Estimation and Test of a Simple Model of Intertemporal Capital Asset Pricing *

Michael J. Brennan, Ashley W. Wang, and Yihong Xia†

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†Michael Brennan is Emeritus Professor, and Ashley Wang is a doctoral candidate at the Anderson School, UCLA. Yihong Xia is an assistant professor at the Wharton School of University of Pennsylvania. Corresponding Address: Brennan and Wang: The Anderson Graduate School of Management; University of California, 110 Westwood Plaza, Los Angeles, CA 90095-1481. Xia: Finance Department, The Wharton School, University of Pennsylvania; 2300 Steinberg Hall-Dietrich Hall; Philadelphia, PA 19104-6367. Phone: (215) 898-3004. Fax: (215) 898-6200. E-mail: yxia@wharton.upenn.edu.

Abstract

A simple valuation model that allows for time variation in investment opportunities is developed and estimated. The model assumes that the investment opportunity set is completely described by two state variables, the real interest rate and the maximum Sharpe ratio, which follow correlated Ornstein-Uhlenbeck processes. The model parameters and time series of the state variables are estimated using data on US Treasury bond yields and expected inflation for the period January 1952 to December 2000, and, as predicted, the estimated maximum Sharpe ratio is shown to be related to the equity premium. In cross-sectional asset pricing tests using the 25 Fama-French size and book-to-market portfolios, both state variables are found to have significant risk premia, which is consistent with the ICAPM of Merton (1973). In contrast to the CAPM and the Fama-French 3-factor model, the simple ICAPM is not rejected by cross-sectional tests using the 25 Fama-French size and B/M sorted portfolios. Returns on the 30 industrial portfolios do not discriminate clearly between the three models. When both sets of portfolios are included as test assets all three models are rejected, but the estimated risk premia for both ICAPM state variables are significant while those associated with the Fama-French arbitrage portfolios are insignificant.

1 Introduction

In the short run, investment opportunities depend only on the real interest rate and the slope of the capital market line, or Sharpe ratio, as in the classic Sharpe-Lintner Capital Asset Pricing Model. The slope of the capital market line depends in turn on the risk premium and volatility of the market return, and there is now strong evidence of time variation both in the equity risk premium and in market volatility, implying variation in the market Sharpe ratio, as well as in the real interest rate. Kandel and Stambaugh (1990), Whitelaw (1997), and Perez-Quiros and Timmermann (2000) have all found significant cyclical variation in the market Sharpe ratio.[1]

The Intertemporal Capital Asset Pricing Model (ICAPM) of Merton (1973) suggests that when there is stochastic variation in investment opportunities, it is likely that there will be risk premia associated with innovations in the state variables that describe the investment opportunities. However, despite this evidence of time variation in investment opportunities, and despite the lack of empirical success of the classic single period CAPM and its consumption based variant, there has been relatively little effort to test models based on Merton's classic framework.[2] One reason for this may have been the tendency to lump the ICAPM and Ross' (1976) Arbitrage Pricing Theory together as simply different examples of "Factor Pricing Models".[3] Yet this is to ignore the distinguishing characteristic of the ICAPM - that the "factors" that are priced are not just any set of factors that are correlated with returns, but are the innovations in state variables *that predict future returns*.[4] In this paper

[1] Other studies that identify significant predictors of the equity risk premium include: Lintner (1975) for interest rates; Campbell and Shiller (1988) and Fama and French (1988) for dividend yield; Fama and French (1989) for term spread and junk bond yield spread; Kothari and Shanken (1999) for Book-to-Market ratio.

[2] An important exception is Campbell (1993).

[3] "The multi-factor models of Merton (1973) and Ross (1976) ... can involve multiple factors and the cross-section of expected returns is constrained by the cross-section of factor loadings .... The multi-factor models are an empiricist's dream ... can accommodate ... any set of factors that are correlated with returns." Fama (1991, p1594).

[4] It is surprising that papers testing conditional versions of the CAPM that allow for time variation

we estimate a simple ICAPM that allows for time-variation in the real interest rate and slope of the capital market line, and evaluate the ability of the model to account for the returns on portfolios sorted according to size and Book-to-market ratio, as well as according to industry.

In the simple ICAPM that we develop, time variation in the instantaneous investment opportunity set is fully described by the dynamics of the real interest rate and the maximum Sharpe ratio. We assume that these two variables follow correlated Ornstein-Uhlenbeck processes; consequently, the current values of these variables are sufficient statistics for all future investment opportunities and are the only state variables that are priced in an ICAPM setting.[5] Then, using the martingale pricing approach, we show how a claim to a future cash flow is valued.[6] With additional assumptions about the stochastic process for the price level, the model is adapted in a simple fashion to the pricing of default-free nominal bonds, and the model parameters, as well as the time series of the real interest rate and the Sharpe ratio, are estimated by Kalman filter on data on US Treasury Bond yields and inflation expectations for the period January 1952 to December 2000. It is shown that, as predicted by the model, the Sharpe ratio estimate is significantly related to the 'ex-post' equity market Sharpe ratio which is measured by the ratio of the excess return on the market index to an estimate of volatility obtained from an EGARCH model.

In order to determine whether the simple ICAPM can account for the cross-section of returns on the 25 Fama-French size and B/M sorted portfolios for the period

in expected returns typically do not allow for the pricing of the state variables they use to describe the investment opportunity set. Jagannathan and Wang (1996) explicitly assume that "the hedging motives are not sufficiently important .... "(to warrant consideration of the ICAPM).

[5]Nielsen and Vassalou (2001) demonstrate formally that investors hedge only against stochastic changes in the slope and the intercept of the instantaneous capital market line, which implies that only variables that forecast the real interest rate and the Sharpe ratio will be priced.

[6]Papers that are related to our general valuation framework in allowing for time-variation in interest rates and risk premia include Ang and Liu (2001) and Bekaert and Grenadier (2000). The valuation model in this paper differs from the models presented in these papers chiefly in its parsimonious specification of the relevant state variables.

from January 1952 to December 2000, we follow a two-stage cross-sectional regression procedure. In the first stage, we regress the 25 portfolio returns on the market excess return, and the estimated innovations in the state variables, the real interest rate and the maximum Sharpe ratio, to obtain the three betas. In the second stage, the sample mean excess returns are regressed on the estimated betas to obtain the risk premia for market risk, the real interest rate risk and the maximum Sharpe ratio risk. A χ^2 test of the pricing restrictions implied by the model cannot reject them for the whole sample period, although the model is rejected using data from the second half of the period. Moreover, the simple ICAPM explains well the returns on both of the Fama-French arbitrage portfolios.[7] The pricing error for SMB is only -0.07% per month, and for HML 0.02% per month, and neither of these pricing errors is significant. On the other hand, when similar tests are conducted for the Fama-French 3-factor model and the CAPM, both models are strongly rejected for the whole sample period as well as for both halves of the sample. As a robustness check, we also test the three models using the one-step Generalized Method of Moments (GMM) Discount Factor approach. Again, the simple ICAPM is not rejected while the other two models are strongly rejected.

Motivated by the data-snooping concerns expressed by Lo and MacKinlay (1990), we also report the results of tests using 30 industrial portfolios instead of the size and book-to-market sorted portfolios. In this case, none of the three models is rejected for any of the sample periods. However, for both the simple ICAPM and the 3-factor model, the point estimates of the risk premia are quite different when the 30 industrial portfolios are used. Therefore we combine all 55 portfolios in a single estimation. The ICAPM risk premia remain significant in the combined sample while the estimated

[7]The ICAPM has been suggested by Fama and French (FF) themselves as one possible reason for the premia that they find to be associated with loadings on the SMB and HML hedge portfolios that are formed on the basis of firm size and book-to-market ratio. In FF (1995) they argue that the premia, "are consistent with a multi-factor version of Merton's (1973) intertemporal asset pricing model in which size and BE/ME proxy for sensitivity to risk factors in returns."

premia associated with the Fama-French arbitrage portfolios become economically and statistically insignificant. However, the pricing restrictions imposed by all three models are rejected. We conjecture that the rejection of the simple ICAPM is due to the omission of significant state variables that forecast future values of the interest rate and maximum Sharpe ratio.

The remainder of the paper is organized as follows. In Section 2 we construct a simple valuation model that allows for a stochastic interest rate and Sharpe ratio and specialize the model to the ICAPM. In Section 3 we describe the data and estimation of the valuation model and the state variables. The main empirical results are reported and discussed in Section 4, and Section 5 concludes.

2 Valuation with Stochastic Investment Opportunities

The value of a claim to a future cash flow depends on both the characteristics of the cash flow itself, its expected value, time to realization, and risk, and on the macroeconomic environment as represented by interest rates and risk premia. Holding the risk characteristics of the cash flow constant, unanticipated changes in claim value will be driven by changes in interest rates and risk premia, as well as by changes in the expected value of the cash flow. Most extant valuation models place primary emphasis on the role of cash flow related risk. However, Campbell and Ammer (1993) estimate that only about 15% of the variance of aggregate stock returns is attributable to news about future dividends. Their results further suggest that news about real interest rates plays a relatively minor role, leaving about 70% of the total variance of stock returns to be explained by news about future excess returns or risk premia. Fama and French (1993)[8] show that there is considerable common variation between bond and stock returns, which also suggests that changes in interest rates and risk

[8]See also Cornell (1999).

premia are important determinants of stock returns. In this section we construct an explicit model for the valuation of stochastic cash flows that takes account of stochastic variation in interest rates and risk premia.

Let V denote the value of a non-dividend paying asset. The absence of arbitrage opportunities implies the existence of a pricing kernel, a random variable, m, such that $E[d(mV)] = 0$.[9] This condition implies that the expected return on the asset can be written as:

$$\mathrm{E}\left[\frac{dV}{V}\right] = -\mathrm{E}\left[\frac{dm}{m}\right] - \mathrm{Cov}\left(\frac{dm}{m}, \frac{dV}{V}\right) \tag{1}$$

Assume that the dynamics of the pricing kernel can be written as a diffusion process:

$$\frac{dm}{m} = -r(X)dt - \eta(X)dz_m \tag{2}$$

where X is a vector of variables that follow a vector Markov diffusion process:

$$dX = \mu_X dt + \sigma_X dz_X \tag{3}$$

Then equations (1) and (2) imply that the expected return on the asset is given by:

$$\mathrm{E}\left[\frac{dV}{V}\right] \equiv \mu_V dt = r(X)dt + \eta(X)\rho_{Vm}\sigma_V dt \tag{4}$$

where $\rho_{Vm}dt = dz_V dz_m$, and σ_V is the volatility of the return on the asset. It follows, first, that $r(X)$ is the risk free rate since it is the return on an asset with $\sigma_V = 0$, and, secondly, that $\eta(X)$ is the risk premium per unit of covariance with the pricing kernel. It is immediate from equation (4) that the Sharpe ratio for any asset, V, is given by $S_V \equiv (\mu_V - r)/\sigma_V = \eta\rho_{Vm}$. Recognizing that ρ_{Vm} is a correlation coefficient, it follows that η is the maximum Sharpe ratio for any asset in the market - it is the slope of the capital market line, or "market" Sharpe ratio. An investor's *instantaneous*

[9] See Cochrane (2001) for a complete treatment.

investment opportunities then are fully described by the vector of the instantaneously riskless rate and the Sharpe ratio of the capital market line, $(r, \eta)'$.

In order to construct a tractable valuation model, we shall simplify, by identifying the vector X with $(r, \eta)'$, and assuming that r and η follow simple correlated Ornstein-Uhlenbeck processes.[10] Then, the dynamics of the investment opportunity set are fully captured by:

$$\frac{dm}{m} = -rdt - \eta dz_m \tag{5.1}$$

$$dr = \kappa_r(\overline{r} - r)dt + \sigma_r dz_r \tag{5.2}$$

$$d\eta = \kappa_\eta(\overline{\eta} - \eta)dt + \sigma_\eta dz_\eta \tag{5.3}$$

Although model (5) is not a structural model since it does not start from the specification of the primitives - the tastes, beliefs and opportunities of investors,[11] it provides a simple basis for consideration of the essential feature of the Intertemporal Capital Asset Pricing Model, the pricing of risk associated with variation in investment opportunities, since it allows for variation in the instantaneous investment opportunity set while limiting the number of state variables to be considered to the two that are required to describe that set. The strong assumption in model (5) is that r and η follow a joint Markov process.

While the pricing model (5) explicitly allows for time-variation in the investment opportunity set, it is not equivalent to Merton's ICAPM without further specification of the covariance characteristics of the pricing kernel: for example, the model will be equivalent to the simple static CAPM if the innovation in the pricing kernel is perfectly correlated with the return on the market portfolio. A specific version of the

[10] Kim and Omberg (1996) also assume an O-U process for the Sharpe ratio.

[11] For a structural model of time variation in investment opportunities that relies on habit formation see Campbell and Cochrane (1999).

ICAPM is obtained by specializing the pricing model (5) so that the innovation in the pricing kernel is an exact linear function of the total wealth portfolio return and the innovations in r and η:

$$\frac{dm}{m} = -rdt - \omega\eta\zeta' dz \tag{6}$$

where $\zeta' = (\zeta_M, \zeta_\eta, \zeta_r)'$, $dz = (dz_M, dz_\eta, dz_r)'$, $\omega \equiv (\zeta'\Omega\zeta)^{-1/2}$, and $\Omega dt = (dz)(dz)'$, where M denotes the portfolio for total wealth.

The structure (5) implies that the riskless interest rate is stochastic, and that all risk premia are proportional to the stochastic Sharpe ratio η. To analyze the asset pricing implications of the system (5), consider a claim to a (real) cash flow, x, which is due at time T. Let the expectation at time t of the cash flow be given by $y(t) \equiv \mathrm{E}\left[x|\Lambda_t\right]$ where Λ_t is the information available at time t, and $y(t)$ follows a driftless geometric Brownian motion with constant volatility, σ_y:[12]

$$\frac{dy}{y} = \sigma_y dz_y \tag{7}$$

Letting ρ_{ij} denote the correlation between dz_i and dz_j, the value of the claim to the cash flow is given in the following theorem.

Theorem 1 *In an economy in which the investment opportunity set is described by (5), the value at time t of a claim to a real cash flow x at time $T \equiv t + \tau$, whose expectation, y, follows the stochastic process (7), is given by:*

$$V(y, \tau, r, \eta) = \mathrm{E}_t^Q\left[x_T \exp^{-\int_t^T r(s)ds}\right] = \mathrm{E}_t^Q\left[y_T \exp^{-\int_t^T r(s)ds}\right] = yv(\tau, r, \eta) \tag{8}$$

where Q denotes the risk neutral probability measure, and

$$v(\tau, r, \eta) = \exp[A(\tau) - B(\tau)r - D(\tau)\eta] \tag{9}$$

with $A(\tau)$, $B(\tau)$ and $D(\tau)$ defined in the Appendix.

[12]The assumption of constant volatility is for convenience only. For example, as Samuelson (1965) has shown, the volatility of the expectation of a future cash flow will decrease monotonically with the time to maturity if the cash flow has a mean-reverting component.

Theorem 1 implies that, $v(\tau, r, \eta)$, the value per unit of expected payoff of the claim, is a function of the maturity, τ, of the cash flow. It also depends on the covariance of the cash flow with the pricing kernel, $\phi_y \equiv \sigma_m \sigma_y \rho_{ym}$, through $A(\tau)$ and $D(\tau)$. Finally, it depends on the current values of the two state variables that fully describe the investment opportunity set, r and η. An increase in the interest rate, r, unambiguously decreases the normalized claim value, $v(\tau, r, \eta)$. An increase in the Sharpe ratio, η, also decreases $v(\tau, r\eta)$ if $\rho_{ym} > 0$, which corresponds to positive cash flow systematic risk, and $\rho_{rm} < 0$, which implies a positive interest rate risk premium. The sensitivities of $v(\tau, r, \eta)$ to r and η, $B(\tau)$ and $D(\tau)$, are closely related to the mean reversion parameters κ_r and κ_η.

Applying Ito's Lemma, the theorem implies that the return on the claim can be written as:

$$\frac{dV}{V} = \mu(r, \eta, \tau)dt + \frac{dy}{y} - B(\tau)\sigma_r dz_r - D(\tau)\sigma_\eta dz_\eta, \tag{10}$$

where

$$\mu \equiv \mu(r, \eta, \tau) = r + (D_\tau(\tau) + \kappa_\eta D(\tau))\eta.$$

Thus, the risk premium of the claim, $\mu - r$, is proportional to the state variable η, and its proportional coefficient depends on the claim's cash flow maturity. The return risk is determined by the innovations in the two state variables, r and η, as well as in the cash flow expectation, y. Note that the innovations in r and η are systematic factors that affect the returns on all securities while $\frac{dy}{y}$ is security-specific. Moreover, the loadings on the systematic risks, $B(\tau)$ and $D(\tau)$, are functions of the cash flow maturity.

Although equation (10) is for claims of a single cash flow, the intuitions can be easily carried over to securities paying multiple cash flows. It implies that securities (cash flow claims) will have different risk exposures to the systematic state variables as long as they have different cash flow durations, and the ICAPM in (6) may have

the potential to explain cross-sectional stock portfolio returns if the portfolios are constructed to have enough dispersions of cash flow durations among them. Growth and value firms have quite different cash flow durations, and Perez-Quiros and Timmermann (2000) show that portfolios of large and small firms have different sensitivities to credit conditions so that we should at least expect them to have different loadings on r. In section 4, we shall empirically examine the ICAPM using 25 size and Book-to-Market sorted portfolios.

The value of a real discount bond is obtained as a special case of Theorem 1 by imposing $x \equiv y \equiv 1$ and $\sigma_y = 0$. The resulting expression generalizes the Vasicek (1977) model for the price of a (real) discount bond to the case in which the risk premium, as well as the interest rate, is stochastic. In order to value *nominal* bonds, it is necessary to specify the stochastic process for the price level, P; this is assumed to follow the diffusion:

$$\frac{dP}{P} = \pi dt + \sigma_P dz_P, \tag{11}$$

where the volatility of inflation, σ_P, is constant, while the expected rate of inflation, π, follows an Ornstein-Uhlenbeck process:

$$d\pi = \kappa_\pi(\overline{\pi} - \pi)dt + \sigma_\pi dz_\pi. \tag{12}$$

Then, noting that the real payoff of the nominal bond is $1/P_T$, the nominal price of a zero coupon bond with a face value of \$1 and maturity of τ, $N(P, r, \pi, \eta, \tau)$, and the corresponding real price, $n(P, r, \pi, \eta, \tau)$, are given in the following theorem.

Theorem 2 *If the stochastic process for the price level P is as described by (11) and (12), the nominal and the real prices of a zero coupon bond with face value of* \$1 *and maturity τ, are given by:*

$$N(P, r, \pi, \eta, \tau) \equiv Pn(r, \pi, \eta, \tau) = \exp[\widehat{A}(\tau) - B(\tau)r - C(\tau)\pi - \widehat{D}(\tau)\eta] \tag{13}$$

where $\widehat{A}(\tau)$, $B(\tau)$, $C(\tau)$, and $\widehat{D}(\tau)$ are given in Appendix A.

Equation (13) implies that the nominal yield on a bond of given maturity is a linear function of the state variables, r, π, and η:

$$-\frac{\ln N}{\tau} = -\frac{\widehat{A}(\tau)}{\tau} + \frac{B(\tau)}{\tau} r + \frac{C(\tau)}{\tau} \pi + \frac{\widehat{D}(\tau)}{\tau} \eta. \tag{14}$$

This equation shows that our valuation model, when applied to bonds, is a special case of the (essentially) affine term structure model considered in Duffee (2002) and is also closely related to the class of (complete) affine structures discussed by Duffie and Kan (1996) and Dai and Singleton (2000).

3 Data and Estimation

The primary data set consists of monthly observations of the yields on eight synthetic constant maturity zero coupon U.S. treasury bonds with maturities of 3, 6 months, and 1, 2, 3, 4, 5, and 10 years for the period from January 1952 to December 2000. Table 1 reports summary statistics for the bond yield data. The sample mean of the bond yields increases slightly with maturity, while the standard deviation remains relatively constant across maturities. The returns on 25 size and book-to-market sorted value weighted portfolios and the nominal short interest rate for the same period are used for the initial cross-sectional pricing tests.[13] The CRSP value weighted market portfolio is used to proxy for the portfolio of total wealth. The tests are repeated using the returns on the 30 Fama-French industrial portfolios which ZZ are re-balanced at the end of June every year using 4-digit SIC codes at that time ZZ.[14]

In principle, it is possible to estimate the parameters of the system (5) by the

[13]We thank Luis Viceira and Robert Bliss for providing the bond yield data. Our data start in January 1952 because the Federal-Treasury Accord that re-asserted the independence of the Fed from the Treasury was adopted in March 1952. Equity market data are taken from the website of Ken French.

[14]The returns are taken from the website of Ken French which describes the formation procedure.

standard Maximum Likelihood Method from the yields on three different bonds using equation (14) for the nominal bond yield. However, the choice of bonds to use in the estimation is arbitrary, and there is no guarantee that the estimates will be consistent with the yields of other bonds. Therefore, to minimize the consequences of possible model misspecification as well as measurement errors in the fitted bond yield data, we allow for errors in the pricing of individual bonds and use a Kalman filter to estimate the time series of the unobservable state variables r, π and η, and their dynamics, from data on yields of eight bonds with different maturities. Details of the estimation are presented in Appendix B. In summary, there are three transition equations for the unobserved state variables, and there are n observation equations based on the yields at time t, $y_{\tau_j,t}$, on bonds with maturities τ_j, $j = 1, \cdots, n$.

The transition equations are the discrete time versions of equations (5.2), (5.3) and (12), the equations that describe the dynamics of the state variables, r, η, and π.

The observation equations are derived from equation (14) by the addition of measurement errors, ϵ_{τ_j}:

$$y_{\tau_j,t} \equiv -\frac{\ln N(t, t+\tau_j)}{\tau_j} = -\frac{\widehat{A}(t,\tau_j)}{\tau_j} + \frac{B(\tau_j)}{\tau_j} r_t + \frac{C(\tau_j)}{\tau_j}\pi_t + \frac{\widehat{D}(\tau)}{\tau}\eta_t + \epsilon_{\tau_j}(t). \quad (15)$$

The measurement errors, $\epsilon_{\tau_j}(t)$, are assumed to be serially and cross-sectionally uncorrelated, and to be uncorrelated with the innovations in the transition equations. To reduce the number of parameters to be estimated, the measurement error variance was assumed to be of the form: $\sigma^2(\epsilon_{\tau_j}) = \sigma_b^2/\tau_j$ where σ_b is a parameter to be estimated. This is equivalent to the assumption that the measurement error variance of the log price of the bonds is independent of the maturity.[15]

[15] In estimating a version of the model that has a constant value of η Brennan and Xia (2002) find that the standard errors of the estimated bond yields decline with maturity out to 5 years. The model was also estimated assuming a maturity independent measurement error variance: convergence was slower but the estimates were similar.

The final observation equation uses the Livingston Survey data on the expected rate of inflation over the next six months,[16] π_{liv}, as a signal of the instantaneous expected rate of inflation π:

$$\pi_{\text{liv}} = \pi + \epsilon_{\text{liv}}. \tag{16}$$

Although equation (16) is not necessary to estimate the model, it helps to identify r and π, which enter the bond yield observation equation (15) in a symmetric way. We also estimated the model using only bond yield data. The estimates of the state variables r, π and η obtained using this additional observation equation are highly correlated with the estimates using only the bond yield data: the correlations between the levels (innovations) of the two sets of series are, for r, 0.92 (0.96); for π, 0.97 (0.77); and for η, 0.95 (0.87). However, the estimated sample means of r and π obtained using the additional observation equation (16) are more reasonable: 2.47% and 3.38% as compared with 4.54% and 1.15%.

4 Empirical Results

In Section 4.1 we show that the time series of nominal bond yields and inflation provide strong evidence of time variation in both the real interest rate and the Sharpe ratio, and that these estimates are associated with the nominal risk free rate and the equity premium. In Section 4.2 we estimate the risk premia associated with innovations in r and η and consider the ability of the model to explain the cross section of returns on, first, 25 size and book-to-market sorted equity portfolios and, secondly, on 30 industry portfolios.

[16]The Livingston survey is carried out twice a year in June and December. The CPI forecast contains the economists' forecast of the CPI level in six and 12 months. We use the mean of the six-month forecast to construct the expected rate of inflation, and then use linear interpolation for the other months. The data are available from the Federal Reserve Bank of Philadelphia: http://www.phil.frb.org/econ/liv.

4.1 State Variable Estimates

In this section we report Kalman filter estimates of the stochastic process for the state variables, r, π, and η, as well as the estimated time series of the state variables. In order to identify the process for the Sharpe ratio, η, it is necessary to impose a restriction that determines the overall favorableness of investment opportunities.[17] The restriction we impose is that $\bar{\eta} = 0.7$. This value was chosen after fitting an EGARCH model to the market excess return and using the resulting times series of volatility estimates to calculate a time series of realized equity market Sharpe ratios. This series has a mean of 0.57.[18] Since η is the maximum Sharpe ratio of the economy, we set $\bar{\eta}$ to 0.7 to allow for the fact that the equity market portfolio is not mean-variance efficient. This normalization affects only estimates of the scale of η and correlations of other variables with η. Finally, to improve the efficiency of estimation, $\bar{\pi}$ was set equal to the sample mean of CPI inflation, $\bar{r}$ was set equal to the difference between the sample mean of the 3-month nominal interest rate and $\bar{\pi}$, and the volatility of unexpected inflation, σ_P, was set equal to 1.16%, the sample volatility of CPI inflation. As a result of predetermining these parameter values, the standard errors of all other parameters reported in Panel B of Table 1 are understated.

Since the bond yield data were constructed by estimating a cubic spline for the spot yield curve from the prices of coupon bonds, the bond yields are measured with error. For estimation purposes the variance of the yield measurement error was assumed to be inversely proportional to maturity τ. The estimated measurement error parameter, σ_b, implies that the standard deviation of the measurement error varies from 16 basis points for the three month maturity to 5 basis points for the ten

[17] Equation (4) shows that the structure of risk premia is invariant up to a scalar multiplication of η and the vector of inverse security correlations with the pricing kernel with typical element, $1/\rho_{Vm}$.

[18] The excess return of the CRSP value weighted market portfolio during the sample period has a mean of about 0.62% and a standard deviation of 4.23% per month, implying a Sharpe ratio of 0.5 if volatility is assumed to be constant. Mackinlay (1995) reports an average Sharpe ratio of around 0.40 for the $S\&P500$ for the period 1981-1992.

year maturity, which is comparable to previous estimates.[19]

The estimated volatility of expected inflation, σ_π, is around 0.75% per year, while κ_π is close to zero, so that the expected rate of inflation rate follows almost a random walk.[20] The estimated volatility of the real interest rate process, σ_r, is 1.11% per year; on the other hand the sample volatility of the innovations to the estimated r series is 0.4% per year, which compares to Campbell and Viceira's (*op. cit.*) estimate of 0.5% per year for a slightly shorter sample period. The estimated mean reversion intensity for the interest rate, κ_r, is 0.074 per year which implies a half life of about 9 years. The volatility of the Sharpe ratio process, σ_η, is 0.31 per year which compares with the imposed long run mean value of 0.70; the mean reversion intensity for the Sharpe ratio is 0.047, which implies a half life of more than 14 years. Thus the real interest rate and the Sharpe ratio are persistent processes. This means that they are important state variables from the point of view of an investor with a long horizon so that we should expect to see significant risk premia associated with these variables.

To place the volatility of the real interest rate and the Sharpe ratio in perspective, suppose that the volatility of the market return, σ_M, is constant at 15%. Then if the correlation between the market and the pricing kernel is, say, 0.7, which corresponds to the point estimate from equation (19) below, the estimated volatility of the expected market return that is due to the volatility of the Sharpe ratio is $0.15 \times 0.31 \times 0.7 = 3.33\%$, which is three times as great as the estimated volatility attributable to the real interest rate (1.11%), so that the variation in η is of much more importance for the variation in the expected return on the equity market than is the variation in r. This is consistent with the finding of Campbell and Ammer (1993) that stock returns are strongly affected by 'news about future excess stock returns', while real interest rates

[19] Babbs and Nowman (1999) report standard deviations of the measurement errors of 10 basis points for a one-year maturity and 6 basis points for an eight year maturity for a 3-factor generalized Vasicek model estimated over a much shorter sample period.

[20] Campbell and Viceira (2001) also find that the expected rate of inflation is close to a random walk in a similar setting, using a model with constant risk premia.

have relatively little impact. Note, however, that the estimated correlation between the innovations in r and in η is -0.53, so that a significant part of the effect of the innovations in these two variables on the expected market return is offsetting. The Wald statistic for the null hypothesis that $\sigma_\eta = \kappa_\eta = 0$ (so that η is a constant) is highly significant, providing strong evidence, given the pricing model, that the Sharpe ratio is time varying. Finally, the t-statistics on $\rho_{\eta m}$ and ρ_{rm} strongly reject the null that the opportunity set state variables, r and η, are unpriced. However, this is not in itself evidence in favor of the ICAPM, because it is possible that the risk premia are due to the correlation of these variables with the market portfolio as in the classical CAPM; we shall investigate this further below. The t-statistics on ρ_{Pm} and $\rho_{\pi m}$ are either not significant or are only marginally significant: thus there does not appear to be a risk premium associated with inflation.

Figures 1 plots the time series of the estimated instantaneous real interest rate r. It reaches a maximum of 6.91% in mid-1984, and a minimum of -2.37% in mid-1980: it is positive for most of the sample period and its average value is 2.5%. The real interest rate tends to fall during recessionary periods which are represented by the shaded areas in the figures, and to rise following cyclical troughs.[21] Figure 2 plots the estimated series for π along with the corresponding Livingston estimates of the expected rate of inflation. The estimated π series tracks the Livingston inflation well, both of them capturing the inflationary episodes of 1974-75 and 1980-81.[22]

Since the real interest rate series in Figure 1 appears very volatile, while the expected inflation series in Figure 2 tracks the Livingston survey quite well, it is of interest to check how well the instantaneous *nominal* interest rate implied by the model tracks the one month nominal interest rate, R_f, which is taken from French's

[21] The period of recession is measured from peak to trough as determined by the National Bureau of Economic Research.

[22] We also estimated the model using only bond yields or using bond yields together with CPI inflation data. The fitted state variable π from these two data sets is also highly correlated with the Livingston inflation, but has a lower mean.

website. The model-implied instantaneous nominal risk free rate is given by

$$R = r + \pi - \sigma_P \rho_{Pm} \eta - \sigma_P^2. \tag{17}$$

Since the estimate of ρ_{Pm} reported in Table 1 is not significantly different from zero and $\sigma_P^2 \approx 0$, we can approximate the model-implied nominal interest rate, R, by $r+\pi$. Figure 3 plots $R \approx r+\pi$ along with the empirical 1-month rate R_f. The two series are almost coincident. Given that the π series tracks the Livingston expected inflation rates well, this implies that our estimated real interest rate series, r, is approximately equal to the real interest rate implied by the 1-month interest rate and the Livingston survey data.

Figure 4 plots the estimated Sharpe ratio, which shows considerable variation over time, reaching a maximum of 2.02 in April 1985 and a minimum of -1.64 in March 1980.[23] Recessions are generally associated with an increasing Sharpe Ratio. Whitelaw (1997) and Perez-Quiros and Timmermann (2000) have found similar cyclical patterns in the Sharpe ratio in the equity market.[24] The correlation between the estimated levels of r and η is about 0.1, but there is strong negative correlation between the innovations in these two variables (-0.53). Cochrane and Piazzesi (2002) have recently constructed a bond excess return forecasting factor as a linear combination of several different bond yields, without imposing any model restrictions. Like our estimate of η, their forecasting factor has high volatility and exhibits a strong cyclical pattern. Therefore, their forecasting factor corresponds to our η in an unrestricted model and its sample correlation with our estimate of η is 0.65.[25] The similarity is not surprising in view of the fact that our model has three state variables and Litterman and Scheinkman (1991) find that three principal components capture

[23] Boudoukh *et. al.*(1993) find evidence that the ex-ante *equity* market risk premium is negative in periods in which Treasury Bill rates are high. In March 1980 the Treasury Bill rate was over 14%.

[24] Fama and French (1989) have also documented common variation in expected returns on bonds and stocks that is related to business conditions.

[25] We thank Monika Piazzesi for making their series available to us.

well over 90% of the variation in bond yields.

Equation (15) implies that the difference in yields of bonds with different maturities, the 'term spread', is a linear function of the state variables r, π, and η. We find that our estimated state variables explain almost 99% of the variance of the term spread as measured by the difference between the yields on one-year and ten-year zero coupon Treasury bonds; this is further confirmation of the model fit. To the extent that the state variable innovations are factors in pricing equity returns as our theory predicts and the empirical results reported below confirm, the innovation to the term spread will also inherit some of their power in pricing equity returns.

The pricing kernel represented by equation (5) implies that the risk premia on all assets vary together with η. In particular, equation (4) implies that the equity market risk premium is related to η by:

$$\mu_M - r = \eta \rho_{Mm} \sigma_M$$

which implies that the equity market Sharpe ratio can be written as:

$$SH_M \equiv \frac{\mu_M - r}{\sigma_M} = \eta \rho_{Mm}. \tag{18}$$

The realized market Sharpe ratio for each month, SH_M^R, was constructed by dividing the market excess return for that month by the EGARCH fitted volatility. To test equation (18), SH_M^R was regressed on η_{-1}. The estimated equation is:

$$\begin{array}{rcccc} SH_M^R & = & 0.156 & + & 0.707\eta_{-1}, \qquad \bar{R}^2 \\ & & (0.71) & & (2.64) \qquad\quad 1.2\% \end{array} \tag{19}$$

where the Newey-West adjusted t-ratios are in the parentheses. Thus, as the model predicts, the Sharpe ratio that we have estimated using data on bond yields and inflation has significant information for normalized excess returns on the stock market.

The estimated coefficient of η_{-1} implies that $\rho_{Mm} = 0.71$,[26] and the lack of significance of the intercept is consistent with the theoretical specification (18). The adjusted R^2 increases from 1.2% to 4% if the realized average Sharpe ratio over the next 12 months is the dependent variable. Note that the filtered estimate of the Sharpe ratio is a linear function of the bond yields; this provides an economic rationale for the empirical success of yield-based predictors of expected returns such as nominal interest rates and yield spreads found by Stambaugh (1988) and Fama and French (1989) among others.

However, the parsimony of the simple ICAPM that we have proposed comes at the cost of possible model mis-specification. In particular, the assumption that r, π, and η follow a correlated O-U process is a strong one. Examination of the estimated innovations to r, π, and η reveals that they are autocorrelated and non-normal. The autocorrelation suggests that there may be additional, potentially priced, state variables, X, that affect the dynamics of r and η. We shall discuss this issue further below.

The significant values of ρ_{rm} (-0.85) and $\rho_{\eta m}$ (0.96) reported in Table 1 imply that there are indeed risk premia associated with innovations in r and η. In order to test whether premia associated with innovations in the investment opportunity set state variables, r and η, provide incremental explanatory power for equity portfolio returns relative to the CAPM, we calculate the innovations in the state variable (sv) estimates, Δr, $\Delta\pi$ and $\Delta\eta$, using the parameter values reported in Table 1:

$$\Delta sv = sv - \bar{sv}\left(1 - e^{-\kappa_{sv}/12}\right) - sv(-1)e^{-\kappa_{sv}/12}, \quad sv = r, \pi, \eta.$$

In the next section, we consider whether the estimated risk premia associated

[26]Note that the estimated correlation depends on the assumed value of $\bar{\eta}$. Shanken (1987) presents evidence that one can reject the hypothesis that the correlation between a linear combination of the CRSP *equal* weighted index and a bond portfolio and the pricing kernel exceeds 0.7. Kandel and Stambaugh (1987) report similar evidence for the *value* weighted index.

with these innovations can account for the observed returns on size and B/M sorted portfolios and on industry portfolios.

4.2 Cross-sectional Pricing

Under the simple ICAPM described in section 2, the pricing kernel is a linear function of the excess return on the market portfolio and the innovations in the state variables r and η so that the unconditional risk premium on asset i may be written as:

$$\begin{aligned} \mu_i - r &= \bar{\eta}\left(\beta_{iM}\rho_{mM}\sigma_M + \beta_{ir}\rho_{rm}\sigma_r + \beta_{i\eta}\rho_{\eta m}\sigma_\eta\right) \\ &= \beta_{iM}\lambda_M + \beta_{ir}\lambda_r + \beta_{i\eta}\lambda_\eta \end{aligned} \tag{20}$$

where the λ's are the unconditional premia for market risk, real interest rate risk and Sharpe ratio risk, and β_{iM} etc. are coefficients from the regression of asset returns on market excess returns and state variable innovations:

$$R_i - R_f = \alpha_i + \beta_{iM}\left(R_M - R_f\right) + \beta_{i\eta}\Delta\eta + \beta_{ir}\Delta r + \epsilon_i. \tag{21}$$

We can directly test the implications of the ICAPM for stock returns by using the two-stage cross-sectional regression procedure described in Chapter 12 of Cochrane (2001). Initially, we use as test returns the monthly excess returns on the 25 Fama-French size and book-to-market sorted portfolios for the period of January 1952 to December 2000. In the first stage, we regress the 25 portfolio returns on the market excess return, and the estimated innovations in the state variables, the real interest rate and the maximum Sharpe ratio, to obtain the three betas, as described in equation (21). In the second stage, we estimate the cross-sectional regression:

$$\overline{R_i - R_f} = \hat{\beta}_{iM}\lambda_M + \hat{\beta}_{ir}\lambda_r + \hat{\beta}_{i\eta}\lambda_\eta + u_i, \quad i = 1, \cdots, 25,$$

which corresponds to (20) with μ_i replaced by portfolio i's sample mean $\bar{R}_i$ and betas

replaced by their estimates from the first stage.

The model mispricing for portfolio i, $\hat{\alpha}_i$, is then defined by:

$$\hat{\alpha}_i \equiv \overline{R_i - R_f} - \left(\hat{\beta}_{iM}\hat{\lambda}_M + \hat{\beta}_{ir}\hat{\lambda}_r + \hat{\beta}_{i\eta}\hat{\lambda}_\eta\right).$$

The variance-covariance matrix of the pricing errors, $\mathbf{\Sigma} \equiv \text{Cov}\,(\hat{\alpha})$, is calculated using Shanken's (1992) correction. The model restrictions are tested using the quadratic form $\hat{\alpha}'\mathbf{\Sigma}^{-1}\hat{\alpha}$, which is distributed as $\chi^2(22)$ under the null hypothesis. For comparison, we also estimate and test the CAPM and the Fama-French 3 factor model using the same approach.

ZZ The estimated risk premia and the χ^2 test statistics for the three models are summarized in Table 2, which also reports the sample statistics for the market excess return, the SMB and HML portfolio returns.ZZ The model mispricing, $\hat{\alpha}$, is reported in Table 3. The χ^2 test statistic does not reject the pricing restrictions implied by the simple ICAPM ($p-$value $\approx$ 20%), while both the CAPM and the Fama-French 3-factor model are strongly rejected ($p-$values $<$ 1%).[27] For the ICAPM, both real interest rate risk and Sharpe ratio risk are significantly priced; moreover the signs of the risk premia are consistent with the estimates of the correlations of the innovations in these variables with the pricing kernel reported in Table 1. For the Fama-French 3-factor model, the factor HML carries a significant 0.4% per month risk premium, which is close to the mean sample return on the HML portfolio of 0.36%per month; the risk premium for the SMB factor is only 0.08% per month and not significant, and the mean return on the SMB portfolio for the sample period is only 0.10% per month and is not significant either. The estimated monthly market risk premium is 0.56% from the ICAPM, 0.58% from the FF 3-factor model, and 0.70% from the CAPM, as compared to the sample mean of the market excess return of 0.61%

[27]The CAPM and Fama-French 3-factor models are also rejected using the Gibbons-Ross-Shanken (1989) (GRS) test for the joint significance of the α's in OLS regressions of portfolio excess returns on the market excess return and (for the 3 factor model) the returns on SMB and HML.

per month. The individual mispricing α's are about the same magnitude under the ICAPM as those under the FF 3-factor model, so that the theoretically motivated ICAPM performs as well as the FF 3-factor model in this regard. Interestingly, there is considerable overlap between the portfolios whose $\alpha's$ have significant t-statistics under the ICAPM and the 3-factor model. Small, high book-to-market, portfolios are significantly mispriced by both models, as are small and big low book-to-market portfolios; however, the t-statistics are more significant for the 3-factor model and there are other portfolios that are well priced by the ICAPM but are mispriced by the 3-factor model. ZZ The tests were repeated for the two halves of the sample period and the results are summarized in Panel B and Panel C of Table 2: the CAPM and the 3-factor model are rejected in both subperiods, while the ICAPM is rejected only for the second sub-period. The estimated risk premia for r and η, and for the Fama-French arbitrage portfolios are qualitatively similar across the two-subperiods. ZZ

When the two special portfolios from the Fama-French 3-factor model, SMB and HML, are added as additional test assets in the cross-sectional test, the ICAPM is still not rejected, and the mispricings for SMB and HML are -0.07% and 0.02% per month respectively, and are statistically insignificant. Thus the ICAPM is able to account for the mean excess returns on the two Fama-French risk factors themselves.

The standard errors used to compute the $t-$ratios reported in Table 2 are calculated using the Shanken (1992) correction to account for errors in the first stage beta estimates. However, the beta estimates from the first stage will be biased to the degree that the factors Δr and $\Delta\eta$ are estimated with error. Then the second stage parameter estimates are biased due to the errors in the estimated betas. Therefore, as a robustness check, we also estimate and test the three models using the one-step Generalized Method of Moments Discount Factor (GMM/DF) approach. A discrete time version of the pricing model (5) implies that $\mathrm{E}\left[\psi(\tilde{R} - R_f)\right] = 0$ where ψ can

be written as a linear function of the three ICAPM factors:

$$\psi = 1 + b_M R_M + b_r \Delta r + b_\eta \Delta \eta.$$

Define $g_T(b) = \frac{1}{T}\sum_{t=1}^{T} \psi_t (R_t - R_f \mathbf{1}_{25})$ where $b = (b_1, b_2, b_3)'$, R_t is a (1×25) vector of portfolio returns, and $\mathbf{1}_{25}$ is a vector of ones with length 25. The GMM/DF approach is carried out by minimizing the quadratic form $g_T(b) W g_T(b)'$ with respect to b, where the weighting matrix W is set to be the inverse of the variance covariance matrix of the pricing errors, S^{-1}. The same approach is also applied to the FF 3-factor model and the CAPM. Under the null hypothesis of the ICAPM, $TJ_T \equiv T g_T(\hat{b}) S^{-1} g_T(\hat{b})'$ is distributed as $\chi^2(22)$. Again for the whole sample period, the simple ICAPM is not rejected ($p-$value =6.04%) while the 3-factor model and the CAPM are rejected at significance levels of $< 1\%$. The estimated coefficients of the ICAPM pricing kernel are (t-ratios in parentheses): $\hat{b}_1 = -4.40$ (3.1), $\hat{b}_2 = -69.48$ (1.4), and $\hat{b}_3 = -7.51$ (3.8). When the tests are repeated for the two halves of the sample period, the ICAPM is not rejected for the first half sample with $p-$value of 11.2% but is rejected at better than 1% significance level for the second sub-period. The FF and the CAPM are rejected for both sub periods at better than 1% significance level. The estimated coefficients of the pricing kernel are (t-ratios in parentheses): $\hat{b}_1 = -3.07$ (1.9), $\hat{b}_2 = -77.14$ (1.3), and $\hat{b}_3 = -7.06$ (3.2) for the first sub-period, and $\hat{b}_1 = -6.89$ (3.8), $\hat{b}_2 = -57.02$ (1.6), and $\hat{b}_3 = -4.35$ (2.9) for the second sub-period. Thus, the GMM results confirm the cross-sectional pricing results.

Lo and MacKinlay (1990) advise caution in drawing inferences from samples of characteristic sorted data such as the size and book-to-market sorted portfolios. Therefore, as a robustness check we repeat the analysis using the returns on ZZ 30 Fama-French industrial portfolios.ZZ The results are reported in Table 4, which contains the test statistics and the estimated factor risk premia for the whole sample period and two sub-periods, and Table 5, which contains the pricing errors for the whole sample period. Now, none of the three models is rejected by the two stage

cross-sectional test, either for the whole period or for the subperiods, and the pricing errors of the models are similar. The estimates of the risk premium associated with the market factor of 0.64-0.75% per month are similar to those obtained with the 25 size and B/M portfolios, and are broadly consistent with the sample mean excess return on the market portfolio of 0.62% per month. However, the signs of the risk premium estimates associated with η for the ICAPM, and with both SMB and HML for the 3-factor model, are reversed when the industry data are used, and are generally not significant. This is true both for the whole sample period and for the two sub-periods. Moreover, for the 3-factor model the estimated risk premia for the SMB (HML) factors of -0.30% (-0.38%) per month are quite different from the sample mean returns on the corresponding portfolios of 0.10% (0.36%) per month. As a result, a one-pass GRS F test, which includes in the null hypothesis the equality of the factor risk premia and the corresponding portfolio risk premia, strongly rejects the 3-factor model for the whole sample period and for the two halves with p-values well under 1%. The CAPM is also rejected under the one-pass GRS test with a p-value of about 4%. The ICAPM does not constrain the factor risk premia so the GRS test is not directly applicable to this model. However, if factor-mimicking portfolios[28] are constructed for the innovations in r and η then the GRS F-test, which use these portfolios as factors, does not reject the ICAPM (p-value = 8.3%) in constrast to the rejection of both FF 3-factor model and the CAPM. We do not place much weight on the mimicking portfolio results, however, since the portfolio weights of the mimicking portfolios are estimated with considerable error and in any case, as we shall see, the ICAPM is rejected by the two-stage cross-sectional tests reported below when the test assets include both the industrial portfolios and the size and book-to-market sorted portfolios. It is interesting to note that for all three models it is the tobacco industry, 'Smoke', that is the most mispriced - its return is from 3.6% to 6.2% *under*-predicted by the models. ZZ

[28]See Breeden (1979) and Breeden, Gibbons and Litzenberger (1989) for details.

Given the contrasting point estimates of the factor premia from the two sets of data, we examine how the model performs when the 55 portfolio returns are combined. The results are reported in Table 6. First, the estimated market risk premium is quite similar across the three models and the two subperiods at around $0.6-0.7\%$ per month which is close to the sample mean excess return on the market portfolio. For the 3-factor model, the risk premia associated with SMB and HML are close to zero and insignificant: for SMB the point estimate is around 25 basis points per year, and for HML around 144 basis points, in both cases well below the sample mean returns on the corresponding portfolios. On the other hand, both ICAPM factor risk premia are highly significant in the full sample and, though not significant, have the same sign in both sub-periods. All three models are rejected at better than the 1% level for the three sample periods. The GMM/DF tests yield similar rejections except that the ICAPM is not rejected in the first subperiod.

The fact that the simple ICAPM is rejected is not entirely surprising in view of the evidence of model mis-specification that we reported in Section 4.1: since r and η do not follow a joint Markov process there must be additional state variables in the stochastic process for these two variables that describe the instantaneous investment opportunity set, and in general we should expect them to be priced. Moreover, in specifying the empirical version of the simple ICAPM we have treated the stock market index as the market portfolio and the resulting mismeasurement of the return on aggregate wealth could also account for the model rejection.

5 Conclusion

In this paper we have developed, estimated, and tested a simple model of asset valuation for a setting in which real interest rates and risk premia vary stochastically. The model implies that zero-coupon nominal bond yields are linearly related to the state variables r and η, the real interest rate and the maximal Sharpe ratio, as well as

to the expected rate of inflation, π. Data on bond yields and expected inflation were used to provide estimates of the state variables and of the parameters of their joint stochastic process. The estimated real interest rate and Sharpe ratio both show strong business cycle related variation: the Sharpe ratio rising, and the real interest rate falling, during recessions. The Sharpe ratio estimate was shown to be related to the excess return on the equity market portfolio, consistent with the model predictions. However, there was evidence of model mis-specification in that the filtered series for r and η did not possess the hypothesized Markov property.

The model was tested first on the 25 size and book-to-market portfolios. The estimated risk premia for both r and η were significant, and the model pricing restrictions were not rejected for the whole sample period, although they were rejected for one half of the period. In contrast, both the CAPM and the Fama-French 3-factor model were rejected for the whole sample period and the two subperiods.

Lo and Mackinlay (1990) have warned against testing asset pricing models on the returns of portfolios that have been formed on the basis of some characteristic which is known to be associated with returns. Since the size and book-to-market portfolios seem to meet this criterion, we also tested the model using the returns on 30 industry portfolios. None of the three models was rejected using these returns. However, the point estimates of the risk premia associated with r and η and with SMB and HML were quite different from those obtained using the 25 portfolios.

Finally, the models were estimated and tested using the combined sample of 55 portfolios. With these data the market risk premium was significant for all three models; and, while neither of the premia associated with the Fama-French arbitrage portfolios were significant, the premia associated with both r and η were highly significant. Despite this, the pricing restrictions of all three models were rejected. It is hypothesized that the rejection of the simple ICAPM could be due either to mismeasurement of the return on aggregate wealth or to the failure of the assumption that

the two investment opportunity set state variables follow a Markov process. Nevertheless, these results with a highly simplified ICAPM are sufficiently encouraging to warrant further empirical investigation of the ICAPM. We stress the need in empirical implementation of the ICAPM to pay careful attention to the selection of state variables: the ICAPM is not just another "factor model"; the state variables of the model must be limited to those that predict future investment opportunities.

Appendix

A. Proof of Theorems 1 and 2

The real part of the economy is described by the processes for the real pricing kernel, the real interest rate, and the maximum Sharpe ratio (5.1)-(5.3), while the nominal part of the economy is described by the processes for the price level and the expected inflation rate (11)-(12). Under the risk neutral probability measure Q, we can write these processes as:

$$dr = \kappa_r(\bar{r} - r)dt - \sigma_r \rho_{mr} \eta dt + \sigma_r dz_r^Q \tag{A1}$$

$$d\pi = \kappa_\pi(\bar{\pi} - \pi)dt - \sigma_\pi \rho_{m\pi} \eta dt + \sigma_\pi dz_\pi^Q \tag{A2}$$

$$d\eta = \kappa_\eta^*(\bar{\eta}^* - \eta)dt + \sigma_\eta dz_\eta^Q \tag{A3}$$

where $\kappa_\eta^* = \kappa_\eta + \sigma_\eta \rho_{m\eta}$ and $\bar{\eta}^* = \frac{\kappa_\eta \bar{\eta}}{\kappa_\eta^*}$.

Let y, whose stochastic process is given by (7), denote the expectation of a nominal cash flow at a future date T, X_T. The process for $\xi \equiv y/P$, the deflated expectation of the nominal cash flow, under the risk neutral probability measure can be written as:

$$\frac{d\xi}{\xi} = \left[-\pi - \sigma_y \sigma_P \rho_{yP} + \sigma_P^2 - \eta(\sigma_y \rho_{ym} - \sigma_P \rho_{Pm})\right] dt + \sigma_y dz_y^Q - \sigma_P dz_P^Q. \tag{A4}$$

The real value at time t of the claim to the nominal cash flow at time T, X_T, is given by expected discounted value of the real cash flow under Q:

$$\begin{aligned} V(\xi, r, \pi, \eta, T-t) &= \mathrm{E}_t^Q \left[\frac{X_T}{P_T} \exp^{-\int_t^T r(s)ds}\right] = \mathrm{E}_t^Q \left[\frac{y_T}{P_T} \exp^{-\int_t^T r(s)ds}\right] \\ &= \mathrm{E}_t^Q \left[\xi_T \exp^{-\int_t^T r(s)ds}\right] \end{aligned} \tag{A5}$$

Using equation (A4), we have

$$\begin{aligned}\xi_T &= \xi_t \exp\left\{\left(-\frac{1}{2}\sigma_y^2+\frac{1}{2}\sigma_P^2\right)(T-t)-(\sigma_y\rho_{ym}-\sigma_P\rho_{Pm})\int_t^T \eta(s)ds\right.\\ &- \left.\int_t^T \pi(s)ds+\sigma_y\int_t^T dz_y^Q-\sigma_P\int_t^T dz_P^Q\right\}. \end{aligned} \tag{A6}$$

Tedious calculations from equations (A1), (A2), and (A3) easily lead to results for $\int_t^T \eta(s)ds$, $\int_t^T \pi(s)ds$, and $\int_t^T r(s)ds$. Substituting the expressions for $\int_t^T \eta(s)ds$, $\int_t^T \pi(s)ds$, and $\int_t^T r(s)ds$ into equation (A5) yields

$$V(\xi, r, \pi, \eta, T-t) = \xi_t G \mathrm{E}_t^Q\left[\exp^{\varphi}\right], \tag{A7}$$

where G is given by

$$G = \exp\left\{E(\tau)-B(\tau)r_t-C(\tau)\pi_t-D(\tau)\eta_t\right\} \tag{A8}$$

and

$$B(\tau) = \frac{1-e^{-\kappa_r(T-t)}}{\kappa_r} \tag{A9}$$

$$C(\tau) = \frac{1-e^{-\kappa_\pi(T-t)}}{\kappa_\pi} \tag{A10}$$

$$D(\tau) = d_1+d_2e^{-\kappa_\eta^*\tau}+d_3e^{-\kappa_r\tau}+d_4e^{-\kappa_\pi\tau} \tag{A11}$$

$$\tag{A12}$$

with

$$d_1 = -\frac{\sigma_P\rho_{mP}-\sigma_y\rho_{my}}{\kappa_\eta^*}-\frac{\sigma_r\rho_{mr}}{\kappa_r\ \kappa_\eta^*}-\frac{\sigma_\pi\rho_{m\pi}}{\kappa_\pi\kappa_\eta^*} \tag{A13}$$

$$\begin{aligned} d_2 &= -\frac{\sigma_y\rho_{my}}{\kappa_\eta^*}-\frac{\sigma_r\rho_{mr}}{(\kappa_\eta^*-\kappa_r)\kappa_\eta^*}-\frac{\sigma_\pi\rho_{m\pi}}{(\kappa_\eta^*-\kappa_\pi)\kappa_\eta^*} \\ &= -d_1-d_3-d_4 \end{aligned} \tag{A14}$$

$$d_3 = \frac{\sigma_r\rho_{mr}}{(\kappa_\eta^*-\kappa_r)\kappa_r} \tag{A15}$$

$$d_4 = \frac{\sigma_\pi\rho_{m\pi}}{(\kappa_\eta^*-\kappa_\pi)\kappa_\pi} \tag{A16}$$

The stochastic variable φ is a linear function of the Brownian motions:

$$\begin{aligned}\varphi &= \sigma_\eta \int_t^T \left[d_2\left(1-e^{-\kappa_\eta^*(T-s)}\right)+d_3\left(1-e^{-\kappa_r(T-s)}\right)+d_4\left(1-e^{-\kappa_\pi(T-s)}\right)\right] dz_\eta^*(s) \\ &- \frac{\sigma_r}{\kappa_r}\int_t^T \left(1-e^{-\kappa_r(T-s)}\right) dz_r^*(s) - \frac{\sigma_\pi}{\kappa_\pi}\int_t^T \left(1-e^{-\kappa_\pi(T-s)}\right) dz_\pi^*(s) \\ &+ \sigma_y \int_t^T dz_y^*(s) - \sigma_P \int_t^T dz_P^*(s). \end{aligned} \tag{A17}$$

Since φ is normally distributed with mean zero, V is given by

$$V(\xi, r, \pi, \eta, T-t) = \xi_t G_1 \exp\left\{\frac{1}{2}\mathrm{Var}_t(\varphi)\right\} \tag{A18}$$

Calculating $\mathrm{Var}_t(\varphi)$ and collecting terms, we get that

$$V(\xi, r, \pi, \eta, T-t) = \xi_t \exp\left\{A(\tau) - B(\tau)r_t - C(\tau)\pi_t - D(\tau)\eta_t\right\} \tag{A19}$$

where

$$\begin{aligned}A(\tau) &= a_1\tau + a_2\frac{1-e^{-\kappa_r\tau}}{\kappa_r} + a_3\frac{1-e^{-\kappa_\pi\tau}}{\kappa_\pi} + a_4\frac{1-e^{-\kappa_\eta^*\tau}}{\kappa_\eta^*} \\ &+ a_5\frac{1-e^{-2\kappa_r\tau}}{2\kappa_r} + a_6\frac{1-e^{-2\kappa_\pi\tau}}{2\kappa_\pi} + a_7\frac{1-e^{-2\kappa_\eta^*\tau}}{2\kappa_\eta^*} \\ &+ a_8\frac{1-e^{-(\kappa_\eta^*+\kappa_r)\tau}}{\kappa_\eta^*+\kappa_r} + a_9\frac{1-e^{-(\kappa_\eta^*+\kappa_\pi)\tau}}{\kappa_\eta^*+\kappa_\pi} + a_{10}\frac{1-e^{-(\kappa_r+\kappa_\pi)\tau}}{\kappa_r+\kappa_\pi}. \end{aligned} \tag{A20}$$

Define $a_0 \equiv \frac{\sigma_{r\eta}}{\kappa_r} + \frac{\sigma_{\pi\eta}}{\kappa_\pi} + \sigma_{P\eta} - \sigma_{y\eta} - \kappa_\eta^*\bar{\eta}^*$, $\bar{r}^* \equiv \bar{r} - \frac{\sigma_{Pr}-\sigma_{yr}}{\kappa_r}$, and $\bar{\pi}^* \equiv \bar{\pi} - \frac{\sigma_{P\pi}-\sigma_{y\pi}}{\kappa_\pi}$,

then $a_1, \ldots, a_{10}$ are expressed as

$$
\begin{aligned}
a_1 &= \sigma_P^2 - \sigma_{yP} + \frac{\sigma_r^2}{2\kappa_r^2} + \frac{\sigma_\pi^2}{2\kappa_\pi^2} + \frac{\sigma_{r\pi}}{\kappa_r\kappa_\pi} + \frac{\sigma_\eta^2}{2}d_1^2 - \bar{r}^* - \bar{\pi}^* + a_0 d_1 && \text{(A21)}\\
a_2 &= \bar{r}^* - \frac{\sigma_r^2}{\kappa_r^2} - \frac{\sigma_{r\pi}}{\kappa_r\kappa_\pi} - \frac{\sigma_{r\eta}}{\kappa_r}d_1 + a_0 d_3 + \sigma_\eta^2 d_1 d_3 && \text{(A22)}\\
a_3 &= \bar{\pi}^* - \frac{\sigma_\pi^2}{\kappa_\pi^2} - \frac{\sigma_{r\pi}}{\kappa_r\kappa_\pi} - \frac{\sigma_{\pi\eta}}{\kappa_\pi}d_1 + a_0 d_4 + \sigma_\eta^2 d_1 d_4 && \text{(A23)}\\
a_4 &= a_0 d_2 + \sigma_\eta^2 d_1 d_2 && \text{(A24)}\\
a_5 &= \frac{\sigma_r^2}{2\kappa_r^2} + \frac{\sigma_\eta^2}{2}d_3^2 - \frac{\sigma_{r\eta}}{\kappa_r}d_3 && \text{(A25)}\\
a_6 &= \frac{\sigma_\pi^2}{2\kappa_\pi^2} + \frac{\sigma_\eta^2}{2}d_4^2 - \frac{\sigma_{\pi\eta}}{\kappa_\pi}d_4 && \text{(A26)}\\
a_7 &= \frac{\sigma_\eta^2}{2}d_2^2 && \text{(A27)}\\
a_8 &= -\frac{\sigma_{r\eta}}{\kappa_r}d_2 + \sigma_\eta^2 d_2 d_3 && \text{(A28)}\\
a_9 &= -\frac{\sigma_{\pi\eta}}{\kappa_\pi}d_2 + \sigma_\eta^2 d_2 d_4 && \text{(A29)}\\
a_{10} &= \frac{\sigma_{r\pi}}{\kappa_r\kappa_\pi} - \frac{\sigma_{\pi\eta}}{\kappa_\pi}d_3 - \frac{\sigma_{r\eta}}{\kappa_r}d_4 + \sigma_\eta^2 d_3 d_4 && \text{(A30)}
\end{aligned}
$$

Theorems 1 and 2 follow as special cases of equation (A19). Theorem 1 is obtained by setting σ_P and the parameters in the expected inflation process (A2) to zero. Theorem 2 is obtained by setting σ_y to zero.

B. Details of Kalman Filter

The yield-based estimates of the state variable dynamics are derived by applying a Kalman filter to data on bond yields and inflation using equation (15). The transition equations for the state variables, r, π and η are derived by discretizing equations (5.2), (5.3), and (12):

$$\begin{pmatrix} r_t \\ \pi_t \\ \eta_t \end{pmatrix} = \begin{pmatrix} e^{-\kappa_r \Delta t} & 0 & 0 \\ 0 & e^{-\kappa_\pi \Delta t} & 0 \\ 0 & 0 & e^{-\kappa_\eta \Delta t} \end{pmatrix} \begin{pmatrix} r_{t-\Delta t} \\ \pi_{t-\Delta t} \\ \eta_{t-\Delta t} \end{pmatrix} + \begin{pmatrix} \bar{r}\left[1 - e^{-\kappa_r \Delta t}\right] \\ \bar{\pi}\left[1 - e^{-\kappa_\pi \Delta t}\right] \\ \bar{\eta}\left[1 - e^{-\kappa_\eta \Delta t}\right] \end{pmatrix} + \begin{pmatrix} \epsilon_r(t) \\ \epsilon_\pi(t) \\ \epsilon_\eta(t) \end{pmatrix} \tag{B1}$$

where the vector of innovations is related to the standard Brownian motions, dz_r, dz_π and dz_η, by

$$\begin{pmatrix} \epsilon_r(t) \\ \epsilon_\pi(t) \\ \epsilon_\eta(t) \end{pmatrix} = \begin{pmatrix} \sigma_r e^{-\kappa_r t} \int_{t-\Delta t}^{t} e^{\kappa_r \tau} dz_r(\tau) \\ \sigma_\pi e^{-\kappa_\pi t} \int_{t-\Delta t}^{t} e^{\kappa_\pi \tau} dz_\pi(\tau) \\ \sigma_\eta e^{-\kappa_\eta t} \int_{t-\Delta t}^{t} e^{\kappa_\eta \tau} dz_\eta(\tau) \end{pmatrix}, \tag{B2}$$

and the variance-covariance matrix of the innovations is

$$Q = \begin{pmatrix} \frac{\sigma_r^2}{2\kappa_r}\left[1 - e^{-2\kappa_r \Delta t}\right] & \frac{\sigma_r \sigma_\pi \rho_{r\pi}}{\kappa_r + \kappa_\pi}\left[1 - e^{-(\kappa_r + \kappa_\pi)\Delta t}\right] & \frac{\sigma_r \sigma_\eta \rho_{r\eta}}{\kappa_r + \kappa_\eta}\left[1 - e^{-(\kappa_r + \kappa_\eta)\Delta t}\right] \\ \frac{\sigma_r \sigma_\pi \rho_{r\pi}}{\kappa_r + \kappa_\pi}\left[1 - e^{-(\kappa_r + \kappa_\pi)\Delta t}\right] & \frac{\sigma_\pi^2}{2\kappa_\pi}\left[1 - e^{-2\kappa_\pi \Delta t}\right] & \frac{\sigma_\pi \sigma_\eta \rho_{\pi\eta}}{\kappa_\pi + \kappa_\eta}\left[1 - e^{-(\kappa_\pi + \kappa_\eta)\Delta t}\right] \\ \frac{\sigma_r \sigma_\eta \rho_{r\eta}}{\kappa_r + \kappa_\eta}\left[1 - e^{-(\kappa_r + \kappa_\eta)\Delta t}\right] & \frac{\sigma_\pi \sigma_\eta \rho_{\pi\eta}}{\kappa_\pi + \kappa_\eta}\left[1 - e^{-(\kappa_\pi + \kappa_\eta)\Delta t}\right] & \frac{\sigma_\eta^2}{2\kappa_\eta}\left[1 - e^{-2\kappa_\eta \Delta t}\right] \end{pmatrix}. \tag{B3}$$

The first n observation equations assume that the observed yields at time t, $y_{\tau_j,t}$, on bonds with maturities τ_j, $j = 1, \cdots, n$, are given by equation (15) plus a measurement error terms, ϵ_{τ_j}:

$$y_{\tau_j,t} \equiv -\frac{\ln V(t, t+\tau_j)}{\tau_j} = -\frac{A(t,\tau_j)}{\tau_j} + \frac{B(\tau_j)}{\tau_j} r_t + \frac{C(\tau_j)}{\tau_j} \pi_t + \frac{D(\tau_j)}{\tau_j} \eta + \epsilon_{\tau_j}(t). \tag{B4}$$

The measurement errors, $\epsilon_{\tau_j}(t)$, are assumed to be serially and cross-sectionally uncorrelated and are uncorrelated with the innovations in the transition equations. The $n+1$ observation equation uses the Livingston estimate of the rate of inflation:

$$\pi_{\text{liv}} = \pi + \epsilon_{\text{liv}}. \tag{B5}$$

where ϵ_{liv} is assumed to be uncorrelated with the yield measurement errors and the innovations in the transition equation.

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Table 1

Summary Statistics on Bond Yields and Model Parameter Estimates

Panel A of the table reports summary statistics for the bond yield data. The bond data are monthly observations on constant maturity zero coupon U.S. Treasury yields for the period from January 1952 to December 2000. Panel B reports the summary statistics for the inflation data. CPI inflation is calculated using CPI data for the same sample period while the Livingston inflation is calculated using the Livingston survey data. Panel C reports estimates of the parameters of the stochastic process of the investment opportunity set, equations (5.2) to (5.3), obtained from a Kalman filter applied to the inflation and bond yield data with $\bar{r} = 1.62\%$, $\bar{\pi} = 3.85\%$ and $\bar{\eta} = 0.7$, where $\bar{r}$ and $\bar{\pi}$ are the sample means, and $\bar{\eta}$ is 20% higher than the CRSP value weighted market index Sharpe ratio. η is the Sharpe ratio. r is the real interest rate. m is the pricing kernel, π is the expected rate of inflation, and P is the price level. Asymptotic t-ratios are in parentheses.

A. Bond Yields (% per year)

Bond Maturity (years)	0.25	0.5	1	2	3	4	5	10
Mean	5.47	5.70	5.90	6.12	6.26	6.36	6.44	6.64
Std. Dev.	2.86	2.90	2.88	2.83	2.79	2.77	2.76	2.72

B. Inflation (% per year)

	CPI Inflation	Livingston Inflation
Mean	3.85%	3.15%
Std. Dev.	1.16%	0.68%

C. Parameter Estimates

	σ_b	σ_r	σ_π	σ_η	κ_r	κ_π	κ_η
Estimate	0.16%	1.11%	0.75%	30.79%	0.074	0.000	0.047
t-ratio	(86.75)	(11.76)	(12.58)	(8.52)	(14.15)	(0.75)	(5.44)
	$\rho_{r\pi}$	$\rho_{r\eta}$	ρ_{rm}	$\rho_{\pi\eta}$	$\rho_{\pi m}$	$\rho_{\eta m}$	ρ_{Pm}
Estimate	0.114	-0.528	-0.852	-0.283	-0.219	0.962	0.383
t-ratio	(1.30)	(6.27)	(11.21)	(3.52)	(1.05)	(4.98)	(1.89)

	$\bar{r}$	$\bar{\pi}$	$\bar{\eta}$	σ_P	σ_{liv}	ML
Pre-set Value	1.62%	3.85%	0.700	1.16%	0.35%	30,962.63

Table 2

Joint Test of ICAPM, FF and CAPM for 25 Size and Book-to-Market Sorted Portfolios: $\overline{\mathbf{R} - R_f} = \hat{\beta}\lambda + \epsilon$

This table reports the two-stage cross-sectional regression results under the ICAPM. In the first stage, the excess returns on 25 size and book-to-market sorted portfolios are regressed on the market risk premium, $R_M - R_f$, and the innovations in the state variables, Δr and $\Delta\eta$ to obtain estimates of the loadings on the three factors, $\hat{\beta}$. In the second stage, the sample mean of the excess returns is regressed on the beta without the intercept: $\overline{\mathbf{R} - R_f} = \hat{\beta}\lambda + \epsilon$, and then the mispricing is calculated as $\hat{\alpha} = \overline{\mathbf{R} - R_f} - \hat{\beta}\hat{\lambda}$. A χ^2 test is then formed on $\hat{\alpha}$ to test the joint significance of the mispricing. The individual t-ratios and the variance covariance matrix, $\mathbf{\Sigma}$, are calculated using Shanken's adjustment. The sample period is from January 1952 to December 2000.

Panel A: January 1952 to December 2000

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_M	λ_r	λ_η	λ_M	λ_{SMB}	λ_{HML}	λ_M	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.56%	-0.43%	13.59%	0.58%	0.08%	0.40%	0.70%	0.62%	0.10%	0.36%
t-ratios	3.11	2.39	3.44	3.28	0.62	3.48	3.75	3.55	0.82	3.26
$\alpha'\mathbf{\Sigma}^{-1}\alpha$		27.31			70.04		94.43			
$p-$value		19.97%			$< 1\%$		$< 1\%$			

Panel B: January 1952 to June 1976

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_M	λ_r	λ_η	λ_M	λ_{SMB}	λ_{HML}	λ_M	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.43%	-0.15%	7.53%	0.53%	0.03%	0.44%	0.62%	0.55%	0.06%	0.39%
t-ratios	1.75	1.43	2.67	2.23	0.21	3.23	2.41	2.32	0.72	4.68
$\alpha'\mathbf{\Sigma}^{-1}\alpha$		28.83			46.16		57.39			
$p-$value		14.98%			$< 1\%$		$< 1\%$			

Panel C: June 1976 to December 2000

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_M	λ_r	λ_η	λ_M	λ_{SMB}	λ_{HML}	λ_M	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.69%	-0.43%	15.93%	0.64%	0.13%	0.34%	0.78%	0.68%	0.14%	0.34%
t-ratios	2.60	2.17	2.57	2.47	0.68	1.84	2.87	2.65	0.74	1.90
$\alpha'\mathbf{\Sigma}^{-1}\alpha$		41.94			89.10		108.95			
$p-$value		$< 1\%$			$< 1\%$		$< 1\%$			

Table 3

Mispricing under ICAPM, FF and CAPM for 25 Size and Book-to-Market Sorted Portfolios: January 1952 to December 2000

This table reports the model mispricing, $\hat{\alpha}_i$, in the two-stage cross-sectional regression for 25 Size and Book-to-Market Sorted Portfolios. For the ICAPM $\hat{\alpha}_i \equiv \overline{R_i - R_f} - \left(\hat{\beta}_{iM}\hat{\lambda}_M + \hat{\beta}_{ir}\hat{\lambda}_r + \hat{\beta}_{i\eta}\hat{\lambda}_\eta\right)$. For the 3-factor model $\hat{\alpha}_i \equiv \overline{R_i - R_f} - \left(\hat{\beta}_{iM}\hat{\lambda}_M + \hat{\beta}_{iSMB}\hat{\lambda}_{SMB} + \hat{\beta}_{iHML}\hat{\lambda}_{HML}\right)$. For the CAPM $\hat{\alpha}_i \equiv \overline{R_i - R_f} - \left(\hat{\beta}_{iM}\hat{\lambda}_M\right)$. The sample period is from January 1952 to December 2000.

	Book-to-Market					Book-to-Market				
	Low	2	3	4	High	Low	2	3	4	High
Size	$\alpha(\%)$					$t(\alpha)$				
ICAPM										
small	-0.44	-0.20	-0.17	0.07	0.16	1.67	1.25	1.28	0.84	1.85
2	-0.25	-0.25	0.07	0.02	0.16	1.62	2.00	0.74	0.23	1.51
3	-0.01	-0.09	0.04	0.16	0.03	0.08	0.82	0.41	1.74	0.25
4	-0.11	-0.18	0.19	0.13	0.09	1.02	2.13	1.91	0.98	0.53
big	0.28	0.04	0.17	0.13	0.28	2.25	0.37	1.80	0.79	1.35
Fama-French 3-Factor Model										
small	-0.25	-0.10	-0.08	0.16	0.18	1.69	0.88	0.91	2.56	2.46
2	-0.14	-0.04	0.17	0.03	0.15	1.48	0.47	2.32	0.43	1.85
3	0.05	0.07	0.04	0.17	-0.12	0.63	0.99	0.46	2.42	1.52
4	0.03	-0.15	0.17	0.06	-0.07	0.48	1.97	2.43	0.72	0.66
big	0.16	-0.08	0.08	-0.17	-0.25	1.75	1.16	0.91	2.03	2.15
CAPM										
small	-0.45	-0.17	-0.07	0.23	0.35	2.20	1.19	0.57	2.28	2.77
2	-0.49	-0.16	0.15	0.14	0.34	3.45	1.67	1.87	1.84	3.28
3	-0.29	-0.02	0.05	0.31	0.12	2.37	0.28	0.67	3.81	1.12
4	-0.27	-0.21	0.24	0.22	0.18	2.18	2.25	2.60	2.20	1.34
big	-0.10	-0.11	0.16	0.07	0.09	0.61	0.85	1.19	0.47	0.58

Table 4

Joint Test of ICAPM, FF and CAPM for 30 Industrial Portfolios: $\overline{\mathbf{R} - R_f} = \hat{\beta}\lambda + \epsilon$

This table reports the two-stage cross-sectional regression results under the ICAPM. In the first stage, the excess returns on 30 industrial portfolios are regressed on the market risk premium, $R_M - R_f$, and the innovations in the state variables, Δr and $\Delta\eta$ to obtain estimates of the loadings on the three factors, $\hat{\beta}$. In the second stage, the sample mean of the excess returns is regressed on the beta without the intercept: $\overline{\mathbf{R} - R_f} = \hat{\beta}\lambda + \epsilon$, and then the mispricing is calculated as $\hat{\alpha} = \overline{\mathbf{R} - R_f} - \hat{\beta}\hat{\lambda}$. A χ^2 test is then formed on $\hat{\alpha}$ to test the joint significance of the mispricing. The t-ratios and the variance covariance matrix, $\mathbf{\Sigma}$, are calculated using Shanken's adjustment. Results for the whole sample period of from January 1952 to December 2000 and two equal sub samples are reported.

Panel A: January 1952 to December 2000

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_M	λ_r	λ_η	λ_M	λ_{SMB}	λ_{HML}	λ_M	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.69%	-0.08%	-4.01%	0.75%	-0.30%	-0.38%	0.64%	0.62%	0.10%	0.36%
t-ratios	3.81	1.09	1.45	4.18	1.82	2.51	3.52	3.55	0.82	3.26
$\alpha^{'}\mathbf{\Sigma}^{-1}\alpha$		18.73			18.80		25.59			
$p-$value		87.95%			87.71%		64.72%			

Panel B: January 1952 to June 1976

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_M	λ_r	λ_η	λ_M	λ_{SMB}	λ_{HML}	λ_M	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.65%	0.04%	-2.24%	0.69%	-0.08%	-0.25%	0.62%	0.55%	0.06%	0.39%
t-ratios	2.69	0.61	1.12	2.88	0.44	1.35	2.52	2.32	0.72	4.68
$\alpha^{'}\mathbf{\Sigma}^{-1}\alpha$		20.91			18.80		22.54			
$p-$value		79.04%			87.71%		79.73%			

Panel C: July 1976 to December 2000

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_M	λ_r	λ_η	λ_M	λ_{SMB}	λ_{HML}	λ_M	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.71%	-0.09%	-7.82%	0.79%	-0.86%	-0.49%	0.66%	0.68%	0.14%	0.34%
t-ratios	2.60	0.76	1.75	3.01	2.85	2.08	2.46	2.65	0.74	1.90
$\alpha^{'}\mathbf{\Sigma}^{-1}\alpha$		19.18			25.15		32.99			
$p-$value		86.33%			56.59%		27.82%			

Table 5

Mispricing under ICAPM, FF and CAPM for 30 Industrial Portfolios: January 1952 to December 2000

This table reports the model mispricing, $\hat{\alpha}_i$, in the two-stage cross-sectional regression for 30 industrial portfolios. For the ICAPM $\hat{\alpha}_i \equiv \overline{R_i - R_f} - \left(\hat{\beta}_{iM}\hat{\lambda}_M + \hat{\beta}_{ir}\hat{\lambda}_r + \hat{\beta}_{i\eta}\hat{\lambda}_\eta\right)$. For the 3-factor model $\hat{\alpha}_i \equiv \overline{R_i - R_f} - \left(\hat{\beta}_{iM}\hat{\lambda}_M + \hat{\beta}_{iSMB}\hat{\lambda}_{SMB} + \hat{\beta}_{iHML}\hat{\lambda}_{HML}\right)$. For the CAPM $\hat{\alpha}_i \equiv \overline{R_i - R_f} - \left(\hat{\beta}_{iM}\hat{\lambda}_M\right)$. The sample period is from January 1952 to December 2000.

Industry	ICAPM		FF		CAPM	
	$\alpha(\%)$	$t(\alpha)$	$\alpha(\%)$	$t(\alpha)$	$\alpha(\%)$	$t(\alpha)$
Food	0.18	1.74	0.14	1.37	0.26	2.23
Beer	0.20	1.26	0.09	0.63	0.12	0.83
Smoke	0.30	1.55	0.42	2.25	0.52	2.63
Games	0.07	0.46	0.07	0.52	-0.01	0.03
Books	0.10	0.78	0.12	1.08	0.10	0.94
Hshld	0.11	0.97	-0.07	0.80	0.15	1.41
Clths	-0.19	1.15	-0.01	0.04	-0.20	1.43
Hlth	0.12	0.83	0.02	0.14	0.28	2.01
Chems	-0.11	0.86	-0.22	2.17	-0.11	1.05
Txtls	0.04	0.29	0.10	0.81	-0.19	1.36
Cnstr	-0.21	2.20	-0.11	1.39	-0.15	1.83
Steel	-0.10	0.77	-0.21	1.51	-0.31	2.17
FabPr	-0.17	1.68	-0.17	1.84	-0.18	1.97
ElcEq	0.09	0.56	-0.03	0.24	0.02	0.15
Autos	0.02	0.13	0.00	0.01	-0.01	0.07
Carry	0.01	0.09	0.16	1.09	0.11	0.75
Mines	-0.01	0.07	-0.11	0.57	-0.25	1.25
Coal	0.03	0.14	0.01	0.06	-0.08	0.35
Oil	0.07	0.39	-0.01	0.09	0.15	0.91
Util	-0.04	0.41	0.15	1.37	0.19	1.50
Telcm	0.03	0.20	0.03	0.25	0.15	1.09
Servs	0.06	0.37	0.05	0.50	0.05	0.34
BusEq	0.21	1.44	-0.04	0.34	0.12	0.81
paper	-0.11	0.96	-0.15	1.55	-0.04	0.43
Trans	-0.17	1.29	-0.08	0.66	-0.17	1.44
Whlsl	-0.09	0.81	0.05	0.55	-0.07	0.70
Rtail	0.10	0.72	0.01	0.11	0.07	0.63
Meals	0.05	0.31	0.15	1.07	0.05	0.37
Fin	-0.16	1.72	0.00	0.01	0.06	0.64
Other	-0.25	2.05	-0.14	1.21	-0.19	1.68

Table 6

Joint Test of ICAPM, FF and CAPM using 25 size and B/M sorted and 30 Industrial Portfolios: $\overline{\mathbf{R} - R_f} = \hat{\beta}\lambda + \epsilon$

This table reports the two-stage cross-sectional regression results under the ICAPM. In the first stage, the excess returns on 55 portfolios are regressed on the market risk premium, $R_M - R_f$, and the innovations in the state variables, Δr and $\Delta\eta$ to obtain estimates of the loadings on the three factors, $\hat{\beta}$. In the second stage, the sample mean of the excess returns is regressed on the beta without the intercept: $\overline{\mathbf{R} - R_f} = \hat{\beta}\lambda + \epsilon$, and then the mispricing is calculated as $\hat{\alpha} = \overline{\mathbf{R} - R_f} - \hat{\beta}\hat{\lambda}$. A χ^2 test is then formed on $\hat{\alpha}$ to test the joint significance of the mispricing. The t-ratios and the variance covariance matrix, $\mathbf{\Sigma}$, are calculated using Shanken's adjustment. Results for the whole sample period of from January 1952 to December 2000 and two equal sub samples are reported.

Panel A: January 1952 to December 2000

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_{mkt}	λ_r	λ_η	λ_{mkt}	λ_{SMB}	λ_{HML}	λ_{mkt}	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.64%	-0.25%	5.81%	0.65%	0.02%	0.12%	0.67%	0.62%	0.10%	0.36%
t-ratios	3.54	3.23	2.31	3.63	0.17	1.03	3.67	3.55	0.82	3.26
$\alpha'\mathbf{\Sigma}^{-1}\alpha$		116.39			161.95		186.09			
$p-$value		$< 1\%$			$< 1\%$		$< 1\%$			

Panel B: January 1952 to June 1976

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_{mkt}	λ_r	λ_η	λ_{mkt}	λ_{SMB}	λ_{HML}	λ_{mkt}	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.59%	-0.02%	1.36%	0.65%	-0.01%	0.15%	0.61%	0.55%	0.06%	0.39%
t-ratios	2.46	0.33	0.66	2.71	0.03	1.00	2.47	2.32	0.72	4.68
$\alpha'\mathbf{\Sigma}^{-1}\alpha$		110.37			109.74		120.91			
$p-$value		$< 1\%$			$< 1\%$		$< 1\%$			

Panel C: July 1976 to December 2000

	ICAPM			FF 3-factor Model			CAPM	Sample Mean		
	λ_{mkt}	λ_r	λ_η	λ_{mkt}	λ_{SMB}	λ_{HML}	λ_{mkt}	$\overline{R_M - R_f}$	$\overline{SMB}$	$\overline{HML}$
Estimate	0.74%	-0.22%	1.55%	0.69%	0.01%	0.02%	0.71%	0.68%	0.14%	0.34%
t-ratios	2.79	2.10	0.46	2.63	0.07	0.10	2.69	2.65	0.74	1.90
$\alpha'\mathbf{\Sigma}^{-1}\alpha$		180.21			209.66		229.59			
$p-$value		$< 1\%$			$< 1\%$		$< 1\%$			

Figure 1
Time Series of Real Interest Rate Estimates

The figure plots the estimated real interest rate series from January 1952 to December 2000, which is filtered out from the bond yield and inflation data. Shaded area indicates periods of U.S. recessions.

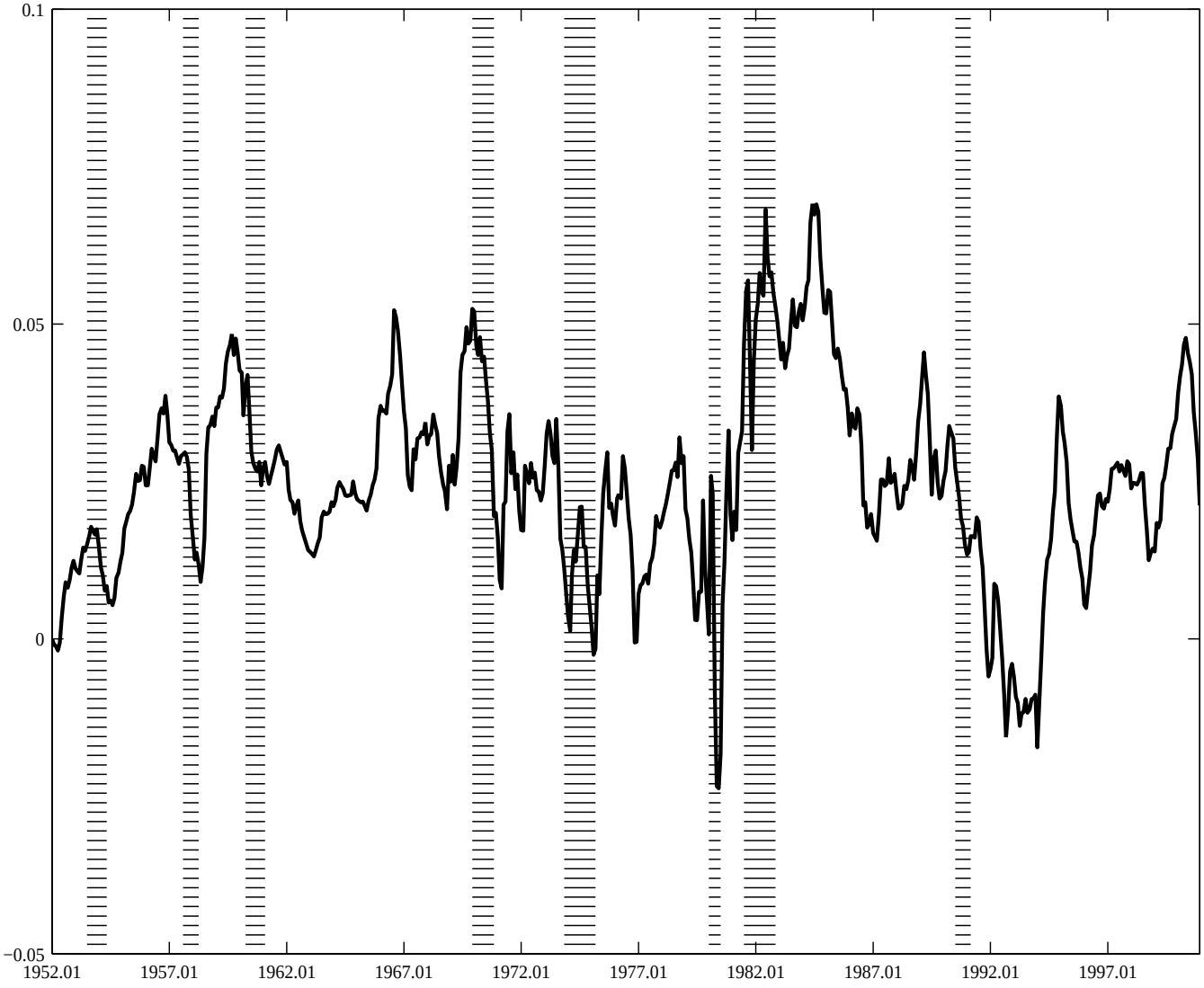

Figure 2
Time Series of Expected Inflation Rate Estimates and the Livingston Expected Inflation Rates

The figure plots the estimated expected inflation rates, π, against the expected inflation rate from the Livingston Survey, π_{liv}, from January 1952 to December 2000. The expected inflation rate is filtered out from the bond yield and Livingston inflation data. The Livingston Survey expected inflation rate, π_{liv}, is constructed from the economists' six-month ahead forecast of CPI level. The monthly data is constructed using linear interpolation. The data in the figure is in percent per month. Shaded area indicates periods of U.S. recessions.

Legend: Solid line - π; dash-dot line - π_{liv}

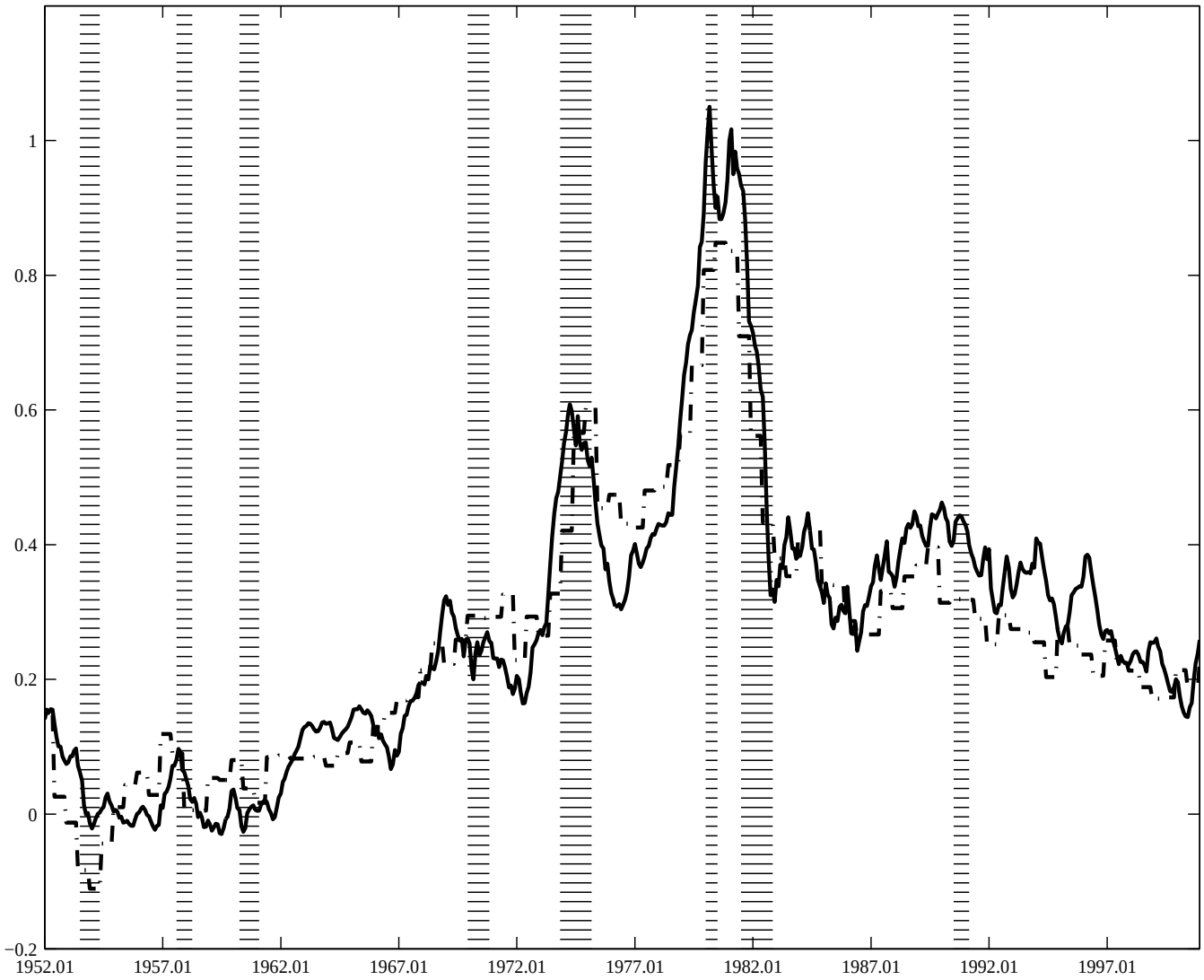

Figure 3
Time Series of Nominal Interest Rate Estimates and the Realized One-month T-bill Rate

The figure plots the estimated nominal interest rate series, $R \equiv r + \pi$, against the realized one-month T-bill rate, R_f, from January 1952 to December 2000. The real interest rate r and the expected inflation π are filtered out from the bond yield and inflation data. Shaded area indicates periods of U.S. recessions.

Legend: Solid line - $r + \pi$; dash-dot line - R_f

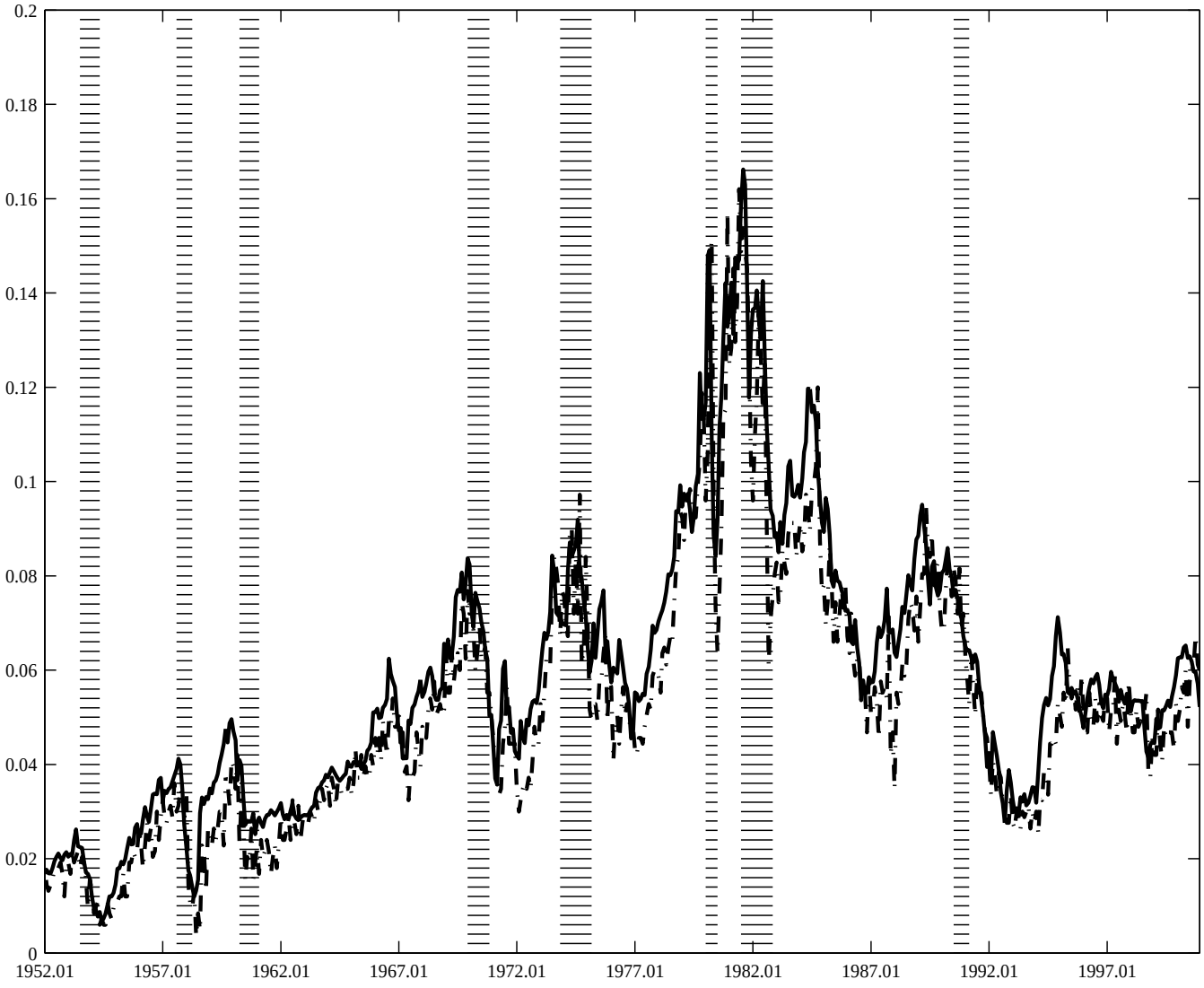

Figure 4
Time Series of Sharpe Ratio Estimates

The figure plots the estimated Sharpe ratio series from January 1952 to December 2000, which are filtered out from the bond yield and inflation data. Shaded area indicates periods of U.S. recessions.