A Decision Support System for Planning Manufacturers' Sales Promotion Calendars

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Abstract

A common event in the consumer packaged goods industry is the negotiation between a manufacturer and a retailer of the sales promotion calendar. Determining the promotion calendar involves a large number of decisions regarding levels of temporary price reductions, feature ads, and in-store displays, each executed at the level of individual retail accounts and brand SKUs over several months or a year. Though manufacturers spend much of their marketing budget on trade promotions, they lack decision support systems to address the complexity and dynamics of promotion planning. Previous research has produced insights into how to evaluate the effectiveness of promotional events, but has not addressed the planning problem in a dynamic environment. This paper develops a disaggregate-level econometric model to capture the dynamics and heterogeneity of consumer response. By modeling the purchase incidence (timing), choice and quantity decisions of consumers we decompose total sales into incremental and nonincremental (baseline plus borrowed).

The response model forms the basis of a market simulator that permits us to search for the manufacturer's optimal promotion calendar (subject to a set of constraints, some of them imposed by the retailer) via the simulated annealing algorithm. Calendar profits are the net result of the contribution from incremental sales minus the opportunity cost from giving away discounts to nonincremental sales and the fixed costs associated with implementing promotional events (e.g., retagging, features, displays). Incremental sales result from promotion-induced switching, the acceleration and quantity promotion effects on those switchers, increased consumption and the carryover effect from purchase event feedback.

We applied our approach to the promotion-planning problem of a large consumer-packaged goods company in a nonperishable, staple product category suggested by company executives (canned tomato sauce). Subject to a retailer pass-through constant rate of 80%, provided to us by the collaborating firm, the optimal promotion calendar produced by the modeling system followed a pattern of frequent and shallow temporary price reductions with no feature or display activity. We also analyze how that result would change under different retailer pass-through scenarios.

Our findings indicated that the manufacturer could substantially improve the profitability of its sales promotion activity and that there would be a concurrent positive effect on retailer profit and volume levels. Management reported to us that the insights from the use of the system were implemented in their promotion-planning process and produced positive results. A validation analysis on follow-up data for one market showed that promotion activity could be significantly reduced, as recommended, with no adverse effect on the brand's market share, as predicted.

To generalize the model beyond the specific category where it was implemented, we conducted a sensitivity analysis on the profile of the calendar (i.e., frequency, depth, and duration of deals) with respect to changes in market response, competitive activity, and retailer pass-through. First, we found that the optimal depth, frequency, and timing of discounts is stable for price elasticities ranging from near zero to around four (in absolute magnitude). We also found no systematic impact of competitive promotions on the profile of the optimal calendar. For example, variation in competitive activity did not affect the optimal depth or frequency of discounts. Lastly, we found changes in retailer pass-through to have a significant effect on the optimal depth and number of weeks of trade promotion that a manufacturer should offer. This emphasizes the importance to manufacturers of having accurate estimates of pass-through for purposes of promotion budgeting and planning.

(Trade Promotion; Brand Management; Decision Support Systems; Scanner Data)

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Introduction

Trade promotion continues to be the largest single spending category in the marketing mix budget of U.S. packaged goods companies (e.g., Kotler 1997). Improving the productivity of trade promotion dollars therefore remains a high priority in the consumer products industry. A significant opportunity to enhance the efficiency and effectiveness of marketing spending lies in the implementation of trade promotion policies. This process involves a very large number of tactical decisions regarding desired levels of temporary price reductions, feature ads, and in-store displays, each executed at the level of individual retail accounts and brand SKUs. When viewed over several months or a year, these decisions collectively make up what is known as the *sales promotion calendar*.

The promotion calendar reflects not only numerous decisions but inherently complex ones. Each should take into account a variety of factors, including marketing mix effects, the dynamics of consumer response, competition, and retailer behavior. In this environment, a decision support system (e.g., Little 1979) offers the potential to improve decision making (cf. Hoch and Schkade 1996) and, of course, to save an enormous amount of time. By programming the system to produce "win-win" promotion calendars (i.e., where both manufacturer and retailer come out ahead), a manufacturer's gains need not come at the expense of the retailer. By presenting "win-win" calendars, backed up by forecasts of category profitability, sales representatives should be able to streamline trade promotion discussions with retailers and devote more time to brand-building activities.

Our paper shows how scanner modeling technology and optimization methods can make it possible to begin to automate the process of planning the promotion calendar. First, we develop and demonstrate the benefits of an implementable decision support system for the tactical decisions that comprise the sales promotion calendar. Second, we provide insight about the profile of the promotion calendar and what is robust with respect to variations in the marketing environment and rates of retail pass-through. Finally, we show how to apply simulated annealing (Kirkpatrick et al. 1983) to solve the complex optimization problem (large number of control variables, nonlinear function, and multiple local optima) involved in determining the promotion calendar over a one year time horizon.

During the development of the decision support system, we worked closely with senior management at the firm with whom we collaborated. The extensive interaction allowed us to develop a system that was compatible with the trade promotion practices of the firm, thereby enhancing management understanding and acceptance of the system. Based on the insights provided by the model, the collaborating firm made significant changes to its trade promotion policy in the product category we analyzed and obtained positive results. (In a subsequent section, we present follow-up data that validates our model.) A stated long-term goal of management was to have a portable version of the system installed in the laptops of the sales representatives to support the sales force in its negotiations with retailers.

Sales Promotion Planning

Our model is based on a dynamic view of the promotion decision-making process. Manufacturers' current practice often pays little consideration to the future impact of promotional offerings. For example, at the packaged goods company with which we collaborated, planners based their promotion budget allocations on response elasticities that were assumed to be fixed over time and did not take into account the effects of previous marketing activity. Our approach allows manufacturers to consider the dynamic effects of sales promotion on consumer response (e.g., household inventory, increased category usage or consumption, and purchase event feedback) and to adopt a longer-run view of the promotion calendar.

This paper focuses on what promotions the manufacturer would like to see in front of the consumer over a prespecified time horizon. What the consumer sees, however, is the result of the way the retailer implements the manufacturer's promotional offerings. Thus, a decision-support system for this problem must include the role of the retailer. Our approach assumes that the manufacturer has good knowledge of the nature of retailer response. Managers at our collaborating firm had historical data on how each retailer had responded to different promotional offerings, from

which the retailer's response function could be approximated. Our model is designed to work by then searching for the promotional offering to the trade that would result in the desired calendar in front of the consumer at the point of purchase. (We also show how variable rates of retail pass-through can be incorporated into the decision support system.)

Our system is intended to be used as a planning and negotiating tool by a manufacturer's sales representatives in the field. To avoid inefficiencies in the supply chain, such as inventory build-ups by retailers, manufacturers need to design promotional offerings that reward the retailer for execution of the program rather than just forward buying. Our model is based on a "pay-for-performance" environment where the retailer has no incentive to forward buy. In this arrangement, the retailer is paid a promotional allowance based on the volume sold during the promotion period. This is how the collaborating firm operated in the product category we analyzed. This type of agreement is already a significant part of trade dealing, in part because it places the focus on consumer demand as the driving force for promotional decisions and thereby attempts to minimize inventory held in the channel.

One key premise of our approach is that the manufacturer wants the right calendar in front of the consumer. We therefore optimize manufacturer profit over sales to the consumer, taking into account the pass-through response function of the retailer. Our focus is to determine how to structure the offer to the retailer so as to obtain the desired effects at the consumer level.

A second key premise is that the market response model should be based on disaggregate data. This enables the "truly incremental" sales due to promotion to be separated from not only baseline sales, but also from borrowed sales (i.e., purchases that would have been made in the future but were accelerated due to promotion). Such a capability provides a critical distinction from promotion evaluation models that are based on aggregate-level data. By using a disaggregate model of demand, we can also naturally incorporate consumer heterogeneity into the planning of the promotion calendar. We note that extensive previous research (e.g., Stiglitz 1977, Varian 1980, Narasimhan 1984, Jeuland and Narasimhan 1985, Narasimhan 1988, Raju et al. 1990) has shown that sales promotion can work as a price discrimination device, i.e., a marketing tool that takes advantage of consumer heterogeneity.

Literature

A number of recent articles are closely related to our work. Abraham and Lodish (1993) described a method to measure the effectiveness of promotional events using store-level data. Because the approach uses store-level data, it does not decompose the promotional lift (i.e., the volume of sales above baseline) into sales that are truly incremental versus those that are borrowed. (Our approach uses panel data to estimate the purchase acceleration and/or stockpiling induced by a promotion.) Their approach also does not incorporate dynamics in consumer response or the effects of previous marketing activity on future promotional events.

Midgley et al. (1997) use genetic algorithms to analyze marketing strategies under oligopolistic competition. In their approach, demand is represented by an aggregate or market-level model. Again, this does not permit a decomposition of the promotional "bump" into incremental and borrowed sales. Neslin et al. (1995) develop an aggregate model with three players: the manufacturer, the retailer, and a set of consumers. The manufacturer's profit is maximized over sales to the retailer, not over sales to the consumer. A potential drawback of this approach is that sales could end up in inventory build-ups in the channel, potentially overstating the true profitability of a promotion.

In contrast to the work just described, Tellis and Zufryden (1995) develop a retail promotion planning model that is based on a disaggregate consumer response model. Their work differs from our approach on several dimensions. First, they address the retailer's problem, taking the manufacturer's behavior as given, i.e., the objective function is retailer category profits given full knowledge of manufacturers' trade promotions. In contrast, the goal of our system is to determine manufacturers' promotion programs, taking into account the managerially calibrated response of the retailer. Second, their demand model does not segment consumers in terms of their responsiveness to marketing activity. This omits a key driver of price promotions, i.e., the ability to identify consumer segments with different demand functions (cf. Stiglitz 1977).

Lastly, their optimization and sensitivity analyses are based on mean household values. While this greatly simplifies the optimization problem, it implies that the procedure does not take into account any preference or response heterogeneity among panelists

Overview of Decision Support System

Our system (see Figure 1) combines a disaggregate market response model with an optimization procedure that searches for the promotion calendar providing the greatest increment to the manufacturer's profit. To estimate a promotion's incremental impact on profit, the consumer response model computes expected sales and the proportion that is truly incremental from the promotion. Truly incremental sales are (1) units sold to consumers who bought the brand as a consequence of its promotional status and who would *not* have bought it otherwise (now or in the future), (2) any promotionally-induced consumption increase, and (3) any positive carryover effect from purchase event feedback. The sales promotion "bump" (Figure 2) that is routinely observed in sales data also contains sales to consumers who accelerated their purchases or bought more units than usual (stockpiled), but would have bought the brand at the regular price (now or in the future) had the promotion not been run. These units (less any that may be attributed to a consumption increase; see Ailawadi and Neslin 1998) are sales borrowed from the future, and are not truly incremental for the manufacturer.

To decompose the promotional "bump," we require a modeling approach that captures the source of consumer response: switching, acceleration and/or stockpiling. Previous research (e.g., Neslin et al. 1985, Krishnamurthi and Raj 1988, Currim and Schneider 1991, Bucklin and Gupta 1992, and Bell et al. 1999), has shown that acceleration and stockpiling play significant roles in consumer response to promotional activity. Our response model captures consumers' purchase incidence, choice and quantity decisions and handles consumer heterogeneity by using latent-class analysis. The system then uses parameter estimates from the response model, together with household specific variables (e.g., brand loyalty, consumption, and purchase rates) and environment variables (e.g., competitive activity, retailer pass-through, and mark-up) to simulate the purchase decisions made by a large sample of panelists.

Figure 1 Overview of Decision Support System

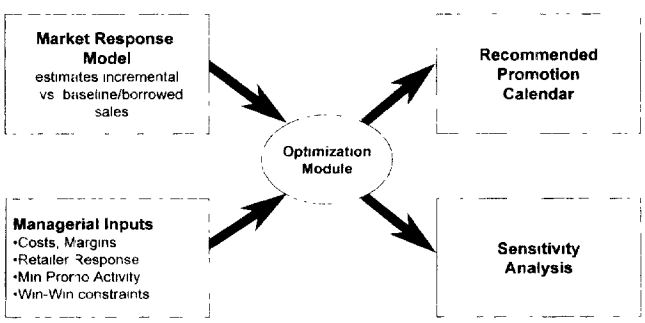
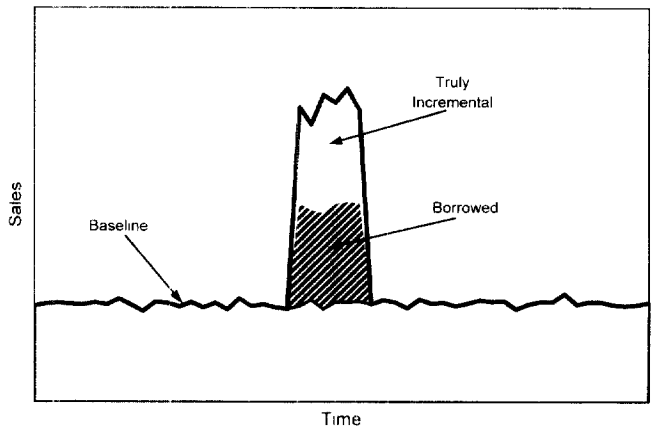


Figure 2 Truly Incremental and Borrowed Sales



We link the market response model that simulates household decisions to an optimization module that uses the simulated annealing algorithm (Kirkpatrick et al. 1983) to search for the set of decisions over the planning horizon that maximizes manufacturer profit. Those decisions include when, for how long, and, in the case of temporary price reductions, how deep to run promotional events. The optimization procedure is constrained to the set of schedules that would produce acceptable levels of expected category profit for the retailer. Comparative statics analyses can be performed by simulating different market characteristics (e.g., by varying response parameter values, segment sizes, competitive activity, retailer pass-through and mark-up) and examining the changes, if any, in the optimal promotion calendar. This feature of the model

also may be used by manufacturers to search for robust strategies.

Market Response Model and Incremental Sales Estimation

Using purchase histories, we compute for each household the probability of visiting each store in the market area. Conditional on a store visit, the consumer then decides whether to buy in the target category. Given a decision to purchase in the category, the consumer then chooses a brand-size alternative. (If the promotion planning is to be performed at the UPC level, the model could be modified with the procedure developed by Fader and Hardie 1996.) Finally, given a category purchase and a brand-size choice, the consumer decides how many units of the brand-size alternative to purchase (cf. Ailawadi and Neslin 1998, Bucklin et al. 1998). The household-level demand model, conditional on a store visit, is given by

$$E(Q_{it}^h) = E(Q_{it}^h \mid Q_{it}^h > 0) \times P_t^h(i \mid inc) \times P_t^h(inc), \text{ where} \quad (1)$$

$E(Q_{it}^h \mid Q_{it}^h > 0)$ = the expected number of units that household h will buy of brand-size alternative i at time t given that household h has decided to buy brand-size alternative i at time t (i.e., given that $Q_{it}^h > 0$),

$P_t^h(i \mid inc)$ = the probability that household h chooses brand-size alternative i at time t, given that it has decided to purchase in the product category (i.e., given purchase incidence), and

$P_t^h(inc)$ = the probability that household h decides to make a category purchase at time t, given a store visit.

The Appendix gives details of the response model specification. We now describe how the response model can be used to estimate incremental vs. nonincremental sales from promotions, thereby producing the inputs needed for the calendar optimization.

Using the Response Model to Measure a Manufacturer's Incremental Sales

A promotion calendar is composed of a sequence of events that result in "bumps" in sales volume (see Figure 2). The challenge is to estimate how much of the observed "bump" is truly incremental for the manufacturer. A traditional approach (e.g., Abraham and Lodish 1987, 1993) is to estimate the volume of sales that would have been achieved had the promotional event not been run (baseline sales) and define as incremental all the volume above the baseline. As we illustrate below, this approach has several limitations (cf. Abraham and Lodish 1993, p. 250, para. 2).

The Loyal Consumer. Consider the case of a hard-core loyal (Colombo and Morrison 1989) consumer of brand A. Let us refer to that consumer as household 1. Household 1 only buys brand A and will not consider buying any competitive brands, even if they are on promotion. Assume that brand A is on promotion in period t. Household 1 may take advantage of that promotion by accelerating the timing of its purchase (incidence effect) and/or buy more units than usual (quantity effect). Thus, brand A's promotion would result in household 1 buying more units in period t than it would had the promotion not been run. Following Abraham and Lodish's approach, those additional units bought by the hard-core loyal consumer of brand A would be computed as incremental volume for the manufacturer.

Consider now that household 1 has a constant consumption rate, regardless of its inventory level. For example, household 1 always consumes one unit of brand A per week, regardless of how many units it has in its pantry. In that case, the additional units bought by household 1 in period t will cannibalize future sales of brand A and will not be truly incremental for the manufacturer in the long run. Therefore, those units should be considered "borrowed" and not incremental.

Assume that household 2 is also a hard-core loyal of brand A. It follows a purchase behavior similar to household 1, with the exception that holding additional inventory motivates household 2 to increase its consumption rate. To simplify the example, assume that all of the incremental household inventory becomes incremental consumption for household 2. In that case, the extra units bought when brand A was on promotion will be incremental for the manufacturer. Between household 1 and household 2, we can think of a continuum of hard-core loyal households for

whom the extra inventory will result in some increase in consumption but will also result in some cannibalization of future sales. The computation of incremental sales should capture these effects.

The Brand Switching Consumer. Let us now consider the case of a brand switching consumer. In the absence of any promotional activity, household 3 (a hypothetical brand switcher) has a 50-50 chance of buying brand A or a competitor's brand. When brand A is on promotion the choice probabilities shift to 1 for the promoted brand A and 0 for the unpromoted competitor's brand (and vice-versa when the competitor's brand is on promotion). Assume that when no brand is on promotion, household 3 would make a category purchase of 4 units when its inventory reaches 0. When a brand is on promotion, household 3 would buy enough units of the promoted brand to reach a household inventory level of 8 units (storage constraint)

Consider the scenario where household 3's category inventory reaches 0 at time t and no brand is on promotion. In this instance, the household would make a category purchase of 4 units. With 0.50 choice probabilities for both brands, the expected quantities are 2 units for brand A and 2 units for the competing brand. These units would be baseline sales (i.e., they occur in the absence of promotional activity).

Now consider the scenario where brand A is on promotion and household 3's inventory reaches 0. Here, household 3 would buy 8 units of brand A and 0 units of the competing brand. The Abraham and Lodish (1987, 1993) approach would compute incremental units for brand A by subtracting the baseline sales (2 units) from the total sales on promotion (8 units), giving incremental sales of 6 units. Truly incremental sales, however, are likely to be less than 6 units. To see this, begin by noting that those 6 units can be decomposed into 2 units due to switching (0 units for the competitor's brand vs. 2 units in the baseline case) and 4 units due to stockpiling (a category purchase of 8 units versus 4 units in the baseline case). Of the 4 units stockpiled, 2 are incremental for brand A, because they can be attributed to the promotion's effect on brand choice. (With a 0.50 baseline choice probability the inventory is expected to be 50% brand A and 50% the competitor's brand.) Thus, 4 out of the 6 units making up the sales bump for brand A are clearly incremental. Of the remaining 2 units, some, all, or none will be incremental, depending on how many become extra household consumption and how many cannibalize future sales. Our approach initially classifies these two units as *borrowed* sales. All, none, or part of the *borrowed* units will ultimately be incremental for the manufacturer, depending upon the extent to which the additional household inventory prompts an increase in consumption.

We summarize below the preceding sales decomposition according to how it would be computed from the incidence, choice, and quantity response models at the level of an individual household. To simplify this example, we take the household's initial inventory to be 0 Thus, the purchase incidence probability will be 1 regardless of the promotional status of brand A.

Brand A Total Sales (Brand A on Promotion)	
Incidence Probability	1
Choice Probability	1
Quantity Model	8
Expected Total Quantity	8
Brand A Baseline Sales (Assuming No Promotions on Brand A)	
Incidence Probability	1
Choice Probability	0.5
Quantity Model	4
Expected Baseline Quantity	2
Brand A Baseline + Borrowed Sales (Assuming No Choice Effect)	
Incidence Probability	1
Choice Probability	0.5
Quantity Model	8
Expected Baseline + Borrowed Quantity	4

The Abraham and Lodish (1987, 1993) approach would estimate an incremental volume of 6 units (total sales minus baseline sales). This volume, however, is truly incremental if and only if the 2 units of "borrowed sales" all end up in incremental consumption for household 3 (i.e., there is no cannibalization of future sales). At the time of a promotion, we do not know how many (if any) of those 2 borrowed units (4 baseline and borrowed minus 2 baseline) will become incremental consumption for the household. (Note that

baseline plus borrowed sales include baseline sales plus those sales that will turn out to be either stockpiled or extra consumption.) Therefore, our approach takes the difference between total sales (8 units) and baseline plus borrowed sales (4 units) to be incremental at that time. We then determine how much, if any, of the 2 borrowed units to classify as incremental by estimating whether there is additional consumption by the household in future periods (see the Appendix, Equation (A10)). Those units are then credited as incremental to the period in which the added consumption is realized.

The response model in Equation (1) provides an estimate of total sales, $E(Q_{it}^h)$. To obtain an estimate of incremental sales, we simulate baseline plus borrowed sales, $E(BBQ_{it}^h)$. Incremental sales are then given by the difference between these two quantities. We note that simulated baseline sales, $E(BQ_{it}^h)$, should be a function of the inventory and consumption levels that would have been realized in the absence of promotional activity for the target brand (brand-size alternative i). Similarly, simulated baseline plus borrowed sales should be a function of the inventory and consumption levels that would have been realized if promotions on the target brand resulted only in borrowed sales (via purchase acceleration and/or stockpiling). Thus, in computing simulated baseline plus borrowed sales, we remove the choice effect of brand-size i's promotions.

We then compute how many of those *borrowed* units result in incremental consumption in future periods. For this computation we use the difference between the consumption rate for the simulated baseline plus borrowed sales, $CRBB_t^h$, and the consumption rate pertaining to baseline sales, CRB_t^h. Thus, the *borrowed* volume that results in incremental consumption can be estimated period by period by the difference between the following two consumption rates, $CRBB_t^h - CRB_t^h$. Those units are then added to the incremental sales for period t. Thus, our model captures the incremental component, if any, of the "borrowed" sales from a promotion by summing the consumption carryover effect across future periods. This is given by $\Sigma_{\tau=t+1}^{\infty} CRBB_\tau^h - CRB_\tau^h$.

Our model also includes the carryover effects of purchase feedback. These effects result in an increase or decrease in choice probabilities in future periods as a consequence of previous choice decisions. This is captured via the last brand purchased (*LBP*) variable in the choice model (see the Appendix, Equation (A2)). Thus, a promotion for brand A which results in a purchase of brand A will increase the choice probability for brand A at the subsequent choice occasion. We compute this carryover effect by simulating baseline plus borrowed sales with the *LBP* variable set to the value it would have had, had no promotions for brand A been offered (see the Appendix, Equations (A13) and (A14)).

Putting all of the above together, from the manufacturer's perspective, a promotional event can produce truly incremental sales under three conditions: (1) consumers switch to the target brand (choice effect), (2) a temporary increase in consumption takes place (consumption effect), or (3) future sales increase due to purchase event feedback. Nonincremental volume (baseline plus borrowed) is simulated by setting promotional variables to zero in the choice model, incremental consumption to zero, and removing purchase feedback effects from the choice probabilities. Mathematically, this is expressed as

$$E(BBQ_{it}^h) = E(Q_{it}^h \mid Q_{it}^h > 0)\mid_{INV_t^h = INVBB_t^h} \times P_t^h(i \mid inc)\mid_{\substack{NO\text{-}PROMO \\ NO\text{-}PURCH\ FEEDBACK}} \times P_t^h(inc)\mid_{INV_t^h = INVBB_t^h} \quad (2)$$

where BBQ is baseline plus borrowed volume, $INVBB_t^h$ is the household's inventory given that the choice effect of promotions for brand-size i is removed, *NO-PROMO* sets promotional variables (e.g., temporary price reduction, feature, display) to zero, and *NO-PURCH.FEEDBACK* removes purchase event feedback effects from the choice model. For example, the three factors in Equation (2) respectively correspond to the magnitudes 8, 0.5, and 1 in the computation of the 4 baseline plus borrowed units in our previous example. Note that this number may be significantly higher than the expected baseline sales given by:

$$E(BQ_{it}^h) = E(Q_{it}^h \mid Q_{it}^h > 0)\mid_{\substack{NO\text{-}PROMO, \\ INV_t^h = INVB_t^h}} \times P_t^h(i \mid inc)\mid_{\substack{NO\text{-}PROMO \\ NO\text{-}PURCH\ FEEDBACK}} \times P_t^h(inc)\mid_{\substack{NO\text{-}PROMO, \\ INV_t^h = INVB_t^h}} \quad (3)$$

where BQ refers to baseline volume and $INVB_t^h$ is an

estimate of what the household's inventory level would be if no promotions on brand-size i had been run. (The three factors in Equation (3) would respectively be the magnitudes 4, 0.5, and 1 in the computation of the 2 baseline units in our previous example.)

The expected *incremental* number of units (for the manufacturer) of brand-size alternative i sold to household h at time t is then obtained by (1) subtracting baseline plus borrowed sales from total expected units, and (2) adding back any previously "borrowed" sales that resulted in incremental consumption in period t ($\Delta CR_t^h = CRBB_t^h - CRB_t^h$). Incremental units are given by

$$E(\Delta Q_{it}^h) = E(Q_{it}^h) - E(BBQ_{it}^h) + \Delta CR_t^h. \quad (4)$$

The expected number of units at the household level is estimated conditional on a shopping trip taking place. To obtain the expected number of units for a given store or chain, we multiply those expected number of units by the probability that the panelist makes a shopping trip to that store or chain. This probability is obtained from the household's history of store visits. We assume that the marketing activity of the individual product category does not affect consumers' store choice decisions (i.e., the category is not a traffic builder, cf. Tellis and Zufryden 1995). The final inputs to the optimization model are

$$E(\Delta Q_{it}) = \frac{\sum_h \tau^h \times E(\Delta Q_{it}^h)}{H}, \text{ and} \quad (5)$$

$$E(BBQ_{it}) = \frac{\sum_h \tau^h \times E(BBQ_{it}^h)}{H}, \text{ where} \quad (6)$$

H = the number of panelists in the sample, and
τ^h = the probability that panelist h makes a shopping trip to the store.

Once the components of the consumer response model (choice, incidence, and quantity) are calibrated, parameter values together with the household specific variables (e.g., brand and size loyalties, inventory, etc.) are used to compute the expected number of incremental (Equation (5)) and nonincremental (Equation (6)) units sold.

Unlike manufacturers, retailers can increase their sales via the acceleration and quantity effects of promotional events. For product categories which do not play the role of traffic builders, store competition involves capturing a higher share of the consumer's category purchases, once the shopper enters the store. Consequently, sales that are borrowed for the manufacturer can be incremental for the retailer as a result of this indirect type of store competition (see Bucklin and Lattin 1992, Abraham and Lodish 1993, p. 250)

Latent Class Analysis

Heterogeneity in consumer response parameters is addressed with latent segment analysis (e.g., Bucklin et al. 1998). In the latent segment model, Equation (1) is modified to

$$E(Q_{it}^h) = \sum_s \pi_s E(Q_{sit}^h \mid Q_{sit}^h > 0) \times P_{st}^h(i \mid inc) \times P_{st}^h(inc), \quad (7)$$

where π_s equals the size of segment s ($0 < \pi_s < 1$). Here $P_{st}^h(i \mid inc)$ is the brand-size choice probability, $P_{st}^h(inc)$ is the category purchase incidence probability, and $E(Q_{it}^h = q_{sit}^h \mid Q_{sit}^h > 0)$ is the expected number of units bought given that household h bought item i and given that household h is a member of segment s. To determine the number of latent segments to retain, we compare models according to the Bayesian Information Criterion (BIC) and select the number of segments that maximizes the BIC.[1]

Optimization Module

The purpose of the optimization is to search for the promotion calendar (i.e., the sequence of weekly price discounts, features, and displays) that maximizes net incremental contribution for the manufacturer's target brand-size i, given retailer response and competitive activity. Below, we present the mathematics for the objective function, constraints, and variable definitions. We then describe them and discuss their properties.

[1]Rust et al. (1995) have shown that the BIC (Schwarz 1978) is the best overall model selection criterion. The BIC is given by $LL - (k/2) \ln(n)$ where LL is the log likelihood fit of the model, k is the number of parameters, and n is the sample size. Thus, models which add parameters without substantially improving the log likelihood will be rejected.

Manufacturer's Objective Function

$$\max_{K_{it}, \mathrm{FEAT}_{it}, \mathrm{DISP}_{it}} \left[\begin{aligned} &\sum_{t=1}^{T} \delta^t \times N \times E(\Delta Q_{it}) \times (BPRICE_{it} \\ &\quad \times (1 - K_{it} \times DSTEP) - MCOST_{it}) \\ &- \sum_{t=1}^{T} \delta^t \times N \times E(BBQ_{it}) \\ &\quad \times (BPRICE_{it} \times K_{it} \times DSTEP) \\ &- \sum_{t=1}^{T} \delta^t \times (DISC_{it} \times TCOST_{it} + FEAT_{it} \\ &\times FCOST_{it} + DISP_{it} \times DCOST_{it}) \\ &+ \sum_{t=T+1}^{T+13} \delta^t \times N \times \\ &\quad E(\Delta Q_{it} \mid_{RPRICF_{it} = \overline{RPRICE_t}, PC_{it}=0, FEAT_{it}=0, DISP_{it}=0,}) \\ &\quad \times (\overline{BPRICE_i} - \overline{MCOST_i}), \end{aligned} \right] \quad (8)$$

where $t = 1, 2, \ldots, T$ is the planning horizon and $t = T + 1, T + 2, \ldots, T + 13$ are the subsequent 13 weeks over which carryover effects are computed.

Constraints

$$K_{it} \text{ are integer } \forall t, \quad \text{where } K_{it} = 0, 1, 2, \ldots, 10 \ \forall t, \quad (9)$$

$$DISC_{it} \text{ are binary } 0/1 \text{ variables } \forall t, \quad (10)$$

$$FEAT_{it} \text{ are binary } 0/1 \text{ variables } \forall t, \quad (11)$$

$$DISP_{it} \text{ are binary } 0/1 \text{ variables } \forall t \quad (12)$$

$$\sum_t PROMO_{it} \geq E, \quad (13)$$

$$\sum_t PROMO_{it} \leq \frac{T}{2}, \quad (14)$$

$$\sum_j \sum_t BPRICE_{jt} [MKUP + K_{jt} \times DSTEP \times (1 - PTHRU_{jt} \times (1 + MKUP))] \geq \Pi_R, \quad (15)$$

$$PTHRU_{it} = f(K_{it}). \quad (16)$$

Definitions and Relationships Among Variables

t = week number.

i = manufacturer's target brand-size.

j = a brand-size alternative in the category.

δ = discount rate.

N = average number of category consumers that shop in the store or chain (a scaling factor).

$E(\Delta Q_{it})$ = expected number of incremental units of brand-size alternative i bought by a consumer that made a store visit in week t, i.e., units bought by a consumer who would not have bought the brand-size had not it been on promotion (see Equations (5)).

$E(BBQ_{it})$ = expected number of baseline plus borrowed sales, i.e., units of brand-size alternative i bought by a consumer that made a store visit in week t and would have chosen brand-size i, regardless of its promotional status (see Equation (6)).

$BPRICE_{it}$ = wholesale base (regular depromoted) price of brand-size i in week t.

K_{it} = discount multiplier for brand-size i in week t. If $K_{it} = 0$, brand-size i is sold at the base price in week t. When the manufacturer offers a discount, it is computed as a multiple of a discount step level, e.g., 5%.

$DSTEP$ = depth of discount step (or base discount) according to manufacturer's policy (e.g., 5%).

$MCOST_{it}$ = manufacturer's marginal cost of brand-size i in week t.

$PRICE_{it}$ = shelf price of brand-size i in week t. Related to the previous variables by the equation

$$PRICE_{it} = BPRICE_{it} \times (1 - PTHRU_{it} \times K_{it} \times DSTEP) \times (1 + MKUP). \quad (17)$$

$RPRICE_{it}$ = regular (depromoted) retail price of brand-size i, at time t [$= BPRICE_{it} \times (1 + MKUP)$].

PC_{it} = retail price cut of brand-size i, at time t [$= BPRICE_{it} \times PTHRU_{it} \times K_{it} \times DSTEP$], so that shelf price can also be expressed as $PRICE_{it} = RPRICE_{It} - PC_{it}$.

$PTHRU_{it}$ = retailer's pass-through, i.e., the proportion of manufacturer's discount the retailer passes to the consumer.

$MKUP$ = retailer's markup in percent.

$$DISC_{it} = \begin{cases} 0 & \text{if } K_{it} = 0, \text{ i.e., if no discount is offered,} \\ 0 & \text{if } K_{i,t-1} \geq 1, \text{ i.e., if a discount started in a previous week,} \\ 1 & \text{otherwise.} \end{cases} \quad (18)$$

$TCOST_{it}$ = tagging cost, i.e., the fixed cost charged by the supermarket chain to discount the price of brand-size i in week t. This cost is charged only once during a promotion period.

$$FEAT_{it} = \begin{cases} 1 & \text{if a feature advertisement is offered for brand-size } i \text{ in week } t, \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

$FCOST_{it}$ = fixed cost charged by the chain to run a feature advertisement for brand-size i in week

$$DISP_{it} = \begin{cases} 1 & \text{if an in-store display is offered for brand-size } i \text{ in week } t, \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

$DCOST_{it}$ = fixed cost charged by the chain to set up a display for brand-size i in week t.

$$PROMO_{it} = \begin{cases} 1 & \text{if } FEAT_{it} = 1 \text{ or } DISP_{it} = 1 \text{ or } K_{it} > 1, \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

$\overline{BPRICE}_i$ = average wholesale base (regular depromoted) price of brand-size i during the planning period ($t = 1, 2, \ldots, T$).

$\overline{RPRICE}_i$ = average regular (depromoted) retail price of brand-size i during the planning period ($t = 1, 2, \ldots, T$).

$\overline{MCOST}_i$ = average manufacturer's marginal cost of brand-size i during the planning period ($t = 1, 2, \ldots, T$).

E = minimum number of promotional "events" required by the retailer during the planning period, $E \geq 0$.

T = number of weeks or periods in the planning horizon.

Π_R = minimum category profit level required by the retailer to support the promotion program.

Discussion of Optimization Model Formulation

We begin by noting that our specification of the objective function in Equation (8) is based on incremental profits, not total profits. This means that the objective function does not include profits from baseline sales or profits from borrowed sales. Instead, it includes the *opportunity costs* associated with discounting prices on those sales. Consumption and purchase feedback effects impact future periods. Following Neslin et al. (1995, p. 755) we extend the time horizon for the optimization procedure to include a 13-week period subsequent to the planning horizon ($t = T + 1, T + 2, \ldots, T + 13$). In this way, we allow the carryover effects of promotions at the end of the planning period to be realized, but we do not permit promotions beyond the planning period ($t = 1, 2, \ldots, T$). We note that in all the applications of the model, carryover effects were negligible beyond the 9th week following a promotion event.

Turning to the specifics of Equation (8), note that the manufacturer has three decision variables for each week: K_{it}, $FEAT_{it}$, and $DISP_{it}$. K_{it} specifies the level of discount offered each week. Manufacturers typically offer temporary price reductions that are a multiple of a discount step, e.g., 5%.[2] For example, a TPR could range from 0% to 50% in steps of 5%.[3] This behavior is captured by defining $TPR_{it} = K_{it} \times DSTEP$, and by constraining K_{it} with Equation (9). $FEAT_{it}$, and $DISP_{it}$ are 0/1 variables (see Equations (11) and (12)) that indicate whether a feature or display is run in week t.

The objective function has four components, each of which is one term in Equation (8): (1) the expected contribution from incremental units, (2) the expected opportunity cost of selling at a discount to consumers who would have bought the brand at the regular price, (3) the fixed costs associated with promotion decisions, and (4) the carryover effects from consumption and purchase feedback over a 13-week period subsequent

[2]Evidence for this decision behavior comes from our discussions with the collaborating consumer goods company and from contacts with marketing research firms.

[3]Note that here we are referring to discount levels offered by the manufacturer to the retailer. The discount that the consumer will be exposed to will depend on the retailer pass-through decision (see Equation (16)).

to the planning horizon. The contribution from incremental units is expected incremental units times dollar contribution margin per unit. The expected incremental number of units sold in week t is given by $N \times E(\Delta Q_{it})$. The contribution margin per unit results from deducting the unit marginal cost ($MCOST_{it}$) from the wholesale price, i.e., the base (or regular) price minus the discount, $BPRICE_{it} \times (1 - K_{it} \times DSTEP)$.

The opportunity cost of discounts on baseline plus borrowed sales is given by the volume sold to those consumers ($N \times E(BBQ_{it})$) times the dollar unit discount, i.e., $BPRICE_{it} \times K_{it} \times DSTEP$. Our model computes the opportunity cost of borrowed sales in the period in which those sales are generated. Three fixed costs of promotion decisions are also incorporated in the model. These are (1) the cost charged by the retailer for changing prices when a discount is offered ($TCOST_{it}$), (2) the cost charged for running a feature advertisement ($FCOST_{it}$), and (3) the cost for setting up a display ($DCOST_{it}$). The variables $DISC_{it}$, $FEAT_{it}$, and $DISP_{it}$ control when to apply those fixed costs (see Equations (18), (19), and (20)).

Turning to the constraints, we permit the retailer to require the manufacturer to run a minimum number of events (E) during the period in order to continue offering shelf space and support to the brand (Equations (13) and (21)). Furthermore, the retailer may insist on a minimum level of category profits (Π_R), assured by (15). The retailer may pass to the consumer only a proportion of the trade discount offered by the manufacturer (Equation (17)). We assume that the manufacturer knows the respective pass-through function for each chain from historical data (Equation (16)).

If previous exposure to a brand on promotion is high, consumers may come to think of the brand as a "promoted brand," and expect to find it on promotion in future purchase occasions (cf. Kalwani et al. 1990, Lattin and Bucklin 1989). Thus, the frequency of promotional events should lie below the threshold that would make consumers perceive the brand as a "promoted brand."[4] Based on empirical findings by Lattin and Bucklin (1989) and Kalwani and Yim (1992), we set an upper bound of 50% on the number of weeks on deal (Equation (14)). We expect this type of limit may also be imposed by the retailer.

[4]For example, Kalwani and Yim (1992) suggest that "consumers who are exposed to price promotions on a brand to a degree beyond a prescribed threshold come to expect a discount every time they buy that brand."

Approach to the Optimization Problem

Our optimization problem is nonlinear with integer variables. (Note that the expressions $E(\Delta Q_{it})$ and $E(BBQ_{it})$ are highly nonlinear and are a function of the integer decision variables.) Our approach follows the simulated annealing method proposed by Kirkpatrick et al. (1983) on the basis of the Metropolis et al. (1953) algorithm. This method asymptotically converges with probability one to a globally optimal configuration (cf. Johnson et al. 1986, Laarhoven and Aarts 1987, Laarhoven 1988, Otten and Van Ginneken 1989, Lawler et al. 1990). In marketing, Borin et al. (1994) and Borin and Farris (1995) have used simulated annealing for retailer shelf management problems. In simulated annealing there is a nonzero probability of moving to a "worse" solution from the current one. This feature allows the system to explore different regions of the solution space to find the global optimum.

Application

Data

The product category used for the application was canned tomato sauce. The category has two major national brands (A and B), two minor national brands (C and D), and there is extensive participation by private labels. Store scanner and panelist "wand" data for a medium-sized, Midwest city for 1993–1994 was provided by ACNielsen. The store scanner data provided the marketing environment information while the panelist data provided households' shopping trip and purchase histories. The first 30 weeks were used for initializing model variables and the last 75 weeks for the calibration of the response model. Households were included in the analysis if they made at least one category purchase in the initialization period and at least two in the calibration period. Over the calibration period, the 368 households that qualified for inclusion made 55,489 shopping trips to food stores and 3,359 purchases in the product category. On average, households purchased in the category 6.3 times per year and bought 2.14 units per purchase.

The brands included in the study account for 98.2% of category sales. A list of these brands, their market shares, and average prices in cents per ounce follow:

Brand	Market Share	Average Price (¢/oz)
National Brand *A*	0.136	4.33
National Brand *B*	0.447	4.30
National Brand *C*	0.018	3.20
National Brand *D*	0.010	4.29
Private Label Chain 130	0.188	2.78
Private Label Chain 151	0.078	2.49
Private Label Chain 25	0.082	2.90
Private Label Chain 221	0.023	2.59

These brands were available in three different sizes: 7.75/8 oz. (size 1), 15/15.25 oz. (size 2), and 28/29 oz. (size 3). Size 1 accounted for 28.8% of the volume sold, size 2 for 52.1%, and size 3 for 19.1%.

A concern sometimes raised about the use of panel data is how well it represents aggregate-level sales. We investigated this by correlating total sales volumes computed from the panel data with total sales volumes computed from the store-level data. Across the brands used in our analysis and optimization, the correlation of the two sales volume time series was 0.76. This indicates that panel data tracks aggregate sales reasonably well in our data set. In their study of the representativeness of panel data, Gupta et al. (1996) found that inferences on the effectiveness of marketing activity from panel and store data are close, despite significant differences in demographics and brand market shares. Given that panel data is needed to decompose the promotional sales "bump" into truly incremental and borrowed sales, we believe that its use is appropriate in this setting. A worthwhile topic for future research would be to develop an approach that combines the use of panel data with store-level data for promotion planning.

Response Model Calibration and Parameter Estimates

The choice, incidence, and quantity components (see the Appendix, Equations (A1), (A4), and (A8)) of the model were simultaneously calibrated by maximum likelihood. Table 1 summarizes the results for latent class analysis. The best-fitting model by the Bayesian Information Criterion (BIC) was the two-segment model with flexible consumption (see Table 2 for parameter estimates).[5] With a few exceptions, none of which were statistically significant, all coefficients have the correct signs. Note that the flexible consumption coefficient values are relatively close to 1, an indication of low consumption flexibility (Ailawadi and Neslin 1998). This is an expected result, as tomato sauce is an ingredient and, as such, consumers are less likely to increase consumption due to inventory effects. This is also consistent with the Ailawadi and Neslin (1998) result for ketchup. We also note that management did not expect that "loading" consumers with inventory in this category would lead to greater product usage

Application of the Optimization Model

The optimization model was used to determine the promotion calendar for national brand *A*.[6] A large supermarket chain in which brand *A*'s share is similar to its overall market share was picked for analysis. For the purposes of this application, the following assumptions were made: (1) the set of alternatives was restricted to national brands *A* and *B* and the store private label, which together accounted for more than

[5]For the application at the collaborating firm, we used the model labeled "Constrained" in Table 1 (i.e., the best-fitting model with constant consumption which was a two-segment choice, incidence, and quantity model in which coefficients for marketing variables were constrained to zero in one segment of each behavior) Details are available from the authors upon request Calendar results from the constrained model were identical to the ones presented here for both TPR depth and the number of weeks on promotion. There were some differences in the recommended timing of the promotions; the constrained model used for the firm application recommended promoting largely out of phase with competition, while the model presented here recommended a mix of on and out-of-phase promotions with competition. In practice, manufacturers generally do not know precisely when competitors will promote nor can they control the precise period in which promotion will occur at the retail level. Thus, precise model recommendations for timing are of less practical usefulness than the recommendations for depth and number of weeks.

[6]Even though the optimization is performed at the brand-size level, we restricted our search to calendars in which all sizes for the brand were promoted concurrently. This restriction was requested by the management of the collaborating firm because the low unit prices and volumes in tomato sauce made it hard for promotional events instrumented at the size level to cover fixed implementation costs

Table 1 Fit of Alternative Consumer Response Models

Model	Number of Parameters	Log Likelihood	U^2	BIC
Null Model[1]	2	−10,854 37	—	−10,863.96
Latent Class Results[2]				
One Segment—Constant Consumption	40	−8,497 45	0 2171	−8,689.27
One Segment—Flexible Consumption	41	−8,417 07	0.2245	−8,613 69
Two Segments—Constant Consumption	61	−8,364.18	0 2294	−8,656.71
Two Segments—Flexible Consumption[3]	63	−8,212 70	0.2434	−8,514.82
Three Segments—Constant Consumption	82	−8,283.92	0 2368	−8,677 16
Three Segments—Flexible Consumption	85	−8,166 01	0 2477	−8,573 63
Constrained Model[4]	49	−8,415.10	0 2247	−8,650 08

[1]Equal choice probabilities, one constant term for incidence and one constant term for quantity.

[2]Except constrained model, see note 4

[3]Best fitting model.

[4]Model used for the company application (best fitting model with no consumption effect). Two segments for choice, incidence, and quantity. Coefficients for marketing variables constrained to 0 in one segment of each model. Sequential estimation of choice, incidence, and quantity.

90% of category sales at the chain, (2) the retailer did not charge a fixed cost for changing shelf prices, (3) pass-through was 80%,[7] and (4) retailer markup was 20%. These assumptions are based on information supplied by the collaborating manufacturer. Historical records showed that retailer pass-through for this product category ranged from 60% to 100%, with 80% the modal value for large supermarket chains. The modal markup for this product category was 20%.

Feature and display were originally part of the optimization application, but their respective costs were much greater than their respective contributions from incremental units. We discussed this result with the collaborating firm. Management showed us a consulting study reporting similar findings and recommended that our analysis focus on temporary price reductions (TPR). Thus, we dropped feature and display from subsequent iterations of the optimization procedure. Nevertheless, the model remains formulated to include feature and display in the promotion calendar.

The optimization of the promotion calendar for a quarterly planning horizon, involving price cuts only, can be performed by enumeration, albeit with a substantial computational time.[8] This allows us to test how well the simulated annealing algorithm approximates the global optimum by comparing its results with those obtained by full enumeration. For each quarter of 1994, we searched for the optimal calendar by full enumeration of the possible solutions. Next, we used simulated annealing to search for the optimal calendar. To set the initial value of the control parameter, we generated 1,000 random calendars. For the configurations that resulted in a worse solution than the current calendar, we computed the average decrease in net incremental contribution. This figure was used to compute the starting level of the control parameter that would result in an initial 80% acceptance ratio (cf. Laarhoven and Aarts 1987).

For each value of the control parameter, the algorithm was repeated 100 times. This is a more conservative approach than suggested by Kirkpatrick et al. (1983), who recommend that the number of iterations equal the number of variables in the problem, or 39 for a 13-week period. The control parameter was decreased at the end of each Markov chain (i.e., at the end of each set of 100 iterations) by a factor of 0.10 (i.e.,

[7]We analyze the impact of alternative pass-through specifications in the sensitivity analysis section. Note that we are assuming a pay-for-performance environment, which (albeit at a certain pass-through rate) implies a 100% participation of the retailer.

[8]It took 19 hours and 50 minutes of CPU time on an HP9000 model 735.

Table 2 Parameter Estimates for Selected Consumer Response Model

Parameter	Segment 1	Segment 2
Choice Model		
National Brand A	0.4537	
	(1.0431)	
National Brand B	0 8096	
	(1.9280)	
National Brand C	1 5822	
	(2.5964)	
National Brand D	0 1806	
	(0.2917)	
Private Label Chain 130	0 5850	
	(1.5691)	
Private Label Chain 151	0 8068	
	(1.9992)	
Private Label Chain 25	0 5616	
	(1.2281)	
Size 1 (7.75/8 oz)	- 0.4521	
	(– 3 0109)	
Size 2 (15/15.25 oz)	0.2752	
	(2.7265)	
Brand Loyalty	2.0791	1 9085
	(3.8621)	*(4.1677)*
Last Brand Purchased	0.6424	0 9154
	(2 3609)	*(3 8312)*
Size Loyalty	2 0403	2 5672
	(5 6264)	*(8 6413)*
Last Size Purchased	0 5473	0 3876
	(3 4140)	*(2 4232)*
Regular Price	- 0.3658	- 0 4156
	(– 2.7943)	*(– 2 8235)*
Temporary Price Reduction	0.5643	0 4752
	(2 0132)	*(2 3703)*
Feature	0 8698	1 2259
	(1.7898)	*(3 3990)*
Display	- 0 2757	1.1042
	(- 0 4353)	*(1 6141)*
Incidence Model		
Intercept	- 2.6335	-5.2562
	(- 3.9728)	*(14 4632)*
Purchase Frequency	5 6330	5 2590
	(6 8198)	*(10.8906)*
Inventory	0 0216	0 0201
	(- 16 7810)	*(11 8041)*
Category Value	- 0.1413	0 3338
	(1 1874)	*(5.8117)*

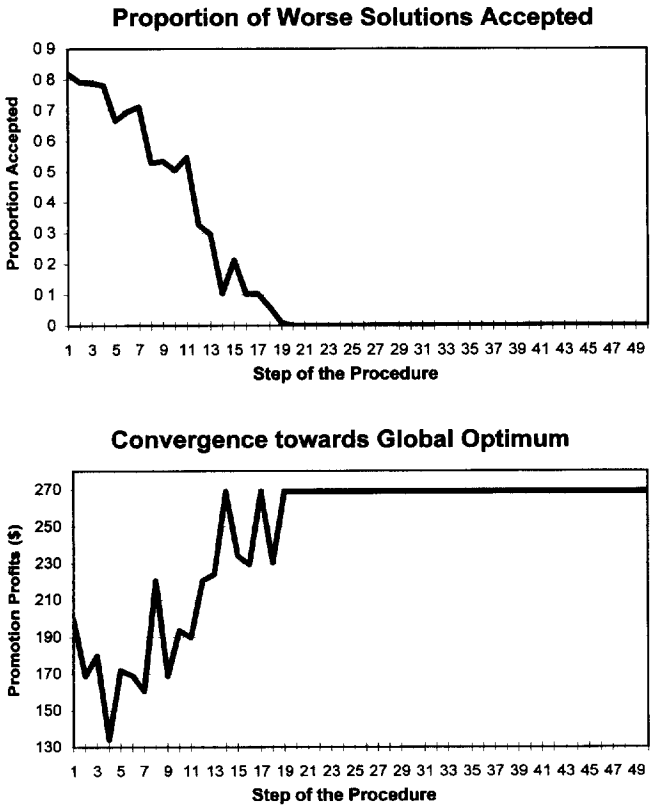
Parameter	Segment 1	Segment 2
Quantity Model		
National Brand A	0.0140	
	(0.0336)	
National Brand B	– 0 1356	
	(– 0.3363)	
National Brand C	– 0.6122	
	(– 0.6140)	
National Brand D	– 0 6191	
	(– 0.6329)	
Private Label Chain 130	– 0 0746	
	(– 0.2114)	
Private Label Chain 151	– 0 0104	
	(– 0.0294)	
Private Label Chain 25	– 0 3775	
	(– 0.9638)	
Private Label Chain 221	– 0 0392	
	(– 0.0628)	
Size 1 (7 75/8 oz)	– 0 0146	
	(– 0.0337)	
Size 2 (15/15 25 oz)	– 0.5907	
	(– 1.2753)	
Size 3 (28/29 oz)	– 1 2144	
	(– 2.6191)	
Purchase Rate	0 2909	0.3153
	(10.4870)	*(12.5617)*
Inventory	- 0 0017	- 0 0097
	(– 1 8626)	*(- 8.2779)*
Brand Loyalty	0 6523	0 0428
	(4 2288)	*(0.4031)*
Size Loyalty	0.7075	– 0 3135
	(3.2366)	*(- 2 5779)*
Regular Price	– 0.1781	- 0.0770
	(– 1.6530)	*(- 0 6844)*
Temporary Price Reduction	0.1588	0.3239
	(0.9249)	*(2.9528)*
Feature	0 0096	0 5517
	(0.0275)	*(5 4419)*
Display	0 3700	– 0.0686
	(0.5742)	*(- 0.4440)*
Flexible Consumption	1 1463	0.7566
	(52 0817)	*(43.3554)*
Segment Size	0.3498	0.6502
Log Likelihood	– 8,212.70	
BIC	– 8,514.82	
U^2	0.2434	

Asymptotic *t*-statistics in parentheses

$c_{k+1} = \alpha \cdot c_k$, where $\alpha = 0.90$). The final value of the control parameter was set to 0.20 (i.e., close to 0), a value for which almost no deteriorations in the objective function were accepted. This implies repeating the procedure for 50 values of the control parameter. In other words, the procedure consists of 50 steps, each of which iterates the algorithm 100 times.

Simulated annealing converged to the global optimum found by full enumeration for each of the four quarters of 1994. The computational time required was 2 hours 41 minutes of CPU time on an HP9000 model 735, compared to 19 hours and 50 minutes for full enumeration. Figure 3 summarizes the performance of the simulated annealing procedure along the 50 steps of the control parameter. We note that the "cooling" schedule used is very conservative. In fact, the global optimum was achieved by step 19, i.e., with only 19 decrements of the control parameter. These results are encouraging for the application of the simulated annealing algorithm to longer planning horizons (e.g., 26 or 52 weeks) for which full enumeration is computationally infeasible.

Figure 3 **Application of Simulated Annealing (Weeks 1 to 13 of 1999)**



We then focused on the annual (i.e., 52 weeks) promotion calendar. Summary comparative results from the actual and model-proposed calendar are presented below.[9]

	Actual Calendar	Model-Proposed Calendar
Brand *A* Net Incremental Contrib. ($)	219	1,018
Brand *A* Market Share (%)	12.2	12.4
Retailer Category Profit ($)	19,501	25,531
Retailer Category Sales (oz)	3,011,382	3,937,127

Table 3 presents additional details. There are substantial differences between the actual and model-proposed calendar in the number of weeks on promotion and the depth of price discounts. In this case, the decision support system recommends (Case I) shallower discounts and a greater number of weeks on promotion. Figure 4 presents a graphical comparison between actual and model-proposed programs.

As noted before, the use of panel data allows us to decompose consumer response into its purchase incidence, choice and quantity components and thus obtain better measures of incremental sales. Aggregate (i.e., store-level) models consider all sales above baseline as incremental (e.g., Abraham and Lodish 1993) and hence overstate incremental sales (and understate nonincremental sales) by the volume of borrowed sales. A disaggregate model also allows us to incor-

[9]In the brand choice model, two variables, last brand purchased and last size purchased, are a function of the panelists' previous simulated choice We tested whether an unlikely simulated choice could have distorted the results by performing a Monte Carlo procedure with 1,000 iterations on both the optimal and actual calendars. Results indicated very tight bounds on the estimated net incremental contributions. Adding a Monte Carlo procedure to each estimation of the promotion calendar would substantially slow the optimization and, given the tight bounds on the means, appears to have little effect on the outcome Fader et al. (1995) propose an analytic method with a more efficient alternative that could be incorporated in future versions of the system We thank an anonymous reviewer for pointing out this issue.

Table 3 Promotion Calendars Comparative Results

					Manufacturer's Results		Retailer's Results	
Program[1]	Weeks on Promotion	Minimum Depth (%)	Maximum Depth (%)	Average Depth (%)	Promotion Profit ($)[2]	Market Share (%)	Category Profits ($)	Category Sales (oz)
Actual	18	12.4	13.8	13.2	219	12.2	19,501	3,011,382
Case I Borrowed Sales Nonincremental, Carryover Effects (Purchase Feedback and Consumption); Constant Pass-Through = 80%, (Base Optimal Program)	26	5.0	5.0	5.0	1,018	12.4	25,531	3,937,127
Case II All Lift Incremental, Carryover Effects (Purchase Feedback and Consumption), Constant Pass-Through = 80%	26	10.0	15.0	11.5	765	13.0	25,665	3,945,506
Case III Borrowed Sales Nonincremental, No Carryover Effects, Constant Pass-Through = 80%	23	5.0	5.0	5.0	884	12.3	25,527	3,937,270
Case IV All Lift Incremental, No Carryover Effects; Constant Pass-Through = 80%	25	10.0	10.0	10.0	740	12.8	25,662	3,943,708

[1]"Optimal" calendars resulting from the respective case definition of "incremental" and "nonincremental" sales.

[2]In all cases, promotion profits were computed based on truly incremental sales and carryover effects (Case I definition)

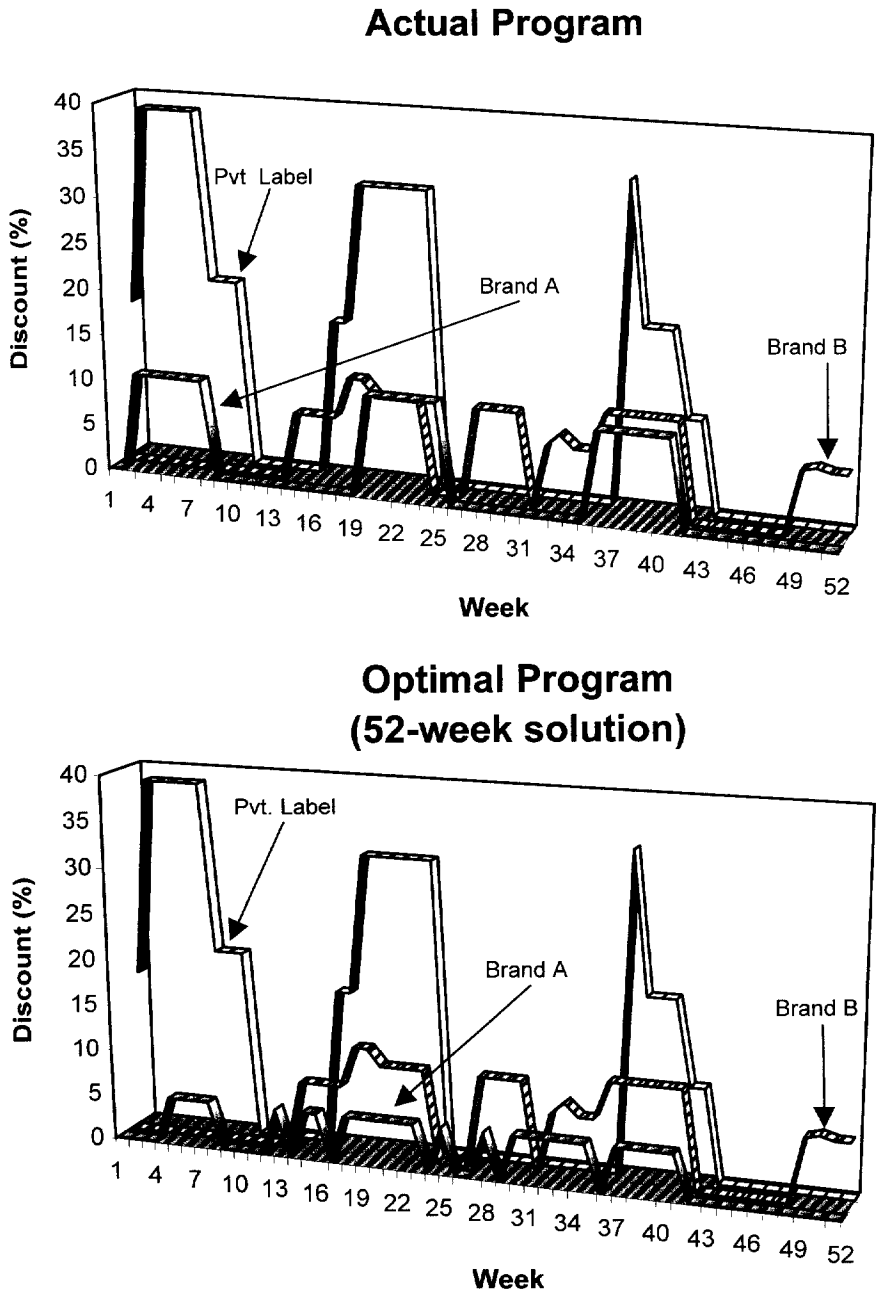
porate consumer dynamics via inventory, consumption, and purchase feedback effects.

We tested the contribution of using panel data in the following way. We reformulated the optimization model to consider the entire lift above baseline sales as incremental and searched for the "optimal" calendar under such a specification. For the proposed calendar, we then computed the manufacturer's program profits with the correct split between incremental and nonincremental sales. The results are reported in Table 3 (Case II).

We note that misclassifying borrowed sales as incremental would have resulted in a calendar with deeper discounts. Brand *A* would have increased its share to 13% but at the cost of a substantial reduction in manufacturer's profits (about one third).

We also tested the impact of not including consumer dynamics (i.e., the carryover effects from inventory, consumption, and purchase feedback). We did so in a similar way to the previous test, by searching for the optimal calendar with those effects constrained to zero. The results are also reported in Table 3 (Case III). Including carryover effects increases the number of weeks on promotion and program profit rises 15%. Case IV in Table 3 completes the analysis by presenting the results when the optimization is run considering all sales above baseline as incremental and not including carryover effects.

Figure 4 **Comparative Results for Annual Calendar (Weeks 1 to 52 of 1994)**



Implementation and Validation

The insights and recommendations produced by the decision support system were presented to and extensively discussed with the management of the collaborating firm. Based on the results of our decision support system,[10] management reduced the depth of its

[10]As noted in footnote 5, these results pertained to the constrained model without carryover effects.

deals offered to the trade and reported no adverse effects on sales. While we were not provided information on profitability, with lower promotion expenses and comparable sales, it appears likely that profitability improved. Management also told us that their goal was to have a "fast" version of the system implemented in the laptops of their sales reps to be used when negotiating promotion calendars with retail chains. It would also be used at the management level to determine general guidelines for trade promotion policy and planning.

The qualitative insights of the work were also presented to the sales representatives for use in their individual negotiations with retailers. This has been especially useful to determine the optimal depth and for avoiding the tendency to offer deeper discounts when retailers restrict the number of weeks on deal. Thus, we can report actual implementation of a reduction in discount depth plus qualitative implementation of key insights. Management was also involved in all stages of model development, vetting the response model, the nature of segmentation, the objective function, and the assumptions about retailer behavior.

To attempt to further validate our model, we requested specific follow-up data for a market where our approach was implemented. We were provided with market share and price promotion information summarized from ACNielsen store-level scanner data. The data cover the eight quarterly periods (two years) following the presentation of our model-based recommendations to management. We have information for both Brand *A* and Brand *B* for the category's largest selling size (size 2). The data also come from the same city in which our original model had been calibrated. Thus, we have a good opportunity to assess the practical results of the decision support system. We present the follow-up data in Table 4.

During the first two quarters, before the recommendations were implemented, the market share for brand *A*, size 2, was 4.74% and 5.16% with average TPR levels of 15.2% and 12.6% respectively. The model-based recommendations were implemented over the following four quarters. Average TPR levels dropped to 7.6%, 7.1%, 3.7%, and 4.7%. (Recall that our model-based recommendation was for TPR levels of 5%.) Market share for the brand also went up slightly over these quarters to 5.8%, 5.5%, 5.5%, and 4.8%. Note that these share results are consistent with our response model predictions.

A management change in the operating division of the collaborating firm responsible for the brand occurred mid-way through the test period. This resulted in returning to previous promotion policies for the last two quarters. Average TPR went to 16.9% and 17.3% and the respective market share levels were 4.9% and 5.4%. We note that during the entire eight-quarter follow-up period there was no feature or display activity. As Table 4 shows, the major competitor (national brand *B*) also retained its usual deep discount level (average 21%) and a market share of 47%. We remark in passing that brand *B* appears to be starting to *decrease* its TPR in quarter numbers seven and eight, perhaps in response to the reduced promotion activity by brand *A* over the previous year.

Sensitivity Analysis

The optimal calendar is a function of the specific characteristics of the market in which brand *A* is competing. Those characteristics are captured by the parameter values in the consumer response model. Sensitivity analysis involves varying some of those parameters over reasonable value ranges and exploring the effect on the profile of the optimal calendar. This enables us to learn about the relationships between market characteristics and the profile of the optimal calendar, and to explore the robustness of the solution found by analyzing the parameter ranges over which it remains stable. Below, we summarize the results and discuss key findings.

Retailer Pass-Through

The application was based on a constant retailer pass-through rate of 80%. This was the modal rate according to the collaborating firm's historical data and also the typical rate for the store chain we used. We analyzed the impact of different levels of constant retailer pass-through (from 50% to 90%) and the case in which retailer pass-through is a (concave) function of the manufacturer's offered price discount.[11] The results are reported in Table 5.

[1] We thank the editor and area editor for suggesting this analysis.

Table 4 Model Validation

	Brand *A* (Size 2)[1]			Brand *B* (Size 2)[1]		
Period	Weeks on Promotion	Average TPR (%)[2]	Average Weekly Share[3]	Weeks on Promotion	Average TPR (%)	Average Weekly Share
1. Oct.–Dec. 95	10	15.2	4 74	12	23 0	22 72
2. Jan –Mar 96	11	12 6	5.16	11	18.9	22 25
3 Apr –June 96[4]	5	7 6	5.81	13	23.2	23.61
4 July–Sept. 96[4]	8	7.1	5.55	9	20.7	23 73
5 Oct –Dec 96[4]	7	3 7	5.46	13	20.4	23 09
6 Jan.–Mar. 97[4]	11	4.7	4 79	13	20 5	23 82
7 Apr.–June 97	12	16 9	4.89	12	12.2	23 79
8 July–Sept. 97	6	17.3	5.36	12	13 0	21.68

[1]Results for the market provided by ACNielsen

[2]Average TPR was computed over promoted weeks only

[3]Average weekly share represents volume share of total category sales and was computed over all (promoted and nonpromoted) weeks.

[4]TPR % is statistically significantly lower for periods 3, 4, 5, and 6 than for periods 1 and 2 (t = 6.7) and periods 7 and 8 (t = 9 1) Market Share for brand *A*, size 2 is statistically significantly greater in periods 3, 4, 5, and 6 than for periods 1 and 2 (t = 2 4)

Table 5 Effect of Retailer Pass-Through on Optimal Promotion Calendar

					Manufacturer's Results		Retailer's Results	
Retailer Pass-Through	Weeks on Promotion	Minimum Depth (%)	Maximum Depth (%)	Average Depth (%)	Promotion Profit ($)	Market Share (%)	Category Profit ($)	Category Sales (oz)
Constant Pass-Through = 80% (Base Optimal Program)	26	5.0	5 0	5 0	1,018	12 4	25,531	3,937,127
Constant Pass-Through = 50%	13	10.0	10.0	10.0	786	12.0	29,192	3,933,431
Constant Pass-Through = 60%	18	5.0	10 0	6 2	838	12 1	27,542	3,933,899
Constant Pass-Through = 70%	21	5 0	5 0	5 0	938	12 3	26,402	3,935,389
Constant Pass-Through = 90%	26	10.0	10.0	10 0	1,196	13 0	24,874	3,946,286
Pass-Through: Concave Function of Depth [PTHRU = 0.95(1-exp(−15 × *MDISC*))]	26	15 0	20 0	16 2	1,145	13 7	24,997	3,957,193

Lower constant pass-through rates result in fewer weeks on promotion as the manufacturer finds it more costly and less effective to promote. When the pass-through rate is 50% to 60%, the manufacturer needs to offer a deeper discount to the trade in order to obtain a meaningful impact on consumer response. Compared to the base case of 80 percent pass-through, the lower pass-through rates produce lower manufacturer profits and higher retailer profits. A higher (e.g., 90%) constant pass-through rate results in deeper discounts and higher manufacturer's profits, but lower retailer profits. When the retailer's pass-through rate is modeled as a function of manufacturer's discount depth, the manufacturer can now jointly choose depth and pass-through. This additional flexibility allows manufacturers to increase profits and market share. Conversely, the retailer's profits may be lower than in the 80% base case.

Consistent with Kim and Staelin (1999), these results show that even in the case of low pass-through rates,

it would be optimal for manufacturers to offer price discounts to the trade. Moreover, they suggest that retailers may be better off using a policy of constant pass-through rates because allowing pass-through to vary with discount depth has the effect of giving the manufacturer greater flexibility. These examples are intended to illustrate the impact of different retailer pass-through policies on the manufacturer's optimal promotion calendar. A solution of the retailer pass-through decision problem would require both manufacturer and store competition to be modeled (e.g., Kim and Staelin 1999). Nevertheless, these results should emphasize the importance to manufacturers of good estimates of pass-through for purposes of promotion budgeting and planning.

Consumer Sensitivity to Price Discounts

A second sensitivity analysis was to vary the value of the *Temporary Price Reduction* coefficient in the choice model from 0.10 to 2.00 (the fitted value was 0.56 for segment 1 and 0.48 for segment 2), which corresponds to a price elasticity (due to discount) range of -0.27 to -6.07 [12] The results are presented in Figure 5. The optimal price discount depth remains at 5% over a wide range of price elasticities, holding all else constant. Only for elasticity levels higher (in absolute value) than -4 (a very unusual case) is the optimal price discount affected and, in certain periods, it jumps to a 30% level. In our model deeper discounts result in greater incremental volume (though at a lower margin) and higher opportunity costs (more borrowed sales at a greater discount). Thus, very high consumer sensitivity to price discounts is needed to produce sufficient expansion of incremental sales to compensate for margin losses on borrowed sales. We contrast this result with Neslin et al. (1995) and Tellis and Zufryden (1995), which both employ a smoother functional relationship between price discounts, consumer sensitivity, and profits (e.g., Neslin et al. 1995, p. 751, especially Equations (5), (6), and (7). In those models, a more continuous positive relationship between consumer sensitivity and size of promotional discounts is produced (e.g., Neslin et al. 1995, Table 5a, p. 761; Tellis and Zufryden 1995, Figure 7).

Competition

The effect of varying competitive promotional activity on the profile of brand *A*'s optimal calendar was studied by analyzing different competitive scenarios. The results are summarized in Figure 6. Scenario 1 is the previously discussed optimal calendar under the actual competitive environment that brand *A* faced during the 52 weeks of 1994.

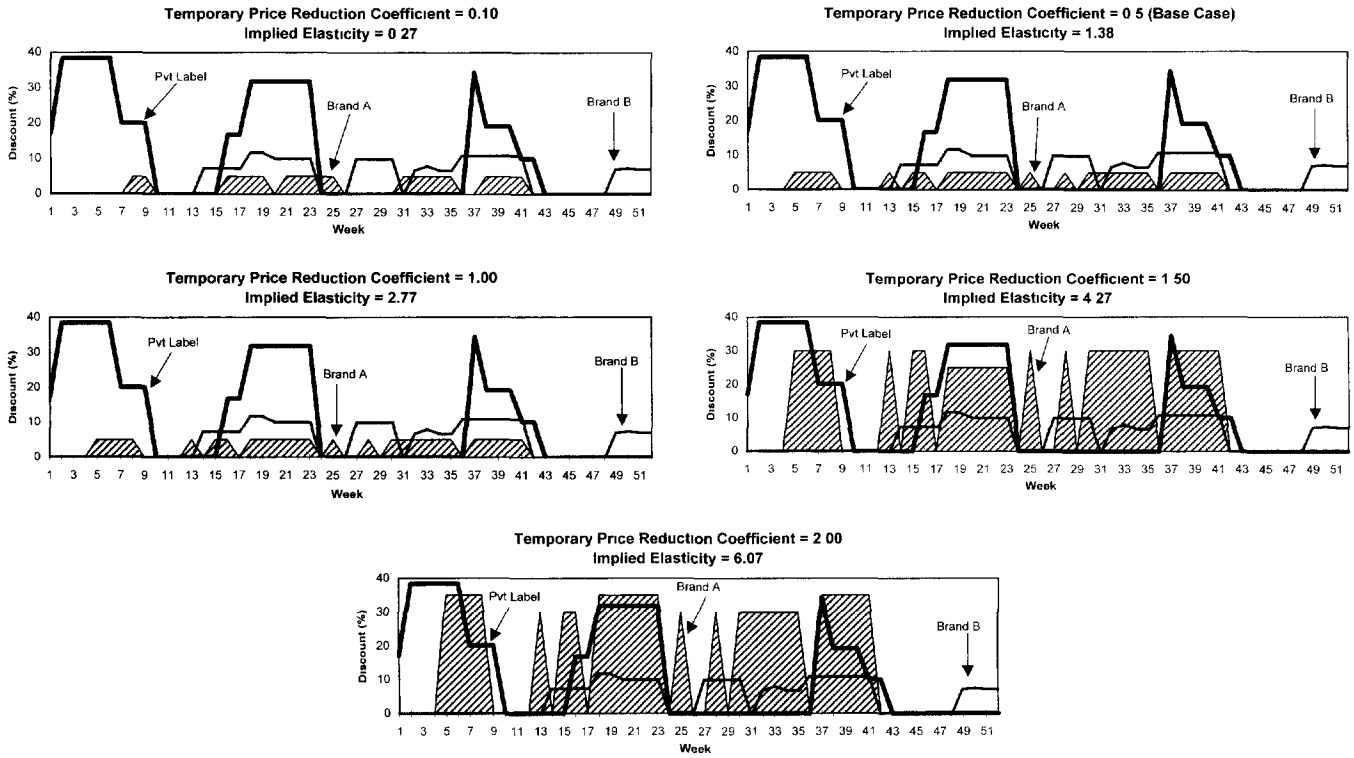
Scenario 2 presents the optimal calendar when brand *A* is facing no promotional competitive activity. Competitors' price discounts were set to 0 and regular prices were set at their respective mean values over the 52-week planning period. In scenario 3, brand *B* (the largest national brand) offers 10%[13] price reductions in the 26 even-numbered weeks. The private label offers no promotion during the 52-week period and regular prices are set to their respective annual mean levels. In scenario 4, brand *B* and the private label alternate their promotions. Brand *B* offers 10% discounts (its approximate mean actual level) in the odd-numbered weeks and the private label offers 30% discounts (its approximate actual mean level) in the even-numbered weeks. Scenario 5 is similar to scenario 4, except that brand *B* offers deep discounts (30%) and the private label shallow discounts (10%).

The results show no effect on the optimal depth and number of weeks on promotion for Brand *A* (the variables the manufacturer is more likely to influence). There are also no systematic effects on timing. This result is somewhat surprising because previous literature (e.g., Lal 1990a, Lal 1990b) has found it optimal to promote out-of-phase with competition. The difference in results is due to the implementation of carry-over (purchase feedback, consumption) in our model. Without those effects, it would be optimal to promote out-of-phase, especially when competition is offering deep discounts (due to the high cost of fighting for a small share of the pool of switchers). Purchase feedback and consumption increases, however, can extend

[12]In his meta-analysis of 367 price elasticities from 220 different brands/markets, Tellis (1988) found that mean price elasticity is about -2.50 and price elasticities below -4 were rare. It should be noted that his study is about price elasticities, not price elasticities due to discount.

[13]This is approximately the mean price discount offered by brand *B* during the 52-week period

Figure 5 Effect of Consumer Choice TPR Sensitivity on Optimal Calendar for Brand *A*



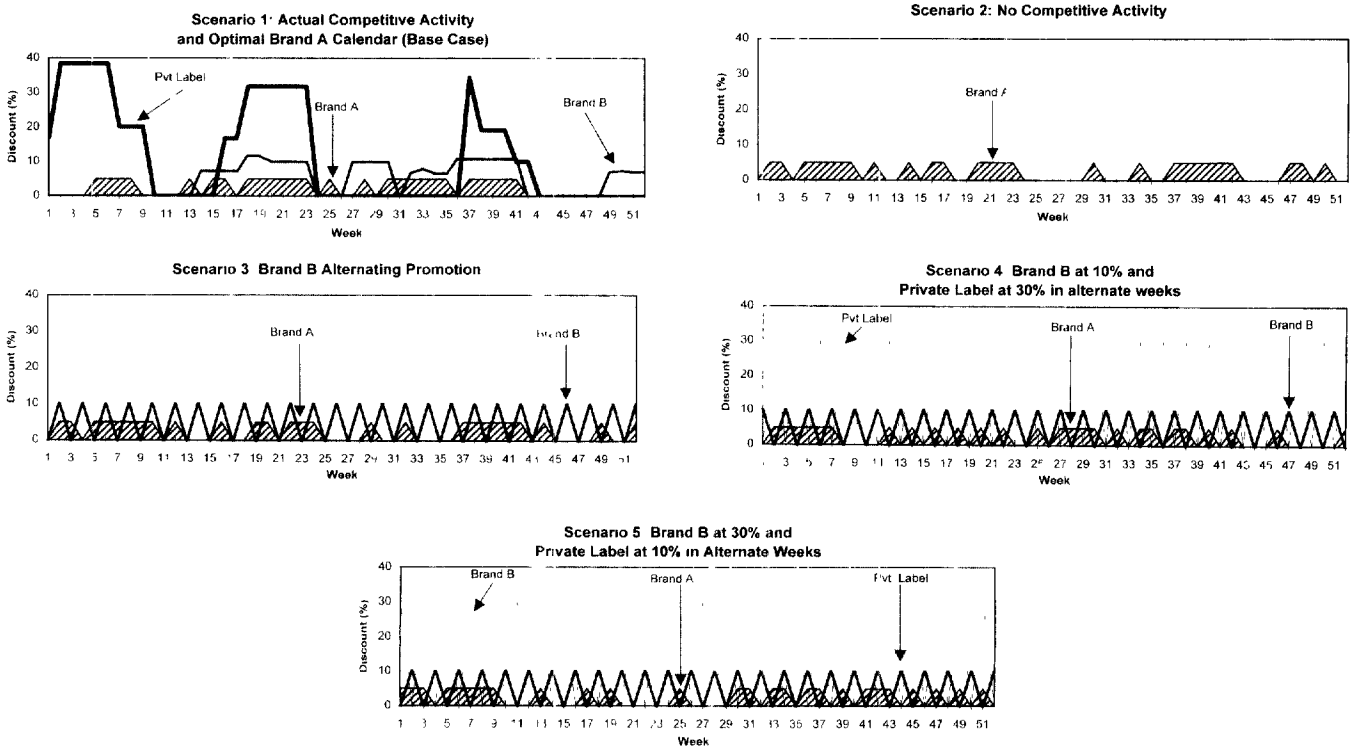
the effect of a promotion over subsequent periods and may make it preferable to promote on-phase so as to prevent a detrimental effect on future sales. As pointed out by Tellis and Zufryden (1995, H2, p. 296), alternating promotions among brands is more profitable for retailers than promoting several brands at the same time. This could also explain the tendency for promotions to be run out-of-phase.

Conclusion

This paper deals with a core marketing problem for consumer products manufacturers: how to make trade promotion programs more productive. The objective of this paper was to develop a decision support system to determine the manufacturer's sales promotion calendar, thereby improving routine promotional decisions and potentially enhancing the efficiency of the marketing function. The first component of our system is a consumer response model that allows us to measure the truly incremental component of sales during a promotional period. We use a disaggregate model to decompose the consumer purchase decision into choice, incidence, and quantity. This enables us to tease out the true incremental effect due to brand switching from the apparent, but not truly incremental, effects of purchase acceleration and stockpiling (net of their impact on purchase feedback and consumption). The second component of our approach is an optimization module that uses the information from the consumer response model to search for efficient promotion calendars. These calendars are determined subject to constraints on retailer's category profitability, pass-through, and markup behaviors.

The system was applied to the promotion planning problem for a national brand in the tomato sauce category. Comparative results between the actual and the model-based promotion programs show that the collaborating firm's brand could achieve a substantially higher profitability level if, instead of its actual program, it had adopted the calendar recommended by the model. Moreover, the increase in profitability did

Figure 6 Effect of Competitive Activity on Optimal Calendar for Brand *A*



not come at the expense of the brand's market share and the retailer would be better off in terms of category profits and sales.

Model Results Validated in a Subsequent Test Market

Results from a market where the collaborating firm applied our approach are consistent with our findings.[14] In that market, the firm implemented shallower price reductions. Consistent with the predictions of the model, market share was not affected and we believe that it is reasonable to infer that the firm's profitability improved. The collaborating firm's stated objective was to provide its salesforce with a "fast" laptop version of the model which they could use to discuss "win-win" promotional plans with the retailer. More strategically, the system can also be used to help set overall promotion spending levels for a brand because the optimal spending level is produced simultaneously with the optimal promotion calendar.

Understanding that Retailer Pass-Through is Critical for Manufacturers

We found that whether pass-through is constant or functionally related to discount depth may substantially impact the optimal discount depth and number of weeks on promotion offered to the trade.[15] Consistent with Kim and Staelin (1999), these results show that even in the case of low pass-through rates, it would be optimal for manufacturers to offer price discounts to the trade. However, as summarized in Table 5, the optimal program to be offered by the manufacturer to the trade, and its profitability, is significantly

[14] As previously noted (see footnote 5), the implementation was based on the constrained model (Table 1)

[15] These results are based on a pay-for-performance system which (albeit at a certain pass-through rate) implies a 100% participation for the retailer

affected by retailer pass-through. These results emphasize the importance to manufacturers of accurate estimates of pass-through for promotion budgeting and planning.

For a Given Pass-Through Rate, Sensitivity Analysis Shows the Results to be Robust

Sensitivity analysis allowed us to generalize the results to other marketing environments beyond that represented by the specific application presented here. For example, for a wide range of price sensitivity levels, frequent shallow deals are preferred to deeper discounts. Greater discount depth is optimal only for unusually high levels of price elasticity (greater than 4 in absolute value). This corroborates Simester (1997), who found, using a game-theoretic approach, that deeper discounts are preferable for markets with high levels of price sensitivity and/or a high proportion of switchers.

Our sensitivity analysis also showed that there were no systematic effects from varying competitive promotion activity. Competition does not affect the optimal level of discount or the frequency of promotion activity. In terms of timing, if there were no carryover effects, it would be optimal for the manufacturer to promote out-of-phase with competition. But our analysis suggests that the dynamic effects due to purchase feedback and consumption could make it preferable for a manufacturer to sometimes promote on-phase with competition.

Limitations and Extensions

One potential limitation of our system is the availability of disaggregate (panel-level) data. Current policy at both IRI and ACNielsen (as confirmed by personal communication) is generally not to make raw panel data available to clients. Both IRI and ACNielsen emphasized to us, however, that this policy in no way precludes clients from implementing a decision support system based on panel data. IRI, for example, permits clients to conduct analyses on panel data using an on-line interface through their time-sharing mainframe. Thus, the only practical limitation is that the estimation of the consumer response model component of the system would need to be implemented with the involvement of IRI or ACNielsen.

Scanner or wand panel data are also not collected in all markets and product categories and may have small sample sizes in many cases where they are collected. An interesting opportunity for future research is to develop a methodology to combine store-level data with panel data to overcome the limitations of panel-data availability, while avoiding the drawbacks of store-level data. One idea would be to relate panel data, where available, to demographic variables and store-level data (e.g., brand shares, promotion intensity, etc.). Those relationships would be used to construct a synthetic panel based on demographics and store data for those markets where panel data are not available or are of limited sample size.

A second limitation is that our model does not address the consumer store choice decision. This could be an issue for products that play a significant role as retail traffic builders. Even though sensitivity analysis allows us to examine several competitive scenarios, a topic for future research is to simultaneously solve the optimization problem for all the manufacturers. The use of genetic algorithms (cf. Midgley et al. 1997) may be a promising way of investigating this.

The retailer plays an important role in this model. Nevertheless, a third limitation of our system is that the role is passive. We assume that the manufacturer knows how the retailer responds to promotional offerings and that information is provided to the model as managerial input. We cannot be sure if that information corresponds to the retailer's optimal behavior. Thus, another important generalization of this model would be to address the optimization problem of the manufacturer and the retailer simultaneously.

Finally, many manufacturers have product offerings in several product categories. This research does not consider promotional offerings across categories. The modeling of this problem is challenging because the manufacturer will face different competitors in the various categories and retailers are likely to respond differently across categories (e.g., in terms of pass-through and markup).

In sum, we have addressed a complex and important problem for consumer packaged goods manufacturers: the planning and budgeting of promotional calendars over a long time horizon. We developed an approach that overcomes many limitations of previous work in

this area (e.g., Abraham and Lodish 1993). The recommendations from our decision support system were successfully implemented by the collaborating firm and improved their trade promotion programs.[16]

Appendix Market Response Model

Model 1: Response Model with Constant Consumption

Choice Model. The choice model estimates the probability that household h will choose brand-size alternative i given a category purchase decision at time t. The brand-size choice probability is given by the multinomial logit model (e.g., Guadagni and Little 1983):

$$P_t^h(i \mid inc) = \frac{\exp(U_{it}^h)}{\sum_k \exp(U_{kt}^h)}, \tag{A1}$$

where $P_t^h(i \mid inc)$ is the choice probability for brand-size i given category purchase and store visit at t. U_{it}^h is the deterministic component of utility associated with brand i for panelist h and is modeled as

$$\begin{aligned} U_{it}^h = {} & u_{bi} + u_{si} + \beta_1 BLOY_i^h + \beta_2 LBP_{it}^h + \beta_3 SLOY_i^h \\ & + \beta_4 LSP_{it}^h + \beta_5 RPRICE_{it} + \beta_6 PC_{it} \\ & + \beta_7 FEAT_{it} + \beta_8 DISP_{it}, \text{ where} \end{aligned} \tag{A2}$$

$BLOY_i^h$ = loyalty of household h to brand of brand-size i, computed over initialization period.
LBP_{it}^h = 1 if i was last brand purchased, 0 otherwise.
$SLOY_i^h$ = loyalty of household h to size of brand-size i, computed over initialization period
LSP_{it}^h = 1 if i was last size purchased, 0 otherwise.
$RPRICE_{it}$ = regular (depromoted) retail price of brand-size i, at time t.
PC_{it} = temporary price cut of brand-size i, at time t, so that shelf price results from

$$PRICE_{it} = RPRICE_{it} - PC_{it}. \tag{A3}$$

$FEAT_{it}$ = 1 if brand-size i appeared in a feature ad at time t, 0 otherwise
$DISP_{it}$ = 1 if brand-size i was specially displayed at time t, 0 otherwise
$\{u_{bi}\}$ = brand constant for brand-size alternative i to be estimated
$\{u_{si}\}$ = size constant for brand-size alternative i to be estimated
$\{\beta_1, \beta_2, \ldots, \beta_8\}$ = parameters to be estimated

Our approach to $BLOY_i^h$, $SLOY_i^h$, LBP_{it}^h, and LSP_{it}^h follows Bucklin and Lattin (1991) cf. (Ailawadi et al. 1999) and our approach to $\{u_{bi}\}$ and $\{u_{si}\}$ follows Fader and Hardie (1996).

Purchase Incidence Model. The purchase incidence model estimates the probability that a household h decides to buy in the category given a store visit at time t. The buy/no-buy outcome is modeled with a binary nested logit (e.g., Guadagni and Little 1998):

$$P_t^h(inc) = \frac{\exp(V_t^h)}{1 + \exp(V_t^h)}, \tag{A4}$$

where $P_t^h(inc)$ is the category purchase incidence probability given store visit at t. V_t^h is the deterministic component of utility associated with household h at time t (Guadagni and Little 1998):

$$V_t^h = \gamma_0 + \gamma_1 PFRQ^h + \gamma_2 INV_t^h + \gamma_3 CV_t^h, \text{ where} \tag{A5}$$

$PFRQ^h$ = category purchase frequency for household h, i.e., the proportion of shopping trips in which household h makes a category purchase.
INV_t^h = mean-centered inventory estimate for household h, at time t

$$CV_t^h = \ln \sum_k \exp(U_{kt}^h). \tag{A6}$$

$\{u_{bi}\}$ = brand constant for brand-size alternative i estimated in the choice model
$\{u_{si}\}$ = size constant for brand-size alternative i estimated in the choice model.
$\{\gamma_0, \gamma_1\}$ = parameters to be estimated.

The inventory of household h at time t is given by (Bucklin and Lattin 1991)

$$INV_t^h = INV_{t-1}^h + \sum_j (Q_{j,t-1}^h \times OZ_j) - CR^h, \text{ where} \tag{A7}$$

$Q_{j,t-1}^h$ = number of units of brand-size alternative j bought at time $t - 1$ by household h.
OZ_j = weight in ounces of brand-size alternative j.
CR^h = rate of category consumption for household h (ounces/week)

Note that the deterministic component of utility includes household purchase frequency, inventory, and an indicator of category value (e.g., Guadagni and Little 1998). This model formulation has the appealing property that a household's purchase incidence decision will be affected more by the marketing activity of the preferred brands than by the marketing activity of the less preferred brands (Bucklin and Gupta 1992).

Quantity Model. Given a purchase of brand-size alternative i, the probability that household h buys q_{it}^h units of brand-size i at time t can be captured by a Poisson model with truncation of the zero outcome (e.g., Ailawadi and Neslin 1998, Bucklin et al. 1998). The mathematical formulation is

$$P(Q_{it}^h = q_{it}^h \mid Q_{it}^h > 0) = \left[\frac{\exp(-\lambda_{it}^h)(\lambda_{it}^h)^{q_{it}^h}}{(1 - \exp(-\lambda_{it}^h))q_{it}^h!}\right]. \tag{A8}$$

The purchase rate of the household h for brand i at time t is modeled as

[16]The authors would like to thank Rick Staelin, the Area Editor and three anonymous reviewers for their suggestions and insightful comments as well as the collaborating firm and the ACNielsen Co for providing scanner and wand data used in this study

$$\lambda_{it}^h = \exp\begin{pmatrix} \lambda_{bi} + \lambda_{si} + \varphi_1 PR^h + \varphi_2 INV_t^h \\ + \varphi_3 BLOY_t^h + \varphi_4 SLOY_t^h \\ + \varphi_5 RPRICE_{it} + \varphi_6 PC_{it} + \varphi_7 FEAT_{it} \\ + \varphi_8 DISP_{it} \end{pmatrix}, \quad \text{(A9)}$$

where

PR^h = purchase rate for household h, i.e., average number of units bought by household h per category purchase occasion.[17]

INV_t^h = mean-centered inventory estimate for household h, at time t.

$\{\lambda_{bi}\}$ = brand constant for brand-size alternative i to be estimated.

$\{\lambda_{si}\}$ = size constant for brand-size alternative i to be estimated.

$\{\varphi_1, \varphi_2, \ldots, \varphi_8\}$ = parameters to be estimated.

Model 2: Response Model with Flexible Consumption

In some product categories, promotions may impact consumption via household inventory pressure (Assunção and Meyer 1993) which may result from the acceleration of purchase timing or stockpiling quantity. In that case, following Ailawadi and Neslin (1998), we model consumption rate as

$$CR_t^h = INV_t^h \left[\frac{\overline{CR^h}}{\overline{CR^h} + (INV_t^h)^\phi}\right], \text{ where} \quad \text{(A10)}$$

$\overline{CR^h}$ = mean rate of category consumption for household h (ounces/week).

$\{\phi\}$ = parameter to be estimated.

Thus, Equation (A7) becomes

$$INV_t^h = INV_{t-1}^h + \sum_i (Q_{i,t-1}^h \times OZ_i) - CR_{t-1}^h. \quad \text{(A7a)}$$

Model 3: Latent Segment Model

In this model, the response parameters, β in Equation (A2), γ in Equation (A5), φ in Equation (A9), and ϕ in Equation (A10), become segment-specific (e.g., Bucklin et al 1998, Eq. (7), p. 192).

Expected Incremental and Non-Incremental Sales

The expected number of units of brand i purchased by household h at time t, conditional on brand-size i choice, is obtained from the expected value of the truncated Poisson:[18]

$$E(Q_{it}^h \mid Q_{it}^h > 0) = \frac{\lambda_{it}^h}{1 - \exp(-\lambda_{it}^h)}. \quad \text{(A11)}$$

[17]Note that households in this category are highly size loyal; 87% of the purchases are made on the respective preferred size.

[18]Note that we are using the expected number of units. Using a Monte Carlo procedure to draw from the (truncated) Poisson distribution would be impractical due to the computation time required when combining that procedure with simulated annealing. In a similar way, we are using the purchase incidence probability instead of drawing from the binary logit.

Using Equations (A11), (A1), and (A4), Equations (1), (2), and (4) can respectively be expressed as

$$E(Q_{it}^h) = \left[\frac{\lambda_{it}^h}{1 - \exp(-\lambda_{it}^h)}\right] \times \left[\frac{\exp(U_{it}^h)}{\sum_k \exp(U_{kt}^h)}\right] \times \left[\frac{\exp(V_t^h)}{1 + \exp(V_t^h)}\right], \quad \text{(A12)}$$

$$E(BBQ_{it}^h) = \left[\frac{\lambda_{it}^h \mid_{INV_t^h = INVBB_t^h}}{1 - \exp(-\lambda_{it}^h \mid_{INV_t^h = INVBB_t^h})}\right] \times \left[\frac{\exp(U_{it}^h \mid_{PC_{it}=0, FEAT_{it}=0, DISP_{it}=0, LBP_{it}^h = LBPB_{it}^h})}{\sum_k \exp(U_{kt}^h \mid_{PC_{it}=0, FEAT_{it}=0, DISP_{it}=0, LBP_{it}^h = LBPB_{it}^h})}\right] \times \left[\frac{\exp(V_t^h \mid_{INV_t^h = INVBB_t^h})}{1 + \exp(V_t^h \mid_{INV_t^h = INVBB_t^h})}\right], \text{ and} \quad \text{(A13)}$$

$$E(\Delta Q_{it}^h) = \left[\frac{\lambda_{it}^h}{1 - \exp(-\lambda_{it}^h)}\right] \times \left[\frac{\exp(U_{it}^h)}{\sum_k \exp(U_{kt}^h)}\right] \times \left[\frac{\exp(V_t^h)}{1 + \exp(V_t^h)}\right] - \left[\frac{\lambda_{it}^h \mid_{INV_t^h = INVBB_t^h}}{1 - \exp(-\lambda_{it}^h \mid_{INV_t^h = INVBB_t^h})}\right] \times \left[\frac{\exp(U_{it}^h \mid_{PC_{it}=0, FEAT_{it}=0, DISP_{it}=0, LBP_{it}^h = LBPB_{it}^h})}{\sum_k \exp(U_{kt}^h \mid_{PC_{it}=0, FEAT_{it}=0, DISP_{it}=0, LBP_{it}^h = LBPB_{it}^h})}\right] \times \left[\frac{\exp(V_t^h \mid_{INV_t^h = INVBB_t^h})}{1 + \exp(V_t^h \mid_{INV_t^h = INVBB_t^h})}\right] + CRBB_t^h - CRB_t^h, \quad \text{(A14)}[19]$$

where

$LBPB_{it}^h$ = the value of LBP variable for household h, brand-size alternative i, if there had not been any promotional activity for brand-size alternative i before time t.

$INVBB_t^h$ = mean-centered category inventory level (in ounces) for household h at time t, given that the choice effect of promotions for brand-size i is removed, i.e., if household h purchase decisions would have followed the baseline plus borrowed volume per Equation (A13).

$CRBB_t^h$ = consumption rate for household h at time t if household h purchase decisions would have followed the baseline plus borrowed volume per Equation (A13). $CRBB_t^h$ is obtained by substituting $INVBB_t^h$ for INV_t^h in Equation (A10).

[19]The results reported in this article were obtained with incremental consumption defined as $CR_t^h - CRB_t^h$. That specification may count as incremental consumption some of the units that were already computed as incremental when the promotional purchase took place. Because tomato sauce is a category with little or no consumption effect, we found no material change in the results from changing the specification. We thank the Area Editor for pointing this out.

CRB_t^h = consumption rate for household h at time t if there had not been any promotional activity for brand-size alternative i before time t. CRB_t^h is obtained by substituting $INVB_t^h$ for INV_t^h in Equation (A10), where $INVB_t^h$ is an estimate of what the mean-centered category inventory level (in ounces) for household h would be at time t if no promotions on brand-size i had been run.

In the latent-class model, Equation (A12) is modified to

$$E(Q_{it}^h) = \sum_s w_s^h \left[\frac{\lambda_{sit}^h}{1 - \exp(-\lambda_{sit}^h)}\right] \times \left[\frac{\exp(U_{sit}^h)}{\sum_k \exp(U_{skt}^h)}\right] \times \left[\frac{\exp(V_{sit}^h)}{1 + \exp(V_{sit}^h)}\right], \quad \text{(A12a)}$$

and Equations (A13) and (A14) are modified in a similar way.

The household-level segment membership probabilities (w_s^h) and the segment size parameters π_s, are not estimated directly but as logit functions, following the procedure in Kamakura and Russell (1989). The latent segment models are also estimated by maximum likelihood.

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