relation.[2] We conclude that in the dynamic setting, the multivariate regression of returns on lagged returns and lagged order flow implies negative coefficients for both, and returns continue to pick up the belief reversal (the risk aversion coefficient R does not occur in (13)), whereas the coefficient on order flow picks up a combination of belief reversal and the inventory

[2]With regard to the price move across dates 1 and 2 the following results hold. First the covariance between $P_2 - P_1$ and $P_1 - P_0$ is $-k^{'2}v_z$ and is negative. The covariance between $P_2 - P_1$ and the date 1 order flow, z_1 is

$$-\frac{R^3v_\theta^4v_z(v_c - v_\theta)}{\left[R^2v_\theta^3v_z + v_c(v_c + v_\theta)\right]\left[v_c + v_\theta\right]}$$

and is negative so long as v_θ is small relative to v_c. The multivariate regression of $P_2 - P_1$ on both $P_1 - P_0$ and the date 1 order flow is multicollinear because both the regressors are linear in z_1 and no other random variable. Generally, the notion that price changes load both on lagged price changes and order flow still holds.

effect.

Overall, our analysis indicates that an order flow-return relation is necessary for inventory effects to cause short-horizon return reversals. This is simply because the inventory effect is a premium demanded by the market makers for bearing inventory risk, and this premium is necessarily a function of the order flow. The additional insight yielded by the analysis is that in a multivariate regression of returns on lagged returns and lagged order flow, the latter dominates the former if it contains the trades of biased investors as well as exogenous order flow. However, if biased investors take the other side of the order flow, then the coefficient on order flow captures the reversal due to inventory effects, while the coefficient on returns principally captures the reversal in beliefs. One way of potentially shedding light on the two types of explanations for short-horizon return reversals is to examine the three-way relation between current returns, lagged returns, and lagged order flows. In the remainder of the paper, we focus on these empirical relations. To preserve normality and hence tractability, we analyze price changes in the model, which is standard practice in the microstructure literature on informed trading. However, as per empirical convention, and to preserve comparability in the cross-section, we analyze returns in our tests to follow. This distinction, of course, is of no material consequence in that the economic forces in the model apply equally to price changes and returns.[3]

[3]See for instance, Hong and Stein (1999) who also model price changes but draw implications for returns that are tested in Hong, Lim, and Stein (2000).

3 Data

The transactions data sources are the Institute for the Study of Securities Markets (ISSM) and the NYSE Trades and Automated Quotations (TAQ) databases. The ISSM data cover 1988-1992 inclusive while the TAQ data are for 1993-1998. We use only NYSE stocks to avoid any possibility of the results being influenced by differences in trading protocols.

3.1 Inclusion Requirements

Stocks are included or excluded depending on the following criteria:

1. To be included in any given year, a stock had to be present at the beginning and at the end of the year in both the Center for Research in Security Prices (CRSP) and the intraday databases.

2. If a firm changed exchanges from Nasdaq to NYSE during the year (no firms switched from the NYSE to the Nasdaq during our sample period), it is dropped from the sample for that year.

3. Since their trading characteristics might differ from those for ordinary equities, assets in the following categories are also expunged: certificates, American Depositary Receipts, shares of beneficial interest, units, companies incorporated outside the U.S., Americus Trust components, closed-end funds, preferred stocks and Real Estate Investment Trusts.

4. To avoid the influence of unduly high-priced stocks, if the price at any month-end

during the year was greater than $999, the stock was deleted from the sample for the year.

5. Stock-days on which there are stock splits, reverse splits, stock dividends, repurchases or a secondary offering are eliminated from the sample.

Next, intraday data were purged for one of the following reasons: trades out of sequence, trades recorded before the open or after the closing time, and trades with special settlement conditions (because they might be subject to distinct liquidity considerations). Our preliminary investigation revealed that auto-quotes (passive quotes by secondary market dealers) were eliminated in the ISSM database but not in TAQ. This caused the quoted spread to be artificially inflated in TAQ. Since there is no reliable way to filter out auto-quotes in TAQ, only BBO (best bid or offer)-eligible primary market (NYSE) quotes are used. Quotes established before the opening of the market or after the close were discarded. Negative bid-ask spread quotations, transaction prices, and quoted depths were discarded. Following Lee and Ready (1991), any quote less than five seconds prior to the trade is ignored and the first one at least five seconds prior to the trade is retained.

3.2 Imbalance and Return Data

We sign trades using the Lee and Ready (1991) procedure: if a transaction occurs above the prevailing quote mid-point, it is regarded as a purchase and vice versa. If a transaction occurs exactly at the quote mid-point, it is signed using the previous transaction price according to the tick test (i.e., buys if the sign of the last non-zero

price change is positive and vice versa). For each stock we then define OIBVOL, the estimated monthly buyer-initiated minus seller-initiated dollar volume of transactions.

We recognize that our imbalance measures are approximate, because we only sign market orders. Nevertheless, we believe that our regressions shed useful light on the source of short-horizon return predictability. We also note that our method allows us to sign only market orders, so that our net imbalance measures the aggregate demand of agents that require immediacy. While this caveat is worth mentioning, we believe that the standard microstructure paradigm is of patient market makers (which include limit order traders) who absorb the demands of traders that have relatively urgent needs to trade. As such, we believe that it is hard to argue that inventory effects, if any, would not manifest themselves in premia required to bear the imbalance caused by submitters of market orders.

Of course, short-horizon return computations are subject to the well-known bid-ask bounce bias. We therefore do *not* use returns obtained from CRSP data in our empirical analysis. Instead, we use a series which calculates the daily returns using quote midpoints associated with the last transaction on a particular day (which are not necessarily the same as closing quote midpoints). Throughout, these midpoint returns are used in the analysis.

Since we run time-series as well as cross-sectional regressions in the paper, we need enough observations to estimate the parameters reliably. We impose the requirement that a stock have at least 48 monthly observations of returns as well as imbalances to be included in the database. Table 1 presents the summary statistics for the pooled time-

series, cross-sectional sample of 156,752 firm-months. The average monthly return in the sample is 1.3%, whereas the average imbalance is $ 9.8 million. The mean proportional imbalance is about 0.40%. The average of the absolute level of the proportional imbalance is quite high, about 17.8%. The standard deviations indicate adequate variations relative to the mean to allow us to capture relationships between returns and order flow.

4 Empirical Results

In performing our analysis, we have the choice of aggregating estimates from cross-sectional or time-series regressions. Aggregation of the estimates obtained from time-series regressions presents problems because the estimates are likely to be cross-correlated due to a systematic component in the independent variable (i.e., imbalance). We initially choose to focus on cross-sectional regressions because they allow us to use Fama and MacBeth (1973) technique to aggregate estimates. At the same time, we note that we use cross-sectional analysis even though our theoretical model is for a single security. In Appendix B, we show that under certain plausible conditions, the sign of the coefficient estimate obtained from the two aggregation methods will be identical. To confirm this, we also report some estimates using the time-series approach, as well as a panel data approach, and find that the conclusions are broadly unaltered.

We first present the results of performing Fama-Macbeth regressions involving regressing the current month's cumulative returns on the past three months' returns (Table 2). Note that our use of mid-point returns in these regressions mitigates con-

cern about bid-ask bounce affecting our results. Hence the need to omit a certain time-period between months (Jegadeesh, 1990) is also reduced. As can be seen, in this part of the analysis the first lag of the cumulative return is significantly and negatively related to the past month's return, which is consistent with the analysis of Jegadeesh (1990) and Lehmann (1990). The other lags of the return are not significant, suggesting that reversals in the cross-section obtain principally at the short horizon.[4]

We now explore the relation between monthly returns and lagged imbalances.[5] Of course, one problem in regressing returns on lagged imbalances is the multicollinearity caused by autocorrelations in imbalance (see Chordia, Roll, and Subrahmanyam, 2002).[6] To address this issue, we calculate imbalance innovations by regressing imbalances on twelve lags each of past returns and past imbalances. We next regress returns on three lags of past imbalance innovations (Table 3).[7] As can be seen there is a significant and negative relation between imbalance innovations one and two months ago. This seems to indicate that imbalance shocks take longer to be reversed than the

[4]We do not include other variables such as book/market ratio in our regression, as it would complicate the analysis without adding further insight. Haugen and Baker (1996) show that the monthly reversal is the strongest effect in the cross-section of monthly returns and survives a host of controls.

[5]Monthly returns are calculated by cumulating the daily mid-point returns.

[6]For our sample, the cross-sectional averages of the autocorrelations for the first three lags of imbalance measured in number of transactions are 0.419, 0.296, and 0.055, whereas those for the imbalance measured in dollars are 0.112, 0.066, and 0.055, respectively. All are significant at the 5% level.

[7]How many lags to include is a matter of judgment under the constraint that we are using large cross-section and need consistency across stocks. Return reversal has be shown to obtain over monthly horizons, so including three lags of imbalance innovations appears to be a reasonable judgment call to capture lagged adjustments to order flow. Further, while the usage of twelve monthly lags to back out imbalance innovations imposes a stringent requirement on the continuity of the imbalance series for a stock, this exercise would seem to capture most of the lagged dependence in imbalance. Indeed, the absolute value of the average autocorrelation in dollar imbalances remains at or below 0.02 from the third lag through the twelfth lag, suggesting that longer lags would not materially assist in computing imbalance innovations more accurately.

monthly horizon at which return reversals obtain. This also indicates that, consistent with Hasbrouck and Sofianos (1993), and Madhavan and Smidt (1993), inventory effects take a longer time to play out in stock returns than the standard intraday horizons on which microstructure research is focused.

Next, we include both returns and imbalance innovations in our return regression to examine evidence of return reversals independent of imbalance reversals. Such a return reversal would be consistent with the notion that the beliefs of agents about the stock reverse in the following month. The results are presented in Table 4. Interestingly, we find the coefficient of the first lag of return is not materially affected by the inclusion of imbalance innovations. The magnitude of the coefficient changes by about 11%, and remains significant. The second and third lag of imbalance innovations are negative and significant, consistent with inventory pressures influences stock prices.

We also stratify our sample by size terciles. Specifically, we rank all stocks each month based on their average market capitalization during the month. We then divide these stocks into three groups. In Table 5, we present the regression results stratified by size. Not surprisingly, the one month reversal is strongest for the small firm group. It is also present in the mid-cap group. While the coefficient for the large firm group is negative, it is not significant. Interestingly, however, imbalance innovations are most strongly significant for the large firm group, suggesting that inventory pressures due to high volume are more of an issue in large stocks, as opposed to small stocks.

Next, we present our results in a different way by performing individual stock time-series regressions and averaging the time-series coefficients. In this case, we need to

account for the notion that the residuals could be cross-correlated across stocks, thus contaminating inferences drawn from the t statistic for the average coefficient, which assumes independence across regressions. To address this issue we proceed as follows. First we identify the 470 firms that were present every month in the sample. For this set of firms we run a regression with three lags each of returns and imbalance innovations, with the contemporaneous and three lags of returns on the S&P 500. The average cross-correlation of residuals from this regression is a fairly small 0.015, suggesting that cross-correlation of residuals is not a serious issue. However, using the formula in Chordia, Roll, and Subrahmanyam (2000), we adjust the t-statistics of the regressions and report these together with the coefficient estimates in Table 6.[8] As can be seen, the coefficients on the first two lags of returns are quite significant, while the imbalance innovations are not significant in any case.[9]

We next apply the same technique to examine time-series regressions for the full sample of firms,[10] imposing the restriction that at least 48 months of data should be present for the firm to be included in the sample. The analog of the results in Table 6 thus obtained is reported in Table 7. As can be seen, while there is strong evidence of reversals, there is not much evidence of a relationship between lagged imbalance innovations and returns. Thus, based on this table, order flow innovations

[8]The formula (in footnote 8 of Chordia, Roll, and Subrahmanyam, 2000) suggests that an approximation of the amount by which the standard error is inflated is given by $\sqrt{1+\rho(N-1)}-1$ (the formula contains an erroneous numeral 2), where N is the number of firms (470) and ρ is approximated by the average correlation (0.015), leading to a deflation factor for the t-statistic of 1.83. All t-statistics are scaled by this deflation factor.

[9]It is also worth mentioning that 64% of the coefficients on the first lag of the return are negative, 19% are negative and significant at the 10% level, whereas only 9% are positive and significant. In contrast, 49% of the coefficients on the first lag of the imbalance innovation are negative and none are significant.

[10]For the next two tables, the same deflation factor is applied as that for Table 6.

do not get reversed in the return series.[11] Note that in Table 5, the absolute t-statistic obtained from averaging of the times-series coefficients across stocks undoubtedly is upwardly biased because of the cross-sectional correlation in the estimates. Hence, the true significance of the imbalance coefficients is probably even lower than what Table 5 suggests. Table 8 presents the time-series results stratified by size. The return reversals are uniformly apparent across the terciles, but the impact of imbalance innovations is not strong.

Finally, to confirm the robustness of our results, and to more completely account for cross-correlation in the regressors, we perform a panel data regression for the sample of firms that were present every day in the sample. We estimate a two-way random effects model, which allows for firm-specific shocks for each time-period, an overall firm-specific shock across time, and an overall time-specific shock across firms (see Fuller and Battese, 1974, or Hsiao, 2002 for estimation details). The results appear in Table 9. Again, the first lag of returns is significant, but no significance is found for the imbalance innovations. The panel regressions thus more closely resemble the cross-sectional regressions, where longer lags of returns are not significant.

Overall, our analysis supports the notion that at least part of the monthly return reversal occurs in a manner orthogonal to the order flow. The cross-sectional regressions, on the other hand, suggest that longer lags of imbalance (beyond a month) are negatively related to returns. Since our measure of order flow requires the estimation

[11] A version of the regression in Table 7 with returns omitted does not materially change the coefficients of imbalance innovations: they remain insignificant in this scenario as well. Furthermore, the imbalance variables remain insignificant when imbalance innovations are replaced by raw imbalances, whether one lag at a time, or all three lags together.

of the sign of an order, one cannot definitively state on the basis of our results that inventory-related phenomena do not get reflected in the monthly reversals. At the same time, it is highly unlikely that inventory-induced reversals would not manifest themselves in a relation between returns and estimated dollar imbalances measured in market order terms. Hence, we conclude that inventory effects alone do not appear to account for the monthly reversals in stock returns.

5 Conclusion

We have attempted to shed light on an important regularity in stock return data: namely, reversals at the monthly horizon. An explicit model allows us to obtain implications for the relation between current returns, past returns, and past order flows. The model indicates that inventory effects require a relation between current returns and past order flows, whereas no such relation is necessitated by the behavioral theory of overreaction and correction.

Our empirical analysis does confirm the monthly return reversal effect illuminated by Lehmann (1990) and Jegadeesh (1990), but does not obtain a consistently strong relation between current returns and lagged order flows at the monthly horizon. This indicates that inventory effects do not appear to completely account for the return reversal at this horizon. However, imbalance innovations at lags of two and three months are indeed cross-sectionally related to current returns, suggesting that microstructure effects take at least three months to be fully reversed out of stock prices.

A puzzle that is raised by our paper is the issue of how beliefs evolve and are re-

flected in returns at various horizons. For example, Chordia, Roll, and Subrahmanyam (2003) show that serial correlation at intradaily horizons is virtually zero. However, there are reversals at monthly horizons (as documented in our work and in Cootner, 1964, Fama, 1964, Jegadeesh, 1990, and Lehmann, 1990), cross-sectional momentum at six monthly and annual horizons (Jegadeesh and Titman, 1993), and reversal at a three- to five-year horizon (DeBondt and Thaler, 1985). Obviously, one cannot appeal to inventory effects in trying to explain predictability at annual and three-year horizons, and behavioral models such as Daniel, Hirshleifer, and Subrahmanyam (1998), Barberis, Shleifer, and Vishny (1998), and Hong and Stein (1999) have been developed to address these phenomena. However, these models are specifically designed to explain short-term continuation and long-term reversal and raise the issue of why there is no predictability in the very short-term (intradaily and daily horizons). In general, why markets show an alternating pattern of reversal and continuation, and how they transit from no predictability in the very short-run to predictability in the longer run is an interesting challenge for future empirical and theoretical research.

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Appendix A

Proofs of Lemma 1 and Proposition 1: The mean-variance optimization owing to exponential utility implies that the demand of the overconfident agents, denoted by x, is given by

$$x = \frac{E(\theta|\theta+\epsilon) - P_2}{R\text{var}(\theta|\theta+\epsilon)}$$

where the expectation and the variance are calculated under overconfidence. Market clearing implies that

$$\frac{E(\theta|\theta+\epsilon) - P_2}{R\text{var}(\theta|\theta+\epsilon} + z_1 = 0$$

Solving for P, we have Eq. (1).

Note now that P_1 is non-stochastic. Hence the covariance $\text{cov}(\theta - P_2, P_2 - P_1)$ becomes

$$\text{cov}[\theta - k(\theta+\epsilon) - Rkv_\theta z_1, k(\theta+\epsilon+Rkv_\theta z_1]$$

which reduces to (2). □

Proofs of Propositions 2 and 3: Proposition 2 follows directly from Eq. (1). For proving Proposition 3, we use the well-known result that if there exist random vectors $\upsilon_{\mathbf{1}}$ and $\upsilon_{\mathbf{2}}$ such that

$$(\upsilon_{\mathbf{1}}, \upsilon_{\mathbf{2}}) \sim \mathbf{N}\left[(\mu_{\mathbf{1}}, \mu_{\mathbf{2}}), \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right]$$

then the conditional distribution of $\upsilon_{\mathbf{1}}$ given $\upsilon_{\mathbf{2}} = \mathbf{X_2}$ is normal with a mean given by the vector

$$E(\upsilon_{\mathbf{1}}|\upsilon_{\mathbf{2}} = \mathbf{X_2}) = \mu_{\mathbf{1}} + \mathbf{\Sigma_{12}\Sigma_{22}^{-1}}(\mathbf{X_2} - \mu_{\mathbf{2}}) \tag{15}$$

and a variance given by

$$\mathrm{var}(\upsilon_{\mathbf{1}}|\upsilon_{\mathbf{2}} = \mathbf{X_2}) = \mathbf{\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}} \tag{16}$$

In our case, $\upsilon_1 = P_2 - P_1$ and $\upsilon_2 = [Q_1, Q_2]$, and the relevant unconditional means are all zero. A straightforward application of (15) yields the result that the coefficients α and β in the conditional expectation (3) are given by Eqs. (4) and (5). □

Proof of Proposition 5: Let $k_c = v_\theta/(v_\theta + v_c)$, and $v_o = v_\theta v_c/(v_\theta + v_c)$. Further, let k_1 and k_2 be the coefficients of $\theta + \epsilon$ and z in Eq. (6). Then, we find that in the conditional expectation of (8), Σ_{12} in (15) has the elements

$$k_1((1-k_1)v_\theta - k_1^2 v_\epsilon - k_2^2 v_z$$

and

$$\frac{(1-k_1)(k_c-k_1)v_\theta}{Rv_o} - \frac{k_1(k_c-k_1)v_\epsilon}{Rv_o} - k_2 v_z\left(1 - \frac{k_2}{Rv_o}\right)$$

while Σ_{22} is comprised of the three elements (in order)

$$k_1^2(v_\theta + v_\epsilon) + k_2^2 v_z$$

$$(k_1 v_\theta + k_2 v_\epsilon)\left(\frac{k_c - k_1}{Rv_o}\right) + k_2 v_z\left(1 - \frac{k_2}{Rv_o}\right)$$

$$\left(\frac{k_c - a_1}{Rv_o}\right)^2 (v_\theta + v_\epsilon) + \left(1 - \frac{k_2}{Rv_o}\right)^2 v_z$$

Computing $\Sigma_{12}\Sigma_{22}^{-1}$ yields the expressions in (9) and (10). □

Derivation of Eq. (12): Let $\bar{P}_2$ be the mean of P_2, conditional on the date 1 information set. Then, we have

$$\begin{aligned} W^E &= \frac{(\theta - P_2)}{Rv}(\theta + \epsilon) - \frac{(\theta - P_2)}{Rv}P_2 - x_1(P_1 - P_2) + B_0 \\ &= \frac{(\theta - P_2)^2}{Rv} + \frac{(\theta - P_2)\epsilon}{Rv} - x_1(P_1 - P_2) + B_0 \end{aligned}$$

Now, from the formula for the characteristic function of a normal distribution, it follows that if $u \sim N(\mu, \sigma^2)$, then $E(exp(vu)) = exp(\mu v + (1/2)\sigma^2 v^2)$. In our case, setting $u = W^E$, $v = -R$, we have

$$E(-exp(-RW^E)|\phi_2) = -exp\{-R[-x_1P_1 + x_1P_2 + (\theta - P_2)^2/(2Rv)]\} \tag{17}$$

It follows that at date 1, the early-informed traders maximize the derived expected utility of their date 2 wealth

$$E[[-exp\{-R[x_1P_1 + x_1P_2 + (\bar{F} + \theta - P_2)^2/(2Rv)]\}]|\phi_1]. \tag{18}$$

Now, (18) can be written as

$$-[2\pi\sigma_{P_2}^2]^{-\frac{1}{2}} \int_{-\infty}^{\infty} exp\left\{-R\left[x_1P_1 + x_1P_2 + \frac{(\bar{F} + \theta - P_2)^2}{(2Rv)}\right]\right.$$
$$\left. -\frac{1}{2}\frac{(P_2 - \bar{P}_2)^2}{v_{P2}}\right\} d(P_2 - \bar{P}_2). \tag{19}$$

Completing squares, the expression within the exponential above can be written as

$$-\left[\frac{1}{2}w^2 s + hw + l\right], \tag{20}$$

where

$$\begin{aligned} w &= P_2 - \bar{P}_2 \\ h &= Rx_1 - \tfrac{(\theta - \bar{P}_2)}{v} \\ s &= \tfrac{1}{\sigma_{P_2}^2} + \tfrac{1}{v} \\ l &= Rx_1(\bar{P}_2 - P_1) + \tfrac{(\theta - \bar{P}_2)^2}{2v} + RB_0, \end{aligned}$$

Define $u \equiv \sqrt{s}w + h/\sqrt{s}$. Then, expression (20) becomes $-(1/2)u^2 + (1/2)h^2/s - l$. The Jacobian of the transformation from w to u is $s^{-\frac{1}{2}}$, and thus the integral (19) becomes

$$-[2\pi\sigma_{P_2}^2 s]^{-\frac{1}{2}} \int_{-\infty}^{\infty} exp\left(-\frac{1}{2}u^2 + \frac{1}{2}\frac{h^2}{s} - l\right) du$$

$$= -\frac{1}{(\sigma_{P_2}^2 s)^{\frac{1}{2}}} exp\left(\frac{1}{2}\frac{h^2}{s} - l\right). \tag{21}$$

Solving for the optimal x_1 by maximizing the above objective, we obtain (12). □

Derivation of Eqs. (13) and (14): In this case, we again apply Eq. (16). We have that Σ_{12} has the elements $k(1-k)v_\theta - k^2 v_\epsilon - Rkk'v_\theta v_z$ and $-Rkv_\theta v_z$. The symmetric matrix $\Sigma_2 2$ has the three elements $k^2(v_\theta + v_\epsilon) + k^{'2} v_z$, $k'v_z$, and v_z. Applying (16) and performing some tedious algebra yields the expressions (13) and (14). □

Appendix B

In this appendix, we briefly compare and contrast two methods of estimation: the first performs individual stock time-series regressions and aggregates the time-series estimates, whereas the second performs Fama-Macbeth cross-sectional regressions and aggregates the estimates through time.

Letting c denote a cross-sectional expectation and t denote a time-series one, the Fama-Macbeth approach produces the following univariate estimate for a regression of y on x:

$$E_c\left[\frac{E_t(xy) - E_t(x)E_t(y)}{E_t(x - E_t(x))^2}\right]$$

The time-series approach produces an estimate

$$E_t\left[\frac{E_c(xy) - E_c(x)E_c(y)}{E_c(x - E_c(x))^2}\right]$$

Comparing the two approaches we find that whether the signs of the two expressions coincide depends, in part on the cross-sectional variation in $E_t(x - E_t(x))^2$, and the time-series variation in $E_c(x - E_c(x))^2$. If these are vanishingly small, for example, then the denominator in each case can be pulled out of the expectation and signs of the two estimates will coincide. While no general results are available beyond this, we choose to adopt the Fama-Macbeth approach because that allows us to report the simple t-statistic that assumes independence. The time-series estimates, on the other hand, are correlated across stocks so this approach is not possible.

When a variable Y is regressed on a vector of variables X, then the two centered

expressions above can be written in matrix notation as

$$E_c\left[\Sigma_{tYX}\Sigma_{tXX}^{-1}\right]$$

and

$$E_t\left[\Sigma_{cYX}\Sigma_{cXX}^{-1}\right]$$

where Σ_{AB} denotes the covariance vector of the variable A with the vector B, Σ_{BB} is the variance-covariance matrix of the vector B, and the subscripts t and c are defined as before. A similar intuition holds in this case. If Σ_{tXX}^{-1} and Σ_{cXX}^{-1} are non-stochastic, they can be pulled out of the respective expectations and in that case, the signs of the two estimate vectors will coincide on an element-by-element basis.

Table 1: Summary Statistics

This table presents the summary statistics associated with the pooled cross-section and monthly time-series of NYSE stocks used in the analysis. The time-period is January 1988 to December 1998. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. Order imbalance is measured by the difference in dollar buys and dollar sells (in millions of dollars). Proportional order imbalance is obtained from dividing order imbalance by the total dollar values of buys and sells.

Variable	Mean	Median	Standard deviation
Return	0.0128	0.0090	0.1122
Absolute return	0.0746	0.0527	0.0849
Order imbalance	9.846	0.2224	60.41
Absolute value of order imbalance	17.81	3.02	58.56
Proportional imbalance	0.0040	0.0243	0.2139
Absolute value of proportional imbalance	0.1560	0.1161	0.1464

Table 2: Monthly Cross-Sectional Regressions for Lagged Returns

This table presents the results of cross-sectional Fama-Macbeth type regressions for monthly returns of NYSE stocks on three lags of monthly returns. The time-period is January 1988 to December 1998. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. LRET, L2RET, and L3RET denote the first three lags of the monthly return. The coefficients are multiplied by a factor of 100.

Variable	Coefficient	t-statistic
LRET	−2.71	−3.18
L2RET	−0.227	−0.32
L3RET	0.804	1.27

Table 3: Monthly Cross-Sectional Regressions for Lagged Imbalances

This table presents the results of cross-sectional Fama-Macbeth type regressions for monthly returns of NYSE stocks on three lags of monthly imbalances. The time-period is January 1988 to December 1998. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. LROIB, L2ROIB, and L3ROIB represent the first three lags of imbalance innovations obtained from regressing imbalances on twelve lags each of returns and imbalances. All coefficients are multiplied by a factor of 10^4.

Variable	Coefficient	t-statistic
LROIB	−0.283	−3.19
L2ROIB	−0.238	−2.65
L3ROIB	−0.131	−1.67

Table 4: Monthly Cross-Sectional Regressions for Lagged Returns and Imbalance Innovations

This table presents the results of cross-sectional Fama-Macbeth type regressions for monthly returns of NYSE stocks on three lags of monthly returns and three lags of imbalance innovations. The time-period is January 1988 to December 1998. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. LRET, L2RET, and L3RET denote the first three lags of the monthly return. LROIB, L2ROIB, and L3ROIB represent the first three lags of imbalance innovations obtained from regressing imbalance on twelve lags each of returns and imbalances. Three lags of total volume are included in the regressions, but their coefficients are not reported. All return and imbalance coefficients are multiplied by factors of 10^2 and 10^4, respectively.

Variable	Coefficient	t-statistic
LRET	−2.55	−2.79
L2RET	−0.218	−0.28
L3RET	0.874	1.08
LROIB	−0.108	−1.37
L2ROIB	−0.241	−3.22
L3ROIB	−0.266	−3.49

Table 5: Monthly Cross-Sectional Regressions for Lagged Returns and Imbalance Innovations, Stratified by Market Capitalization

This table presents the results of cross-sectional Fama-Macbeth type regressions for monthly returns of NYSE stocks on three lags of monthly returns and three lags of imbalance innovations, stratified by monthly market capitalization. The time-period is January 1988 to December 1998. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. LRET, L2RET, and L3RET denote the first three lags of the monthly return. LROIB, L2ROIB, and L3ROIB represent the first three lags of imbalance innovations obtained from regressing imbalance on twelve lags each of returns and imbalances. Three lags of total volume are included in the regressions, but their coefficients are not reported. All return and imbalance coefficients are multiplied by factors of 10^2 and 10^4, respectively.

Panel A: Small firm tercile

Variable	Coefficient	t-statistic
LRET	−4.66	−4.46
L2RET	−0.530	0.49
L3RET	−0.993	−0.94
LROIB	−0.111	−0.50
L2ROIB	−0.327	−1.64
L3ROIB	−0.290	−1.38

Panel B: Mid-cap firm tercile

Variable	Coefficient	t-statistic
LRET	−2.27	−2.17
L2RET	−2.69	−2.63
L3RET	1.20	1.25
LROIB	−0.638	−1.23
L2ROIB	−0.350	−0.59
L3ROIB	−1.42	−2.66

Panel C: Large firm tercile

Variable	Coefficient	t-statistic
LRET	−1.56	−1.25
L2RET	−0.741	−0.61
L3RET	0.567	0.57
LROIB	−0.006	−0.09
L2ROIB	−0.233	−3.19
L3ROIB	−0.207	−2.91

Table 6: Monthly Time-Series Regressions for Lagged Returns and Imbalance Innovations

This table presents the results of individual stock time-series regressions for monthly returns of NYSE stocks on three lags of monthly returns and three lags of imbalance innovations. The time-period is January 1988 to December 1998. Regressions are run for the 470 firms that were present every month in the sample period. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. LRET, L2RET, and L3RET denote the first three lags of the monthly return. LROIB, L2ROIB, and L3ROIB represent the first three lags of imbalance innovations obtained from regressing imbalance on twelve lags each of returns and imbalances. Three lags of total volume as well as the contemporaneous and three lags of the S&P 500 monthly return are included in the regressions, but their coefficients are not reported. Cross-sectional averages of the coefficients from the time-series regressions are reported, along with the t-statistic (corrected using an approximation for cross-correlation in the regressors) for the mean being different from zero. The coefficients are multiplied by a factor of 100.

Variable	Coefficient	t-statistic
LRET	−4.66	−4.89
L2RET	−4.53	−4.93
L3RET	−0.773	−0.83
LROIB	0.007	0.11
L2ROIB	−0.024	−0.39
L3ROIB	0.005	−0.09

Table 7: Monthly Time-Series Regressions for Lagged Returns and Imbalance Innovations

This table presents the results of individual stock time-series regressions for monthly returns of NYSE stocks on three lags of monthly returns and three lags of imbalance innovations. The time-period is January 1988 to December 1998. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. LRET, L2RET, and L3RET denote the first three lags of the monthly return. LROIB, L2ROIB, and L3ROIB represent the first three lags of imbalance innovations obtained from regressing imbalance on twelve lags each of returns and imbalances. Three lags of total volume as well as the contemporaneous and three lags of the S&P 500 monthly return are included in the regressions, but their coefficients are not reported. Cross-sectional averages of the coefficients from the time-series regressions are reported, along with the t-statistic (corrected using an approximation for cross-correlation in the regressors) for the mean being different from zero. The coefficients are multiplied by a factor of 100.

Variable	Coefficient	t-statistic
LRET	−6.15	−8.67
L2RET	−5.73	−8.47
L3RET	−1.15	−1.68
LROIB	−0.795	−0.83
L2ROIB	−0.144	−0.24
L3ROIB	0.527	0.33

Table 8: Monthly Time-Series Regressions for Lagged Returns and Imbalance Innovations, Stratified by Market Capitalization

This table presents the results of individual stock time-series regressions for monthly returns of NYSE stocks on three lags of monthly returns and three lags of imbalance innovations, stratified by monthly market capitalization. The time-period is January 1988 to December 1998. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. LRET, L2RET, and L3RET denote the first three lags of the monthly return. LROIB, L2ROIB, and L3ROIB represent the first three lags of imbalance innovations obtained from regressing imbalance on twelve lags each of returns and imbalances. Three lags of total volume as well as the contemporaneous and three lags of the S&P 500 monthly return are included in the regressions, but their coefficients are not reported. Cross-sectional averages of the coefficients from the time-series regressions are reported, along with the t-statistic (corrected using an approximation for cross-correlation in the regressors) for the mean being different from zero. The coefficients are multiplied by a factor of 100.

Panel A: Small firm tercile

Variable	Coefficient	t-statistic
LRET	−5.67	−4.23
L2RET	−3.81	−2.83
L3RET	−1.85	−1.45
LROIB	−2.61	−0.87
L2ROIB	−0.760	−1.15
L3ROIB	1.82	0.37

Panel B: Mid-cap firm tercile

Variable	Coefficient	t-statistic
LRET	−7.02	−5.02
L2RET	−7.85	−6.33
L3RET	−0.877	−0.70
LROIB	0.029	0.69
L2ROIB	0.036	0.88
L3ROIB	−0.048	−1.30

Panel C: Large firm tercile

Variable	Coefficient	t-statistic
LRET	−6.63	−5.34
L2RET	−7.23	−6.56
L3RET	−1.55	−1.25
LROIB	0.002	0.36
L2ROIB	−0.001	−0.17
L3ROIB	0.003	0.50

Table 9: Panel Data Regressions for Lagged Returns and Imbalance Innovations

This table presents the results of individual stock panel data regressions for monthly returns of NYSE stocks on three lags of monthly returns and three lags of imbalance innovations. The time-period is January 1988 to December 1998. Regressions are run for the 470 firms that were present every month in the sample period. Returns are cumulated using daily mid-points of the quoted spreads associated with the last transaction of each trading day. LRET, L2RET, and L3RET denote the first three lags of the monthly return. LROIB, L2ROIB, and L3ROIB represent the first three lags of imbalance innovations obtained from regressing imbalance on twelve lags each of returns and imbalances. Three lags of total volume as well as the contemporaneous and three lags of the S&P 500 monthly return are included in the regressions, but their coefficients are not reported. Cross-sectional averages of the coefficients from the time-series regressions are reported, along with the t-statistic for the mean being different from zero. All return and imbalance coefficients are multiplied by factors of 10^2 and 10^4, respectively.

Variable	Coefficient	t-statistic
LRET	−1.70	−3.92
L2RET	0.352	0.81
L3RET	−0.664	−1.51
LROIB	−0.100	−1.57
L2ROIB	−0.043	−0.52
L3ROIB	−0.054	−0.64