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Covariance Risk, Mispricing, and the Cross Section of Security Returns

Kent D. Daniel*

David Hirshleifer**

Avanidhar Subrahmanyam***

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* Finance Department, Kellogg School, Northwestern University, Evanston, IL 60208-2006; (847) 491-4113; *kent@ kent.kellogg.nwu.edu*; This working paper is available at *http://kent.kellogg.nwu.edu/*

** Fisher College of Business, The Ohio State University, 740A Fisher Hall, 2100 Neil Avenue, Columbus, OH 43210-1144; (614) 292-5174; *hirshleifer.2@ osu.edu*

***Anderson Graduate School of Management, University of California at Los Angeles, Los Angeles, CA 90095-1481; (310) 825-5355; *subra@ anderson.ucla.edu*

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- *Abstract* -

This paper offers a multisecurity model in which prices reflect both covariance risk and misperceptions of firms' prospects, and in which arbitrageurs trade to profit from mispricing. We derive a pricing relationship in which expected returns are linearly related to both risk and mispricing variables. The model thereby implies a multivariate relation between expected return, beta, and variables that proxy for mispricing such as fundamental/price ratios or market value. With many securities, mispricing of idiosyncratic components of value tends to be arbitraged away but systematic mispricing is not. The theory is consistent with several empirical findings regarding the cross-section of equity returns, including: the observed ability of fundamental/price ratios to forecast aggregate and cross-sectional returns, and of market value but not non-market size measures to forecast returns cross-sectionally; and the ability in some studies of fundamental/price ratios and market value to dominate traditional measures of security risk. The model also offers several untested empirical implications for the cross-section of expected returns and for the relation of volume to subsequent volatility.

The classic theory of securities market equilibrium beginning with Sharpe, Lintner, and Black is based on the interaction of fully rational optimizing investors. In recent years, several important studies have explored alternatives to the premise of full rationality. One approach models market misvaluation as a consequence of noise or positive feedback trades (see, e.g., Black (1986), De Long, Shleifer, Summers, and Waldman (1990a, 1990b), Campbell and Kyle (1993)). Another branch of literature makes assumptions about imperfect rationality of individuals to derive trades and misvaluation.[1]

The above studies provide many valuable insights in settings with risk-neutral investors and/or a single risky security. In this study we build upon this literature to examine how the cross-section of expected security returns is related to *risk* and *investor misvaluation.* To address these issues we therefore examine a setting with both risk aversion and multiple securities.

Many empirical studies attempt to predict security stock returns using not just risk measures like CAPM beta, but also variables such as book/market that are open to multiple interpretations as either proxies for factor risk or for market misvaluation. The debate over empirical results has been pursued in the absence of an explicit model of the ability of different proxies to predict returns when there are both misvaluation effects and risk effects among a cross-section of securities.

Furthermore, it has often been argued that mispricing effects will tend to be 'arbitraged away' by smart traders. Such arbitrage strategies may include diversification by trading portfolios of mispriced securities, and hedging away of factor risk. The risk and profitability of multi-security arbitrage strategies, and the extent to which these do indeed eliminate mispricing are issues that have yet to be explored in the literature.

This paper offers a model in which investors who are both imperfectly rational and risk averse solve a multisecurity portfolio problem, and in which arbitrageurs can profit by trading against mispricing. Based on extensive psychological evidence[2], our premise is that investors are overconfident about their abilities, and hence overestimate the quality of information signals they have generated about security values. Other individuals exploit the pricing errors introduced by the trading of the informed overconfident individuals, but do not eliminate all mispricing because of risk aversion. The theory offers

[1]See Shiller (1981), De Long, Shleifer, Summers, and Waldmann (1991), Benartzi and Thaler (1995), Hirshleifer, Subrahmanyam, and Titman (1994), Kyle and Wang (1997), Odean (1998), Wang (1998), Benos (1998), Caballé and Sákovics (1996), Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999); see also Barberis, Huang, and Santos (1999) on time-inconsistent, loss-averse investor preferences.

[2]See for example, the discussions and references in DeBondt and Thaler (1995), Odean (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998).

an explanation for some known cross-sectional empirical patterns in the predictability of security returns, describes the effects of rational risk arbitrage activity upon factor and idiosyncratic of mispricing, and offers new empirical implications relating fundamental price ratios, volume, future returns, and future volatility.

Our analysis differs from previous models of investor overconfidence[3] in examining how covariance risk and misvaluation jointly determine the cross-section of expected security returns. Our specification of overconfidence is most similar to those of Kyle and Wang (1997), Odean (1998), and Daniel, Hirshleifer, and Subrahmanyam (1998). The latter paper assumed risk neutrality and a single risky security in order to examine the dynamics of confidence as a result of biased self-attribution, and the possibility of either over- or under-reaction.

In contrast, this paper examines only static overconfidence in a single period. We can thereby allow tractably for risk aversion, multiple risky securities, and can examine how prices balance the views of overconfident traders and arbitrageurs. Our focus is on providing a cross-sectional asset pricing model when there is long-run overreaction and correction; the analysis does not address the patterns of short-term versus long-term return autocorrelations studied in Daniel, Hirshleifer, and Subrahmanyam (1998), Barberis, Shleifer, and Vishny (1998) and Hong and Stein (1999).[4]

The model addresses a variety of cross-sectional empirical findings (see Appendix B), including: (1) the ability of fundamental/price ratios (dividend yield, earnings/price, and book/market) to predict cross-sectional differences in future returns, incrementally to, and in some tests dominating, the effect of market beta, (2) the ability of firm size to predict future returns when size is measured by market value, but not when measured by non-market proxies such as book value, (3) greater ability of book/market than firm size to predict future returns in both univariate and multivariate studies, and (4) the positive association between aggregate fundamental-scaled price measures and future aggregate stock market returns.

[3]See, for example, De Long, Shleifer, Summers, and Waldmann (1991), Hirshleifer, Subrahmanyam, and Titman (1994), Kyle and Wang (1997), Odean (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Gervais and Odean (1999), Benos (1998), Wang (1998), Caballé and Sákovics (1996), and Hirshleifer and Luo (1999).

[4]Put differently, we look at overreaction and its correction, but do not model extra dates in which overreaction can temporarily become more severe, and in which overreaction can be only partially corrected. Such a dynamic pattern can lead to short-term positive return autocorrelations ('momentum') as well as long term negative autocorrelation ('reversal'). Recently, Jegadeesh and Titman (1999) have provided evidence that momentum, though often interpreted as a simple underreaction, results from a process of overreaction and correction.

Some recent papers have attempted to explain these patterns with rational asset pricing models. The challenges faced by risk-based explanations are significant (see Appendix B for details). Within the standard asset-pricing framework, the high Sharpe ratios achieved by trading strategies based on these patterns would imply extreme variation in marginal utility, especially given that returns to such strategies seem to have low correlations with plausible risk factors. While we cannot rule out explanations based on risk or market imperfections, it is reasonable to consider alternative explanations based on imperfect rationality.

In the model, investors receive private information about, and misvalue, both systematic factors and firm-specific payoffs.[5] In equilibrium, expected security returns are linearly increasing in the beta of the security with an adjusted market portfolio, as perceived by the overconfident investors. However, expected returns also depend on current mispricing, so returns can be predicted better by conditioning on proxies for misvaluation. A natural ingredient for such a proxy is the market price itself, since price reflects misvaluation. For example, following a favorable information signal, the price of the security overreacts, so price is too high. A misvaluation proxy that contain price in the denominator therefore decreases. (It helps to scale the proxy in the numerator with a non-market fundamental measure such as book value, earnings or dividends.) In this setting, firms with low (high) fundamental/price ratios are overvalued (undervalued), and tend to have low (high) expected future returns.

A prevalent intuition is that whenever the market makes valuation errors, high fundamental/price ratios predict high returns.[6] This intuition is just that if the market overvalues a stock, both its fundamental/price ratio and the future return tend to be low. This argument implicitly assumes that the pricing error is uncorrelated with shifts in the firm's true conditional expected value. However, as we show, this premise is not in general true if errors arise from misinterpretation of new information. In our setting, informed investors generally have favorable private information about low book/market

[5]The existence of an active industry selling macroeconomic forecasts to investors suggests that, rightly or wrongly, investors believe that something akin to private information about aggregate factors exists. Consistent with genuine private information about aggregate factors, several studies have provided evidence that aggregate insider trading forecasts future industry and aggregate stock market returns (see, e.g., Lakonishok and Lee (1998)). In addition, there are many market timers who trade based on what they perceive to be information about market aggregates, and investors looking for industry plays such as internet or biotech stocks.

[6]Berk (1995a) derives an explicit set of statistical conditions under which a price-related variable such as size has incremental power to predict future returns. Here we offer an equilibrium model in order to explore the economic conditions under which this occurs.

firms. If the informed were to systematically underreact to this private information (because of *under*confidence) then a low book/market ratio would forecast *high* future returns. Thus, the simpler intuition is not correct. The empirical direction of these effects supports theories based on overreaction and correction (such as the psychological theory proposed here) over theories based on pure underreaction.

The model implies that even when covariance risk is priced, fundamental-scaled price measures can be better forecasters of future returns than covariance risk measures such as beta (Appendix B describes existing evidence). Intuitively, the reason that fundamental/price ratios have incremental power to predict returns is that a high fundamental/price ratio (e.g., high book/market) can arise from high risk and/or overreaction to an adverse signal. If price is low due to a risk premium, on average it rises back to unconditional expected value. If there is an overreaction to genuine adverse information, then the price will on average recover only part way toward the unconditional expected terminal cash flow. Since high book/market reflects both mispricing and risk, whereas beta reflects only risk, book/market can be a better predictor of returns.

In general, knowing the level of covariance risk (beta) helps disentangle risk and mispricing effects. This is consistent with the findings of several (but not all) empirical studies (discussed in Appendix B) that beta positively predicts future returns after controlling for fundamental/price ratios or size. However, interestingly, there is a special case in which, even though risk is priced and beta is a perfect proxy for risk, beta does not have any incremental explanatory power. Thus, such a test may create the appearance that market risk is not priced even if it were fully priced. Subsection 1.3.1 provides a numerical illustration of the basic intuition for these implications.[7]

The theory has other implications about the ability of alternative misvaluation proxies to predict future returns. First, since the market value of the firm reflects misvaluation, firm size as measured by market value will predict future returns but non-market measures of firm size will not. Price (or market value) of course varies for reasons other than misvaluation. Such differences in scale (unconditional expected firm payoff) are noise to an econometrician who seeks to extract misvaluation from market price. Scaling prices by fundamental measures (e.g., book value, earnings, or dividends) tends to

[7] An alternative approach to securities pricing is offered by Shefrin and Statman (1994), who analyze the effect of mistaken beliefs on equilibrium in stock, option and bond markets. Their model allows for general beliefs, and therefore for a wide range of possible patterns. However, their focus is not on empirically predicting the direction of pricing errors or addressing evidence on the cross-section of security returns. In a contemporaneous paper, Shumway (1998) examines the effects of loss-aversion on securities prices. He does not, however, examine whether this approach can explain the known patterns in the cross-section of securities prices.

improve predictive power by filtering out irrelevant scale variability. Thus, a variable like book/market should predict future returns better than size. Nevertheless, if the fundamental proxy measures expected future cash flows with error, market value will still have some incremental ability beyond the fundamental/price ratio to predict future returns. To filter out industry-wide noise in fundamental measures, it can also be useful to use a price-related industry normalization (such as measuring a firm's fundamental/price ratio relative to industry ratios). Such filtering will, however, unavoidably remove not just noise, but the industry-related component of mispricing.

The theory also offers empirical implications that are untested or that have received ex post confirmation. The theory predicts that fundamental/price ratios should better forecast risk-adjusted returns for businesses that are hard to value (e.g., R&D-intensive firms comprised largely of intangible assets). Recent empirical research has provided evidence consistent with this implication (see Section 3). The theory also offers implications about the cross-sectional dispersion in fundamental/price ratios and their power to predict future returns in relation to market-wide levels of fundamental/price ratios.

Further untested empirical implications relate to current volume as a predictor of future market return volatility. High volume indicates extreme signals values and strong disagreement between overconfident traders and arbitrageurs. High volume therefore predicts a larger future correction. This leads to an increasing quadratic relation between current volume and future market volatility. Furthermore, periods in which the volume/volatility relation is strong are associated with a strong relation between the market fundamental/price ratio and future returns.

As noted above, a common objection to models of price anomalies based upon imperfect rationality is that smart traders should be able to trade against mispricing, and thereby eliminate it. In our setting, risk aversion limits the extent of such risk arbitrage activity. However, there is an incentive for arbitrageurs to invest in value or small-cap funds–diversified portfolios that are long in stocks with high fundamental/price ratios or low market values, and short in stocks with the opposite characteristics. With many securities, arbitrageurs are able to almost eliminate the idiosyncratic mispricing, but do *not* eliminate the systematic mispricing.

A further objection to models with imperfect rationality is that if such trading causes wealth to flow from irrational to smart traders, eventually the smart traders may dominate price-setting. In our setting, arbitrageurs exploit the mispricing, but do not earn riskless profits. Furthermore, as in De Long, Shleifer, Summers, and Waldmann (1990a), overconfident individuals invest more heavily in risky assets, and thereby may earn higher

or lower expected profits than the arbitrageurs.

The remainder of the paper is structured as follows. Section 1 presents a pricing model based on investor psychology. Section 2 examines forecasting future returns using both mispricing measures and traditional risk-based return measures (such as the market beta), and develops further empirical implications. Section 3 examines further empirical implications relating to the circumstances affecting the degree of overconfidence. Section 4 examines volume and future future volatility. Section 5 examines the profitability of trading by arbitrageurs and overconfident individuals.

1 The Model

1.1 The Economic Setting

We have argued that the psychological basis for overconfidence is that people overvalue their own expertise. This suggests that people will tend to be overconfident about private signals. A signal that only a subset of individuals receive presumably reflects special expertise on the part of the recipients. We therefore examine a setting in which some traders possess private information and some do not. The traders who possess private information are *overconfident*: they overestimate the precision of their signals. The uninformed traders have no signals to be overconfident about.[8]

The analysis has two other equivalent interpretations. First, the uninformed class of investors could instead be viewed as a set of fully rational uninformed investors. The second interpretation relies on the fact that, in equilibrium, prices are fully revealing, so that the 'uninformed' traders end up making full use of the private information of the informed. In consequence, some or all of the uninformed traders can instead be viewed as being fully rational *informed* arbitrageurs. All three interpretations lead to identical results. We refer to the signals the informed individuals receive as 'private'.[9] Individuals

[8] A purely rational trader would disagree with the overconfident investors as to posterior payoff variances. This suggests that there may be profit opportunities for trading in options markets. If the model were extended to continuous time using the stylized assumptions of arbitrage-based option pricing (smooth diffusion of information, non-stochastic volatility), then rational traders would be able to obtain large riskfree profits by forming hedge portfolios of options, stocks and bonds. However, as options professionals are well aware, information arrives in discrete chunks such as earnings reports, and volatility evolves stochastically. Thus, even a trader who has a better assessment of volatility cannot make riskfree profits. In other words, a reasonable dynamic extension of the model would provide risky profit opportunities, but not arbitrage opportunities, to rational agents.

[9] Overconfident investors recognize that other overconfident investors perceive a similarly high precision for the signal. Since they share this perception, they do not regard the others as overconfident.

who receive a private signal about a factor or about a security's idiosyncratic payoff component are referred to as the overconfident informed with respect to that signal. Individuals who do not receive a given signal are referred to as arbitrageurs with respect to that signal.[10]

1.1.1 Timing

A set of identical risk averse individuals who are each endowed with baskets containing shares of $N+K$ risky securities and of a riskfree consumption claim with terminal (date 2) payoff of 1. At each date individuals can trade the consumption claims for shares. There are three dates. At date 0, individuals hold identical prior beliefs about the risky security payoffs. At date 1 some, but not all, individuals receive noisy private signals about the risky security payoffs. Whether or not an individual receives a signal affects his belief about the precision of that signal. Individuals then trade securities based on their beliefs. At date 2, conclusive public information arrives, the $N+K$ securities pay liquidating dividends of $\boldsymbol{\theta} = (\theta_1, ...\theta_{N+K})'$, the risk-free security pays 1, and all consumption takes place. Appendix A provides a guide to the model notation.

1.1.2 Individuals and the Portfolio Problem

All individuals have identical preferences. Individual j selects his portfolio to maximize $E_j[-exp(-A\tilde{c}_j)]$, where $\tilde{c}_j$, date 2 consumption, is equal to his portfolio payoff. The j subscript here denotes that the expectations are taken using individual j's beliefs, conditional on all information available to j as of date 1. Let $\mathbf{P}$ denote the date 1 vector of prices of each security relative to the riskfree security, and $\mathbf{x}_j$ denote the vector of risky security demands by individual j, and let $\bar{\mathbf{x}}_j$ be the vector of individual j's security endowment. Let $\boldsymbol{\mu}_j \equiv E_j[\boldsymbol{\theta}]$ denote the vector of expected payoffs, and $\boldsymbol{\Omega}_j \equiv E_j[(\boldsymbol{\theta} - E_j[\boldsymbol{\theta}])(\boldsymbol{\theta} - E_j[\boldsymbol{\theta}])']$ denote the covariance matrix of security payoffs.

Since all asset payoffs are normally distributed, individual j solves:

$$\max_{\mathbf{x}_j} \ \mathbf{x}_j'\boldsymbol{\mu}_j - \frac{A}{2}\mathbf{x}_j'\boldsymbol{\Omega}_j\mathbf{x}_j \quad \text{subject to} \quad \mathbf{x}_j'\mathbf{P} = \bar{\mathbf{x}}_j'\mathbf{P}.$$

All individuals act as price takers. Differentiating the Lagrangian with respect to $\mathbf{x}_j'$ gives the first order condition:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}_j'} = \boldsymbol{\mu}_j - A\boldsymbol{\Omega}_j\mathbf{x}_j - L\mathbf{P} = 0.$$

[10] We therefore allow for the possibility that an individual is overconfident with respect to one signal, but acts rationally to arbitrage mispricing arising from a different signal.

The condition that the price of the riskfree security in terms of itself is 1 implies that the Lagrangian multiplier $L = 1$, so

$$\mathbf{P} = \boldsymbol{\mu}_j - A\boldsymbol{\Omega}_j \mathbf{x}_j. \tag{1}$$

1.1.3 Risky Security Payoffs - The Factor Structure

Before any information signals are received, the distribution of security payoffs at date 2 are described by the following K-factor structure:

$$\theta_i = \bar{\theta}_i + \sum_{k=1}^{K} \beta_{ik} f_k + \epsilon_i, \tag{2}$$

where β_{ik} is the loading of the ith security on the kth factors, f_k is realization of the kth factor, and ϵ_i is the ith residual. As is standard with factor models, we specify w.l.o.g. that $E[f_k] = 0$, $E[f_k^2] = 1$, $E[f_j f_k] = 0 \ \forall i \neq j$, $E[\epsilon_i] = 0$, $E[\epsilon_i f_k] = 0 \ \forall i, k$. The values of $\bar{\theta}_i$ and β_{ik} are common knowledge, but the realizations of f_k and ϵ_i are not revealed until date 2. Let V_i^ϵ denote $Var(\epsilon_i)$.

With many securities, K mimicking portfolios can be formed that correlate arbitrarily closely with the K factors and diversify away the idiosyncratic risk. As a convenient approximation, we assume that each of the first K securities is a factor-mimicking portfolios for factor K, and therefore that each of these assets has zero residual variance, has a loading of 1 on factor k, and zero on the other $K - 1$ factors.

1.1.4 An Equivalent Maximization Problem

Since an individual can, by means of the K mutual funds, hedge out the factor risk of any individual asset, he can construct a portfolio with arbitrary weights on the K factors and N residuals. Therefore, the individual's utility maximization problem is equivalent to one in which the investor directly chooses his portfolio's loadings on the K factors and N residuals, and his holdings of the risk-free asset. This can be viewed as the problem that arises when the risky securities are replaced with a set of $N + K$ uncorrelated risky porfolios each of which has a expected payoff (at date 0) of zero and a loading of one on the relevant factor or residual and zero on all others. That is, the k'th factor portfolio ($k = 1, \ldots, K$) has a date 2 payoff of f_k, and the n'th residual portfolio ($n = 1, \ldots, N$) has a date 2 payoff of ϵ_n.

Since this set of portfolios spans the same space as the original set of securities, optimizing the weights on these portfolios generates the same overall consumption portfolio.

We solve for the market prices of these portfolios, and then for the market prices of the original securities. One unit of the i'th original asset (as described in equation (2)) can therefore be reproduced by holding $\bar{\theta}_i$ of the risk-free asset, one unit of the i'th residual portfolio, and $\beta_{i,k}$ units of each of the $k = 1, ..., K$ factor portfolios. At any date, the price of this security is the sum of the prices of these components. From this point on we number assets so that the first K risky assets ($i = 1, \ldots, K$) in the equivalent setting are the K factor portfolios, and the remaining N ($i = K+1, \ldots, K+N$) are the N residual portfolios.

1.1.5 Date 1 Signals

Some individuals receive signals at date 1 about the K factors and N residuals. We assume that it is common knowledge that a fraction $\phi_i, i = 1, ..., K+N$ of the population receives a signal about the payoff of the ith asset. For $i = 1, ..., K$ the signal is about a factor realization and for $i = K+1, ..., K+N$ it is about a residual. Before receiving signals, the individuals are essentially identical (under exponential utility, differences in wealth do not affect demands for risky assets); the only thing that differentiates them is whether signals they receive about asset payoffs. The key is not the mere *possession* of the information — in the model all private signals are revealed through prices. Without overconfidence, all individuals would therefore have identical posterior beliefs. The heterogeneity instead arises because individuals who obtain a signal are overconfident about that signal, while individuals who only *infer* the signal through prices are not overconfident about it.

We assume that all individuals who receive a signal about a factor or residual receive precisely the same signal.[11] The noisy signals about the payoff of the k'th factor portfolio and i'th residual portfolio take the form

$$s_k^f = f_k + e_k^f \quad \text{and} \quad s_i^\epsilon = \epsilon_i + e_i^\epsilon.$$

The true variance of the signal noise terms e_k^f e_i^ϵ are V_k^{Rf} and $V_i^{R\epsilon}$, respectively (R denotes "Rational"), but because the informed investors are over$C$onfident, the mistakenly believes the variance to be lower: $V_k^{Cf} < V_k^{Rf}$, and $V_i^{C\epsilon} < V_i^{R\epsilon}$. In much of the analysis,

[11] Our assumption that all individuals receive exactly the same signal is not crucial for the results, but signal noise terms must be correlated. Some previous models with common private signals include Grossman and Stiglitz (1980), Admati and Pfleiderer (1988), and Hirshleifer, Subrahmanyam, and Titman (1994). If, as is true in practice, some groups of analysts and investors use related information sources to assess security values, and interpret them in similar ways, the errors in their signals will be correlated.

it will be more convenient to use the precision, $\nu \equiv 1/V$. Thus we define $\nu_k^{Cf} \equiv 1/V_k^{Cf}$, $\nu_k^{Rf} \equiv 1/V_k^{Rf}$, $\nu_i^{C\epsilon} \equiv 1/V_i^{C\epsilon}$, and $\nu_i^{R\epsilon} \equiv 1/V_i^{R\epsilon}$.

Finally, we assume independence of this signal errors, i.e., $cov(e_i^\epsilon, e_{i'}^\epsilon) = 0$ for $i \neq i'$, $cov(e_k^f, e_{k'}^f) = 0$ for $k \neq k'$, and $cov(e_i^\epsilon, e_k^f) = 0$ for all i, k. This set of assumptions makes the model tractable and is without loss of generality.

1.1.6 Expectations and Variances of Portfolio Payoffs

Let the superscript $\bullet = C, R$ denote a quantity that can be calculated with respect to either overconfident (C) or rational (R) expectations. To solve for price in terms of exogenous parameters, we first calculate the expectation of portfolio i's terminal value given all of the signals.

For convenience we will now slightly abuse the notation by letting the variable μ refer to means for the factor and residual portfolios instead of the original assets, and x the number of shares of the factor and residual portfolios. Since all variables are jointly normally distributed, the posterior distributions for f_k and ϵ_i are also normal. We define the "$\bullet$" notation next to an expectation operator as implying conditioning upon all signals available to the individual. The posterior mean and variance of payoffs of factor and residual portfolios are

$$\mu_i^\bullet \equiv E^\bullet[\delta_i] = \frac{\nu_i^\bullet s_i}{\nu_i + \nu_i^\bullet}; \quad E^\bullet\left[(\delta_i - \mu_i^\bullet)^2\right] = \frac{1}{\nu_i + \nu_i^\bullet} \quad \text{for } i = 1, \ldots, N+K \tag{3}$$

where δ_i is the payoff of the ith asset in the equivalent setting, and $\delta_k = f_k$ for $k = 1, \ldots, K$, and $\delta_i = \epsilon_{i-K}$ for $i = K+1, \ldots, N+K$, and where ν_i denotes the prior precision of the portfolio (*i.e.*, $1/V_i$, where $V_i = V_i^\epsilon$ for a residual, and $V_i = 1$ for a factor). Since the precision of the prior on f is 1 by assumption, $\nu_k = 1$ for $k = 1, \ldots, K$.

1.1.7 Prices and Portfolio Holdings

Because the payoffs of the K factor portfolios and N residual portfolios are uncorrelated, the covariance matrix $\mathbf{\Omega}$ in equation (1) is diagonal, and we can rewrite equation (1) on an element-by-element basis as

$$P_i = \mu_i^\bullet - \left(\frac{A}{\nu_i + \nu_i^\bullet}\right) x_i^\bullet,$$

or

$$x_i^\bullet = \frac{1}{A}(\nu_i + \nu_i^\bullet)(\mu_i^\bullet - P_i) = \frac{\nu_i + \nu_i^\bullet}{A} E^\bullet[R_i] \qquad \text{for } i = 1, \ldots, N+K, \tag{4}$$

where $x_i^\bullet$ denotes the number of shares of portfolio i an individual would hold. Also, since individuals have constant absolute risk aversion, it is convenient to define the date 1-2 'return' of portfolio i as the terminal payoff minus the price, $R_i \equiv \delta_i - P_i$.

In this setting each individual knows the others' preferences, initial endowments, the number of individuals who receive signals, and how other individuals interpret their signals. There is no noise trading or shock to security supply. In consequence, uninformed individuals can infer all the signals perfectly from market prices. The uninformed end up with the same information as the informed traders, but use it differently as they are not overconfident about these signals (see the discussion in Subsection 1.1.5).

We impose the market clearing condition that that the average holding of each asset equal the number of endowed shares per individual, ξ_i, of each factor or residual portfolio (This is the number of shares that would be required to construct the market portfolio using just the $N+K$ factor and residual portfolios, divided by the number of individuals.) Recall that ϕ_i denotes the fraction of the population which receives information about, and is overconfident about, portfolio i. Then by equation (4),

$$\begin{aligned} \xi_i &= \phi_i x_i^C + (1-\phi_i) x_i^R \\ &= \frac{1}{A}\left[\phi_i(\nu_i + \nu_i^C)(\mu_i^C - P_i) + (1-\phi_i)(\nu_i + \nu_i^R)(\mu_i^R - P_i)\right]. \end{aligned}$$

Using the Bayesian expression for $\mu_i^\bullet$ in (3), the above equation yields:

$$P_i = \begin{bmatrix} \nu_i^R + \phi_i(\nu_i^C - \nu_i^R) \\ \nu_i + \nu_i^R + \phi_i(\nu_i^C - \nu_i^R) \end{bmatrix} s_i - \begin{bmatrix} A \\ \nu_i + \nu_i^R + \phi_i(\nu_i^C - \nu_i^R) \end{bmatrix} \xi_i. \tag{5}$$

Let ν_i^A be the *consensus precision* (A for Average), or, more formally, the population-weighted average assessment of signal precision for signal i:

$$\nu_i^A \equiv \phi_i \nu_i^C + (1-\phi_i)\nu_i^R. \tag{6}$$

Then (5) can be rewritten as

$$P_i = \left(\frac{\nu_i^A}{\nu_i + \nu_i^A}\right) s_i - \left(\frac{A}{\nu_i + \nu_i^A}\right) \xi_i. \tag{7}$$

This expression shows that prices are set as if all agents were identically overconfident and assessed the signal precision to be ν^A. We further define

$$\lambda_i \equiv \frac{\nu_i^A}{\nu_i + \nu_i^A}, \quad \lambda_i^R \equiv \frac{\nu_i^R}{\nu_i + \nu_i^R} \quad \text{and} \quad \lambda_i^C \equiv \frac{\nu_i^C}{\nu_i + \nu_i^C}, \qquad \text{for } i = 1, \ldots, N+K. \tag{8}$$

λ_i is the actual response of the market price of asset i to a unit increase in the signal s_i; λ_i^R is what the response would be if all individuals in the population behaved rationally, and λ_i^C is what the response would be were all individuals overconfident.

If there is a mixture of arbitrageurs and overconfident informed individuals in the population, $0 < \phi < 1$, then $\lambda_i^C > \lambda_i > \lambda_i^R$. Prices respond too strongly to private signals, but not as strongly as they would were there no arbitrageurs to trade against the overconfidence-induced mispricing. The higher the fraction of overconfident informed agents ϕ_i, the greater the amount of overreaction to the private signal.

Substituting these definitions into (7), we can calculate the price and expected return of asset i (expectation at date 1 of the date 1-2 price change), conditional on the signal.

$$P_i = \lambda_i^R s_i + (\lambda_i - \lambda_i^R)s_i - \left(\frac{A}{\nu_i + \nu_i^A}\right)\xi_i \tag{9}$$

$$E^R[R_i] = \mu_i^R - P_i = -(\lambda_i - \lambda_i^R)s_i + \left(\frac{A}{\nu_i + \nu_i^A}\right)\xi_i, \quad \text{for } i = 1, \ldots, N+K. \tag{10}$$

The price equation has three terms. The first, $\lambda_i^R s_i = E^R[\delta_i]$, is the expected payoff of the security from the perspective of a rational investor. The extra term, $(\lambda_i - \lambda_i^R)s_i$, is the extra price reaction to the signal s_i due to overconfidence. The right-hand term of the equation is the price-discount for risk. The (rationally assessed) expected return on portfolio i depends only on the i-signal. Intuitively, with constant absolute risk aversion news about other components of wealth does not affect the premium individuals demand for trading in the ith portfolio.

The expected return then consists of two terms: the correction of the extra price reaction to the signal, and the a risk premium that compensates for the risk of the portfolio. The risk premium is proportional to the coefficient of risk aversion, A, share endowment ξ_i, and the consensus-variance $1/(\nu_i + \nu_i^A)$ (the representative individual's assessed cash-flow variance of asset i). Recall that an overconfident informed individual always thinks that the security is less risky than it really is. Hence, the greater the fraction of overconfident individuals in the population (the greater ϕ), and the greater their overconfidence, the lower its risk premium.

Equation (10) gives the expected return, as assessed by a rational arbitrageur. The more general expression that gives the expected return as assessed by either overconfident informed traders or arbitrageurs is

$$E^\bullet[R_i] = \mu_i^\bullet - P_i = (\lambda_i^\bullet - \lambda_i)s_i + \left(\frac{A}{\nu_i + \nu_i^A}\right)\xi_i, \quad \text{for } i = 1, \ldots, N+K, \tag{11}$$

Since under overconfidence, $\lambda_i^C > \lambda_i > \lambda_i^C$, the first term of this equation shows that the arbitrageurs and the informed overconfident traders disagree on whether securities are over- or under-priced. Ignoring the risk-premium (the last term), if s_i is positive, than an arbitrageur thinks that the price is too high by $(\lambda_i - \lambda_i^R)s_i$, and an overconfident thinks that the price is too low by approximately $(\lambda_i^C - \lambda_i)s_i$. Because of these differing beliefs, the arbitrageurs and informed overconfident traders, whose holdings are given by equation (4), take opposing positions following a signal.[12]

The expressions for the price and expected return can be expressed more compactly with the following rescaling:

$$S_i \equiv \lambda_i^R s_i, \quad \text{and} \quad \omega_i \equiv \frac{\lambda_i - \lambda_i^R}{\lambda_i^R} \quad \text{for } i = 1, \ldots, N+K. \tag{12}$$

Here S_i is rescaled so that a unit increase in the signal would cause a unit increase in the price, were all agents rational. However, with overconfident traders there is excess sensitivity to the signal: ω_i denotes the fractional excess sensitivity for the ith signal. Given these definitions, equations (9) and (10) become:

$$P_i = (1+\omega_i)S_i - \frac{A}{\nu_i + \nu_i^A}\xi_i \tag{13}$$

$$E^R[R_i] = -\omega_i S_i + \frac{A}{\nu_i + \nu_i^A}\xi_i, \qquad \text{for } i = 1, \ldots, N+K. \tag{14}$$

1.1.8 The Adjusted Market Portfolio

Using equation (14), we can write the returns on each of the $N+K$ portfolios as:

$$R_i = E^R[R_i] + u_i \qquad \text{for } i = 1, \ldots, N+K,$$

where by rationality of the true expectation $E^R[u_i] = 0$ and $E^R[R_i u_i] = 0$. Also, from (3), $E^R[u_i^2] = 1/(\nu_i + \nu_i^R)$. And, as discussed previously $E^\bullet[u_i u_j] = 0$ for $i \neq j$ and $\bullet = \{C, R\}$, since rational and overconfident agree that the $N+K$ assets are uncorrelated with one another.

Let the true (per capita) market return be the return on the porfolio with security weights equal to the endowed number of shares of each security per individual (*i.e.*, the weights are the total market portfolio weights divided by the population size):

$$R_m = \sum_{i=1}^{N} \xi_i R_i,$$

[12]The overconfident also think that the security is less risky than do the arbitrageurs. Hence, they are willing to hold a larger position. For a favorable signal, the return and risk effects are reinforcing, but for an adverse signal they are opposing.

and let the *adjusted market portfolio* M be the portfolio with weights

$$\xi_i' = \xi_i \begin{pmatrix} \nu_i + \nu_i^R \\ \nu_i + \nu_i^A \end{pmatrix}, \quad i = 1, \ldots, N+K. \tag{15}$$

The adjustment factor in parentheses is the ratio of the asset's consensus variance $1/[\nu_i + \nu_i^A]$ to the true variance $1/(\nu_i + \nu_i^R)$. The rationally assessed covariance between the asset i return and the adjusted market return is $cov^R(R_i, R_M) = \xi_i/(\nu_i + \nu_i^A)$. Substituting this into equation (14) gives

$$E^R[R_i] = -\omega_i S_i + A\ cov^R(R_i, R_M) \qquad \text{for } i = 1, \ldots, N+K. \tag{16}$$

Thus, the expected return is the sum of a mispricing component and a risk component which is based on the covariance with the adjusted market portfolio.

1.2 Pricing Relationships

The previous subsection derived expressions for prices and expected returns for the factor and residual portfolios. For the original $N+K$ assets, equation (16) implies the following asset pricing relationship. We now let ω_i^ϵ and S_i^ϵ, for $i = 1, ..., N$, and ω_k^f and S_k^f, for $k = 1, ..., K$, denote the fractional overreaction (ω_i) and scaled signal (S_i) for the N residuals and K factors.

Proposition 1 *If risk averse investors with exponential utility are overconfident about the signals they receive regarding K factors and about the idiosyncratic payoff components of N securities, then securities obey the following relationships:*

$$P_i = \bar{\theta}_i - \alpha\beta_{iM} + (1+\omega_i^\epsilon)S_i^\epsilon + \sum_{k=1}^{K} \beta_{ik}(1+\omega_k^f)S_k^f \tag{17}$$

$$E^R[R_i] = \alpha\beta_{iM} - \omega_i^\epsilon S_i^\epsilon - \sum_{k=1}^{K} \beta_{ik}\omega_k^f S_k^f, \tag{18}$$

for all $i = 1, \ldots, N+K$, where

$$\beta_{iM} \equiv cov(R_i, R_M)/var(R_M).$$

Equation (18) implies that true expected return decomposes additively into a risk premium (the first term) and components arising from mispricing (the next two terms).

Mispricing arises from the informed's overreaction to signals about the factors and the idiosyncratic payoff components. The mispricing due to overreaction to factor information is proportional to the security's sensitivity to that factor. In addition, the securities expected return includes a premium for market risk.[13] If there were no overconfidence ($\lambda_i = \lambda_i^R$ and $\lambda_k = \lambda_k^R$), this equation would be identical to the CAPM with zero riskfree rate.

We now use the fact that the expected value of the signals is zero to derive the optimal predictor of future returns for an observer who does not condition on current market prices nor on any other proxy for investors' private signals. The following corollary follows by taking the rational expectation of (18); and then taking a weighted sum of security expected returns to show that $\alpha = E[R_M]$.

Corollary 1 *Conditioning only on β_{iM}, expected security returns obey the pricing relationship:*

$$E[R_i] = E[R_M]\beta_{iM}, \quad i = 1, \ldots, N + K,$$

where $E[R_M]$ is the expected return on the adjusted market portfolio, and β_{iM} is the security's beta with respect to the adjusted market portfolio.

This is identical in form to the CAPM security market line (with zero riskfree rate). However, here M is the adjusted market porfolio. This relationship holds for the true market portfolio m as well in the natural benchmark case in which investors are *equally overconfident* about all signals, *i.e.*, the ratio

$$\frac{\nu_i + \nu_i^R}{\nu_i + \nu_i^A} = \frac{\nu_j + \nu_j^R}{\nu_j + \nu_j^A} \tag{19}$$

is equal for all factors and residuals i and j.[14] This can be seen by substituting equation (19) into equation (15).[15]

Although consistent with the univariate evidence that mean returns are increasing with beta, the model implies that there are better ways to predict future returns than the CAPM security market line. Proposition 1 implies that better predictors can be obtained by regressing not just on beta, but on proxies for market misvaluation.

[13]The coefficient β_{iM} is a price-change beta, not the CAPM return beta. That is, β_{iM} is the regression coefficient in $\theta_i - P_i = \alpha_i + \beta_{iM}(\theta_M - P_M)$ where P_i and P_M are known. The CAPM return beta is the coefficient in the regression $(\theta_i - P_i)/P_i = \alpha_i^R + \beta_{iM}^{\text{Ret}}(\theta_M - P_M)/P_M$, and is equal to $(P_M/P_i)\beta_{iM}$.

[14]If there are many securities, as discussed in Section 5, all that is required is that this ratio be equal across factors, but not necessarily across residuals.

[15]Empirically, as discussed in Appendix B, the evidence is supportive of a positive univariate relation between market beta and future returns, although estimates of the strength and significance of the effect varies across studies.

1.3 Proxies for Mispricing

Since mispricing is induced by signals that are not directly observable by the econometrician, it is important to examine how expected returns are related to observable proxies for mispricing, and with measures of risk. We begin with a simple numerical illustration of the basic reasoning.

1.3.1 A Simple Example

We illustrate here (and formalize later) three points:

1. High fundamental/price ratios (henceforth in this subsection, F/P's) predict high future returns if investors are overconfident, and low future returns if investors are sufficiently underconfident.

2. Regressing on β as well as an F/P yields positive coefficients on both β and the F/P.

3. If overconfidence about signals is extreme, even though β is priced by the market, β has no incremental power to predict future returns over an F/P.

To understand parts (1)-(3), suppose for presentational simplicity that there is only information about idiosyncratic risk, and consider a stock that is currently priced at 80. Suppose that its unconditional expected cash flow is known to be 100. The fact that the price is below the unconditional fundamental could reflect a rational premium for factor risk, an adverse signal, or both.

Suppose now that investors are overconfident, and consider the case in which $\beta = 0$. Then the low price ($80 < 100$) must be the result of an adverse signal. Since investors overreacted to this signal, the true conditional expected value is greater than 80— the stock is likely to rise. Thus the above-average $F/P(100/80)$ is associated with a positive abnormal expected return. Of course, the reverse reasoning indicates that a below-average value (e.g, 100/120) predicts a negative expected return. Thus, when investors are overconfident, a high fundamental/price ratio predicts high future returns, consistent with a great deal of existing evidence.

If instead investors were underconfident, then the price of 80 would be an *under*reaction to the signal, so price could be expected to fall further. Thus, the high fundamental/price ratio would predict *low* returns, inconsistent with the evidence.

If we allow for differing β's, there is a familiar interfering effect (Berk (1995a)): a high beta results in a higher risk discount and hence also results in a high F/P. Thus,

even if there is no information signal, a high F/P predicts high returns. This illustrates point (1) above. The confounding of risk and mispricing effects suggests regressing future returns on beta as well as on F/P.

This confounding leads to point (2). Thus, if the econometrician knew that the discounting of price from 100 to 80 was purely a risk premium, then the expected terminal cash flow would still be 100. In contrast, if this is a zero-β (zero risk premium) stock, the true conditional expected value (90, say) would lie between 80 and 100: the signal is adverse ($90 < 100$), and investors overreact to it ($80 < 90$). Thus, the true expected return is positive, but not as large as in the case of a pure risk premium. By controlling for β as well as the F/P, the econometrician can disentangle whether the price will rise to 100 or only to 90.

To understand point (3), consider now the extreme case where overconfidence is strong, in the sense that the 'signal' is almost pure noise and investors greatly underestimate this noise variance. In this case, even when the price decreases to 80 purely because of an adverse 'signal,' it will still on average recover to 100. This leads to exactly the same expected return as when the price of 80 is a result of a high beta. Both effects are captured fully and equally by F/P, whereas beta captures only the risk effect. So even though beta is priced, the F/P completely overwhelms beta in a multivariate regression.

Our reasoning is *not* based on the notion that if investors were so overconfident that they thought their signals were perfect, they would perceive risk to be zero, which would cause β to be unpriced. Even if investors are only overconfident about idiosyncratic risk, so that covariance risk is rationally priced, β has no incremental power to predict returns. In contrast, the effect described here is not founded on weak pricing of risk.

The Part (3) scenario, while extreme, does offer an explanation for why the incremental beta effect can be weak and therefore hard to detect statistically. The following subsections develop these insights formally, and provide further implications about the usefulness of alternative mispricing proxies.

1.3.2 Noisy Fundamental Proxies

Consider an econometrician who wishes to forecast returns. Since prices reflect misvaluation, it is natural to include a price-related predictor. However, it is hard to disentangle whether a low price arises because $\bar{\theta}_i$, its unconditional expected payoff, is low, or because the security is undervalued. The econometrician can use a fundamental measure as a noisy proxy for the unconditional expected value. We examine here how well scaled price variables can predict returns when the fundamental proxy is the true expected

cash flow plus noise,

$$F_i = \bar{\theta}_i + e_i^F. \tag{20}$$

Here e_i^F is *i.i.d.* normal noise with zero mean, and $V^F \equiv E[(e^F)^2]$ is the variance of the error in the fundamental measure of a randomly selected security.

Suppose that the econometrician randomly draws a security, observes the fundamental-scaled price variable $F_i - P_i$, and uses this to predict the future return. We let variables with i subscripts *omitted* denote random variables whose realizations include a stage in which a security is randomly selected (i.e., there is a random selection of security characteristics). This stage determines security parameters such as $\bar{\theta}$ or ω. Other random variables, such as the price, return and signal variables R, P, and S, require a second stage in which signal and price outcomes are realized.

Security expected payoffs $\bar{\theta}_i$, $i = 1, \ldots N$ are assumed to be distributed normally from the econometrician's perspective,

$$\bar{\theta} \sim \mathcal{N}(\bar{\bar{\theta}}, V_{\bar{\theta}}), \tag{21}$$

where $\bar{\bar{\theta}}$ is the cross-sectional expectation of the unconditional expected values, and $V_{\bar{\theta}}$ is the variance of $\bar{\theta}$.

We denote the moments of the factor loading distribution β_k (the loading of a randomly selected security on factor k) by $E[\beta_k], V_{\beta_k}$; of β_M (the beta of a randomly selected security with the adjusted market price change) by $E[\beta_M], V^{\beta_M}$; of ω^ϵ (the excess sensitivity of price to a unit increase in the signal about idiosyncratic risk for a randomly selected security) by $E[\omega^\epsilon], V^{\epsilon\omega}$; of ω_k^f (the excess sensitivity of prices to a unit increase in the signal about factor k) by $E[\omega^f], V^{f\omega}$ (moments assumed to be independent of k); of S_k^f (the normalized signal about factor k) by V_k^{Sf}; and of S^ϵ (the normalized signal about idiosyncratic risk for a randomly selected security i) by $V^{S\epsilon}$ (by our earlier assumptions, the last two random variables have means of zero). Further, we assume that the choice of firm is independent of the signal realization, so that signal realizations are uncorrelated with the ω's and β.[16]

Now, consider the linear projection of the security return R onto $F - P$,

$$R = a + b_{F-P}(F - P) + e,$$

[16]The restriction on exogenous parameters that achieves this is that the signal realizations be uncorrelated with factor loadings and with variances of signals and noises.

where e is mean-zero independent noise. The slope coefficient value that minimizes error variance is

$$b_{F-P} = \frac{cov(R, F-P)}{var(F-P)}. \tag{22}$$

Let

$$cov_{OC} \equiv -\left(E[\omega^\epsilon] + E[(\omega^\epsilon)^2]\right) E[(S^\epsilon)^2] - \left(E[\omega^f] + E[(\omega^f)^2]\right) \sum_{k=1}^{K} E[\beta_k^2] E[(S_k^f)^2] \tag{23}$$

$$var_{OC} \equiv \left[(1+E[\omega^\epsilon])^2 + V^{\epsilon\omega}\right] E[(S^\epsilon)^2] + \left[(1+E[\omega^f])^2 + V^{f\omega}\right] \sum_{k=1}^{K} E[\beta_k^2] E[(S_k^f)^2]. \tag{24}$$

We will show that the quantities cov_{OC} and var_{OC} are the contributions of the individuals' private information to the covariance and variance respectively in equation (22). We then have the following proposition (The proof is presented in Appendix C).

Proposition 2 *The regression of the return R on the fundamental-scaled price $F-P$ yields the following coefficient:*

$$b_{F-P} = \frac{\alpha^2 V^{\beta_M} - cov_{OC}}{\alpha^2 V^{\beta_M} + var_{OC} + V^F} = \left(\frac{var(\bar{\theta}-P)}{var(F-P)}\right) b_{\bar{\theta}-P}, \tag{25}$$

where

$$b_{\bar{\theta}-P} \equiv \frac{\alpha^2 V^{\beta_M} - cov_{OC}}{\alpha^2 V^{\beta_M} + var_{OC}} \tag{26}$$

is the regression coefficient when the fundamental proxy is perfect ($\bar{\theta}$ instead of F). The coefficient is positive if:

1. *Investors are on average overconfident, $E[\omega^\epsilon] \geq 0$ and $E[\omega^f] \geq 0$, with at least one inequality strict, or*

2. *Investors are rational, and not all security betas are equal, i.e., $E[\omega^\epsilon] = E[\omega^f] = 0$, and $V^{\beta_M} \neq 0$.*

If we restrict the distribution of ω^f and ω^ϵ so that investors are not overconfident about any signal and are underconfident about at least one signal ($\omega_k^f \leq 0$, $\omega_i^\epsilon \leq 0$, with at least one inequality strict), and if V^{β_M} is sufficiently small, then the coefficient is negative. The R^2 of the regression is

$$R^2_{F-P} = \frac{[cov(R, F-P)]^2}{var(R)var(F-P)} = \left(\frac{var(\bar{\theta}-P)}{var(F-P)}\right) R^2_{\bar{\theta}-P}.$$

Intuitively, overconfident investors overreact to their private signals, so a high price (low $F - P$) probably means too high a price, and consequently that expected risk-adjusted returns are negative.[17] Any cross-sectional variation in beta contributes further to the tendency of high price to predict low future returns. High beta implies low current price and a high future expected return.[18]

On the other hand, if investors are on average *under*-confident then a high fundamental/price ratio is associated with a low expected future return (if V^{ω} is not too large and if risk effects, reflected in V^{β_M}, are small). Intuitively, if investors are underconfident and hence underreact to their private signals, then a high price means that the price is likely to increase still more.

The slope coefficient b_{F-P} in (25) is smaller than that in (26) because F_i is a noisy proxy for $\bar{\theta}_i$, so that $var(F - P) > var(\bar{\theta} - P)$; similarly, the regression R^2 is also lower. Any adjustment of the fundamental proxy F that decreases the measurement error variance improves R^2_{F-P}. One method of doing so is to adjust fundamental ratios relative to industry values. Accounting measures of value differ across industry for reasons that do not reflect differences in fundamental value. For example, different businesses have differing importance of intangible assets, which are imperfectly reflected in accounting measures of value. This suggests that industry adjustment, by filtering out measurement noise that is correlated for firms within an industry, can improve the ability of fundamental measures to predict return. This is consistent with the evidence of Cohen and Polk (1995) and of Dreman and Lufkin (1996).

We have assumed that the public information signal is conclusive. Similar results would apply with noisy public information arrival. Basically, a noisy public signal only partly corrects the initial overreaction to the private signal. So high fundamental/price still indicates undervaluation (even though the public signal on average has partly corrected the market price upwards). However, as analyzed in Daniel, Hirshleifer, and Subrahmanyam (1998), in a setting with dynamic overconfidence overreactions can temporarily continue before eventually being corrected. If we were to combine such dynamics with our assumptions here of risk aversion and multiple risky securities, since any misvaluation must eventually be corrected, it is intuitively reasonable to conjecture that

[17] In equation (26), $var_{OC} > 0$ by its definition in (24). If the $E[\omega]$'s are positive for all signals, then cov_{OC} is negative, so the regression coefficient is positive.

[18] Equation (26) shows that even with no overconfidence, there is still be a risk-based relationship between $\bar{\theta} - P$ and R arising from risk effects. The V^{β_M} term in the numerator and denominator reflects cross-sectional variation in risk and risk premia. If there is no overconfidence, then $E[\omega]$ and V^{ω} are zero for all signals, which implies $cov_{OC} = 0$. In standard pricing models such as the CAPM, low price firms are those that are discounted heavily, i.e., high beta/high return firms (see Berk (1995a)).

the cross-sectional relationship between fundamental/price ratios and future returns in such a setting will still be positive.

1.4 Aggregate Fundamental-Scaled Price Variable Effects

This subsection examines the special case of a single risky security ($N = 0, K = 1$), which we interpret as the aggregate stock market portfolio. The implications for the aggregate market are obtained from the general model by deleting all i subscripts.

The model then predicts that future aggregate market returns will be predicted by variables of the form F/M, where F is a publicly observable *non*-market measure of expected fundamental value and M is aggregate market value. Four examples are aggregate dividend yield, aggregate earnings/price ratio, the aggregate book/market ratio, and the reciprocal of market value (where the numerator $F \equiv 1$ is a constant). Noisy public information may moderate, but does not eliminate this effect. These deviations will gradually be corrected, though this correction may be slow. Thus, the model explains the empirical finding of a dividend yield effect, and predicts aggregate earnings yield and book/market effects as well. Thus, the model is consistent with the profitable use of asset allocation strategies wherein arbitrageurs tilt their portfolios toward investment toward either the riskfree asset or toward the stock market depending upon whether variables such as the market dividend/earnings yield are high or low.

2 Risk Measures versus Fundamental-Scaled Price Ratios

2.1 Analytical Results

Often tests of return predictability look simultaneously at standard risk measures and measures of mispricing. These regressions usually involve the market β, and variables such as market value, or a fundamental-scaled variable such as the book-to-market (see, *e.g.*, Fama and French (1992) and Jagannathan and Wang (1996)). Firm 'size' or market value (P) is a special case of a fundamental scaled measure in which the fundamental proxy is a constant.[19]

[19] P can be interpreted as either a per-share price or a total firm market value. Here we assume that θ_i is proportional to the total value of the firm. However, the analysis is equally valid on a per-share basis, and is therefore consistent with the empirical evidence that, cross-sectionally, share price is negatively correlated with future returns. A fuller analysis of this topic would include the number of shares relative to total firm value as a source of cross-sectional noise, so that firm value versus share price could have

In our setting, if the expected fundamental value is measured with noise, as in Section 1.3.2, the fundamental-scaled variable is an imperfect proxy for the private signal, and both size (P) and a fundamental-scaled variable ($F_i - P_i$), in addition to the risk measure β, predicts future returns. To see this, consider the linear projection of R onto β_M, $F-P$ and P:

$$R = a + b^{\dagger}_{\beta}\beta_M + b^{\dagger}_{F-P}(F-P) + b^{\dagger}_{P}P + e. \tag{27}$$

The optimal coefficients come from the standard matrix equation (see Appendix C).

Proposition 3 *The regression of the return R on β_M, the price-scaled fundamental $P-F$, and the price P yields the following set of coefficients:*

$$b^{\dagger}_{\beta} = \alpha \begin{pmatrix} var_{OC} + cov_{OC} + K_1 \\ var_{OC} + K_1 \end{pmatrix} \tag{28}$$

$$b^{\dagger}_{F-P} = -\frac{V_{\bar{\theta}}}{V_{\bar{\theta}} + V^F} \begin{pmatrix} cov_{OC} \\ var_{OC} + K_1 \end{pmatrix} \tag{29}$$

$$b^{\dagger}_{P} = \frac{V^F}{V_{\bar{\theta}} + V^F} \begin{pmatrix} cov_{OC} \\ var_{OC} + K_1 \end{pmatrix} \tag{30}$$

where

$$K_1 \equiv \frac{V_{\bar{\theta}} V^F}{V_{\bar{\theta}} + V^F}$$

is half the harmonic mean of $V_{\bar{\theta}}$ and V^F, and $K_1 = 0$ if either $V^F = 0$ or $V_{\bar{\theta}} = 0$. Under overconfidence, the coefficient on the fundamental/price ratio is positive and on the price is negative.

This proposition provides a theoretical motivation for the use of book/market ratios in the well-known regressions (and cross-classifications) of Fama and French (1992) and Jagannathan and Wang (1996).

If individuals are on average overconfident, $E[\omega^f] > 0$, then equation (23) shows that cov_{OC} is negative. Also, provided the fundamental measure is not perfect (*i.e.*, $V^F > 0$), these equations show that: (1) the regression coefficient on β is positive but less than α (the CAPM 'market price of risk'); (2) the coefficient on size (P) is negative; and (3) the coefficient on a $F - P$ variable (such as book/market) is positive. These results are thus consistent with the evidence of Fama and French (1992).

If the fundamental proxy is a noiseless indicator of unconditional expected returns, $V^F = 0$, then the coefficient on P is zero, because $F - P$ captures mispricing perfectly

different degrees of predictive power for future returns.

whereas P reflects not just mispricing but scale variability. Alternatively, if there is no variability in fundamental values across firms (if $V_{\bar{\theta}} = 0$), then P_i is a perfect proxy for the signal s_i, and the fundamental-scaled price variable has no additional explanatory power for future returns. Further, if a multiple regression is run with any number of fundamental-scaled price variables, such as book/market and price/earnings ratios, and if the errors of the different fundamental proxies are imperfectly correlated, so that each proxy adds some extra information about $\bar{\theta}_i$, the coefficient on each variable is non-zero. If the errors are independent, then the coefficient on price is negative and the coefficient on each regressor that contains price inversely (such as book/market) is positive.

As the variability in unconditional expected cash flows across securities becomes large, $V_{\bar{\theta}} \to \infty$, $b_P^\dagger \to 0$ and $b_{F-P}^\dagger$ does not. Thus, if the variance of expected cash flows across securities is large relative to the noise in the fundamental proxy, the size variable is dominated by the fundamental/price variable in the multiple regression. More broadly, when $V_{\bar{\theta}}$ is large, the coefficient on size will be less significant than the coefficient on a fundamental/price variable such as book market. Intuitively, if securities have very different expected cash flows (as is surely the case), it becomes very important to find a proxy to filter out scale variation in order to locate mispricing effects. Two recent studies find that in a multivariate regression or cross-classification, book/market is more significant than size (Fama and French (1992), Brennan, Chordia, and Subrahmanyam (1998)); see also Davis, Fama, and French (2000).

The model implies that future returns should be related to market-value, but not to non-price measures of size. Non-price measures of size such as number of employees or book-value are unrelated to the error in the informed's signal e_i, and are therefore also unrelated to the future return on the security. This is consistent with the empirical findings of Berk (1995b).

The analysis suggests that the relationship between book/market and future returns is a valid one rather than an ex post relationship arising from data-snooping. However, there is no implication that there exists any meaningful book/market factor, nor that sensitivities with respect to a factor constructed from book/market portfolios can be used to price assets. In this regard, our analysis is consistent with the evidence of Daniel and Titman (1997) that the book-market effect is associated with the book/market characteristic, not an underlying factor (distinct from the market return). Specifically, our analysis suggests that book/market captures a combination of market risk and mispricing.[20]

[20]To see this, consider the special case in which there is just a single factor (essentially the market) and

Corollary 2 *If the fundamental measure is noiseless, $V^F = 0$, if the expected tendency to overreact approaches infinity, $E[\omega^\epsilon], E[\omega^f] \to \infty$, and variability in overconfidence (as reflected in $V^{\epsilon\omega}$ and $V^{f\omega}$) remains finite then the regression coefficient on β, $b^\dagger_\beta$ in Proposition 3 approaches zero, whereas the coefficient on the fundamental-scaled price approaches unity.*

The limiting case of infinite tendency to overreact occurs with λ^C constant and $\lambda^R \to 0$. In other words, signals become close to pure noise and investors drastically underestimate this noise.[21] Alternatively, λ^C could approach infinity, but this would lead to infinitely volatile prices. In the limit trading strategies can be viewed as noise trading, because nontrivial price revisions are triggered by very little information. As this occurs, the coefficient on β approaches zero, and the coefficient on $F - P$ approaches 1. Intuitively, if investors push prices away from fundamentals based on pure noise alone, then $F - P$ is equal to the sum of (1) the future price change due to the correction of the mispricing, and (2) the future price change due to risk. Since $F - P$ is a proxy for both terms, beta adds no information, and its coefficient is therefore zero.

With regard to the last point, if investors' private signals are very noisy, the statistical relationship between fundamental/price ratios and expected return will be strong, and the relation between β and expected return will be weak. Thus, the theory is consistent with the differing findings of several studies regarding the existence of a cross-sectional relation between return and β after controlling for book-to-market or for market-value (see Appendix B).

Intuitively, a high fundamental/price ratio (low value of market price relative to unconditional expected terminal cash flow) could arise from adverse private information, or from high risk. In either case, price should rise (because overreaction is reversed, or because of a risk premium), but in general by different amounts. If price is low because of a risk premium, on average it should rise back to the unconditional expected terminal value. But if price is low because of adverse information, then the conditional expected value is below the unconditional expected value. A risk measure such as β in general helps disentangle these two cases, so it has incremental explanatory power.

in which there is no information about the factor, only about idiosyncratic security return components. Clearly there is no book/market factor as distinct from the market factor. Nevertheless, a price-scaled fundamental (such as book/market) predicts future returns, and may dominate beta.

[21] As signals become close to pure noise, the rational reaction to a signal becomes close to zero. By equations (8) and (9), if the consensus signal precision approaches zero at the rate of the square root of the rational precision, price volatility is asympotically proportional to 1 (i.e., i.e., it approaches neither zero nor infinity.

However, when overconfident individuals trade based on pure noise ($V^R \to \infty$), the conditional expected value of θ is equal to the unconditional value. Thus, risk measures such as β provide no incremental explanatory power for future returns. In contrast, fundamental/price does have explanatory power when β is held constant, because the fundamental/price ratio reflects both the risk premium and the mispricing.

Analogous to the discussion in Subsection 1.4, it is sometimes asserted that almost any model with valuation errors would imply that high fundamental/price to predict high returns. However, our analysis shows that this is not the case. The conclusion would reverse in settings where investors *under*reacted to private information (such as an *under*confidence setting). In such a setting, a low fundamental/price ratio could reflect favorable private information, so that in the long run market price needs to rise still further. Thus, the direction of the fundamental/price effect is *not* a general implication of irrationality, but an implication of a specific type of bias (overconfidence).

3 Variations in Confidence: Empirical Implications

There is evidence that individuals tend to be more overconfident in settings where feedback on their information or decisions is slow or inconclusive than where the feedback is clear and rapid (Einhorn (1980)). Thus, mispricing should be stronger for businesses which require more judgment to evaluate, and where the feedback on the quality of this judgment is ambiguous in the short run. This line of reasoning suggests that fundamental/price effects should be stronger for businesses that are difficult to value, as with high-tech industries (as measured by high R&D expenditures) or industries (e.g. service industries) with high intangible assets.[22] Subsequent to our developing this prediction, Chan, Lakonishok, and Sougiannis (1999) investigated this question, and have reported evidence consistent with this empirical implication of the model.[23]

A further implication of our approach is based on the notion that market overconfidence can vary over time. To develop this implication, we begin by stating the following

[22]A low book/market ratio is itself an indicator of high intangible assets, but can also be low for other reasons such as a risk premium or market misvaluation. Thus, conditioning on other intangible measures provides a test of how intangible measures affect the misvaluation-induced relation between fundamental-price ratios and future returns.

[23]Their Table 3 sorts firms into five groups based on R&D expenditures relative to sales. They then sort each of these portfolios into high and low sales/market firms. Sales/market is a fundamental/price variable here. The average return differential between the high and low sales/market firms for the low R&D firms was 3.54% per year over the three post-formation years. For the high R&D firms, this differential was 10.17% per year. This evidence indicates that the fundamental/price ratio effect is far higher for the high R&D firms then for the low R&D firms.

Then by symmetry,

$$E[R|X] = \frac{A}{\nu + (1-\phi)\nu^R + \phi\nu^C},$$

which is independent of X and the same as the unconditional expected return. Intuitively, each value of X corresponds to two equiprobable signal values, one of which is associated with high future return and the other with low.

Conditional return volatility is

$$\begin{aligned} \mathrm{var}(R|X) &= E[R^2|X] \\ &= E[(\gamma s + \eta)^2|X] \\ &= \left(\frac{\gamma}{\kappa}\right)^2 X^2 + \sigma^2(\eta). \end{aligned}$$

Thus, conditional volatility increases quadratically in absolute turnover X.

Proposition 4 *1. The market's future volatility is an increasing quadratic function of its current volume.*

2. A greater mass of overconfident/informed individuals uniformly increases the absolute slope of this relationship.

3. A strong relation between volatility and volume is associated with a strong relation between the market fundamental/price ratio and future returns.

Part 2 follows because the greater is the mass of the overconfident/informed, the greater is the ratio γ/κ. Thus the greater the mass of overconfident agents, the stronger is the convex relationship between future volatility and current volume.[25]

Part 3 follows from Proposition 2 applied to the special case of a time-series regression for a single security whose β with the market is know with certainty (= 1), and for which overconfidence is a constant. This eliminates the V^{β_M} terms in equation (26), and sets the $K = 1$, $\omega^\epsilon \equiv 0$, $V^{\epsilon\omega} = 0$, $E[\omega^f] = \omega$ (a constant), $E[\omega^2] = \omega^2$, and $V^{f\omega} = 0$ in equations (23) and (24). It follows from the proposition that the coefficient of the fundamental price ratio is increasing in ω. From the definition of ω (see equation (12), λ (see equation (8)) and ν^A (see equation (6)), this is increasing in ϕ, the fraction of informed/overconfident investors. Thus, Part 3 obtains. Treating shifts in confidence

[25] Our analysis focuses on volume arising from traders receiving information and taking positions based on this information. In a dynamic setting, it would be important to take into account volume generated by unwinding of trades. Intuitively, such volume will be unrelated to future volatility, because it is not indicative of any *current* market price overreaction. This will make volume a noisier predictor of future volatility, but will not reverse the effect identified here.

as exogenous, this suggests during time periods where a stronger convex relationship between volatility and volume obtains, the effect of fundamental/price ratios on price is particularly strong.

Most past empirical studies of volume and volatility focus on relatively shorter horizons of days or weeks. Our results in this section come from corrections of mispricings. Empirical evidence such as fundamental-scaled price variable effects and return reversals suggest that these effects are important at horizons of several years. Thus, the volatility and volume effects that we describe here are predicted to occur at these longer horizons.[26]

Reasoning very similar to the above demonstrates that Proposition 4 also extends to the relation between securities' idiosyncratic volume and their idiosyncratic volatility. This relationship holds even when the number of securities becomes large. However, with many securities, the fraction of idiosyncratically informed traders per security becomes very small, so that information-induced volatility becomes extremely small. The result for individual securities is therefore not very interesting in our setting. However, a broader interpretation of our approach is that market imperfections limit arbitrage of idiosyncractic mispricing (see the discussion near the end of Section 5 regarding this arbitrage strategy). This suggests that there could be a substantial fraction of idiosyncratically informed investors who substantially influence price. In such a broader setting we conjecture that a similar result will apply relating idiosyncratic volume to idiosyncratic volatility for individual securities.

5 Profitability of Trading, Mispricing, and Diversified Arbitrage Strategies

This section analyzes the profitability of trading for the overconfident informed, and the extent to which arbitrageurs can eliminate mispricing through diversified trading

[26]Numerous theoretical papers have analyzed determinants of volume and volatility. However, most focus on a contemporaneous relation between the two. Odean (1998) and Daniel, Hirshleifer, and Subrahmanyam (1998) theoretically examined the relation of overconfidence to volatility and/or volume. However, in Odean's single-period setting volatility is non-stochastic; and Daniel, Hirshleifer, and Subrahmanyam (1998) did not focus on volume. In contrast, here we examine how current volume is correlated with stochastic shifts in subsequent long-horizon volatility. Empirical researchers have identified a contemporaneous volume/volatility relation (see, e.g., Daigler and Wiley (1999)). We are not aware of any empirical work relating variations in volume to subsequent volatility at a long horizon.

strategies.[27,28] To allow for diversified trading strategies, we analyze what occurs in the limit as the number of available securities becomes large. An argument sometimes put forth in favor of efficient markets is that if a large number of securities were mispriced, a portfolio that is long on underpriced stocks and short on overpriced stocks would diversify risk and thus achieve near-infinite Sharpe ratios. This would imply near-infinite volume of trade and a nearly infinite flow of wealth from imperfectly rational traders to arbitrageurs. Thus, it is argued that such mispricing should not exist in equilibrium.

This conclusion does not follow in our setting. Based on a reasonable specification of information arrival, we show that the mispricing of most residuals approaches zero, but factor mispricing does not. Thus, although all securities are mispriced, and trading against mispricing is profitable, there are no riskfree arbitrage opportunities, and volume of trade remains finite. Indeed, for the reasons analyzed by De Long,Shleifer, Summers, and Waldman (1990a, 1991) (DSSW), the overconfident informed in our setting may make as much or greater expected profits than the arbitrageurs. Intuitively, if people believe they have information about a 'new economy' factor (for example), they may misprice an entire industry that loads upon this factor. Playing an arbitrage game based on such a mispriced factor could be profitable, but is certainly risky.

Using the equivalent setting presented in Subsection 1.1.4, we analyze on a security-by-security basis how much an arbitrageur gains as a result of the overconfidence of others in relation to the relative numbers of arbitrageurs versus informed overconfident traders. We then examine overall portfolio profitability, and examine how profitability changes as the number of available securities becomes large.

We compare the expected profit of an individual who is overconfident about every security to that of an arbitrageur whose beliefs about all securities are rational. The expressions derived generalize easily to give the expected return of an individual who receives (and is overconfident about) private information on only a subset of securities. The true expected returns (based on rational beliefs) of an investor's optimal portfolios can be derived from (4) by substituting for expected returns from equation (11), taking

[27]Several previous papers have argued that there are limits to the degree that arbitrage reduces market inefficiencies, and that imperfectly rational or overconfident traders can earn higher expected profits than fully rational traders and therefore can be influential in the long run; see, e.g., De Long, Shleifer, Summers, and Waldmann (1990a), Kyle and Wang (1997), and Shleifer and Vishny (1997). Also, even if overconfident traders make less money, it may also be the case that those traders who make the most money become more overconfident (see Daniel, Hirshleifer, and Subrahmanyam (1998) and Gervais and Odean (1999)).

[28]As described earlier, these arbitrageurs can be viewed either as fully rational uninformed traders, fully rational informed traders, or as overconfident uninformed traders, without affecting the analysis.

expectations over s_i, and simplifying:

$$E^R[R_p^R] = \sum_{i=1}^{N+K} x_i^R E^R[R_i] = \frac{1}{A} \sum_{i=1}^{N+K} \left[\phi_i^2 C_i + \frac{(\nu_i + \nu_i^R)}{(\nu_i + \nu_i^A)^2} (A\xi_i)^2 \right] \tag{32}$$

$$E^R[R_p^C] = \sum_{i=1}^{N+K} x_i^C E^R[R_i] = \frac{1}{A} \sum_{i=1}^{N+K} \left[-\phi_i(1-\phi_i) C_i + \frac{(\nu_i + \nu_i^C)}{(\nu_i + \nu_i^A)^2} (A\xi_i)^2 \right], \tag{33}$$

where

$$C_i \equiv \frac{(\nu_i^C - \nu_i^R)^2 \nu_i^2}{\nu_i^R (\nu_i + \nu_i^R)(\nu_i + \nu_i^A)^2}. \tag{34}$$

The first term in the brackets in equations (32) and (33) is the expected return gain (or loss) that results from the trading on the overconfidence-induced mispricing. The population-weighted sum of the mispricing terms (with weights ϕ_i and $1 - \phi_i$) is zero, i.e., the mispricing results in a wealth transfer from the overconfident/informed to the arbitrageurs.

The second term in the brackets above is larger for the overconfident/informed than for the arbitrageurs (because $\nu_i^C > \nu_i^R$). Since an overconfident informed individual underestimates risk, he holds a larger position in riskier assets than does an arbitrageur, and thereby captures a greater risk premium (the right-hand-side term in brackets); see DSSW. Thus, whether the overconfident make more or less money than would a rational 'arbitrageur' depends on a balance of effects. There is no presumption that rational traders will drive out overconfident ones.

In order to analyze mispricing a limiting economy with many securities, we need to be more specific about how many signals of different kinds individuals obtain. We assume first that each individual receives a finite number of signals about firm-specific payoffs. Implicitly this reflects the notion that it is costly to obtain information about a very large number of firms. Second, each individual receives at least one factor signal, with no factor shunned by all individuals. This reflects in a simple way the intuitive notion that even if individuals were to study only individual firms, since the aggregate market is the sum of its constituent firms, such study provides information about the fixed set of market factors. Third, we assume that who observes what factor is independent of the number of securities. Assumptions 2 and 3 imply that as the number of securities N grows, a fixed positive fraction of the population continues to receive a signal about any given factor. (Assumption 3 is not important for the result, but allows simple presentation.)

Mathematically, the assumption that each individual observes only a finite number of signals implies that for all but a finite number of the N securities, the fraction of individuals who are informed about the residual component is $O(1/N)$ (i.e., the fraction approaches zero proportional to $1/N$). This is consistent with a finite number of firms receiving a great deal of attention; for example, even if there are many internet stocks, a few like Amazon.com or Yahoo may garner attention from a substantial portion of the public. In contrast, the assumption that everyone observes a factor signal (and no factor is shunned) in a fixed observation structure implies that as N varies the fraction of individuals informed about any given factor is a positive constant.

Now, equation (10) can be rewritten as:

$$E^R[R_i] = -\phi_i \begin{pmatrix} \nu_i^C - \nu_i^R \\ \nu_i + \nu_i^R \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_i + \nu_i^A \end{pmatrix} s_i + \begin{pmatrix} A \\ \nu_i + \nu_i^A \end{pmatrix} \xi_i$$

giving the expected return for each portfolio. The first term is the extra return from mispricing, and is proportional to ϕ_i. When the fraction of overconfident informed investors is small, the rational investors compete to drive away almost all mispricing. This removes the component of return deriving from correction of mispricing (the term multiplied by s_i). When the number of securities N is large, and since for residuals, $\phi_i^\epsilon = O(1/N)$, the mispricing term tends to zero at the rate $1/N$. In contrast, factor mispricing in *not* arbitraged away; for factors, $\phi_i^f = O(1)$, i.e., ϕ_i^f does not approach zero as N becomes large.

Consider how much an arbitrageur gains as a result of mispricing. Rewriting equation (32) in terms of factors and residuals gives

$$E^R[R_p^R] = \frac{1}{A}\sum_{i=1}^{K} \phi_i^2 C_i + \frac{1}{A}\sum_{i=K+1}^{N+K} \phi_i^2 C_i + \frac{1}{A}\sum_{i=1+1}^{N+K} \frac{(\nu_i + \nu_i^R)}{(\nu_i + \nu_i^A)^2}(A\xi_i)^2. \tag{35}$$

The last term is the risk premium. The second term is the portfolio return gain for the arbitrageur resulting from mispricing of residuals, and is proportional to $(\phi_i^\epsilon)^2$. Since $\phi_i^\epsilon = O(1/N)$, and there are N residuals, this term approaches zero at the rate $1/N$ (i.e., it is $O(1/N)$) so long as the cross-sectional variation in C_i is not too large (e.g., the C_i's are bounded above by a finite number):

$$\frac{1}{A}\sum_{i=K+1}^{N+K} (\phi_i^\epsilon)^2 C_i = O\left(\frac{1}{N}\right).$$

The arbitrageur's portfolio variance remains above zero, even in the limit as $N \to \infty$, so the Sharpe ratios of the arbitrageurs' portfolios do not explode, even for large N.

Despite disagreement, volume of trade in each security remains finite. Under the assumption of Section 4 that the date 0 endowments of all agents are identical (and therefore equal to ξ_i), an arbitrageur's expected date 1 trade in security i is

$$E[x_i^R - \xi_i] = -\phi_i \begin{pmatrix} \nu_i^C - \nu_i^R \\ \nu_i + \nu_i^A \end{pmatrix} \xi_i.$$

This is negative because, on average, an overconfident individual underestimates the risk of a security he has information about, and therefore tends to purchase it.

Beyond this non-stochastic risk-sharing component of volume is the volume induced by the value of the signal realization. This extent of this stochastic volume is measured by the variability of the trade. The variance of the trade is obtained by substituting equations (8) and (11) into equation (4), and taking expectations over s_i:

$$var(x_i^R - \xi_i) = \phi_i^2 \nu_i \begin{pmatrix} \nu_i + \nu_i^R \\ \nu_i^R \end{pmatrix} \begin{pmatrix} \nu_i^R - \nu_i^C \\ A(\nu_i + \nu_i^A) \end{pmatrix}^2 = O\begin{pmatrix} 1 \\ N^2 \end{pmatrix}$$

for residual (but not factor) portfolios. In the limit, the variability of the arbitrageur's trade in each residual portfolio approaches zero. So does the risk-shifting component of volume mentioned earlier. It follows that expected per-capita volume $E(|x_i^R - \xi_i|)$ also approaches zero for residual portfolios. Intuitively, as the number of securities grows large, if each individual only receives information about a finite number of securities, residual mispricing becomes very small so arbitrageurs take vanishingly small bets on each residual. Thus, even with many securities the volume of trade based on residual mispricing remains bounded. We summarize this analysis as follows.

Proposition 5 *Suppose that C_i as defined in (34) is bounded above, that each individual receives a finite number of signals about firm-specific payoffs, that each individual receives at least one factor signal with no factor shunned by all individuals, and that who observes what factor is independent of the number of securities. Then as the number of securities N grows large:*

1. *The idiosyncratic mispricing of all but a finite number of residuals approaches zero.*
2. *The systematic mispricing of each of the K factors is bounded above zero*
3. *Per capita volume in every residual approaches zero.*

Going beyond the formal model, transaction costs of trading could limit arbitrage activity enough to allow substantial idiosyncratic mispricings to persist. For example,

if a set of residuals are underpriced, an arbitrageur must buy the underpriced securities and short the correct amount of each of the relevant factor portfolios, each of which is constructed from many securities. In a dynamic world with evolving factor sensitivities, the weights on securities within each portfolios and the weights placed upon each of these portfolios would have to be readjusted each period. Taking into account the cognitive costs of identifying mispriced securities and of calculating optimal arbitrage strategies would widen the bounds for possible mispricing. Thus, the conclusion that idiosyncratic mispricing vanishes may be sensitive to our assumptions of perfect markets and near-perfect rationality. In contrast, the conclusion that factor mispricing persists is robust.

6 Conclusion

This paper offers a multisecurity theory of asset pricing based on investor risk aversion and overconfidence. In the model covariance risk and mispricing jointly determine the cross-section of expected security returns. Several insights derive from this approach:

- Generally, beta and price-related misvaluation measures jointly predict future returns. However, when investors are overconfident about pure noise and the fundamental measure is perfect, fundamental/price ratios completely dominate beta–even though covariance risk is priced. These results are consistent with the joint effects found in several empirical studies.

- Existing evidence on the psychology of the individual is consistent with capital market evidence of predictability of returns based on size and fundamental/price ratios. The pervasive psychological bias of overconfidence implies stock market overreaction and correction. This leads to a positive fundamental/price effect and a negative size effect. Pure underreaction, as would be implied by underconfidence, would reverse the directions of these effects, inconsistent with the evidence.

- The analysis provides a conceptual basis for choosing between alternative measures of mispricing as predictors of future returns. Normalizing price by a fundamental measure such as book value helps filter out variations in market value that arises from differences in scale rather than mispricing (book-to-market versus firm size). However, a fundamental measure such as book value measures scale (unconditional expected payoffs) with error, and thereby introduces its own noise. Adjusting a mispricing measure by examining deviations from industry levels can filter out

industry-related noise in the fundamental measure, at the cost of filtering out some industry-level mispricing as well. The theory therefore offers predictions about the relative predictive power of firm size (market value), fundamental-scaled variables such as book/market or p/e, variables based on deviations from industry averages, and variables that use constructed accounting measures of fundamentals.

- The theory provides additional empirical implications regarding the strength of fundamental/price effects in stocks that are difficult to value, about the cross-sectional dispersion and predictive power of fundamental/price ratios as a function of aggregate levels of fundamental/price ratios, and about the relation of volume to subsequent returns. One of these implications has received some ex post validation (see Section 3), but most remain to be tested.

- When there are many securities, owing to the activities of 'risk arbitrageurs', misvaluations of most idiosyncratic components of security payoffs approach zero. In contrast, misvaluation of industry or market-wide factors persists. In our model size and value funds can be built to exploit factor mispricing. Such funds do not provide arbitrage profits because they load on systematic risk factors.

It is likely that the relative numbers of overconfident individuals and fully rational 'arbitrageurs' varies with stock characteristics such as liquidity and firm size. Explicit modeling of market imperfections, such as fixed setup costs of trading in a given security, may offer further implications for the cross-section of expected security returns.

The effects of risk and mispricing in our model separate additively into a 'beta' term and a set of 'mispricing' terms, where factor mispricing is inherited by securities according to their factor sensitivities. The challenge for implementing the model empirically is to identify good proxies for security and/or factor mispricing. Our main focus has been on fundamental/price ratios as possible proxies for market mispricing. However, fundamentals such as dividends, earnings and book value are very crude proxies for unconditional expected value. An accounting index in which some of the noise is removed should be a better forecaster of future returns. Abarbanell and Bushee (1999) and Frankel and Lee (1996) show that such an approach can be effective.

Another possible proxy for market misvaluation could be publicly disclosed insider trading; Lakonishok and Lee (1998) provide evidence that imitation of insider trades for up to about two years after disclosure is a profitable strategy even after controlling for size and book/market. Another set of possible proxies for market misvaluation

involve corporate actions such as aggregate new-issue versus repurchase activity. Indeed, Loughran and Ritter (1995) explicitly propose that managers time new issues in order to exploit market misvaluation.

It would be interesting to extend our approach to address the issues of market segmentation and closed-end fund discounts, in the spirit of De Long, Shleifer, Summers, and Waldmann (1990a), and Lee, Shleifer, and Thaler (1991)). In the pure noise approach, discounts reflect mispricing and therefore forecast future stock returns. In our approach, since the mispricing arises from overreaction to genuine information, changes in fund discounts should predict not just future stock performance, but also future fundamentals such as accounting performance. Swaminathan (1996) finds such predictive power for future fundamentals, which he interprets as tending to support a rational risk premium hypothesis as opposed to a noise/sentiment approach. His evidence at lags of greater than one year is surprising, because high discounts predict both low future accounting profits and high future stock returns. This evidence is consistent with an overconfidence approach, wherein genuine adverse information is associated with large discounts and low future profitability, yet high future stock returns as the market corrects its overreaction.

An important question is whether the misvaluation effects identified in our model should persist in the long run. We mentioned models in which biased learning can cause traders, based on experience, to become more overconfident instead of converging toward rationality. In our model (see Section 5), the overconfident can make greater expected profits than rational traders, a possibility demonstrated in several papers cited earlier.

Stepping beyond the model, suppose initially arbitrageurs are not sure whether there are overconfident traders in the market, and that some sort of noise prevents an arbitrageur from inferring instantly and perfectly the information of overconfident traders. Over time, by statistical analysis of the history of fundamentals and prices, arbitrageurs will learn that other players were in fact overconfident. This encourages more aggressive contrarian trading strategies. (One might interpret these arbitrageurs as 'quants'.) Thus, one interpretation of the high predictability of stock returns over the last several decades is that some investors are overconfident, and this was not fully recognized by other investors who could have exploited this. This interpretation suggests that as arbitrageurs' expectations become more accurate, anomalous predictability of returns should diminish but not vanish. However, arbitrageurs themselves could be overconfident about their abilities to identify statistical patterns or be too attached to the patterns they have identified. If so, then mispricing effects could fluctuate dynamically over time.

Appendices

A A Guide to the Model Notation

Symbol	Definition	Description
θ_i	$\bar{\theta}_i + \sum_k \beta_{ik} f_k + \epsilon_i$	Security i payoff at date 2; price at date 2
$\mu_i^\bullet$	$\equiv E^\bullet[\theta_i]$	Expected payoff based on all information at date 1, $\bullet \in \{C, R\}$
P_i		Date 1 Security Price
R_i	$\equiv \theta_i - P_i$	date 1-2 price change
θ_i		Fundamental value, or expected security i payoff based on all info
$\bar{\bar{\theta}}, V_{\bar{\theta}}$		Cross-sectional mean and variance of $\bar{\theta}_i$.
V_k^f, V_i^ϵ	$= E[(f_k)^2], E[(\epsilon_i)^2]$	factor k/residual i variance
A		Investor coefficient of absolute risk aversion
$x_i^\bullet$		Number of shares of portfolio i held by informed (over$C$onfident) agent or uninformed (Rational) agent, $\bullet \in \{C, R\}$.
ϕ_i		Fraction of population informed about portfolio i
ξ_i		Per-capital endowment of portfolio i, or market weight of i
ξ_i'	see eqn. (15)	Adjusted per-capital market weight of portfolio i
R_M	$\equiv \sum_i \xi_i'(\theta_j - P_j)$	Adjusted Market price change
s_i^ϵ, s_k^f	$= \epsilon_i + e_i^\epsilon, \quad f_k + e_k^f$	factor/residual signal received by informed at date 1,
$V_i^{\bullet\epsilon}, V_k^{\bullet f}$	$= E^\bullet[(e_i^\epsilon)^2], E^\bullet[(e_k^f)^2]$	$\bullet \in \{C, R\}$ assessment of residual or factor signal error variance
F_i	$= \bar{\theta}_i + e_i^F$	Fundamental measure – a noisy proxy for $\bar{\theta}_i$
V^F	$= E\left[(e^F)^2\right]$	Variance of the fundamental measure of a randomly selected security
ν_i	$= 1/V_i$	The prior precision of factor or residual i
$\nu_i^\bullet$	$= 1/V_i^\bullet$	I's over$C$onfident or true (Rational) assessment of i'th signal's precision
ν_i^A	$\equiv \phi_i \nu_i^C + (1-\phi_i)\nu_i^R$	Consensus assessment of precision of i'th signal
λ_i	$= \nu_i^A/(\nu_i + \nu_i^A)$	Weight placed by the market on signal i
$\lambda_i^\bullet$	$= \nu_i^\bullet/(\nu_i + \nu_i^\bullet)$	Weight placed by Over$C$onfident I or ARbitrageur on signal i
$s_i^\bullet$		$(\bullet \in \{f, \epsilon\})$ - signal about factor or residual i
S_i	$\equiv \lambda_i^R s_i$	Normalized signal i
ω_i	$\equiv (\lambda_i^C - \lambda_i^R)/\lambda_i^R$	Informed's relative overconfidence for signal i
α	$\equiv A\text{var}^C(R_M)$	Coefficient of absolute risk aversion times the market variance
β_{iM}	$= \frac{cov(\theta_i, R_M)}{var(R_M)}$	price-change beta of firm i with respect to the adjusted market
$E[\beta_k], V^{\beta_k}$		The first and second moments of the loading of a randomly selected security on factor k
$E[\beta_M], V^{\beta_M}$		Moments of the adjusted market beta of a randomly selected security
$E[\omega^\bullet], V^{\bullet\omega}$		Moments of the excess sensitivity to residual variance $(\bullet \in \{f, \epsilon\})$.
$V_i^{S\bullet}$		second moment of the normalized factor/residual signal $(\bullet \in \{f, \epsilon\})$
cov_{OC}	eqn (23), page 19	Covariance between R and $P - \theta$ attributable to Is overconfidence in their private signals
var_{OC}	eqn (24), page 19	Variance of $P - \bar{\theta}$ that can be attributed to signals received by Is
$b_{\bar{\theta}-P}$	eqn (22), page 19	Coefficient in linear projection of R on $\bar{\theta} - P$
b_{F-P}	eqn (25)	Coefficient in linear projection of R on $F - P$
$b_\beta^\dagger, b_{P-F}^\dagger, b_P^\dagger$	eq. (28)-(30), p. 22	Coefficients on β_M, $F-P$, and P in linear projection of R on β_M, $F-P$ and P
γ, η		Coefficient and error term in projection of R on s
Π	$\equiv \frac{(1-\phi)\nu^R + \phi\nu^C}{\nu + (1-\phi)\nu^R + \phi\nu^C}$	Coefficient on s in the equilibrium formula for price P
κ	$\equiv -\frac{2(1-\phi)(\nu+\nu^R)\gamma}{A\nu}$	Coefficient in formula for signed turnover in terms of s in (31)
X		Volume (absolute signal induced turnover)

C Proofs

Proof of Proposition 2 Since the signals in the model are mean zero, by equation (18), $E[R] = E[\bar{\theta} - P] = \alpha E[\beta_M]$. From equations (17) and (18) (and applying the law of iterated expectations),

$$cov(R, \bar{\theta} - P) = E\left[\omega^\epsilon(1+\omega^\epsilon)(S^\epsilon)^2 + \sum_{k=1}^{K} \beta_k^2 \omega_k^f (1+\omega_k^f)(S_k^f)^2 + \alpha^2 \beta_M^2\right] - \alpha^2 E[\beta_M]^2.$$

Now, applying $V^{\epsilon\omega} \equiv var(\omega^\epsilon) = E[(\omega^\epsilon)^2] - (E[\omega^\epsilon])^2$ and $V^{\beta_M} \equiv E\left[(\beta_M)^2\right] - (E[\beta_M])^2$ we have

$$\begin{aligned} cov(R, \bar{\theta} - P) &= \alpha^2 V^{\beta_M} - cov_{OC} && (36) \\ var(\bar{\theta} - P) &= \alpha^2 V^{\beta_M} + var_{OC} && (37) \end{aligned}$$

where cov_{OC} and var_{OC} are as defined in the text. ||

Proof of Proposition 3 Let the vector $X \equiv [\beta_M, F - P, P]$ and define

$$\Sigma_{YX} \equiv [\text{cov}(R, \beta_M),\ \text{cov}(R, F - P),\ \text{cov}(R, P)].$$

Further, let Σ_{XX} denote the variance-covariance matrix of $\beta_M, P - F$ and P. Then the OLS predictor of R is $\Sigma_{YX}\Sigma_{XX}^{-1}X$. The vector of regression coefficients in (27) can therefore be written as

$$[b_\beta^\dagger,\ b_{F-P}^\dagger,\ b_P^\dagger] = \Sigma_{YX}\Sigma_{XX}^{-1}. \tag{38}$$

By (36) and (37), the covariances and variances required to calculate Σ_{YX} and Σ_{XX} are

$$\begin{aligned} cov(R, P) &= -\alpha^2 V^{\beta_M} \\ cov(R, F - P) &= -cov_{OC} + \alpha^2 V^{\beta_M} \\ var(P) &= var_{OC} + \alpha^2 V^{\beta_M} + V_{\bar{\theta}} \\ var(F - P) &= var_{OC} + \alpha^2 V^{\beta_M} + V^F \\ var(\beta_M) &\equiv V^{\beta_M} \\ cov(R, \beta) &= \alpha V^{\beta_M} \\ cov(\beta, F - P) &= \alpha V^{\beta_M}. \end{aligned}$$

Explicitly calculating these coefficients and substituting into (38) yields the expressions in Proposition 3. ||

Proof of Corollary 2: In equation (28), since $V^F = 0$, $K_1 = 0$. The term in parentheses is therefore $1 + (cov_{OC}/var_{OC})$. As $E[\omega^\epsilon], E[\omega^f] \to \infty$ in equations (23) and (24), and noting that $E[(\omega^\epsilon)^2] = V^{\epsilon\omega} + E[\omega^\epsilon]^2$ and $E[(\omega^f)^2] = V^{f\omega} + E[\omega^f]^2$, we see that the terms containing $E[(\omega^f)^2]$ and $E[(\omega^\epsilon)^2]$ dominate constants (1 or V^ω) and the linear terms containing $E[\omega^f]$ or $E[\omega^\epsilon]$. It follows that in the limit, $cov_{OC}/var_{OC} = -1$, so the term in parentheses in (28) approaches zero, proving the result. ||

Proof of Corollary 3: As shown in the Proof of Proposition 3:

$$var(F - P) = var_{OC} + \alpha^2 V^{\beta_M} + V^F$$

Under the assumptions of Corollary 2, and from the definition of var_{OC} in equation (37), $var_{OC} \to \infty$ as $E[\omega^\epsilon], E[\omega^f] \to \infty$. This proves part 1.

Next, from equations (20), (2), and (17),

$$R_i \equiv \theta_i - P_i = (F_i - P_i) - e_i^F + S_i^\epsilon + \sum_{k=1}^{K} \beta_{ik} S_k^f + (\epsilon_i - E[\epsilon_i]) + \sum_{k=1}^{K} \beta_{ik}(f_k - E[f_k])$$

Since, the expectations in this expression are rational,

$$var(R_i - [F_i - P_i]) \leq var\left(-e_i^F + S_i^\epsilon + \sum_{k=1}^{K} \beta_{ik} S_k^f + \epsilon_i + \sum_{k=1}^{K} \beta_{ik} f_k\right)$$

Since the cross-sectional variance of everything on the RHS of this expression, with the exception of $F - P$, remains finite, but $var(F - P) \to \infty$ as $E[\omega^\epsilon], E[\omega^f] \to \infty$, this means that $var(R)/var(F - P) \to 1$. Since the R^2 of the regression of R on $F - P$ is given by

$$R^2_{F-P} = \frac{[cov(R, F - P)]^2}{var(R)var(F - P)},$$

and since, as shown in the proof of Corollary 2, in the limit, $cov_{OC}/var_{OC} \to -1$, it follows that $R^2 \to 1$, proving part 2. ||

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