Advance Booking Programs for Managing Supply, Demand, and Price Risks

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Abstract: While advance booking programs have been shown to be effective for firms to manage uncertain demand, the effectiveness of such programs is unclear when supply, demand, and price risks are present in a supply chain. Motivated by an advance booking program for managing these three types of risks in a flu vaccine supply chain, we present a two-stage Stackelberg game model to examine the dynamic interactions between a manufacturer and a retailer over two stages. In each stage, both firms enter a Stackelberg game: the manufacturer sets his wholesale price and the retailer determines her order quantity. However, when making the decisions in the second stage, both firms take into account the decisions chosen in the first stage as well as the information about supply and demand revealed after the first stage. Our analysis shows that the advance booking program is always beneficial to the manufacturer but not to the retailer especially when a supply shortage is likely to occur. Interestingly, we find that supply uncertainty and demand uncertainty affect the firms' profits in an opposite manner under the advance booking program: the firms' expected profits tend to decrease in supply uncertainty, but they tend to increase in demand uncertainty.

Key words: advance booking, dynamic game, healthcare, supply chain risk management

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1 Introduction

In the classical newsvendor problem with uncertain demand, a firm places a single order prior to the start of the selling season by considering the trade-off between over-stocking and under-stocking. The problem is to determine an order quantity that maximizes the expected profit when the procurement cost and the selling price are fixed. As an innovative way to manage uncertain demand in the apparel industry, Fisher et al. (1994) examine an idea in which a firm can place two orders in the newsvendor problem setting: a pre-book order well in advance of the selling season, and a regular order closer to the selling season. The pre-book order enables the firm to secure some

supply at a lower cost and the regular order allows the firm to postpone her ordering decision until more accurate information about demand becomes available. Fisher and Raman (1996) develop a formal model to examine the benefit of the advance booking program. This seminal paper has motivated researchers to develop different models to explore the notion of advance booking in various contexts. (In this paper, we refer to "advance booking" or "pre-booking" as an order placed prior to a regular order, and we refer to "advance booking program" as an ordering system in which both pre-book and regular orders are allowed.)

While advance booking programs were originally developed for managing uncertain demand, many firms have used this program to manage supply chains that face supply risk arising from production yield uncertainty, demand risk due to market uncertainty, and price risk caused by the imbalance of supply and demand. We offer two examples of such programs observed in practice. First, consider an Integrated Circuit (IC) manufacturer that produces IC chips for its customers (electronics device manufacturers). Besides demand risk due to rapid technological innovations, the IC manufacturer faces the inherent yield uncertainty associated with IC fabrication. In addition, there is a price risk in the IC market that is caused by the imbalance of supply and demand. For example, due to a severe shortage of dynamic RAM (DRAM) chips in 1988, many computer manufacturers lamented the surge in the price of DRAM chips. However, after a decline in sales of personal computers in 2000, there was an over-supply of DRAM chips in 2000; Kanellos (2000) reported the significant drop in the market price of DRAM chips. In order to deal with the problem of using uncertain supply to meet uncertain demand, many IC manufacturers such as Hynix and Xilinx now offer advance booking programs to their customers. Under these programs, each customer can place a pre-book order at a known price, and a regular order later. However, the regular order is subject to uncertain price and its fulfillment is subject to availability. Because pre-book orders are filled first before regular or spot orders, customers are eager to place their pre-book orders even without advance booking discounts (Brown 2009).

Second, consider a flu vaccine supply chain that faces challenges in matching demand with supply. The challenges in this industry are highlighted in the following quote from an expert at the Centers for Disease Control and Prevention (CDC) (Williams 2005): *"We know flu is unpredictable. We've learned that flu vaccine supply is unpredictable, and what we're discovering now, of course, is that the demand for a flu vaccine is also very unpredictable."* Because of these challenges, the industry had experienced significant mismatches of demand and supply in the past. For example, in the 2006-7 flu season, there was an over-supply of 18.4 million doses that were discarded after

the market experienced a supply of 120.9 million doses and a demand of 102.5 million doses (Health Industry Distributors Association 2007). The imbalance of supply and demand can trigger price fluctuations as well. For example, during the 2003-4 flu season, the supply of flu vaccines in the U.S. was cut by nearly half from 100 million doses to 55 million doses due to contamination at the Chiron's plant that produces flu vaccines for the U.S. market (Offit 2005). Due to this severe shortage, it was reported that the price went up by 10 times from $8-9 a dose to $80-90 a dose in some areas (Flaherty 2004). As a way to reduce the imbalance of supply and demand, manufacturers (e.g., Sanofi Aventis) and distributors (e.g., FFF Enterprises, McKesson) have developed advance booking programs for healthcare providers or retailers. Under these programs, pre-book orders are usually placed between February and April when production is in progress, and the regular orders are placed between September and December during the flu season after the production process is almost complete. Once the supply of flu vaccine becomes available, it is first used to fill pre-book orders and the remaining supply, if any, is used to fill regular orders. According to Yadav (2009), the wholesale price for pre-book orders can be lower but the wholesale price for regular orders can fluctuate a lot depending on the actual supply and demand.

While advance booking programs have been shown to be effective for managing demand risk in the literature, it is not clear if such programs are advantageous for managing supply, demand, and price risks in a supply chain. Although demand risk is probably the most prevalent risk that is getting all the attention, one cannot neglect the significance of supply risk and price risk in supply chains. As an initial attempt to examine advance booking programs under different types of risks, we develop a two-stage dynamic Stackelberg game model of a two-level supply chain that faces all three types of the aforementioned risks. Our base model entails one manufacturer and one retailer, while we discuss its extension to multiple retailers. The first stage occurs during the 'speculation period' that takes place before uncertain yield and demand are realized. (Throughout this paper, the terms 'stage' and 'period' are used interchangeably.) In this stage, the manufacturer ('he') and the retailer ('she') enter a Stackelberg game in which the manufacturer acts as the leader by setting his pre-book wholesale price and then the retailer acts as the follower by determining her pre-book order quantity. At the end of the first stage, the actual yield is realized, and the manufacturer is obligated to use his actual supply to fill the pre-book order. After the retailer receives her allocation associated with her pre-book order, the actual market demand is realized. This instant marks the beginning of the second stage that takes place during the 'reaction period'. In the second stage, the manufacturer and the retailer enter another Stackelberg game that is similar to the one in the

first stage in terms of their roles and decisions. However, when making the decisions in the second stage, both firms would take into account the decisions chosen in the first stage and the observed information. The Stackelberg setting is appropriate in the industry such as the flu vaccine industry where a large pharmaceutical company sells flu vaccines to a smaller healthcare provider possessing a lower bargaining power[1].

We analyze our model using backward induction: we first solve the second stage Stackelberg game conditioning on the realized supply and demand and the decisions chosen in the first stage. Then we solve the first stage Stackelberg game by embedding the equilibrium outcomes of the second stage game in the firms' expected profits in the first stage. This involves solving the decisions of two players in each of two stages, which turns out to be quite complex. To develop a tractable analysis, we impose the following assumptions that are best described and justified in the context of flu vaccine supply chains.

- *Uncertain Supply:* The production capacity is known and fixed in each flu season because building or expanding the capacity is costly and time-consuming (3-5 years) due to the stringent requirements of the FDA approval process (Matthews 2006). However, owing to the biological nature of vaccine and the current production method of using chicken eggs, production yield is highly uncertain with a wide range of values being equally plausible. Thus, following Cho (2009), we shall use a proportional random yield model with fixed capacity.

- *Uncertain Demand.* The primary factor that determines the demand for flu vaccines is the prevalence and severity of flu activities during the flu season which lasts usually from October to March in the following year. As mentioned above by the CDC expert, the spread and evolution of flu viruses are fundamentally uncertain.

- *Uncorrelated Supply and Demand.* The process yield of the manufacturing process determines the available supply, whereas the flu activities during a flu season primarily determine the demand for flu vaccines. Thus, we shall assume the uncertain supply and the uncertain demand are not correlated.[2]

[1] In a typical year, healthcare providers procure flu vaccines directly from distributors or manufacturers and the proportion of orders from the federal government in the U.S. is only about 5% (Health Industry Distributors Association 2007). However, in the 2009-2010 flu season, healthcare providers acquire vaccines through the government because of unusually high demand in a pandemic situation and anticipated delay and shortage of seasonal flu vaccine and H1N1 supplementary vaccine.

[2] We do not consider a situation in which insufficient supply creates an unexpected surge in demand due to public fear or herd effect.

- *No Demand Forecast Updating.* Because of the long production lead time of 6-8 months, the pre-book orders of flu vaccines are placed long before the actual flu season. As such, neither the manufacturer nor the retailer can obtain more accurate forecast for consumer demand during the first stage. However, for tractability, we shall assume that the second stage takes place after all uncertainties have been resolved.[3]

- *Non-Cancellable Pre-book Orders.* According to the pre-book order forms provided by FFF Enterprises and McKesson, the pre-book orders of flu vaccines are not cancellable[4]. However, due to the uncertain supply, the manufacturer may not be able to fill all pre-book orders. For such unfilled pre-book orders, the manufacturer provides full refund. We do not consider the time value of money.

- *Uncertain Market Price.* Based on anecdotal evidence (e.g., Fine 2004, Flaherty 2004), the retail price of flu vaccines in the market appears to be higher (or lower) than average when the actual demand is higher (or lower) than the actual supply. Hence, we shall assume a linear down-sloping demand curve to model the impact of the imbalance of demand and supply on the market price.

Our model captures the following key trade-offs that the retailer and the manufacturer face under advance booking programs. If the retailer does not pre-book and postpones her ordering decision until the uncertainties of supply and demand are resolved, she can avoid over-stocking but she takes on the risk of paying a higher wholesale price in the event of a shortage (i.e., the actual supply is lower than the actual demand). When there are multiple retailers, the retailer bears the additional risk of not getting her regular order filled because the pre-booked orders placed by the other retailers are filled first. On the other hand, if the retailer places a pre-book order, then she reduces her price risk by committing the pre-book order at the known pre-book wholesale price. However, she bears the risk of over-stocking and over-paying in the event when the actual demand is lower than the actual supply. Therefore, it is unclear if the retailer should pre-book and if so, how much the retailer should pre-book. The trade-offs faced by the manufacturer are essentially the opposite of the retailer. The retailer's pre-book order reduces the manufacturer's demand risk

[3] This assumption is commonly used in the supply chain literature (see, e.g., van Mieghem and Dada (1999), Cachon (2004), and Erhun et al. (2008)).

[4] See the pre-booking form used by FFF Enterprises at http://www.mmd.admin.state.mn.us/mmcap/pdf/1%20FFF%20Enterprises%20Attachemnt%201.pdf. In the 2010-2011 pre-booking form, McKesson specifies that orders may not be cancelled after August 1, 2010 (see http://www.supplymanagementonline.com/flushots/fluvaccineprebook.asp). Note that information about uncertain demand and supply is seldom available prior to August 1.

because it cannot be cancelled. However, it increases his price risk in the event when he can charge a much higher regular wholesale price when the actual demand is higher than the actual supply.

By analyzing the above trade-offs between the firms in the supply chain, we aim to answer the following research questions:

(1) Who will benefit from the advance booking program? The manufacturer, the retailer, or both? If not both, when is this program not beneficial to a member of the supply chain? What is the source of the benefits?

(2) What are the impacts of production capacity, supply uncertainty, and demand uncertainty on the manufacturer's pricing decisions, the retailer's ordering decisions, and the resulting profits? Are the effects of supply uncertainty and demand uncertainty the same?

The remainder of this paper is organized as follows. In section 2 we review related literature. In section 3 we present our model. In section 4 we derive the equilibria associated with three different games in which only pre-booking, only regular ordering, or both pre-booking and regular ordering are allowed, respectively. In section 5 we compare the equilibrium outcomes of the three games to answer our first research question about the benefits of advance booking programs. In section 6 we present comparative statics to answer our second research question about the impacts of capacity, supply uncertainty and demand uncertainty. In section 7 we discuss an extension of our model to multiple retailers. Section 8 concludes our paper. All proofs are provided in Appendix A1.

2 Literature Review

Our paper is related to two streams of operations management literature. The first stream deals with the issue of advance booking, which is akin to quick response (QR). We provide a brief review of QR models and refer the reader to Choi and Sethi (2010) for a comprehensive review. The second stream deals with price postponement.

To our knowledge, Fisher et al. (1994) and Fisher and Raman (1996) are the first to introduce the idea of advance booking programs in the retail industry. Iyer and Bergan (1997) analyze the benefit of QR in the supply chain consisting of one manufacturer and one retailer. In their model, the retailer places an order once, but in the QR system, she can place the order after observing partial demand information. They show that the manufacturer may not be better off under QR. Gurnani and Tang (1999) analyze a model that deals with a retailer's two ordering decisions. In their model, the wholesale price for the pre-book order is known, but the wholesale price for the

regular order is uncertain. They examine the trade-off between a more accurate forecast and a potentially higher cost of the regular order. Donohue (2000) analyzes advance booking programs in a supply chain consisting of one manufacturer and one retailer in which the manufacturer incurs a higher production cost for the regular order. She examines how the wholesale price and returns policy can achieve channel coordination. Brown and Lee (2003) analyze advance booking programs in which a buyer can reserve the supplier's capacity in advance and purchase an additional amount later at an extra price after observing demand information. Weng and Parlar (1999), Tang et al. (2004), and McCardle et al. (2004) study the benefits of advance-purchase discounts from retailers to customers. Özer and Wei (2006) study advance purchase contracts between a manufacturer and a retailer when there is asymmetric information about demand. They show that it is optimal for the manufacturer to charge a higher wholesale price for the advance order than the regular order.

Ferguson (2003) and Ferguson et al. (2005) study the procurement decision of an end-product manufacturer from a part supplier. They examine how the part price and the timing of production and ordering decisions affect the distribution of supply chain profits between the buyer and the supplier. Erhun et al. (2008) extend their work to a two-period model and investigate the impact of the timing of the supplier's ordering decisions and the additional demand information on the manufacturer's pricing and capacity decisions. By considering the case when demand follows a binary distribution, they obtain closed-form equilibrium outcomes and also discuss additional insights through numerical examples.

While the above papers analyze the value of advance booking in the context of demand uncertainty or private demand information, Cachon (2004) studies the value of advance booking in the context of risk allocation between members in a supply chain as in our paper. He shows that the manufacturer can shift the risk of having excessive inventory to the retailer by offering a price discount for pre-book orders to the retailer.

Our work contributes to this stream of research in the following sense. First, we analyze the benefits of advance booking programs in the presence of supply, demand and price risks. Much of the previous effort has been spent on analyzing the benefits of those programs when uncertain demand is the only source of risk. However, in practice, firms in various industries use advance booking to manage various types of risk. Our effort is analogous to the numerous extensions of lot sizing models under uncertain demand to those in the presence of random yields. Second, we examine the dynamic interactions between the manufacturer's pricing decisions and the retailer's ordering decisions over two periods for the case when supply and demand uncertainties follow

general probability distributions. With the exception of a few papers, most papers in the literature assumes that only one firm in a supply chain utilizes information obtained between two decision instants and make his/her decisions dynamically, while the other firm's decisions are exogenous or determined prior to observing new information. In particular, the manufacturer's wholesale prices to the retailer are given exogenously or set much in advance of the selling season, and the pre-book price is assumed lower than the regular price. However, due to long production lead time, it is common for a manufacturer to postpone his pricing decision for the regular order after uncertainties are resolved. In this setting, because the regular wholesale price is not known to the retailer in advance, the retailer has an incentive to place pre-book orders even without a price discount (Brown 2009). Third, based on the Newsboy model, most models in the literature assume exogenous market price and demand. In contrast, we adopt a price-sensitive demand model which reflects the imbalance of supply and demand. This enables us to analyze the causal relationships among demand, supply and price.

Similar in spirit to our paper, the second stream of literature departs from the first stream by considering price-sensitive demand and by allowing a firm to postpone its price decision after demand uncertainty is resolved. Van Mieghem and Dada (1999) present a two-stage model in which a firm makes three decisions: capacity, price, and production quantity. To evaluate the benefits of postponements, they analyze various scenarios in which the firm can postpone part or all of those decisions after demand uncertainty is resolved in the second stage. Motivated by the product postponement concept examined in Lee and Tang (1997), Chod and Rudi (2005) extend Van Mieghem and Dada (1999) to the two-product case. They study the value of resource flexibility and price postponement (also called responsive pricing). Recently, Tang and Yin (2007) analyze the issue of price postponement in a setting where the demand function is known but the supply yield is uncertain. This stream of work deals with a single firm's one-time decision under either demand uncertainty or supply uncertainty. In contrast, our emphasis is on studying dynamic interactions between two firms in a supply chain over two periods under supply, demand, and price uncertainties.

3 The Model

Consider a two-level supply chain that is comprised of one manufacturer and one retailer. The manufacturer M uses his plant with a fixed capacity k to produce a single product, and sells this product through the retailer R over a short selling season. Due to the long production lead time,

the manufacturer needs to start his production long before the start of the selling season. After the season is over, unsold products are discarded. The objective of the manufacturer (or the retailer) is to maximize his (or her) expected profit $E\Pi_M$ (or $E\Pi_R$).

The manufacturer fully utilizes his capacity k but his actual output is subject to a random production yield $\tilde{\theta}$. Using a proportional random yield model (e.g., Yano and Lee 1995), we represent the manufacturer's actual supply as $k\tilde{\theta}$, where $\tilde{\theta}$ follows a cumulative distribution F with support over $[\underline{\theta}, \bar{\theta}]$ where $\underline{\theta} \geq 0$. We normalize the mean yield to 1, i.e., $E\tilde{\theta} = 1$, and allow $\tilde{\theta}$ to be greater than one, which is common in the production process having the inputs and outputs measured in different units such as the flu vaccine manufacturing process (Cho 2009). Without loss of generality, we shall assume that the unit production cost is zero.

The retailer faces a down-sloping demand curve $p = 1 + \tilde{\varepsilon} - q$, where p represents a retail price to consumers, $\tilde{\varepsilon}$ represents a random demand component, and q represents the quantity available for sales by the retailer. Because q depends on the manufacturer's actual supply $k\tilde{\theta}$, the retail price p is random and is affected by both supply uncertainty $\tilde{\theta}$ and demand uncertainty $\tilde{\varepsilon}$. We assume that $\tilde{\varepsilon}$ follows a cumulative distribution G with support over $[\underline{\varepsilon}, \bar{\varepsilon}]$, where $E\tilde{\varepsilon} = 0$ and $\underline{\varepsilon} \geq -1$. Thus, the expected market potential, $E[1 + \tilde{\varepsilon}]$, is normalized to 1. We assume that $\tilde{\varepsilon}$ and $\tilde{\theta}$ are independent.

The two-stage dynamic Stackelberg game between the manufacturer and the retailer begins soon after the manufacturer starts his production. Figure 1 illustrates the sequence of different events associated with this two-stage game. In this game, advance booking takes place at the beginning of the first stage and regular ordering occurs at the beginning of the second stage. The first stage occurs during the 'speculation period' when both supply and demand are uncertain. In this stage, the retail price p is also uncertain because it is affected by demand and supply uncertainties. The manufacturer acts as the leader who sets his pre-book wholesale price w_1, and the retailer acts as the follower who determines her pre-book order quantity x_1. At the end of the first stage, the production yield $\tilde{\theta}$ is realized, and the manufacturer uses his actual supply $k\tilde{\theta}$ to allocate $a_1 = \min\{k\tilde{\theta}, x_1\}$ to fill the pre-book order x_1. If there is remaining supply after allocating a_1 (i.e., when $k\tilde{\theta} - a_1 > 0$), then the game proceeds to the second stage; otherwise, the game ends.

The second stage occurs during the 'reaction period' when the uncertain demand component $\tilde{\varepsilon}$ is also realized. Upon observing the remaining supply $(k\tilde{\theta} - a_1)$ and the realized demand component $\tilde{\varepsilon}$, the manufacturer acts as the leader by setting his regular wholesale price w_2, and the retailer acts as the follower by determining her regular order quantity x_2. As we show later, whether or not the

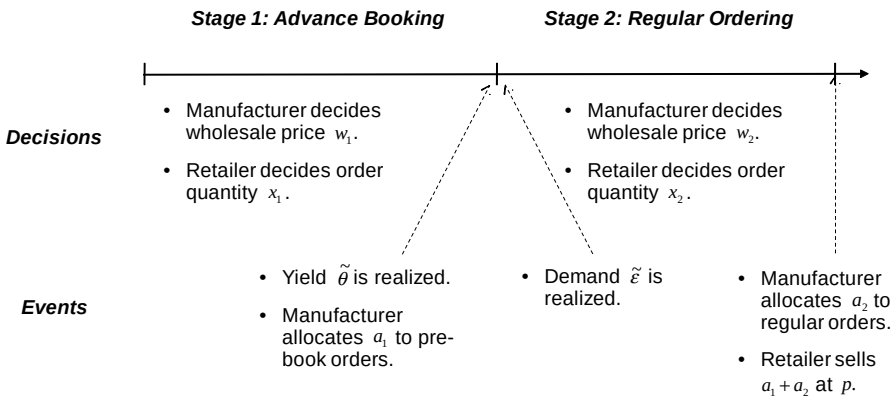

Figure 1: Timeline of Decisions and Events in the Model

actual supply $k\tilde{\theta}$ is observable by the retailer does not affect the retailer's ordering decision. At the end of the second stage, the manufacturer allocates $a_2 = \min\{k\tilde{\theta} - a_1, x_2\}$ to fill the regular order by using his remaining supply $(k\tilde{\theta} - a_1)$. In this way, the manufacturer assigns a higher priority to the pre-book order x_1 than the regular order x_2. The quantity available for sales by the retailer is $q = a_1 + a_2$, and the corresponding retailer price is $p = 1 + \tilde{\varepsilon} - (a_1 + a_2)$. Note that the price p reflects imbalance between demand $(1 + \tilde{\varepsilon})$ and supply $(a_1 + a_2)$. By discarding all unsold units at zero value at the end of the second stage, the manufacturer determines his actual profit.

4 Equilibrium Analysis

To analyze the benefits of advance booking programs in which both advance booking and regular ordering are allowed, we first analyze two benchmark systems in which either advance booking or regular ordering is used. In the first system, both the manufacturer and the retailer make their decisions *before* uncertainties in supply and uncertain are resolved, hence we shall refer to this game as the "speculative" Stackelberg game A. In the second system, the game between the two firms takes place *after* all uncertainties are resolved, hence we shall refer to this game as the "reactive" Stackelberg game B. The analysis of these two games prepares us to analyze the two-stage "dynamic" Stackelberg game AB presented in the previous section. We use backward induction to derive subgame-perfect Nash equilibrium. To denote equilibrium outcomes in each of these three games, we use the following notation: Π_i^j denote the equilibrium profit of firm i $(= M$ or $R)$ in game j $(= A, B$ or $AB)$, w_1^j and w_2^j denote the equilibrium pre-book wholesale price and regular wholesale price in game j, respectively, and x_1^j and x_2^j denote the equilibrium pre-book

order quantity and regular order quantity in game j, respectively.

4.1 Advance Booking Only: Speculative Stackelberg Game A

Consider the speculative Stackelberg game A that takes place *before* the supply and demand uncertainties are resolved. In this game, each player makes a single decision: the manufacturer acts as the leader by setting a pre-book wholesale price w_1, and the retailer acts as the follower by placing a pre-book order x_1. We identify a subgame-perfect equilibrium by using backward induction: we first find the retailer's optimal pre-book order quantity $x_1^A(w_1)$ for any given pre-book wholesale price w_1 and then find the manufacturer's optimal pre-book wholesale price w_1^A.

Suppose the retailer has received allocation a_1 at the wholesale price w_1 after yield $\tilde{\theta}$ is realized. Then she can sell this quantity at the retail price $(1 + \tilde{\varepsilon} - a_1)$ and obtain an expected profit

$$E_{\tilde{\varepsilon}}\left[\Pi_R(a_1, w_1)\right] = E_{\tilde{\varepsilon}}[\{(1 + \tilde{\varepsilon} - a_1) - w_1\}\, a_1] = (1 - a_1 - w_1)a_1. \tag{1}$$

From the first-order condition of (1), $\frac{1-w_1}{2}$ is the 'ideal' allocation of a_1 that the retailer would like to receive when $w_1 \leq 1$. However, the retailer does not decide on the allocation a_1, but she decides on the order quantity x_1, where $a_1 = \min\{k\tilde{\theta}, x_1\}$. Despite the fact that the retailer does not have direct control over a_1, we claim that it is optimal for the retailer to order $x_1 = \frac{1-w_1}{2}$. Our claim is based on the following arguments. First, suppose the retailer orders $x_1 > \frac{1-w_1}{2}$. If the actual supply $k\tilde{\theta} > \frac{1-w_1}{2}$, then she is worse off than ordering $\frac{1-w_1}{2}$ because her actual allocation is higher than $\frac{1-w_1}{2}$. Conversely, if $k\tilde{\theta} \leq \frac{1-w_1}{2}$, then her expected profit is the same as that of ordering $\frac{1-w_1}{2}$. Thus, ordering $x_1 = \frac{1-w_1}{2}$ dominates ordering $x_1 > \frac{1-w_1}{2}$. By using the same argument, we can show that the retailer cannot be better off by setting her order $x_1 < \frac{1-w_1}{2}$. Hence, we have proved the following proposition.

Proposition 1 *In the speculative game A, the retailer will set her pre-book order quantity* $x_1^A(w_1) = \frac{1-w_1}{2}$ *in equilibrium for any pre-book wholesale price* $w_1 \leq 1$.

Let $a_1^A(w_1) = \min\{k\tilde{\theta}, x_1^A(w_1)\}$ denote the allocation that the retailer will actually receive when ordering $x_1^A(w_1)$. Then, the retailer's expected profit and the manufacturer's expected profit can

be expressed respectively as follows:

$$E_{\tilde{\theta}}\left[\Pi_R(w_1)\right] = E_{\tilde{\theta}}[(1 - a_1^A(w_1) - w_1)a_1^A(w_1)], \tag{2}$$

$$E_{\tilde{\theta}}\left[\Pi_M(w_1)\right] = E_{\tilde{\theta}}[w_1 \cdot a_1^A(w_1)] = E_{\tilde{\theta}}\left[\min\left\{k\tilde{\theta}w_1, \frac{w_1 - w_1^2}{2}\right\}\right]. \tag{3}$$

By proving that $E_{\tilde{\theta}}\left[\Pi_M(w_1)\right]$ in (3) is a concave function of w_1, we show in the proof that there exists a unique optimal value w_1^A that maximizes $E_{\tilde{\theta}}\left[\Pi_M(w_1)\right]$. By substituting w_1^A into $x_1^A(w_1)$ in Proposition 1, we can also characterize the equilibrium outcome of the retailer. The following proposition summarizes the results.

Proposition 2 *The equilibrium of the speculative game A satisfies the following:*
(i) If $k\underline{\theta} \geq 0.25$*, then* $w_1^A = 0.5$*,* $E\Pi_M^A = \frac{1}{8}$*,* $x_1^A(w_1^A) = 0.25$*, and* $E\Pi_R^A = \frac{1}{16}$*;*
(ii) If $k\underline{\theta} < 0.25$*, then* $w_1^A > 0.5$*,* $E\Pi_M^A < \frac{1}{8}$*,* $x_1^A(w_1^A) < 0.25$*, and* $E\Pi_R^A < \frac{1}{16}$*.*

Proposition 2 can be interpreted as follows. First, consider case (i) when the supply chain operates as if there were no supply constraints (i.e., as if $a_1 = x_1$). In this case, the manufacturer sets his pre-book wholesale price $w_1^A = 0.5$ and the retailer pre-books $x_1^A(w_1^A) = 0.25$. Consequently, the actual allocation is $a_1^A(w_1^A) = \min\{k\tilde{\theta}, x_1^A(w_1^A)\} = 0.25$ for any $\tilde{\theta}$ and the expected retail price is $E_{\tilde{\varepsilon}}[1 + \tilde{\varepsilon} - a_1^A(w_1^A)] = 0.75$. This is the classic double marginalization result: the manufacturer's profit is twice of the retailer's profit. Next, consider case (ii) when the capacity k is lower and/or yield $\tilde{\theta}$ is more uncertain than case (i). In this case, a supply shortage can occur with the actual supply $k\tilde{\theta}$ being lower than the retailer's desired allocation of 0.25. To compensate for potential low sales volume, the manufacturer increases his pre-book wholesale price by setting $w_1^A > 0.5$. This causes the retailer to reduce her pre-book order quantity so that $x_1^A(w_1^A) < 0.25$. Because of the potential shortage, both firms obtain lower expected profits in case (ii) than in case (i).

4.2 Regular Ordering Only: Reactive Stackelberg Game *B*

Consider the reactive Stackelberg game B that takes place *after* the uncertain supply $k\tilde{\theta}$ and the uncertain demand $\tilde{\varepsilon}$ are realized. Let $l = k\tilde{\theta}$ denote the realized supply and $m = 1 + \tilde{\varepsilon}$ denote the realized market potential (or maximum retail price). In this game, each player makes a single decision: the manufacturer acts as the leader by setting the regular wholesale price w_2, and the retailer acts as the follower by selecting the regular order quantity x_2. We identify a subgame-perfect equilibrium by using backward induction as in the previous section.

Suppose the retailer can decide on the allocation a_2 for any given wholesale price w_2, and sells this allocated quantity at retail price $p = (m - a_2)$. Then the retailer's profit is $\Pi_R(a_2, w_2) = \{(m - a_2) - w_2\} a_2$. When $w_2 \leq m$, the retailer's 'ideal' allocation is $a_2 = \frac{m-w_2}{2}$. By the same argument as presented in the previous subsection, the optimal regular order quantity is $x_2^B(w_2) = \frac{m-w_2}{2}$ regardless of whether the actual supply l is observable by the retailer.[5] When the retailer orders $x_2^B(w_2)$, the retailer will receive an allocation $a_2^B(w_2) = \min\{l, x_2^B(w_2)\}$.

Anticipating the retailer's best response $x_2^B(w_2)$, the manufacturer will set his regular wholesale price w_2 that maximizes his profit:

$$\Pi_M(w_2) = w_2 \cdot a_2^B(w_2) = \begin{cases} w_2 l & \text{if } w_2 \leq m - 2l \\ w_2(\frac{m-w_2}{2}) & \text{if } m - 2l < w_2 \leq m. \end{cases} \tag{4}$$

By observing that (4) has a structure similar to (3), we can use the same approach as in the proof of Proposition 2 to establish the following results:

Proposition 3 *The equilibrium of the reactive game B satisfies the following:*
(i) If $l \geq \frac{m}{4}$, then $w_2^B = \frac{m}{2}$, $\Pi_M^B = \frac{m^2}{8}$, $x_2^B(w_2^B) = \frac{m}{4}$, and $\Pi_R^B = \frac{m^2}{16}$;
(ii) If $l < \frac{m}{4}$, then $w_2^B = m - 2l \ \left(> \frac{m}{2}\right)$, $\Pi_M^B = (m - 2l)l \ \left(< \frac{m^2}{8}\right)$, $x_2^B(w_2^B) = l \ \left(< \frac{m}{4}\right)$, and $\Pi_R^B = l^2 \ \left(< \frac{m^2}{16}\right)$.

We can interpret the above results in a similar fashion as in the previous subsection. First, when the actual supply is sufficiently high (i.e., $l = k\tilde{\theta} \geq \frac{m}{4}$) in case (i), Proposition 3 implies the double marginalization outcome in which the manufacturer earns the twice of the retailer. However, when there is a supply shortage (i.e., $l < \frac{m}{4}$) in case (ii), Proposition 3 asserts that the manufacturer will increase his regular wholesale price above $\frac{m}{2}$ so as to entice the retailer to set her regular order quantity below $\frac{m}{4}$. In this case, the profits of both firms decrease as the supply decreases.

In a later section, we shall compare the equilibrium outcomes of the reactive game B with those of the other games. To make this comparison meaningful, we need to identify the *ex-ante* equilibrium outcomes of game B. By applying Proposition 3 with the fact that $l = k\tilde{\theta}$ and $m = 1 + \tilde{\varepsilon}$, we can express the ex-ante expected wholesale price in equilibrium as:

$$E_{\tilde{\theta},\tilde{\varepsilon}}\left[w_2^B\right] = \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max\left(\frac{1+\tilde{\varepsilon}}{2}, 1 + \tilde{\varepsilon} - 2k\tilde{\theta}\right) dG(\tilde{\varepsilon}) dF(\tilde{\theta}). \tag{5}$$

[5] If the retailer can observe the actual supply l and $l \leq \frac{m-w_2}{2}$, then $\frac{m-w_2}{2}$ is not the only optimal quantity. This is because the retailer will receive l for any $x_2 \geq l$; hence, both firms will obtain the same profits.

Similarly, we can obtain the manufacturer's ex-ante expected profit $E\Pi_M^B$, the retailer's ex-ante expected regular order quantity Ex_2^B, and the retailer's ex-ante expected profit $E\Pi_R^B$ in equilibrium.

4.3 Advance Booking and Regular Ordering: Two-Stage Dynamic Game *AB*

In this section, we analyze the two-stage dynamic game AB in which the manufacturer and the retailer play the speculative Stackelberg game in the first stage *before* uncertainties in supply and demand are resolved, and then play the reactive Stackelberg game in the second stage *after* observing actual supply and demand. We first solve the second stage Stackelberg game conditioning on the realized supply and demand and the decisions chosen in the first stage, and then solve the first stage Stackelberg game by embedding the equilibrium outcomes of the second stage game into the firms' expected profits in the first stage.

Let us first consider the retailer's problem in the second stage for any given regular wholesale price w_2. In the second stage, the retailer has the information about the pre-book wholesale price w_1, the allocation $a_1 = \min\{k\tilde{\theta}, x_1\}$ associated with the pre-book order x_1, and the realized demand component $\tilde{\varepsilon}$. Notice that the games ends when there is no available supply to fill the regular order. Hence, it is sufficient for us to focus on the case when the remaining supply after the first stage $l' \equiv k\tilde{\theta} - a_1 > 0$, hence $a_1 = x_1$. Given the available information, the retailer determines her regular order quantity x_2 that maximizes her profit over both stages. If the retailer receives the allocation $a_2 = x_2$ for her order x_2, the retailer's profit over both stages satisfies

$$\begin{aligned}\Pi_R(x_2, w_2; x_1, w_1) &= \{(1 + \tilde{\varepsilon} - x_1 - x_2) - w_2\}\, x_2 + \{(1 + \tilde{\varepsilon} - x_1 - x_2) - w_1\}\, x_1 \\ &= -x_2^2 + (m' - w_2)x_2 + (1 + \tilde{\varepsilon} - x_1 - w_1)x_1,\end{aligned}$$

where $m' \equiv 1 + \tilde{\varepsilon} - 2x_1$. For any given pre-book order x_1 and the regular wholesale price w_2, we can use the same argument presented in the previous subsection to determine the retailer's optimal regular order, getting:

Corollary 1 *In the dynamic game AB, the retailer's regular order quantity in equilibrium is* $x_2^{AB}(x_1, w_2) = \max\left\{0, \frac{m'-w_2}{2}\right\}$.

By noting that $m' \equiv 1 + \tilde{\varepsilon} - 2x_1$, the retailer's regular order quantity $x_2^{AB}(x_1, w_2)$ decreases in the pre-book quantity x_1 and the regular wholesale price w_2 as expected. Once the retailer places her regular order x_2^{AB}, she will actually receive an allocation of $a_2^{AB} = \min\{l', x_2^{AB}\}$.

In anticipation of the retailer's regular order quantity $x_2^{AB}(x_1, w_2)$, the manufacturer sets his regular wholesale price w_2. In the second stage, the manufacturer has information about the realized yield $\tilde{\theta}$ and the realized demand $\tilde{\varepsilon}$ (hence l' and m') as well as the pre-book decisions (w_1, x_1) made in the first stage. By noting that a_2^{AB} takes on different values depending on the value of m', l' and w_2, we can express the manufacturer's profit over both stages as a function of w_2 as follows:

$$\Pi_M(w_2; x_1, w_1, m', l') = w_2 a_2^{AB} + w_1 a_1 = \begin{cases} w_1 x_1 & \text{if } m' \leq 0 \text{ or } w_2 > m' \\ w_2 l' + w_1 x_1 & \text{if } m' > 0 \text{ and } w_2 \leq m' - 2l' \\ w_2 \frac{m' - w_2}{2} + w_1 x_1 & \text{if } m' > 0 \text{ and } m' - 2l' < w_2 \leq m'. \end{cases} \tag{6}$$

Notice that (6) has the structure similar to (4). As such, using Proposition 3, we can derive the following results:

Corollary 2 *The equilibrium of the dynamic game AB satisfies the following:*
(i) If $m' > 0$ *and* $l' \geq \frac{m'}{4}$, *then* $w_2^{AB}(x_1) = \frac{m'}{2}$, $\Pi_M(x_1, w_1) = \frac{m'^2}{8} + w_1 x_1$, $x_2^{AB}(x_1) = \frac{m'}{4}$, *and* $\Pi_R(x_1, w_1) = \frac{m'^2}{16} + (1 + \tilde{\varepsilon} - x_1 - w_1)x_1$. *The manufacturer has leftovers of* $l' - \frac{m'}{4}$ *at the end of the season.*
(ii) If $m' > 0$ *and* $l' < \frac{m'}{4}$, *then* $w_2^{AB}(x_1) = m' - 2l' \left(> \frac{m'}{2}\right)$, $\Pi_M(x_1, w_1) = (m' - 2l')l' + w_1 x_1 \left(< \frac{m'^2}{8} + w_1 x_1\right)$, $x_2^{AB}(x_1) = l' \left(< \frac{m'}{4}\right)$, *and* $\Pi_R(x_1, w_1) = l'^2 + (1 + \tilde{\varepsilon} - x_1 - w_1)x_1$ $(< \frac{m'^2}{16} + (1 + \tilde{\varepsilon} - x_1 - w_1)x_1)$. *The manufacturer has no leftovers at the end of the season.*
(iii) If $m' \leq 0$, *then* w_2^{AB} *can be any positive value;* $\Pi_M(x_1, w_1) = w_1 x_1$, $x_2^{AB}(x_1) = 0$, *and* $\Pi_R(x_1, w_1) = (1 + \tilde{\varepsilon} - x_1 - w_1)x_1$. *The manufacturer has leftovers of* l' *at the end of the season.*

The above results resemble Proposition 3 in the reactive game B. The difference is that the market potential $m = 1 + \tilde{\varepsilon}$ is replaced with $m' = 1 + \tilde{\varepsilon} - 2x_1$ and the available supply $l = k\tilde{\theta}$ is replaced with $l' = k\tilde{\theta} - x_1$. This is because the pre-book order is filled prior to the regular order. In addition, if $m' \leq 0$, Corollary 1 implies that $x_2^{AB} = 0$ for any positive w_2 ; i.e., the retailer orders nothing in the second stage when she has pre-booked and received too many units in the first stage in relation to the realized demand.

Given the equilibrium outcomes of the second stage of the game, let us examine the retailer's problem in the first stage of the game. This takes place *before* uncertainties in supply and demand are resolved. As shown in Corollary 2, the retailer's profit in equilibrium depends on m' and l', which in turn depend on random $\tilde{\theta}$ and $\tilde{\varepsilon}$. Recall that the second stage is reached only when the manufacturer has some remaining supply $l' = k\tilde{\theta} - a_1 > 0$ to fill the retailer's regular order. When

$l' \leq 0$, the retailer receives an allocation of $a_1 = k\tilde{\theta}$, the game ends after the first stage, and the retailer earns a profit of $\{(1+\tilde{\varepsilon}-k\tilde{\theta})-w_1\}k\tilde{\theta}$. Combining this observation with the results stated in Corollary 2 for the case when $l' > 0$, we can compute the expected profit of the retailer as a function of (x_1, w_1) as follows:

$$\begin{aligned} E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1,w_1)\right] &= E_{\tilde{\theta},\tilde{\varepsilon}}\left[\{(1+\tilde{\varepsilon}-a_1)-w_1\}a_1\right] + E_{\tilde{\theta},\tilde{\varepsilon}}\left[\frac{m'^2}{16}|m'>0, l' \geq \frac{m'}{4}\right] \\ &\quad + E_{\tilde{\theta},\tilde{\varepsilon}}\left[l'^2|m'>0, 0<l'<\frac{m'}{4}\right]. \end{aligned} \tag{7}$$

Similarly, by utilizing the results stated in Corollary 2, we can compute the manufacturer's expected profit as a function of (x_1, w_1) as follows:

$$\begin{aligned} &E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_M(x_1,w_1)\right] \\ &\quad = E_{\tilde{\theta},\tilde{\varepsilon}}\left[w_1 a_1\right] + E_{\tilde{\theta},\tilde{\varepsilon}}\left[\frac{m'^2}{8}|m'>0, l' \geq \frac{m'}{4}\right] + E_{\tilde{\theta},\tilde{\varepsilon}}\left[(m'-2l')l'|m'>0, 0<l'<\frac{m'}{4}\right]. \end{aligned} \tag{8}$$

One can show that the retailer's expected profit $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1,w_1)\right]$ given in (7) is a piecewise-continuous function of x_1 over the closed interval $[0, \frac{1+\bar{\varepsilon}}{2}]$ for any given w_1. Hence, we can compute the retailer's optimal pre-book order quantity $x_1^{AB}(w_1)$. By substituting $x_1^{AB}(w_1)$ into $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_M(x_1,w_1)\right]$ in (8), we obtain $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_M(x_1^{AB}(w_1),w_1)\right]$. It is also possible to compute the manufacturer's optimal pre-book wholesale price w_1^{AB} by conducting a search over the closed interval $[0, 1+\bar{\varepsilon}]$. In a later section, we shall present an efficient computational procedure for determining $x_1^{AB}(w_1)$ and w_1^{AB} when the yield $\tilde{\theta}$ and the demand $\tilde{\varepsilon}$ follow specific probability distributions.

For general probability distributions of $\tilde{\theta}$ and $\tilde{\varepsilon}$, there are no closed-form expressions for $x_1^{AB}(w_1)$ and w_1^{AB}. Even so, we are able to make analytical comparisons between the equilibrium outcomes associated with the dynamic game AB and those of the speculative game A and the reactive game B in the next section.

5 Comparisons of Equilibrium Outcomes in Games A, B and AB

In the last section, we have determined the equilibrium outcomes associated with three separate games: the speculative game A, the reactive game B, and the dynamic game AB that involves both games A and B sequentially. In section 5.1, we first show how the equilibrium outcomes capture the trade-offs that each firm faces. Then we compare the equilibrium outcomes of game

AB with those of games A and B. While our main results are proved in section 5.2 under general probability distributions of yield $\tilde{\theta}$ and demand $\tilde{\varepsilon}$, we conduct our numerical study in section 5.3 by assuming their specific probability distributions. These comparisons will enable us to answer our first research question regarding the benefits of advance booking programs.

5.1 Firms' Trade-offs in Equilibrium Outcomes

Suppose the retailer pre-books nothing and places only a regular order as in the reactive game B. Then she avoids the risk of overstocking but bears the risk of a higher wholesale price when there is a supply shortage. This trade-off is formally captured in the equilibrium wholesale prices as stated in Proposition 3. Let us consider the case when the actual supply is low relative to the realized demand (i.e., $k\tilde{\theta} = l < \frac{m}{4} = \frac{1+\tilde{\varepsilon}}{4}$). Statement (ii) of Proposition 3 states that the regular wholesale price $w_2^B = m - 2l = 1 + \tilde{\varepsilon} - 2k\tilde{\theta}$ $(> 0.5 + 0.5\tilde{\varepsilon})$, which is *ex-ante* uncertain and increases as the actual yield $\tilde{\theta}$ decreases or the realized demand $\tilde{\varepsilon}$ increases. On the other hand, the pre-book wholesale price w_1 is known to the retailer in the speculative game A or the dynamic game AB. For example, in game A, Proposition 2 reveals that the pre-book wholesale price w_1^A is 0.5 or a constant that is greater than 0.5. Thus, $w_2^B > w_1^A$ when the supply turns out to be low relatively to the demand. This confirms that the retailer bears the price risk when she pre-books nothing.

Next, suppose the retailer only pre-books without placing a regular order as in the speculative game A. Then she avoids the price risk by paying the known pre-book wholesale price w_1^A but she bears the risk of overstocking, which leads to a lower retail price p. This trade-off is also captured in the equilibrium outcomes. Consider the speculative game A as follows. Statement (i) of Proposition 2 shows that the retailer pre-books $x_1^A = 0.25$ when $k\underline{\theta} \geq 0.25$. However, if the retailer knew that the demand is going to be low, she would have pre-booked less than 0.25. For example, in the reactive game B, statement (i) of Proposition 3 shows that, after observing the demand $\tilde{\varepsilon}$, the retailer would order $x_2^B = \frac{m}{4} = 0.25(1 + \tilde{\varepsilon}) < x_1^A$ when $\tilde{\varepsilon} < 0$ and $k\tilde{\theta} \geq \frac{1+\tilde{\varepsilon}}{4}$. Hence, the retailer bears the risk of overstocking when she only pre-books.

Using the same approach, one can utilize the results presented in Propositions 2 and 3 to highlight the trade-offs the manufacturer faces when deciding his pre-book wholesale price and regular wholesale price. To avoid repetition, we omit the details.

5.2 Analytical Comparisons of Equilibrium Outcomes

We now compare the equilibrium outcomes of the dynamic game AB presented in Section 4.3 with those of the reactive game B presented in Section 4.2. Even though there are no closed-form expressions for the manufacturer's optimal pre-book wholesale price w_1^{AB} and for the retailer's optimal pre-book order quantity $x_1^{AB}(w_1)$, we are able to establish the following results:

Theorem 1 *(a) In equilibrium, the manufacturer's and the retailer's expected profits in the dynamic game AB are greater than or equal to their respective ex-ante expected profits in the reactive game B; i.e., $E\Pi_M^{AB} \geq E\Pi_M^B$ and $E\Pi_R^{AB} \geq E\Pi_R^B$, where the equalities hold when $x_1^{AB} = 0$.*
(b) In equilibrium, the regular wholesale price in the dynamic game AB is lower than or equal to that in the reactive game B; i.e., $w_2^{AB} \leq w_2^B$ for any realized yield $\tilde{\theta}$ and demand $\tilde{\varepsilon}$.
(c) In equilibrium, the total order quantity in the dynamic game AB is larger than or equal to that in the reactive game B, while the regular order quantity in the dynamic game AB is smaller than or equal to that in the reactive game B; i.e., $x_2^{AB} \leq x_2^B \leq x_1^{AB} + x_2^{AB}$ for any realized yield $\tilde{\theta}$ and demand $\tilde{\varepsilon}$.

Relative to the case when the manufacturer allows the retailer to place only a regular order as in the reactive game B, Theorem 1(a) shows that both the manufacturer and the retailer will obtain higher expected profits when both pre-book and regular orders are allowed as in the dynamic game AB. To explain this result intuitively, notice that game B is a special case of game AB in which the retailer pre-books nothing, i.e., $x_1 = 0$, so the retailer has an additional option to place a pre-book order x_1 in game AB. Thus, for any given pre-book wholesale price w_1, the retailer can do at least as well in game AB as in game B by setting her pre-book quantity $x_1^{AB}(w_1) = 0$. This additional option enables the retailer to obtain a higher expected profit in game AB. Similarly, the manufacturer has an option to set his pre-book wholesale price w_1 in game AB in addition to his regular wholesale price w_2 in game B. Because the manufacturer as a Stackelberg leader can always induce the retailer to choose $x_1^{AB}(w_1) = 0$ by setting w_1 sufficiently high, the manufacturer also obtains a higher expected profit in game AB. Therefore, relative to the case when only regular ordering is allowed, the manufacturer can create a win-win situation by offering advance booking in addition to regular ordering. By mitigating the demand, supply and price risks, the advance booking program improves the overall supply chain performance.

Theorem 1(b) states that the manufacturer charges a lower regular wholesale price in equilibrium under the dynamic game AB than under the reactive game B. To see this, observe from

Corollary 2(i) that the regular wholesale price $w_2^{AB}(x_1)$ in game AB decreases in the pre-book quantity x_1 when the actual supply is sufficiently large. Combining this observation with the fact that the regular wholesale price w_2^B in game B is equal to $w_2^{AB}(0)$ in game AB for the case when $x_1 = 0$, we obtain the result as stated in Theorem 1(b). This reveals two benefits of the advance booking program to the retailer: a known pre-book wholesale price as opposed to an uncertain regular wholesale price, and a potentially lower regular wholesale price. These benefits provide an intuitive explanation about why the retailer can obtain a higher profit when the manufacturer offers advance booking as well as regular ordering.

In addition, Theorem 1(c) asserts that the manufacturer can entice the retailer to order more in total by offering advance booking in addition to regular ordering. Since the expected supply is constant ($= k$), this implies that the expected leftovers after the selling season will be reduced. Hence, advance booking is beneficial to the manufacturer.

Next, by comparing the equilibrium outcomes of the dynamic game AB presented in Section 4.3 with those of the speculative game A presented in Section 4.1, we establish the following results:

Theorem 2 *(a) In equilibrium, the manufacturer's expected profit in the dynamic game AB is greater than or equal to that in the speculative game A; i.e., $E\Pi_M^{AB} \geq E\Pi_M^A$ where the equality holds when $x_1^{AB} \geq \min\left\{k\bar{\theta}, \frac{1+\bar{\varepsilon}}{2}\right\}$.*
(b) For any given pre-book wholesale price w_1, the retailer's pre-book quantity in the dynamic game AB is smaller than or equal to that in the speculative game A; i.e., $x_1^{AB}(w_1) \leq x_1^A(w_1)$ $\forall w_1$. Furthermore, if $w_1^{AB} \geq w_1^A$, then $x_1^{AB}(w_1^{AB}) \leq x_1^A(w_1^A)$.

Relative to the case when the manufacturer allows only a pre-book order from the retailer as in the speculative game A, Theorem 2(a) shows that the manufacturer obtains a higher expected profit in the dynamic game AB. In view of the result stated in Theorem 1(a), it suffices to prove that $E\Pi_M^B \geq E\Pi_M^A$. Our proof is based on two observations. First, in game B, the manufacturer can utilize information about the realized demand and supply to set the regular wholesale price accordingly. Better information enables the manufacturer to extract more surplus from the retailer. Second, observe from Proposition 3 that the manufacturer's *ex-post* profit Π_M^B in game B is increasing and convex in uncertain demand component $\tilde{\varepsilon}$. By using Jensen's inequality, it is easy to check that the manufacturer will obtain higher *ex-ante* expected profit in game B as demand uncertainty increases (see Appendix A1 for details). Combining these two observations with Theorem 1(a), we can conclude that $E\Pi_M^{AB} \geq E\Pi_M^B \geq E\Pi_M^A$. Next, consider the case when the retailer's pre-book

quantity x_1^{AB} is sufficiently high (i.e., $x_1^{AB} \geq \min\left\{k\bar{\theta}, \frac{1+\bar{\varepsilon}}{2}\right\}$). In this case, the retailer will never exercise the additional option of placing a regular order. Therefore, both firms obtain the same expected profits in both games AB and A.

The result in Theorem 2(b) is intuitive because the retailer has an additional opportunity to place a regular order in the dynamic game AB after observing demand and supply. Since the second result in Theorem 2(b) relies on the condition $w_1^{AB} \geq w_1^A$, we shall investigate this condition numerically in the next section.

5.3 Numerical Comparisons of Equilibrium Outcomes

To complement our analytical findings presented earlier, we conduct extensive numerical experiments. For our numerical study, we assume that $\tilde{\theta}$ is uniformly distributed between $1-r$ and $1+r$, where $r \in (0,1]$ represents the degree of supply uncertainty, and that $\tilde{\varepsilon} = e$ (representing high demand) with probability 0.5 and $\tilde{\varepsilon} = -e$ (representing low demand) with probability 0.5, where $e \in (0,1]$ represents the degree of demand uncertainty[6]. We have constructed 810 scenarios by varying capacity k from 0.1 to 1 and by varying both the level of demand uncertainty e and the level of supply uncertainty r from 0.1 to 0.9 with an increment of 0.1. For each scenario of (k, e, r), we compute equilibrium outcomes in games A, B and AB. (See Appendix A2 for details about the computational method.) By comparing these equilibrium outcomes, we have verified the results stated in Theorem 1 regarding game AB versus game B. In addition, we draw the following observations regarding game AB versus game A that augment the results stated in Theorem 2. To illustrate, we present the results of 27 scenarios in Table 1.

Observation 1 *(a) In equilibrium, the pre-book wholesale price in the dynamic game AB is higher than or equal to that in the speculative game A, i.e., $w_1^{AB} \geq w_1^A$.*
(b) In equilibrium, the retailer's expected profit in the dynamic game AB can be greater or smaller than that in the speculative game A, i.e., $E\Pi_R^{AB} - E\Pi_R^A$ can be either positive or negative.
(c) In equilibrium, the expected total order quantity in the dynamic game AB can be larger or smaller than that in the speculative game A; i.e., $[(x_1^{AB} + Ex_2^{AB}) - x_1^A]$ can be either positive or

[6] We have chosen simple probability distributions that reasonably approximate the flu vaccine supply chain in the following sense. The uniform distribution of the yield reflects the fact that a wide range of yields are equally plausible in producing flu vaccines (see Cho (2009) and references therein). Due to the fact that a flu is either contained locally or spreads across regions, a flu season is typically either mild (low demand) or severe (high demand). The binary demand distribution is also used for tractability in the literature (e.g., Erhun et al. (2008), Anand and Mendelson (2009)).

negative.

Observation 1(a) verifies the condition stated in Theorem 2(b); hence, $x_1^{AB} \leq x_1^A$. Because the retailer pre-books less in game AB, one may suspect the manufacturer may become worse-off by giving the retailer the additional option. However, in the dynamic game AB, the manufacturer also has an additional option to set his regular wholesale price after uncertainties in supply and demand are resolved but prior to receiving a regular order from the retailer. Due to this option of postponing his pricing decision, the manufacturer can afford to set a higher pre-book wholesale price in game AB so that $w_1^{AB} \geq w_1^A$ as shown in Observation 1(a). Also, our numerical result suggests that $w_1^{AB} \geq Ew_2^{AB}$ (omitted). Hence, when there are two chances to set the wholesale price as in the dynamic game AB, it is optimal for the manufacturer to set a higher wholesale price in the first stage, and then switch to a lower price (in expectation) in the second stage. This result is not uncommon due to strategic interactions between two parties over time. For instance, Donohue (2000) shows that the profit margin in the first stage must be set higher than that in the second stage to coordinate a two-level supply chain.

In contrast to the result stated in Theorem 1(a), Observation 1(b) reveals that the retailer can be worse off in the dynamic game AB than in the speculative game A (i.e., $E\Pi_R^{AB} < E\Pi_R^A$) especially when the capacity k is sufficiently low, say $k = 0.1$. When $k = 0.1$, the last column of Table 1 reveals that the remaining supply is likely to be unavailable (i.e., $l' \leq 0$) or low relative to the desired allocation associated with the regular order (i.e., $0 < l' < \frac{m'}{4}$). When the remaining supply is sufficiently low, the regular wholesale price can be much higher than the pre-book wholesale price. For example, consider the case when $(k, e, r) = (0.1, 0.5, 0.5)$, one can apply Corollary 2(ii) to show that the highest possible value of w_2^{AB} occurs when $\tilde{\varepsilon} = e$ (high demand) and $\tilde{\theta} = 1 - r$ (low yield) so that $w_2^{AB}(= 1.4) > w_1^{AB}(= 0.81) > w_1^A(= 0.75)$. In addition, Table 1 shows that the expected total order quantity is smaller in game AB when $k = 0.1$ so that $(x_1^{AB} + Ex_2^{AB}) < x_1^A$ (Observation 1(c)). Hence, when the capacity is sufficiently low, the retailer can be better off under the speculative game A by pre-booking a larger quantity at a lower price than under the dynamic game AB. In short, 'speculation' could pay off for the retailer when the risk of a supply shortage is high!

Table 1. Numerical Comparisons of Equilibrium Outcomes between Game AB and Game A.

Parameters			Equilibrium Outcome Comparisons					
k	e	r	$E\Pi_M^{AB}-E\Pi_M^A$	$E\Pi_R^{AB}-E\Pi_R^A$	$w_1^{AB}-w_1^A$	$x_1^{AB}-x_1^A$	$x_1^{AB}+Ex_2^{AB}-x_1^A$	Prob*
0.1	0.1	0.1	0.001	-0.001	0.010	-0.105	-0.005	1.000
0.1	0.1	0.5	0.006	-0.003	0.049	-0.124	-0.024	1.000
0.1	0.1	0.9	0.008	-0.003	0.086	-0.143	-0.043	1.000
0.1	0.5	0.1	0.001	-0.001	0.010	-0.105	-0.005	1.000
0.1	0.5	0.5	0.006	-0.004	0.058	-0.102	-0.025	0.930
0.1	0.5	0.9	0.008	-0.005	0.110	-0.128	-0.047	0.840
0.1	0.9	0.1	0.007	-0.006	0.088	-0.092	-0.039	0.500
0.1	0.9	0.5	0.012	-0.008	0.127	-0.112	-0.059	0.500
0.1	0.9	0.9	0.016	-0.009	0.166	-0.133	-0.078	0.556
0.4	0.1	0.1	0.017	0.012	0.062	-0.125	0.063	0.000
0.4	0.1	0.5	0.013	0.013	0.043	-0.132	0.051	0.258
0.4	0.1	0.9	0.015	0.009	0.069	-0.137	0.024	0.354
0.4	0.5	0.1	0.037	0.019	0.080	-0.192	0.023	0.275
0.4	0.5	0.5	0.035	0.015	0.063	-0.149	0.026	0.274
0.4	0.5	0.9	0.034	0.013	0.085	-0.148	0.010	0.346
0.4	0.9	0.1	0.095	0.018	0.078	-0.237	-0.034	0.500
0.4	0.9	0.5	0.087	0.022	0.099	-0.191	-0.020	0.373
0.4	0.9	0.9	0.081	0.023	0.128	-0.177	-0.025	0.326
0.7	0.1	0.1	0.017	0.012	0.062	-0.125	0.063	0.000
0.7	0.1	0.5	0.017	0.012	0.062	-0.125	0.063	0.000
0.7	0.1	0.9	0.016	0.011	0.066	-0.131	0.044	0.185
0.7	0.5	0.1	0.047	0.027	0.062	-0.125	0.063	0.000
0.7	0.5	0.5	0.045	0.026	0.065	-0.136	0.055	0.059
0.7	0.5	0.9	0.040	0.020	0.075	-0.137	0.035	0.183
0.7	0.9	0.1	0.114	0.058	0.095	-0.154	0.059	0.000
0.7	0.9	0.5	0.109	0.051	0.092	-0.171	0.037	0.118
0.7	0.9	0.9	0.097	0.039	0.116	-0.168	0.013	0.175

*Prob represents $\Pr\{l' \leq 0 \text{ or } 0 < l' \leq \frac{m'}{4}\}$ in the dynamic game AB.

6 Comparative Statics

We now examine our second research question: what are the effects of capacity, demand uncertainty, and supply uncertainty on the manufacturer's pricing decisions, the retailer's ordering decisions, and the resulting profits? As it turns out, these factors have several effects that are not necessarily unidirectional. We take a numerical approach to explore the dominant effects of these factors and to compare comparative statics across all three games A, B and AB. (In Appendix A3, we analyze comparative statics associated with games A and B using stochastic ordering relations under general probability distributions of yield and demand.)

By using the same set of parameter values for (k, r, e) as presented in Section 5.3, we obtain the results as reported in Table 2. We examine the impact of each factor as follows. For capacity k, we compute the difference in the equilibrium outcomes associated with the adjacent values of k

for any given (r, e). Because there are 81 possible pairs of (r, e) and 9 increments of k, there are 729 scenarios for which we can examine whether the equilibrium outcome increases or decreases with an increase of k. For example, the first three entries in the first row of Table 2 characterize the impact of capacity k. As k increases, the pre-book quantity x_1^A in game A has increased in 432 scenarios, decreased in 0 scenario, and unchanged in 297 scenarios (out of 729 scenarios). Similarly, we examine the impact of supply uncertainty r and demand uncertainty e in 720 scenarios. We now use our results presented in Table 2 to discuss the impact of k, r, and e on the equilibrium outcomes across all three games.

Table 2. Comparative Statics

		k			r			e		
		$\uparrow$	$\downarrow$	$\cdot$	$\uparrow$	$\downarrow$	$\cdot$	$\uparrow$	$\downarrow$	$\cdot$
Game A	x_1^A	432	0	297	126	306	288	0	0	720
	w_1^A	0	432	297	306	126	288	0	0	720
	$E\Pi_R^A$	432	0	297	72	360	288	0	0	720
	$E\Pi_M^A$	432	0	297	0	432	288	0	0	720
Game B	Ex_2^B	542	0	187	0	460	260	0	494	226
	Ew_2^B	0	542	187	460	0	260	494	0	226
	$E\Pi_R^B$	542	0	187	148	389	183	526	171	23
	$E\Pi_M^B$	542	0	187	0	537	183	697	0	23
Game AB	x_1^{AB}	550	9	170	221	331	168	76	519	125
	Ex_2^{AB}	521	70	138	132	423	165	179	428	113
	$x_1^{AB}+Ex_2^{AB}$	591	0	138	41	512	167	41	565	114
	w_1^{AB}	63	512	154	431	111	178	502	78	140
	Ew_2^{AB}	0	591	138	470	83	167	583	23	114
	$E\Pi_R^{AB}$	591	0	138	179	441	100	514	183	23
	$E\Pi_M^{AB}$	591	0	138	5	582	133	697	0	23

6.1 Comparative Statics: Advance Booking Only in Game A

In the speculative game A, Table 2 shows that, as capacity k increases, the expected profits of both firms increased in 432 scenarios out of 729 scenarios. These scenarios correspond to the case when $k\underline{\theta} = k(1-r) < 0.25$ in Proposition 2(ii). In this case, there is a positive probability that supply will be lower than the retailer's 'ideal' allocation. Proposition 4 in Appendix A3 asserts that, as capacity k increases, the manufacturer would push the product to the retailer by lowering his pre-book wholesale price. Responding to a lower wholesale price, the retailer would increase her ordering quantity as shown in Proposition 1. As a result, both firms earn more expected profits. The remaining 297 scenarios correspond to the case when $k(1-r) \geq 0.25$. Since there is always enough supply in this case, the equilibrium outcomes do not change with an increase of capacity.

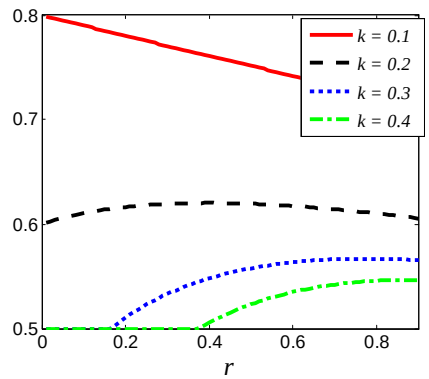

Figure 2: Pre-book Wholesale Price w_1^A in the Speculative Game A

Table 2 reveals that supply uncertainty r will always hurt the manufacturer's expected profit $E\Pi_M^A$. This is intuitive because as r increases, the manufacturer bears more risk in game A where the manufacturer receives all orders before observing uncertain demand and supply. Interestingly, however, supply uncertainty does not always hurt the retailer's expected profit $E\Pi_R^A$. To see why this is the case, let us first examine the impact of r on the pre-book wholesale price w_1^A. Observe from Proposition 2 that if $k(1-r) \geq 0.25$, $w_1^A = 0.5$; otherwise, w_1^A is varying with r. It turns out that w_1^A is not monotonic in r, as illustrated in Figure 2. When the minimum supply $k(1-r)$ is lower than but close to 0.25 (e.g., $k = 0.3$ or 0.4, and $0.4 \leq r \leq 0.6$), the supply risk is low. In this case, as the supply risk increases with an increase of r, a supply shortage is more likely to occur. To compensate for potential low sales volume, the manufacturer would increase his pre-book wholesale price. In contrast, when $k(1-r)$ is much lower than 0.25 (e.g., $k = 0.1$ or 0.2, and $0.6 \leq r \leq 0.9$), the supply risk is high. As the supply risk becomes more eminent with an increase in r, the manufacturer would reduce his pre-book wholesale price w_1^A in order to secure more pre-book orders from the retailer. Since the retailer would acquire a higher pre-book order quantity at a lower price on average, a higher level of supply uncertainty r could improve the retailer's expected profit[7].

Demand uncertainty e does not affect the equilibrium outcomes of game A. This is because the retailer maximizes her expected profit which does not depend on the level of demand uncertainty e as shown in (1).

[7] In Table 2, there are 72 scenarios in which $E\Pi_R^A$ is increasing in r, although there are 126 scenarios in which w_1^A is decreasing in r and x_1^A is increasing in r. This difference is simply due to the fact that $E\Pi_R^A$ is neither monotonic in w_1^A nor in x_1^A.

6.2 Comparative Statics: Regular Ordering Only in Game B

In the reactive game B, both firms make their decisions after observing the realized demand and supply. Thus, we examine the comparative statics of the *ex-ante* equilibrium outcomes. Appendix A2 presents the closed-form expressions of the *ex-ante* equilibrium outcomes and the signs of their first derivatives with respect to r, e and k under various conditions. Based on these detailed analyses, we next discuss the results summarized in Table 2.

Table 2 shows that the effect of capacity k in the reactive game B is the same as that in the speculative game A, except that there are 187 scenarios when k has no impact on the equilibrium outcomes as compared to 297 scenarios in game A. Such scenarios in game B correspond to the case when the *ex-ante* probability of a supply shortage is zero, i.e., $\Pr\{l \geq m/4\} = \Pr\{k\tilde{\theta} \geq \frac{1+\tilde{\varepsilon}}{4}\} = 0$ (see Proposition 3).

The effect of supply uncertainty r in the reactive game B is also similar to that in the speculative game A. In both games, supply uncertainty will always hurt the manufacturer but not the retailer. However, unlike in game A, the regular wholesale price Ew_2^B as well as the regular order quantity Ex_2^B change monotonically in game B. (Recall from Section 6.1 that the non-monotonicity of w_1^A and x_1^A in game A is due to the fact that when facing high supply risk, the manufacturer reduces his pre-book wholesale price w_1^A in order to induce the retailer to pre-book more.) Unlike in game A, each firm chooses an action that maximizes his/her *ex post* profit in game B after uncertainties are resolved. Thus, the supply risk in game A that leads the manufacturer to reduce his pre-book wholesale price with an increase of r does not exist in game B.[8]

The effect of demand uncertainty e on the ex-ante expected profit of the manufacturer $E\Pi_M^B$ is opposite to that of supply uncertainty r. Specifically, Table 2 suggests that supply uncertainty r hurts the manufacturer, whereas demand uncertainty e benefits the manufacturer. On the surface, it appears that both types of uncertainties should affect the manufacturer's expected profit negatively. To investigate this counter-intuitive result, let us examine the manufacturer's *ex post* profit Π_M^B presented in Proposition 3. As illustrated in Figure 3(a), Π_M^B is nondecreasing and concave in the yield $\tilde{\theta}$. This is because the marginal benefit of yield $\tilde{\theta}$ is positive but decreasing in the interval I_1 (in which supply is short relative to the desired quantity) because additional units will be sold to customers with low reservation prices; whereas it is zero in the interval I_2 (in which supply

[8] Note in Table 2 that $E\Pi_R^B$ and $E\Pi_M^B$ remain unchanged in 183 scenarios, while Ex_2^B and Ew_2^B remain unchanged in 260 scenarios. This is because a change in r can affect the *ex-ante* expected profits even when it does not alter any firm's decision. For example, when a supply shortage occurs with probability 1, Appendix A2 shows that $E\Pi_R^B$ increases in r and $E\Pi_M^B$ decreases in r although Ex_2^B and Ew_2^B do not change.

is sufficient relative to the desired quantity) because additional units will not be sold (see Figure 3(c)). On the other hand, Figure 3(b) shows that Π_M^B is strictly increasing and convex in demand $\tilde{\varepsilon}$. This is because the marginal benefit of demand $\tilde{\varepsilon}$ is increasing in the interval I_2 because additional units will be sold to customers at a higher price; whereas it is a positive constant in the interval I_1 because no additional units will be sold but units will be sold at a higher price (see Figure 3(d)). Jensen's inequality implies that the manufacturer's *ex-ante* expected profit $E\Pi_M^B$ increases with demand uncertainty e due to the convexity of Π_M^B in $\tilde{\varepsilon}$, and that $E\Pi_M^B$ decreases with supply uncertainty r due to the concavity of Π_M^B in $\tilde{\theta}$. The former result is consistent with Van Mieghem and Dada (1999), who show that the benefit of price postponement increases as demand uncertainty increases. Our results extend Van Mieghem and Dada (1999) by showing that the benefit of price postponement decreases as supply uncertainty increases.

Table 2 reveals that the effect of demand uncertainty e and supply uncertainty r on the *ex-ante* expected profit of the retailer $E\Pi_R^B$ is not unidirectional. We can see this from Proposition 3, which shows the retailer's *ex post* profit Π_R^B is neither convex nor concave in $\tilde{\theta}$ or $\tilde{\varepsilon}$. Note that the retailer's ex post profit remains constant as demand or supply increases beyond a certain point, whereas the manufacturer can always take advantage of a higher demand by extracting more consumers' surplus (see Figure 3(b)). Because the manufacturer has the pricing power as the Stackelberg leader, the retailer does not share the extra surplus.

6.3 Comparative Statics: Advance Booking and Regular Ordering in Game *AB*

In the dynamic game AB, both firms make their decisions before *and* after supply and demand uncertainties are resolved. We observe from Table 2 that most equilibrium outcomes do not change monotonically with a change of any parameter. This is primarily due to the non-monotonicity of the pre-book wholesale price w_1^{AB} with a change of any parameter value. (To illustrate, we provide numerical examples in Appendix A4.)

Despite the prevalent non-monotonicity in the comparative statics of game AB, Table 2 reveals that the general patterns observed in game AB are quite similar to those in games A and B. Specifically, we observe the following dominant effects of each factor:

(a) As capacity k increases, the manufacturer tends to reduce his (pre-book and regular) wholesale prices, the retailer tends to increase her (pre-book and regular) order quantities, and both firms tend to obtain higher expected profits.

(b) As supply uncertainty r increases, the manufacturer tends to increase his wholesale prices, the

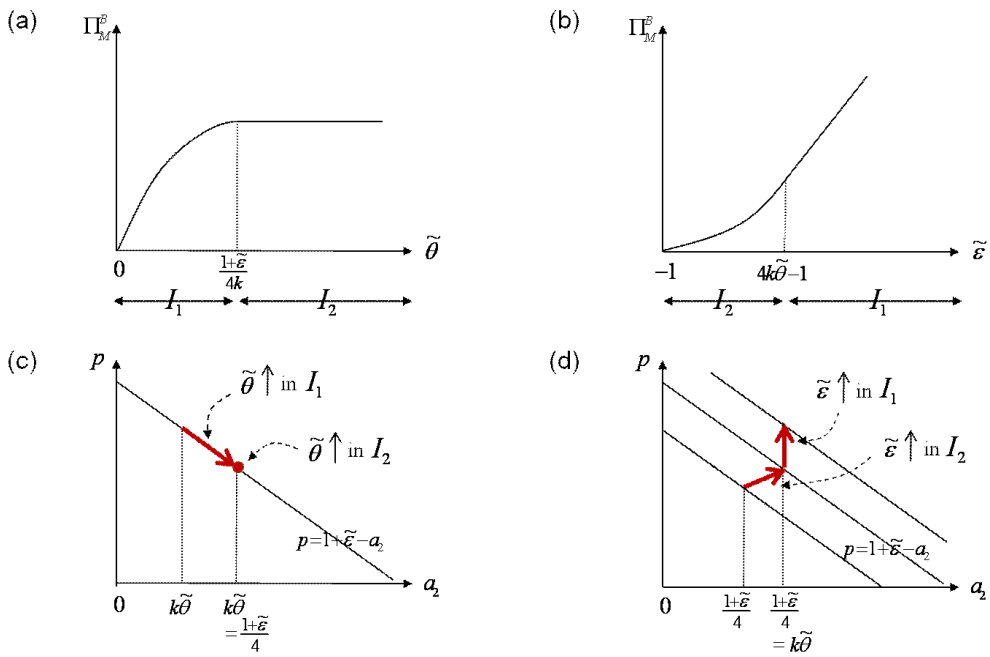

Figure 3: (a) Π_M^B with Respect to Yield $\tilde{\theta}$; (b) Π_M^B with Respect to Demand Component $\tilde{\varepsilon}$; (c) Demand Curve when Yield $\tilde{\theta}$ Increases; (d) Demand Curves when Demand Component $\tilde{\varepsilon}$ Increases

retailer tends to reduce her order quantities, and both firms tend to obtain lower expected profits. (c) As demand uncertainty e increases, the manufacturer tends to increase his wholesale prices, the retailer tends to reduce her order quantities, and both firms tend to obtain higher expected profits.

7 Extension to Multiple Retailers

So far, our analysis is based on a supply chain with a single retailer who faces the demand, supply, and price risks. In a supply chain with multiple retailers, each retailer has to compete for potentially scarce supply with the other retailers. In addition to demand, supply and price risks, each retailer faces an additional 'allocation risk' which arises when there is insufficient supply to fulfill the orders placed by the retailers. Without pre-booking some units, a retailer may not get her regular order filled because the other retailers' pre-book orders could have taken up the entire supply. Therefore, our intuition suggests that a retailer would pre-book more in the multiple-retailer supply chain than in the one-retailer supply chain. Below we formalize this intuitive argument by extending the results presented in Cachon and Lariviere (1999a&b) for a one-stage setting to a two-stage setting in our model.

Before the game begins, suppose that the manufacturer announces the mechanism for allocating

his supply in the event when the sum of all retailer orders in each stage exceeds the available supply. Three popular mechanisms are proportional, linear and uniform allocations. When a supply shortage occurs, proportional allocation gives each retailer the same fraction of her order, while linear allocation gives each retailer her order minus a common deduction. Thus, both mechanisms ensure that when limited supply is allocated, every retailer receives less than her order but can receive more by ordering a larger quantity. In contrast, under uniform allocation, the manufacturer equally divides the available supply among retailers; if a retailer orders less than her equal share, she receives her order and the remaining supply is allocated equally among the other retailers. Thus, a retailer cannot increase her allocation by ordering a larger quantity under uniform allocation of limited supply. See Cachon and Lariviere (1999a&b) for more details.

The sequence of decisions and events in our model is the same as before except that the manufacturer uses the allocation mechanism when a shortage occurs in each stage. Specifically, at the end of the first stage after the random yield is realized, the manufacturer allocates available supply to the pre-book orders placed earlier by multiple retailers. If the total pre-book orders exceed available supply, the manufacturer allocates the supply to retailers according to the pre-announced mechanism and then the game ends. Otherwise, the manufacturer delivers what each retailer has pre-booked and the game proceeds to the second stage. The remaining supply after allocating the supply to pre-book orders is common knowledge in the second stage. At the end of the second stage, if total regular orders exceed the remaining supply, the manufacturer allocates the supply to retailers according to the pre-announced mechanism; otherwise, the manufacturer delivers what each retailer has ordered. In this manner, the manufacturer gives a higher priority to pre-book orders than regular orders.

We assume that, as in Cachon and Lariviere (1999a&b), the retailers' demands are independent, i.e., they are local monopolists. Retailer n $(= 1, 2, ..., N)$ faces a demand curve, $p_n = 1 + \tilde{\varepsilon}_n - q_n$, where p_n represents a retail price to consumers, $\tilde{\varepsilon}_n$ represents a random demand component, and q_n represents the quantity available for sales by the retailer. We assume that $\tilde{\varepsilon}_n$ is independent and identically distributed. Each retailer knows the distribution of the others' random demands but does not observe their realizations. We also assume that the capacity of Nk is available, so each retailer's (ideal) share of capacity is k as in the one-retailer supply chain.

We consider the dominant strategy equilibrium concept in which each retailer has the ideal allocations for her pre-book and regular orders that maximize her profit regardless of the orders of the other retailers (Cachon and Lariviere 1999a). We are particularly interested in whether or not,

in a dominant equilibrium, all retailers order the optimal quantities x_1^{AB} and x_2^{AB} obtained earlier in the one-retailer supply chain without the allocation risk. Let us first consider retailers' regular ordering decisions in the second stage. Given the regular wholesale price and the fixed supply, each retailer places her regular order after observing her demand. This setting is identical to those in Cachon and Lariviere (1999a&b), so their results apply here as follows. Under proportional or linear allocation mechanism, all retailers ordering their ideal allocations is not a dominant equilibrium. In contrast, under uniform allocation mechanism, all retailers order their ideal allocations in equilibrium. This is because, unlike the first two mechanisms, retailers do not receive more by inflating their orders when a shortage occurs.

Next, let us consider retailers' pre-book decisions in the first stage prior to observing uncertain demand and yield. It is easy to see that proportional and linear allocations will induce retailers to inflate their orders as in the second stage, so we focus our discussion on the retailers' decisions under uniform allocation. Suppose a dominant equilibrium exists in which every retailer pre-books her ideal pre-book allocation. Then, when a supply shortage occurs in the first stage, each retailer cannot increase her allocation by increasing her pre-book order because of the property of the uniform allocation rule. However, when there is no supply shortage in the first stage, the manufacturer has remaining supply to fill the regular orders. In this case, any retailer can increase her allocation by pre-booking more than her ideal pre-book allocation because pre-book orders have a higher fulfillment priority than regular orders. These observations suggest that ordering the optimal quantities x_1^{AB} and x_2^{AB} derived from the one-retailer supply chain is not, in general, a dominant equilibrium in the multiple-retailer supply chain. Hence, in the presence of the allocation risk, retailers have more incentives to participate in the advance booking program so that they can secure the quantities available to them by pre-booking more.

8 Concluding Remarks

Advance booking programs are mechanisms to align the manufacturer's and retailers' incentives for matching supply and demand. While operations management researchers have shown that such programs are effective when uncertain demand is the only source of risk, it has not been studied whether or not the advance booking programs are also effective in the presence of demand, supply and price risks. Motivated by an advance booking program for managing these three types of risks in the flu vaccine supply chain, we have examined the benefits of the advance booking program in a

two-level supply chain in which a manufacturer and a retailer play a two-stage dynamic Stackelberg game.

By considering the case when the random yield and the random demand follow general probability distributions, we have shown that the advance booking program always benefits the manufacturer. Specifically, the manufacturer earns the highest expected profit ($E\Pi_M^{AB}$) when allowing the retailer to place both advance booking and regular orders as in the dynamic game AB, the second highest ($E\Pi_M^B$) when offering only regular orders as in the reactive game B, and the worst ($E\Pi_M^A$) when offering only advance booking as in the speculative game A; i.e., $E\Pi_M^{AB} \geq E\Pi_M^B \geq E\Pi_M^A$.

The first result $E\Pi_M^{AB} \geq E\Pi_M^B$ is due to the nonnegative value of an option of receiving pre-book orders in addition to regular orders. As the Stackelberg leader, the manufacturer can always induce the retailer to pre-book some units by charging a lower pre-book wholesale price than he would have charged for a regular order without advance booking. The retailer responds by increasing her total order quantity (in expectation). Because the retailer possesses the option of pre-booking or not, the retailer also earns a higher expected profit under the advance booking program (i.e., $E\Pi_R^{AB} \geq E\Pi_R^B$). As a result of the improved match between supply and demand, the expected leftovers at the end of the selling season are also lower under this program. This improvement in supply chain performance is due to a reduction in the market risk that both firms face.

The second result $E\Pi_M^B \geq E\Pi_M^A$ is due to two factors: (i) *price postponement*: the manufacturer's ability to set his regular wholesale price after observing the realized demand and supply; and (ii) *convex profit function and Jensen's inequality*: the manufacturer's *ex post* profit Π_M^B has an increasing marginal return with respect to the realized $\tilde{\varepsilon}$. In contrast, the retailer can be better off when only advance booking is allowed (i.e., $E\Pi_R^B < E\Pi_R^A$ and $E\Pi_R^{AB} < E\Pi_R^A$ are possible), especially when a supply shortage is likely to occur due to the low capacity.

Our analysis indicated that the effect of demand uncertainty is opposite to that of supply uncertainty under the advance booking program. Specifically, more supply uncertainty tends to decrease the manufacturer' expected profit, whereas more demand uncertainty tends to increase the manufacturer's expected profit. The primary reason for this contrasting result is due to their different impacts on the manufacturer's *ex post* profit: the marginal benefit of additional supply is nonincreasing but the marginal benefit of additional demand is nondecreasing. Although the effects of both types of uncertainties on the retailer's profit are not unidirectional, our numerical experiments suggested that their effects are also opposite in most scenarios.

We have extended our model to the case when there are multiple retailers in a supply chain

competing for potentially scarce supply. In this setting, in addition to those three aforementioned risks, each retailer faces the allocation risk which arises when the total order quantity from all retailers exceeds the available supply. Because pre-book orders have higher priorities over regular orders, we have shown that a retailer has a stronger incentive to pre-book more in a multiple-retailer supply chain than in a single-retailer supply chain.

Although our model captures the key trade-offs associated with the firms' decisions under advance booking programs, one can make further extensions to enrich our findings. First, while we have shown that all retailers ordering the optimal quantities as in the one-retailer supply chain is not a dominant equilibrium in a multiple-retailer supply chain, we are unable to compute the equilibrium in the multiple-retailer supply chain model mainly because there are two many different cases to analyze. For tractable analysis, one may focus on retailers' ordering decisions between two periods for a pre-specified wholesale pricing scheme. Second, based on our observation of industry practices, we have assumed that pre-book orders are not cancellable. However, it is of interest to examine a contract that allows cancellation of some pre-book orders with or without penalties. Lastly, our two-stage dynamic Stackelberg game model may serve as a building block for others to develop models that capture the competitive dynamics between multiple manufacturers and multiple retailers over multiple time periods.

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Appendix

A1. Proofs of Analytical Results

Proof of Proposition 1. The proof is presented in the main text.

Proof of Proposition 2. Since $\Pi_M(w_1)$ in (3) is concave in w_1 for any $\tilde{\theta}$, $E_{\tilde{\theta}}[\Pi_M(w_1)]$ is also concave in w_1, hence a unique w_1^A exists. As illustrated in Figure 4, if $k\tilde{\theta} \geq 0.25$, $\arg\limits_{w_1}\left[\min\left\{k\tilde{\theta}w_1, \frac{w_1-w_1^2}{2}\right\}\right] = 0.5$ and otherwise, $\arg\limits_{w_1}\left[\min\left\{k\tilde{\theta}w_1, \frac{w_1-w_1^2}{2}\right\}\right] = 1-2k\tilde{\theta} > 0.5$. Thus, if $k\tilde{\theta} \geq 0.25\ \forall\tilde{\theta}$, $w_1^A = 0.5$ and otherwise, $w_1^A > 0.5$. The result for $x_1^A(w_1^A)$ follows from Proposition 1. By substituting w_1^A and $x_1^A(w_1^A)$ into (2) and (3), we obtain the results for $E\Pi_R^A$ and $E\Pi_M^A$. $\square$

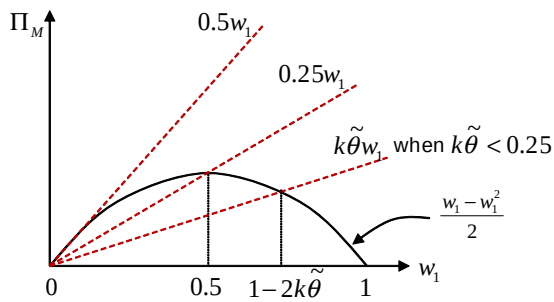

Figure 4: Manufacturer's Ex Post Profit Π_M as a Function of the Pre-book Wholesale Price w_1 in the Speculative Game A.

Proof of Proposition 3. The proof is similar to the proof of Proposition 2, hence we omit the proof.

Proof of Theorem 1. For any given (w_1, x_1), $E\Pi_R(w_1, x_1)$ in (7) and $E\Pi_M(w_1, x_1)$ in (8) under the dynamic game AB can be re-written respectively as follows:

$$E\Pi_R(w_1, x_1) = f_R^A(w_1, x_1) + f_R^B(x_1) \tag{9}$$

$$E\Pi_M(w_1, x_1) = f_M^A(w_1, x_1) + f_M^B(x_1) \tag{10}$$

$$\text{where } f_R^A(w_1, x_1) \equiv E_{\tilde{\theta},\tilde{\varepsilon}}\left[(1+\tilde{\varepsilon}-a_1-w_1)a_1\right] \tag{11}$$

$$f_R^B(x_1) \equiv E_{\tilde{\theta},\tilde{\varepsilon}}\left[\frac{m'^2}{16}|m'>0, l' \geq \frac{m'}{4}\right] + E_{\tilde{\theta},\tilde{\varepsilon}}\left[l'^2|m'>0, 0<l'<\frac{m'}{4}\right] \tag{12}$$

$$f_M^A(w_1, x_1) \equiv E_{\tilde{\theta},\tilde{\varepsilon}}\left[w_1 a_1\right] \tag{13}$$

$$f_M^B(x_1) \equiv E_{\tilde{\theta},\tilde{\varepsilon}}\left[\frac{m'^2}{8}|m'>0, l' \geq \frac{m'}{4}\right] + E_{\tilde{\theta},\tilde{\varepsilon}}\left[(m'-2l')l'|m'>0, 0<l'<\frac{m'}{4}\right] \tag{14}$$

Note that $f_R^A(w_1, x_1)$ is the retailer's expected profit for any given (w_1, x_1) under the speculative

game A, and that $f_R^B(0)$ is the ex-ante expected profit of the retailer in equilibrium under the reactive game B, i.e., $E\Pi_R^B$. Similarly, $f_M^A(w_1, x_1)$ is the manufacturer's expected profit for any given (w_1, x_1) under the speculative game A and $f_M^B(0)$ is the ex-ante expected profit of the manufacturer in equilibrium under the reactive game B, i.e., $E\Pi_M^B$.

(a) We first prove $E\Pi_R^{AB} \geq E\Pi_R^B$. From the definition of f_R^A in (11), $f_R^A(w_1, 0) = 0$, hence from (9), $E\Pi_R(w_1, 0) = f_R^A(w_1, 0) + f_R^B(0) = E\Pi_R^B$. For any given w_1, $E\Pi_R\left(w_1, x_1^{AB}(w_1)\right) = \max_{x_1 \geq 0} E\Pi_R(w_1, x_1) \geq E\Pi_R(w_1, 0) = E\Pi_R^B$. Therefore, $E\Pi_R^{AB} = E\Pi_R\left(w_1^{AB}, x_1^{AB}(w_1^{AB})\right) \geq E\Pi_R^B$ where the equality holds when $x_1^{AB} = 0$.

Next, we show $E\Pi_M^{AB} \geq E\Pi_M^B$. From the definition of f_M^A in (13), $f_M^A(w_1, 0) = 0$, hence from (10), $E\Pi_M(w_1, 0) = f_M^A(w_1, 0) + f_M^B(0) = E\Pi_M^B$. If $w_1 \geq 1$, then $x_1^{AB}(w_1) \equiv \arg\max_{x_1} E\Pi_R(w_1, x_1) = \arg\max_{x_1} \left\{f_R^A(w_1, x_1) + f_R^B(x_1)\right\} = 0$ because $\arg\max_{x_1} f_R^A(w_1, x_1) = 0$ by Proposition 1 and $f_R^B(x_1)$ is decreasing in x_1 from the observation of (12). Thus, if $w_1 \geq 1$, $E\Pi_M\left(w_1, x_1^{AB}(w_1)\right) = E\Pi_M(w_1, 0) = E\Pi_M^B$. Therefore, $E\Pi_M^{AB} = E\Pi_M^{AB}(w_1^{AB}, x_1^{AB}(w_1^{AB})) = \max_{w_1 \geq 0} E\Pi_M(w_1, x_1^{AB}(w_1)) \geq \max_{w_1 \geq 1} E\Pi_M(w_1, x_1^{AB}(w_1)) = E\Pi_M^B$.

(b) From Proposition 3, $w_2^B = \max\{\frac{1+\tilde{\varepsilon}}{2}, 1 + \tilde{\varepsilon} - 2k\tilde{\theta}\}$ (≥ 0). From Corollary 2, $w_2^{AB}(x_1) = \max\left\{\frac{1+\tilde{\varepsilon}-2x_1}{2}, 1 + \tilde{\varepsilon} - 2k\tilde{\theta}, 0\right\}$. By comparing these equations, we obtain the following result: for any $x_1 \geq 0$, $w_2^{AB}(x_1) \leq w_2^B$ $\forall \tilde{\theta}, \tilde{\varepsilon}$.

(c) From Proposition 3, $x_2^B = \min\left\{\frac{1+\tilde{\varepsilon}}{4}, k\tilde{\theta}\right\}$ (≥ 0 because $1 + \tilde{\varepsilon} \geq 0$ and $k\tilde{\theta} \geq 0$). From Corollary 2, $x_2^{AB}(x_1) = \max\left\{\min\left(\frac{1+\tilde{\varepsilon}-2x_1}{4}, k\tilde{\theta} - x_1\right), 0\right\}$ and $x_1 + x_2^{AB}(x_1) = \max\left\{\min\left(\frac{1+\tilde{\varepsilon}+2x_1}{4}, k\tilde{\theta}\right), x_1\right\}$. By comparing these equations, we obtain the following result: for any $x_1 \geq 0$, $x_2^{AB}(x_1) \leq x_2^B \leq x_1 + x_2^{AB}(x_1)$ $\forall \tilde{\theta}, \tilde{\varepsilon}$. ■

Proof of Theorem 2. (a) Since $E\Pi_M^{AB} \geq E\Pi_M^B$ from Theorem 1(a), it suffices to show that $E\Pi_M^B \geq E\Pi_M^A$. From (3), $E\Pi_M^A$ can be expressed as follows:

$$\begin{aligned} E\Pi_M^A &= \max_{w_1} \int_{\underline{\theta}}^{\bar{\theta}} w_1 \cdot \min\left\{k\tilde{\theta}, \frac{1 - w_1}{2}\right\} dF(\tilde{\theta}) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} w_1^A \cdot \min\left\{k\tilde{\theta}, \frac{1 - w_1^A}{2}\right\} dF(\tilde{\theta}). \end{aligned} \tag{15}$$

From (4), $E\Pi_M^B$ can be expressed as follows:

$$\begin{aligned} E\Pi_M^B &= \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \max_{w_2} \left\{w_2 \cdot \min\left(k\tilde{\theta}, \frac{1 + \tilde{\varepsilon} - w_2}{2}\right)\right\} dG(\tilde{\varepsilon}) dF(\tilde{\theta}) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} h(\tilde{\theta}, \tilde{\varepsilon}) dG(\tilde{\varepsilon}) dF(\tilde{\theta}), \end{aligned}$$

where $h(\tilde{\theta},\tilde{\varepsilon})$ is defined from Proposition 3 as follows: if $l \geq \frac{m}{4}$, i.e., $\tilde{\varepsilon} \leq 4k\tilde{\theta}-1$, $h(\tilde{\theta},\tilde{\varepsilon}) = \frac{m^2}{8} = \frac{(1+\tilde{\varepsilon})^2}{8}$; and otherwise, $h(\tilde{\theta},\tilde{\varepsilon}) = (m-2l)l = (1+\tilde{\varepsilon}-2k\tilde{\theta})k\tilde{\theta}$. As illustrated in Figure 3(b), $h\left(\tilde{\theta},\tilde{\varepsilon}\right)$ is strictly increasing and convex in $\tilde{\varepsilon}$ because $(1+\tilde{\varepsilon}-2k\tilde{\theta})k\tilde{\theta}$ is tangent to $\frac{(1+\tilde{\varepsilon})^2}{8}$ at $\tilde{\varepsilon} = 4k\tilde{\theta}-1$. Then, from Jensen's inequality,

$$\begin{aligned} E\Pi_M^B &\geq \int_{\underline{\theta}}^{\bar{\theta}} \max_{w_2} \left\{ w_2 \cdot \min\left(k\tilde{\theta}, \frac{1+E\left[\tilde{\varepsilon}\right]-w_2}{2}\right)\right\} dF(\tilde{\theta}) \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \max_{w_2} \left\{ w_2 \cdot \min\left(k\tilde{\theta}, \frac{1-w_2}{2}\right)\right\} dF(\tilde{\theta}). \end{aligned} \tag{16}$$

Since the integrand of (16) is greater than or equal to that of (15) for any $\tilde{\theta}$, $E\Pi_M^B \geq E\Pi_M^A$.

From (10), $E\Pi_M^{AB} = E\Pi_M^A$ if and only if $f_R^B(x_1) = 0\ \forall \tilde{\varepsilon}, \tilde{\theta}$. We see from (14) that this occurs when $m' \leq 0$ or $l' \leq 0$ with probability 1. Therefore, if $x_1^{AB} \geq \min\left\{k\bar{\theta}, \frac{1+\bar{\varepsilon}}{2}\right\}$, $E\Pi_M^{AB} = E\Pi_M^A$.

(b) From (9), $\left.\frac{\partial E\Pi_R(w_1,x_1)}{\partial x_1}\right|_{x_1=x_1^A(w_1)} = \left.\frac{\partial f_R^A(w_1,x_1)}{\partial x_1}\right|_{x_1=x_1^A(w_1)} + \left.\frac{\partial f_R^B(x_1)}{\partial x_1}\right|_{x_1=x_1^A(w_1)}$. Because $x_1^A(w_1)$ maximizes $f_R^A(w_1,x_1)$ and $f_R^A(w_1,x_1)$ is concave in x_1, $\left.\frac{\partial f_R^A(w_1,x_1)}{\partial x_1}\right|_{x_1=x_1^A(w_1)} = 0$ and $\frac{\partial f_R^A(w_1,x_1)}{\partial x_1} < 0$ for $x_1 > x_1^A(w_1)$. From (12), we observe that $f_R^B(x_1)$ decreases in x_1. Therefore, $\left.\frac{\partial E\Pi_R(w_1,x_1)}{\partial x_1}\right|_{x_1=x_1^A(w_1)} \leq \left.\frac{\partial E\Pi_R(w_1,x_1)}{\partial x_1}\right|_{x_1=x_1^{AB}(w_1)} = 0$, hence $x_1^A(w_1) \geq x_1^{AB}(w_1)\ \forall w_1$.

If $w_1^{AB} \geq w_1^A$, then $x_1^{AB}(w_1^{AB}) \leq x_1^A(w_1^{AB}) \leq x_1^A(w_1^A)$ where the first inequality is due to the earlier result that $x_1^A(w_1) \geq x_1^{AB}(w_1)\ \forall w_1$, and the second inequality follows from Proposition 1 which shows that $x_1^A(w_1)$ decreases in w_1. ■

A2. Equilibrium Outcomes under Specific Probability Distributions

In our numerical study, we have assumed that $\tilde{\theta}$ is uniformly distributed between $1-r$ and $1+r$, and that $\tilde{\varepsilon} = e$ with probability 0.5 and $\tilde{\varepsilon} = -e$ with probability 0.5. This section presents the closed-form expressions of equilibrium outcomes under these probability distributions in games A and B, and describe the efficient procedure to compute equilibrium outcomes in game AB.

In the speculative game A, $a_1^A(w_1)$ depends on the random yield $\tilde{\theta}$ in the following manner: if $\frac{1-w_1}{2k} \leq \tilde{\theta} \leq 1+r$, $a_1^A(w_1) = \frac{1-w_1}{2}$ and if $1-r \leq \tilde{\theta} \leq \frac{1-w_1}{2k}$, $a_1^A(w_1) = k\tilde{\theta}$. Thus, we can express $E_{\tilde{\theta}}\left[\Pi_M(w_1)\right]$ in (3) as:

$$E_{\tilde{\theta}}\left[\Pi_M(w_1)\right] = E_{\tilde{\theta}}[w_1 \cdot a_1^A(w_1)] = \int_{\theta_a}^{1+r} w_1 \frac{1-w_1}{2}\ \frac{1}{2r}\ d\theta + \int_{1-r}^{\theta_a} w_1 k\theta\ \frac{1}{2r}\ d\theta, \tag{17}$$

where $\theta_a \equiv \min\left\{1+r, \max\left(1-r, \frac{1-w_1}{2k}\right)\right\}$. By noting that θ_a can take on three different values

depending on w_1, (17) can be rewritten as:

$$E_{\tilde{\theta}}\left[\Pi_M(w_1)\right] = \begin{cases} kw_1 & \text{if } 0 \le w_1 \le 1-2k(1+r) \\ -\frac{w_1}{16rk}\left[w_1 - \{1-2k(1+r+2\sqrt{r})\}\right] & \text{if } 1-2k(1+r) < w_1 < 1-2k(1-r) \\ \cdot\left[w_1 - \{1-2k(1+r-2\sqrt{r})\}\right] & \\ \frac{w_1(1-w_1)}{2} & \text{if } 1-2k(1-r) \le w_1 \le 1. \end{cases} \tag{18}$$

Proposition 2 has shown that if $k(1-r) \ge 0.25$, $w_1^A = 0.5$. If $k(1-r) < 0.25$, we can show

$$w_1^A = \frac{2}{3}(1-2k-2kr) + \frac{1}{3}\sqrt{4k^2(1+14r+r^2) - 4k(1+r) + 1}. \tag{19}$$

The proof of the above result is as follows. From (18) , let $g_1(w_1) \equiv kw_1$, $g_2(w_1) \equiv -\frac{w_1}{16rk}[w_1 - \{1-2k(1+r+2\sqrt{r})\}]\,[w_1 - \{1-2k(1+r-2\sqrt{r})\}]$, and $g_3(w_1) \equiv \frac{w_1(1-w_1)}{2}$. Note that g_2 is a cubic function of w_1 and its domain $[1-2k(1+r), 1-2k(1-r)]$ is contained in $[1-2k(1+r+2\sqrt{r}), 1-2k(1+r-2\sqrt{r})]$. Since $\frac{d^3 g_2}{dw_1^3} < 0$ and $1-2k(1+r-2\sqrt{r}) > 0$, g_2 is unimodal with its maximum at a larger root of $\frac{dg_2(w_1)}{dw_1} = 0$, which is equal to the right hand side of (19). Also, g_3 is concave in w_1 with its maximum at 0.5. Since $\left.\frac{dg_1}{dw_1}\right|_{w_1=1-2k(1+r)} = \left.\frac{dg_2}{dw_1}\right|_{w_1=1-2k(1+r)} = k > 0$, w_1^A does not exist in the first interval of w_1. Suppose $k(1-r) \ge 0.25$. Then, $\left.\frac{dg_2}{dw_1}\right|_{w_1=1-2k(1-r)} = -\frac{1}{2} + 2k(1-r) \ge 0$, hence $w_1^A = 0.5 \in [1-2k(1-r), 1]$. Next, suppose $k(1-r) < 0.25$. Then, $\left.\frac{dg_2}{dw_1}\right|_{w_1=1-2k(1+r)} > 0$, $\left.\frac{dg_2}{dw_1}\right|_{w_1=1-2k(1-r)} < 0$ and $\left.\frac{dg_2}{dw_1}\right|_{w_1=1-2k(1-r)} < 0$, hence w_1^A in (19) is optimal. In this case, $w_1^A > 0.5$ because $\left.\frac{dg_2}{dw_1}\right|_{w_1=0.5} = \frac{1}{16kr}\left[\left(\frac{1}{2}\right)^2 - \{2k(1-r)\}^2\right] > 0$. By substituting $w_1 = w_1^A$ into $x_1^A(w_1)$ in Proposition 1, we can obtain the closed-form expressions for $x_1^A(w_1^A)$ and similarly for $E\Pi_M^A$ and $E\Pi_R^A$.

For the reactive game B, using the *ex post* equilibrium outcomes given in Proposition 3, we compute the *ex-ante* equilibrium outcomes. From Proposition 3, by noting that the condition $l \ge \frac{m}{4}$ can be rewritten as $\tilde{\theta} \ge \frac{1+\tilde{\varepsilon}}{4k}$, we can rewrite the *ex-ante* expected wholesale price in (5) as:

$$\begin{aligned} E_{\tilde{\theta},\tilde{\varepsilon}}\left[w_2^B\right] &= \Pr(\tilde{\varepsilon} = e)\left[\Pr(\tilde{\theta} \ge \theta_b)\frac{1+e}{2} + \int_{1-r}^{\theta_b}\left[(1+e) - 2k\theta\right]\frac{d\theta}{2r}\right] \\ &\quad + \Pr(\tilde{\varepsilon} = -e)\left[\Pr(\tilde{\theta} \ge \theta_c)\frac{1-e}{2} + \int_{1-r}^{\theta_c}\left[(1-e) - 2k\theta\right]\frac{d\theta}{2r}\right] \\ &= \frac{1}{4r}\left[-1 + 3r + \frac{1}{2}\left\{(1+e)\theta_b + (1-e)\theta_c\right\} - k\left\{\theta_b^2 + \theta_c^2 - 2(1-r)^2\right\}\right], \end{aligned}$$

where $\theta_b \equiv \min\left\{1+r, \max\left(1-r, \frac{1+e}{4k}\right)\right\}$ and $\theta_c \equiv \min\left\{1+r, \max\left(1-r, \frac{1-e}{4k}\right)\right\}$. Similarly, we can compute other ex-ante equilibrium outcomes such as $E\Pi_M^B$, Ex_2^B, and $E\Pi_R^B$. Table 3 shows that there are six possible cases of having different pairs of θ_b or θ_c. Table 4 displays the probabilities of a supply shortage in each case. For example, in Case 1 when $\theta_b = 1+r$ and $\theta_c = 1+r$, the probability of a supply shortage is 1 for both demand states. Note that Case 1 represents the lowest level of capacity with the highest probabilities of a supply shortage, whereas Case 6 represents the highest level with the lowest probabilities; Cases 2-5 represent the intermediate level. Below we present the closed-form expressions of the *ex-ante* equilibria in each of these cases. By evaluating their first derivatives with respect to r, e and k both analytically and numerically, we obtain comparative statics results as summarized in Table 5.

Case 1: $\theta_b = 1+r,\ \theta_c = 1+r$

$Ex_2^B = k$

$Ew_2^B = 1-2k$

$E\Pi_R^B = \frac{k^2(r^3+3)}{3}$

$E\Pi_M^B = k - \frac{2k^2(r^3+3)}{3}$

Case 2: $\theta_b = 1+r,\ \theta_c = \frac{1-e}{4k}$

$Ex_2^B = -\frac{1}{128rk}\left(-8k-2e-96k^2r+16k^2r^2+8ke-8kr+e^{2\times1}+16k^2+8kre+1\right)$

$Ew_2^B = \frac{1}{64rk}\left(-8k-2e-96k^2r+16k^2r^2+8ke+56kr+e^{2\times1}+16k^2+8kre+1\right)$

$E\Pi_R^B = \frac{1}{384rk}(6k+3e+6ke^{2\times1}+288k^3r-96k^3r^2+96k^3r^3-12ke+6kr-3e^{2\times1}+e^{3\times1}-32k^3+6kre^{2\times1}-12kre-1)$

$E\Pi_M^B = -\frac{1}{384rk}(-12k-3e-12ke^{2\times1}-48k^2e-288k^2r+576k^3r+48k^2r^2-192k^3r^2+192k^3r^3+24ke-12kr+3e^{2\times1}-e^{3\times1}+48k^2-64k^3-12kre^{2\times1}-96k^2re-48k^2r^2e+24kre+1)$

Case 3: $\theta_b = 1+r,\ \theta_c = 1-r$

$Ex_2^B = \frac{k}{2} + \frac{1-e}{8}$

$Ew_2^B = \frac{3+e}{4} - k$

$E\Pi_R^B = \frac{(1-e)^2}{32} + \frac{k^2}{2} + \frac{k^2r^2}{6}$

$E\Pi_M^B = \frac{(1-e)^2}{16} + \frac{(1+e)k}{2} - \frac{k^2(r^2+3)}{3}$

Case 4: $\theta_b = \frac{1+e}{4k},\ \theta_c = \frac{1-e}{4k}$

$Ex_2^B = -\frac{1}{64}\frac{-8k+e^2-32k^2r+16k^2r^2-8kr+16k^2+1}{kr}$

$Ew_2^B = \frac{1}{32}\frac{-8k+e^2-32k^2r+16k^2r^2+24kr+16k^2+1}{kr}$

$E\Pi_R^B = \frac{1}{192}\frac{6k-3e^2+96k^3r-96k^3r^2+32k^3r^3+6ke^2+6kr-32k^3+6kre^2-1}{kr}$

$E\Pi_M^B = -\frac{1}{192}\frac{-12k+3e^2-96k^2r+192k^3r+48k^2r^2-192k^3r^2+64k^3r^3-12ke^2-12kr+48k^2-64k^3-12kre^2+1}{kr}$

Case 5: $\theta_b = \frac{1+e}{4k},\ \theta_c = 1-r$

$Ex_2^B = -\frac{1}{128}\frac{-8k+2e+e^2-32k^2r+16k^2r^2-8ke-24kr+16k^2+8kre+1}{kr}$

$Ew_2^B = \frac{1}{64}\frac{-8k+2e+e^2-32k^2r+16k^2r^2-8ke+40kr+16k^2+8kre+1}{kr}$

$E\Pi_R^B = \frac{1}{384}\frac{6k-3e-3e^2-e^3+96k^3r-96k^3r^2+32k^3r^3+12ke+6ke^2+18kr-32k^3-12kre+18kre^2-1}{kr}$

$E\Pi_M^B = -\frac{1}{384rk}(-12k+3e+3e^2+e^3+48k^2e-96k^2r+192k^3r+48k^2r^2-192k^3r^2+64k^3r^3-24ke-12ke^2-36kr+48k^2-64k^3-96k^2re+48k^2r^2e+24kre-36kre^2+1)$

Case 6: $\theta_b = 1-r,\ \theta_c = 1-r$

$Ex_2^B = \frac{1}{4}$

$Ew_2^B = \frac{1}{2}$

$E\Pi_R^B = \frac{1+e^2}{16}$

$E\Pi_M^B = \frac{1+e^2}{8}$

Table 3. Six Possible Cases of θ_b and θ_c in Game B

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
θ_b	$1+r$	$1+r$	$1+r$	$\frac{1+e}{4k}$	$\frac{1+e}{4k}$	$1-r$
θ_c	$1+r$	$\frac{1-e}{4k}$	$1-r$	$\frac{1-e}{4k}$	$1-r$	$1-r$

Table 4. Probabilities of Supply Shortage in Six Cases of θ_b and θ_c in Game B

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
high demand ($\tilde{\varepsilon} = e$)	1	1	1	(0, 1)	(0, 1)	0
low demand ($\tilde{\varepsilon} = -e$)	1	(0, 1)	0	(0, 1)	0	0

Table 5. Effect of k, e and r on the Ex-ante Equilibrium Outcomes in Game B

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
$\frac{\partial Ex_2^B}{\partial k}$	$+$	$+$	$+$	$+$	$+$	0
$\frac{\partial Ew_2^B}{\partial k}$	$-$	$-$	$-$	$-$	$-$	0
$\frac{\partial E\Pi_R^B}{\partial k}$	$+$	$+$	$+$	$+$	$+$	0
$\frac{\partial E\Pi_M^B}{\partial k}$	$+$	$+$	$+$	$+$	$+$	0
$\frac{\partial Ex_2^B}{\partial r}$	0	$-$	0	$-$	$-$	0
$\frac{\partial Ew_2^B}{\partial r}$	0	$+$	0	$+$	$+$	0
$\frac{\partial E\Pi_R^B}{\partial r}$	$+$	$\pm$	$+$	$\pm$	$\pm$	0
$\frac{\partial E\Pi_M^B}{\partial r}$	$-$	$-$	$-$	$-$	$-$	0
$\frac{\partial Ex_2^B}{\partial e}$	0	$-$	$-$	$-$	$-$	0
$\frac{\partial Ew_2^B}{\partial e}$	0	$+$	$+$	$+$	$+$	0
$\frac{\partial E\Pi_R^B}{\partial e}$	0	$-$	$-$	$\pm$	$\pm$	$+$
$\frac{\partial E\Pi_M^B}{\partial e}$	0	$+$	$+$	$+$	$+$	$+$

Note: $\pm$ indicates that the sign of the derivative can be $+$ or $-$.

For the dynamic game AB, we have presented in Corollary 2 the closed-form expressions of the equilibrium outcomes in the second stage game. As we show below, the determination of the retailer's equilibrium pre-book quantity in the first stage involves the analysis of 39 cases. Thus, instead of finding the closed-form expressions for each case, we devise an efficient procedure to compute the equilibrium outcomes in the first stage.

First, we consider the retailer's problem in the first stage. Note that the conditions provided in Corollary 2 can be re-written as follows: $l' \geq \frac{m'}{4}$ as $\tilde{\theta} \geq \frac{1+\tilde{\varepsilon}+2x_1}{4k}$, $l' > 0$ as $\tilde{\theta} > \frac{x_1}{k}$, and $m' > 0$ as $x_1 < \frac{1+\tilde{\varepsilon}}{2}$. Define threshold numbers $\theta_d \equiv \min\left\{1+r, \max\left(1-r, \frac{1+e+2x_1}{4k}\right)\right\}$, $\theta_e \equiv \min\left\{1+r, \max\left(1-r, \frac{1-e+2x_1}{4k}\right)\right\}$, and $\theta_f \equiv \min\left\{1+r, \max\left(1-r, \frac{x_1}{k}\right)\right\}$; and the indicator function $I(y) = 1$ if y is true and $I(y) = 0$, otherwise. Then $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ in (7) can be rewritten as:

$$\begin{aligned} E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right] &= \int_{\theta_f}^{1+r} (1 - x_1 - w_1)\, x_1 \, \frac{d\theta}{2r} + \int_{1-r}^{\theta_f} (1 - k\theta - w_1)\, k\theta \, \frac{d\theta}{2r} \\ &+ \Pr(\tilde{\varepsilon} = e) I\left(x_1 < \frac{1+e}{2}\right)\left[\frac{1}{16}\int_{\theta_d}^{1+r} (1+e-2x_1)^2 \, \frac{d\theta}{2r} + \int_{\theta_f}^{\theta_d} (k\theta - x_1)^2 \, \frac{d\theta}{2r}\right] \\ &+ \Pr(\tilde{\varepsilon} = -e) I\left(x_1 < \frac{1-e}{2}\right)\left[\frac{1}{16}\int_{\theta_e}^{1+r} (1-e-2x_1)^2 \, \frac{d\theta}{2r} + \int_{\theta_f}^{\theta_e} (k\theta - x_1)^2 \, \frac{d\theta}{2r}\right]. \end{aligned} \tag{20}$$

Depending on the values of θ_d, θ_e, θ_f, $I\left(x_1 < \frac{1+e}{2}\right)$, and $I\left(x_1 < \frac{1-e}{2}\right)$, the retailer's expected profit $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ in (20) takes on different functional forms of x_1. Note that θ_d, θ_e or θ_f is either a constant or a linear function of x_1, and $I\left(x_1 < \frac{1+e}{2}\right)$ or $I\left(x_1 < \frac{1-e}{2}\right)$ is either 0 or 1. Hence, in each interval of x_1 having a different combination of θ_d, θ_e, θ_f, $I\left(x_1 < \frac{1+e}{2}\right)$, and $I\left(x_1 < \frac{1-e}{2}\right)$, $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ is at most a cubic function of x_1. The pre-book order quantity $x_1^{AB}(w_1)$ that maximizes $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ for any given w_1 is either a boundary point between any two intervals or an interior point at which the first order condition is satisfied. There are potentiality 39 candidates for $x_1^{AB}(w_1)$ which consist of 9 boundary points between two intervals of x_1 and 30 interior optimal points within any interval of x_1. The boundary points are: 0, $\frac{1+e}{2}$, $\frac{1-e}{2}$, $\frac{4k(1+r)-(1+e)}{2}$ (at which $1+r = \frac{1+e+2x_1}{4k}$), $\frac{4k(1-r)-(1+e)}{2}$ (at which $1-r = \frac{1+e+2x_1}{4k}$), $\frac{4k(1+r)-(1-e)}{2}$ (at which $1+r = \frac{1-e+2x_1}{4k}$), $\frac{4k(1-r)-(1-e)}{2}$ (at which $1-r = \frac{1-e+2x_1}{4k}$), $k(1+r)$ (at which $1+r = \frac{x_1}{k}$), and $k(1-r)$ (at which $1-r = \frac{x_1}{k}$). To find the interior optimal points, we first find the expression of $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ in (20) for each of the following 30 cases:

(i) If $x_1 \geq \frac{1+e}{2}$, $I\left(x_1 < \frac{1+e}{2}\right) = I\left(x_1 < \frac{1-e}{2}\right) = 0$, so $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ can have 3 different expres-

sions when $\theta_f = 1+r,\ 1-r$ or $\frac{x_1}{k}$;

(ii) If $\frac{1-e}{2} \leq x_1 < \frac{1+e}{2}$, $I\left(x_1 < \frac{1+e}{2}\right) = 1$ and $I\left(x_1 < \frac{1-e}{2}\right) = 0$, so $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ can have 9 different expressions when $\theta_f = 1+r,\ 1-r$ or $\frac{x_1}{k}$, and $\theta_d = 1+r,\ 1-r$ or $\frac{1+e+2x_1}{4k}$;

(iii) If $x_1 < \frac{1-e}{2}$, $I\left(x_1 < \frac{1+e}{2}\right) = I\left(x_1 < \frac{1-e}{2}\right) = 1$, so $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ can have 18 different expressions when $\theta_f = 1+r,\ 1-r$ or $\frac{x_1}{k}$, $\theta_d = 1+r,\ 1-r$ or $\frac{1+e+2x_1}{4k}$, and $\theta_e = 1+r,\ 1-r$ or $\frac{1-e+2x_1}{4k}$ (note: 18 cases exist instead of 27 cases because $\theta_d \geq \theta_e$).

For each of the above 30 cases, we can easily obtain an interior optimal point from the first order condition (which we omit here). By comparing $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_R(x_1, w_1)\right]$ at these 39 candidates, we can find the retailer's best response $x_1^{AB}(w_1)$ for any given w_1.

Next, we examine the manufacturer's decision of his pre-book wholesale price w_1 in the first stage. Similar to (20), we can rewrite $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_M(x_1, w_1)\right]$ in (8) as:

$$
\begin{aligned}
E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_M(x_1, w_1)\right] = & \int_{\theta_f}^{1+r} (w_1 x_1)\ \frac{d\theta}{2r} + \int_{1-r}^{\theta_f} (w_1 k\theta)\ \frac{d\theta}{2r} \\
& + \Pr(\tilde{\varepsilon} = e) I\left(x_1 < \frac{1+e}{2}\right)\left[\frac{1}{8}\int_{\theta_d}^{1+r} (1+e-2x_1)^2\ \frac{d\theta}{2r} + \int_{\theta_f}^{\theta_d} (1+e-2k\theta)(k\theta - x_1)\ \frac{d\theta}{2r}\right] \\
& + \Pr(\tilde{\varepsilon} = -e) I\left(x_1 < \frac{1-e}{2}\right)\left[\frac{1}{8}\int_{\theta_e}^{1+r} (1-e-2x_1)^2\ \frac{d\theta}{2r} + \int_{\theta_f}^{\theta_e} (1-e-2k\theta)(k\theta - x_1)\ \frac{d\theta}{2r}\right].
\end{aligned}
$$

We can efficiently compute the pre-book wholesale price w_1^{AB} that maximizes $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_M(x_1^{AB}(w_1), w_1)\right]$ as follows. We first compute the retailer's pre-book quantity $x_1^{AB}(w_1)$ as a function of w_1 and identify boundary points between any two adjacent intervals of w_1 at which $x_1^{AB}(w_1)$ switches from one of the 39 candidate points to another. In each interval of w_1, $E_{\tilde{\theta},\tilde{\varepsilon}}\left[\Pi_M(x_1^{AB}(w_1), w_1)\right]$ is a continuous function, hence its local maximum is attained at either a boundary point or an interior point at which the first order condition is satisfied. By comparing local maxima, we can identify a global optimal point w_1^{AB}.

A3. Comparative Statics in Games *A* and *B*

Proposition 4 *In the speculative game A, suppose capacity k increases. Then,*

(i) If $k\underline{\theta} \geq 0.25$, then the equilibrium outcomes remain unchanged;

(ii) Otherwise, w_1^A decreases, $E\Pi_M^A$ increases, $x_1^A(w_1^A)$ increases, and $E\Pi_R^A$ increases.

Proof. (i) The result follows from the proof of Proposition 2(i).

(ii) From the proof of Proposition 2(ii), if $k\tilde{\theta} \geq 0.25$, $\arg\limits_{w_1}\left[\min\left\{k\tilde{\theta}w_1, \frac{w_1 - w_1^2}{2}\right\}\right] = 0.5$, and otherwise,

$\arg\limits_{w_1}\left[\min\left\{k\tilde{\theta}w_1, \frac{w_1-w_1^2}{2}\right\}\right] > 0.5$. As k increases, $\Pr\{k\tilde{\theta} \geq 0.25\}$ decreases, hence w_1^A decreases. Then, by Proposition 1, $x_1^A(w_1^A)$ increases in k. $E\Pi_M^A$ increases because $E_{\tilde{\theta}}\left[\Pi_M(w_1)\right]$ in (3) increases in k for all w_1. From (2), if $a_1^A(w_1^A) = k\tilde{\theta}$, $\Pi_R(w_1^A) = -\left(k\tilde{\theta} - \frac{1-w_1^A}{2}\right)^2 + \frac{(1-w_1^A)^2}{4}$ and otherwise, $\Pi_R(w_1^A) = \frac{(1-w_1^A)^2}{4}$. In the former case, $\Pi_R(w_1^A)$ increases in k because $a_1^A(w_1^A) = k\tilde{\theta} \leq \frac{1-w_1^A}{2}$. In the latter case, $\Pi_R(w_1^A)$ increases in k because $w_1^A < 1$ and w_1^A decreases in k. Thus, $E\Pi_R^A$ increases in k. ■

Proposition 5 *In the speculative game A, suppose $\tilde{\theta}_1$ dominates $\tilde{\theta}_2$ in the sense of second-order stochastic dominance, i.e., $\tilde{\theta}_1 \succ_{SSD} \tilde{\theta}_2$. Then, $E_{\tilde{\theta}_1}[\Pi_M^A] \geq E_{\tilde{\theta}_2}[\Pi_M^A]$ where the equality holds when $\Pr\{\tilde{\theta}_1 \leq \frac{x_1^A(w_1^A)}{k}\} = \Pr\{\tilde{\theta}_2 \leq \frac{x_1^A(w_1^A)}{k}\} = 1$ or $\Pr\{\tilde{\theta}_1 > \frac{x_1^A(w_1^A)}{k}\} = \Pr\{\tilde{\theta}_2 > \frac{x_1^A(w_1^A)}{k}\} = 1$.*

Proof. First, note that $\Pi_M(w_1) = \min\left\{k\tilde{\theta}w_1, \frac{w_1-w_1^2}{2}\right\}$ in (3) is concave in $\tilde{\theta}$ for any given w_1. By the well-known property of the second-order stochastic dominance, $E_{\tilde{\theta}_1}\left[u(\tilde{\theta}_1)\right] \geq E_{\tilde{\theta}_2}\left[u(\tilde{\theta}_1)\right]$ for any concave function u. Thus, $E_{\tilde{\theta}_1}\left[\Pi_M(w_1)\right] \geq E_{\tilde{\theta}_2}\left[\Pi_M(w_1)\right]$ $\forall w_1$. Therefore, $E_{\tilde{\theta}_1}[\Pi_M^A] = \max\limits_{w_1} E_{\tilde{\theta}_1}\left[\Pi_M(w_1)\right] \geq \max\limits_{w_1} E_{\tilde{\theta}_2}\left[\Pi_M(w_1)\right] = E_{\tilde{\theta}_2}[\Pi_M^A]$. When $a_1^A(w_1^A) = k\tilde{\theta}$ $\forall\tilde{\theta}$ or $a_1^A(w_1^A) = x_1^A(w_1^A)$ $\forall\tilde{\theta}$, $\Pi_M(w_1)$ is linear or constant with respect to $\tilde{\theta}$, so $E_{\tilde{\theta}}\left[\Pi_M(w_1)\right]$ is independent of the distribution of $\tilde{\theta}$; hence $E_{\tilde{\theta}_1}[\Pi_M^A] = E_{\tilde{\theta}_2}[\Pi_M^A]$. ■

Proposition 6 *In the reactive game B, suppose capacity k increases. Then,*
(i) If $\Pr\{l \geq \frac{m}{4}\} = \Pr\{\tilde{\theta} \geq \frac{1+\tilde{\varepsilon}}{4k}\} = 1$, then the equilibrium outcomes remain unchanged;
(ii) Otherwise, w_2^B decreases, $E\Pi_M^B$ increases, $x_2^B(w_2^B)$ increases, and $E\Pi_R^B$ increases.

Proof. The proof proceeds similar to the proof of Proposition 4. ■

Proposition 7 *In the reactive game B, suppose $\tilde{\theta}_1 \succ_{SSD} \tilde{\theta}_2$. Then,*
(a) $E_{\tilde{\theta}_1}[\Pi_M^B] \geq E_{\tilde{\theta}_2}[\Pi_M^B]$ where the equality holds when $\Pr\{\tilde{\theta}_1 \geq \frac{1+\tilde{\varepsilon}}{4k}\} = \Pr\{\tilde{\theta}_2 \geq \frac{1+\tilde{\varepsilon}}{4k}\} = 1$;
(b) $E_{\tilde{\theta}_1}[w_2^B] \leq E_{\tilde{\theta}_2}[w_2^B]$ where the equality holds when $\Pr\{\tilde{\theta}_1 \geq \frac{1+\tilde{\varepsilon}}{4k}\} = \Pr\{\tilde{\theta}_2 \geq \frac{1+\tilde{\varepsilon}}{4k}\} = 1$ or $\Pr\{\tilde{\theta}_1 < \frac{1+\tilde{\varepsilon}}{4k}\} = \Pr\{\tilde{\theta}_2 < \frac{1+\tilde{\varepsilon}}{4k}\} = 1$;
(c) $E_{\tilde{\theta}_1}[x_2^B] \geq E_{\tilde{\theta}_2}[x_2^B]$ where the equality holds when $\Pr\{\tilde{\theta}_1 \geq \frac{1+\tilde{\varepsilon}}{4k}\} = \Pr\{\tilde{\theta}_2 \geq \frac{1+\tilde{\varepsilon}}{4k}\} = 1$ or $\Pr\{\tilde{\theta}_1 < \frac{1+\tilde{\varepsilon}}{4k}\} = \Pr\{\tilde{\theta}_2 < \frac{1+\tilde{\varepsilon}}{4k}\} = 1$.

Proof. (a) From Proposition 3, the manufacturer's *ex post* profit Π_M^B is increasing and concave in $\tilde{\theta}$ for $\tilde{\theta} < \frac{1+\tilde{\varepsilon}}{4k}$ and is constant for $\tilde{\theta} \geq \frac{1+\tilde{\varepsilon}}{4k}$. Thus, Π_M^B is nondecreasing and concave in $\tilde{\theta}$. By the same argument as presented in the proof of Proposition 5, $E_{\tilde{\theta}_1}[\Pi_M^B] \geq E_{\tilde{\theta}_2}[\Pi_M^B]$, where the equality holds when Π_M^B is constant for all $\tilde{\theta}_1 \geq \frac{1+\tilde{\varepsilon}}{4k}$ and $\tilde{\theta}_2 \geq \frac{1+\tilde{\varepsilon}}{4k}$.

(b) From Proposition 3, w_2^B is linearly decreasing in $\tilde{\theta}$ for $\tilde{\theta} < \frac{1+\tilde{\varepsilon}}{4k}$ and is constant for $\tilde{\theta} \geq \frac{1+\tilde{\varepsilon}}{4k}$. Thus, w_2^B is nonincreasing and convex in $\tilde{\theta}$. By using the same method as in (a), we obtain the result.

(c) From Proposition 3, x_2^B is linearly increasing in $\tilde{\theta}$ for $\tilde{\theta} < \frac{1+\tilde{\varepsilon}}{4k}$ and is constant for $\tilde{\theta} \geq \frac{1+\tilde{\varepsilon}}{4k}$. Thus, x_2^B is nondecreasing and concave in $\tilde{\theta}$. By using the same method as in (a), we obtain the result. ■

Proposition 8 *In the reactive game B, suppose* $\tilde{\varepsilon}_1 \succ_{SSD} \tilde{\varepsilon}_2$. *Then,*

(a) $E_{\tilde{\varepsilon}_1}[\Pi_M^B] \leq E_{\tilde{\varepsilon}_2}[\Pi_M^B]$ *where the equality holds when* $\Pr\{\tilde{\varepsilon}_1 > 4k\tilde{\theta} - 1\} = \Pr\{\tilde{\varepsilon}_2 > 4k\tilde{\theta} - 1\} = 1$;

(b) $E_{\tilde{\varepsilon}_1}[w_2^B] \leq E_{\tilde{\varepsilon}_2}[w_2^B]$ *where the equality holds when* $\Pr\{\tilde{\varepsilon}_1 > 4k\tilde{\theta} - 1\} = \Pr\{\tilde{\varepsilon}_2 > 4k\tilde{\theta} - 1\} = 1$ *or* $\Pr\{\tilde{\varepsilon}_1 \leq 4k\tilde{\theta} - 1\} = \Pr\{\tilde{\varepsilon}_2 \leq 4k\tilde{\theta} - 1\} = 1$;

(c) $E_{\tilde{\varepsilon}_1}[x_2^B] \geq E_{\tilde{\varepsilon}_2}[x_2^B]$ *where the equality holds when* $\Pr\{\tilde{\varepsilon}_1 > 4k\tilde{\theta} - 1\} = \Pr\{\tilde{\varepsilon}_2 > 4k\tilde{\theta} - 1\} = 1$ *or* $\Pr\{\tilde{\varepsilon}_1 \leq 4k\tilde{\theta} - 1\} = \Pr\{\tilde{\varepsilon}_2 \leq 4k\tilde{\theta} - 1\} = 1$.

Proof. (a) From Proposition 3, Π_M^B is increasing and convex in $\tilde{\varepsilon}$ for $\tilde{\varepsilon} \leq 4k\tilde{\theta} - 1$ and is linearly increasing for $\tilde{\varepsilon} > 4k\tilde{\theta} - 1$. Thus, Π_M^B is increasing and and convex in $\tilde{\varepsilon}$. By using the same method as in the proof of Proposition 7, we obtain the result.

(b) and (c) The proofs are similar to (a), hence we omit them. ■

A4. Non-monotonicity in Comparative Statics of Game *AB*

As discussed in Appendix A3, the retailer's expected profit $E_{\tilde{\theta},\tilde{\varepsilon}}[\Pi_R(x_1, w_1)]$ in (20) is a piecewise continuous function of x_1, and the optimal quantity $x_1^{AB}(w_1)$ which maximizes $E_{\tilde{\theta},\tilde{\varepsilon}}[\Pi_R(x_1, w_1)]$ is either a boundary point between any two intervals or an interior point at which the first order condition is satisfied. The non-monotonicity of x_1^{AB} and w_1^{AB} is due to the fact that $x_1^{AB}(w_1^{AB})$ can move from an interior optimal point in one interval of x_1 to an interior optimal point in the other interval as w_1^{AB} varies with a change of any parameter value. To illustrate, we provide the following numerical examples:

Example 1. When $(k,\ e,\ r) = (0.2,\ 0.3,\ 0.1)$, $x_1^{AB} = 0.036$, $Ex_2^{AB} = 0.159$, $w_1^{AB} = 0.662$, $Ew_2^{AB} = 0.609$, $E\Pi_R^{AB} = 0.036$, $E\Pi_M^{AB} = 0.122$.

Example 2. When $(k,\ e,\ r) = (0.3,\ 0.3,\ 0.1)$, $x_1^{AB} = 0.087$, $Ex_2^{AB} = 0.172$, $w_1^{AB} = 0.547$, $Ew_2^{AB} = 0.481$, $E\Pi_R^{AB} = 0.063$, $E\Pi_M^{AB} = 0.139$.

Example 3. When $(k,\ e,\ r) = (0.4,\ 0.3,\ 0.1)$, $x_1^{AB} = 0.084$, $Ex_2^{AB} = 0.208$, $w_1^{AB} = 0.611$, $Ew_2^{AB} = 0.416$, $E\Pi_R^{AB} = 0.074$, $E\Pi_M^{AB} = 0.149$.

In the above examples, as k increases from 0.2 to 0.3 and to 0.4 for the fixed values of (e, r), neither w_1^{AB} nor x_1^{AB} has changed monotonically (see also Figure 5). We find that x_1^{AB} in each example is attained in a different interval of x_1 in which one of θ_d, θ_e and θ_f differs as follows:

- In Example 1, x_1^{AB} is the interior optimal point at which $\theta_d = 1+r$, $\theta_e = \frac{1-e+2x_1^{AB}}{4k}$, and $\theta_f = 1-r$;
- In Example 2, x_1^{AB} is the interior optimal point at which $\theta_d = 1+r$ and $\theta_e = \theta_f = 1-r$;
- In Example 3, x_1^{AB} is the interior optimal point at which $\theta_d = \frac{1+e+2x_1^{AB}}{4k}$ and $\theta_e = \theta_f = 1-r$.

For the same reason, in some instances, we observe that both w_1^{AB} and x_1^{AB} have increased with an increase of k. Although the order quantities in each stage may increase or decrease in k, Table 2 shows that the expected total order quantity is always nondecreasing in k.

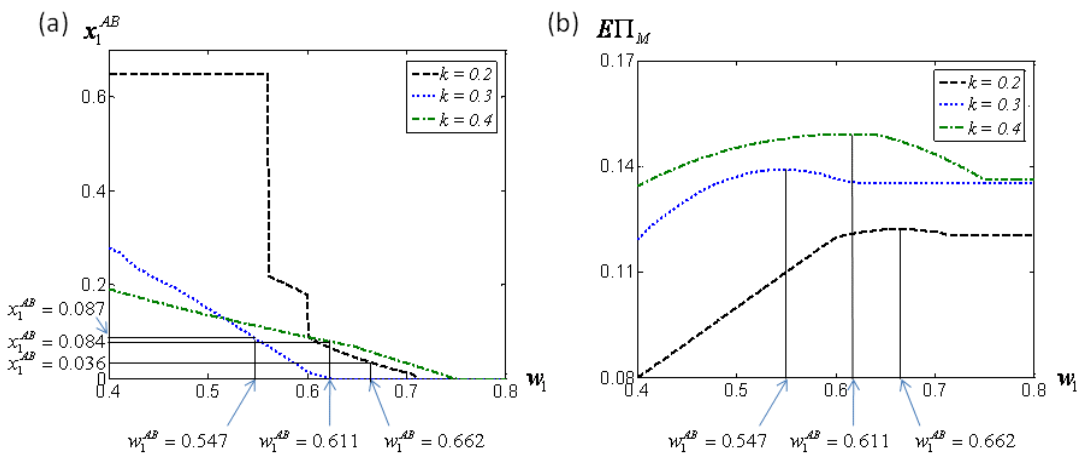

Figure 5: Examples 1, 2 and 3: (a) x_1^{AB} as a Function of w_1 and (b) $E\Pi_M$ as a Function of w_1.