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Impulse Responses of Exchange Rate and Prices under Purchasing Power Parity: Japanese Evidence from An Extracted Inflation-Based Study

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Abstract

The longtime perplexing purchasing power parity (PPP) puzzle has been recently resolved empirically by using the pure price inflation rates extracted and estimated by a pioneering financial-asset pricing approach. Applying the same extracted inflation rates, we estimate a vector error-correction (VEC) model of prices and the Japanese yen per U.S. dollar exchange rate, and find strong evidence supportive of (i) the PPP restriction which yields *the equilibrium error in the form of a real exchange rate*. Documented, further, under the PPP relationship so detected are (ii) the impulse responses of exchange rate to prices and those of prices to exchange rate that would imply exchange rates channeling inflation of one country into another country. Together, the findings lend, in the VEC context, desired PPP-theoretic content to the pure inflation rate estimates used.

Contents

1 Introduction

Lying at the heart of the purchasing power parity (PPP) puzzle are such inconsistencies between the PPP hypothesis and the empirical findings as the empirical gap in the speeds of adjustment between official price indices and exchange rates: official price indices such as the CPI and WPI (macroeconomic variables) tend to move slowly, whereas an exchange rate (a financial asset) moves much faster. The puzzle was studied in the open macroeconomic framework by Dornbusch (1976) who proposed the now popular (Dornbusch) overshooting model. More recently, the puzzle has been explored, now in the financial-asset context, by Chowdhry, Roll and Xia (2005) (C-R-X) in a novel way that bridges the gap in the speeds of adjustment between prices and exchange rates. Emphasizing that the gap in the speeds of adjustment is due to official price indices moving too slowly and that, to bridge the gap, price indices of *financial-asset* nature should be considered, they extract estimates for realized pure price inflation rate from stock returns. The estimates so extracted turn out to be sufficiently volatile that, using their extracted inflation rates, they document evidence in support of the short-run, relative PPP, thereby resolving the PPP puzzle.

Still another difficult question that remains is whether prices determine exchange rates. Ito (2005, pp.4-5) argues that "Even when the PPP is shown to hold, it is often difficult to determine whether domestic prices adjust to exchange rate changes or the exchange rate is determined by the gap between domestic and foreign prices. That is, proving that the PPP holds does not automatically prove the causality from prices to exchange rates. ... If the causality runs from the exchange rates to the domestic prices, as is feared in currency crisis countries, the estimated PPP relationship is not a theory of exchange rate determination. ... This process occurred in Indonesia, Russia and Argentina." Ito (p.7) adds that "The episode of the hyperinflation resulting in the exchange rate adjustment of a similar magnitude is the best example of the PPP relationship."

Yet, empirically, exchange rate moving fast as a financial variable can *lead* changes in goods prices: exchange rate effects on goods prices are indeed studied extensively in the empirical (and theoretical) literature on exchange rate pass-through. One of recent studies on pass-through is Landon and Smith (2006) who estimate the exchange rate effects on the industry-level investment good prices, for a panel of OECD countries, finding that an exchange rate depreciation [appreciation] leads to a significant rise [fall] in the prices of the investment goods used by most sectors.[1]

The present paper thus focuses on and explores the impulse responses of exchange rate and prices (i.e.,

[1]Further, Taylor (2000) suggests, and presents evidence on, a hypothesis that a low inflationary environment results in a low degree of exchange rate pass-through to domestic prices. See also Marston (1990) and Kojima (1995) on two critical patterns of corporate pricing behavior, i.e., exchange rate pass-through and pricing-to-market.

impulse responses of exchange rate to prices, those of prices to exchange rate, and those between prices) as well as the PPP relationship, and, in so doing, applies C-R-X's pure price inflation rate estimates and a vector error-correction (VEC) framework. Specifically, twofold research objectives are set:

(i) to explore the long-run structure of the yen per dollar exchange rate and C-R-X's extracted price indices, which is a VEC-based cointegration test of the PPP; and

(ii) to conduct the analyses of variance decomposition and impulse response functions in a cointegrated system of the yen per dollar rate and the extracted price indices, which is a study of the short-run structure of the time series, given the long-run structure estimated in (i).[2]

To my knowledge, the paper is the very first attempt to apply C-R-X's extracted inflation rates in investigating these issues for the yen per dollar rate in the VEC model. In the past PPP literature, long-run analyses of real exchange rate have been the main focus of study. For example, a test of the long-run PPP was conducted by Ito (1997) using the real effective yen exchange rate, and the unit-root test was employed to test the long-run constancy for the 1879 through 1995 period, whose results vary depending on the price indices used, the CPI or the WPI.[3] Another important, relevant past work here is Roll (1979) presenting an innovative, efficient markets view of the PPP that the real exchange rate should follow a random walk process. The present paper attempts to contribute to the literature by newly building and estimating a *multivariate* system of exchange rate and C-R-X's extracted price indices and interpreting its estimated short- as well as long-run structures in the contexts of PPP and impulse responses. Investigating whether the real exchange rate defined with C-R-X's extracted price indices still obeys a random walk process constitutes another important research topic and this is studied by Kojima (2006b), a companion paper.

To be specific, the paper considers a system of three economic variables (all logged): a nominal exchange rate s_t of a home currency (Japanese yen) against a foreign currency (U.S. dollar), a foreign price index p_t^* and a home price index p_t, where the price indices are constructed from C-R-X's extracted inflation rates. We will proceed by analyzing a series of the following problems:

First (as part of a preliminary analysis), we ask whether the two series $s_t + p_t^*$ and p_t have a strong positive association at all. If they do, does the series $e_t = s_t + p_t^* - p_t$ appear to be a stationary process?

[2] Note that we do not test for the Granger non-causality null hypothesis in the VEC framework; rather, impulse response functions of exchange rate and prices will be computed and studied within the short-run structure. Test for the Granger non-causality in our cointegrated system will be remarked subsequently in section 3.2.3.

[3] For a brief useful survey of existing studies on PPP, see Hausman, Panizza and Rigobon (2006, p.95). For example, the panel data-based unit-root tests for real exchange rates, as a test of the long-run PPP, are conducted by Wu (1996); Kojima (1993) attempts, by a univariate time series analysis, to detect structure changes in the yen per dollar real exchange rate movement that likely induce nonstationarity.

(If it does, then the two series $s_t + p_t^*$ and p_t will be cointegrated.) A preliminary analysis of these questions is important in the context of the PPP relationship, since, with the real exchange rate defined as a particular linear combination $r_t \equiv s_t + p_t^* - p_t$, absolute PPP asserts that $s_t + p_t^* = p_t$, that is, $r_t = 0$.[4] Based on the preliminary results obtained, we will next carry out a formal cointegration analysis of the entire three series (s_t, p_t^* and p_t). (The preliminary analysis here is only briefly summarized in the present paper and its details are reported in Kojima 2006a, section 3.1.2.)

Second, are the entire three series (s_t, p_t^* and p_t) cointegrated? In the cointegration analysis of the three series, a particular restriction motivated by our economic arguments will be the (strong) PPP restriction $\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$, which is in fact a vector of coefficients on the right-hand side of the real exchange rate's definition above. If the PPP restriction is supported by the data, then C-R-X's estimated inflation rates will turn out to have desired PPP-theoretic content, here in the VEC context. This would add to C-R-X's evidence of the same theoretical content documented in their single equation-based analysis of the PPP relationship.

Third, if the three series are cointegrated to have a long-run structure characterized as a PPP relationship, then we next turn to their short-run structure and ask whether the short-run Δs_t equation in the cointegrated system contains any statistically significant short-run effects of Δp_{t-l}^* and Δp_{t-l} with $l > 0$. If it does, the responses of exchange rate to prices will be likely observed. Also, if the short-run price equations contain effects of $\Delta s_{t-l}, \Delta p_{t-l}^*$ and Δp_{t-l} with $l > 0$, then exchange rate effects (i.e., the responses of prices to exchange rate) and the responses between the prices are likely detected.

Finally, to further explore responses of exchange rate and prices, the variance decomposition and impulse response functions are computed and studied. Specifically, if the variance of the one-step forecast error for s_t is accounted for by innovations of price indices p_t^* and/or p_t, rather than by own innovations, the responses of exchange rate to prices are likely confirmed. In the impulse response functions analysis, we will interprete the confidence bands as indicating the degree of uncertainty about the shape of impulse responses estimated. Based on the shape of the impulse responses, we will study two simple flows of impact, $p_t^* \to s_t \to p_t$ and $p_t \to s_t \to p_t^*$, to investigate the possible role of exchange rates as channeling inflations into countries.

The remaining of the paper is organized as follows: section 2 summarizes the data including C-R-X's inflation rates extracted from stock returns. The cointegration analysis of the yen per dollar rate and the extracted price indices is conducted in a VEC framework in section 3. Given the long-run structure (the

[4] Relative PPP requires, in terms of percentage, that $\Delta s_t + \Delta p_t^* = \Delta p_t$ where Δ is the first difference operator. See, for example, MacDonald and Marsh (1994, pp.24-25) and Hausman, Panizza and Rigobon (p.94). C-R-X investigate the PPP puzzle, focusing on the failure of relative PPP and successfully resolve the puzzle in the short run by using their extracted inflation rates as Δp_t^* and Δp_t (C-R-X, pp.260-261).

PPP relationship) estimated in section 3, an analysis of short-run effects is carried out in section 4, and the impulse responses of exchange rate and prices are further explored in section 5, along with variance decomposition. The final section gives some concluding remarks and a summary of findings.

2 Data and Extracted Inflation Rates

The system of monthly exchange rate determination to be investigated in the paper is a system of three economic variables, s_t, p_t^* and p_t (Japanese yen per U.S. dollar nominal rate, U.S. price index and Japanese price index, respectively). The underlying vector autoregressive (VAR) model for our system will be eq. (1) in section 3.1.1. All the data the present paper uses for the system are those of C-R-X, and compiled and tabulated by Kojima (2006a, Tables 33 and 34), along with their means and standard deviations to do the data replication check with C-R-X (Tables 1 and 6, ps.262 and 266, in particular). The data period is May 1983 through December 1999.

The source of the data for the yen per dollar exchange rate is the same as that in C-R-X (pp. 261-262): the Database Retrieval System (v2.11), available at http://pacific.commerce.ubc.ca/xr/. The monthly percentage changes are computed between the *ends* of two adjacent months as $(s_t - s_{t-1}) \times 100$ with s_t denoting logged end-of-month exchange rate; the monthly percentage changes and the raw (unlogged) month-end yen per dollar rates are provided, respectively, under columns "jpusfx"and "jpusfxr" in Kojima (2006a, Table 33) .

C-R-X extract a proxy for realized pure inflation rates from stock returns, which they call the "extracted risk-free rate," denoted by $\hat{R}_{ft}$. The time-series of the extracted risk-free rate is, however, not explicitly displayed in any tabular or graphical format in C-R-X. Kojima (2006a, Tables 33 and 34) thus constructs the tables to provide a full set of the extracted risk-free rate time series for Japan and the U.S. ($\hat{R}_{ft}^i, i = J, U$).

The Japanese and U.S. price indices (p_t and p_t^*) are logs of price indices constructed from (and hence *implied* by) $\hat{R}_{ft}^i, i = J, U$, and the Japanese and U.S. CPIs (cpi_t and cpi_t^*) are those constructed from the Japanese and U.S. CPI inflation rates $\pi_{CPI,t}^i, i = J, U$, with price indices at month April 1983 being set equal to unity. These price indices and CPI inflation rates are also provided in Kojima (2006a, Tables 33 and 34). Drawn in Figures 1 through 8 are a set of plots for $\hat{R}_{ft}^i$ and $\pi_{CPI,t}^i, i = J, U$, and another set for p_t, cpi_t, p_t^* and cpi_t^*. (See Table 1 for the symbols used in the graphs. Throughout the paper, multiplying $\hat{R}_{ft}^i$ and $\pi_{CPI,t}^i, i = J, U$, by 100 gives percent-per-month figures.)

The data source for $\pi_{CPI,t}^i, i = J, U$, is also exactly the same as that used by C-R-X (p.261). While the official CPI inflation rates are being saved in one of the data files downloadable from the American

Economic Association (AEA) Website, C-R-X's estimated pure inflation rates extracted from the stock returns are not and must be computed and saved by one of the program files downloaded from the AEA Website. Details of the files involved in the latter are given in Kojima (2006a, section 2.1).

The time series features of C-R-X's extracted inflation rates and how they are related to the official inflation rates are investigated by Kojima (2006a, sections 2.1 and 2.2), using the data and program files downloaded from the AEA Website. Replicated there are C-R-X's Tables 1 (for data summary statistics), 6 (for summary statistics for extracted risk-free rate differentials, CPI inflation differentials, and foreign exchange rate changes) and 11 (for cointegration relation between the CPI and the constructed price index), though with some minor differences found;[5] and yet those differences are negligible enough to proceed with the present study relying on the extracted inflation rates and price indices as compiled in Kojima (2006a, Tables 33 and 34).

3 Long-run Structure

To explore the long-run structure of the exchange rate and extracted price indices requires a test for unit roots, which is in the present paper equivalent to a multivariate cointegration testing of PPP. A better basis for examining the number of unit roots in a vector of variables is given by the multivariate cointegration methodology of Johansen; specifically, the multivariate form of the augmented Dickey-Fuller (ADF) test will be used, with a null of stationarity (rather than a null of nonstationarity).

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3.1 Models and preliminary study

3.1.1 The underlying VAR model and its VEC model

Let the vector $\boldsymbol{y}_t = (s_t, p_t^*, p_t)'$, each element of which is a potentially endogenous variable[6] and assumed to be integrated of order 1, $I(1)$.[7] To conduct multivariate cointegration tests of PPP, we consider the VAR model including a constant and augmented with centered seasonal dummies:[8]

[5]Perfect matches are observed with C-R-X's Table 6.

[6]It could turn out wealky exogenous, as will be evidenced in section 3.2.2. The rationale behind choosing the ordering (s_t, p_t^*, p_t) instead of, for example, its reverse (p_t, p_t^*, s_t) is given in section 5.2.

[7]Hansen and Juselius (1995, p.1) remark that "... we assume that $\boldsymbol{y}_t$ is at most $I(1)$ ... However not all the individual variables included in $\boldsymbol{y}_t$ need be $I(1)$, as is often incorrectly assumed. To find cointegration between nonstationary variables, only two of the variables have to be $I(1)$."

[8]For *centered* seasonal dummies, see Hansen and Juselius (p.8) and Doan (*RM*, p.84, pp.367-368); Harris (1995, p.81) remarks that "Seasonal dummies are centered to ensure that they sum to zero over time, and thus they do not affect the

$$\boldsymbol{y}_t = \sum_{l=1}^{L} \boldsymbol{\Phi}_l \boldsymbol{y}_{t-l} + \boldsymbol{\mu} + \boldsymbol{\Psi} \boldsymbol{D}_t + \boldsymbol{u}_t. \tag{1}$$

The underlying VAR model is reformulated in the error-correction form as the VEC model:

$$\Delta \boldsymbol{y}_t = \sum_{l=1}^{L-1} \boldsymbol{\Phi}_l^{\Delta} \Delta \boldsymbol{y}_{t-l} + \boldsymbol{\Pi} \boldsymbol{y}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\Psi} \boldsymbol{D}_t + \boldsymbol{u}_t \tag{2}$$

where: Δ is the first-difference operator; the *short-run* matrices $\boldsymbol{\Phi}_l^{\Delta}$ represent the short-run dynamics/adjustment to past change in $\boldsymbol{y}_t$, $\Delta \boldsymbol{y}_{t-l}$;[9] and the *long-run* marix $\boldsymbol{\Pi}$ represents long-run adjustment. The initial assumptions include, in particular, the white noise $\boldsymbol{u}_t \sim IN(\boldsymbol{0}, \boldsymbol{\Sigma})$ or $\boldsymbol{u}_1, ..., \boldsymbol{u}_T$ are $niid(\boldsymbol{0}, \boldsymbol{\Sigma})$; the dependence is allowed among the white-noise disturbance terms $u_{1t_1}, u_{2t_2}, u_{3t_3}$ for any $t_i, i = 1, 2, 3$. For monthly data, L may be set at 12; it will be far smaller for our set of the data, however, as shown later.

Short-run effects/dynamics/matrices $\boldsymbol{\Phi}_l^{\Delta}$, the short-run dynamics/adjustment to past changes in $\boldsymbol{y}_t$, and their estimates are crucial in our analysis of short-run PPP, for C-R-X has shown, using the pure inflation rate they extracted from stock returns, that the short-run PPP is strongly supported.

Note that the analysis of the short-run structure (consisting of short-run effects $\boldsymbol{\Phi}_l^{\Delta}, l = 1, ..., L-1$) will be made *after* the modeling of the long-run structure is completed: the estimated cointegration vectors in the long-run structure will be considered as given or known, in the subsequent short-run analysis (in section 4).

Long-run adjustment If the long-run marix $\boldsymbol{\Pi}$ is either zero or non-zero and full-rank, it is of no use to write the VAR in form (2) rather than (1), to begin with; if it is non-zero but less than full-rank, then it is usefully written as[10]

$$\boldsymbol{\Pi} = \boldsymbol{\alpha} \boldsymbol{\beta}' \tag{3}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $3 \times r$ matrices, with r being the rank of $\boldsymbol{\Pi}$.[11] Following Engle and Granger's (1987) definition of equilibrium error,[12] $\boldsymbol{\beta}' \boldsymbol{y}_{t-1} \neq 0$ in eq. (2) is interpreted as an equilibrium error, with $\boldsymbol{\beta}$

underlying asymptotic distributions upon which tests (including tests for cointegration rank) depend."

[9]The terms "short-run matrices" and "short-run dynamics" are those used by Hansen and Juselius (ps.29, 71).

[10]See Doan (*UG*, p.360). In this case, eq. (2) *without* the term $\boldsymbol{\Pi} \boldsymbol{y}_{t-1}$ would be a misspecified model.

[11]$\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are matrices of full rank (see Hansen and Juselius, p.2). The decomposition in eq. (3) is not unique; where r is one, it is unique up to a scale factor in the two parts (see Doan, *UG*, p.360).

[12]The very beginning of Engle-Granger's formal analysis is to consider a set of n economic variables in long-run equilibrium when $\sum_{i=1}^{n} \beta_i y_{it} = 0$; the equilibrium error is a disequilibrium defined as a deviation from long-run equilibrium and given by $e_t \equiv \sum_{i=1}^{n} \beta_i y_{it}$; in the long run, $e_t = 0$.

being a matrix representing long-run coefficients such that the term $\boldsymbol{\beta}'\boldsymbol{y}_{t-1}$, the deviation from long-run equilibrium embedded in eq. (2), represents up to $(n-1)$ cointegration relationships in the multivariate model which ensure that the $\boldsymbol{y}_t$ converge to their long-run steady-state solutions. The rank r indicates the number of cointegration relations $\boldsymbol{\beta}'\boldsymbol{y}_{t-1}$. Assuming $\boldsymbol{y}_t$ is a vector of nonstationary $I(1)$ variables, then all the terms in (2) which involve $\Delta\boldsymbol{y}_t$ are $I(0)$, while $\boldsymbol{\Pi}\boldsymbol{y}_{t-1}$ must also be stationary for $\boldsymbol{u}_t \sim I(0)$ to be white noise (Harris, p.79).[13]

$\boldsymbol{\alpha}$ is a matrix representing a measure of the average speed of convergence towards the long-run equilibrium (i.e., the speed of adjustment to disequilibrium).[14] The elements of $\boldsymbol{\alpha}$ will be shown in section 3.2.3 to indicate how rapidly a current deviation from PPP is offset in the future.

$\boldsymbol{\Pi}$, the long-run adjustment, has been the major topic of interest in the cointegration and error-correction model analysis of PPP. In a way, this is due to the lack of short-run support for PPP in the past PPP literature. Now that C-R-X have found strong support for PPP in the short-run and made available more appropriate inflation rate data for the first time, it is an interesting and meaningful work, using their extracted price data, to statistically examine the short-run dynamics based on the VAR model and its error-correction representation.

Conditional/partial version If one (or more), say s_t, of the three variables turns out to be weakly exogenous to the system under study, then the conditional (or partial) version of eq. (2), conditioned on the weakly exogenous variable s_t, is written as

$$\Delta\boldsymbol{z}_t = \sum_{l=0}^{L-1} \boldsymbol{\Theta}_l^{\Delta}\Delta s_t + \sum_{l=1}^{L-1} \tilde{\boldsymbol{\Phi}}_l^{\Delta}\Delta\tilde{\boldsymbol{y}}_{t-l} + \tilde{\boldsymbol{\Pi}}\tilde{\boldsymbol{y}}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\Psi}\boldsymbol{D}_t + \boldsymbol{u}_t, \tag{4}$$

where $\boldsymbol{z}_t = (p_t^*, p_t)', \tilde{\boldsymbol{y}}_t = (p_t^*, p_t, s_t)'$ with s_t being the *last* element now and $\tilde{\boldsymbol{\Pi}} = \tilde{\boldsymbol{\alpha}}\tilde{\boldsymbol{\beta}}'$ where $\tilde{\boldsymbol{\alpha}} = \boldsymbol{\alpha}$ with the very *last* row vector being $\boldsymbol{0}$ and the *last* element of $\tilde{\boldsymbol{\beta}}$ corresponds to the exchange rate s_t. The conditional version of the model (4) will be investigated later in section 4.2.

3.1.2 Preliminary analysis

Carried out at the outset is a step-by-step preliminary analysis (which will be later followed by the cointegration test of PPP). It is rather long and not reported here; it is detailed in Kojima (2006a, section 3.1.2). Just remarks on setting L and infering r in the preliminary analysis are in order: Kojima (2006a, Table 12) sets $L = 4$ in such a way that the number of eigenvalues of the companion matrix that

[13] That the deviation from long-run equilibrium is stationary means that the deviation is temporary in nature. The stationarity requirement imposed on $\boldsymbol{\Pi}\boldsymbol{y}_{t-1}$ is investigated by Kojima (2006b).

[14] See Hansen and Juselius (pp.2-3) and Harris (pp.77-78).

are close to unity is sensible. The number of roots close to unity is found to be two, which is supposed to be equal to $3-r$, yielding $r=1$; there is then found present only one cointegration relation/vector $\boldsymbol{\beta}$, a third-order column vector.

3.2 Multivariate cointegration testing of PPP

With all the preliminary results detailed in Kojima (2006a, section 3.1.2), we next test for unit roots of each of the three variables by the Johansen procedure.

3.2.1 Testing for unit roots with the Johansen procedure

The null hypothesis to be tested in the Johansen procedure is the *stationarity* of a variable, not its nonstationarity. The stationarity null for the ith variable is formulated as the null that $\boldsymbol{e}_i$, whose elements are all zero except for the ith element being unity, is in the cointegrating space:[15]

$$\boldsymbol{\beta} = (\mathbf{H}_i, \boldsymbol{\varphi}) \tag{5}$$

where a $(3\times r_1)$ matrix $\mathbf{H}_i = \boldsymbol{e}_i$, to test for a unit root for the ith variable, and $\boldsymbol{\varphi}$ is a $(3\times r_2)$ matrix, with $r = r_1 + r_2$ (Harris, p.106). In our case, $r = r_1 = 1$ with $r_2 = 0$, for there exists only one cointegrating vector:

$$\mathbf{H}_0 : \boldsymbol{\beta} = (\mathbf{H}_i). \tag{6}$$

Tests of a null of type $\mathbf{H}_0$ are performed for each variable i, whose likelihood ratio (LR) test results, with degrees of freedom equal to $(3-r)r_1 = (3-1)1 = 2$, are reported in Table 2, in which no restrictions are yet imposed on either $\boldsymbol{\beta}$ or $\boldsymbol{\alpha}$. As desired, the null of stationarity is rejected strongly at any conventional level of significance, for every variable. This is much in line with a preliminary observation that, for Japan, all the roots of the companion matrix are inside the unit circle, which is in turn consistent with each variable being $I(1)$.[16]

We futher go on to testing for restrictions on cointegration relation(s) $\boldsymbol{\beta}$ and on $\boldsymbol{\alpha}$ in the VEC system (2).

[15]For $r=1=i$, for example, in eq. (2), $\boldsymbol{\Pi y}_{t-1} = \boldsymbol{\alpha\beta' y}_{t-1} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \end{bmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{bmatrix} s_{t-1} \\ p^*_{t-1} \\ p_{t-1} \end{bmatrix} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \end{bmatrix} s_{t-1}$. Notice that eq. (2) is indeed seen as a multivariate form of the univariate ADF regression equation and thus the Johansen approach is essentially "a particular type of unit-root test using a multivariate form of the ADF test with the null of stationarity" (Harris, p.107).

[16]Moreover, the result is consistent with the unit-root tests by the Engle-Granger single-equation two-step cointegration method: see Kojima (2006a, section 2.2.2 and Table 6).

3.2.2 Restricting cointegration relation $\boldsymbol{\beta}$ and the speed of adjustment $\boldsymbol{\alpha}$

Restricting only the cointegrating vector(s) in $\boldsymbol{\beta}$ We first test for linear hypotheses on cointegration relation(s). Since we only have ($r =$) one cointegration vector, no identification problem occurs[17] and the (transposed) vector

$$\boldsymbol{\beta}' = \begin{pmatrix} \beta_{11} & \beta_{21} & \beta_{31} \end{pmatrix}. \tag{7}$$

A general type of restriction we impose here is

$$\mathbf{R}'\boldsymbol{\beta} = \mathbf{0}, \tag{8}$$

and a particular restriction motivated by our economic arguments is the (strong) *PPP restriction* on $\boldsymbol{\beta}'$ in (7)

$$\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}.^{18} \tag{9}$$

This is in fact a vector of coefficients on the right-hand side of the real exchange rate's definition (in section 1) and implies two homogeneity restrictions:

$$\beta_{11} + \beta_{31} = 0 \tag{10}$$

$$\beta_{21} + \beta_{31} = 0. \tag{11}$$

Thus, in eq. (8),

$$\mathbf{R}' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}. \tag{12}$$

Note that normalizing on β_{11} leads readily to the PPP restriction (9) if $\boldsymbol{\beta}'$ in (7) has either of signs $\begin{pmatrix} + & + & - \end{pmatrix}$ and $\begin{pmatrix} - & - & + \end{pmatrix}$.

Under column "Restricting Only $\boldsymbol{\beta}$" in Table 3, note the italic figures which indicate (i) two homogeneity restrictions (10) and (11) implied by the (strong) PPP restriction on $\boldsymbol{\beta}$, in panel "Eigenvector(s) (transposed)," and (ii) normalizing on the first element of the cointegration vector (7), in panel "$\boldsymbol{\beta}'$."

[17] When $r > 1$, any linear combination of two cointegration relations will preserve the stationarity property, in which case one must go on to test whether the vectors of $\boldsymbol{\beta}$ are identified. See, for example, Harris (p.104) and Hansen and Juselius (p.3).

[18] Weaker PPP restrictions are as follows (MacDonald and Marsh, Table 4, p.35):

$\begin{pmatrix} 1 & \beta_{21} & -\beta_{21} \end{pmatrix}$ where β_{21} is free with its sign being constrained and, in eq. (8), one-row $\mathbf{R}' = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}$ (with element 1, s_t, being normalized on);

$\begin{pmatrix} 1 & \beta_{21} & -1 \end{pmatrix}$ where β_{21} is free and, in eq. (8), one-row $\mathbf{R}' = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$; and

$\begin{pmatrix} 1 & 1 & \beta_{31} \end{pmatrix}$ where β_{31} is free and, in eq. (8), one-row $\mathbf{R}' = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$.

The table reports the LR test results for the null hypothesis of the PPP restriction: the null of the PPP restriction is not rejected (or is readily accepted). The VEC model (2) under the PPP restriction which yields *the equilibrium error in the form of a real exchange rate* may now be formally called a *PPP-based* system of exchange rate and prices. C-R-X's extracted inflation rates have thus been shown to have desired theoretical content, here in the VEC model of the PPP relationship; this adds to C-R-X's evidence of the same theoretical content documented in their single equation-based analysis of the PPP relationship.

Restricting only the speed of adjustment $\boldsymbol{\alpha}$ Toward the end of the preliminary analysis (in Kojima 2006a, section 3.1.2) it is shown that α_{11} for the first element s_t is not statistically significant, suggesting the weak exogeneity of s_t. A restriction is now imposed on $\boldsymbol{\alpha}$ and tested, without imposing any restrictions on $\boldsymbol{\beta}$.

For one restriction on the first element, one-row

$$\mathbf{B}' = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, \tag{13}$$

and

$$\mathbf{B}'\boldsymbol{\alpha} = \mathbf{0}. \tag{14}$$

Under column "Restricting Only $\boldsymbol{\alpha}$" in Table 3, note (i) no restrictions on $\boldsymbol{\beta}$, in panel "Eigenvector(s) (transposed)," (ii) normalizing on the first element of the cointegration vector (7), in panel "$\boldsymbol{\beta}'$," and (iii) the italic figures indicating the zero restriction on the first element of $\boldsymbol{\alpha}$, in panel "$\boldsymbol{\alpha}$." The LR test results reported in the table show that the null of the weak exogeneity is not rejected. We will later estimate the conditional version (4), which is conditioned on the weakly exogenous variable s_t.[19]

Restricting, jointly, $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ Under column "Restricting Both" in Table 3, notice the italic figures indicating (i) two homogeneity restrictions (10) and (11) implied by the (strong) PPP restriction on $\boldsymbol{\beta}$, in panel "Eigenvector(s) (transposed)," (ii) normalizing on the first element of the cointegration vector (7), in panel "$\boldsymbol{\beta}'$," and (iii) a zero restriction on the first element of $\boldsymbol{\alpha}$, in panel "$\boldsymbol{\alpha}$." The LR test results in the table show that the null of the joint restrictions on $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ is not rejected (or is readily accepted).

Further, shown in Table 4 is a list of the eigenvalues of the companion matrix computed under the joint restrictions: two unit roots are obtained; this is, as desired, consistent with two eigenvalues close to unity remarked in section 3.1.2.

[19] Harris (pp.103-104) presents similar weak-exogeneity evidence for the U.K. (effective) exchange rate in the PPP and UIP model of Johansen and Juselius (1992).

3.2.3 Summary: Long-run structure, real exchange rate, weak exogeneity, and causality direction

Long-run structure and real exchange rate The estimated results under column "Restricting Both" in Table 3 strongly suggest the following features of the long-run structure of the yen per dollar rate and the prices: in the context of PPP-based VEC model (2) with $\boldsymbol{y}_t = (s_t, p_t^*, p_t)'$, there is *not* observed any long-run cointegration relation in the short-run equation for Δs_t, while there is for two other (price change) equations. That is, the short-run change in the nominal exchange rate Δs_t will *not* adjust to the equilibrium error (i.e., a previous real exchange rate $s_{t-1}+p_{t-1}^*-p_{t-1}$), while two short-run price changes Δp_t^* and Δp_t do adjust, respectively, in a negative and positive direction, but in almost the same speed, to the previous real exchange rate.

Weak exogeneity and causality direction s_t is found weakly exogenous to the system of equations under study. Does the weak exogeneity of s_t make sense in the PPP context? Ito (2005) argues that the direction of causality *from prices to exchange rates* is consistent with the PPP hypothesis. Also, in a cointegrated system, $\{y_t\}$ (e.g., price) does not Granger cause $\{z_t\}$ (e.g., exchange rate) if (i) lagged values Δy_{t-l} do not enter the short-run Δz_t equation *and* if (ii) $\{z_t\}$ is weakly exogenous to the system of the equations (i.e., does not respond to the deviation from long-run equilibrium) (Enders 2004, p.334). In our present empirical results (as will be shown in the next section), (i) does not apply, although (ii) does;[20] therefore, no such Granger *non*-causality from prices to exchange rate seems to be detected.

The weak-exogeneity evidence of the yen per dollar rate alone would suggest no direction of causality. We do not test for the Granger non-causality null for our cointegrated system, however; rather, impulse response functions of exchange rate and prices will be computed and studied within the short-run structure of the cointegrated system, in the subsequent section.

4 Short-run Structure and Conditional Model

We now accept the long-run structure as estimated under column "Restricting Both" (including the strong PPP restriction) in Table 3, as a reasonable yen against dollar behavior during the sample time period, and move on to the short-run study. Later, we will turn to the conditional version of the model, to attempt another multivariate PPP analysis based on eq. (4), assuming explicitly the weak exogeneity of the exchange rate.

[20]See the Δs_t equation in eq. (15) or the Δs_t row in its summary Table 6 there.

4.1 Short-run structure

4.1.1 The complete PPP-based VEC model and the short-run effects

With the long-run structure as estimated under column "Restricting Both" in Table 3 in the preceding section, the short-run matrices are estimated and shown in eq. (15) which is a numerically written, complete PPP-based VEC model (2) with (3) being substituted.[21] The corresponding t-values are reported in Table 5,[22] and the statisitically significant short-run effects as asterisked in (15) are summarized in Table 6.[23]

$$
\begin{aligned}
\begin{bmatrix} \Delta s_t \\ \Delta p^*_t \\ \Delta p_t \end{bmatrix} =& \begin{bmatrix} 0.028 & -0.112^{***} & 0.000 \\ 0.100 & 0.064 & -0.082^{*} \\ 0.014 & -0.189^{***} & 0.033 \end{bmatrix} \begin{bmatrix} \Delta s_{t-1} \\ \Delta p^*_{t-1} \\ \Delta p_{t-1} \end{bmatrix} + \begin{bmatrix} 0.157^{***} & 0.023 & 0.010 \\ \underline{0.109} & -0.055 & -0.016 \\ 0.023 & -0.133^{*} & -0.073 \end{bmatrix} \begin{bmatrix} \Delta s_{t-2} \\ \Delta p^*_{t-2} \\ \Delta p_{t-2} \end{bmatrix} \\
&+ \begin{bmatrix} 0.022 & 0.005 & -0.091^{***} \\ 0.180^{*} & 0.045 & -0.056 \\ 0.023 & \underline{-0.105} & -0.053 \end{bmatrix} \begin{bmatrix} \Delta s_{t-3} \\ \Delta p^*_{t-3} \\ \Delta p_{t-3} \end{bmatrix} + \begin{bmatrix} 0.000 \\ -0.095^{***} \\ 0.098^{***} \end{bmatrix} \begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \begin{bmatrix} s_{t-1} \\ p^*_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} -0.003 \\ 0.497^{***} \\ -0.510^{***} \end{bmatrix} \\
&+ \begin{bmatrix} -0.013 & -0.018^{*} & -0.008 & -0.006 & 0.001 & -0.016 & -0.008 & -0.014 & -0.004 & -0.009 & -0.012 \\ 0.004 & -0.052^{***} & -0.020 & -0.019 & -0.052^{***} & -0.058^{***} & -0.023 & -0.061^{***} & -0.013 & -0.053^{***} & -0.014 \\ 0.028 & 0.020 & 0.019 & 0.042^{*} & -0.011 & -0.011 & -0.034^{*} & 0.035^{*} & 0.018 & 0.041^{*} & 0.022 \end{bmatrix} \boldsymbol{sdum} \qquad (15)
\end{aligned}
$$

where: *** and * denote significance at 1 and 10% levels, respectively; the underlined figures are large in absolute value but statistically insignificant (as reported in Tables 5 and 6); and the column vector

$$\boldsymbol{sdum} = \begin{pmatrix} sdum_2 & sdum_3 & sdum_4 & sdum_5 & sdum_6 & sdum_7 & sdum_8 & sdum_9 & sdum_{10} & sdum_{11} & sdum_{12} \end{pmatrix}'.$$

The short-run effects As also seen from Table 6, the short-run Δs_t equation in the estimated system (15) contains *strongly* statistically significant short-run effects of $\Delta s_{t-2}(+)$,[24] $\Delta p^*_{t-1}(-)$ and $\Delta p_{t-3}(-)$, while s_t itself is weakly exogenous to the system. Then the responses of exchange rate to prices appear to be detected here.

Notice, too, that neither short-run effects of one-month lagged Δs_{t-1} and Δp_{t-1} are found significant in the Δs_t equation.[25] This may appear puzzling, and some refinement of the model could be required. Instead, here, we estimate a univariate, 3rd-order autoregressive model, AR[3], to check if Δs_{t-1} would become statistically significant in a simple time series model:

[21] The residual analysis for the jointly restricted model here is carried out in Kojima (2006a, section 4.1.2).

[22] Hansen and Juselius' *CATS in RATS* does not compute p-values here.

[23] The statistical significance for $\boldsymbol{\alpha}$ is reported under column "Restricting Both" in Table 3; recall that the (strong) PPP restriction (9) is imposed on $\boldsymbol{\beta}'$.

[24] The sign in parantheses indicates the sign of the effect.

[25] A similar observation (with regard to Δs_{t-1}, in particular) is documented in MacDonald and Marsh (Table 5, pp.36-39) for the U.K. and Germany.

$$\Delta s_t = c + \sum_{i=1}^{3} \phi_i \Delta s_{t-i} + u_t \tag{16}$$

where c and u_t are, respectively, a constant and a white noise. As readily seen from Table 7, Δs_{t-1} is still not statistically significant.[26] The statistical *in*significance of Δs_{t-1} (and Δp_{t-1} as well) could be then due to the estimated VEC model (15) consisting of *growth-rate* (or first-difference) equations rather than level equations: the statistical significance/insignificance in the model should be interpreted in terms of growth rate. We may thus take the statistical insignificance of Δs_{t-1} in VEC model (15) to be reasonable in the growth-rate form and consistent with the widely documented random-walk nature of s_t.

The short-run Δp_t^* (U.S. price change) equation contains weakly statistically significant short-run effects of $\Delta s_{t-3}(+)$ and $\Delta p_{t-1}(-)$, along with large, though insignificant, effects from the previous change in exchange rate $\Delta s_{t-2}(+)$: some degree of exchange rate effect and response of U.S. price to Japanese price appear to be detected here.

On the other hand, the short-run Δp_t (Japanese price change) equation contains statistically significant short-run effects only of $\Delta p_{t-l}^*(-), l = 1, 2$: no exchange rate effects but only the responses of Japanese price to U.S. price appear to be present.

The analysis here is only preliminary; those positive and negative short-run effects detected here will be graphically displayed, respectively, as upward and downward kinks in the impulse response functions, later in section 5.

Another set of short-run effects comprises seasonal dummies (Harris, p.83): notice in eq. (15) that both price change equations contain statistically significant effects of seasonal dummies: several, strongly statistically significant effects are observed in the Δp_t^* equation, while weakly statistically significant effects in the Δp_t equation.

---

[26]Examining the regression (16) with a GARCH error process, Ito (2005, Table 2, pp.12-13) also obtains similar evidence for the sample period from April 1991 through December 2003 that *none* of the three autoregressive parameters are statistically significant for monthly data. His result indeed evidences the random-walk nature of the nominal exchange rate level s_t. (See MacDonald and Marsh, pp.25-28 for three approaches to testing a random walk hypothesis, one of which estimates such a model as (16).)

In Kojima (2006a, Table 6) we already infer that s_t itself has a unit root. Only for reference purposes, however, Table 7 also reports the estimated AR[3] model for s_t

$$s_t = c + \sum_{i=1}^{3} \phi_i s_{t-i} + u_t; \tag{17}$$

the table shows that s_{t-1} is statistically significant with its estimated ϕ_1 being slightly greater than unity, which violates the sationarity condition of the AR model (and thus is consistent with the non-rejection of the unit-root null for s_t in Kojima 2006a, Table 6). The random walk hypothesis is studied by Kojima (2006b).

4.2 Conditional version of the model

With the weak exogeneity of s_t, we now turn to and estimate eq. (4), a conditional version of the model, where $\boldsymbol{z}_t = (p_t^*, p_t)'$ and $\boldsymbol{y}_t = (p_t^*, p_t, s_t)$. The estimated results are shown in Kojima (2006a, section 4.2) and some main results are summarized as follows:

The LR test of the PPP-restriction null leads to not rejecting, or readily accepting, the null. This is exactly the same decision we reached for the (unconditional) model (2). Also, the short-run structure as well remains unchanged, except that there is no longer a Δs_t equation; as is clear from eq. (4), Δs_t appears as a separate term in each endogenous variable equation. Moreover, compared with the diagnostic tests of the residuals for the unconditional model (2), the diagnostic tests for the conditional model indicate only slight or little change.[27] One may then prefer the unconditional model (2) which will provide valuable information on the short-run dynamics of the exchange rate behavior, which is missing in the conditional/partial system (4).

5 Impulse Responses in the Cointegrated System

The previous section has investigated the responses of exchange rate and prices only by examining the short-run parameters' statistical significance in the estimated PPP-based VEC model (15). Here we will study the same problem, by employing more refined statistical methods of variance decomposition and impulse response functions.

The estimated short-run effects as already given in eq. (15) may be directly used to perform the analyses of variance decomposition and impulse response functions in the cointegrating system (2), with (3) rewritten as

$$\boldsymbol{\Pi} = \begin{pmatrix} \alpha_{11} & \alpha_{21} & -\alpha_{31} \end{pmatrix}' \tag{18}$$

under the (strong) PPP restriction $\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$ on $\boldsymbol{\beta}'$ being supported by the data in the earlier section. For a purely technical programming reason, however, we will estimate once again the short-run effects for (2).

5.1 Setting a lag length and estimation

We again start out with selecting a lag length L for the underlying VAR model (1). At least a year's worth of lags is usually recommended (Doan, *UG*, p.332). In the preliminary analysis by Kojima (2006a,

[27]Harris (pp.115-117) illustrates similar evidence for the U.K. PPP and UIP model, suggesting both unconditional and conditional models (2) and (4) exhibit very similar residual behavior.

Table 12) we attempted to select L, based on the observation of the roots of the companion matrix. Here, we consider several pairs of lags to test a null (longer) length of lags, based on Tables 8 and 9.[28]

Although the sequential likelihood ratio tests in Table 8 show that there are indeed several possible lags that can be appropriate for the model, we only concentrate on $L \leq 5$ for the reason based on the roots of the companion matrix given in Kojima (2006a, Table 12). Also, based on the first set of tests in Table 9, the null of $L = 12$ is not rejceted and yet it is not our chosen lag length for the same reason.

It is clear from Table 8 that the null of $L = 5$ is rejected at a 10% significance level (p-value=0.059), with the alternative of $L - 1 = 4$ being accepted. The decision here does support the earlier one made based on Kojima (2006a, Table 12). The second test in Table 9, however, could lead to an opposite decision; we will ignore this test result, for both Table 8 and the third set in Table 9 do lead to a decision to set $L = 4$.

By Table 8 and the third and the fourth sets in Table 9 combined together, one could make L even shorter, such as three months. $L = 4$ and $L - 1 = 3$ will, however, be our final choice, respectively, for the underlying VAR model (1) and the VEC model (2), since too short a length could cause a serial correlation of the residuals of the model.

We now again estimate the short-run effects for the cointegrating system (2) with (18); those newly estimated short-run matrices/effects are reported in Table 10.[29] Note, in the table, that the dependent variable indicated is a level variable (such as s_t) and yet the model actually estimated by the cointegrated least squares is a cointegrated system (2).

5.2 Variance decomposition analysis

Information on interactions among the variables is now studied. In variance decomposition and impulse response function analyses, different orderings in the vector $\boldsymbol{y}_t = (s_t, p_t^*, p_t)'$ could yield different results on the interactions.[30] One criterion for ordering, as proposed by Sims (1980), is that contemporaneous causes come first. For example, the ordering (s_t, p_t^*, p_t) can be taken as a causality from exchange rate to prices, whereas its reversed ordering (p_t, p_t^*, s_t) as a causality from prices to exchange rate. Ito (2005) may prefer the latter (as quoted in section 1), but we will continue with the former that has been analyzed

[28]See Doan (*UG*, p.336) for the test statistics used in the tables.

[29]Those estimates of short-run matrices are indeed close in magnitude to, and exactly the same in terms of sign and statistical siginificance as, those estimates in eq. (15) and t-values in Table 5. This does not apply to the deterministic terms (including a constant). This does not in any way affect the estimates of impulse response functions, however, as will be confirmed by comparing Figure 10 and Table 6.

[30]This will be most likely in the general case where the disturbance terms in the VEC model (2) are contemporaneously correlated. See, for example, Kojima (1996, section 3.3.2).

in the previous sections, since s_t is found to be weakly exogenous.[31]

The decomposition of variance for a level variable s_t in Table 11, which is plotted in Figure 9, is consistent with the weak exogeneity of the yen per dollar exchange rate as detected in section 3.2: the variance of the one-step forecast error for s_t is accounted for nearly by own innovations. Notice, in Table 11, that the variance of the one-step forecast error for the U.S. extracted price index p_t^*, at the longer steps, is less explained by own innovations and more by the Japanese extracted price index p_t, but only slightly by the yen per dollar exchange rate s_t. On the other hand, the variance of the one-step forecast error for the Japanese extracted price index p_t, at the longer steps, is less explained by own innovations and more by the yen per dollar exchange rate s_t, but only moderately by the U.S. extracted price index p_t^*.

The observed variance decomposition here will be combined later with impulse response functions, to infer robust responses of prices and exchange rate.

5.3 Impulse response functions

The impulse response functions and their two-standard-deviation confidence bands are computed and drawn in Figure 10, to complement the above variance decomposition analysis. Those kinks in the impulse response functions observed in the figure coincide, in sign though not in magnitude, with the statistically significant "short-run effects" in Table 6 (or in eq. (15)) earlier: the positive and negative short-run effects detected there are graphically displayed, respectively, as upward and downward kinks in the impulse response functions; for example, $\Delta s_{t-2}(+)$ in the short-run Δs_t equation corresponds to the upward kink in the top left impulse response function in Figure 10. We interpret the impulse responses in the figure, following Sims and Zha (1999, esp. p.1148, 1150-1153), and the confidence bands are interpreted as indicating the degree of uncertainty about the shape of impulse responses estimated.

Stock and Watson (2003, p.798) note that "Exchange rates are a channel through which inflation can be imported in open economies." The role of exchange rate as such a channel may be drawn, based on absolute PPP, as two simple flows of impact: (I) $p_t^* \rightarrow s_t \rightarrow p_t$ and (II) $p_t \rightarrow s_t \rightarrow p_t^*$. Clearly, the first half of each impact flow is the causality direction from prices to exchange rate that is claimed to be in line with PPP by Ito (2005) (as quoted in section 1), and the second half is the exchange rate effects on prices. Focusing on the three-month long horizon, the estimated responses as drawn in Figure 10 are now interpreted based on these impact flows.

One remark is in order on impulse responses plotted in Figure 10. Impulse response functions are

[31] Doan (*UG*, p. 353) recommends that an exogenous variable be put first in the ordering, and this is indeed satisfied by our chosen ordering (s_t, p_t^*, p_t). For weak exogeneity of s_t, see section 3.2.2.

computed in such a way that their contemporaneous (i.e., time-zero) values in the *lower off-diagonal* of Figure 10 are all set equal to zero.[32] Therefore, in the following sections, no interpretations are given to these contemporaneously zero-valued responses in the lower off-diagonal of the figure. Even so, we will be able to draw economic insights from the remaining, later impulse responses.

5.3.1 From U.S. price to exchange rate and to Japaenese price: (I) $p_t^* \to s_t \to p_t$

$p_t^* \to s_t$: Positive shocks to the monthly U.S. price index p_t^* induce (as shown in the second row of Figure 10)

- one to three months later a *negative* response of the exchange rate (i.e., an *appreciation* of the Japanese yen), with the (strongly) statistically significant kink $\Delta p_{t-1}^*(-)$ in the Δs_t equation in Table 6.

$s_t \to p_t$: Positive shocks to the monthly exchange rate level s_t induce (as shown in the first row of Figure 10)

- contemporaneously and up to three months later a *strong positive* response of Japanese price index p_t,[33] though without any statistically significant kink $\Delta s_{t-l}, l > 0$ in the Δp_t equation in Table 6.

Those exchange rate effects on the price changes here (and in (II) in the next subsection) are in line with those documented in the recent empirical study of exchange rate pass-through by Landon and Smith (2006),[34] and accord with our casual anticipation of the *Japanese yen depreciation* shocks leading to the *higher* Japanese yen price of the U.S. goods imported to Japan. They are also consistent with the variance decomposition for p_t due to the exchange rate's innovation gradually *rising* (above that due to the p_t^* innovation) (see Figure 9).

$p_t^* \to s_t \to p_t$: The impulse response mechanism for (I) is then summarized as a flow over the three-month horizon of impact:

$$\begin{aligned} &\text{positive shock to the U.S. price}\, p_t^* \to \text{negative response of the yen per dollar rate}\, s_t \\ &\qquad\qquad\qquad\qquad\qquad\quad \to \text{negative response of the Japanese price}\, p_t. \end{aligned}$$

There is indeed observed an unambiguous flow from the U.S. price to the Japanese price, channeled through exchange rate.

[32] See Kojima (1996, section 3.3.2).

[33] The lower confidence band is above zero almost throughout the horizon.

[34] See section 1 for the literature on exchange rate pass-through.

Note that this impact flow from U.S. price to Japanese price (via exchange rate) is in part consistent with the impulse responses between the two prices in Figure 10: positive shocks to the monthly U.S. price index p_t^* induce (as shown in the second row of Figure 10)

- no contemporaneous response, and then gradually the *negative* response possibly over the three-month horizon of the Japanese price index p_t, with the strongly statistically significant kink $\Delta p_{t-1}^*(-)$ and marginally significant kink $\Delta p_{t-2}^*(-)$ in the Δp_t equation in Table 6.

5.3.2 From Japanese price to exchange rate and to U.S. price: (II) $p_t \to s_t \to p_t^*$

$p_t \to s_t$**:** Positive shocks to the monthly Japanese price index p_t induce (as shown in the third row of Figure 10)

- one to two months later *no* (or, equally likely positive or negative) response and a *negative* response three months later of the exchange rate, the latter of which is due to the (strongly) statistically significant kink $\Delta p_{t-3}(-)$ in the Δs_t equation in Table 6.

$s_t \to p_t^*$**:** Positive shocks to the monthly exchange rate level s_t induce (as shown in the first row of Figure 10)

- the *negative* response of U.S. price index p_t^*, contemporaneously and one to two months later, with gradually smaller absolute magnitude due to the (marginally) statistically significant kink $\Delta s_{t-3}(+)$ in the Δp_t^* equation in Table 6.

The exchange rate effects on the price changes here accord with our casual anticipation of the *Japanese yen depreciation* shocks leading to the *lower* U.S. dollar price of the Japanese goods exported to the U.S.

$p_t \to s_t \to p_t^*$**:** The impulse response chain for (II) here is summarized as follows:

positive shock to the Japanese price p_t

$\to$ initially ambiguous but 3 months later negative response of the yen per dollar rate s_t

$\to$ (initially ambiguous but) later positive response of the U.S. price p_t^*.

In contrast to the unambiguous flow for (I), the (opposite) flow of impact from the Japanese price to the U.S. price, via exchange rate, is somewhat ambiguous to the extent that, initially, no or ambiguous Japanese price effects are observed on the exchange rate. Later, though, the yen appreciates (i.e., U.S. dollar depreciates), and then the U.S. price will rise, possibly in part, due to higher dollar prices of Japanese goods imported to the U.S.

Note that this flow from Japanese price to U.S. price (via exchange rate) is consistent with the impulse responses between the two prices in Figure 10: positive shocks to the monthly Japanese price index p_t induce (as shown in the third row of Figure 10)

- one month later no response and the *positive* response two and three months later of the U.S. price index p_t^*, although with (marginally) statistically significant kink $\Delta p_{t-1}(-)$ in the Δp_t^* equation in Table 6.

Analyzing two impact flows, (I) and (II), we thus find that the U.S. price and Japanese price effects on the yen per dollar rate turn out *both* negative (after three months). How could they be both negative? Under relative PPP, $\Delta s_t = \Delta p_t - \Delta p_t^*$, positive shock to the Japanese price would be expected to *contemporaneously* have a *positive* impact on Δs_t (through Δp_t), while that to the U.S. price a negative impact (through $-\Delta p_t^*$). It is not immediately clear how, with these two contemporaneous, opposing impacts derived under relative PPP, "both negative" effects could still result three months after the shocks to the prices. An attempt to resolve the apparent puzzling conflict may require refinements in testing. One obvious refinement is to test for the Granger non-causality null hypothesis in our cointegrated system (2) (as remarked in section 3.2.3). This will be a future extension of the present study.

5.3.3 Summary

The U.S. price, initially, appears to have more definitive (negative) effects on the yen per dollar exchange rate than the Japanese price would have on the exchange rate;[35] but, later (over the horizon of three months), the observed exchange rate changes seem to unambiguously channel inflation of one country into another country. The observed exchange rate effects on prices are found to be in accord with both our casual anticipation and the recent empirical result in the exchange rate pass-through literature.

6 Concluding Remarks

Relying on the estimates of Japanese and U.S. inflation rates extracted from stock returns by Chowdhry, Roll and Xia (2005), our exploration starts out with the cointegration analysis of the yen per U.S. dollar nominal exchange rate (s_t) and Japanese and U.S. prices (p_t, p_t^*) to study their long-run structure (i.e., their PPP relationship), and then, based on the estimated PPP-based vector error-correction model, we

[35] It should be noted that the possible responses of exchange rate to prices as detected in (I) and (II) may be somewhat ambiguous, since more than 90% of the exchange rate's variance decomposition is accounted for by *own* innovation (see Figure 9). The ambiguity here is consistent, in particular, with two confidence bands having zero in between for the Japanese price index (as drawn in the left most in the third row of Figure 10).

conduct analyses of variance decomposition and impulse response functions to examine the short-run structure (i.e., the impulse responses of exchange rate and prices). Strong evidence is documented in support of (i) the PPP restriction which yields the equilibrium error in the form of a real exchange rate. Evidenced, further, under the PPP relationship so documented are (ii) the impulse responses of exchange rate to prices and those of prices to exchange rate that would imply exchange rates channeling inflations into countries.

Several main findings that lead to the results (i) and (ii) are summarized as follows:

First, in the cointegration analysis with $\boldsymbol{y}_t = (s_t, p_t^*, p_t)'$, the null of a (strong) PPP restriction $(\ 1 \quad 1 \quad -1\ )$ on the cointegration vector $\boldsymbol{\beta}'$ is readily accepted by the LR test. C-R-X's estimated inflation rates are thus shown to have desired PPP-theoretic content, here in the VEC context. This adds to C-R-X's evidence of the same theoretical content documented in their single equation-based analysis of the PPP relationship. The same inference is derived for the conditional version of the cointegrating system which takes into account the detected weak exogeneity of the exchange rate.

Second, the estimated speed of adjustment $\boldsymbol{\alpha}$ shows that s_t appears weakly exogenous to the system of equations (2). That is, the short-run change Δs_t will not adjust to the previous equilibrium error (i.e., the previous real exchange rate $s_{t-1} + p_{t-1}^* - p_{t-1}$), while two short-run price changes Δp_t^* and Δp_t do adjust, respectively, in a negative and positive direction, but in almost the same speed, to the equilibrium error.

Third, the short-run Δs_t equation in the estimated PPP-based system (15) contains strongly statistically significant short-run effects of $\Delta s_{t-2}, \Delta p_{t-1}^*$ and Δp_{t-3}: the responses of exchange rate to prices appear to be detected here. Inferred at the same time are some responses of prices to exchange rate (i.e., exchange rate effects on prices) in the Δp_t^* equation (but not in the Δp_t equation): exchange rate as a financial variable may lead changes in U.S. goods prices.

Finally, from the impulse response functions and their confidence bands computed, we infer two flows over the three-month long horizon of impact from one price to exchange rate and, further *via exchange rate*, on to another price: (I) positive shock to the U.S. price $p_t^* \rightarrow$ negative response of the yen per dollar rate $s_t \rightarrow$ negative response of the Japanese price p_t; and (II) positive shock to $p_t \rightarrow$ initially ambiguous but 3 months later negative response of $s_t \rightarrow$ (initially ambiguous but) later positive response of p_t^*. That is, the U.S. price, initially, appears to have more definitive (negative) effects on the yen per dollar exchange rate than the Japanese price would have on the exchange rate, but, later (over the horizon of three months), the observed exchange rate changes seem to unambiguously channel inflation of one country into another country. We also find that the observed exchange rate effects on prices (i.e., the second half of each impact flow) are in accord with both our casual anticipation and the recent empirical result in the exchange rate pass-through literature.

C-R-X also extracted the U.K. and German inflation rates from the assoicated stock returns. Would the (strong) PPP restriction be satisfied by the VEC models for the two countries as well? What would the impulse responses look like? Would testing for the Granger non-causality null hypothesis in our cointegrated system (2) lead to results different from, or consistent with, the impulse responses? These are the topics that deserve further research, in an attempt to provide additional international evidence on PPP, impulse responses and Granger causality in the VEC framework.

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Table 1 Notation

Notation in the Graphs		Notation Following C-R-X[a]
eRf1, ERF1		$\hat{R}^{J}_{ft}$
eRf2, ERF2		$\hat{R}^{U}_{ft}$
gp1_c, GP1_C		$\pi^{J}_{CPI,t}$
gp2_c, GP2_C		$\pi^{U}_{CPI,t}$
p1, P1	p_t	log of $P^{J}_{R,t}$
p2, P2	p^*_t	log of $P^{U}_{R,t}$
p1_c	cpi_t	log of $P^{J}_{CPI,t}$
p2_c	cpi^*_t	log of $P^{U}_{CPI,t}$
e12, E12	s_t	log of month-end yen per dollar exchange rate

[a]Superscripts, J and U, denote, respectively, Japan and U.S.

Table 2 Cointegration Analysis:[a] The Johansen Approach to Testing for Unit Roots; Unrestricted Model (2) with $r = 1$

In $\mathbf{H}_0$ in eq. (6):	$i = 1\,(s_t)$			$i = 2\,(p^*_t)$			$i = 3\,(p_t)$		
Re-normalisation of the eigenvectors									
Eigenvector(s) (transposed)									
	s_t	p^*_t	p_t	s_t	p^*_t	p_t	s_t	p^*_t	p_t
	3.696	0.000	0.000	0.000	4.698	0.000	0.000	0.000	5.058
The LR test:[b]									
χ^2 (2)	17.08 (0.00)[c]			14.94 (0.00)			16.06 (0.00)		
$\boldsymbol{\beta}'$									
	s_t	p^*_t	p_t	s_t	p^*_t	p_t	s_t	p^*_t	p_t
	1.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	1.000
$\boldsymbol{\alpha}$									
Δs_t	-0.017			0.020			-0.007		
Δp^*_t	-0.004			-0.043			0.004		
Δp_t	-0.003			0.007			-0.061		
T-values for $\boldsymbol{\alpha}$									
	-2.016			1.820			-0.609		
	-0.250			-2.065			0.174		
	-0.170			0.290			-2.308		
$\mathbf{\Pi}$									
	s_t	p^*_t	p_t	s_t	p^*_t	p_t	s_t	p^*_t	p_t
Δs_t	-0.017	0.000	0.000	0.000	0.020	0.000	0.000	0.000	-0.007
Δp^*_t	-0.004	0.000	0.000	0.000	-0.043	0.000	0.000	0.000	0.004
Δp_t	-0.003	0.000	0.000	0.000	0.007	0.000	0.000	0.000	-0.061
T-values for $\mathbf{\Pi}$									
	-2.016	NA	NA	NA	1.820	NA	NA	NA	-0.609
	-0.250	NA	NA	NA	-2.065	NA	NA	NA	0.174
	-0.170	NA	NA	NA	0.290	NA	NA	NA	-2.308

[a]Neither $\boldsymbol{\beta}$ nor $\boldsymbol{\alpha}$ is under any restrictions yet.

[b]The null of stationarity of each (potentially) endogenous variable is tested.

[c]P-value.

Table 3 Cointegration Analysis: Hypothesis Testing of the Restrictions; Restricted Model (2) with $r = 1$

	Restricting Only $\boldsymbol{\beta}$ (PPP Restriction)[a]			Restricting Only $\boldsymbol{\alpha}$			Restricting Both		
Re-normalisation of the eigenvectors									
Eigenvector(s) (transposed)	s_t	p_t^*	p_t	s_t	p_t^*	p_t	s_t	p_t^*	p_t
	-6.262	*-6.262*	*6.262*	6.242	6.001	-6.564	*-6.262*	*-6.262*	*6.262*
The LR test:									
χ^2 (2)[b]	0.16 (0.92)[c]			0.10 (0.76)			0.21 (0.98)		
$\boldsymbol{\beta}'$									
	s_t	p_t^*	p_t	s_t	p_t^*	p_t	s_t	p_t^*	p_t
	1.000	*1.000*	*-1.000*	1.000	0.961	-1.052	*1.000*	*1.000*	*-1.000*
$\boldsymbol{\alpha}$									
Δs_t	-0.003			*0.000*			*0.000*		
Δp_t^*	-0.094			-0.092			-0.095		
Δp_t	0.097			0.102			0.098		
T-values for $\boldsymbol{\alpha}$									
	-0.222			0.000			0.000		
	-3.464			-3.397			-3.502		
	2.983			3.145			3.010		
$\boldsymbol{\Pi}$									
	s_t	p_t^*	p_t	s_t	p_t^*	p_t	s_t	p_t^*	p_t
Δs_t	-0.003	-0.003	0.003	0.000	0.000	0.000	0.000	0.000	0.000
Δp_t^*	-0.094	-0.094	0.094	-0.092	-0.088	0.097	-0.095	-0.095	0.095
Δp_t	0.097	0.097	-0.097	0.102	0.098	-0.107	0.098	0.098	-0.098
T-values for $\boldsymbol{\Pi}$									
	-0.222	-0.222	0.222	NA	NA	NA	NA	NA	NA
	-3.464	-3.464	3.464	-3.448	-3.448	3.448	-3.555	-3.555	3.555
	2.983	2.983	-2.983	3.168	3.168	-3.168	3.031	3.031	-3.031

[a]See the italic figures under this column; the same applies to the remaining two restrictions.

[b]Degrees of freedom= $(3 - r)r_1 = (3 - 1)1 = 2$, where $r = r_1 + r_2$ with $r_2 = 0$ (Hansen and Juselius, p.40).

[c]P-value.

Table 4 Cointegration Analysis: The Eigenvalues of the Companion Matrix; (Jointly) Restricted Model (2) with $r = 1$

real	complex	modulus	argument
1.000	-0.000	1.000	-0.000
1.000	0.000	1.000	0.000
0.722	0.144	0.736	0.196
0.722	-0.144	0.736	-0.196
0.357	0.350	0.500	0.775
0.357	-0.350	0.500	-0.775
0.082	0.489	0.496	1.404
0.082	-0.489	0.496	-1.404
-0.259	0.408	0.483	2.137
-0.259	-0.408	0.483	-2.137
-0.436	-0.163	0.466	-2.784
-0.436	0.163	0.466	2.784

Table 5 Cointegration Analysis: T-values of the Estimated Short-run Matrices and the Deterministic Variables in Eq. (15)

The short-run matrices $\mathbf{\Phi}_l^{\Delta}$:

Time: $t-1$			Time: $t-2$			Time: $t-3$		
0.379	-2.921	0.014	2.183	0.570	0.319	0.315	0.125	-2.825
0.745	0.907	-1.377	0.825	-0.754	-0.275	1.378	0.612	-0.941
0.084	-2.212	0.452	0.144	-1.510	-1.017	0.145	-1.180	-0.734

$\mathbf{\Psi}$ for the deterministic variables:

SEA(1)	SEA(2)	SEA(3)	SEA(4)	SEA(5)	SEA(6)	SEA(7)	SEA(8)	SEA(9)	SEA(10)	SEA(11)	CONST
-1.086	-1.482	-0.673	-0.533	0.110	-1.387	-0.671	-1.195	-0.331	-0.751	-1.007	-0.044
0.189	-2.388	-0.925	-0.871	-2.348	-2.634	-1.051	-2.713	-0.555	-2.418	-0.648	3.518
1.081	0.775	0.739	1.642	-0.408	-0.420	-1.295	1.291	0.664	1.550	0.831	-3.004

Table 6 Summary Results: Statistically Significant Effects in Eq. (15)[a]

Short-run equation	Short-run effects of[b]	Long-run cointegration relation
Δs_t	$\Delta s_{t-2}(+)^{***}, \Delta p^*_{t-1}(-)^{***}, \Delta p_{t-3}(-)^{***}$	Not observed
Δp^*_t	$\Delta s_{t-3}(+)^{*}, \Delta p_{t-1}(-)^{*}$; large but insignificant[c] $\Delta s_{t-2}(+)$	Observed
Δp_t	$\Delta p^*_{t-1}(-)^{***}, \Delta p^*_{t-2}(-)^{*}$; large but insignificant $\Delta p^*_{t-3}(-)$	Observed

[a]*** and * (attached to the right paranthesis) denote significance at 1 and 10% levels, respectively. The sign in parantheses indicates the sign of the effect.

[b]The statistically significant "short-run effects" here coincide, in sign though not in magnitude, with those kinks in the impulse response functions observed in Figure 10: the positive and negative short-run effects detected there are graphically displayed, respectively, as upward and downward kinks in the impulse response functions; for example, $\Delta s_{t-2}(+)$ in the short-run Δs_t equation corresponds to the upward kink in the top left impulse response function in Figure 10.

[c]See the underlined figures in eq. (15).

Table 7 Box-Jenkins - Estimation by Gauss-Newton: Two AR[3] Models

Dependent Variable	Eq. (16): Δs_t	Eq. (17): s_t
Constant	-0.005 (0.131)[a]	4.702 (0.000)
Δs_{t-1}	0.037 (0.613)	
Δs_{t-2}	0.134 (0.062)	
Δs_{t-3}	-0.018 (0.808)	
s_{t-1}		1.020 (0.000)
s_{t-2}		0.010 (0.334)
s_{t-3}		-0.135 (0.057)
Monthly Data	1983:09 To 1999:12	1983:08 To 1999:12
Usable Observations	196	197
Degrees of Freedom	192	193
Adjusted R^2	0.983	0.983
Residual Standard Deviation	0.035	0.035
Regression F(3,192); F(3,193)	3704.201 (0.000)	3873.769 (0.000)
Durbin-Watson Statistic	1.984	1.998
Q(36-3)	29.595 (0.637)	30.369 (0.599)
Ljung-Box Q-Statistics for Residuals SACF[b]	17.607 (0.347)	18.334 (0.305)

[a]P-value.

[b]Sample autocorrelation function.

Table 8 Setting Lag Length [1]:[a] Testing the Null of l versus the Alternative of $l-1$

Lags l	AIC	SBC	LR Test	P-Value
1	-17.059*	-16.285*		
2	-17.039	-16.1100	14.227	0.115
3	-16.988	-15.903	8.380	0.496
4	-16.961	-15.722	12.929	0.166
5	-16.953	-15.558	16.420	0.059
6	-16.920	-15.370	11.804	0.225
7	-16.925	-15.221	19.094	0.024
8	-16.868	-15.009	7.220	0.614
9	-16.805	-14.790	6.062	0.734
10	-16.801	-14.632	17.291	0.044
11	-16.7690	-14.445	12.003	0.213
12	-16.703	-14.224	5.575	0.782

[a]See Doan (*UG*, p.336) for the test statistics used.

Table 9 Setting Lag Length [2]: Test of l_0 versus l_1 Lags

l_0	l_1	χ^2 Statisitc
12	4	71.093[a] (0.508)[b]
5	4	13.705[c] (0.133)
4	3	11.604 (0.237)
3	2	7.836 (0.551)

[a]The degrees of freedom is 72.

[b]P-value.

[c]The degrees of freedom is 9, which also applies to the tests below.

Table 10 Estimation by Cointegrated Least Squares: Model (2) with (18)

Dependent Variable	s_t	p_t^*	p_t
Δs_{t-1}	0.030 (0.693)[a]	0.099 (0.487)	0.014 (0.933)
Δs_{t-2}	0.160 (0.037)	0.109 (0.442)	0.024 (0.888)
Δs_{t-3}	0.025 (0.744)	0.179 (0.197)	0.023 (0.888)
Δp_{t-1}^*	-0.111 (0.007)	0.064 (0.397)	-0.188 (0.039)
Δp_{t-2}^*	0.024 (0.566)	-0.056 (0.474)	-0.132 (0.157)
Δp_{t-3}^*	0.007 (0.878)	0.045 (0.568)	-0.105 (0.269)
Δp_{t-1}	-0.001 (0.970)	-0.082 (0.199)	0.032 (0.675)
Δp_{t-2}	0.009 (0.801)	-0.016 (0.802)	-0.073 (0.336)
Δp_{t-3}	-0.093 (0.007)	-0.056 (0.380)	-0.053 (0.486)
Constant	0.014 (0.867)	0.491 (0.001)	-0.506 (0.005)
C_SEASON{-10}[b]	-0.005 (0.710)	-0.041 (0.095)	0.023 (0.434)
C_SEASON{-9}	-0.008 (0.524)	-0.002 (0.937)	0.004 (0.882)
C_SEASON{-8}	0.004 (0.760)	0.013 (0.600)	-0.018 (0.531)
C_SEASON{-7}	-0.009 (0.500)	0.017 (0.484)	0.010 (0.719)
C_SEASON{-6}	-0.013 (0.290)	-0.040 (0.091)	0.002 (0.933)
C_SEASON{-5}	-0.004 (0.760)	-0.008 (0.746)	0.001 (0.963)
C_SEASON{-4}	-0.002 (0.874)	-0.006 (0.792)	0.024 (0.391)
C_SEASON{-3}	0.005 (0.664)	-0.039 (0.094)	-0.029 (0.304)
C_SEASON{-2}	-0.012 (0.337)	-0.045 (0.058)	-0.029 (0.309)
C_SEASON{-1}	-0.004 (0.758)	-0.011 (0.651)	-0.052 (0.066)
C_SEASON	-0.010 (0.423)	-0.048 (0.044)	0.017 (0.559)
EC1{1}[c]	-0.003 (0.834)	-0.094 (0.001)	0.097 (0.005)
Monthly Data	1983:09 To 1999:12	1983:09 To 1999:12	1983:09 To 1999:12
Usable Observations	196	196	196
Degrees of Freedom	174	174	174
Adjusted R^2	0.038	0.084	0.063
Residual Standard Deviation	0.035	0.064	0.077
Durbin-Watson Statistic	1.952	2.005	2.001

[a]P-value.

[b]Following Doan (*UG*, p.46 and *RM*, p.368), the 11 centered seasonal dummies C_SEASON{-10 to 0} correspond, respectively, to $sdum_2$ (February dummy) to $sdum_{12}$ (December dummy) in eq. (15).

[c]EC1{1}$\equiv \boldsymbol{y}_{t-1} = (\ s_{t-1} \quad p_{t-1}^* \quad p_{t-1}\ )'$ and their corresponding coefficients are those α's in the right-hand side of (18).

Table 11 Decomposition of Variance[a]

For Series	s_t				p_t^*				p_t			
Step	Std Error[b]	s_t	p_t^*	p_t	Std Error	s_t	p_t^*	p_t	Std Error	s_t	p_t^*	p_t
1	0.033	100.000	0.000	0.000	0.061	2.944	97.056	0.000	0.073	1.418	0.075	98.507
2	0.048	98.024	1.975	0.001	0.084	2.870	97.120	0.010	0.100	2.357	0.166	97.477
3	0.063	98.044	1.935	0.021	0.098	2.641	96.921	0.438	0.116	3.554	0.412	96.034
4	0.075	97.370	1.926	0.704	0.109	2.231	96.817	0.952	0.126	5.022	0.550	94.427
5	0.086	97.125	1.852	1.022	0.117	2.062	95.886	2.052	0.134	6.346	0.495	93.159
6	0.095	96.984	1.742	1.274	0.124	2.026	94.228	3.746	0.140	7.662	0.705	91.633
7	0.103	96.961	1.644	1.394	0.131	2.118	92.268	5.614	0.146	9.059	1.212	89.729
8	0.111	96.952	1.595	1.453	0.136	2.312	89.916	7.772	0.152	10.593	1.953	87.453
9	0.118	96.941	1.580	1.478	0.142	2.578	87.347	10.076	0.157	12.189	2.850	84.961
10	0.125	96.930	1.588	1.482	0.148	2.889	84.716	12.395	0.162	13.775	3.830	82.395
11	0.131	96.919	1.610	1.472	0.153	3.222	82.136	14.642	0.167	15.307	4.829	79.864
12	0.138	96.906	1.639	1.455	0.158	3.559	79.683	16.758	0.172	16.756	5.802	77.442
13	0.143	96.892	1.672	1.435	0.163	3.886	77.401	18.713	0.177	18.108	6.722	75.170
14	0.149	96.879	1.707	1.414	0.168	4.197	75.304	20.499	0.182	19.356	7.575	73.069
15	0.154	96.866	1.741	1.393	0.173	4.487	73.395	22.118	0.186	20.502	8.357	71.141

[a]See Figure 9.

[b]The standard error of forecast (or, more precisely, the variance of the one-step forecast error) for the "Series" indicated.

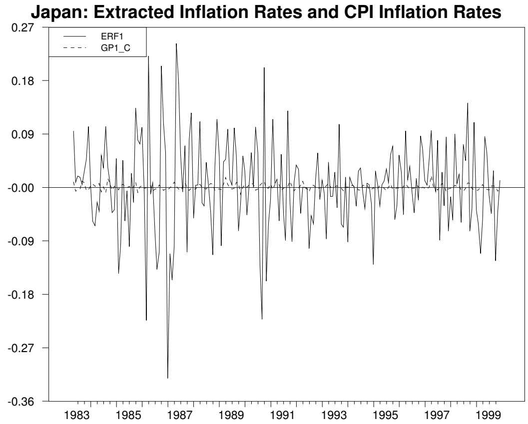

Figure 1 Japan: Extracted Inflation Rates $\hat{R}^J_{ft}$ (eRf1) and CPI Inflation Rates $\pi^J_{CPI,t}$ (gp1_c); 1983:5 - 1999:12. See also Table 1 for the notation.

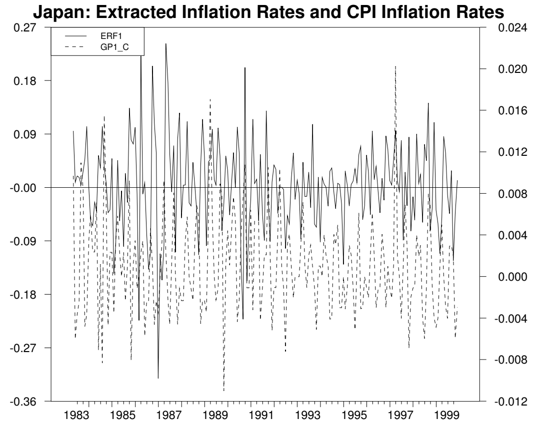

Figure 2 Japan: Extracted Inflation Rates and CPI Inflation Rates, with $\pi^J_{CPI,t}$ (gp1_c) on the Right-side Scale. See Figure 1 for the notation.

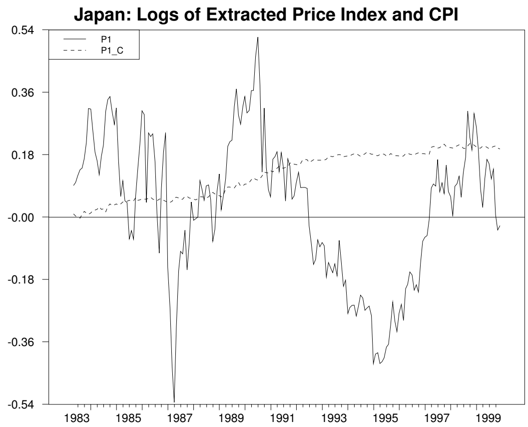

Figure 3 Japan: Logs of Extracted Price Index p_t (p1) and CPI cpi_t (p1_c); 1983:5 - 1999:12. See also Table 1 for the notation.

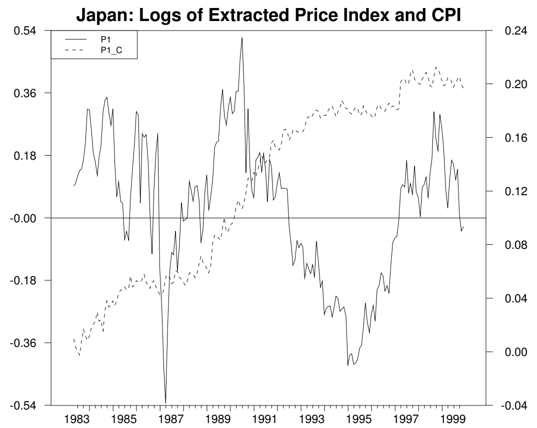

Figure 4 Japan: Logs of Extracted Price Index and CPI, with cpi_t (p1_c) on the Right-side Scale. See Figure 3 for the notation.

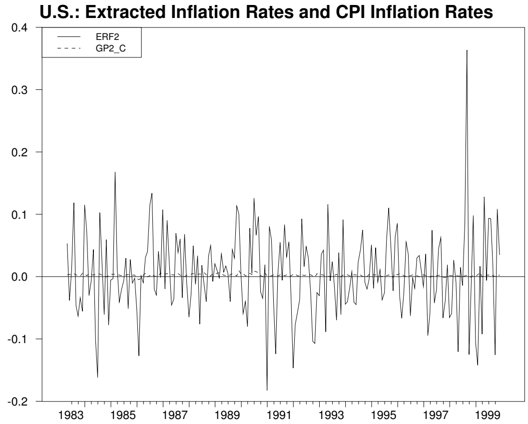

Figure 5 U.S.: Extracted Inflation Rates $\hat{R}^U_{ft}$ (eRf2) and CPI Inflation Rates $\pi^U_{CPI,t}$ (gp2_c); 1983:5 - 1999:12. See also Table 1 for the notation.

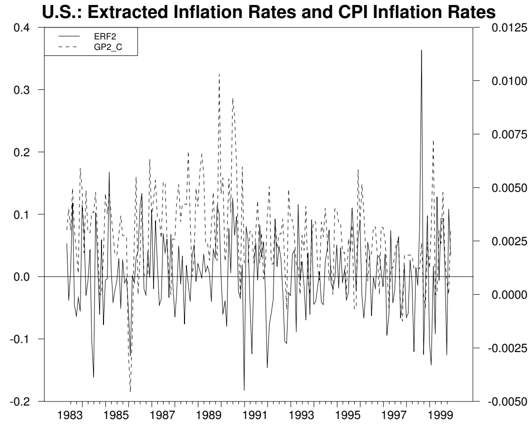

Figure 6 U.S.: Extracted Inflation Rates and CPI Inflation Rates, with $\pi^U_{CPI,t}$ (gp2_c) on the Right-side Scale. See Figure 5 for the notation.

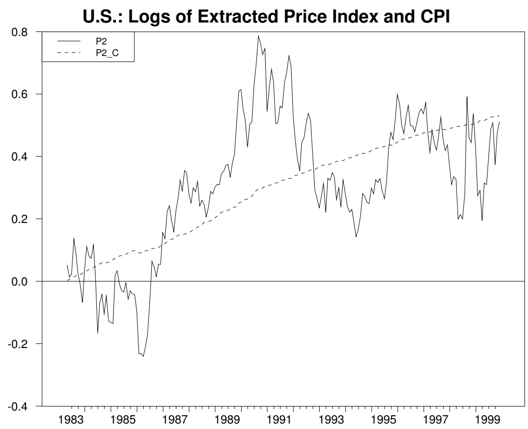

Figure 7 U.S.: Logs of Extracted Price Index p^*_t (p2) and CPI cpi^*_t (p2_c); 1983:5 - 1999:12. See also Table 1 for the notation.

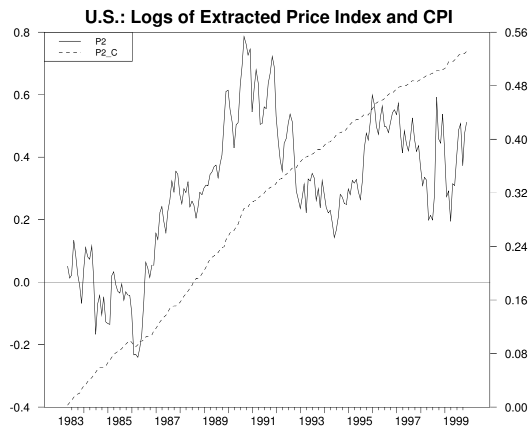

Figure 8 U.S.: Logs of Extracted Price Index and CPI, with cpi^*_t (p2_c) on the Right-side Scale. See Figure 7 for the notation.

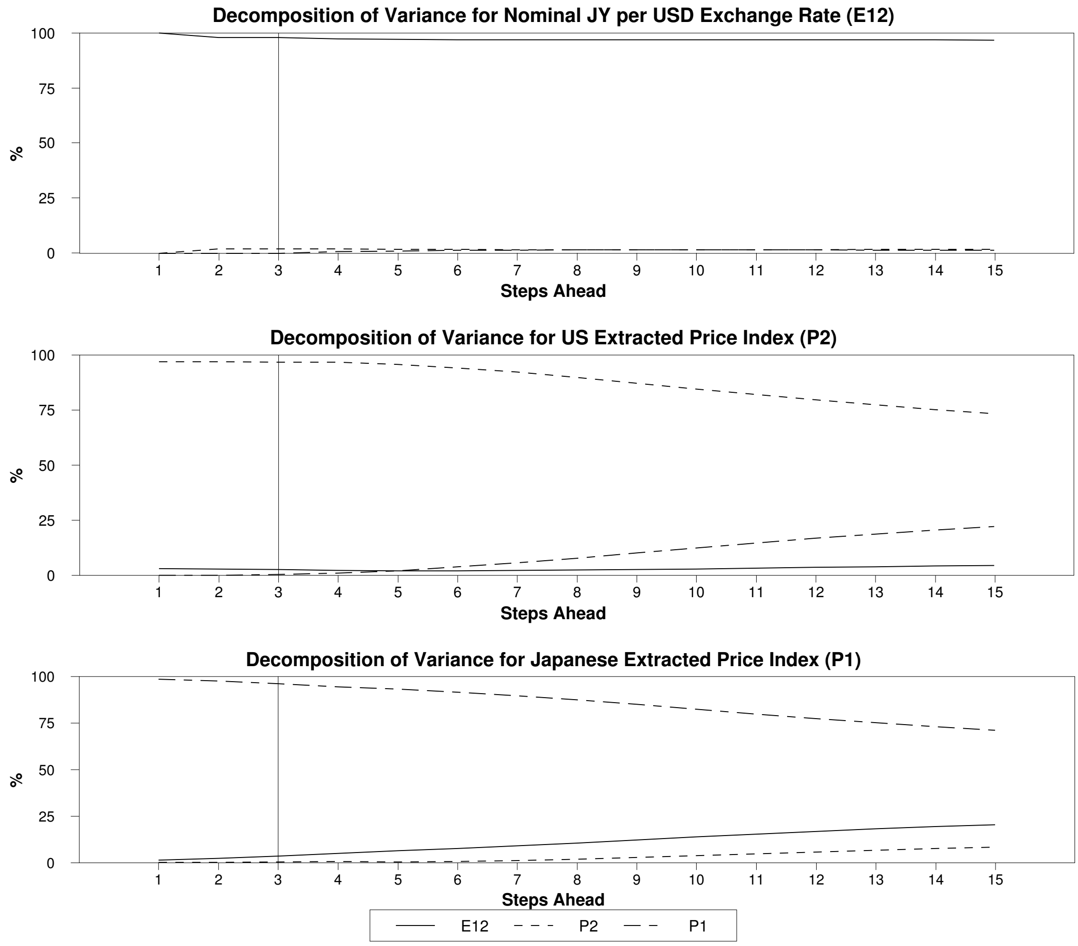

Figure 9 Variance Decomposition for s_t (E12), p_t^* (P2), and p_t (P1). The vertical grid line is drawn at $L - 1 = 3$, the lag length of model (2); see Table 11. See Table 1 for notation of the symbols in the graph.

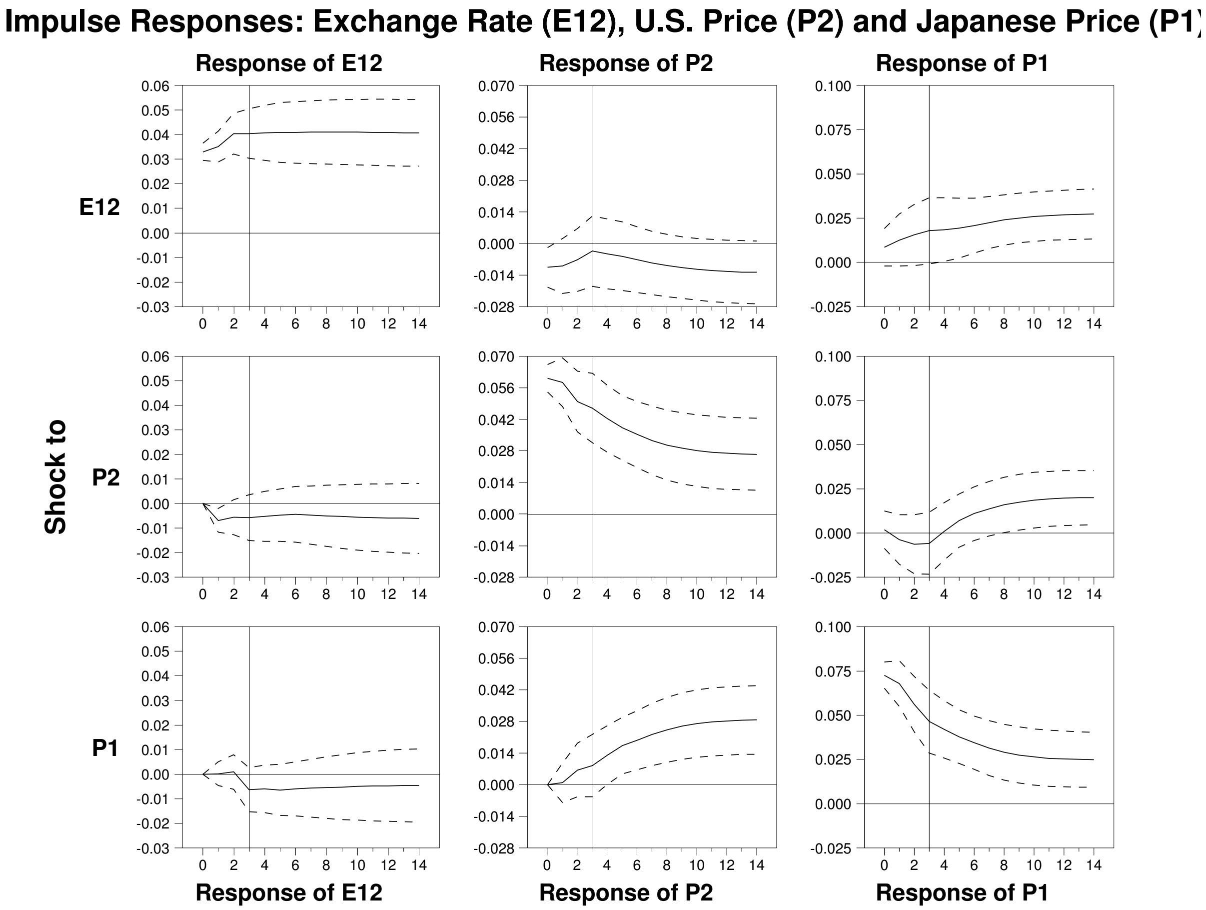

Figure 10 Impulse Responses of s_t (E12), p_t^* (P2), and p_t (P1). Forecast origin and horizon are, respectively, 0 and from 1 to 14 months. See Doan (*UG*, pp.350-355) for technicalities of computing the impulse response functions. The two-standard-deviation confidence bands (the dotted lines) for impulse responses are computed by Monte Carlo integration; see Doan (*UG*, ps.351, 472, 486) and Sims and Zha (p.1127). The vertical grid line is drawn at $L-1=3$, the lag length of model (2). For details on the kink(s) in each plot, see column “Short-run effects of” in Table 6. See Table 1 for notation of the symbols in the graph.