On the Right-of-first-refusal*

by

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Abstract

When the seller of an asset grants a right-of-first-refusal to a buyer, this special buyer has the opportunity to purchase the asset at the best price the seller can obtain from the other potential buyers. We show that the right-of-first-refusal is inefficient, and it benefits the special buyer at the expense of the seller and other buyers. In a private values model, the benefit from granting a right-of-first-refusal to the special buyer equals the cost to the seller. When buyers' valuations are correlated, the presence of a special buyer exacerbates the winner's curse on regular buyers. Consequently, if some of the regular buyers do not participate in the price discovery stage, the expected gain from granting a right-of-first-refusal to the special buyer is often less than the expected loss to the seller.

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1 Introduction

New price setting mechanisms come into being to solve extant problems. For example, posted prices became common in the early part of the nineteenth century to solve, in part, the agency problems engendered by the confluence of bargaining and the advent of large corporations and multi-location merchandising. (See Arnold and Lippman [1998] for a comparison of posted prices and bargaining.) Prompted by advances in technology and the need to serve geographically dispersed bidders, auctions have greatly increased in visibility in the last fifteen years. These three price setting mechanisms are joined by other contracting mechanisms to solve additional problems such as externalities, risk sharing, and the alleviation of market failures. Having come into existence, price setting mechanisms and other economic institutions survive only if they continue to fulfill some purpose. One such price setting mechanism is the right-of-first-refusal. This right, awarded by the seller of an asset, grants to a special buyer the ability to purchase the asset in question at the highest price offered to the seller by any other buyer. The practice of granting a right-of-first-refusal is most common in real estate transactions, in the purchase of a partnership interest (by one of the extant partners), in professional sports, and in the right to employ artistic talent in the entertainment industry (books, movies, music).

The reasons for the granting of such a right-of-first-refusal appear, at first blush, valid. The current tenant would like the opportunity of becoming his own landlord should the current landlord seek to sell the property; the original partners might seek to avoid taking in a new partner and wish the right to purchase the exiting partner's interest at the price to be paid by a potential new partner; and the firm taking the risk on new talent or a new talent-based project seeks to appropriate the full benefits of any spin-off or synergy from such a venture.

Nevertheless, some anecdotal evidence suggests that it may not be in the best interests of the seller to grant a right-of-first-refusal [see Bulow (1995) and Brandenburger and Nalebuff (1996)]. In 1994, the Miami Dolphins football team was sold to Wayne Huizenga, the founder of Blockbuster Video, at a price that was thought to be considerably below its valuation. Mr. Huizenga had a right-of-first-refusal on the sale of the Dolphins. Unfortunately for the owners, the Dolphins' sale attracted only one other buyer who offered a very low price. Mr. Huizenga exercised his right to purchase at that price.

The goal of this paper is to investigate the impact of a right-of-first-refusal on the seller and the potential buyers. All buyers in our model, including the special buyer, have identical probability distributions over their valuations and information. The seller employs a sealed-bid second-price auction to price the asset. The asset is allocated to the highest bidder in the auction, modulo the special buyer's ability to match the allocation price. Our assumption of *ex ante* identical buyers enables us to investigate whether it is profitable for a seller to grant a right-of-first-refusal when faced with buyers who are similar to each other.

We show in Section 3 that a right-of-first-refusal increases the special buyer's expected profits for two reasons. First, the special buyer might purchase the asset even when her valuation for the object is not the highest among all potential buyers. Thus, the outcome under a right-of-first-refusal is inefficient. Second, when buyer valuations are correlated, the right-of-first-refusal exacerbates the winner's curse for the regular (i.e., non-special) buyers, causing them to bid less aggressively.

These facts also imply that the seller places himself in a disadvantageous position by awarding the special buyer this right. Presumably, the special buyer compensates the seller, in some manner, at the time the seller grants the special buyer a right-of-first-refusal. The seller can be adequately compensated only if the sum of the benefits to this pair, seller and special buyer, is positive when the seller grants this right to the special buyer. That is, a right-of-first-refusal should be granted only if the magnitude of the seller's loss due to the right-of-first-refusal is less than the special buyer's gain. In Section 4, we investigate whether the seller and special buyer can mutually benefit from this option.

We first investigate whether there exist mutual gains to trade from a right-of-first-refusal in two extreme cases: the private values model, where there is an efficiency loss due to the right-of-first-refusal but there is no winner's curse, and the pure common values model, where there is a severe winner's curse but no efficiency loss results from granting a right-of-first-refusal.

In a private values setting, it is always a dominant strategy for regular buyers to bid their valuations. The price at which the asset is offered to the special buyer is the second highest value of the regular buyers. Hence, the special buyer exercises her option whenever her value is greater than the second highest value of the others. Consequently, the allocation is inefficient if her value is between the highest and second highest values of the regular buyers. The gain from this option to the special buyer equals the loss to the seller (Section 4.1). Hence, there are no gains to the pair from granting a right-of-first-refusal to the special buyer.

In a pure common values setting, allocation of the asset to any buyer is efficient: there is no efficiency loss from granting the right-of-first-refusal. Moreover, the winner's curse for the regular buyers is so severe that they submit very low bids, and, in equilibrium, the special buyer always exercises her right-of-first-refusal (Section 4.2). Because the right-of-first-refusal allows the seller and special buyer to capture all the surplus from the sale of the asset, the granting of this option is advantageous to the pair.

The results of these two polar cases may lead one to conjecture the following: As the correlation between the buyers' valuations increases (i.e., as one moves from a private values to a common values model), the fraction of the surplus captured by the seller and special buyer through this option increases. However, the situation is complicated (Section 4.3): while we are able to delineate instances in which the benefit to the pair from granting a right-of-first refusal is positive, we also find instances in which the pair's benefit is negative.

It is unrealistic to assume that all regular buyers elect to participate in the auction when there is a special buyer with a right-of-first refusal – recall that in the Dolphins' sale there was little interest from buyers other than Wayne Huizenga. We investigate in Section 4.4 the impact of non-participation by one or two regular buyers due to the granting of a right-of-first-refusal. In most cases the benefit to the seller-special buyer pair is negative when the presence of a special buyer causes one or two regular buyers to drop out of the second-price auction. This is consistent with Bulow and Klemperer (1996) who show that the seller makes more money with $n+1$ bidders in an English auction with no reserve price than he does with an optimally designed selling mechanism for n bidders.

In summary, the special buyer usually cannot adequately compensate the seller for the right-of-first-refusal. We discuss possible reasons for the prevalence of the right-of-first-refusal arrangement in Section 5.[1]

2 The Model

There are $n+1$ potential buyers for an indivisible object. The $(n+1)^{\text{st}}$ buyer has been granted a right-of-first-refusal (henceforth, ROFR) on the sale of the object. Initially, the seller conducts a second-price auction in which buyers $1, 2, ..., n$ participate, $n \geq 2$.[2] The auction determines a potential winner and the selling price P of the object. Buyer $n+1$ is presented with the opportunity of buying the object at the price P. If she decides not to exercise her option, then the winner in the auction buys the object and pays the price P. All this is common knowledge among the buyers and the seller. We refer to buyers $1, 2, ..., n$ as regular buyers and to the buyer with the ROFR as the special buyer.

Buyer i has reservation value V_i for the object, and buyer i privately observes a signal X_i about his valuation V_i before bidding, $i = 1, 2 \dots n+1$. Let $[\underline{X}, \overline{X}]$ denote the support (of the marginal distribution) of X_i; without loss of generality, assume $\underline{X} = 0$. We assume that the joint probability density for these random variables exists and is denoted $f(v_1, v_2, ..., v_{n+1},\ x_1, x_2, ..., x_{n+1})$. We also assume that the buyers are all symmetric in their signals and valuations. That is, the density function satisfies the following symmetry condition:

$$f(v_1, v_2, ..., v_{n+1},\ x_1, x_2, ..., x_{n+1}) \equiv f(v_{j_1}, v_{j_2}, ..., v_{j_{n+1}},\ x_{j_1}, x_{j_2}, ..., x_{j_{n+1}})$$

[1]When a buyer of products or services grants his current supplier the right to match the price of another (potential) supplier, it is called a meet-or-release provision or a meet-the-competition clause. This is essentially the mirror-image of the right-of-first-refusal, with the roles of buyer and seller reversed. Similar issues arise in the analysis of a meet-or-release provision. There are exact counterparts of our results in a model of the meet-or-release option.

[2]The seller might elicit several bids from each regular buyer, giving them an opportunity to exceed the current high bid; such a price determination method is conveniently modeled as a second-price auction.

where $(j_1, j_2, ..., j_{n+1})$ is a permutation of $(1, 2, ..., n+1)$. Define

$$V(x_1; x_2, ..., x_{n+1}) \equiv E[V_1 | X_1 = x_1,\ X_2 = x_2, ..., X_{n+1} = x_{n+1}]. \quad (1)$$

Then $V(\cdot)$ is symmetric in the last n arguments. Moreover, because the distribution of all the X's and V's has a density, $V(x_1; x_2, ..., x_{n+1})$ is continuous in all arguments.

Lastly, we assume that the random variables $(V_1, V_2, ..., V_{n+1}, X_1, X_2, ..., X_{n+1})$ are weakly affiliated. That is, for any two points $(\underline{v}, \underline{x}) = (v_1, ..., v_{n+1},\ x_1, ..., x_{n+1})$ and $(\underline{v}', \underline{x}') = (v_1', ..., v_{n+1}',\ x_1', ..., x_{n+1}')$,

$$f(\underline{v}, \underline{x}) + f(\underline{v}', \underline{x}') \quad \leq \quad f(\underline{v} \vee \underline{v}',\ \underline{x} \vee \underline{x}') + f(\underline{v} \wedge \underline{v}',\ \underline{x} \wedge \underline{x}'), \quad (2)$$

where $\vee$ indicates maximum and $\wedge$ indicates minimum. An implication of affiliation is that $V(x_1; x_2, ..., x_{n+1})$ is non-decreasing in all its arguments. Further, we assume that $V(x_1; x_2, ..., x_{n+1})$ is strictly increasing in x_1.[3] A sufficient condition for $V(x_1; x_2, ..., x_{n+1})$ to be strictly increasing in its first argument is that the affiliation inequality (2) is strict whenever $(v_i, x_i) \neq (v_i', x_i')$. See Milgrom and Weber (1982) for more on affiliation (which is same as association or monotone positivity due to Karlin and Rinott (1980)).

Throughout we assume, other things being equal, higher values of one's own signal is as at least as good news about the object as higher values of some other buyer's signal. This is formalized by

Assumption A0: $V(x_1; x_2, ..., x_{n+1}) \geq V(x_2; x_1, ..., x_{n+1}), \qquad \forall x_1 \geq x_2.$

This is a mild assumption. A consequence of A0 is that it is efficient to allocate the object to the buyer with the highest signal.[4]

In the next section, we look at the existence and uniqueness of equilibrium strategies for the buyers. In particular we find a symmetric Bayesian Nash equilibrium for this game subsequent to the grant of a ROFR to buyer $n+1$.

3 Symmetric Equilibrium

Buyers $1, 2, ..., n$ participate in a sealed-bid second-price auction which determines the selling price P for the object. After the auction, the price P is revealed to buyer $n+1$ who then decides whether to buy the object at that price. If buyer $n+1$ decides not to buy, then the highest bidder in the auction buys the object at price P.

The strategy of buyer i, $i = 1, 2, ..., n$, is a function $b_i : [0, \overline{X}] \to \Re$ which maps i's private signal X_i to a bid $b_i(X_i)$. The strategy of buyer $n+1$ maps the selling price

[3] This is the non-degeneracy assumption of Milgrom and Weber (1982).

[4] This is a slightly stronger version of the single-crossing condition in Maskin (1992) and other papers. The single-crossing condition is necessary for a second-price auction to be efficient.

P and her own signal X_{n+1} to a *Buy/Refuse* decision. Suppose that the selling price determined in the auction is p. By affiliation, the special buyer's expected value of the object conditional on p and on her signal realization x is increasing in x. Therefore, given $P = p$, if the special buyer exercises her ROFR when $X_{n+1} = x$, then she would also exercise her ROFR for $X_{n+1} = x'$, $\forall x' > x$. Thus, a rational strategy for the special buyer can be described by a cutoff function $c(\cdot)$, where buyer $n+1$'s decision is to buy if and only if $X_{n+1} > c(P)$.

In a Bayesian Nash equilibrium, each buyer ($i = 1, 2, ..., n+1$) uses a best response to the others' equilibrium strategies. The strategy $b_i(\cdot)$ of buyer i, $i = 1, 2, ..., n$, is *increasing* if $x > x'$ implies that $b_i(x) > b_i(x')$.[5] When buyers $1, 2, ..., n$ use the same strategy, we say that the equilibrium is symmetric. We restrict attention to symmetric Bayesian Nash equilibria in which the regular buyers use an increasing strategy.

At a symmetric equilibrium in which all regular buyers use an increasing strategy, the signal of the second highest bidder (who, along the equilibrium path, has the second highest signal realization among buyers $1, 2, ..., n$) can be inferred from the price P. Buyer $n+1$'s strategy is represented by a cutoff function, $h_* : [0, \overline{X}] \to [0, \overline{X}]$, from the second highest signal among the regular buyers to a realization of X_{n+1} at which she is indifferent between buying or not. Let $b_*(\cdot)$ be each (of the first n) buyer's symmetric equilibrium strategy. Thus, the second highest signal among buyers $1, 2, ..., n$ is inferred to be $b_*^{-1}(P)$. Buyer $n+1$'s equilibrium strategy is to buy if and only if $X_{n+1} \geq h_*(b_*^{-1}(P))$.

Let $Z_{r,k}$, $1 \leq r \leq k$, be the r^{th} highest order statistic of the collection $\{X_1, X_2, ..., X_k\}$. Define

$$W(x, z) \;\equiv\; E[V_{n+1} | X_{n+1} = x,\; Z_{2,n} = z].$$

$W(x, z)$ is the expected value of the object for the special buyer when her signal is x and (she infers that) z is the second highest signal for the first n buyers. Observe that if each regular buyer uses the same increasing bid function $b(\cdot)$, then the best-response strategy for buyer $n+1$ is to use the cutoff function

$$h_b(z) \;\equiv\; \min\left\{u \in [0, \overline{X}] \;:\; W(u, z) - b(z) \geq 0\right\}. \tag{3}$$

Define

$$\begin{aligned} U(x, y, u) \;&\equiv\; E[V_n | X_n = x,\; Z_{1,n-1} = y,\; X_{n+1} \leq u], \\ \phi(x, z, u) \;&\equiv\; W(x, z) - U(z, z, u). \end{aligned} \tag{4}$$

The function $U(x, y, u)$, which describes the expected value of the object for buyers $1, 2, \ldots, n$ given certain signal values, is important in determining the equilibrium bids for these buyers. We shall see that the function ϕ is the expected profit

[5] There might be an equilibrium in which some buyer employs a strategy which is not non-decreasing.

of buyer $n+1$ when she exercises her ROFR, the second highest signal realization among buyers $1,2,...,n$ is z, the regular buyers believe that buyer $n+1$ will exercise her ROFR at this auction price if and only if $X_{n+1} \geq u$, and $X_{n+1} = x$.

Our assumptions imply that U and W are nondecreasing and continuous in all arguments; moreover, U and W are increasing in their first argument. Define

$$\begin{aligned} h_*(z) &\equiv \min\left\{u \in [0, \overline{X}] \,:\, \phi(u, z, u) \geq 0\right\} \qquad (5) \\ &= \min\left\{u \in [0, \overline{X}] \,:\, W(u, z) - U(z, z, u) \geq 0\right\}. \end{aligned}$$

As we shall see in Proposition 1, h_* is an equilibrium cutoff function for buyer $n+1$. The following lemma is useful in proving Proposition 1 and subsequent results.

Lemma 1
(i) h_* is well-defined.
(ii) $h_*(z) \leq z$ for all $z \in [0, \overline{X}]$.
(iii) If $h_*(z) > 0$, then

$$\begin{aligned} W(h_*(z), z) &= U(z, z, h_*(z)) \qquad (6) \\ \textit{i.e.,}\quad E[V_{n+1}|X_{n+1} = h_*(z),\, Z_{2,n} = z] &= E[V_n|X_n = z,\, Z_{1,n-1} = z,\, X_{n+1} \leq h_*(z)]. \end{aligned}$$

Proof: Observe that for all $z \in [0, \overline{X}]$

$$\begin{aligned} \phi(z, z, z) &= W(z, z) - U(z, z, z) \\ &= E[V_{n+1}|X_{n+1} = z,\, Z_{2,n} = z] - E[V_n|X_n = z,\, Z_{1,n-1} = z,\, X_{n+1} \leq z] \\ &= E[V_{n+1}|X_{n+1} = z,\, Z_{2,n} = z] - E[V_{n+1}|X_{n+1} = z,\, Z_{1,n-1} = z, X_n \leq z] \\ &= E[V_{n+1}|X_{n+1} = z,\, Z_{2,n} = z] - E[V_{n+1}|X_{n+1} = z,\, Z_{1,n} = z] \\ &\geq 0. \end{aligned}$$

The inequality follows from affiliation, and the second-to-last equality uses the symmetry of the distribution of the X_i's and V_i's. Because $\phi(z, z, z) \geq 0$ and ϕ is continuous in all its arguments, $h_*(\cdot)$ is well-defined, piece-wise continuous, and for all $z \in [0, \overline{X}]$, we have $h_*(z) \leq z$.

If $h_*(z) > 0$, then (6) follows immediately from (5). ∎

Suppose that the special buyer uses the cutoff function $h(z)$ and the regular buyers use the same bidding strategy. Then each of the regular buyers knows that the most the object can be worth to him, when his signal is z and he obtains the object, is

$$U(z, z, h(z)) \;=\; E\left[V_n|X_n = z, Z_{1,n-1} = z, X_{n+1} \leq h(z)\right].$$

If buyer n, say, wins the auction with a signal of z, then $Z_{1,n-1} \leq z$; further, if buyer $n+1$ does not exercise her ROFR, then $X_{n+1} \leq h(z)$. Therefore, a regular buyer with signal z should not bid more than $U(z, z, h(z))$. Below we show that under certain conditions (i) $U(z,z,h(z))$ is symmetrically the best-response to the cutoff function $h(z)$ and (ii) when $h(z) = h_*(z)$, this bid function forms a symmetric Nash equilibrium with $h_*(z)$. To this end define b_* by

$$b_*(x) \equiv U(x,x,h_*(x)). \tag{7}$$

If $b_*(x)$ is increasing, then $b_*^{-1}(\cdot)$ is well-defined and the special buyer can infer $Z_{2,n}$ from the auction price P.

Proposition 1 *Suppose that $b_*(x)$, defined by (7), is increasing in x with $h_*(\cdot)$ defined by (5). Then the following is a symmetric Nash equilibrium:*

- *For $i = 1,2,...,n$, buyer i with signal X_i bids $b_*(X_i)$ in the second-price auction.*
- *If P is the (random) price in the auction, buyer $n+1$ buys the object at this price if and only if $X_{n+1} \geq h_*(b_*^{-1}(P))$.*

Proof: Suppose that $X_{n+1} \geq h_*(b_*^{-1}(P))$. If buyer $n+1$ exercises her ROFR, then her payoff is

$$\begin{aligned} W(X_{n+1}, b_*^{-1}(P)) - U(b_*^{-1}(P), b_*^{-1}(P), h_*(b_*^{-1}(P))) &\geq \phi(b_*^{-1}(P), b_*^{-1}(P), h_*(b_*^{-1}(P))) \\ &\geq 0. \end{aligned}$$

(The second $\geq$ may be replaced by $=$ if $h_*(b_*^{-1}(P)) > 0$.) If instead, $X_{n+1} < h_*(b_*^{-1}(P))$, then the first $\geq$ above is replaced by $<$, and the second $\geq$ is replaced by $=$. Thus, buyer $n+1$'s strategy is a best response.

Suppose that buyer n is informed of $Z_{1,n-1}$. We show that buyer n would not want to change his bid even with this additional information. If he bids high enough to win the auction and the special buyer does not exercise her ROFR, then buyer n's payoff will be:

$$\begin{aligned} U(X_n, Z_{1,n-1}, h_*(Z_{1,n-1})) &- b_*(Z_{1,n-1}) = \\ U(X_n, Z_{1,n-1}, h_*(Z_{1,n-1})) &- U(Z_{1,n-1}, Z_{1,n-1}, h_*(Z_{1,n-1})). \end{aligned}$$

The above quantity is positive if and only if $\{X_n > Z_{1,n-1}\}$. If buyer n does not change his equilibrium bid $b_*(X_n)$ after learning $Z_{1,n-1}$, then he will win only if $\{X_n > Z_{1,n-1}\}$ and $\{X_{n+1} < h_*(Z_{1,n-1})\}$; his profit conditional upon winning is $U(X_n, Z_{1,n-1}, h_*(Z_{1,n-1})) - U(Z_{1,n-1}, Z_{1,n-1}, h_*(Z_{1,n-1}))$. Therefore, he cannot do better by deviating from his equilibrium bid. ∎

We emphasize several consequences associated with granting a ROFR. First, because the special buyer purchases the item whenever $X_{n+1} \geq h_*(Z_{2,n})$ and we have $h_*(z) \leq z$, a ROFR converts the second price auction into something better than a third price auction for the special buyer. Upon winning the object, she pays the second highest among the others' bids; moreover, she may win the object even if her signal is less than the second highest signal of the other buyers. Second, from a regular buyer's standpoint, the presence of a special buyer converts a second price auction with $n+1$ buyers into something worse than a second price auction with n buyers. When a regular buyer, say buyer 1, wins the object, he pays the highest among the other (regular) buyers' bids; but he wins only if his signal is higher than the signals of buyers 2,3,...,n, *and* is sufficiently higher than the special buyer's signal. Third, the allocation of the object may be inefficient because, as already noted, the special buyer may purchase the object even when she does not have the highest signal, i.e., when $X_{n+1} \in [h_*(Z_{2,n}), Z_{1,n})$.[6]

A regular buyer will win only if the special buyer does not exercise her ROFR, i.e., when, after drawing inferences from the auction price about regular buyers' information and also based on her own private information, the special buyer concludes that the object is over-priced. Thus, if valuations of buyers are correlated, then a ROFR exacerbates the winner's curse for the regular buyers. The regular buyers will bid less aggressively and the average selling price will be lower than if the seller did not grant a ROFR to buyer $n+1$.

Let $\hat{b}(x)$ denote the symmetric equilibrium bid for a buyer with signal x in a second-price auction with $n+1$ participants and no ROFR. We show that $\hat{b}(x) \geq b_*(x)$. From Matthews (1977) and Milgrom (1981) we know that

$$\hat{b}(x) \;=\; E\left[V_{n+1}\middle|X_{n+1}=x, Z_{1,n}=x\right]. \tag{8}$$

Thus, recalling that $h_*(x) \leq x$ and the buyers' signals and valuations are symmetrically distributed,

$$\begin{aligned}
b_*(x) \;&=\; E\left[V_n\middle|X_n=x, Z_{1,n-1}=x, X_{n+1}\leq h_*(x)\right] \\
&\leq\; E\left[V_n\middle|X_n=x, Z_{1,n-1}=x, X_{n+1}\leq x\right] \\
&=\; E\left[V_{n+1}\middle|X_{n+1}=x, Z_{1,n}=x,\right] \;=\; \hat{b}(x).
\end{aligned}$$

If P_{ROFR} and $\hat{P}$ are the selling prices with and without a ROFR, respectively, then

$$P_{ROFR} \;=\; b_*(Z_{2,n}) \;\leq\; \hat{b}(Z_{2,n}) \;\leq\; \hat{b}(Z_{2,n+1}) \;=\; \hat{P}.$$

Moreover, because $\text{Prob}[Z_{2,n+1} > Z_{2,n}] > 0$ and $\hat{b}$ is strictly increasing, we have $E[P_{ROFR}] < E[\hat{P}]$. The special buyer benefits from her ROFR. First, she wins more often than before because $\{X_{n+1} \geq h_*(Z_{2,n})\} \supseteq \{X_{n+1} \geq Z_{1,n}\}$. Second, she pays a smaller price whenever she wins: $E[P_{ROFR}] < E[\hat{P}]$. Hence, we have:

[6]It is clear that if, instead of a second-price auction, another market institution determines the best price from regular buyers, (e.g., a first-price auction or sequential search), the ROFR is still inefficient. Moreover, it confers an advantage on the special buyer.

Corollary 1 *Granting a ROFR to buyer $n+1$*
(i) reduces the expected price obtained by the seller
(ii) increases the payoff to the special buyer
(iii) is inefficient (except in the pure common values case).

The impact of a ROFR on regular buyers is ambiguous. Regular buyers win less often (than they would if buyer $n+1$ did not have a ROFR), but the price they pay upon winning is lower. The next result establishes that when buyers' signals are i.i.d., regular buyers are worse off.[7] The proof of Proposition 2 is given at the beginning of the appendix.

Proposition 2 *For any valuation/signal structure in which the signals $X_1, X_2, \ldots, X_{n+1}$ are independent, granting a ROFR to a special buyer reduces the expected profits of the regular buyers.*

3.1 Existence and Uniqueness

We now turn to conditions under which a symmetric equilibrium exists (i.e., sufficient conditions for the right hand side of (7) to be increasing) and is unique.

Without further assumptions on the distribution of the X_i's and V_i's or on $h_*(\cdot)$, there is nothing to guarantee that $b_*(x)$, as it is defined, is increasing. If $h_*(\cdot)$ is non-decreasing, then affiliation implies that $b_*(x) = U(x, x, h_*(x))$ is increasing. It seems natural that $h_*(\cdot)$ would be non-decreasing. Example 1 reveals that this is not always the case.

Example 1: Let $V_i = aV + (1-a)X_i$ with $a \in (0,1)$ and $V = Z_{1,n+1}$. The signals $X_1, X_2, \ldots, X_{n+1}$ are i.i.d. random variables with

$$P(X \geq x) \;=\; \frac{e}{(x+e)\left[\log(x+e)\right]^{1+\alpha}} \qquad \text{for } x \in [0,\infty),$$

where e is the exponential constant and $\alpha > 0$. Thus, the buyers' valuations are correlated, while their signals are not. Then

$$E\left[X | X \geq z\right] \;=\; z + \left(\frac{z+e}{\alpha}\right)\log(z+e).$$

It can be shown (using equation (12) of Section 4.3) that

$$h_*(z) \;=\; \left(z - \frac{a}{1-a} E\left[X - z | X \geq z\right]\right) \vee 0.$$

Thus, we have

$$h_*(z) \;=\; z - \frac{a}{1-a} E\left[X - z | X \geq z\right] \;=\; z - \frac{a}{\alpha(1-a)}(z+e)\log(z+e),$$

[7] In Section 4 it is established that a ROFR makes regular buyers worse off in the private values and common values cases, whether or not their signals are i.i.d.

if the expression on the right-hand side is positive; otherwise $h_*(z) = 0$. It may be verified that the right-hand side expression is an increasing function for $z < z^* \equiv e^{\frac{\alpha(1-a)}{a}-1} - e$; for $z > z^*$, it is decreasing. If a is close to 1 then the right-hand side is negative for all z, so that $h_*(z) = 0$ for all z. To ensure that $h_*(z^*) > 0$ and $z^* > 0$, restrict a so that $a < \frac{\alpha}{C+\alpha}$, where C solves $e^{C-2} = C$ [$C \approx 3.1462$]. Then $h_*(z)$ is positive and increasing on the interval $[0, z^*)$ and decreasing for $z > z^*$ until it hits zero, remaining there for all larger values of z.

The key to Example 1 is the fat tail of the X-distribution. As the second-highest signal $Z_{2,n}$ (which in this case equals the auction price) rises, the expectation of the largest signal rises faster than any linear function of $Z_{2,n}$. This raises the special buyer's estimate of the value of the object, which, in some circumstances, more than makes up for the higher price. In fact if a is too large, this effect is so pronounced that the special buyer never declines to buy the object. Only with small a does the rise in price at first outweigh the subsequent rise in valuation. But, no matter what the value of a is in this example, if $Z_{2,n}$ becomes large enough, buyer $n+1$ will exercise her ROFR, regardless of the value of her own signal.

However, the bid function $b_*(x) \equiv U(x, x, h_*(x))$ is increasing. Hence, we have demonstrated that having $h_*(\cdot)$ increasing (though it is sufficient) is not necessary for $b_*(x)$ to be increasing. $\triangle$

To ensure the existence of the Nash equilibrium described in Proposition 1, we need to restrict the class of distributions on the X_i's and V_i's. We make the natural assumption that

Assumption A1: $\phi(u, z, u)$ *is non-increasing in* z *for all* u.

In other words, raising a price-determining regular buyer's signal has at least as much impact on this regular buyer's valuation when he assumes that he wins the object, as it does on the valuation of buyer $n+1$.

With b_* defined in (7) and h_* defined in (5), we have

Proposition 3 *Under assumption A1, $h_*(\cdot)$ is non-decreasing. Thus, $b_*(x)$ is increasing, and the strategies $(b_*, b_*, \ldots, b_*; h_*)$ form a Nash equilibrium.*

Proof: Fix $0 \le z < z' \le \overline{X}$. If $h_*(z) = 0$, there is nothing to prove. If $h_*(z) > 0$, then the definition of $h_*(z)$ and (6) yield, for all $u < h_*(z)$,

$$\phi(u, z', u) \;\le\; \phi(u, z, u) \;<\; \phi(h_*(z), z, h_*(z)) \;=\; 0.$$

By definition $h_*(z')$ must be $\ge h_*(z)$. This implies that $b_*(x)$ is increasing. This, in turn, implies that Proposition 1 holds. ∎

We would like to show that $(b_*, b_*, \ldots, b_*; h_*)$ is the unique symmetric solution to this problem. However, looking at the proof of Proposition 1, we see that any pair of functions $(b(\cdot), h(\cdot))$ that satisfy

C1. $\phi(h(z), z, h(z)) \geq 0$ for all z,

C2. $\phi(h(z), z, h(z)) = 0$ whenever $h(z) > 0$, and

C3. $b(x) \equiv U(x, x, h(x))$ increases with x

form a symmetric equilibrium. The function $h_*(\cdot)$ is the smallest h-function that is in an equilibrium pair (b, h). But there is nothing in the structure of $\phi(x, z, u)$ that precludes there being another such function. For instance,

$$h^*(z) \;\equiv\; \max\left\{u \in [0, \overline{X}] \mid \phi(u, z, u) \leq 0\right\}.$$

Therefore, if we are to have any hope of establishing (b_*, h_*) as the unique equilibrium pair, we must restrict the class of distributions on the X_i's and V_i's so that the function $k(\,\cdot\,; z) \equiv \phi(\,\cdot\,, z, \,\cdot\,)$ has at most one zero. To this end, we make another logical assumption:

Assumption A2: *$\phi(u, z, u)$ is (strictly) increasing in u for all z.*

Recall that $\phi(u, z, u)$ is the expected profit of buyer $n+1$ when she exercises her ROFR, $Z_{2,n} = z$, $X_{n+1} = u$, and the regular buyers believe that buyer $n+1$ will exercise her ROFR at the auction price $b_*(z)$ if and only if $X_{n+1} \geq u$. Under A2, it is obvious that for each z

$$\phi(u, z, u) > 0, \quad \forall u > h_*(z).$$

Therefore, for every z there is at most one point $u\,[= h_*(z)]$ at which $\phi(u, z, u)$ can be equal to 0. In this case $h_*(\cdot)$ is the only function that satisfies conditions C1-C3.

Proposition 4 *Given A1 and A2, suppose (b_0, h_0) is a pair of strategies that form a symmetric Nash equilibrium. If b_0 is increasing and h_0 is non-decreasing, then $h_0(z) = h_*(z)$, $\forall z$ and*

$$\begin{aligned} b_0(x) &\leq W(0, x), \ \forall x \leq \underline{z}, \\ b_0(x) &= b_*(x), \ \text{for almost all } x > \underline{z}, \end{aligned}$$

where $\underline{z} \;\equiv\; \sup\{z \in [0, \overline{X}] | h_*(z) = 0\}$.

When $X_i \leq \underline{z}$, $i \leq n$, regular buyer i will not win the object even if he is the highest bidder in the auction because the special buyer will exercise her ROFR as $h_*(Z_{2,n}) \leq h_*(X_i) = 0$. Hence, a regular buyer's bid function is not unique for signals at which he will not win.

Finally, we discuss the restrictions imposed by A1 and A2 when each buyer's valuation is a convex combination of a private value and a common value: $V_i = aV + (1-a)X_i$, $a \in [0,1]$. We have

$$\begin{aligned}\phi(u,z,u) \;\equiv\; & (1-a)(u-z) + \\ & a\Big(E[V|X_{n+1}=u, Z_{1,n} \geq z, Z_{2,n}=z] \;-\; E[V|X_{n+1} \leq u, Z_{1,n}=z, Z_{2,n}=z]\Big)\end{aligned} \tag{9}$$

In the private values case ($a = 0$), A1 and A2 are satisfied. A1 requires that the expression multiplying a is non-increasing in z; A2 ensures that this expression is increasing in u. For any $a \in (0,1]$, Example 1 does not satisfy A1 and $h_*(\cdot)$ is not non-decreasing.

4 The value of the right-of-first-refusal

We now investigate whether the seller has an incentive to grant a ROFR to buyer $n+1$. We have shown that granting the ROFR to a special buyer reduces the selling price of the object in a second-price auction: the seller is always worse off when a buyer has been granted the ROFR. Does the benefit that the ROFR extends to the special buyer outweigh the loss of auction revenue that it costs the seller? To answer this question we compare the expected profit to the seller and the special buyer (buyer $n+1$) as a pair with and without a ROFR. We assume that if buyer $n+1$ does not have a ROFR, she will participate in the auction. Moreover, unless otherwise stated, regular buyers participate in the auction whether or not the special buyer is granted a right-of-first-refusal. We are able to delineate instances in which the benefit to the pair of granting this option is positive. But we also find instances wherein the pair's benefit is negative.

In Section 4.1 we examine the case when buyers have private valuations for the object. The pure common values and the correlated values cases are analyzed in Sections 4.2 and 4.3, respectively. Finally, in Section 4.4, we consider the effect of a few of the regular buyers not participating in the presence of the ROFR.

4.1 Private values

In this situation, $V_i = X_i$ for all i whence $W(x,z) = x$, $U(x,y,u) = x$, and $\phi(u,z,u) = u-z$. Thus, $h_*(z) \equiv z$ and $b_*(x) = x$. In other words, the bids of regular buyers in the second-price auction are unaffected by the presence of the special buyer. Indeed, it is a dominant strategy for regular buyers to bid their valuations, and the special buyer should never purchase the object when the price is more than her valuation/signal. Nevertheless, because the special buyer with a ROFR has the option of buying at a price equal to the second highest valuation of the other buyers, the ROFR imparts a strictly positive benefit to her.

In an auction without a ROFR three separate outcomes can occur: buyer $n+1$ has the highest signal, buyer $n+1$ has the second highest signal, or the signal of buyer $n+1$ is less than $Z_{2,n}$. In the first two cases the seller/special buyer pair's profit is X_{n+1} because either buyer $n+1$ obtains the object ($X_{n+1} \geq Z_{1,n}$), or one of buyers 1 through n wins the auction and pays the second highest bid, X_{n+1}, to the seller, ($Z_{1,n} > X_{n+1} \geq Z_{2,n}$). In the third case, ($Z_{2,n} > X_{n+1}$), one of the regular buyers purchases the item at a price of $Z_{2,n}$ and the pair receives $Z_{2,n}$. Thus, their profit is $\max\{X_{n+1}, Z_{2,n}\}$.

When buyer $n+1$ has a ROFR, only two things can happen: either the special buyer obtains the object ($X_{n+1} \geq Z_{2,n}$), or the special buyer does not obtain the object and the seller receives $Z_{2,n}$ ($X_{n+1} < Z_{2,n}$). Therefore, the pair's profit is $\max\{X_{n+1}, Z_{2,n}\}$, and we have proved

Proposition 5 *Under private values, with probability one the gains from trade (between the seller and the special buyer) of a ROFR are zero.*

A ROFR gives the special buyer the option to buy at a price equal to the third-highest among all $n+1$ buyers' values; the special buyer will exercise this option if hers is the second-highest or highest value. The special buyer wins more often and pays a lower price (conditional upon winning) compared to the benchmark case in which she competes in a second-price auction with the other n buyers. This gain exactly offsets the lower price (equal to the 3rd highest rather than the 2nd highest of $n+1$ signals) that the seller obtains with a ROFR.

If buyer $n+1$ exercises her ROFR, then the allocation will not be Pareto optimal when $X_{n+1} \in [h_*(Z_{2,n}), Z_{1,n}] = [Z_{2,n}, Z_{1,n}]$. Thus, the expected surplus strictly decreases when a ROFR is granted. Because the amount of the surplus extracted jointly by the seller and the special buyer remains unchanged, we have:[8]

Corollary 2 *Under private values, a buyer is strictly worse if any one of the other buyers is granted a ROFR.*

4.2 Common values

When all the buyers' valuations are identical ($V_i = V$), the effect of the ROFR upon the auction and upon the determination of the winner is dramatic. In this case the special buyer always exercises her ROFR, without regard to her signal. To see this, note that for any $z \in [0, \overline{X}]$

$$\begin{aligned}\phi(0, z, 0) &= W(0, z) - U(z, z, 0) \\ &= E[V|X_{n+1} = 0,\, Z_{2,n} = z] - E[V|X_n = z,\, Z_{1,n-1} = z,\, X_{n+1} = 0]\end{aligned}$$

[8]Corollary 2 does not assume that the X_i's are independently distributed; hence, it is not a special case of Proposition 2.

$$
\begin{aligned}
&\geq E[V|X_{n+1} = 0,\, Z_{2,n} = z] - E[V|X_n \geq z,\, Z_{1,n-1} = z,\, X_{n+1} = 0\,] \\
&= E[V|X_{n+1} = 0,\, Z_{2,n} = z] - E[V|X_{n+1} = 0,\, Z_{2,n} = z] \\
&= 0.
\end{aligned}
$$

Therefore, $h_*(z) \equiv 0$: buyer $n+1$ buys the object regardless of the price set in the auction, and buyers 1,2,...,n make zero profit in this equilibrium. Hence, we have,

Proposition 6 *Under pure common values, if the seller grants buyer $n+1$ a ROFR, the pair appropriates the entire surplus and the regular buyers never buy the object.*

When buyer $n+1$ has a ROFR, the sum of the surplus of the seller and the special buyer is equal to the full value of the object. In an auction without the ROFR, this pair obtains less than the full value of the object: when buyer $n+1$ does not have the largest signal, the buyer of the object (one of buyers $i = 1, 2, ..., n$) will extract a profit. Thus, the seller/special buyer pair is better off with a ROFR.

Milgrom and Weber (1981) showed that in a pure common value auction, if some buyer A's information partition is finer than some other buyer B's information partition, then B's expected profit is zero. We have shown that when one buyer (the special buyer) observes, in addition to her own signal, the second order statistic of other buyers' signals, then the regular buyers' profit is zero with probability one. This may appear to be a strengthening of the result in Milgrom and Weber (1981). However, in their model both buyers move simultaneously whereas in our model the special buyer moves after observing the second highest of the others' bids; this intensifies the winner's curse for the regular buyers (our counterpart of Milgrom and Weber's less well-informed buyer B).

4.3 Correlated values

In the private values case, the inefficiency resulting from the ROFR is borne entirely by the regular buyers as the seller/special buyer pair is equally well-off with or without a ROFR. The fraction of the surplus captured by the pair due to the granting of a ROFR is zero. Under common values, there is no inefficiency associated with a ROFR. However, the winner's curse for the regular buyers is severe enough that the special buyer always wins and the pair captures the entire surplus; therefore the fraction of the surplus captured by the pair due to the granting of a ROFR is at its upper bound (equal to the fraction of the surplus captured by regular buyers in the absence of a ROFR).

From this one might be tempted to conjecture that the fraction of the surplus captured by the pair through a ROFR increases as the degree of correlation between buyer valuations increases. However, things are somewhat more complicated. The value of the ROFR depends not only on the degree of correlation between buyer valuations and on the number of buyers but also on the functional form of V. For general V, there is little that one can say about the value of granting the ROFR. This inconclusiveness is not due to our inability to find general results but rather to the non-existence of general results.

Under correlated values, the ROFR leads to an exacerbated winner's curse for the regular buyers, as in the common values case, and an inefficiency, as in the private values case. Assume for now that all regular buyers participate even when there is a ROFR. The analysis below hinges on the following four possible cases:[9]

I. $X_{n+1} \geq Z_{1,n}$. The special buyer receives the object whether or not she has the ROFR. There is no net gain to the pair; moreover, the outcome with or without the ROFR is Pareto optimal.

II. $Z_{1,n} > X_{n+1} \geq Z_{2,n}$. The special buyer obtains the object with the ROFR and sets the price that the winner pays the seller without the ROFR. The outcome with the ROFR is not Pareto optimal (except in the common values case). However, the slice of the pie that the pair extracts never decreases (and usually increases) when they trade the ROFR. This is because the special buyer's expected valuation of the object is never less than $\hat{b}(X_{n+1})$, her equilibrium bid without the ROFR.

III. $Z_{2,n} > X_{n+1} \geq h_*(Z_{2,n})$. Here the pair receives the value of the object to the special buyer when the ROFR is granted, and they receive $\hat{b}(Z_{2,n})$ when it is not. With a ROFR the surplus shrinks even further than in II as the special buyer does not even have the second highest signal, let alone the highest. The smaller total surplus can adversely affect the pair's surplus on this set. Often, the value V_{n+1} of the object to the special buyer will be less than $\hat{b}(Z_{2,n})$.

IV. $h_*(Z_{2,n}) > X_{n+1}$. With or without a ROFR, the special buyer does not purchase the object. The pair receives $b_*(Z_{2,n})$ with a ROFR and $\hat{b}(Z_{2,n})$ without. The outcome is always Pareto optimal. But the surplus for the pair with a ROFR is non-positive as $b_*(z) \leq \hat{b}(z)$ for all z.

Thus, only in cases I and IV is the object allocated to the same buyer, with or without a ROFR; the allocation in cases II and III is inefficient. We summarize the gains to the seller/special buyer pair in the following diagram:

$E[b_*(Z_{2,n}) - \hat{b}(Z_{2,n}) \mid IV] \leq 0$		$E[V_{n+1} - \hat{b}(Z_{2,n}) \mid III] = ?$		$E[V_{n+1} - \hat{b}(X_{n+1}) \mid II] \geq 0$		$E[V_{n+1} - V_{n+1} \mid I] = 0$
IV	$h_*(Z_{2,n})$	**III**	$Z_{2,n}$	**II**	$Z_{1,n}$	**I**

Diagram 1: The seller/buyer pair's expected profit in trading the ROFR, when X_{n+1} is in either set I, II, III, or IV.

[9] Recall that $\hat{b}(\cdot)$ is the symmetric equilibrium strategy in the auction with $n+1$ buyers and no ROFR and $b_*(\cdot)$ is the symmetric equilibrium strategy in the auction for each of the n regular buyers when there is a ROFR. An implication of the exacerbated winner's curse for the regular buyers is that $b_*(\cdot) \leq \hat{b}(\cdot)$.

We can understand the results of Sections 4.1 and 4.2 in terms of Diagram 1. In the private value case, set III disappears, as $h_*(z) \equiv z$. Furthermore, because buyers always bid there valuations, regardless of the presence of a ROFR, the expected surplus from the presence of a ROFR is zero on sets II and IV.

In the common value case, set IV disappears, as $h_*(z) \equiv 0$. Thus, there is no loss of auction revenue from this possibility. In addition, because buyers have the same valuation *ex post*, the total surplus does not shrink on set III or set II when the ROFR is present. The expected surplus from the ROFR, therefore, is non-negative on III and is positive on II. Thus, the pair always benefits from the ROFR.

In order to analyze the gains to the seller/special buyer pair from a ROFR when valuations are correlated, we need to limit the scope of our inquiry. Therefore, in the remainder of this section, we only shall look at a subset, albeit an important one, of the possible affiliated distributions of the V_i's. We assume that buyer i's valuation V_i takes the form

$$V_i \;=\; aV + (1-a)X_i, \quad \text{where } a \in [0,1] \tag{10}$$

and V is the common component of buyers' valuations.[10] We have already examined the case $a = 0$ (Section 4.1) and $a = 1$ (Section 4.2). Next, we consider the case when (i) $a \in (0,1)$, (ii) the X_i's are independently distributed, and (iii) V has specific functional forms with respect to the X_i's.

Let R_1 be the expected profit to the seller/buyer pair without the ROFR, and let R_2 be the expected profit to the seller/buyer pair with the ROFR:

$$\begin{aligned} R_1 &\equiv E\left[V_{n+1}\, 1_{\{I\}} + \hat{b}(X_{n+1})\, 1_{\{II\}} + \hat{b}(Z_{2,n})\, 1_{\{III \cup IV\}}\right] \\ R_2 &\equiv E\left[V_{n+1}\, 1_{\{I \cup II \cup III\}} + b_*(Z_{2,n}) 1_{\{IV\}}\right], \end{aligned}$$

where $1_{\{S\}}$ is the indicator function of the set S. Thus, the increase in expected profit to the pair due to granting of the ROFR is

$$\begin{aligned} E[R_2 - R_1] &= E\left[\{V_{n+1} - \hat{b}(X_{n+1})\}1_{\{II\}}\right] \\ &+ E\left[\{V_{n+1} - \hat{b}(Z_{2,n})\}1_{\{III\}}\right] + E\left[\{b_*(Z_{2,n}) - \hat{b}(Z_{2,n})\}1_{\{IV\}}\right]. \end{aligned} \tag{11}$$

In Section 4.3.1 below we consider an example where the signals are uniformly distributed and V is the average of the X_i's. We show $E[R_2 - R_1]$ can be positive or negative: the sign depends on the values of a in (10) and n. In Section 4.3.2 we show that when V is a (non-decreasing) function of $Z_{1,n+1}$ and $Z_{2,n+1}$ only, $E[R_2 - R_1]$ is positive, regardless of the distribution of the X_i's and the values of a and n. On the other hand, when V is a function of the $Z_{k,n+1}$'s for $k \geq 2$ but not a function of $Z_{1,n+1}$, we show that the pair never benefits from a ROFR.

[10] Note that $\mathrm{Cov}[V_i, V_j] = a^2\,\mathrm{Var}[V] + 2a(1-a)\mathrm{Cov}[X_i, V] + (1-a)^2\mathrm{Cov}[X_i, X_j]$. Thus, if the signals are independent and $\mathrm{Var}[V] \approx \mathrm{Var}[X_i]$, then the level of correlation between buyers' values increases with a.

Let us turn briefly to the calculation of h_*. From (5) we know that either (i) $h_*(z) = 0$ and $\phi(0, z, 0) \geq 0$ or (ii) $h_*(z) > 0$ and $\phi(h_*(z), z, h_*(z)) = 0$. In the latter case (9) implies that $h_*(z)$ must satisfy

$$\begin{aligned} & z - h_*(z) && (12) \\ = & \frac{a}{1-a}\Big(E[V|X_{n+1} = h_*(z),\, Z_{2,n} = z] - E[V|X_{n+1} \leq h_*(z),\, Z_{1,n} = z,\, Z_{2,n} = z]\Big). \end{aligned}$$

We use (11) and (12) in the remainder of the paper.

4.3.1 The average of independent uniform signals case

In general nothing definitive can be said about the value of the ROFR to the seller and special buyer. To illustrate this indeterminacy, below we look at the value of the ROFR when $V_i = aV + (1-a)X_i$, V is the average of all the buyer's signals, and the X_i's are i.i.d. uniform r.v.'s on $[0, 1]$. We analyze this example for two reasons. First, the calculations are straightforward. Second, and more important, for this valuation structure the expected value of the common part V of the value of the object to each buyer is the same (1/2), regardless of the number $n + 1$ of buyers. A desirable and basic consistency condition for comparative statics on the number of buyers is that the expectation of V, the common part of the object's value, is the same for all sets of buyers.

Suppose that $h_*(z) > 0$. Then using the fact that the X_i's are i.i.d. uniform on the unit interval and inserting $V = \frac{1}{n+1}\sum_{i=1}^{n+1} X_i$ in (12), we have

$$\begin{aligned} z - h_*(z) &= \frac{a}{(1-a)}\frac{1}{n+1}\Big\{h_*(z) + E[Z_{1,n}|Z_{2,n} = z] - (E[X_{n+1}|X_{n+1} \leq h_*(z)] + z)\Big\} \\ &= \frac{a}{(1-a)}\frac{1}{n+1}\left(h_*(z) + \frac{1+z}{2} - \frac{h_*(z)}{2} - z\right). \end{aligned}$$

Thus,

$$h_*(z) \equiv \max\Big(z - \underline{z},\, 0\Big), \tag{13}$$

where

$$\underline{z} \equiv \frac{a}{1 + (2n+1)(1-a)}. \tag{14}$$

Consistent with earlier results, $h_*(z) \equiv z$ in the private values case ($a = 0$) and $h_*(z) \equiv 0$ in the common values case ($a = 1$).

We claim that

$$\begin{aligned} & E\Big[(R_2 - R_1)\Big] \\ = \; & \frac{a}{(n+1)^2}\left(\frac{1}{n+2} + \underline{z} - \frac{n+1}{2}\underline{z}^n + (n-1)\underline{z}^{n+1} - \frac{n(n-1)}{2(n+2)}\underline{z}^{n+2}\right) \\ & - (1-a)\left(\frac{1}{2}\underline{z}^2 - \frac{n}{n+1}\underline{z}^{n+1} + \frac{n-1}{n+2}\underline{z}^{n+2}\right). \end{aligned} \tag{15}$$

The proof of this equation is in the appendix. We can, via (15), assess the pair's benefit when a ROFR is granted.

The expected size of the surplus, assuming that the outcome of the auction is Pareto optimal, is $E[aV + (1-a)Z_{1,n+1}] = \frac{(2-a)n+2}{2(n+2)}$. This is the most that the pair can possibly garner. For each (a,n)-pair with $a \in [0,1]$ and $n \geq 2$, set

$$\Delta(a,n) \;=\; \frac{E[R_2 - R_1]}{E[aV + (1-a)Z_{1,n+1}]}.$$

Thus $\Delta(a,n)$ is the proportion of the total surplus that the pair extracts via the ROFR. Using $\Delta(a,n)$, we can study the effect that varying a and n has on the expected returns from the ROFR. Because $\underline{z} = 0$ when $a = 0$, $\Delta(0,n) = 0$ for all n, as expected. Also, as expected, $\Delta(1,n) > 0$ for all n. In fact, $\underline{z} = 1$ when $a = 1$, so $\Delta(1,n) \;=\; 2n/[(n+1)^2(n+2)]$. For each n this value is the global maximum of $\Delta(\cdot,n)$. Likewise, for each a, $\Delta(a,2)$ is the global maximum of $\Delta(a,\cdot)$. Therefore, our interest lies with the value of $\Delta(a,n)$ when $a \in (0,1)$ and $n > 2$.

Figure 1 shows the contour map of $\Delta(a,n)$ for a between 0 to 1 and n between 0 to 30. This map has 7 contour lines that indicate level sets of $\Delta(a,n)$ with values ranging from -0.0015 (the inner most curve) to 0.0015 (the outer most). From this contour map we see that for $n \leq 6$ the pair benefits from the ROFR, regardless of the value of $a > 0$. However, for each $n > 6$ there is a range of a values for which their benefit is negative. As n grows, this range widens.

As n increases, $\Delta(a,n)$ generally decreases. Interestingly, for $n > 15$, $\Delta(\cdot,n)$ achieves its minimum at very high values of a, i.e. at $a \in (0.95,1)$. Thus, a slight decrease in a from 1 to something just less than 1 can have a large effect on the benefit of the ROFR.

4.3.2 Gains from right-of-first-refusal and the nature of the common value

In this section, we show that the seller/special buyer pair's benefit from a ROFR depends critically on the nature of the common value V. In particular, if V is a function of $Z_{1,n}$ and $Z_{2,n}$ (but not $Z_{k,n}$, $k \geq 3$), then $E[R_2 - R_1] \geq 0$ for all a and n and any distribution of the X_i's. Instead, if V does not depend on $Z_{1,n}$, then necessarily $E[R_2 - R_1] \leq 0$.

Without loss of generality, write the correlated values model as

$$V_i \;=\; ag(Z_{1,n+1}, Z_{2,n+1}, ..., Z_{n+1,n+1}) + (1-a)X_i$$

where g is non-decreasing.[11]

[11] In Section 4.3.1, $g(Z_{1,n+1}, Z_{2,n+1}, ..., Z_{n+1,n+1}) = \frac{1}{n+1}\sum_{k=1}^{n+1} Z_{k,n}$. More generally, we can replace V by its conditional expectation $g(Z_{1,n+1}, Z_{2,n+1}, ..., Z_{n+1,n+1}) \equiv E[V|X_1, X_2, ..., X_{n+1}]$. In many economically interesting settings, the function g is symmetric in its $n+1$ arguments; however, as per the discussion after Proposition 7, this need not be the case.

Proposition 7

(i) If $V = g(Z_{1,n+1}, Z_{2,n+1})$ and g is a non-decreasing function in both of its arguments, then $E[R_2 - R_1] \geq 0$.

(ii) If, in addition, g is non-constant in its first argument (and $a > 0$), then $E[R_2 - R_1] > 0$.

The proof of this proposition (see appendix) establishes a stronger result: the gains to the pair are zero on sets I and IV and positive on II and III (where the sets I–IV are defined in Diagram 1). A buyer with signal x proceeds as if his signal is the highest and is tied with another buyer's signal: he presumes that the value of the object is $ag(x, x) + (1 - a)x$. Hence, $b_*(x) \equiv \hat{b}(x) = ag(x, x) + (1 - a)x$, and there is no loss of revenue on set IV.

One setting in which V is a function of $Z_{1,n+1}$ alone and not of $(Z_{k,n+1})_{k\geq 2}$, and therefore Proposition 7 applies, is an art-object auction. Consider the auction of a painting that is known to be authentic. Each buyer knows his private value X_i for the painting. The X_i's are individual estimates of aesthetic worth and, thus, are independent but assumed to be taken from a common distribution. Each buyer's total valuation of the painting is a combination of his private value and the market value because the painting is valued for both its private worth and its resale value. Because the painting is known to be authentic, low private estimates of its worth do not directly affect the resale value. Therefore, a good proxy for the resale value of the painting is the largest of the $n + 1$ private values. In this case a reasonable valuation model for a painting to be auctioned off is the correlated values model with $V = Z_{1,n+1}$. Here a indicates how heavily buyers weigh the resale value when considering the purchase of the painting.

If V depends only on $Z_{2,n+1}$, then it is easy to show that $h_*(z) = z$ and that $b_*(x) = \hat{b}(x) = ag(x) + (1 - a)x$ for all x. Thus, the benefit to the pair from a ROFR equals zero. This foreshadows the next result.

Proposition 8 *When $V = g(Z_{2,n+1}, Z_{3,n+1}, \ldots, Z_{n+1,n+1})$ and g is a non-decreasing function in each of its arguments, the expected gains from trade of a ROFR are always non-positive. Furthermore, if $g(z, \cdot, \ldots, \cdot)$ is not equal to a constant almost everywhere for a z-set of positive F-measure (i.e., if there is a non-trivial dependence on the $Z_{k,n}$'s for $k \geq 3$), then the expected gains are negative.*

The outline of the proof (see details in the appendix) is as follows. Recall that we only have to look at the expected gains from trade of ROFR on sets II and III because the expected gains are $= 0$ and ≤ 0 on sets I and IV, respectively. Because the value of the object does not depend on the highest signal, the equilibrium bid of the buyer with the second-highest signal in a regular second-price auction is equal to his expected value of the object. So on II, the expected gain is zero. On set III the pair makes V_{n+1} with the ROFR and $\hat{b}(Z_{2,n})$ without it. But $\hat{b}(Z_{2,n})$ is equal to the expected value of the object to the buyer with the signal $Z_{2,n}$. Because $Z_{2,n} \geq X_{n+1}$ on this set, by affiliation this buyer's expected value on III (i.e., $E[\hat{b}(Z_{2,n})|\text{III}]$) must be greater than or equal to $E[V_{n+1}|\text{III}]$. Therefore, the expected gain on III is non-positive.

4.4 Effect of non-participation by regular buyers

Proposition 2 and Corollary 2 showed that regular buyers are worse off when a ROFR is granted to the special buyer. If regular buyers incur (at least) a small cost of bid preparation and information gathering and if the reduction in expected profit due to the presence of a ROFR is large enough, some, if not all, of the regular buyers might decide not to participate in the second-price auction. We now show that if a few of the regular buyer do not participate in the auction, then, except in the pure common values case, the profits accruing from a ROFR to the pair can vanish rapidly.

Private Values: By Proposition 5, the gains from a ROFR to the pair is zero, the pair's profit with and without an ROFR being $\max\{X_{n+1}, Z_{2,n}\}$. If the ROFR leads to non-participation by one regular buyer, the pair's profit decreases to $\max\{X_{n+1}, Z_{2,n-1}\}$. Thus, depending on the price paid by the special buyer for the ROFR, either the seller or the special buyer will be strictly worse off with the ROFR.

Common Values: The pair is strictly better off with a ROFR, even if some or all regular buyers do not participate (Proposition 6). It would be naive to assume that a regular buyer will participate in an auction in which he will never purchase the object. If most or all of the regular buyers stay away they do not assist in price discovery: buyer $n+1$ and the seller may not be able to agree on a price. If the seller does not foresee this possibility, he might fail to extract a sufficiently high price to cover the implicit cost associated with granting the ROFR. Mr. Huizenga obtained his right-of-first-refusal for the Miami Dolphins in 1990, four years before its sale, when he purchased a 15% equity stake in the company. The family which owned the Miami Dolphins at that time did not receive adequate compensation for the ROFR from Mr. Huizenga (see Bulow (1995) and Brandenburger and Nalebuff (1996)).

Correlated Values: First, consider the example in Section 4.3.1 where V is the average of uniform signals. Figure 1 shows that for some values of a and n, the pair benefits from a ROFR, provided that all n regular bidders participate in the auction. The effect of non-participation by just one of the regular buyers (when there is a special buyer with a ROFR) is dramatic. As shown in Figure 2, for all values of $a \leq 0.9$, the gain associated with a ROFR is negative if one regular buyer drops out.

Next, we turn to the case when $V = g(Z_{1,n+1}, Z_{2,n+1})$, where Proposition 7 showed that $E[R_2 - R_1]$, the benefit to the pair if all regular buyers participate, is positive. Consider an example where $V = Z_{1,n+1}$ and the distribution of X_i is uniform on $[0, 1]$. Figure 3 shows the region of profitability in the presence of a ROFR as a function of n, a, and the number of regular buyers who fail to participate when there is a ROFR. When $a \leq 0.5$, a ROFR is never profitable for the pair if at least 2 regular buyers drop out.

We know that if V is not a function of $Z_{1,n+1}$, the pair experiences no gains from a ROFR. Usually, one would expect V to depend on all the buyers' signals (as in the example in Section 4.3.1). Therefore, in general one cannot draw definitive conclusions about $E[R_2 - R_1]$. However, if one or two regular buyers do not participate, we do not expect there to be gains associated with a ROFR.

5 Concluding remarks

When the seller awards a special buyer the ROFR, he confers upon her a distinct advantage: she is more likely to purchase the asset from him, and she pays a lower price than she would in the absence of possessing this option. Concomitantly, the seller places himself in an inferior position by granting such an option. Presumably, the special buyer compensates the seller, in some manner, at the time he grants her the ROFR. Thus, it is in the interest of the seller to grant a ROFR to a special buyer only if it is jointly beneficial to them. We show that under private values, the benefit from a ROFR to the special buyer is exactly equal to the cost to the seller; further, the social cost of a ROFR, as measured by the reduction in gains from trade of the object, is borne entirely by the regular buyers. When buyers' valuations are correlated, the presence of a special buyer exacerbates the winner's curse on regular buyers. In either case, if the costs of bid preparation for (at least some of the) regular buyers are higher than their expected profits, some of the regular buyers may not participate in the price discovery stage. Consequently, the special buyer's expected gain from the ROFR is usually less than the expected loss to the seller.

In short, the net benefit to the seller and special buyer is usually negative. Why then does this economic arrangement survive? To answer this question, we take our model as a benchmark and informally discuss two departures from our model under which granting a ROFR can be profitable to the seller.

First, the ROFR might have been granted in a preceding period when no other buyers were interested in the seller's product. For instance, this special buyer may have used her monopsony position to extract a ROFR on future sales by the seller. Only after the quality of the seller's product became obvious to other (regular) buyers did they become interested in purchasing the item. For instance, publishers and music studios, who promote and sell the first pieces of work of authors and rock bands, respectively, extract a ROFR on future works by these artists. Something similar happened when Coke and Pepsi granted NutraSweet a meet-the-competition clause (MCC) for the supply of aspartame in the 1980's; at that time there was no other supplier of aspartame as it was patented by NutraSweet. If lack of initial interest from other buyers [or initial availability of other sellers] is the reason for the ROFR [or MCC], then our analysis indicates that at subsequent stages, when other buyers [sellers] are also interested in transacting, the seller [buyer] would prefer not to have granted a ROFR [MCC].

A second reason for the ROFR is that it may be industry norm, the justification being specific investments made by the buyer in order to consume previous purchases. Again, the example of authors (or rock stars) is pertinent here. The publisher takes a risk in promoting a first-time author; if the book succeeds, then the publisher should have the option of publishing the author's next book(s). In this story, as with patents, there is a tension between *ex ante* and *ex post* efficiency. Perhaps it is *ex ante* efficient if the publisher of the first book of an unknown author elicits a ROFR at the time of the first sale in order to compensate the publisher for promoting a book that has a high probability of failure. Without the ROFR the publisher's expected profit from the book might be negative, but a ROFR provides sufficient inducement to publish the book. However, if the book sells millions, then *ex post* social surplus might be larger if the next book by this author is with another publisher with a wider distribution network.

Nevertheless, there are markets in which the practice of granting a ROFR remains a puzzle to us. One example is residential real estate. Landlords sometimes grant a ROFR to tenants who lease a house or apartment from them. If at a future date the landlord decides to sell the property, the ROFR entitles the tenant to buy at the best price others are willing to pay. This is a market in which it is easy to find other buyers (tenants). The leasing contract forbids the tenant from "investing" in (i.e., making improvements to) the property. Hence, none of the two justifications for a ROFR discussed above apply. Further, a ROFR seems particularly detrimental to the landlord because a tenant who is interested in exercising her ROFR does not have an incentive to show the property in its best condition to other potential buyers.

Regardless of the possible reason for the existence of a ROFR, it is clear that this is a benefit that must not be conferred lightly by the seller to a buyer. Furthermore, as a ROFR is *ex post* inefficient, there is a case to be made against contractual arrangements that grant a ROFR [or MCC] in perpetuity.

6 Appendix

6.1 Proofs of section 3 results

Proof of Proposition 2: Let $\hat{V}(x,y,z) \equiv E[V_n | X_n = x, Z_{1,n-1} = y, X_{n+1} = z]$. Without a ROFR the regular buyers as a whole make

$$E[\Pi_{\overline{ROFR}}] = E\left[\left(\hat{V}(Z_{1,n}, Z_{2,n}, X_{n+1}) - \hat{b}(Z_{2,n} \vee X_{n+1})\right) 1_{\{X_{n+1} < Z_{1,n}\}}\right]$$

where $\hat{V}(Z_{1,n}, Z_{2,n}, X_{n+1}) - \hat{b}(Z_{2,n} \vee X_{n+1})$ is the difference between the expected value of the object to the regular buyer with the largest signal and the price that this buyer must pay the seller. When the special buyer has a ROFR, the regular buyers' expected profit is

$$E[\Pi_{ROFR}] = E\left[\left(\hat{V}(Z_{1,n}, Z_{2,n}, X_{n+1}) - b_*(Z_{2,n})\right) 1_{\{X_{n+1} < h_*(Z_{2,n})\}}\right].$$

We show that $E[\Pi_{\overline{ROFR}}] > E[\Pi_{ROFR}]$. To this end, define

$$\Lambda(z_1, z_2, u) \equiv E\left[\left\{\hat{V}(z_1, z_2, X_{n+1}) - E\left[V_n \middle| X_n = z_2, Z_{1,n-1} = z_2, X_{n+1} \leq u\right]\right\} 1_{\{X_{n+1} < u\}}\right].$$

Clearly,

$$E[\Pi_{\overline{ROFR}}] > E\left[\left(\hat{V}(Z_{1,n}, Z_{2,n}, X_{n+1}) - \hat{b}(Z_{2,n})\right) 1_{\{X_{n+1} < Z_{2,n}\}}\right] = \Lambda(Z_{1,n}, Z_{2,n}, Z_{2,n}). \tag{16}$$

To simplify $\Lambda(z_1, z_2, u)$ note that

$$\begin{aligned} E\left[V_n \middle| X_n = z_2, Z_{1,n-1} = z_2, X_{n+1} \leq u\right] &= \int_0^u \hat{V}(z_2, z_2, x) \frac{dP\left(X_{n+1} \leq x \middle| X_n = z_2, Z_{1,n-1} = z_2\right)}{P\left(X_{n+1} \leq u \middle| X_n = z_2, Z_{1,n-1} = z_2\right)} \\ &= \int_0^u \hat{V}(z_2, z_2, x) dP\left(X_{n+1} \leq x \middle| X_{n+1} \leq u\right). \end{aligned}$$

The second equation uses the fact that the signals are independent. With this we see that $\Lambda(z_1, z_2, u)$ equals

$$\int_0^u \hat{V}(z_1, z_2, x) dP(X_{n+1} \leq x) - \int_0^u \hat{V}(z_2, z_2, x) dP\left(X_{n+1} \leq x \middle| X_{n+1} \leq u\right) P(X_{n+1} \leq u)$$

$$= \int_0^u \left[\hat{V}(z_1, z_2, x) - \hat{V}(z_2, z_2, x)\right] dP(X_{n+1} \leq x).$$

By affiliation $\hat{V}(z_1, z_2, x) \geq \hat{V}(z_2, z_2, x)$. Therefore, $\Lambda(z_1, z_2, u)$ is an non-decreasing function of u for every z_1 and z_2.

To complete the proof, we note that $\hat{b}(x) = E\left[V_n \middle| X_n = x, Z_{1,n-1} = x, X_{n+1} \leq x\right]$ and equation (16) together with the definitions of $E[\Pi_{ROFR}]$ and $b_*(x)$ yields

$$E[\Pi_{\overline{ROFR}}] > \Lambda(Z_{1,n}, Z_{2,n}, Z_{2,n}) \geq \Lambda(Z_{1,n}, Z_{2,n}, h_*(Z_{2,n})) = E[\Pi_{ROFR}].$$

∎

Proof of Proposition 4: Let

$$z_0 \equiv \sup\{z \in [0, \overline{X}] \mid h_0(z) = 0\}.$$

As h_0 is non-decreasing, $h_0(z) = 0$ for all $z < z_0$ and $h_0(z) > 0$ for all $z > z_0$. Define

$$b_1(x) \equiv U(x, x, h_0(x)).$$

As h_0 is non-decreasing, affiliation implies that b_1 is increasing. Consequently, b_1 must be continuous almost everywhere in $[0, \overline{X}]$.

The proof follows directly from three key lemmas for the equilibrium increasing/non-decreasing strategy pair (b_0, h_0).

Lemma 2 *At every point $x > z_0$ for which the function b_1 is continuous, we have $b_0(x) = b_1(x)$.*

Proof of Lemma 2: Consider an $x > z_0$ (therefore $h_0(x) > 0$) at which b_1 is continuous. Suppose that $b_0(x) > b_1(x)$. Then, by continuity of b_1, there exists $\epsilon > 0$ such that $b_0(x) > b_1(x+\epsilon)$. Assume that $X_n = y$, where $x < y < x+\epsilon$; hence $b_0(y) > b_0(x) > b_1(y)$. If the highest bid from the first $n-1$ buyers $P \equiv \max_{1\leq i\leq n-1} b_0(X_i)$ is greater than $b_0(y)$, then buyer n does not obtain the object with a bid of $b_0(y)$ or $b_1(y)$. Likewise, if $P \leq b_1(y)$ and $X_{n+1} \leq h\left(b_0^{-1}(P)\right)$, then buyer n receives the object whether he bids $b_0(y)$ or $b_1(y)$ and in each instance pays price P.

On the other hand, if $P \in (b_1(y), b_0(y)]$ and $X_{n+1} \leq h_0\left(b_0^{-1}(P)\right)$, then buyer n obtains the object only when he bids $b_0(y)$. In this case the expected value of the object to buyer n is

$$\begin{aligned} E\left[V_n \middle| X_n = y, Z_{1,n-1} = b_0^{-1}(P), X_{n+1} \leq h_0\left(b_0^{-1}(P)\right)\right] &\leq E\left[V_n \middle| X_n = y, Z_{1,n-1} = y, X_{n+1} \leq h_0(y)\right] \\ &= b_1(y) < P, \end{aligned}$$

where we use affiliation together with the fact that $y \geq b_0^{-1}(P)$ and h_0 is a non-decreasing function. As $h_0(x) > 0$, we know that the event $\{X_{n+1} \leq h_0\left(b_0^{-1}(P)\right)\} \cap \{P \in (b_1(y), b_0(y))\}$ has positive probability. Lastly, we note that Prob$[X_n \in (x, x+\epsilon)] > 0$. Clearly, buyer n would strictly refer to bid $b_1(y)$ rather than $b_0(y)$. Therefore, we must have $b_0(x) \leq b_1(x)$.

A symmetric argument establishes that $b_0(x) \geq b_1(x)$. Thus, $b_0(x) = b_1(x)$ at any point $x > z_0$ where b_1 is continuous. △

At points of discontinuity for b_1 a best-response function to h_0 may differ from b_1. If x is such a point, then $b_0(x') < b_1(x^-)$, $\forall x' < x$, and $b_0(x') > b_1(x^+)$, $\forall x' > x$. As the set of discontinuities of (the increasing function) b_1 has probability measure zero, the differences on this discontinuity set have no effect on the outcome of the bidding. Any symmetric best response $b_0(x)$ to $h_0(x)$ must equal $b_1(x)$ for almost every x for which $h_0(x) > 0$.

Thus, b_0 is specified by h_0 for $x > z_0$. The next lemma considers b_0 for $x < z_0$ and shows that h_0 and h_* have identical zeroes (i.e., $z_0 = \underline{z}$).

Recall that $\underline{z} \equiv \sup\{z \in [0, \overline{X}] \,|\, h_*(z) = 0\}$. By A1, $h_*(z) = 0$ for all $z < \underline{z}$ and $h_*(z) > 0$ for all $z > \underline{z}$. By Proposition 3 we know that (b_*, h_*) form an equilibrium.

Lemma 3 *(i) $b_0(x) \leq W(0, x)$, $\forall x < \underline{z}$. (ii) $z_0 = \underline{z}$.*

Proof of Lemma 3: (i) Suppose instead that $b_0(x) > W(0, x)$, for some $x < \underline{z}$. Because $W(x, z)$ is continuous in both of its arguments and b_0 is an increasing function, there exists an interval $I \subset [0, \underline{z})$ such that for all $z \in I$, $b_0(z) > W(0, z)$. As noted previously in (3), in a symmetric Nash equilibrium with bid function b_0 the special buyer's cutoff function must be

$$h_0(z) \;=\; \min\left\{u \in [0, \overline{X}] \mid W(u, z) - b_0(z) \geq 0\right\}. \tag{17}$$

From the fact that $b_0(z) > W(0, z)$ for all $z \in I$ we know two things: (a) $h_0(z) > 0$ and (b) $W(h_0(z), z) = b_0(z)$ for all $z \in I$.[12] Lemma 2 and (a) imply that if b_0 is to be a best response to h_0 then $b_0(z) = U(z, z, h_0(z))$ for almost all $z \in I$. Recalling that $h_*(z) = 0$ for all $z < \underline{z}$, we see that $W(y, z) > U(z, z, y)$ for all $y > 0$, where strict inequality follows from A2. Thus, $W(h_0(z), z) > U(z, z, h_0(z)) = b_0(z)$ for almost all $z \in I$. But this contradicts (b). Thus, $b_0(x) \leq W(0, x)$, $\forall x < \underline{z}$.

(ii) By (i), $W(0, z) - b_0(z) \geq 0$ for all $z < \underline{z}$. Thus, (17) implies that $h_0(z) = 0$ for all $z < \underline{z}$. Next, suppose that $h_0(z') = 0$ for some $z' > \underline{z}$. As h_0 is non-decreasing, $h_0(z) = 0$, $\forall z \in (\underline{z}, z']$. Thus, by (17), $W(0, z) \geq b_0(z)$, $\forall z \in (\underline{z}, z']$. From the definition of $\underline{z}$, we know that $W(0, z) < U(z, z, 0)$, $\forall z \in (\underline{z}, z']$. Set $b_1(z) = b_0(z)$ for $z < \underline{z}$ and $= U(z, z, h_0(z))$ for $z \geq \underline{z}$. It is easy to see that b_1 leads to a higher expected payoff b. Therefore, we have a contradiction. △

We have established that $h_0(z) = h_*(z)$ for all $z < \underline{z}$. The next lemma implies that $h_0(z) = h_*(z)$ for all $z \geq \underline{z}$.

Lemma 4 *Let $b_1(x) = U(x, x, h_0(x))$. The special buyer's best-response cutoff function, denoted h_1, to b_1 has the following properties:*

1. *$h_1(z) \in (h_0(z), h_*(z)]$ at any point z where $h_0(z) < h_*(z)$, and*
2. *$h_1(z) \in [h_*(z), h_0(z))$ at any point z where $h_0(z) > h_*(z)$.*

[12] The continuity of $W(\cdot, z)$ is also used to conclude (b).

Proof of Lemma 4: If the first n buyers are using the bid function b_1, then when $Z_{2,n} = z$ and $X_{n+1} = x$ buyer $n+1$ makes $W(x,z) - b_1(z)$ in profit, if she decides to buy the object. Therefore, by (3), her optimal response to b_1 is the cutoff function

$$h_1(z) \equiv \min\{u : W(u,z) - b_1(z) \geq 0\} = \min\{u : \phi(u,z,h_0(z)) \geq 0\}.$$

If $h_0(z) < h_*(z)$ at z, then, by the definitions of b_* and b_1, $b_1(z) \leq b_*(z)$. This implies that $h_1(z) \leq h_*(z)$. However, $h_1(z)$ must be strictly greater than $h_0(z)$, because with $h_0(z) < h_*(z)$, $W(h_0(z),z) - U(z,z,h_0(z)) = \phi(h_0(z),z,h_0(z)) < 0$, where the strict inequality follows from A2. If, on the other hand, $h_0(z) > h_*(z)$ at z, then $b_1(z) \geq b_*(z)$, which implies that $h_1(z) \geq h_*(z)$. However, $h_1(z)$ must be strictly less than $h_0(z)$. This holds because, with $h_0(z) > h_*(z)$, assumption A2 implies that $W(h_0(z),z) - b_1(z) = \phi(h_0(z),z,h_0(z)) > 0$. △

To complete the proof of Proposition 4 observe that if (b_0, h_0) forms a symmetric Nash equilibrium, then for $x \leq \underline{z}$, $b_0(x) \leq W(0,x)$ and $h_0(x) = 0$.

Next consider $x > \underline{z}$. We know that $h_0(x) > 0$ in this range. Therefore, by Lemma 2, $b_0(x) = U(x,x,h_0(x))$ almost everywhere. If (b_0,h_0) is to form a Nash equilibrium, then Lemma 4 implies that $h_0(x)$ must equal $h_*(x)$. Thus, for $x > \underline{z}$, $b_0(x) = b_*(x)$, almost everywhere. ∎

6.2 Proofs of section 4 results

The following lemma is useful:

Lemma 5 *When the signals are independent and $V_i = (1-a)X_i + aV$, $\forall i$,*

$$\begin{aligned} E[R_2 - R_1] &= aE\left[\left\{V - E\left[V \middle| X_{n+1}, Z_{2,n}, Z_{1,n} = Z_{2,n} \vee X_{n+1}\right]\right\} 1_{\{II \cup III\}}\right] \\ &\quad - (1-a)E\left[(Z_{2,n} - X_{n+1}) 1_{\{III\}}\right], \end{aligned}$$

where sets II and III are defined as in Section 4.3.

Proof of Lemma 5: Observe that

$$\begin{aligned} \hat{b}(x) &= (1-a)x + aE\left[V \middle| X_{n+1} = x, Z_{1,n} = x\right], \quad \text{and} \\ b_*(x) &= (1-a)x + aE\left[V \middle| X_n = x, Z_{1,n-1} = x, X_{n+1} \leq h_*(x)\right]. \end{aligned}$$

Rewriting (11), we have

$$\begin{aligned} E[R_2 - R_1] &= E\left[\{V_{n+1} - \hat{b}(X_{n+1})\} 1_{\{II\}}\right] \\ &+ E\left[\{V_{n+1} - \hat{b}(Z_{2,n})\} 1_{\{III\}}\right] + E\left[\{b_*(Z_{2,n}) - \hat{b}(Z_{2,n})\} 1_{\{IV\}}\right]. \end{aligned} \tag{18}$$

The first term on the right-hand side above simplifies to

$$aE\left[\left\{V - E\left[V\middle|X_{n+1}, Z_{1,n} = X_{n+1}\right]\right\} 1_{\{II\}}\right].$$

The second term in (18) can be written as

$$(1-a)E\left[(X_{n+1} - Z_{2,n})1_{\{III\}}\right] + aE[V1_{\{III\}}] - aE\left[E\left[V\middle|Z_{2,n}, Z_{1,n+1} = Z_{2,n}\right] 1_{\{III\}}\right].$$

The last term in (18) simplifies to

$$aE\left[E\left[V\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}, IV\right] 1_{\{IV\}}\right] - aE\left[E\left[V\middle|Z_{2,n}, Z_{1,n+1} = Z_{2,n}\right] 1_{\{IV\}}\right].$$

Next, we note that

$$\begin{aligned}
&E\left[E\left[V\middle|Z_{2,n}, Z_{1,n+1} = Z_{2,n}\right] 1_{\{III\}}\right] + E\left[E\left[V\middle|Z_{2,n}, Z_{1,n+1} = Z_{2,n}\right] 1_{\{IV\}}\right] \\
&= E\left[E\left[V\middle|Z_{2,n}, Z_{1,n+1} = Z_{2,n}\right] \{1_{\{III\}} + 1_{\{IV\}}\}\right] \\
&= E\left[E\left[V\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}, III \cup IV\right] \{1_{\{III\}} + 1_{\{IV\}}\}\right] \\
&= E\left[E\left[E\left[V\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}, III \cup IV\right] \{1_{\{III\}} + 1_{\{IV\}}\}\middle|Z_{2,n}\right]\right] \\
&= E\left[E\left[V\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}, III \cup IV\right] P(III \cup IV|Z_{2,n})\right] \\
&= E\left[E\left[V\{1_{\{III\}} + 1_{\{IV\}}\}\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}\right]\right].
\end{aligned}$$

The last equality uses the independence of $Z_{1,n}$ and X_{n+1}. Similarly,

$$\begin{aligned}
E\left[E\left[V\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}, IV\right] 1_{\{IV\}}\right] &= E\left[E\left[V\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}, IV\right] P(IV|Z_{2,n})\right] \\
&= E\left[E\left[V1_{\{IV\}}\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}\right]\right].
\end{aligned}$$

The last equality uses the fact that $Z_{1,n}$ and X_{n+1} are independent.

Putting this all together, we see that in summing the latter two terms in (18) the factors multiplied by $1_{\{IV\}}$ cancel out, reducing this sum to

$$-(1-a)E\left[(Z_{2,n} - X_{n+1})1_{\{III\}}\right] + aE\left[V1_{\{III\}} - E\left[V1_{\{III\}}\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}\right]\right].$$

We note that by the independence of the X's and the fact that set III depends only on X_{n+1} and $Z_{2,n}$

$$E\left[E\left[V1_{\{III\}}\middle|Z_{2,n}, Z_{1,n} = Z_{2,n}\right]\right] = E\left[E\left[V\middle|X_{n+1}, Z_{2,n}, Z_{1,n} = Z_{2,n}\right] 1_{\{III\}}\right].$$

Therefore, the sum of the latter two terms in (18) equals

$$-(1-a)E\left[(Z_{2,n}-X_{n+1})1_{\{III\}}\right]+aE\left[\left\{V-E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=Z_{2,n}\right]\right\}1_{\{III\}}\right].$$

Finally, using the fact that the set $\{X_{n+1}=Z_{1,n}\}=\{X_{n+1}=Z_{1,n}\}\cap II$ and that, conditioned on the value of X_{n+1} and the fact that $Z_{2,n}\leq X_{n+1}$, the values of $Z_{2,n}$ and $Z_{1,n}$ are independent, we have

$$E\left[E\left[V\middle|X_{n+1},Z_{1,n}=X_{n+1}\right]1_{\{II\}}\right] = E\left[E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=X_{n+1}\right]1_{\{II\}}\right].$$

Putting all of this together with (18) yields

$$\begin{aligned}E[R_2-R_1] &= aE\left[\left\{V-E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=X_{n+1}\right]\right\}1_{\{II\}}\right]-(1-a)E\left[(Z_{2,n}-X_{n+1})1_{\{III\}}\right]\\ &+ aE\left[\left\{V-E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=Z_{2,n}\right]\right\}1_{\{III\}}\right].\end{aligned}$$

Combining like terms above completes the proof. △

Proof of Equation (15): Below we make use of two facts: First, a uniform random variable on $[0,1]$ conditioned on being $\leq$ (or $\geq$) z is a uniform random variable on $[0,z]$ (or $[z,1]$). Second, $Z_{1,n}$ is conditionally independent of the $Z_{k,n}$'s for $k\geq 3$, given $Z_{2,n}$.

From Lemma 5, we know that

$$E\left[(R_2-R_1)1_{\{II\}}\right] = aE\left[\left\{V-E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=X_{n+1}\right]\right\}1_{\{II\}}\right] \tag{19}$$

and

$$\begin{aligned}E\left[(R_2-R_1)1_{\{III\cup IV\}}\right] &= aE\left[\left\{V-E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=Z_{2,n}\right]\right\}1_{\{III\}}\right]\\ &\quad -(1-a)E\left[(Z_{2,n}-X_{n+1})1_{\{III\}}\right].\end{aligned} \tag{20}$$

Focussing first on set II, we note that

$$\begin{aligned}&E\left[\left\{V-E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=X_{n+1}\right]\right\}1_{\{II\}}\right]\\ &= E\left[\left\{E\left[V\middle|X_{n+1},Z_{2,n}\right]-E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=X_{n+1}\right]\right\}1_{\{II\}}\right].\end{aligned}$$

Because V is simply the average of the signals and the signals are independent

$$E\left[V\middle|X_{n+1},Z_{2,n}\right] = \frac{1}{n+1}\left(X_{n+1}+E[Z_{1,n}|X_{n+1},Z_{2,n}]+\sum_{k=2}^{n}E[Z_{k,n}|Z_{2,n}]\right),$$

and

$$E\left[V\middle|X_{n+1},Z_{2,n},Z_{1,n}=X_{n+1}\right] = \frac{1}{n+1}\left(X_{n+1}+X_{n+1}+\sum_{k=2}^{n}E[Z_{k,n}|Z_{2,n}]\right).$$

Therefore, equation (19) reduces to

$$
\begin{aligned}
E\left[(R_2 - R_1)1_{\{II\}}\right] &= \frac{a}{n+1}E\left[\{E[Z_{1,n}|X_{n+1}, Z_{2,n}] - X_{n+1}\}\, 1_{\{II\}}\right] \\
&= \frac{a}{n+1}E\left[\left\{\frac{1+X_{n+1}}{2} - X_{n+1}\right\} 1_{\{II\}}\right] \\
&= \frac{a}{2(n+1)}E\left[(1 - X_{n+1})1_{\{II\}}\right].
\end{aligned}
$$

Conditioned on the value of $Z_{2,n}$ and the set II, X_{n+1} is distributed like the smaller of two independent uniform r.v.'s on the interval $[Z_{2,n}, 1]$. Conditioned on $Z_{2,n}$, $Z_{1,n} \sim$ Uniform$[Z_{2,n}, 1]$. Given the value of $Z_{2,n}$, the probability that II occurs equals

$$
P(X_{n+1} \geq Z_{2,n}|Z_{2,n})P(X_{n+1} < Z_{1,n}|Z_{2,n}, X_{n+1} \geq Z_{2,n}) \;=\; (1 - Z_{2,n})\frac{1}{2}.
$$

With this we have

$$
\begin{aligned}
E\left[(R_2 - R_1)1_{\{II\}}\right] &= \frac{a}{2(n+1)}E\left[E\left[1 - X_{n+1}\middle|Z_{2,n}, II\right] P(II|Z_{2,n})\right] \\
&= \frac{a}{2(n+1)}E\left[\left(1 - \frac{2}{3}Z_{2,n} - \frac{1}{3}\right)(1 - Z_{2,n})\frac{1}{2}\right] \\
&= \frac{a}{6(n+1)}E\left[(1 - Z_{2,n})^2\right] \\
&= \frac{a}{(n+1)^2(n+2)}. \qquad (21)
\end{aligned}
$$

Using a similar line of reasoning:

$$
\begin{aligned}
&E\left[\left\{V - E\left[V\middle|X_{n+1}, Z_{2,n}, Z_{1,n} = Z_{2,n}\right]\right\} 1_{\{III\}}\right] \\
&= E\left[\left\{E\left[V\middle|X_{n+1}, Z_{2,n}\right] - E\left[V\middle|X_{n+1}, Z_{2,n}, Z_{1,n} = Z_{2,n}\right]\right\} 1_{\{III\}}\right] \\
&= \frac{1}{n+1}E\left[\left\{X_{n+1} + E[Z_{1,n}|X_{n+1}, Z_{2,n}] + \sum_{k=2}^{n} E[Z_{k,n}|Z_{2,n}]\right.\right. \\
&\qquad\qquad \left.\left. - \left(X_{n+1} + Z_{2,n} + \sum_{k=2}^{n} E[Z_{k,n}|Z_{2,n}]\right)\right\} 1_{\{III\}}\right] \\
&= \frac{1}{n+1}E\left[\{E[Z_{1,n}|X_{n+1}, Z_{2,n}] - Z_{2,n}\}\, 1_{\{III\}}\right] \;=\; \frac{1}{n+1}E\left[\{Z_{1,n} - Z_{2,n}\}\, 1_{\{III\}}\right].
\end{aligned}
$$

Thus, equation (20) simplifies to

$$
E\left[(R_2 - R_1)1_{\{III \cup IV\}}\right] \;=\; \frac{a}{n+1}E\left[\{Z_{1,n} - Z_{2,n}\}\, 1_{\{III\}}\right] - (1-a)E\left[(Z_{2,n} - X_{n+1})1_{\{III\}}\right]
$$

Given that the X_i's are uniform on $[0, 1]$ and independent,

$$
\begin{aligned}
E\left[(Z_{2,n} - X_{n+1})1_{\{III\}}|Z_{2,n}\right] &= \frac{1}{2}\left(Z_{2,n} \wedge \underline{z}\right)^2, \quad \text{and} \\
E\left[(Z_{1,n} - Z_{2,n})1_{\{III\}}|Z_{2,n}\right] &= \frac{1 - Z_{2,n}}{2}(Z_{2,n} \wedge \underline{z}),
\end{aligned}
$$

where $\underline{z} = a/[1+(2n+1)(1-a)]$ and "$\wedge$" indicates minimum. Thus,

$$\begin{aligned} E\left[(R_2-R_1)1_{\{III\}}\right] &= \frac{a}{2(n+1)}E\left[(1-Z_{2,n})(Z_{2,n}\wedge\underline{z})\right] - \frac{1-a}{2}E\left[(Z_{2,n}\wedge\underline{z})^2\right] \\ &= \frac{a}{(n+1)^2}\left(\underline{z} - \frac{n+1}{2}\underline{z}^n + (n-1)\underline{z}^{n+1} - \frac{n(n-1)}{2(n+2)}\underline{z}^{n+2}\right) \\ &\quad - (1-a)\left(\frac{1}{2}\underline{z}^2 - \frac{n}{n+1}\underline{z}^{n+1} + \frac{n-1}{n+2}\underline{z}^{n+2}\right) \qquad (22) \end{aligned}$$

Adding (21) and (22) we get (15). ∎

Proof of Proposition 7: Using Lemma 5, we can easily show that the expected profit to the pair in this case is positive. Instead, we prove the stronger result that the gains to the pair are zero on the sets I and IV, and positive on II and III.

If $h_*(z) > 0$, then equation (12) yields

$$\begin{aligned} z - h_*(z) &= \frac{a}{1-a}\left(E\left[g(Z_{1,n+1}, Z_{2,n+1})\Big| X_{n+1} = h_*(z), Z_{2,n} = z\right] - g(z,z)\right) \\ &= \frac{a}{1-a}\left(E\left[g(X,z)\Big| X \geq z\right] - g(z,z)\right) \end{aligned}$$

The equations above use the independence of signals, and the fact that $h_*(z) \leq z$ and that $Z_{1,n+1} = Z_{1,n}$ and $Z_{2,n+1} = Z_{2,n}$ if $X_{n+1} \leq Z_{2,n}$. Rearranging terms, we have

$$h_*(z) = \max\left\{z - \frac{a}{1-a}E\left[g(X,z) - g(z,z)|X \geq z\right], 0\right\}. \qquad (23)$$

We have seen that the expected gains from ROFR trade equal zero on set I and are non-negative on set II. Next, we show that if $g(z_1, z_2)$ depends on z_1 on a set of positive measure, then the expected gains on II are positive. Observe that on II

$$\begin{aligned} R_2 &= V_{n+1} = (1-a)X_{n+1} + ag(Z_{1,n+1}, Z_{2,n+1}), \text{ and} \\ R_1 &= \hat{b}(X_{n+1}) = (1-a)X_{n+1} + ag(X_{n+1}, X_{n+1}). \end{aligned}$$

Thus,

$$\begin{aligned} E[(R_2-R_1)1_{\{II\}}] &= aE[\{g(Z_{1,n+1}, Z_{2,n+1}) - g(X_{n+1}, X_{n+1})\}1_{\{II\}}] \\ &= aE[\{g(Z_{1,n}, X_{n+1}) - g(X_{n+1}, X_{n+1})1_{\{Z_{1,n} > X_{n+1} \geq Z_{2,n}\}}\}] > 0. \end{aligned}$$

On III, $X_{n+1} \in [h_*(Z_{2,n}), Z_{2,n})$. Thus

$$\begin{aligned} R_2 &= V_{n+1} = (1-a)X_{n+1} + ag(Z_{1,n+1}, Z_{2,n+1}), \text{ and} \\ R_1 &= \hat{b}(Z_{2,n}) = (1-a)Z_{2,n} + ag(Z_{2,n}, Z_{2,n}). \end{aligned}$$

As $X_{n+1} < Z_{2,n}$, we have $Z_{1,n+1} = Z_{1,n}$, $Z_{2,n+1} = Z_{2,n}$. From (23) we have

$$aE\left[g(Z_{1,n}, Z_{2,n}) - g(Z_{2,n}, Z_{2,n})|Z_{2,n}\right] \geq (1-a)\left(Z_{2,n} - h_*(Z_{2,n})\right).$$

From this we see that

$$\begin{aligned} E[(R_2 - R_1)1_{\{III\}}] &= (1-a)E\left[(X_{n+1} - Z_{2,n})1_{\{III\}}\right] + aE\left[(g(Z_{1,n}, Z_{1,n}) - g(Z_{2,n}, Z_{2,n}))1_{\{III\}}\right] \\ &\geq (1-a)E\left[(X_{n+1} - h_*(Z_{2,n}))1_{\{III\}}\right] \;>\; 0, \end{aligned}$$

when $P(III) > 0$, and $= 0$ otherwise. We note that $P(III) = 0$ only when $g(z_1, z_2)$ does not depend on z_1 almost everywhere. In that case $h_*(z) = z$ for all z.

On set IV, the special buyer wins neither auction. Thus, $R_2 = b_*(Z_{2,n})$ and $R_1 = \hat{b}(Z_{2,n})$. It is easy to see that $b_*(z) = \hat{b}(z) = ag(z,z) + (1-a)z,\ \forall z$. Thus, there are no gains or losses on this set. ∎

Proof of Proposition 8: Because V does not depend on $Z_{1,n+1}$, on the sets II and III we have

$$E\left[V\middle|X_{n+1}, Z_{2,n}, Z_{1,n} = Z_{2,n} \vee X_{n+1}\right] \;=\; E\left[V\middle|X_{n+1}, Z_{2,n}\right].$$

Thus, by Lemma 5

$$\begin{aligned} E[R_2 - R_1] &= aE\left[\left\{V - E\left[V\middle|X_{n+1}, Z_{2,n}\right]\right\}1_{\{II\cup III\}}\right] - (1-a)E\left[(Z_{2,n} - X_{n+1})1_{\{III\}}\right] \\ &= -(1-a)E\left[(Z_{2,n} - X_{n+1})1_{\{III\}}\right] \;\leq\; 0, \end{aligned}$$

completing the first part of the proof.

Clearly, when $h_*(z) \equiv z$, then $P(III) = 0$, which from the above analysis implies that $E[R_2 - R_1] = 0$. If, on the other hand, $h_*(z) < z$ on some set of positive F-measure, then $P(III) > 0$ and $E[R_2 - R_1] < 0$.

If $h_*(z) = z$, then by the definition of h_* the function $\phi(z,z,z) = 0$. By (4) this implies that

$$\begin{aligned} &E\left[g(z, Z_{3,n+1}, \ldots, Z_{n+1,n+1})\middle|Z_{2,n} = z, X_{n+1} = z\right] \;- \\ &\quad E\left[g(z, Z_{3,n+1}, \ldots, Z_{n+1,n+1})\middle|Z_{2,n} = z, X_{n+1} \leq z\right] \;=\; 0. \qquad (24) \end{aligned}$$

But (24) implies that for each z, $g(z, \cdot, \ldots, \cdot)$ is constant almost everywhere. Thus, if g does not depend solely on its first argument almost everywhere, then $h_*(z) < z$ for a set of positive F-measure. ∎

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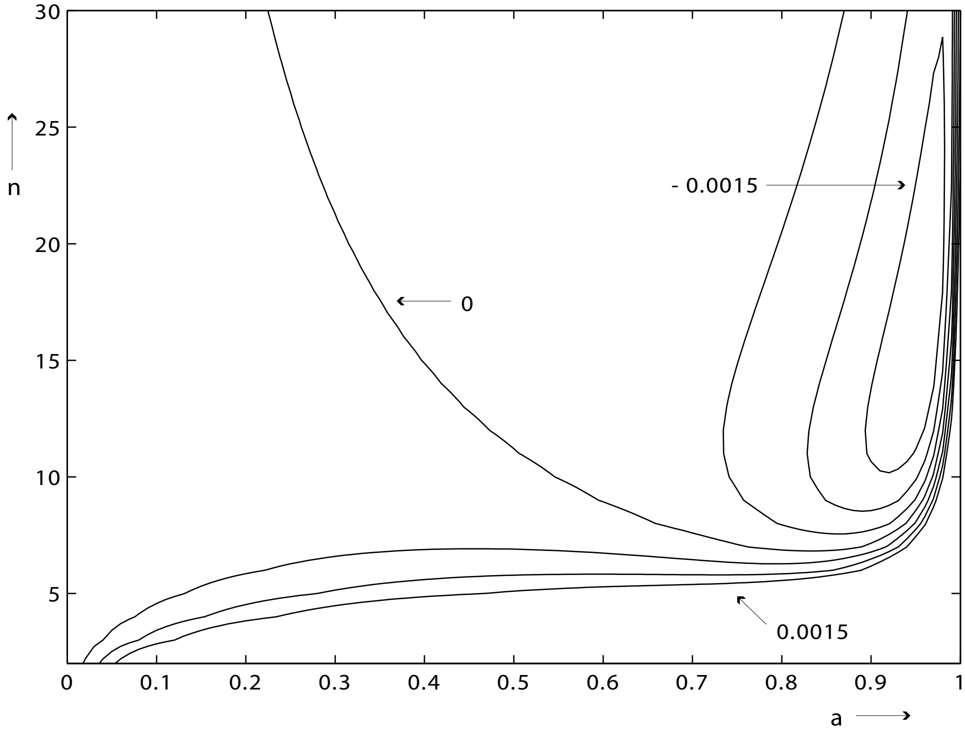
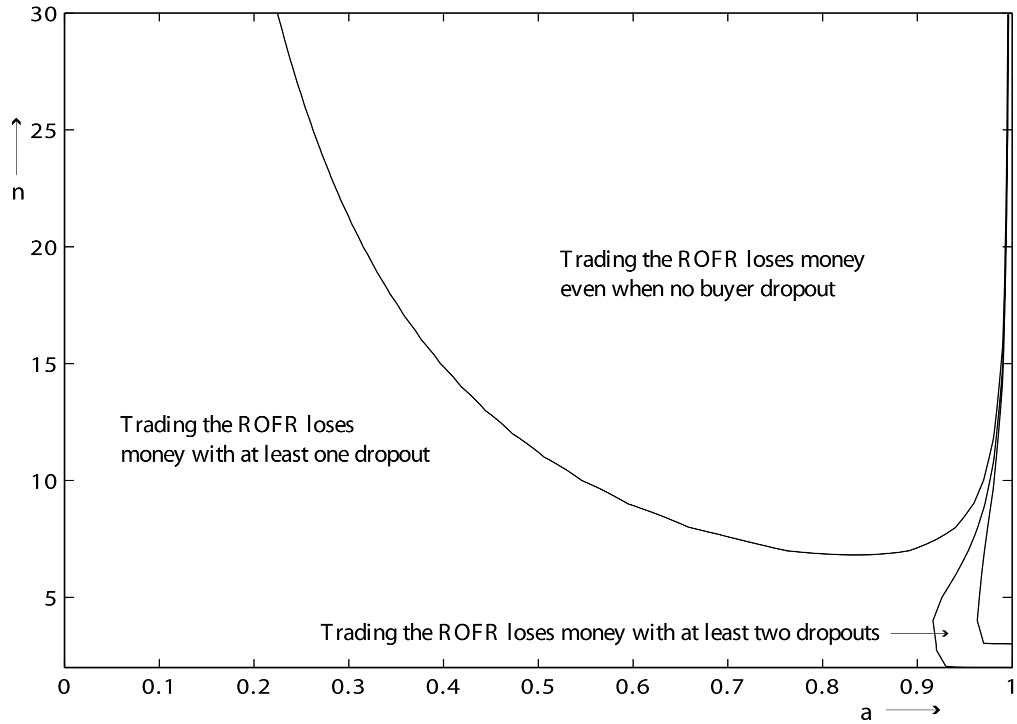


Figure 1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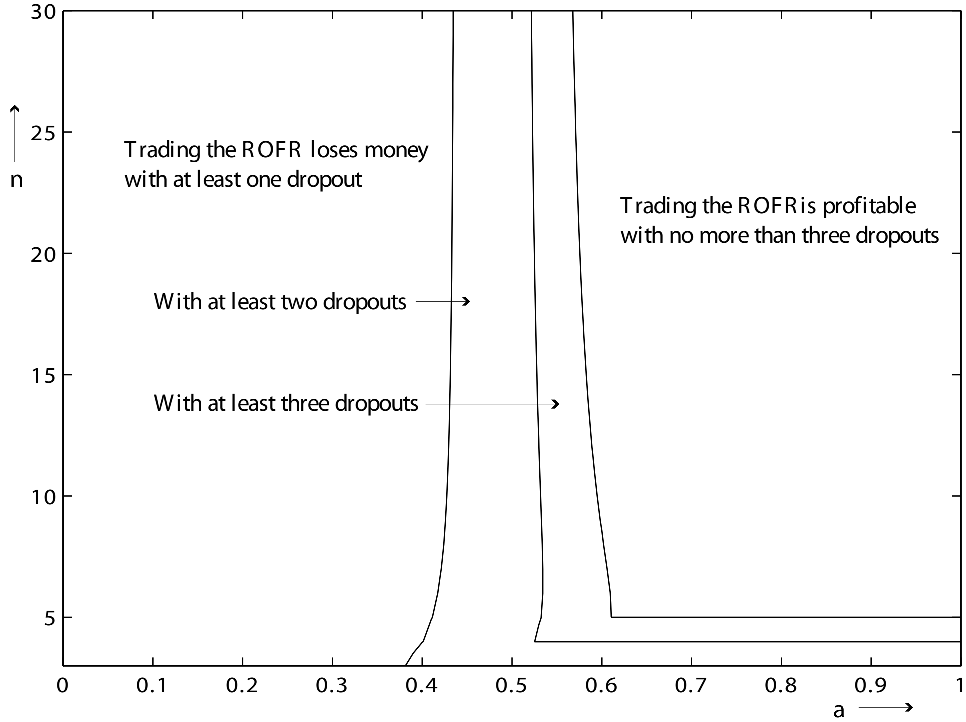
Trading the ROFR loses money even when no buyer dropout
Trading the ROFR loses money with at least one dropout
Trading the ROFR loses money with at least two dropouts

Figure 2



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Trading the ROFR loses money
with at least one dropout
Trading the ROFR is profitable
with no more than three dropouts
With at least two dropouts
With at least three dropouts
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Figure 3