MONTE CARLO EVALUATION MODEL OF AN UNDEVELOPED OIL FIELD*

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ABSTRACT

In this article we develop and implement a model to value an undeveloped oil field and to determine the optimal timing of investment. We assume a two factor model for the stochastic behavior of oil prices for which a closed form solution for futures prices can be obtained. The advantage of this model is that it allows for the term structure of futures prices to be upward sloping (contango), downward sloping (backwardation) and also humped. We use Monte Carlo simulation methods for solving the problem. Since the decision to develop the oil field can be taken at any time until the expiration of the concession, the option to invest is of the American type. This type of options are solved by the numerical solution of the appropriate partial differential equation. If we assume, however, that the decision to invest (exercise the option) can be made at a finite number of points in time instead of continuously, the problem can be solved using simulation methods. Apart from being more intuitive, Monte Carlo simulation methods easily allow for the consideration of many additional random variables such as costs, amount of reserves, etc.

1. INTRODUCTION

An undeveloped oil field has value because it might be developed someday and their oil production sold. The determination of the value that someone would be willing to pay to buy the oil field and the criteria to decide when is the optimal time to develop the field are key questions in finance. The traditional approach to address these issues is the Net Present Value method which essentially discounts the expected net cash flows from the project (value of the oil production minus the costs of production) at a rate that reflects the risks of these cash flows. The value of the project is then the present value of these future cash flows minus the initial investment, and the criteria for optimal investment is to invest if this value is positive. The great advantage of this approach is its simplicity. It has, however, major weaknesses. First, oil prices are very unpredictable so it is not an easy task to estimate future cash flows. Second, the method does not take into account the flexibilities available in a typical project, such as the option to postpone the investment, to increase production in case prices increase and to reduce it if prices decrease, and the option to abandon the project if prices are too low. Third, the estimation of an appropriate discount rate which reflects the risks of the cash flows is also difficult since it should take into account the operating leverage of the project and its flexibilities.

A variation of the Net Present Value approach which can deal with some of the weaknesses discussed above is the Certainty Equivalent approach. The certainty equivalent net cash flow is that net cash flow which would be accepted for certain by the decision maker instead of the risky net cash flow. Since in this method the adjustment for risk is done in the cash flows these can be discounted at the risk free rate of interest. In this method certainty equivalent cash flows are discounted at the risk free rate of interest. In most situations this procedure is not easy to apply since

it is difficult to estimate the certainty equivalent cash flows. When futures markets for the underlying asset exist, however, such as in the case of oil, the futures price of maturity T is the certainty equivalent of the spot to be received at time T. These futures prices can then be used to compute the certainty equivalent cash flows in the future which are discounted at the risk free rate of interest.[1]

Recently, a new approach to the valuation of projects which addresses the weaknesses of the Net Present Value approach and uses the information contained in futures prices has been developed and implemented. It is called the Real Options approach to valuation and is based on the important analogy between financial options and investment projects. In this approach a project is considered as an option on the underlying cash flows and the optimal criteria for investment is the optimal exercise of the option. No assumptions are needed about the future path of oil prices; instead futures prices are used in the calculations. Also, there is no need to calculate a risk adjusted discount rate since options are valued using the risk free rate of interest. Finally, the method allows for the appropriate consideration of the relevant flexibilities or options available.

In this article we use the Real Options approach to value an undeveloped oil field and to determine the optimal timing of investment. Our analysis differs from previous work in two ways. First, we assume a two factor model for the stochastic behavior of oil prices for which a closed form solution for futures prices can be obtained. The advantage of this model is that it allows the futures price curve to be upward sloping (contango), downward sloping (backwardation) and also humped.

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[1] More precisely, the price risk is eliminated. Other types of risk such as those related to costs, volume of production, etc. remain. If we assume that these types of risk are diversifiable, we can still use the risk free rate to discount cash flows.

This is important since empirically we observe all these types of term structures of futures prices.

The second way in which our article differ from previous work is that we use Monte Carlo simulation methods for solving the problem. Since the decision to develop the oil field can be taken at any time until the expiration of the concession, the option to invest is of the American type. This type of options are solved by the numerical solution of the appropriate partial differential equation. If we assume, however, that the decision to invest (exercise the option) can be made at a finite number of points in time instead of continuously, the problem can be solved using simulation methods.[2] Apart from being more intuitive, Monte Carlo simulation methods easily allow for the consideration of many additional random variables such as costs, amount of reserves, etc.

In Section 2 we develop the model first defining the stochastic process for oil prices, then valuing the developed oil field and finally evaluating the undeveloped oil field. Section 3 deals with the implementation issues such as the time-discretization of the stochastic process, estimation of the parameters of the model and estimation of the state variables on the valuation day from the traded futures contracts. Section 4 applies the methodology to the valuation of a specific undeveloped oil field. Section 5 provides some concluding remarks.

2. THE MODEL

We will develop the model in three stages. In the first stage we will make the necessary assumptions about the stochastic process of spot prices which is an essential ingredient to any option model. In the second stage we will determine the value of the developed oil field once the investment to produce has been made. The value of the developed oil field is what is obtained

[2] Financial options that can be exercised at discrete points in time are usually called of the Bermudian type.

when the option to invest is exercised. In the third stage we will develop the Monte Carlo simulation method we use to value the undeveloped oil field and to determine the optimal timing of investment.

2.1 The Stochastic Process

We assume a two factor model for the stochastic behavior of oil spot prices. The spot price follows a geometric Brownian motion with a stochastic convenience yield, which itself follows a mean reverting process. This model was originally proposed by Gibson and Schwartz (1990). Schwartz (1997) shows that this model is able to capture many of the characteristics of the oil and copper term structures of futures prices and volatilities.

The first factor is the spot price of the commodity, S, and the second is the instantaneous convenience yield, δ. These factors are assumed to follow the joint stochastic process:

$$dS = (\mu - \delta) S\,dt + \sigma_1 S\,dz_1 \tag{1}$$

$$d\delta = \kappa(\alpha - \delta)\,dt + \sigma_2\,dz_2 \tag{2}$$

where the increments to standard Brownian motion are correlated:

$$dz_1\,dz_2 = \rho\,dt \tag{3}$$

In this model the commodity is treated as an asset which pays a stochastic dividend yield δ. Then in equation (1) μ would be the 'total return' on the spot price (price appreciation plus convenience yield). In equation (2) the magnitude of the speed of adjustment $\kappa > 0$ measures the degree of mean reversion to the long run mean convenience yield α. ρ is the correlation between the two processes.

The relevant stochastic process for pricing derivatives on the commodity (futures,

options and projects) is not the true process described above, but the risk adjusted process. Since we assume that the commodity is a traded asset the risk adjusted drift of the commodity price process will be r - δ, where r is the continuously compounded risk free rate of interest. Since convenience yield risk cannot be hedged the risk adjusted convenience yield process will have a market price of risk associated with it. The risk adjusted stochastic process for the factors can be expressed as:

$$dS = (r - \delta) S\, dt + \sigma_1 S\, dz_1^* \tag{4}$$

$$d\delta = [\kappa(\alpha - \delta) - \lambda] dt + \sigma_2 dz_2^* \tag{5}$$

$$dz_1^* dz_2^* = \rho\, dt \tag{6}$$

where λ is the market price of convenience yield risk, which is assumed constant. Equation (5) can also be written as:

$$d\delta = \kappa(\hat{\alpha} - \delta) dt + \sigma_2 dz_2^* \tag{7}$$

where:

$$\hat{\alpha} = \alpha - \frac{\lambda}{\kappa} \tag{8}$$

is the risk adjusted long run mean convenience yield.

In Section 3.1 we show how to use a discrete time version of this stochastic process in the Monte Carlo simulations to obtain the value of the undeveloped oil field.

In this model there is a closed form solution for futures prices which considerably facilitates the valuation of oil related cash flows when there are no option components attached to them. The futures price for a contract with maturity T is given by:

$$F(S,\delta,T) = S\exp[-\delta\frac{1-e^{-\kappa T}}{\kappa}+A(T)] \tag{9}$$

where

$$A(T) = (r-\hat{\alpha}+\frac{1}{2}\frac{\sigma_2^2}{\kappa^2}-\frac{\sigma_1\sigma_2\rho}{\kappa})T+\frac{1}{4}\sigma_2^2\frac{1-e^{-2\kappa T}}{\kappa^3}+(\hat{\alpha}\kappa+\sigma_1\sigma_2\rho-\frac{\sigma_2^2}{\kappa})\frac{1-e^{-\kappa T}}{\kappa^2} \tag{10}$$

Note that since we are assuming that interest rates are constant futures and forward prices are identical.

2.2 Value of the Developed Oil Field

We assume that the geological conditions of the field are known so that once the investment has been made the production of oil per year and all the costs needed to take this production to the appropriate market are also known[3]. Then we can use the certainty equivalent

[3] Given the numerical method we employ, these variables could also follow stochastic processes. In reality, here we only assume that we can use the expected value of these variables

approach to obtain the present value of the cash flows from the developed field. The certainty equivalent price of the spot price in the future is the futures price of oil. Once the cash flows are computed using these futures prices they can be discounted at the risk free rate of interest.

Let P_t be the number of barrels of oil to be produced in year t, C_t the total cost of production in year t, D_t the depreciation allowed in year t, and τ the applicable corporate tax rate. If S is the oil spot price and δ the instantaneous convenience yield at the time the valuation is made and assuming that the cash flows are generated at the end of each year, the value of the developed field will be:

$$VD(S,\delta) = \sum_{t=1}^{N} e^{-rt}[(P_t F(S,\delta,t) - C_t)(1-\tau) + \tau D_t] \qquad \textbf{(11)}$$

where N is the life of the oil field once production has started. In this formulation it is assumed that once the oil field is developed all production will be taken according to schedule and that the depreciation tax shield will always be allowed. Both assumptions could be relaxed, but then this part of the analysis would also have to be solved using the simulation methods described in the following section. The assumption that the cash flows are generated at the end of each year could also be trivially relaxed.

2.3 Value of the Undeveloped Oil Field and Optimal Time to Invest

The value of the undeveloped oil field can be considered as an option in which the underlying asset is the value of the developed field and with an exercise price equal to the development investment. That is, the owner of an undeveloped field has the option to pay the

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and that the risks associated with these variables can be diversified away so that the appropriate discount rate is the risk free rate.

development investment and obtain the underlying developed field. Since this investment can be made any time before the concession expires (or any time in the future if there is no expiration date) the option is of the American type.

Very recently a number of Monte Carlo simulation methods have appeared in the literature to price high-dimensional (many underlying factors) American options[4]. In this article we develop a procedure to value the undeveloped oil field based on the Monte Carlo simulation method proposed by Barraquand and Martineau (1995) to value financial options.

The approach consists in partitioning the simulated paths into groups based on the one-dimensional option (developed oil field) payoff space, reducing the high-dimensional problem to one dimension. In this case the problem has two stochastic state variables (the spot price and the instantaneous convenience yield), but the idea is that it can be extended to any number of stochastic state variables such as costs, amount of oil, etc.

First, we assume that the decision to develop the oil field can be made at any discrete time t from t=1 to t=T (it could be once a year or once a month), where T is the expiration of the concession. Then, we do a preliminary simulation to be able at each time t to partition the payoff space into K bins, indexed k=1,....K. For each simulated value of S and δ, we compute the value of the developed oil field (payoff) using equation (11) and we order these payoffs in decreasing order such that each bin contains the same number of simulations.[5] If we do M preliminary simulations, at each time t each bin will contain M/K simulations.

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[4] See Broadie and Glasserman (1997) for a recent survey.

[5] This procedure to construct the bins is suggested in Raymar and Zwecher (1997).

The second step consists in doing a new set of NSIM simulations to estimate the transition probabilities and path payoffs. For each payoff bin k at time t, let $a_t(k)$ record the number of paths that fall into the bin. For each pair of bins k and j at consecutive times t and t+1, let $b_t(k,j)$ record the number of paths that fall into both bins. Finally, for each bin k at time t, let $c_t(k) = \Sigma$ VD(S,δ) record the sum of the payoffs for all the paths that fall into bin k at time t. As each path is generated, the records $a_t(k)$, $b_t(k,j)$ and $c_t(k)$ are incremented as indicated and then the path can be discarded.

The third step is to solve the resulting one-dimensional dynamic programming problem. Let V(t,k) denote the estimated value of the undeveloped field for bin k at time t. Starting at time T, set the estimated value as the maximum of the average payoff of the paths that finish at bin k at time T minus the development investment, I, and zero:

$$V(T,k) = Max[\frac{c_T(k)}{a_T(k)} - I \ , \ 0] \tag{12}$$

Next work backward from time t = T-1 to t = 1. For bin k at time t, the estimated value is the maximum of the average payoff minus the development investment and the estimated continuation value. To estimate the continuation value we first approximate the transition probability from bin k at time t to bin j at time t+1:

$$p_t(k,j) = \frac{b_t(k,j)}{a_t(k)} \tag{13}$$

Then, the recursive step up to t = 1 is

$$V(t,k) = Max[\frac{c_t(k)}{a_t(k)} - I \ , \ e^{-r\Delta t}\sum_{j=1}^{K} p_t(k,j)V(t+1,j)] \tag{14}$$

where Δt is the time between decision points.

Finally, the value of the undeveloped oil field for initial values of the state variables S_0 and δ_0 (at time t=0) is the maximum of developing the field immediately and the continuation value:

$$VU(S_0,\delta_0) = Max[VD(S_0,\delta_0) - I \ , \ e^{-r\Delta t}\sum_{j=1}^{K} \frac{a_1(j)}{NSIM} V(1,j)] \tag{15}$$

This equation also gives the investment criteria: develop the field immediately only if the developed field value is greater than the continuation value.

We have used this "payoff stratification" method because it is easy to implement, fast to run and very flexible. However, in certain situations it may not converge to the correct value especially if the exercise region is composed of two or more disjoint areas. In our case this issue does not seem to be a problem since the exercise area is not disjoint (high values of S and low values of δ). Raymar and Zwecher (1997) have recently developed a new simulation procedure in which many of these problems can be avoided by partitioning on two or more state variables rather that one.

3. IMPLEMENTATION OF THE APPROACH

3.1 Discrete Time Version of the Stochastic Process

To be able to use the continuous time model described in equations (4) to (6) in the

Monte Carlo simulations we first have to obtain the distribution of the (risk adjusted) spot price and convenience yield for a finite period of time in the future. The distribution of the risk adjusted spot price at any time T in the future is log-normal with mean equal to the futures price with maturity T given in equation (7). The variance (of the log price) is obtained by integrating the variance of the rate of return on the futures up to T and is given by:

$$v(T) = (\sigma_1^2 + \frac{\sigma_2^2}{\kappa^2} - \frac{2\rho\sigma_1\sigma_2}{\kappa})T + \frac{\sigma_2^2(1-e^{-2\kappa T})}{2\kappa^3} + 2\sigma_2(\sigma_1\rho - \frac{\sigma_2}{\kappa})\frac{(1-e^{-\kappa T})}{\kappa^2} \tag{16}$$

Since we are simulating values for fixed periods of time, say one year, this expression is a constant with T=1. In practice we simulate the log of the spot price which is normally distributed.

The distribution of the convenience yield at any time T can be obtained by integrating equation (7) form 0 to T. The resulting distribution is normal with mean and variance given by:

$$\delta(T) \sim N[\delta(0)e^{-\kappa T} + (1-e^{-\kappa T})\hat{\alpha}\ ,\ \frac{\sigma_2^2(1-e^{-2\kappa T})}{2\kappa}] \tag{17}$$

The log of the spot price and the convenience yield are then jointly normally distributed. Therefore, to perform the Monte Carlo simulations we simply have to generate a set of correlated normal variates.

3.2 Parameters of the Stochastic Process

We use the parameters estimated in Schwartz (1997) for the joint stochastic process of oil spot prices and convenience yields. These parameters, reproduced in Table 1, were estimated

using the Kalman filter methodology on a proprietary data made available by Enron. The data consisted of daily oil forward curves[6] for the period January 93 to May 96. From this data every week ten forward contracts with maturities up to nine years were used in the estimation. The procedure gives both the parameters of the true and the risk adjusted process.

Period	1/15/93 - 5/16/96
Contracts	10 Forward Contracts
Weeks	163
μ	0.082 (0.120)
κ	1.187 (0.026)
α	0.090 (0.086)
$\sigma 1$	0.212 (0.011)
$\sigma 2$	0.187 (0.012)
ρ	0.845 (0.024)
λ	0.093 (0.101)
r	0.05

Table 1
(standard errors are in parenthesis)
Parameters of the Stochastic Process

In the estimation procedure the state variables, the spot price of oil and the instantaneous convenience yield, are assumed to be unobservable. The observable data used in the estimation is the time series and cross section of futures prices. The Kalman filter methodology

[6] Recall that since we have assumed that interest rates are constant, forward and futures prices are the same.

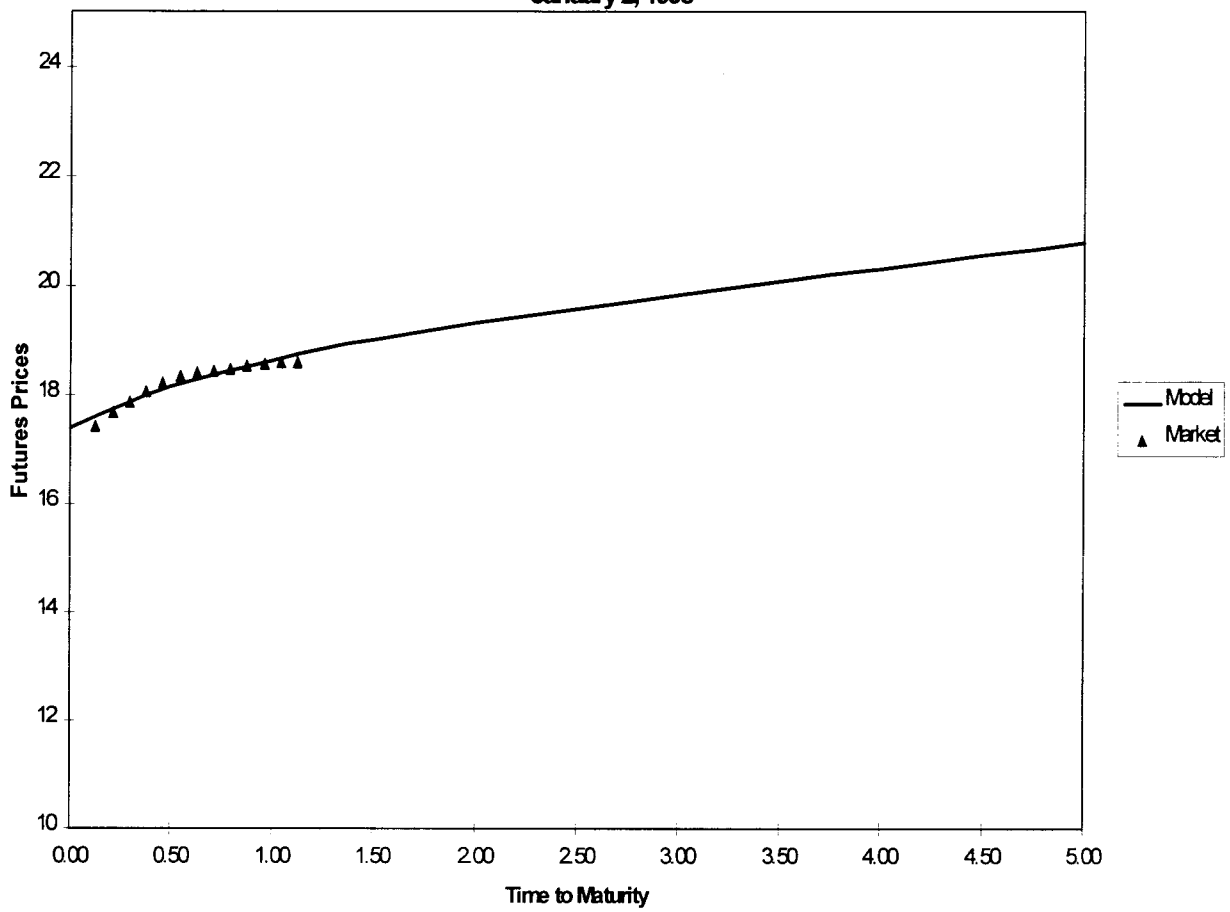
allows for the Maximum Likelihood estimation of the parameters of the model and of the time series of the unobservable state variables. The spot price thus obtained is not intended to fit the futures price closest to maturity, but the whole term structure of futures prices.

3.3 Estimation of Model State Variables

We choose January 2, 1998 as the date in which the evaluation of the undeveloped oil field is done and we use all futures contracts traded that day to estimate the spot price of oil and the instantaneous convenience yield. There were 13 contracts reported in the Wall Street Journal which were traded that day, starting with the February 98 contract and ending with the February 99 contract. There were reported prices for contracts with longer maturities but there was no trading in those contracts so we decided to leave these out of our estimation.

Given the estimated parameters of the model reported in Table 1 and the futures prices with their respective maturities on January 2, 1998, we use the futures pricing equation (9) in a double grid search routine to estimate the state variables, S and δ, which minimize the square deviations between model and market prices. The estimated oil spot price was \$17.39 and the estimated instantaneous convenience yield was -0.057. Figure 1 graphs the market prices of the oil futures prices used in the estimation and the corresponding term structure of oil futures prices implied by the model. Note from the figure that the model futures price for a contract with five years to maturity is \$20.79, whereas the December 2002 price reported in the Wall Street Journal is \$18.63 with no trade.

Figure 1
Market and Model Oil Futures Prices
January 2, 1998



4. CASE STUDY

We analyze an undeveloped oil field concession which, if developed, would produce for 7 years a decaying annual amount of oil ranging from 1.7 million barrels during the first year to 0.07 million barrels during the seventh year. Operating annual cost would also be decreasing from 3.00 million dollars the first year to 0.47 million dollars during the seventh year. Table 2 shows the estimated production and operating costs during the projected life of the developed field. The owner of the concession may exercise the option to develop the oil field at any moment during a five year time-span, after which, if no investment is made, the concession expires. The development investment amounts to 30 million dollars. For purposes of discounting cash flows, both for the

developed and for the undeveloped oil field, we assume that the appropriate discount rate is 0.07, that is 0.02 higher than the risk free rate. This risk premium can be interpreted as the Poisson probability per year of expropriation of the field without compensation to the owner.[7]

Year	Production in millions barrels of oil	Operating costs in million of dollars
1	1.70	3.00
2	1.00	2.00
3	0.55	1.40
4	0.34	1.10
5	0.23	0.97
6	0.16	0.87
7	0.07	0.47

Table 2

Estimated production and operating costs during the life of the oil field.

For simplicity we assume that production is concentrated at the end of each of the production years and that once the development investment has been done production will continue for 7 years independent of the price of oil. We assume a corporate tax rate of 0.35 and full offset of losses.

We assume that the decision to develop the field can be done once a year. That is, the evaluation to decide whether to invest is done at time 0 and the end of each year of the

[7] For this interpretation of "political risk" see Brennan and Schwartz (1985).

concession.[8] In the numerical implementation we do 1000 preliminary simulations to partition the payoff space into 100 bins at each decision point. To estimate the transition probabilities and path payoffs we perform one million simulations.

Given all the parameters of the problem the value of the undeveloped oil field on January 2, 1998 was 7.65 million dollars if we had decided to start production immediately. Its value considering the option to delay the investment, however, was 8.44 million, so given the prevailing prices on that date it was not optimal to develop the oil field.

The procedure also allows us to compute for every year during the life of the concession the average spot price and average instantaneous convenience yield for the bin corresponding to the lowest value of the field for which it is optimal to develop it. Table 3 shows these values. For example, in year one the lowest payoff bin for which it is optimal to invest has an average spot price of $22.30 and an average convenience yield of 0.058. Note that in the last year of the concession the oil field is developed if the spot price is higher than $14.16 which represents the price at which the net present value of the project is positive, because there is no option value left at this point in time.

[8] This decision could be taken at any discrete period of time, say every month.

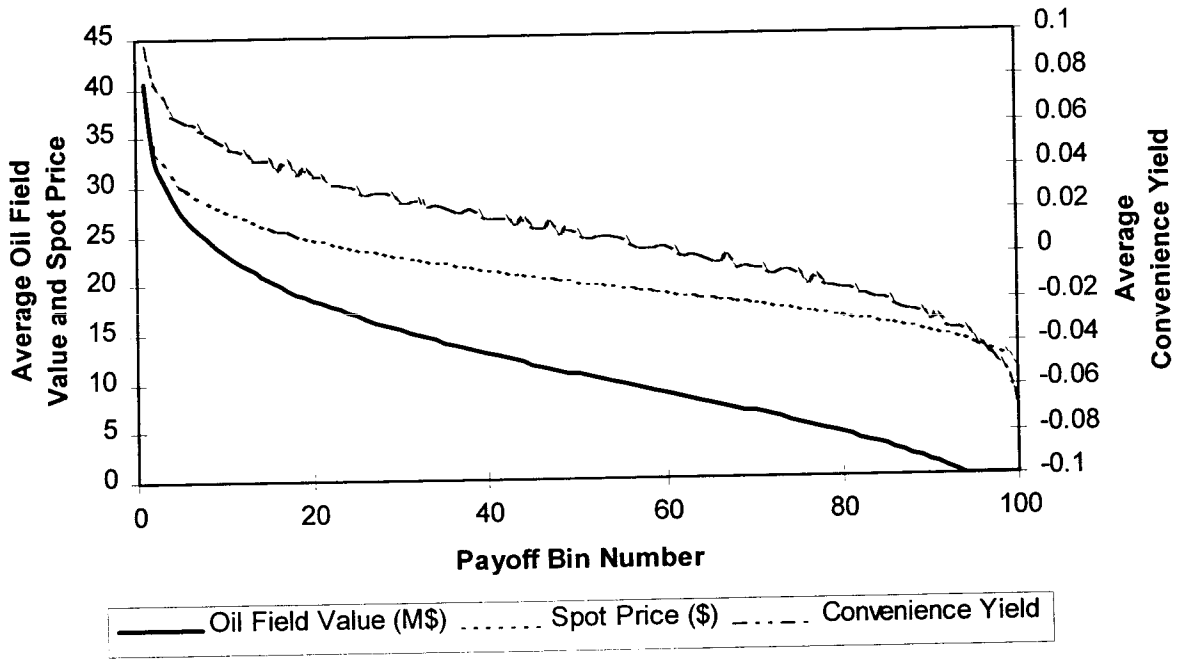
Year	Average Spot Price	Average Convenience Yield
1	22.30	0.0580
2	21.57	0.0364
3	20.93	0.0230
4	20.28	0.0135
5	14.16	-0.0332

Table 3

Average spot prices and convenience yields for bins that trigger investment.

Since the state variables are very highly correlated the payoffs bins are not only in decreasing value of the developed field, but also in decreasing value of the average spot price and instantaneous convenience yield. Figure 2 shows the average undeveloped oil field value, spot price and convenience yield as a function of the payoff bin number at the expiration of the concession. Recall that the bins were constructed so that 1% of the simulated payoffs are contained in each bin in decreasing order of value. Bin 93 is the last one for which the average value is greater than zero (positive net present value) and it corresponds to an average spot price of $14.16 and to and average convenience yield of -0.0332. Note that the convenience yield curve is not perfectly smooth due to the numerical nature of the solution

Figure 2
Average Oil Filed Values, Spot Prices and Convenience Yields at the Expiration of the Concession



.5. CONCLUSION

We develop and implement a model for valuing undeveloped oil fields. The two main features of our approach are that we employ a two factor model for the stochastic process of oil prices and that we use Monte Carlo simulation methods to solve for the American style option imbedded in the problem.

The advantage of using a two factor model for oil prices is that we can capture the varying nature of the term structure of futures prices which sometimes is in contango and sometimes in backwardation, and also that it takes into account the observed mean reversion in spot prices. The two factors or state variables are the spot price of the commodity and the instantaneous convenience yield.

The advantage of using Monte Carlo simulation methods over other numerical procedures is that they allow for the possibility of substantially increasing the number of state variables considered in the analysis. In the case we analyzed we considered only two state variables, but the method could be easily extended to deal with stochastic costs, stochastic reserves, or situations in which two or more different commodities exist in one field (such as oil and natural gas). The main assumption that is made in using simulation methods to value American type options is that the option cannot be exercised at any point in time, but at some discrete pre-specified points in time.

The methodology can also be extended to deal with situations in which there are operational flexibilities such as opening and closing a mine in response to changes in the underlying commodity price as in Brennan and Schwartz (1985).

Given its flexibility and its intuitive appeal we believe that Monte Carlo simulation methods will have an important role to play in the procedures for evaluating projects in the Real Options framework.

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