MANAGING DEVELOPMENT RISK IN PRODUCT DESIGN PROCESSES

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Product development has become the focal point of industrial competition and is the cornerstone of long-term survival for most firms. One of the major management challenges in product development is to deal with development risk in the design process. In this paper we provide a strategic guideline as to how the design process should be managed and controlled. We describe how design reviews and engineering resources can be scheduled as the control mechanisms to operationally manage development risk. The methodologies developed are an integral part of a project to fundamentally restructure product design processes at Rocketdyne Division of Rockwell International, which designs and develops liquid-propellant rocket propulsion systems.

Product development has become the focal point of industrial competition. Product design and development, however, is a very complex and risky process requiring knowledge spanning different disciplines, such as engineering, manufacturing, and marketing. Many empirical studies have documented the high failure rates in new product design and development. In particular, Moore and Pessiner (1993) find that many product development projects fail because of poor management of the design process and product development risk. In this paper we concentrate on managing product development risk, by which we mean the process risk involved in developing a new product. In other words, development risk is the risk of poor technical execution and missing the targets. The degree of product development risk can be determined by such characteristics as product complexity, technological requirements and difficulty, and firm’s product and process development capability. To a large extent development risk can be controlled in the design process, and such controls have direct impact on development lead time, development cost, and product performance.

To optimally configure the product development structure, we concentrate on managerial control tools in development risks: design reviews and design resource allocation at various design stages. These control mechanisms collectively define the product development control structure. Companies that focus on the performance and cost dimensions of the development risk typically conduct many design reviews with various layers of management involvement. These reviews try to minimize development risks at the expense of additional overhead and increased project lead time. It is well known that an inappropriate scheduling of management involvement typically results in extended project completion time. Both under- and over-specification of product development risk result in longer project duration. Consequently, the key question is how we can schedule an ideal level of management control by allocating appropriate engineering resource and choosing the optimal review frequency and review acceptance levels at each design stage. Ha and Porteus (1995) developed a EOQ-like model to determine an optimal review policy. We extend this line of study.

The paper is organized as follows. Section 1 discusses the motivation and provides some background for the research. We use the design process for the turbopumps at Rocketdyne Division of Rockwell International to show and discuss the approach used in the paper. In Section 2, analytical models are developed to determine a design review configuration and resource allocation at each stage of the design process. In Section 3, we discuss various managerial implications drawn from the models. Section 4 concludes the paper.

1. MOTIVATION AND DESIGN SETTING

This research is motivated by, and the methodologies developed are, an integral component of the Rocketdyne Advanced Process Integration Development (RAPID) project at the Rocketdyne Division of Rockwell International. Rocketdyne designs and develops liquid-propellant rocket propulsion systems, including the main engine for the Space Shuttle. Rocketdyne is faced with the challenge of commercializing some of its nonproprietary technologies. Such shifts in business focus have put the organization into a different and more competitive environment. These changes require fundamental restructuring in the business strategic planning process for design and manufacturing. RAPID’s objective is to drastically reduce product development lead time and cost of the rocket engine by managing design development risk. Exhibit 1 shows the Space Shuttle Main Engine developed at Rocketdyne. Next we describe the characteristics of the design process at Rocketdyne’s turbomachinery division and address some of the shortcomings that RAPID is attempting to overcome.
The turbomachinery division designs and tests various turbopumps used in rocket engines. A turbopump is a key component of any rocket engine and consists of three major components: pump, turbine, and mechanical arrangements (see Stangeland 1988). The rocket engine components are typically exposed to extreme, highly dynamic stresses under equally extreme thermal and chemical environments. This operating environment complicates the design and development of the products. Exhibit 2 depicts the existing design process at the turbomachinery division. The process consists of eight design phases as follows: (1) design requirements and ground rules, (2) conceptual, (3) preliminary, (4) detail design, (5) fabrication, (6) development and certification, (7) test and field support, and finally (8) production. Design ground rules and customer design requirements are captured in the first phase. Conceptual design phase results in the development of a cross-sectional view of the turbopump that meets all engine steady-state design requirements. Only the best design concept proceeds to the succeeding design phase. Exhibit 3 shows the type of analysis performed at the conceptual design phase. Preliminary design phase enhances the cross-sectional view to additionally meet all engine pretest, transient, and off-design requirements. Detailed design phase concludes the design process by completing the dimensioned and tolerated detailed component design and the assembly drawing that has projected all system safety failure mode scenarios and meets all producibility requirements. The fabrication phase right after the detail design makes parts and hardware. In the development phase, prototypes are made and tested under various conditions according to the contract.

The existing design process is lengthy and costly. The design phases are primarily based on the levels of detail required. Often, extensive engineering hours are spent and many engineering changes might be required to correct design flaws and improve poor design concepts, which are difficult to detect in the early design phases. Design reviews are usually externally set, are based on the level of funding available for the project, and often do not consider the product development risk inherent in the specific products to be designed. Phase reviews that coincide with the contractual milestones are rigid obstacles to reducing design development time. Transitions to the succeeding phases require approval by high management and government-customer liaison authorities. Various prototypes of the individual components and system are developed and tested while the design is under way to ensure the feasibility of the concept being developed. Formal reports are generated to document the details of the design reviews. The design will progress to the next phase only if the customer concurs with the outcome of the design review and makes further funding available. Conducting formal reviews is a lengthy task and an ineffective means for identifying problems and proposing solution approaches. It is commonly acknowledged that Rocketdyne’s lengthy and costly design process is a major source in meeting today’s challenges. In the next section, we develop models to analyze various trade-offs in managing development risk in the design process.

2. MANAGING DEVELOPMENT RISK

Like production processes, design processes also need to be monitored and controlled in order to achieve high design efficiency and quality. Distinct product development projects require different levels of management involvement and control. Both over-managed and under-managed development processes result in lengthy design lead time and high development cost. The level of management involvement and control in the design process is essentially determined by how the design reviews are organized along the design process and how engineering resources are allocated at each design stage. A design process usually consists of a series of stages, followed by review activities often placed at the end of a design stage. For any given product development project, managers usually identify the ground rules and set the design specifications for each stage, and consequently they determine the target conformance specifications and acceptance level for each design review (see Smith and Reinertsen 1991 for further
Design processes at Turbomachinery Division of Rocketdyne.

2.1. Modeling Framework

We assume that design activities are partitioned and organized into a series of $N$ design blocks (see Ahmadi and Wang 1994, Krishnan et al. 1992, Smith and Eppinger 1997), each of which may be considered as a design stage. The linkages between any two design stages represent information flow and dependency relation. Figure 1 shows the example of a design process with four design stages.

We assume that a design review could be placed only at the end of a design stage, and evaluates only design stages that follow the last review. Final review takes place after the last design stage.

To model the design review problem, we define process confidence as a measure of design conformance quality and the perceived engineering reliability of the product to be designed. Generally, in the design engineering community the process confidence is assumed to be a nondecreasing function of the amount of design resources applied throughout the design process. While design resources include personnel, equipment, technologies, and so on, for our purpose engineering hours is the most critical resource used in the design process. The general form of the functional relationship between process confidence and engineering hours has to be estimated and in our application was provided by the managers at Rocketdyne.

For each design stage, we define stage confidence as a measure of the degree of compatibility and closeness of the design to its target specifications at that stage. Since each design stage provides certain information indispensable to the overall design of the product, the process confidence can therefore be defined as:

$$R = \sum_{i=1}^{N} \alpha_i R_i,$$

where $0 \leq R \leq 1$ denotes the process confidence; $0 \leq R_i \leq 1$ ($i = 1, 2, \ldots, N$) denotes the stage confidence for


Exhibit 3. Conceptual design phase at turbomachinery.

stage $i$; $\alpha_i$ defines the relative importance of design decisions made at stage $i$ and must satisfy $\sum_{i=1}^{N} \alpha_i = 1$. The design quality is ensured by having $R \geq \bar{R}$, where $\bar{R}$ is the desired process confidence level chosen by the project managers according to the design requirement. Stage confidence $R_i$ is determined by two key elements: the amount of confidence obtained by allocating certain amount of engineering hours to that design stage, and the amount of correct design information received from the immediate preceding design stages. The stage confidence is defined as follows:

$$R_i = \lambda_i f_i(x_i) + \sum_{j \in E_i} \lambda_{ji} R_j, \quad \text{for } i = 1, 2, \ldots, N. \quad (2)$$

The first determinant of the stage confidence is captured by function $f_i(x_i)$, where $x_i$ is the design intensity at stage $i$. Assuming that stage confidence $R_i$ is a nondecreasing function of $x_i$, $f_i(x_i)$ can generally be modeled as follows:

$$f_i(x_i) = 1 - e^{-\beta_i x_i}. \quad (3)$$

Parameter $\beta_i > 0$ is determined by the nature of the design project and the complexity of the specific design stage. Higher $\beta_i$ implies a larger improvement margin of design confidence at a given level of $x_i$. Note that $f_i(x_i)$ monotonically converges to 1 as the number of engineering hours allocated increases. To simplify the model representation, we scale $x_i$ such that it is in multiples of the base (or minimum) time required to complete the design at stage $i$. $x_i$ represents the level of design intensity required for the design of stage $i$. The second element of the stage confidence in (2) is captured by the stage confidence achieved from its immediate preceding stages. The higher the preceding stages' confidence levels, the more correct design information they can provide to the design for stage $i$. Weights $\lambda_i$ and $\lambda_{ji}$ in (2) represent the relative impact of the design intensity and the stage confidence of its preceding stages on $R_i$, respectively, and must satisfy $\lambda_i + \sum_{j \in E_i} \lambda_{ji} = 1$. A high dependency level between stages $j$ and $i$ generally leads to a high $\lambda_{ji}$. $E_i$ is the set of stages immediately preceding stage $i$. Finally, we note that the parameters $\lambda_i$, $\lambda_{ji}$, and $\beta_i$ could be efficiently determined through the application of well known techniques such as Analytic Hierarchy Process (see Saaty 1988). Furthermore, performing sensitivity analysis on these parameters enables
the design managers to clearly understand the implications of the parameter set used in the model.

We now present the DESign Review Planning (DERP) model to determine how to configure design reviews, so that the required process confidence level $\tilde{R}$ can be achieved at minimum cost. The DERP model can be depicted through a network, referred to as the review profile network. Figure 2 is the review profile network constructed based on the previous example. Each node of the network represents the position of a review and design stages covered by the review. The node $(i, j)$ indicates that a review is placed at design stage $j$ and covers all stages between $i$ and $j$ (including $j$). The arc from $(i, j)$ to $(j + 1, k)$ represents the linkage between the two consecutive reviews in a review profile. Two dummy nodes $S$ and $T$ represent the start and the end of a review profile. Arcs from $S$ to $(1, i)$, and $(i, N)$ to $T$, for $i = 1, 2, \ldots, N$ are dummy arcs. Any path from the source $S$ to the terminal node $T$ of the network provides a review profile, which specifies where design reviews are placed and which design stages are covered by each review. Associated with each node of the network, there is a cost that specifies the expenses of corresponding to the design review configuration. An optimal review profile and its corresponding design intensity for each stage can be obtained by finding the shortest paths through the network.

A review profile is feasible if the corresponding design intensity for each stage is allocated such that the resulting process confidence satisfies $R \geq \tilde{R}$. We use the following procedure to determine an acceptance level for each design review and corresponding review and design cost for the nodes of the review profile network.

**Procedure to Determine Design Review Acceptance Level**

**Step 1. Determining stage confidence:** In this step, we formulate an optimization problem, say a Design Confidence Allocation (DCA) problem, to define the target confidence for each design stage.

**Step 2. Computing review confidence:** For each review represented by a node of the review profile network, we compute a target review acceptance level, which is simply the weighted sum of stage confidences for the stages covered by the review.

**Step 3. Evaluating design intensity:** For any given design review, we solve a Design Intensity Allocation (DIA) problem to determine the design intensity or the amount of engineering resources required for the stages covered by the review to meet the target review acceptance level. The corresponding review and design cost can be calculated from the solution.

To illustrate the steps of the procedure, we continue with the previous example. Suppose that the required process confidence level to be achieved for the process in Figure 1 is $\tilde{R}$. From the first step of our procedure we get the stage confidence for each design stage $i$ ($\tilde{R}_i$ or each $i$).

Now, for any given node in the review profile network (Figure 2), say $(i, j)$, we compute the target review acceptance level. For instance the review acceptance for node $(2, 4)$ is $\tilde{R}_{2,4} = \alpha_2 \tilde{R}_2 + \alpha_3 \tilde{R}_3 + \alpha_4 \tilde{R}_4$. Finally, in Step 3 of the procedure we solve a DIA problem for each node to determine the intensity of the design needed in each stage covered by the design review. In this particular case, for node $(2, 4)$ we need to determine $x_{2,2}^*, x_{3,3}^*$, and $x_{4,4}^*$ such that $\tilde{R}_{2,4}$ will be satisfied at minimum cost. The cost for the design and review would define the node’s cost in the review profile network, in this case the cost for node $(2, 4)$. The shortest feasible path in the network generates the optimum review profile. We now develop the model for DCA problem.

**2.2. Design Confidence Allocation Problem**

For each design review, we need to determine a review acceptance level in such a way that (1) the process confidence requirement will be satisfied if the acceptance levels are met for the reviews in any review profile; and (2) the resulting design resource for any review profile is efficiently allocated over the design stages. This can be achieved by first solving the DCA problem and then calculating an acceptance level for each review based on the solution of the problem. The DCA problem is to choose an optimal target stage confidence level such that the resulting process confidence will meet the required level $\tilde{R}$. The target stage confidence is determined by examining the trade-off between the development cost and design contribution of each stage to the overall design of the product.

To formulate the DCA problem as a mathematical program, we define $t_i$ to be the base time required for design stage $i$ corresponding to design intensity $x_i = 1$, and let $r_i$ be the average unit pay rate for stage $i$. Furthermore, define $c_i = r_i t_i$ as the average unit design intensity cost, and then the cost associated with performing design activities at stage $i$ is $c_i x_i$. The DCA problem can be formulated as follows:

$$\min \sum_{i=1}^{N} c_i x_i$$

subject to:
\[ R_i = \lambda_i f_i(x_i) + \sum_{j \in E_i} \lambda_{ij} R_j, \quad \text{for } i = 1, 2, \ldots, N, \] (5)

\[ \sum_{i=1}^{N} \alpha_i R_i \geq \tilde{R}, \] (6)

\[ x_i \geq 1, \quad \text{for } i = 1, 2, \ldots, N. \] (7)

The objective function (4) minimizes the total engineering development cost. Constraint set (5) calculates stage confidence. Constraint (6) ensures that the required process confidence level will be achieved, and constraint (7) states that at least a minimum design intensity will be allocated to each stage. Observe that stage confidence \( R_i \), for \( i = 1, 2, \ldots, N \), can be obtained by solving the simultaneous equation set (5). Substituting \( R_i \) into constraint (6), we can combine (5) and (6) into:

\[ \sum_{i=1}^{N} \gamma_i f_i(x_i) \geq \tilde{R}, \] (8)

where \( \gamma_i = g_i(\alpha_i, \lambda_i, \lambda_{i,i+1}), f = i, i + 1, \ldots, N - 1 \). An optimal solution \( x^* = (x_1^*, x_2^*, \ldots, x_N^*) \) for the DCA problem can be found by solving the Kuhn-Tucker conditions. The solution procedure is presented in Appendix A. Once the optimal design intensity level \( x^* = (x_1^*, x_2^*, \ldots, x_N^*) \) is obtained, the stage confidence allocation \( R^*_i = (x_1^*, x_2^*, \ldots, x_N^*) \), \( i = 1, 2, \ldots, N \) is determined. For a design review that is placed at the end of design stage \( n \) and covers stages \( m \) to \( n \), the target review acceptance level is calculated by:

\[ \tilde{R}_{m,n} = \sum_{i=m}^{n} \alpha_i R_i. \] (9)

Later we will show that any review profile, whose review acceptance level determined by (9), is feasible. Stage design confidence levels and review acceptance levels provide guidelines for the project managers to allocate engineering resources throughout design development. The project managers can also modify the target stage confidences and review acceptance levels accordingly to incorporate other managerial considerations. Next we develop the model for DIA problem.

### 2.3. Design Intensity Allocation Program

A design review is a special activity imposed at the end of a design stage. It evaluates one or more design stages to ensure that the design meets the design specifications and quality requirements. Associated with each design review there is a target review acceptance level determined by (9). To model the design review process, we assume that the target review acceptance level captures the design specifications and quality requirements for the design stages covered by the review. Design iteration takes place among the stages covered by the review until the target review acceptance level is satisfied. There are two types of costs involved in the design and review process: the cost of performing the review activities and the cost associated with the engineering resources allocated to the stages. The review cost depends on the number and complexity of design stages evaluated by the review. The DIA problem is to determine the design intensity required for each design stage covered by the review such that the corresponding target review acceptance level is satisfied at the minimum design cost.

We model the iterative review process as a Markov chain. For ease of presentation, we use the simple example in Figure 1 to show the development of the model. Suppose that a design review is placed at the end of stage 3 and covers stages 1 to 3, corresponding to node (1, 3) in the review profile network (Figure 2). Given the dependency levels for the interdependent relation among the three design stages, as shown in Figure 3(a), a Markov chain shown in Figure 3(b) can be constructed to model the iterative review process. \( \{s\} \) and \( \{t\} \) are the dummy states, which represent the start and end of the review process. States \( \{i\} \) for \( i = 1, 2, 3 \) represent the \( i \)-th design stage in the review, and state \( \{r\} \) represents the review stage. The arcs in the Markov process represent the transition, and the number above each arc represents the transition probability. The transition probability \( \rho_{ij} \) for \( i, j = s, 1, 2, 3, r, t \) determines the probability that another iteration of stage \( j \) will be necessary given that stage \( j \) was performed without the knowledge of the latest results from stage \( i \). The transition probability \( \rho_{ri} \) for the arc from state \( \{r\} \) to \( \{i\} \) is determined by review acceptance level \( \tilde{R}_{1,3} \), and is a decreasing function of \( \tilde{R}_{1,3} \). Comparing (a) and (b) in Figure 3, one can see that three backward arcs from state \( \{r\} \) are added in the Markov chain. They represent the review feedbacks. The transition probability from state \( \{r\} \) to state \( \{i\} \) is captured by \( \rho_i \), which is defined as the relative likelihood of design flaws occurring in stage \( i \). \( \rho_i \) for \( i = 1, 2, 3 \) satisfy: \( \Sigma_{i=1}^{3} \rho_i = 1 \). Since the number of iterations in this review process is essentially determined by the review and design feedbacks (e.g., arcs (3, 1) and (2,}

\[ \]
and find the largest $p_{m}^{*}$ that satisfies (11). $p_{m}^{*}$ is the optimal transition probability required to meet the review acceptance level.

Step 4. Calculate design intensity $x_{n}^{*}$ for stage $i = m + 1, \ldots, n$: Stop.

The solution for the DIA problem provides the optimal level of design intensity for each of the design stages covered by the review. The review and design cost associated with the review is then $s_{m,n} + \sum_{i=m}^{n} c_{i} x_{i}^{*}$. This cost will be attached to the node $(m, n)$ of the review profile network.

The shortest feasible path in the network, obtained via a dynamic program, determines the optimum review profile. We note that sometimes a review acceptance level might not be achievable no matter how many iterations are performed. This can occur when a review covers a design stage requiring feedback information from a downstream stage. An unachievable review acceptance level will cause some of the review profiles to be infeasible. An infeasible review profile indicates that the corresponding review profile cannot guarantee the design to achieve the required process confidence. The following proposition shows the existence of a feasible optimal solution.

**Proposition 2.1.** Given any review profile network, we have the following results: (1) There exists at least one feasible review profile; (2) for any review profile in the network, if the design intensity for each design stage is allocated such that the target review acceptance level is satisfied for every review, then the review profile must be feasible.

**Proof.** See Appendix B.

3. MODEL IMPLEMENTATION AND IMPLICATIONS

In the previous section, we developed models to rationalize the review process. In this section, we will use the elements of the design of the turbopump and briefly show the result of the model implementation and draw some managerial insights.

3.1. Implementation at Turbomachinery

We have shown the high level “as-is” design process for the turbopump at Rocketdyne. In this subsection, we will use the design activities defined in the as-is design process, and apply the methodologies developed in the previous sections to form a new rationalized design process. Because of the proprietary nature of the design activities and design parameters, we have disguised the data set used. However, the data set characterizes the actual data. In our implementation we have used 132 design activities from the conceptual, preliminary, and detail design of the as-is process. These activities form the basis of 15 design stages. For each stage, the basic time needed to complete the activities in the stage is computed based on the methodologies given in Ahmadi and Wang (1994). The relationship among the design stages is shown in Exhibit 4(a). The required process confidence for the turbopump design is set at 90%. Next, the corresponding review profile network
is constructed. The shortest path of the network provides the optimal review profile for the design process.

Our solution shows that four design reviews are placed along the design process; consequently, four phases are formed. Each design review sets a milestone for management to control design quality. Design phases 1 to 4 cover design stages 1 to 4, 5 to 7, 8 to 11, and 12 to 15, respectively. The review acceptance levels determined by the DCA problem are 21.28%, 16.36%, 24.25%, and 28.07%, respectively. For each design phase, number of design iterations takes place until the corresponding design review acceptance level is achieved. Exhibit 4(b)-(e) show the Markov chain used in modeling this iterative process for each design phase.

3.2. Managerial Implications
Placing design reviews to properly define milestones can effectively reduce unnecessary engineering changes and shorten design development times. These insights serve as guidelines in helping management to improve the design processes.

Design Review Acceptance Level. In many design processes, design reviews and their criteria are not carefully selected. For example, the design review acceptance levels are selected to meet certain prespecified contract requirements, and often deviate from the optimal level. This practice will significantly lengthen the design lead time. Exhibits 5(b) and (c) show a design process with overspecified and insufficient review acceptance levels for the early phases, respectively, as compared with the design process with optimally constructed design phases and review acceptance levels (Exhibit 5(a)).

Resource Allocation at Design Stage. The design confidence allocation problem not only determines a review acceptance level but also provides a target resource allocation for each design stage. Although the actual resource allocation determined by the design intensity allocation problem might not be the same as the optimum target, the target allocation is very useful in predicting and planning design resource in a design project. Understanding the key elements that affect resource allocation over design stages can help management to effectively manage limited design resources in a project. The design confidence allocation model implies that the stage contribution-cost ratio \( (o_i/c_i) \) is a key element that determines the amount of design resource to allocate to that stage. Generally, the higher the ratio, the more design resource should be allocated.

To show the impact of stage contribution, we divide the design stages in the turbopump example into three groups. The early design stage group contains the first five stages, the middle group contains stages 6 to 10, and the later group contains the last five stages. Figure 4 shows the change of resource allocation among the three groups as we increase or decrease the current contribution-cost ratio for the middle group by 50%. We can see that the total resource does not seem to vary, whereas the resource allocated to the middle design stage group increases rapidly with the ratio. This result is consistent with the intuition, that is, it is more cost effective to put more design effort on an important design stage with less unit design cost.

Impact of Project Complexity on Design Resource Allocation. For projects with different complexity and previous knowledge, the amount of design resource required to complete the design is quite different. To study the impact
of project complexity on the design resource allocation, we take the turbopump example, and vary the dependency level and learning factor to see the corresponding changes on the solution. Figure 5 shows how the design resource allocation is changed with an increase or decrease in the dependency levels and learning factor of the project by 50%. Observe that the total design resource required to complete the project increases rapidly with the complexity of the project, whereas the resource distribution across different stage groups remains almost the same.

4. CONCLUSIONS

Development risk in the product development process is a key source of new product failure. Managing and control-

Figure 4. Resource allocation at design stages.

Figure 5. Impact of project complexity.
If constraint (7) is relaxed, the DCA can be solved very easily by the Lagrangian method. We first solve the relaxed DCA, and then search for an optimal solution with constraint (7) by using the special relation between DCA and its relaxed problem. Relaxing constraint (7) and converting the relaxed DCA into its standard form, we then have:

\[
\max - \sum_{i=1}^{N} c_i x_i \tag{13}
\]

subject to:

\[
- \sum_{i=1}^{N} \gamma_i f_i(x_i) \leq -\tilde{R}, \tag{14}
\]

\[x_i > 0, \quad \text{for } i = 1, 2, \ldots, N. \tag{15}\]

Note that (15) must hold for the problem to be meaningful. Introducing Lagrangian multiplier \(\mu\) for constraint (14), we then have the following Lagrangian function:

\[
L(x, \mu) = - \sum_{i=1}^{N} c_i x_i + \left( \sum_{i=1}^{N} \gamma_i f_i(x_i) - \tilde{R} \right). \tag{16}
\]

Because we will show later that constraint (14) must be binding at optimal, there is an optimal solution that must satisfy the following first order Kuhn-Tucker conditions:

\[
\frac{\partial L}{\partial x_i} = -c_i + \mu \gamma_i f_i(x_i) = 0, \quad \text{for } i = 1, 2, \ldots, N, \tag{16}
\]

\[
\frac{\partial L}{\partial \mu} = \sum_{i=1}^{N} \gamma_i f_i(x_i) - \tilde{R} = 0. \tag{17}
\]

Solving (16), we can find an optimal solution \(\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N)\):

\[
\tilde{x}_i = \frac{\ln(\mu^{\ast} \beta_i \gamma_i) - \ln(c_i)}{\beta_i}, \quad \text{for } i = 1, 2, \ldots, N, \tag{18}
\]

where \(\mu^{\ast}\) can be determined by substituting (18) into (17):

\[
\mu^{\ast} = \frac{\sum_{i=1}^{N} (c_i / \beta_i)}{\sum_{i=1}^{N} \gamma_i} - \frac{\tilde{R}}{\gamma_i}. \tag{19}
\]

The above approach can also be illustrated by Figure 6. Recalling that \(R = \sum_{i=1}^{N} \alpha_i R_i = \sum_{i=1}^{N} \gamma_i f_i(x_i)\), we define ranking function as follows:

\[
Q_1(x_1) = \frac{\partial R}{c_1 \partial x_1} = \frac{\gamma_1}{c_1} f_1(x_1), \quad \text{for } i = 1, 2, \ldots, N. \tag{19}
\]

\(Q_i(x_i)\) provides the units of overall confidence-cost ratio level that can be increased by increasing one unit of design intensity for design stage \(i\). It is always more cost effective to increase the design intensity of a design stage with higher \(Q_i(x_i)\) value. Therefore, an optimal solution \(\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N)\) must satisfy:

\[
Q_1(x_1) = Q_2(x_2) = \cdots = Q_N(x_N). \tag{20}
\]

The problem becomes finding an optimal ranking level \(1 / \mu^{\ast}\) such that the corresponding \(\tilde{x}_i = Q_i^{-1}(1 / \mu^{\ast})\) for \(i = 1, 2, \ldots, N\), satisfies constraint (17).

Having obtained the optimal design intensity \(\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N)\) for the relaxed \((\text{DCA})_R\) problem, two possible cases can occur. The first case is that the solution still satisfies the relaxed constraint (7). In this case, \(\tilde{x}\) is the optimal solution to the DCA problem.

The second possible case is that there exists at least one design stage \(j\), such that \(\tilde{x}_i < 1\). In this case, we try to find an optimal solution for DCA problem by using the relationship between DCA and \((\text{DCA})_R\). We introduce the following proposition.

**Proposition.** Let \(\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N)\) and \(x^{\ast} = (x_1^{\ast}, x_2^{\ast}, \ldots, x_N^{\ast})\) be optimal solutions for \((\text{DCA})_R\) and \((\text{DCA})\), respectively. Suppose \(\tilde{x}\) does not satisfy constraint (7), and define index set \(I = \{1, 2, \ldots, N\}\) and \(E = \{i | \tilde{x}_i < 1, i \in I\}\). Then \(x_i^{\ast} = 1\) for \(i \in E\).

**Proof.** Before proving the proposition, we first show that constraint (14) has to be binding at an optimal solution for DCA. As a result of Assumption (12), there must exist at least an \(i \in I\) such that \(x_i^{\ast} > 1\). If constraint (14) is not binding, we can always find a small enough \(\xi > 0\), such that \(x_i^{\ast} - \xi > 1\) and \(x^{\ast} = (x_1^{\ast}, \ldots, x_i^{\ast} - \xi, \ldots, x_N^{\ast})\) still satisfies (14). We then have \(c_i x^{\ast} - c\tilde{x}^{\ast} = c_i \xi > 0\), a contradiction to \(x^{\ast}\) being an optimal solution.

To prove the proposition, we suppose there exists an \(j \in E\) such that \(x_j^{\ast} > 1\). As \(\tilde{x}_j\) is pushed to increase to \(x_j^{\ast}\), in order to keep constraint (14) binding there exists at least one, \(k \in \{i | \tilde{x}_i < 1, i \in I\}\) such that \(\tilde{x}_k\) decreases to \(x_k^{\ast} > 1\) (see Figure 7). Now we construct another feasible solution \(x' = (x_1^{\ast}, x_2^{\ast} - d x_j, \ldots, x_k^{\ast} + d x_k, \ldots, x_N^{\ast})\) based on \(x^{\ast}\) with only \(j\)th and \(k\)th elements modified, where \(d x_j > 0\) is sufficient small that \(x_j^{\ast} - d x_j > 1\) still holds. In order to keep constraint (14) binding for \(x'\), we can increase \(x_k^{\ast}\) by \(d x_k\), \(d x_k > 0\) is a function of \(d x_k\), and can be obtained by differentiating equation (14):

\[
\frac{dx_k}{dx_j} = \frac{\gamma_j f_j(x_j)}{\gamma_k f_k(x_k)}. \tag{21}
\]
Since $Q_k(x_k) > Q_j(x_j)$ for $x_k, x_j \in [\bar{x}_k, \bar{x}_k]$ as illustrated in Figure 7, we thus have:

$$\frac{\gamma_k f_k(x_k)}{c_k} > \frac{\gamma_j f_j(x_j)}{c_j}.$$  
(21)

Applying (21) to (20), we then have:

$$dx_k c_k - dx_j c_j < 0.$$  
(22)

From (22), we lead to $cx' - cx^* = dx_k c_k - dx_j c_j < 0$, a contradiction to $x^*$ being an optimal solution.

The relationship between $|DCA|$ and $|DCA|_R$ provided by the above proposition, guarantees that a solution obtained by the following algorithm will be optimal for $|DCA|$.

\textbf{Algorithm 1}

\textbf{Step 0}: Solve the relaxed problem for $i \in I$, and let $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_N)$ be the solution. Set $E = \{i|\bar{x}_i < 1, i \in I\}$ and $\bar{R}_0 = \bar{R}$.

\textbf{Step 1}: For $i \in E$, set $x_i^* = 1$, and calculate $\bar{R}' = \sum_{i \in E} \gamma_i f_i(x_i)$.

\textbf{Step 2}: Set $\bar{R} = \bar{R}_0 - \bar{R}'$, and solve the relaxed problem for $i \in I - E$. If solution $\bar{x}_i$ for all $i \in I - E$ satisfies constraint (7), set $x_i^* = \bar{x}_i$ and go to Step 3; otherwise, set $E = E \cup \{i|\bar{x}_i < 1, i \in I - E\}$, and go back to Step 1.

\textbf{Step 3}: Stop, we have obtained optimal solution $x^* = (x_1^*, x_2^*, \ldots, x_N^*)$.

Since the number of activities in set $I - E$ is reduced by at least one at each iteration, the above algorithm will guarantee finding an optimal solution in at most $N$ iterations, which requires solving the relaxed problem at most $N$ times.

\textbf{APPENDIX B: PROOF OF PROPOSITION 3.1}

To prove part (1), we try to show that the review profile with only one review that covers all the design stages in the review profile network is always feasible. Let $R_i^* = R_i(p_n)$ for $i = 1, 2, \ldots, N$, be the stage confidence obtained from solving the DIA problem corresponding to this review. Suppose that $\bar{R}_i$ for $i = 1, 2, \ldots, N$ is the stage confidence obtained from solving the DCA problem. For this special DIA problem, since its simultaneous equations (10) are the same as (5) for the DCA problem, we can choose $p_n$ small enough that the resulting $R_i^* = R_i(p_n)$ from the Markov model satisfies $R_i^* \geq \bar{R}_i$ for $i = 1, 2, \ldots, N$. The feasibility of the review profile follows from:

$$\sum_{i=1}^{N} \alpha_i R_i^* \geq \sum_{i=1}^{N} \alpha_i \bar{R}_i = \bar{R}.$$  

To prove part (2), let $(j_0, j_1, \ldots, j_k, \ldots, j_K)$, where $j_0 = 0$ and $j_K = N$, be a review profile indicating that a review is placed at the end of stage $j_k$, and covers design stages up to stage $j_{k-1} + 1$ for $k = 1, 2, \ldots, K$. The minimum review acceptance level for the $k$th review in the profile is:

$$\bar{R}_{j_{k-1} + 1, j_k} = \sum_{i=j_{k-1} + 1}^{j_k} \alpha_i \bar{R}_i.$$  
(23)

Let $R_i^*$ for $i = j_{k-1} + 1, \ldots, j_k$ and $k = 1, 2, \ldots, K$ be the stage confidence level obtained from solving the DIA problem for $k$th review. The feasibility of the review profile is implied by the following relations:

$$\sum_{i=1}^{N} \alpha_i R_i^* = \sum_{k=1}^{K} \sum_{i=j_{k-1} + 1}^{j_k} \alpha_i \bar{R}_i = \sum_{k=1}^{K} \bar{R}_{j_{k-1} + 1, j_k} = \sum_{i=1}^{N} \alpha_i \bar{R}_i \geq \bar{R},$$

where the second inequality results from (11) in DIA problem, the third equality is from (9), and the fourth inequality from (6) in the DCA problem. This completes the proof of the proposition.

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\textbf{REFERENCES}


