Optimal Pricing and Rebate Strategies in a Two-Level Supply Chain

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Abstract

When selling electronic products, manufacturers and retailers often offer rebates to stimulate sales. Due to certain adverse effects, however, some manufacturers and retailers are contemplating the elimination of their rebate programs. This paper sheds light on the debate about the value of rebate programs by presenting a model for evaluating the conditions under which a firm should offer rebates in a competitive environment. Specifically, we consider a two-level supply chain comprising one manufacturer and one retailer. Each firm makes three decisions: the regular (wholesale or retail) price, whether or not to offer rebates, and the rebate value should the firm decide to launch a rebate program. We determine the equilibrium of a vertical competition game between the manufacturer (leader) and the retailer (follower), and we provide insights about how competition affects the conditions under which a firm should offer rebates in equilibrium.

Keywords: marketing/operations interface, promotion, rebate, supply chain management, vertical competition.

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1 Introduction

Ever since Proctor and Gamble issued rebate coupons to consumers in the 1970s, many manufacturers and retailers have followed suit. Grow (2005) estimates that over 400 million rebate programs are offered in the US with a total face value of US$6 billion each year. According to a report issued by the Food Marketing Institute (www.fmi.org) in 2006, 21% of shoppers use coupons every time they shop at a supermarket. Rebates are even more popular with consumers shopping for home appliances or electronics: NPD Group (www.npd.com) reports that 20% of digital products (cameras, camcorder, etc.) and 33% of computer hardware products were sold with rebates in 2004. The popularity of rebates in electronic products raises an interesting question: why should manufacturers/retailers offer rebates when they can easily mark down their selling prices temporarily? In addition to the fact that manufacturer rebates enable manufacturers to offer discounts to consumers directly, major benefits of rebates include:

1. Rebates can stimulate additional sales via a temporary price drop, while maintaining the marked price. Because the marked price never changes, consumers are less resistant to paying the usually marked price once a rebate program is discontinued (Avila and Avila (1986)). In addition, unlike rebates, price markdowns are often interpreted as a signal for further price drops, which could induce some strategic consumers to delay their purchase (c.f., Elmagrabhy et al. (2008)).

2. Rebates can be more cost effective than price markdowns because the actual redemption rate is less than 100%, due to breakage (customers who either forget to redeem or fail to submit the requisite documentation) and slippage (customers who fail to deposit the rebate checks). According to Grow (2005), over 40% of all rebates are never redeemed. As a case in point, TiVo's profit in 2004 increased by US$5 million when over 50,000 of TiVo's new subscribers failed to redeem their US$100 mail-in coupons.

While rebates are recognized as an effective promotional tool, there are several adverse effects that include:

1. Rebates can generate less additional demand than price markdowns. Because of additional efforts of redeeming rebates and the temporal difference between purchasing products and receiving discounts, consumers may consider a rebate of $1 less valuable than a price discount of $1 (e.g., Soman (1998) and Arcelus et al. (2006)). Customer dissatisfaction with the

complicated process of redemption can also generate negative publicity. For instance, in response to overwhelming on-line complaints about its rebate program, Dell shut down its on-line customer service forums in 2005.

2. Rebates can incur higher advertising and distribution costs associated with the rebate promotion and higher processing costs associated with the redemption process.

In view of the pros and cons of rebates, Darlin (2006) reports that some manufacturers (e.g., Dell and Hewlett-Packard) and retailers (e.g., Best Buy and OfficeMax) are phasing out their rebate programs to focus more on price reductions or instant rebates.[2] However, NPD Group warns that eliminating rebates can be risky because rebates can drive higher sales and higher store traffic. As manufacturers (e.g., Samsung and Sony) and retailers (e.g., Staples and Fry's) continue to offer rebates, Arar (2007) concludes that rebates will not disappear any time soon.

The current debate over rebates has motivated us to examine a basic question: under what conditions should a firm in a supply chain offer rebates? We construct a model for analyzing the pricing and rebate issues arising in a two-level supply chain that comprises one manufacturer (e.g., Panasonic) and one retailer (e.g., Circuit City). The manufacturer sells a single type of product to customers through the retailer over a selling season. Our model entails a Stackelberg game in which the manufacturer acts as the leader and the retailer as the follower. Each firm (manufacturer or retailer) makes three decisions at the beginning of the selling season: (1) the regular (wholesale or retail) price for the entire selling season; (2) whether or not to offer rebates, and (3) the size of the rebates to be offered during a pre-specified promotion period. We denote 'manufacturer rebates' ('retailer rebates') as the rebates offered by the manufacturer (retailer) to consumers.

The pricing and rebate decisions can be recast in a purely pricing context as follows. When a firm decides not to offer rebates, the corresponding effective (wholesale or retail) price will be constant; hence, offering no rebates is similar to an EDLP (Everyday Low Price) format. Alternatively, when a firm offers rebates, the corresponding effective (wholesale or retail) price is low during the promotion period and is high for the remainder of the selling season. Hence, offering rebates is similar to a HILO (Promotional) price format (but not identical due to breakage and slippage). Thus, our model can be interpreted as a vertical competition game in which each player has to adopt either an EDLP or HILO price format.

[2] An instant rebate is akin to a price reduction because the rebate is redeemed at the time of purchase and the redemption rate is nearly 100%. With 100% redemption rate, industry observers such as NPD Group speculate that the instant rebate value would be lower than that of a mail-in rebate.

Before we present our analysis, let us provide a preview of our main results. In our model, each firm evaluates the trade-off between the fixed cost of launching a rebate program and the additional gross profit generated by the rebate program. Let f_R (f_M) denote the retailer's (manufacturer's) fixed promotion cost. A basic intuition is that the retailer (or the manufacturer) would offer rebates if its fixed cost f_R (or f_M) is below some break-even point f_R^* (or f_M^*). Clearly, if each of the firms is non-strategic, in the sense that it does not take the other firm's decision into consideration when making its own decision, then these thresholds f_R^* and f_M^* are constants as illustrated in Figure 1(a). However, in a competitive environment, each player is strategic and takes the other party's decision into consideration. Figure 1(b) shows a typical example of the firms' rebate strategies in equilibrium. As a result of strategic interaction, the structural form of the rebate strategies in equilibrium becomes fairly complex and non-intuitive. For instance, a higher fixed cost does not necessarily discourage the manufacturer from offering rebates; i.e., it is possible that the manufacturer would offer rebates at (f_R, f_M) but not at (f_R', f_M') for some $f_R' < f_R$ and $f_M' \leq f_M$ (e.g., see points A and B in Figure 1(b)). Our analysis also reveals an interesting phenomenon: a more effective manufacturer rebate program (that generate more demand) would lead to a win-win situation in which both firms enjoy higher profits, while a more effective retailer rebate program could lead to a win-lose situation in which the manufacturer enjoys a higher profit and the retailer obtains a lower profit. In addition, we show that, depending on the attributes of each firm's rebate program, the equilibrium can take a structural form which is different from that in Figure 1(b). Our model captures the strategic interactions between firms in a supply chain as well as the specifics of each firm's rebate program.

Insert Figure 1 about here.

This paper is organized as follows. Section 2 provides a brief review of related literature. In Section 3 we first model the demand and profit functions associated with different pricing and rebate strategies, and then we explicate the underlying approach for analyzing the Stackelberg game. Section 4 presents the rebate and pricing strategies of both firms in equilibrium. In Section 5, we conduct a numerical analysis to investigate the impact of various parameters in equilibrium. Section 6 concludes this paper with some suggestions for future research.

2 Literature Review

The literature in sales promotion (e.g., Blattberg and Neslin (1990)) delineates three types of promotions (see Figure 2): *trade promotions* from the manufacturer to the retailer, *retailer promotions* from the retailer to the consumer, and *manufacturer promotions* from the manufacturer to the consumer (which are sometimes called consumer promotions). To entice retailers to increase sales efforts for certain brands, the corresponding manufacturer often offers trade promotions in various forms: off-invoice discounts, discounts based on the retailer's order quantity, discounts based on the retailer's actual sales, etc. As articulated by Buzzell et al. (1990), retailers do not always pass the benefits of the trade promotion on to consumers. This observation has motivated many marketing researchers to analyze the effectiveness of different types of trade promotions (c.f., Lal (1990a), Neslin et al. (1995), Lal et al. (1996), and Tyagi (1999)). Trade deals have also been examined in the context of supply chain contracts. For instance, Taylor (2002) shows that a manufacturer's rebates paid to a retailer for each unit sold beyond a target level can achieve supply chain coordination. Drèze and Bell (2003) show that retailers prefer off-invoice trade promotions, whereas manufacturers prefer price discounts based on the retailer's actual sales quantity during the promotion period. Comprehensive reviews of supply chain contracts can be found in Cachon (2003) and Tang (2006). Our paper differs from this literature in that we do not consider trade promotions; instead, we focus on both retailer and manufacturer rebates.

Insert Figure 2 about here.

Retailer promotions have also received attention among marketing researchers. One common form of retailer promotion is a price discount. By examining different analytical models, researchers find different reasons for offering price discounts including: retailers' inventory holding cost is higher than that of the customers (e.g., Blattberg et al. (1981)), price discrimination among heterogeneous consumers (e.g., Varian (1980), Narasimhan (1988)), competition among multiple retailers (e.g., Lal (1990b), Raju et al. (1990) and Rao (1991)), and reference price effects (e.g., Winer (1986), Greenleaf (1995), Kopalle et al. (1996)). Another common form of retailer promotions is in-store coupons (c.f., Blattberg and Neslin (1990) and Neslin (2002)). This stream of research focuses on retailer promotions in two major settings: horizontal competition among stores, and the interaction between retailers and customers. In contrast, we examine the interaction between retailer promotions and manufacturer promotions in the context of a vertical competition game.

To ensure that customers receive price discounts during the promotion period, manufacturer rebates are becoming a common form of manufacturer promotions in the durable goods markets. First, Gerstner and Hess (1991) consider a model with two market segments in which the customers in one of the segments have a higher reservation price and a higher transaction cost of rebate redemption. By considering 4 different settings – trade deals only, manufacturer rebates only, a combination of trade deals and manufacturer rebates, and retailer rebates only – they examine the conditions under which a particular setting can be used to motivate a retailer to serve both segments effectively. Second, by considering a case in which a retailer needs to determine the order quantity so as to meet uncertain demand, Chen et al. (2007) examine the impact of manufacturer rebates on the manufacturer's and retailer's expected profits. They show that manufacturer rebates always benefit the manufacturer unless the redemption rate is 100%. Third, in a similar setting as considered in Chen et al. (2007), Aydin and Porteus (2008) consider both manufacturer rebates and channel rebates (payments offered by the manufacturer to the retailer based on actual sales). They show that, relative to no rebates, channel rebates yield higher profits for both the retailer and the entire supply chain; however, manufacturer rebates yield higher profits for the retailer but not necessarily for the entire supply chain. Fourth, Baysar et al. (2007) analyze a Stackelberg game between a manufacturer and a retailer and show that manufacturer rebates perform better than trade deals when the market potential is highly uncertain.

Our paper complements the rebate literature in two significant ways. First, we consider a vertical competition game in which each firm (manufacturer or retailer) makes three decisions: the regular (wholesale or retail) price, whether or not to offer a rebate program, and the rebate value should the firm decide to launch a rebate program. Hence, the pricing and rebate strategies for each firm to deploy are determined endogenously. Second, our model explicitly captures a firm-specific fixed cost (advertising, distributing rebate coupons, etc.) that a firm will incur when launching a rebate promotion program. In the context of retailer rebate coupons, Blattberg and Neslin (1990) illustrate that this fixed cost varies between 30% and 66% of the total promotion budget. We evaluate the trade-off between this fixed cost and the additional profit generated by the rebate program, and establish the conditions under which a firm (manufacturer or retailer) should offer rebates in equilibrium.

Lastly, we note that our vertical competition game between a manufacturer and a retailer complements the models of horizontal competition between two retailers, each of which chooses between the EDLP and the HILO price format. Lattin and Ortmeyer (1991) and Lal and Rao

(1997) show that both EDLP and HILO stores can coexist in equilibrium under certain conditions.

3 The Model

Consider a two-level supply chain comprising two players: a manufacturer (leader) and a retailer (follower). The manufacturer, indexed by $i = M$, sells a single type of product through the retailer, indexed by $i = R$, over a short selling season. At the beginning of the selling season, each firm specifies a rebate strategy (i.e., offer rebates to customers or not) and then decides on the regular price and rebate value (if any). Throughout this paper, we shall focus on the case in which each firm will consider using rebates as the only promotion mechanism. We use superscripts j and k to denote the manufacturer's and the retailer's rebate strategies, respectively. As noted before, offering no rebates (offering rebates) is akin to an EDLP (HILO) price format; hence, we let j and k equal E when no rebate is offered and equal H when rebates are offered.

The manufacturer and the retailer enter a Stackelberg game (Figure 3). In stage 1, the manufacturer (leader) decides on the rebate strategy $j \in \{E, H\}$. For any given manufacturer rebate strategy $j \in \{E, H\}$ selected in stage 1, the manufacturer chooses the regular wholesale price w^j in stage 2. If $j = H$, then the manufacturer needs to determine the rebate value r_M^H to be offered over a pre-specified promotion period captured by an exogenously specified parameter $\beta_M \in (0, 1)$.[3] For instance, $\beta_M = 0.2$ means that the manufacturer offers rebates 20% of the time during a selling season. Clearly, if $j = E$, $r_M^E = 0$. Given the manufacturer's pricing and rebate strategy, (w^j, r_M^j), the retailer (follower) first decides on the rebate strategy $k \in \{E, H\}$. Then the retailer chooses a regular retail price p^k. If $k = H$, the retailer needs to specify the rebate value r_R^H to be offered over a pre-specified promotion period captured by an exogenously specified parameter $\beta_R \in (0, 1)$. If $k = E$, $r_R^E = 0$. Once both firms make their rebate and pricing decisions, denoted by (w^j, r_M^j, p^k, r_R^k), the demand function and the profit functions for both firms are determined.

Insert Figure 3 about here.

[3] Pre-specifying a promotion period is not uncommon in practice. For instance, Barilla pre-specifies their promotion periods (c.f., Hammond (1994)), some electronics manufacturers pre-specify their special introductory promotion periods, and some retailers pre-specify their promotion periods (Robinson's Saturday Early Bird special promotion, Nordstrom's Yearly Sales Event, etc). This setting is similar to the mixed equilibrium concept (e.g., Varian (1980), Narasimhan (1988)) where manufacturer/retailer randomize their prices in any given time period.

3.1 Demand and Profit Functions

When neither firm offers rebates (i.e., $j = k = E$), we assume a linear demand function $D = a - bp^E$, where $a > 0$ represents the base demand and $b > 0$ represents the sensitivity of the demand to the regular retail price p^E. To ensure that $D > 0$, we assume the regular retail price $p^k \in (0, \frac{a}{b})$ for $k = E, H$ and the regular wholesale price $w^j \in (0, \frac{a}{b})$ for $j = E, H$.

Our model captures three major effects when firm $i \in \{M, R\}$ offers rebates of value r_i: (i) the demand is increased by $b_i r_i$, where b_i (> 0) denotes the sensitivity of the demand to the rebate value, (ii) the profit margin of firm i is reduced by $\theta_i r_i$, where θ_i ($\in (0, 1]$) denotes the redemption rate of rebates, and (iii) a fixed cost f_i (> 0) is incurred for promoting, distributing, and processing rebates. Below, we elaborate the impact of these three effects on the demand and profits of the firms. Table 1 summarizes our notation.

Insert Table 1 about here.

Rebates increase demand in two ways: rebates may increase the base demand (the size of a market) by inducing customers to consume more, and they may entice new customers to buy the product at a lower rebate-reduced price (see Bell et al. (1999), Bell and Boztug (2007), and the references therein for details). The additional demand generated by rebates of value r_i is captured by the term $b_i r_i$. The parameter b_i is equal to the normal demand sensitivity b times the value of \$1 rebates of firm i perceived by consumers. Consumers may perceive the value of a rebate of \$1 less than a price discount of \$1 because redeeming the rebate requires additional efforts (e.g., Arcelus et al. (2006), Aydin and Porteus (2008), Chen et al. (2007)). The parameters b_R and b_M need not be the same because each value depends on the specifics of the rebate offer. Even if two rebate offers have the same face value, consumers may perceive them differently due to different redemption processes (on-line rebates, mail-in rebates, etc.) and the expected delay until they receive the rebates.

For any rebate value r_M^j chosen by the manufacturer and any pricing strategy (p^k, r_R^k) chosen by the retailer, the total demand over the selling season can be written compactly as:

$$\begin{aligned} D(r_M^j, p^k, r_R^k) &= \beta_M \beta_R (a - bp^k + b_M r_M^j + b_R r_R^k) + \beta_M (1 - \beta_R)(a - bp^k + b_M r_M^j) \\ &\quad + (1 - \beta_M)\beta_R (a - bp^k + b_R r_R^k) + (1 - \beta_M)(1 - \beta_R)(a - bp^k) \qquad (1) \\ &= a - bp^k + \beta_M b_M r_M^j + \beta_R b_R r_R^k \end{aligned}$$

For example, when the manufacturer offers a rebate ($r_M^H > 0$) and the retailer offers no rebate

$(r_R^E = 0)$ (i.e., $j = H$ and $k = E$), the demand is equal to $(a - bp^E + b_M r_M^H)$ during the manufacturer's rebate promotion period and is equal to $(a - bp^E)$ during the remainder of the selling season. Since β_M represents the relative length of the manufacturer rebate promotion period, the total demand over the entire season is equal to $D = \beta_M(a - bp^E + b_M r_M^H) + (1 - \beta_M)(a - bp^E) = a - bp^E + \beta_M b_M r_M^H$.

To model the profit function of each firm, let us consider the profit margin and the fixed cost associated with a rebate program. To simplify our exposition, we assume that the unit manufacturing cost incurred by the manufacturer, the unit operations cost incurred by the retailer, and the unit rebate promotion cost incurred by the manufacturer/retailer are zero. The exact same approach can be used to analyze the case when these unit costs are positive.

Using the redemption rate θ_i, we can represent the manufacturer's effective profit margin as $(w^j - \theta_M r_M^j)$ and the retailer's effective profit margin as $(p^k - w^j - \theta_R r_R^k)$ for any set $(w^j, r_M^j; p^k, r_R^k)$ of rebate and pricing strategies selected by both firms. The parameter θ_i denotes the fraction of consumers who actually redeem their rebate among all consumers who have purchased the product. According to Jolson et al. (1987), 70% of all consumers whose purchases are influenced by mail-in rebates never claim their savings. This suggests that the values of a rebate perceived by consumers at the time of their purchase are different from their actual value. (Soman (1998) explains this phenomenon based on behavioral decision theory and tests several hypotheses experimentally.) Thus, the actual redemption rate θ_i does not affect the demand directly.

A constant marginal cost per unit time $f_i > 0$ is incurred when firm i offers rebates over a promotion period β_i. The cost f_i captures different fixed advertising and processing costs associated with the rebate promotion, and it does not depend on the rebate value nor the number of rebates claimed. Hence, the total fixed cost associated with a rebate program is equal to $\beta_i f_i$.[4]

Then the manufacturer's profit $\Pi_M(w^j, r_M^j, p^k, r_R^k)$ and the retailer's profit $\Pi_R(w^j, r_M^j, p^k, r_R^k)$

[4] While we consider the total fixed cost to be proportional to β_i, the exact same approach can be used to analyze the case when the total fixed cost is independent of β_i.

are:

$$\Pi_M(w^j, r_M^j, p^k, r_R^k) = \begin{cases} \beta_M\beta_R(w^j - \theta_M r_M^j)(a - bp^k + b_M r_M^j + b_R r_R^k) & \\ \quad +\beta_M(1-\beta_R)(w^j - \theta_M r_M^j)(a - bp^k + b_M r_M^j) & \\ \quad +(1-\beta_M)\beta_R w^j(a - bp^k + b_R r_R^k) & \\ \quad +(1-\beta_M)(1-\beta_R)w^j(a - bp^k) - \beta_M f_M & \text{if}\,(j,k) = (H,H), \\ \beta_M(w^j - \theta_M r_M^j)(a - bp^k + b_M r_M^j) & \\ \quad +(1-\beta_M)w^j(a - bp^k) - \beta_M f_M & \text{if}\,(j,k) = (H,E), \\ \beta_R(w^j)(a - bp^k + b_R r_R^k) + (1-\beta_R)(w^j)(a - bp^k) & \text{if}\,(j,k) = (E,H), \\ w^j(a - bp^k) & \text{if}\,(j,k) = (E,E), \end{cases} \tag{2}$$

$$\Pi_R(w^j, r_M^j, p^k, r_R^k) = \begin{cases} \beta_M\beta_R(p^k - w^j - \theta_R r_R^k)(a - bp^k + b_M r_M^j + b_R r_R^k) & \\ \quad +(1-\beta_M)\beta_R(p^k - w^j - \theta_R r_R^k)(a - bp^k + b_R r_R^k) & \\ \quad +\beta_M(1-\beta_R)(p^k - w^j)(a - bp^k + b_M r_M^j) & \\ \quad +(1-\beta_M)(1-\beta_R)(p^k - w^j)(a - bp^k) - \beta_R f_R & \text{if}\,(j,k) = (H,H), \\ \beta_M(p^k - w^j)(a - bp^k + b_M r_M^j) & \\ \quad +(1-\beta_M)(p^k - w^j)(a - bp^k) & \text{if}\,(j,k) = (H,E), \\ \beta_R(p^k - w^j - \theta_R r_R^k)(a - bp^k + b_R r_R^k) & \\ \quad +(1-\beta_R)(p^k - w^j)(a - bp^k) - \beta_R f_R & \text{if}\,(j,k) = (E,H), \\ (p^k - w^j)(a - bp^k) & \text{if}\,(j,k) = (E,E). \end{cases} \tag{3}$$

3.2 Backward Induction Steps

Before we present the analysis of our Stackelberg game, let us describe the decision making process associated with the following backward induction steps. For ease of reference, we use *parentheses* in the superscripts of decision variables and profits to denote their optimal values at the corresponding nodes in Figure 3. For example, at node $(j,k) = (E,E)$, $p^{(E,E)}$ represents the optimal value of the regular retail price p^E. At node (j,k) in stage 4, the retailer uses information (i.e., the manufacturer's rebate strategy j and the corresponding pricing strategy (w^j, r_M^j) and the retailer's rebate strategy k) to determine the optimal regular price $p^{(j,k)}$ and optimal rebate value $r_R^{(j,k)}$ that maximize his profit function $\Pi_R(w^j, r_M^j, p^k, r_R^k)$ given in (3). The retailer solves the following program:

$$(P4) \qquad \Pi_R^{(j,k)}(w^j, r_M^j) = \max_{p^k, r_R^k} \Pi_R(w^j, r_M^j, p^k, r_R^k), \quad \text{for } j,k = E,H. \tag{4}$$

By definition, $r_R^{(j,E)} = 0$. We suppress the fact that $p^{(j,k)}$ and $r_R^{(j,k)}$ depend on (w^j, r_M^j) for ease of exposition.

At node (j) in stage 3, the retailer determines the optimal rebate strategy $k^{(j)}$ by choosing the larger of the two profits $\Pi_R^{(j,E)}(w^j, r_M^j)$ and $\Pi_R^{(j,H)}(w^j, r_M^j)$ given in (4), yielding:

$$(P3) \qquad \Pi_R^{(j)}(w^j, r_M^j) = \max\ \{\Pi_R^{(j,E)}(w^j, r_M^j), \Pi_R^{(j,H)}(w^j, r_M^j)\}, \quad \text{for } j = E, H. \tag{5}$$

The retailer's best response rebate strategy can be prescribed as follows:

$$k^{(j)} = \begin{cases} E & \text{if } (w^j, r_M^j) \in T^{(j,E)} \\ H & \text{if } (w^j, r_M^j) \in T^{(j,H)}, \text{ where} \end{cases} \tag{6}$$

$$\begin{aligned} T^{(j,E)} &= \left\{(w^j, r_M^j) | \Pi_R^{(j,E)}(w^j, r_M^j) \geq \Pi_R^{(j,H)}(w^j, r_M^j)\right\}, \text{ and} \\ T^{(j,H)} &= \left\{(w^j, r_M^j) | \Pi_R^{(j,E)}(w^j, r_M^j) \leq \Pi_R^{(j,H)}(w^j, r_M^j)\right\}. \end{aligned}$$

By definition, $r_M^E = 0$ when $j = E$. Hence, the sets $T^{(E,E)}$ and $T^{(E,H)}$ depend only on w^E.

At node (j) in stage 2, the manufacturer knows his rebate strategy j chosen in stage 1 and anticipates the retailer's best response: $k^{(j)}$, $p^{(j,k^{(j)})}$, and $r_R^{(j,k^{(j)})}$. Hence, the manufacturer can determine his optimal pricing strategy $(w^{(j,*)}, r_M^{(j,*)})$ at node (j) by solving:[5]

$$(P2) \qquad \Pi_M^{(j,*)} = \max\ \{\Pi_M^{(j,E)},\ \Pi_M^{(j,H)}\} \text{ for } j = E, H \quad \text{, where} \tag{7}$$

$$\Pi_M^{(j,k^{(j)})} = \max_{(w^j, r_M^j) \in T^{(j,k^{(j)})}} \Pi_M(w^j, r_M^j, p^{(j,k^{(j)})}, r_R^{(j,k^{(j)})})) \ \text{ for } \ k^{(j)} = E, H, \tag{8}$$

where $\Pi_M(w^j, r_M^j, p^k, r_R^k)$ is given in (2). Given the manufacturer's optimal pricing strategy $(w^{(j,*)}, r_M^{(j,*)})$, we can retrieve the retailer's best response (i.e., $k^{(j,*)}$, $p^{(j,*)}$, and $r_R^{(j,*)}$) from (6) and from the optimal solution to problem (P4) accordingly.

At stage 1, the manufacturer can determine his optimal profit $\Pi_M^{(*)}$ and the corresponding optimal rebate strategy $j^{(*)}$ by choosing the larger of the two profits associated with $j = E$ and $j = H$ given in (7), yielding:

$$(P1) \qquad \Pi_M^{(*)} = \max\ \{\Pi_M^{(E,*)},\ \Pi_M^{(H,*)}\}. \tag{9}$$

Substituting $j = j^{(*)}$ into $k^{(j,*)}$, $(w^{(j,*)}, r_M^{(j,*)})$, and $(p^{(j,*)}, r_R^{(j,*)})$, we obtain the backward-induction outcomes of our Stackelberg game: $(j^{(*)}, k^{(*)})$, $(w^{(*)}, r_M^{(*)})$, and $(p^{(*)}, r_R^{(*)})$.

[5]The additional $*$ in the superscript is intended for ease of reference in a later section.

4 Analysis

This section is organized as follows. In Section 4.1, we analyze the equilibrium of subgame 1 for the case when $j = E$ (i.e., the manufacturer offers no rebates). By following the backward induction steps as described in the last section, we present the results associated with problems (P4), (P3), and (P2), respectively. In Section 4.2, we use the same approach to analyze subgame 2 for the case when $j = H$. By comparing the manufacturer's profits under rebate strategies $j = E$ and $j = H$, we determine the Stackelberg equilibria by solving problem (P1) in Section 4.3.

4.1 Subgame 1: The manufacturer offers no rebates

When the manufacturer offers no manufacturer rebates (i.e., $j = E$), $r_M^E = 0$, so we shall suppress this decision variable throughout this section. By examining the optimal solution to problem (P4) associated with node (E, k) in stage 4 for $k = E$ or H, we have:

Proposition 1 *Consider the case when the manufacturer offers no rebates (i.e., $j = E$).*
(a) Suppose the retailer offers no rebates (i.e., $k = E$). Then, for any given regular wholesale price w^E, problem (P4) yields:

$$p^{(E,E)} = \frac{a + bw^E}{2b}, \text{ and } \quad \Pi_R^{(E,E)} = \frac{(a - bw^E)^2}{4b}. \tag{10}$$

(b) Suppose the retailer offers rebates (i.e., $k = H$) and the following condition holds:

$$1 < \frac{b_R}{b\theta_R} < \frac{2}{\beta_R} - 1. \tag{11}$$

Then, for any given regular wholesale price w^E, problem (P4) yields:

$$p^{(E,H)} = \frac{2b_R\theta_R(a + bw^E) - \beta_R(b\theta_R + b_R)(a\theta_R + b_R w^E)}{A}, \tag{12}$$

$$r_R^{(E,H)} = \frac{(b_R - b\theta_R)(a - bw^E)}{A}, \text{ and} \tag{13}$$

$$\Pi_R^{(E,H)} = \frac{b_R\theta_R(1 - \beta_R)(a - bw^E)^2}{A} - \beta_R f_R, \quad \text{where} \tag{14}$$

$$A = 4bb_R\theta_R - \beta_R(b\theta_R + b_R)^2. \tag{15}$$

Also, $p^{(E,H)}, r_R^{(E,H)}$, and $\Pi_R^{(E,H)}$ are increasing in b_R and decreasing in θ_R.

Proof: All proofs are provided in the Appendix.

Part (a) of Proposition 1 is a standard result when neither firm offers rebates. When the retailer offers rebates, part (b) has the following implications. As consumers show a stronger response to

the retailer rebate program (i.e., when b_R increases) or redeem rebates at lower rates (i.e., when θ_R decreases), the retailer can obtain a higher profit by charging a higher regular retail price $p^{(E,H)}$ and offering a higher rebate value $r_R^{(E,H)}$. Condition (11) is required for the unique equilibrium to exist. It states that the sensitivity of the demand to rebate value, b_R, needs to be within the specified interval of the sensitivity of the demand to regular price, b. This is in line with our basic intuition in the following sense. If b_R is too low (i.e., $\frac{b_R}{b\theta_R} \leq 1$), a rebate program does not generate enough additional demand, so the rebate program should not be launched (i.e., $r^{(E,H)} \leq 0$). On the other hand, if b_R is too high (i.e., $\frac{b_R}{b\theta_R} \geq \frac{2}{\beta_R} - 1$), greater discounts are always preferred and no interior solutions exist. When β_R is small, the range that $\frac{b_R}{b\theta_R}$ can take is fairly wide (e.g., this range is between 1 and 9 when $\beta_R = 0.2$). As such, condition (11) is not restrictive. For the remainder of this paper, we assume it holds.

By examining (10), (12) and (13), we can show that:

Corollary 1 *(a)* $p^{(E,H)} > p^{(E,E)} > p^{(E,H)} - r_R^{(E,H)}$*, and (b)* $\beta_R(p^{(E,H)} - r_R^{(E,H)}) + (1-\beta_R)p^{(E,H)} > p^{(E,E)}$ *if and only if* $b_R > b(2-\theta_R)$*.*

Part (a) shows that the retailer's optimal regular (discount) price under the HILO format (i.e., when $k = H$) is higher (lower) than the optimal regular retail price under the EDLP format (i.e., when $k = E$). Also, part (b) shows that the average retail price under the HILO format is higher than the regular price under the EDLP format when the retailer rebate sensitivity b_R is sufficiently high. These results are consistent with the empirical results presented in Ho et al. (1998).

By comparing the retailer's profits associated with the EDLP and the HILO price formats given in (10) and (14) as in problem (P3), we establish the following result:

Proposition 2 *For any given regular wholesale price* w^E *under the case when* $j = E$*, the retailer's best response rebate strategy in stage 3,* $k^{(E)}$*, satisfies:*

$$k^{(E)} = \begin{cases} E & \text{if } w^E \in T^{(E,E)} = [\tau^E, \frac{a}{b}) \\ H & \text{if } w^E \in T^{(E,H)} = (0, \max\{0, \tau^E\}], \quad \text{where} \end{cases} \tag{16}$$

$$\tau^E = \frac{a}{b} - \frac{2}{b_R - b\theta_R}\sqrt{\frac{A}{b} f_R}. \tag{17}$$

Proposition 2 suggests that the retailer should not offer rebates when the wholesale price w^E is high. This result is intuitive because, when the wholesale price is high, the retailer cannot afford to offer retailer rebates with a meager profit margin. Notice that $\tau^E \leq 0$ when $f_R \geq f_R^a$, where

$$f_R^a = \frac{a^2(b_R - b\theta_R)^2}{4bA}. \tag{18}$$

Hence, the retailer should not offer rebates (i.e., $k^{(E)} = E$) when the retailer's fixed promotion cost f_R exceeds the threshold f_R^a. This result is also intuitive.

Given the retailer's best response rebate strategy $k^{(E)}$ stated in Proposition 2 and the corresponding pricing strategy stated in Proposition 1, we can specify the manufacturer's profit function as follows:

$$\Pi_M(w^E, p^{(E,k^{(E)})}, r_R^{(E,k^{(E)})}) = \begin{cases} \frac{w^E(a-bw^E)}{2} & \text{if } k^{(E)} = E \\ \frac{2b\theta_R b_R(1-\beta_R)w^E(a-bw^E)}{A} & \text{if } k^{(E)} = H. \end{cases} \tag{19}$$

It is easy to check from (16) and (19) that $w^E = \max\{\frac{a}{2b}, \tau^E\}$ is the optimal wholesale price that maximizes the profit function Π_M given in (19) when $w^E \in T^{(E,E)} = [\tau^E, \frac{a}{b})$, and that $w^E = \min\{\frac{a}{2b}, \max\{0, \tau^E\}\}$ is the optimal wholesale price that maximizes the profit function Π_M given in (19) when $w^E \in T^{(E,H)} = [0, \max\{0, \tau^E\})$, respectively. By comparing the corresponding optimal profits $\Pi_M^{(E,E)}$ and $\Pi_M^{(E,H)}$, we have:

Proposition 3 *Suppose the manufacturer offers no rebates (i.e., $j = E$). Then the manufacturer's optimal wholesale price $w^{(E,*)}$ and the resulting profit $\Pi_M^{(E,*)}$ in stage 2 can be expressed as:*

$$w^{(E,*)} = \begin{cases} \frac{a}{2b} & if\ f_R \geq \bar{f}_R^E \\ \frac{a}{b} - \frac{2}{b_R - b\theta_R}\sqrt{\frac{Af_R}{b}} & if\ \underline{f}_R^E < f_R \leq \bar{f}_R^E \\ \frac{a}{2b} & otherwise \end{cases}$$

$$\Pi_M^{(E,*)} = \begin{cases} \frac{a^2}{8b} & if\ f_R \geq \bar{f}_R^E \\ \frac{4ab_R\theta_R(1-\beta_R)}{b_R - b\theta_R}\left\{\sqrt{\frac{bf_R}{A}} - \frac{2bf_R}{a(b_R - b\theta_R)}\right\} & if\ \underline{f}_R^E < f_R \leq \bar{f}_R^E \\ \frac{a^2 b_R\theta_R(1-\beta_R)}{2A} & otherwise \end{cases}$$

where $\bar{f}_R^E = \frac{a^2(b_R - b\theta_R)^2\left\{\beta_R(b_R^2 + b^2\theta_R^2 - 6b\theta_R b_R) + 4b\theta_R b_R + 4(b_R - b\theta_R)\sqrt{b\theta_R b_R \beta_R(1-\beta_R)}\right\}}{64b^2\theta_R b_R(1-\beta_R)A}$, *and* $\underline{f}_R^E = \frac{1}{4}f_R^a$.

By using the optimal wholesale price $w^{(E,*)}$ given in Proposition 3, we can apply the results stated in Propositions 2 and 1 to retrieve the retailer's optimal rebate strategy $k^{(E,*)}$ and optimal pricing strategy $(p^{(E,*)}, r_R^{(E,*)})$, getting:

Corollary 2 *Suppose the manufacturer offers no rebates (i.e., $j = E$). Corresponding to the manufacturer's wholesale price $w^{(E,*)}$, the retailer's best response and the resulting profit are:*

$$k^{(E,*)} = \begin{cases} E \\ H \\ H \end{cases} ;\ p^{(E,*)} = \begin{cases} \frac{3}{4}\frac{a}{b} & if\ f_R \geq \bar{f}_R^E \\ \frac{a}{b} - \frac{2b_R\{2b\theta_R - \beta_R(b_R + b\theta_R)\}}{b_R - b\theta_R}\sqrt{\frac{f_R}{Ab}} & if\ \underline{f}_R^E < f_R \leq \bar{f}_R^E \\ \frac{a\{6b\theta_R b_R - \beta_R(2b\theta_R + b_R)(b_R + b\theta_R)\}}{2bA} & otherwise \end{cases}$$

$$r_R^{(E,*)} = \begin{cases} 0 \\ 2\sqrt{\frac{bf_R}{A}} \\ \frac{a(b_R - b\theta_R)}{2A} \end{cases} ; \ \Pi_R^{(E,*)} = \begin{cases} \frac{a^2}{16b} & if \ f_R \geq \bar{f}_R^E \\ \frac{Af_R}{(b_R - b\theta_R)^2} & if \ \underline{f}_R^E < f_R \leq \bar{f}_R^E \\ \frac{a^2\theta_R b_R(1-\beta_R)}{4A} - \beta_R f_R & otherwise. \end{cases}$$

We now interpret Proposition 3 and Corollary 2 in conjunction with the impact of the fixed cost f_R on the equilibrium outcomes (Figure 4). (The analytical details are provided in the Appendix.) First, consider the 'base case' when the fixed cost f_R is sufficiently high; i.e., $f_R \geq \bar{f}_R^E$. Corollary 2 suggests that the retailer should not offer rebates and that $\Pi_R^{(E,*)} = \frac{1}{2}\Pi_M^{(E,*)}$ (as shown in Proposition 3). The former is intuitive and the latter corresponds to the double marginalization result of a decentralized supply chain in which the manufacturer (leader) earns twice the profit of the retailer (follower) (c.f., Spengler (1950)). Second, consider the case when f_R is sufficiently low; i.e., when $f_R \leq \underline{f}_R^E$. Corollary 2 suggests that the retailer should offer rebates. This result is intuitive. Also, it is easy to check that both firms earn higher profits than those in the base case when $f_R \geq \bar{f}_R^E$. The increase of profit is mainly due to the additional demand generated from the retailer rebates. Hence, lowering the fixed cost f_R can lead to a win-win situation. This observation implies that there is incentive for both firms to develop mechanisms for reducing the fixed cost f_R.

Insert Figure 4 about here.

Next, let us examine the case when the fixed cost $f_R \in [\underline{f}_R^E, \bar{f}_R^E)$. We obtain two counter-intuitive results as depicted in Figure 4: (1) the retailer's profit is increasing in the fixed cost f_R; and (2) the average retail price, $\beta_R(p^{(E,*)} - r_R^{(E,*)}) + (1 - \beta_R)p^{(E,*)}$, is decreasing in the fixed cost f_R. To explain these counter-intuitive results, let us first consider the case when $f_R = \underline{f}_R^E$. In this case, it is easy to check from Proposition 3 that the manufacturer's optimal wholesale price is the same as in the base case, and that the retailer would offer rebates and obtain the same profit as in the base case. Now, consider the case when the retailer's fixed promotion cost f_R exceeds $\underline{f}_R^E$. In this case, the retailer's profit will be below the base case if the retailer offers rebates. This implies that there is no incentive for the retailer to offer rebates unless the manufacturer is willing to lower the wholesale price $w^{(E,*)}$. By anticipating the retailer's rebate strategy, Proposition 3 and Figure 4(b) suggest that, in order to obtain a win-win situation, it is beneficial for the manufacturer to reduce the wholesale price $w^{(E,*)}$ so as to induce the retailer to offer rebates. As a result of this reduced wholesale price, the retailer will offer a lower average retail price, enjoy a higher demand, and obtain a higher profit as the fixed cost f_R increases over $[\underline{f}_R^E, \bar{f}_R^E)$.

Finally, consider the case when the fixed promotion cost $f_R = \bar{f}_R^E$. Figure 4 highlights the fact that there are two equilibria. Specifically, Proposition 3 suggests that the manufacturer may charge $\frac{a}{2b}$ or $\frac{a}{b} - \frac{2}{b_R - b\theta_R}\sqrt{\frac{A\bar{f}_R^E}{b}}$ as these two wholesale prices will lead to the same profit for the manufacturer. It is easy to check from Corollary 2 and Figure 4(a) that the retailer would obtain a higher profit in the latter case.

Notice that the backward-induction outcomes presented in Proposition 2 and Corollary 3 can also be represented by thresholds associated with b_R and θ_R. For instance, we can first obtain the thresholds $\bar{b}_R^E$ and $\underline{b}_R^E$ by expressing $\underline{f}_R^E$ and $\bar{f}_R^E$ in terms of b_R, respectively. Then we can derive the equilibrium according to the intervals as defined by $\underline{b}_R^E$ and $\bar{b}_R^E$ and obtain similar results: the retailer should not offer rebates if the incremental demand generated from rebates b_R is below the threshold $\underline{b}_R^E$. To reduce duplication, we shall focus on our discussion on the impact of the fixed costs f_R and f_M in equilibrium.

4.2 Subgame 2: The manufacturer offers rebates

When the manufacturer offers rebates (i.e., $j = H$), he needs to determine the regular wholesale price w^H and the rebate value r_M^H. We now present our results for subgame 2 by following the same approach as presented in Section 4.1. By examining the solution to problem (P4) associated with the node (H, k) in stage 4 for $k = E$ or H, we have:

Proposition 4 *Consider the case when the manufacturer offers manufacturer rebates (i.e., $j = H$). (a) Suppose the retailer offers no rebates (i.e., $k = E$). Then, for any given regular wholesale price w^H and manufacturer rebate value r_M^H, problem (P4) yields:*

$$p^{(H,E)} = \frac{a + bw^H + \beta_M b_M r_M^H}{2b} \quad \textit{and} \quad \Pi_R^{(H,E)} = \frac{(a - bw^H + \beta_M b_M r_M^H)^2}{4b}. \tag{20}$$

(b) Suppose the retailer offers rebates (i.e., $k = H$). Then, for any given regular wholesale price w^H and manufacturer rebate value r_M^H, problem (P4) yields:

$$p^{(H,H)} = \frac{2b_R\theta_R(a + bw^H + \beta_M b_M r_M^H) - \beta_R(b_R + b\theta_R)\left\{\theta_R(a + \beta_M b_M r_M^H) + b_R w^H\right\}}{A}, \tag{21}$$

$$r_R^{(H,H)} = \frac{(b_R - b\theta_R)(a - bw^H + \beta_M b_M r_M^H)}{A}, \textit{ and} \tag{22}$$

$$\Pi_R^{(H,H)} = \frac{b_R\theta_R(1 - \beta_R)(a - bw^H + \beta_M b_M r_M^H)^2}{A} - \beta_R f_R. \tag{23}$$

Proposition 4 is analogous to Proposition 1 if one treats $a + \beta_M b_M r_M^H$ as a in the demand function and treats w^H as w^E. This is because, when $j = H$, the corresponding total demand is

increased by $\beta_M b_M r_M^H$. Therefore, if one views this as an increase in the base demand a and views the decision variable w^H as w^E, then the retailer's profit functions associated with nodes (E, k) and (H, k) are identical. Similarly, by comparing the optimal retail prices under $k = E$ and under $k = H$, we obtain identical results as shown in Corollary 1 of subgame 1.

By comparing the retailer's profits $\Pi_R^{(H,E)}$ and $\Pi_R^{(H,H)}$ given in (20) and (23), respectively, we can determine the retailer's best response $k^{(H)}$ for problem (P3) as follows:

Proposition 5 *For any given regular wholesale price w^H and rebate value r_M^H under the case when $j = H$, the retailer's optimal rebate strategy in stage 3, $k^{(H)}$, satisfies:*

$$k^{(H)} = \begin{cases} E & \text{if } (w^H, r_M^H) \in T^{(H,E)} = \left\{(w^H, r_M^H) | w^H \geq \tau^H(r_M^H)\right\} \\ H & \text{if } (w^H, r_M^H) \in T^{(H,H)} = \left\{(w^H, r_M^H) | w^H \leq \tau^H(r_M^H)\right\}, \quad \text{where} \end{cases} \tag{24}$$

$$\tau^H(r_M^H) = \frac{a + \beta_M b_M r_M^H}{b} - \frac{2}{b_R - b\theta_R}\sqrt{\frac{A}{b} f_R}. \tag{25}$$

Notice that Proposition 5 is analogous to Proposition 2 if one views $a + \beta_M b_M r_M^H$ as a. As r_M^H increases, the effective base demand $a+\beta_M b_M r_M^H$ increases, the threshold τ^H given in (25) increases, and the size of the corresponding set $T^{(H,H)}$ also increases. Therefore, as the manufacturer offers a higher rebate value, the retailer can afford to offer rebates that would result in a lower margin but higher sales.

We now analyze the manufacturer's pricing decision associated with node (H) in stage 2. In preparation, we substitute the retailer's optimal prices given in Proposition 4 into (2) to obtain the manufacturer's profit function $\Pi_M(w^H, r_M^H,\ p^{(H,E)})$ for the case when $k^{(H)} = E$, and $\Pi_M(w^H, r_M^H, p^{(H,H)}, r_R^{(H,H)})$ for the case when $k^{(H)} = H$. By maximizing the former profit function over the set $T^{(H,E)}$ and the latter over $T^{(H,H)}$, we obtain the the optimal profits $\Pi_M^{(H,E)}$ and $\Pi_M^{(H,H)}$, respectively. By choosing the larger of these two, we determine $\Pi_M^{(H,*)}$ as defined in problem (P2) at node (H). Proposition 6 (see Appendix) establishes the manufacturer's optimal wholesale price $w^{(H,*)}$, manufacturer rebate value $r_M^{(H,*)}$, and the resulting profit $\Pi_M^{(H,*)}$ in stage 2. We can then apply Propositions 5 and 4 to retrieve the retailer's optimal rebate strategy $k^{(H,*)}$, pricing strategies $p^{(H,*)}$ and $r_R^{(H,*)}$, and resulting profit $\Pi_R^{(H,*)}$; Corollary 3 in the Appendix summarizes these results.

The results presented in Proposition 6 and Corollary 3 for the case when $j = H$ possess the same structure as those presented in Proposition 3 and Corollary 2 for the case when $j = E$. Specifically, there exist thresholds $\underline{f}_R^H$ and $\bar{f}_R^H$ that are analogous to the thresholds $\underline{f}_R^E$ and $\bar{f}_R^E$ as defined in Proposition 3. This analogy can be seen as follows. First, the retailer should not launch a rebate

program when the fixed promotion cost f_R is sufficiently high; i.e., $f_R \geq \bar{f}_R^H$. Second, the impact of f_R on the optimal prices and profits as shown in Figure 5 is similar to those as shown earlier in Figure 4. The counter-intuitive results observed from Figure 4 for the case when $f_R \in [\underline{f}_R^E, \bar{f}_R^E)$ in subgame 1 continue to occur for the case when $f_R \in [\underline{f}_R^H, \bar{f}_R^H)$ in subgame 2.

Insert Figure 5 about here.

4.3 Stackelberg Equilibrium

To determine the manufacturer's optimal rebate strategy $j^{(*)}$ in stage 1, we now compare the manufacturer's optimal profit $\Pi_M^{(E,*)}$ for the case $j = E$ (as given in Proposition 3) with $\Pi_M^{(H,*)}$ for the case $j = H$ (as given in Proposition 6 in Appendix). To do so, let us first superimpose Figures 4(a) and 5(a) and then compare these two profit functions in different intervals that depend on the thresholds $\underline{f}_R^E$, $\bar{f}_R^E$, $\underline{f}_R^H$ and $\bar{f}_R^H$. Depending on the ordering of these thresholds, we have different cases to analyze. However, we can use the following Lemma to reduce the number of cases to analyze.

Lemma 1 *The threshold $\underline{f}_R^E$ associated with subgame 1 and the threshold $\underline{f}_R^H$ associated with subgame 2 satisfy: $\underline{f}_R^E < \underline{f}_R^H$.*

Combining Lemma 1 with the fact that $\underline{f}_R^E < \bar{f}_R^E$ and $\underline{f}_R^H < \bar{f}_R^H$, one can check that there are only 3 cases to consider that depend on the value of $\bar{f}_R^E$: (I) $\bar{f}_R^E \in (\underline{f}_R^H, \bar{f}_R^H)$, (II) $\bar{f}_R^E \in (\underline{f}_R^E, \underline{f}_R^H]$, and (III) $\bar{f}_R^E \in [\bar{f}_R^H, \infty)$. Based on our extensive numerical experiments, we have observed that Case (I) is prevalent, Case (II) is rare, and Case (III) is non-existent. For instance, Table 2 shows that out of 243 scenarios, Case (II) has occurred in only 6 scenarios and Case (III) has never occured. Hence, we shall focus our analysis on Case (I) and discuss Case (II) at the end of this section.

Insert Table 2 about here.

To determine the manufacturer's rebate strategy $j^{(*)}$ and the corresponding retailer's rebate strategy $k^{(j^{(*)})}$ (or $k^{(*)}$ in short) in equilibrium for Case (I), we need to compare the manufacturer's profit functions $\Pi_M^{(E,*)}$ and $\Pi_M^{(H,*)}$ over the following five intervals of f_R: $I_1 \equiv (0, \underline{f}_R^E)$, $I_2 \equiv [\underline{f}_R^E, \underline{f}_R^H)$, $I_3 \equiv [\underline{f}_R^H, \bar{f}_R^E)$, $I_4 \equiv [\bar{f}_R^E, \bar{f}_R^H)$, and $I_5 \equiv [\bar{f}_R^H, \infty)$. These comparisons enable us to characterize the manufacturer's rebate strategy $j^{(*)}$ and the retailer's rebate strategy $k^{(*)}$ in equilibrium as follows:

Theorem 1 *Suppose $\underline{f}_R^H < \bar{f}_R^E < \bar{f}_R^H$. Then:*

(a) For any given f_R, there exists a threshold $f_M^*(f_R)$ such that $j^{(*)} = H$ if $f_M \leq f_M^*(f_R)$ and

$j^{(*)} = E$, otherwise.

(b) For any given f_M, there exists a threshold $f_R^*(f_M)$ such that $k^{(*)} = H$ if $f_R \leq f_R^*(f_M)$ and $k^{(*)} = E$, otherwise.

(c) As f_R increases, $f_M^*(f_R)$ is constant over interval I_1, convex and increasing over I_2, increasing linearly over I_3, concave and decreasing over I_4, and is constant over I_5.

(d) The threshold $f_R^*(f_M)$ equals $\bar{f}_R^H$ when $f_M \leq f_M^*(\bar{f}_R^H)$; equals $f_M^{*-1}(f_M)$ when $f_M \in (f_M^*(\bar{f}_R^H), f_M^*(\bar{f}_R^E)]$ (which is concave and decreasing in f_M); and equals $\bar{f}_R^E$ when $f_M > f_M^*(\bar{f}_R^E)$, where $f_M^{*-1}(\cdot)$ denotes the inverse function of $f_M^*(\cdot)$.

(e) When $\beta_M(b_M - b\theta_M) \to 0^+$, $\frac{\partial f_M^*}{\partial f_R} \to 0$ and $\frac{\partial f_R^*}{\partial f_R} \to 0$. In this case, $j^{(*)} = E$ for all f_M; $k^{(*)} = H$ if $f_R \leq \bar{f}_R^E = \bar{f}_R^H$, and $k^{(*)} = E$, otherwise.

Figure 6 illustrates the manufacturer's rebate strategy and the retailer's rebate strategy in equilibrium presented in Theorem 1. In Figure 6, the horizontal axis represents the retailer's fixed cost f_R and the vertical axis represents the manufacturer's fixed cost f_M. Parts (a) and (b) are illustrated in Figure 6: the manufacturer's rebate strategy is divided along the threshold function $f_M^*(f_R)$ and the retailer's rebate strategy is divided along the threshold function $f_R^*(f_M)$, respectively. As a result of these divisions, the entire space of (f_R, f_M) can be partitioned into 4 regions, each of which depicts a specific pair of rebate strategies that both parties will adopt in equilibrium. For example, the upper left region denoted by (E, H) corresponds to the case in which the manufacturer offers no rebates (i.e., $j^{(*)} = E$) and the retailer offers rebates (i.e., $k^{(*)} = H$) in equilibrium.

Insert Figure 6 about here.

The unusual shape of the regions depicted in Figure 6 highlights the effect of competition on the rebate strategies in equilibrium. To elaborate, let us first establish a simple benchmark by considering the case when each party is non-strategic in the sense that each party acts independently without taking the other party's decision into consideration. In this case, the rebate strategy for each party is based on a constant break-even point at which the fixed promotion cost is equal to the additional gross profit generated by the rebate promotion. Consequently, the entire space of (f_R, f_M) is partitioned according to one horizontal line and one vertical line, resulting in 4 different regions that take on rectangular shapes. Next, in a competitive environment, each player is strategic in the sense that each party will take the other party's decision into consideration. As a result, as highlighted in part (a), the manufacturer's rebate strategy in equilibrium $j^{(*)}$ depends on the

retailer's fixed cost f_R via the threshold function $f_M^*(f_R)$. Part (b) can be interpreted in the same manner.

To explain the shape of equilibrium regions in Figure 6, we examine the structural property of the threshold function $f_M^*(f_R)$ as presented in part (c). (The property of threshold function $f_R^*(f_M)$ in part (d) can be explained in a similar fashion.) By definition, when the manufacturer's fixed promotion cost $f_M = f_M^*(f_R)$, the manufacturer is indifferent between offering rebates and not; i.e., $\Pi_M^{(E,*)} = \Pi_M^{(H,*)}$. Let us define a term Π_M^H such that $\Pi_M^{(H,*)} = \Pi_M^H - \beta_M f_M$. Combining this definition with the fact that $\Pi_M^{(E,*)} = \Pi_M^{(H,*)}$ when $f_M = f_M^*(f_R)$, we obtain $f_M^*(f_R) = \frac{1}{\beta_M}\left\{\Pi_M^H - \Pi_M^{(E,*)}\right\}$. By applying the results stated in Propositions 3 and 6 (i.e., as f_R increases, both Π_M^H and $\Pi_M^{(E,*)}$ are initially constant, decrease in a concave fashion, and then stay constant), we can trace the property of $f_M^*(f_R)$ as a function of f_R over those five intervals as stated in part (c).

Since the unusual shape is primarily due to the fact that the threshold function $f_M^*(f_R)$ is increasing in f_R over the interval $I_3 = [\underline{f}_R^H, \bar{f}_R^E)$, we investigate this further. Observe from Figures 4(b) and 5(b) that, regardless of the manufacturer's rebate strategy, the manufacturer would reduce the wholesale price over this interval so as to induce the retailer to offer rebates. However, from above and Propositions 6 and 3, that $\Pi_M^H - \beta_M f_M = \Pi_M^{(H,*)} = \Pi_M^{(E,*)} + B_4 f_R - \beta_M f_M$ in the interval I_3; hence, $f_M^*(f_R)$ satisfies: $f_M^*(f_R) = \frac{1}{\beta_M}\left\{\Pi_M^H - \Pi_M^{(E,*)}\right\} = \frac{B_4}{\beta_M} f_R$ where B_4 (> 0) is defined in Appendix. Even though both $\Pi_M^{(H,*)}$ and $\Pi_M^{(E,*)}$ are decreasing in f_R, $\Pi_M^{(H,*)}$ is decreasing at a slower pace due to the additional positive term $B_4 f_R$. This additional term captures the 'option value' associated with one additional degree of freedom: the manufacturer has the flexibility to select the manufacturer rebate value in addition to the wholesale price when $j = H$. This explains why $f_M^*(f_R)$ is increasing in f_R over I_3 so that the corresponding region for $j^{(*)} = H$ expands. In other words, offering manufacturer rebates is 'relatively more attractive' to the manufacturer over interval I_3.

The structural form of the equilibrium leads to a counter-intuitive result: a higher fixed cost does not necessarily discourage the manufacturer from offering rebates. In all cases, if it is optimal for a firm to offer no rebates, then it is always optimal to offer no rebates with a higher fixed cost of his own as long as the fixed cost of the other party remains constant. However, Figure 6 reveals that it is possible that switching to offering rebates can be optimal for the manufacturer when his fixed cost is higher if there is also an increase in the retailer's fixed cost (see points A and B in Figure 6).

A natural question then is "when do the equilibrium regions described in Figure 6 take a 'usual' rectangular form?" Part (e) in Theorem 1 indicates that, as the 'implicit' benefit of rebate promotion $b_M - b\theta_M$ approaches 0 or promotion frequency β_M approaches 0, the threshold functions $f_M^*(f_R)$ and $f_R^*(f_M)$ become constant. This is a limiting case where the thresholds obtained in the subgames 1 and 2 are approaching each other: $\underline{f}_R^E \to \underline{f}_R^H$ and $\bar{f}_R^E \to \bar{f}_R^H$. In this case, the manufacturer rebate program is approaching break-even or its frequency is approaching zero; thus, the manufacturer becomes indifferent between offering rebates and not. It can be also shown that $f_M^*(f_R) \to 0$ and $f_R^*(f_M) \to \bar{f}_R^E = \bar{f}_R^H$; i.e., the manufacturer would not offer rebates in equilibrium and the retailer would offer rebates in equilibrium if and only if $f_R \leq \bar{f}_R^E = \bar{f}_R^H$.

Having derived the rebate strategies in equilibrium, we now retrieve the pricing strategies and the resulting profits in equilibrium from the outcomes of subgames 1 and 2. To do so, let us define f_M^1, f_M^2 and f_M^3 as follows: $f_M^*(f_R) = f_M^1$ for $f_R \in I_5$, $f_M^*(f_R) = f_M^2$ for $f_R \in I_1$, and $f_M^*(f_R) = f_M^3$ for $f_R = \bar{f}_R^E$ (see Figure 6). In all of our numerical experiments, we have found that $f_M^1 < f_M^2$, so we shall focus our subsequent discussion on this case. When $f_M < f_M^1$, the manufacturer offers rebates in equilibrium (i.e., $j^{(*)} = H$); hence, the prices and profits in equilibrium are equal to those presented in Proposition 6 and Corollary 3 (presented in Appendix) in subgame 2. When $f_M > f_M^3$, the manufacturer offers no rebates in equilibrium (i.e., $j^{(*)} = E$); hence, the prices and profits in equilibrium are equal to those presented in Proposition 3 and Corollary 2 in subgame 1. When $f_M \in [f_M^2, f_M^3]$, the unusual shape of equilibrium regions in Figure 6 leads to an unexpected result: as f_R increases, the manufacturer's rebate strategy $j^{(*)}$ changes from E to H and then from H to E. Figure 7 illustrates the equilibrium profits and prices over f_R for any given $f_M \in [f_M^2, f_M^3]$. Figure 7(b) highlights that there are two equilibria at each threshold $f_M^{*-1}(f_M)$ or $f_R^*(f_M)$, at which the manufacturer or the retailer is indifferent between offering rebates and not. It is interesting to notice from Figure 7 that, unlike others, there are no discontinuities at the threshold $f_M^{*-1}(f_M)$ for the retailer's profit $\Pi_R^{(*)}$ as well as for the retailer rebate value $r_R^{(*)}$. (Obviously, the manufacturer's profit Π_M is continuous over f_R.) We can verify from Corollaries 2 and 3 that $\Pi_R^{(E,*)} = \Pi_R^{(H,*)}$ and $r_R^{(E,*)} = r_R^{(H,*)}$ when $f_R \in I_3 = [\underline{f}_R^H, \bar{f}_R^E)$. This result suggests that the retailer is indifferent whether or not the manufacturer offers rebates over interval I_3. For the remaining case when $f_M \in [f_M^1, f_M^2)$, the manufacturer's rebate strategy $j^{(*)}$ changes from H to E. We omit the details of this case.

Insert Figure 7 about here.

Next, we consider Case (II) where $\bar{f}_R^E \in (\underline{f}_R^E, \underline{f}_R^H]$. Case (II) occurs only when the manufacturer's promotions are so effective that even when the retailer's fixed cost f_R is as high as $\bar{f}_R^E$, the retailer finds it optimal to offer rebates without receiving some discounts in the wholesale price. Instead of the intervals I_2, I_3, and I_4 in Case (I), we need to consider the following three intervals of f_R: $I_2^{'} \equiv [\underline{f}_R^E, \bar{f}_R^E)$, $I_3^{'} \equiv [\bar{f}_R^E, \underline{f}_R^H)$, and $I_4^{'} \equiv [\underline{f}_R^H, \bar{f}_R^H)$. Using the same approach as in Case (I), we can show that the rebate strategies in equilibrium have the structural property depicted in Figure 8. By comparing Figures 6 and 8, we observe two major differences in the equilibrium between Case (I) and Case (II). First, the threshold $f_M^*(f_R)$ is constant in $f_R \in I_3^{'}$ in Case (II), while $f_M^*(f_R)$ is increasing in $f_R \in I_3$ in Case (I). This happens because unlike Case (I), the manufacturer does not have to reduce its wholesale price over the interval $I_3^{'}$ in Case (II) in order to induce the retailer to offer rebates. Second, the threshold $f_R^*(f_M)$ changes more with respect to f_M in Case (II), i.e. $f_R^*(f_M^1) - f_R^*(f_M^3) = \bar{f}_R^H - \bar{f}_R^E$ is larger in Case (II). This suggests that when the manufacturer offers no rebates due to a high cost f_M, the retailer is also less likely to offer rebates because the retailer does not benefit from the increased demand that would have been generated by the manufacturer's promotions to consumers.

Insert Figure 8 about here.

5 Comparative Statics

In this section we investigate the impact of the effectiveness of retailer (and manufacturer) rebate promotion b_R (and b_M) on the rebate and pricing strategies in equilibrium. Since we obtain *all* equilibrium outcomes in closed forms, one can examine the comparative statics analytically. We manage to do so for subgame 1, but the analysis for subgame 2 is highly complex. For this reason, we use numerical examples to better illustrate comparative statics. We start with a base case in which the values of the parameters are given as: $a = 10$, $b = 1$, $b_R = b_M = 0.8$, $\theta_R = \theta_M = 0.5$, and $\beta_R = \beta_M = 0.3$. This set of parameter values yields $\underline{f}_R^H < \bar{f}_R^E < \bar{f}_R^H$, which corresponds to Case (I). We utilize the results stated in Theorem 1 to analyze the impact of b_R and b_M. The main results carry to Case (II), so we shall focus on Case (I) in this section. We have also conducted extensive numerical experiments to analyze the impact of other parameters such as the redemption rate θ_i and the promotion frequency β_i. Our numerical results suggested that θ_i affects the equilibrium outcomes in an opposite manner as b_i and that β_i affects them in the same manner as b_i. This is because a rebate program also becomes more effective as a smaller number of consumers redeem

rebates and (profitable) rebates are offered more often.

5.1 The Impact of Retailer Rebate Promotion

To examine the effect of b_R on the equilibrium, we increase the value of b_R from 0.8 to 0.82. As b_R increases, customers become more responsive to retailer rebates. The changes in the rebate strategies in equilibrium are illustrated in Figure 9. To examine the changes in the profits and the pricing strategies in equilibrium, we have conducted the numerical analysis for different values of f_M and obtained similar results. For illustrative purposes, we present in Figure 10 the case when $f_M = 0.9$; hence, $j^{(*)} = E$ from Figure 9. Upon examining Figures 9 and 10, we make the following observations:

1. The threshold function $f_R^*(f_M)$ increases in b_R for any given value of f_M. However, the threshold function $f_M^*(f_R)$ is not monotone in b_R for any given value of f_R. Moreover, the thresholds $\underline{f}_R^E$, $\bar{f}_R^E$, $\underline{f}_R^H$, $\bar{f}_R^H$, f_M^2 and f_M^3 are increasing in b_R, while f_M^1 is constant in b_R.

2. The retail price $p^{(E,*)}$ and wholesale price $w^{(E,*)}$ are nondecreasing in b_R, but the retail rebate value $r_R^{(E,*)}$ is not monotone in b_R for any given value of f_R.

3. The manufacturer's profit $\Pi_M^{(E,*)}$ is nondecreasing in b_R, but the retailer's profit $\Pi_R^{(E,*)}$ is not monotone in b_R for any given value of f_R.

Insert Figures 9 and 10 about here.

As b_R increases, the additional demand generated by the retailer rebate program increases. As such, offering retailer rebates become more attractive to the retailer. This is reflected in Figure 9 in the following sense: as the threshold function $f_R^*(f_M)$ increases in b_R (for any given value of f_M), the region in which the retailer would offer rebates in equilibrium expands. Intuitively, one would expect that the retailer's profit $\Pi_R^{(E,*)}$ would increase as b_R increases. However, Figure 10(a) shows that this intuition is not generally true. For instance, $\Pi_R^{(E,*)}$ is decreasing in b_R when $f_R \in [\underline{f}_R^E, \bar{f}_R^E)$. This counter-intuitive result arises from the fact that the threshold $\underline{f}_R^E$ (at which the manufacturer would start to reduce the wholesale price) increases as b_R increases. This is because, knowing that the retailer has more (intrinsic) incentive to offer rebates as b_R increases, the manufacturer has less incentive to lower the wholesale price to entice the retailer to offer rebates. This dynamics has caused the retailer's profit to decrease in equilibrium over this interval. By using the same logic, one can show that $f_M^*(f_R)$ and $r_R^{(E,*)}$ are not monotone in b_R.

5.2 The Impact of Manufacturer Rebate Promotion

To examine the effect of b_M on the equilibrium, we increase the value of b_M from 0.8 to 1. As b_M increases, customers become more responsive to manufacturer rebates. The changes in the rebate strategies in equilibrium are illustrated in Figure 11. To examine the changes in the profits and the pricing strategies in equilibrium, we have conducted the numerical analysis for different values of f_M and obtained similar results. For illustrative purposes, we present in Figure 12 for the case when $f_M = 0.3$; hence, $j^{(*)} = H$ from Figure 11. Upon examining Figures 11 and 12, we make the following observations:

1. The threshold function $f_R^*(f_M)$ is nondecreasing in b_M for given value of f_M, and the threshold function $f_M^*(f_R)$ is increasing in b_M for any given value of f_R. Moreover, the thresholds $\underline{f}_R^H$, $\bar{f}_R^H$, f_M^1, f_M^2 and f_M^3 are increasing in b_M, while $\underline{f}_R^E$ and $\bar{f}_R^E$ are constant in b_M.

2. The retail price $p^{(H,*)}$, the wholesale price $w^{(H,*)}$ and the manufacturer rebate value $r_M^{(H,*)}$ are increasing in b_M, and the retail rebate value $r_R^{(H,*)}$ is nondecreasing in b_M for any given value of f_R.

3. The manufacturer's profit $\Pi_M^{(H,*)}$ is increasing in b_M and the retailer's profit $\Pi_R^{(H,*)}$ is nondecreasing in b_M for any given value of f_R.

Insert Figures 11 and 12 about here.

As b_M increases, the additional demand generated by the manufacturer rebate program increases. As such, offering rebates become more attractive to the manufacturer. This is reflected in Figure 11 in the following sense: as the threshold function $f_M^*(f_R)$ increases in b_M (for any given value of f_R), the region in which the manufacturer would offer rebates in equilibrium expands. As b_M increases, the retailer also enjoys a higher demand because the 'effective' base demand $a + \beta_M b_M r_M^H$ is increasing in b_M. As such, offering retailer rebates (in addition to manufacturer rebate) becomes more attractive to the retailer. This is reflected in Figure 11 where the threshold function $f_R^*(f_M)$ is increasing in b_M (but at a slower pace than $f_M^*(f_R)$ as illustrated in Figure 11). As a consequence of more effective rebate promotions offered by the manufacturer, one can observe from Figure 12 that all prices and profits in equilibrium are increasing in b_M except that the retailer's profit $\Pi_R^{(H,*)}$ and the retailer rebate value $r_R^{(H,*)}$ are independent of b_M for the case when $f_R \in [\underline{f}_R^H, \bar{f}_R^H)$ (see Corollary 3 in Appendix).

Our numerical experiments highlight an interesting phenomenon: when b_M increases, both firms enjoy higher profits in equilibrium (Figure 12(a)). However, when b_R increases, the manufacturer's profit always improves but the retailer's profit could suffer for certain values of f_R in equilibrium. This phenomenon seems to suggest that it is in the interest of both firms to focus on increasing the value of b_M instead of b_R. This may explain why many retailers continue to work closely with the manufacturers to promote manufacturer rebate programs.

6 Concluding Remarks

In many product categories, both manufacturers and retailers use rebates to generate additional sales. While rebates have been recognized as an effective promotional tool, firms are debating whether to continue their rebate programs due to certain adverse effects. In this paper, we have analyzed the rebate and pricing decisions of a supply chain comprising a manufacturer and a retailer. Our goal is to determine the conditions under which a firm should offer rebates in a competitive setting. A basic intuition is that the firms' rebate strategies in equilibrium can be specified by a unique pair of thresholds (f_R^*, f_M^*): if the retailer's fixed promotion cost f_R (or the manufacturer's fixed promotion cost f_M) is below the threshold f_R^* (or f_M^*), then the retailer (or the manufacturer) should offer rebates in equilibrium. Clearly, if each of the firms is non-strategic in the sense that it does not take the other firm's decision into consideration when making its own decision, then these thresholds f_R^* and f_M^* are constants. However, as the firms enter a Stackelberg game, they become strategic. This strategic behavior results in more complex forms of the thresholds. Our formal analysis has provided a deeper understanding of the strategic behavior of the firms in equilibrium. Through our analysis, we have obtained the following results:

1. For any given manufacturer's rebate and pricing strategies, we have shown that the retailer would offer rebates if the wholesale price is lower than a certain threshold. This result is intuitive because, when the wholesale price is high, the retailer cannot afford to offer retailer rebates with a meager profit margin. This threshold is even higher when the manufacturer offers manufacturer rebates mainly because the manufacturer rebate program generates additional demand.

2. For any given manufacturer's rebate strategy, we have obtained counter-intuitive results: the retailer's profit is increasing and the average retail price is decreasing in the cost f_R over a certain interval. These results are due to the strategic behavior of the manufacturer who

would reduce the wholesale price in that interval of f_R so as to entice the retailer to offer rebate promotions. This shows that strategic interactions between the two firms can create a win-win situation.

3. We have shown that there are three forms of rebate strategies in equilibrium, depending on the ordering of the thresholds $\underline{f}_R^H$, $\bar{f}_R^H$ and $\bar{f}_R^E$. However, our extensive numerical work suggested that in most cases these thresholds satisfy: $\underline{f}_R^H < \bar{f}_R^E < \bar{f}_R^H$. This observation enables us to focus on this particular ordering of thresholds (i.e., Case (I)). Our analysis shows that the retailer's threshold $f_R^*(f_M)$ is a non-increasing function of f_M. This suggests that if the manufacturer would not offer rebates due to the high promotion cost, then the retailer is also less likely to offer rebate promotions. On the other hand, the manufacturer's threshold $f_M^*(f_R)$ is not a monotonic function of f_R. This leads to a counter-intuitive result: a higher fixed cost of the manufacturer does not necessarily discourage the manufacturer from offering rebates if there is also an increase in the fixed cost of the retailer.

4. Our numerical analyses highlighted an interesting phenomenon: a more effective manufacturer rebate program (i.e., a higher value of b_M) always leads to a win-win situation in which both firms enjoy higher profits; however, a more effective retailer rebate program (i.e., a higher value of b_R) could lead to a win-lose situation in which the manufacturer enjoys a higher profit but the retailer suffers from a lower profit. This phenomenon is due to the strategic behavior of the manufacturer who acts as the leader in a Stackelberg game. Therefore, it is in the interest of both firms to develop initiatives to improve the effectiveness of the manufacturer's rebate program.

Our work provides several avenues for future research. First, the manufacturer and the retailer may want to coordinate their promotions in various ways. For instance, suppose the retailer can choose to synchronize its timing of promotions with the manufacturer's promotions. For the case when $\beta_R = \beta_M = \beta$, our analysis (omitted) shows that the retailer would never synchronize its promotional timing in equilibrium. This result seems to suggest that, unless the coordination of rebate promotions offer additional demand due to a compounding effect or additional savings in the promotional costs to both parties, there is little incentive to coordinate rebate promotional plans. Therefore, one potential avenue of research is to examine the conditions under which both parties would coordinate their rebate promotions in equilibrium. Second, we have assumed that the benefits and costs of promotions are constant and known. In reality, some of these parameters

may depend on each other or may depend on the value of certain decision variables. For instance, the redemption rate θ_i may increase with the rebate value r_i, the promotional frequency β_i may decrease with the rebate value r_i, and the rebate sensitivity b_i and the fixed promotion cost f_i may vary with the frequency of promotions β_i. Several papers reviewed in Section 2 partially address some of these issues; however, to obtain tractable results, these papers rely on certain simplifying assumptions. For instance, Chen et al. (2007) develop a model that captured a positive relationship between θ_i and r_i; however, their model is based on a known fixed rebate value. Also, Baysar et al. (2007) examine the role of uncertainty in parameters; however, their model relies on the assumption that the retailer would use a fixed markup scheme to determine the retail price. Relaxing these assumptions in a more general framework will sharpen our understanding of firms' behavior in equilibrium. Third, we have assumed that the values of all parameters are common knowledge. Information asymmetry could be an interesting extension of the model presented in this paper. Finally, we have focused on vertical competition in this paper. It could be instructive to extend our model by considering both horizontal and vertical competition.

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Appendix

Proof of Proposition 1. (a) The results follow immediately from the first-order condition associated with the problem $\max_{p^E}(p^E - w^E)(a - bp^E)$.
(b) If Π_R is jointly concave with respect to p^H and r_R^H, then the problem $\max_{p^H, r_R^H} \Pi_R(w^E, p^H, r_R^H)$ has a unique pair of optimal solutions $(p^{(E,H)}, r_R^{(E,H)})$ given w^E. For Π_R to be jointly concave, $\frac{\partial^2 \Pi_R}{\partial (p^H)^2} = -2b < 0$, $\frac{\partial^2 \Pi_R}{\partial \left(r_R^H\right)^2} = -2\beta_R \theta_R b_R < 0$, and

$$\frac{\partial^2 \Pi_R}{\partial (p^H)^2} \frac{\partial^2 \Pi_R}{\partial \left(r_R^H\right)^2} - \left(\frac{\partial^2 \Pi_R}{\partial p^H \partial r_R^H}\right)^2 = \beta_R \left\{4bb_R\theta_R - \beta_R(b\theta_R + b_R)^2\right\} (= \beta_R A) > 0. \tag{26}$$

Condition in (26) can be simplified as:

$$\left(\frac{2}{\beta_R} - 1\right) - \sqrt{\left(\frac{2}{\beta_R} - 1\right)^2 - 1} < \frac{b_R}{b\theta_R} < \left(\frac{2}{\beta_R} - 1\right) + \sqrt{\left(\frac{2}{\beta_R} - 1\right)^2 - 1}. \tag{27}$$

Note that the left-hand side of (27) is less than 1 and the right-hand side of (27) is greater than $\frac{2}{\beta_R} - 1$. Thus, condition (27) holds when condition (11) holds.

By considering the first-order conditions $\frac{\partial \Pi_R}{\partial p^H} = 0$, and $\frac{\partial \Pi_R}{\partial r_R^H} = 0$ simultaneously, we obtain (12) and (13). As $\frac{b_R}{b\theta_R} > 1$, $r_R^{(E,H)} > 0$. By substituting (12), (13), and $\beta_M = 0$ into (3), we obtain the retailer's optimal profit in (4). As $\frac{b_R}{b\theta_R} < \frac{2}{\beta_R} - 1$, $p^{(E,H)}$ is increasing in w^E, hence $p^{(E,H)} < \frac{a}{b}$.

Comparative statics are obtained by examining the corresponding first derivatives. We omit the details. □

Proof of Corollary 1. The results follow immediately from (10), (12) and (13). □

Proof of Proposition 2. Observe from (10) and (14) that both $\Pi_R^{(E,E)}$ and $\Pi_R^{(E,H)}$ are decreasing and convex in w^E. Also,

$$\Pi_R^{(E,H)} - \Pi_R^{(E,E)} + \beta_R f_R = \frac{\beta_R(b\theta_R - b_R)^2(a - bw^E)^2}{4bA} > 0. \tag{28}$$

Therefore, $\frac{d\Pi_R^{(E,H)}}{dw^E} < \frac{d\Pi_R^{(E,E)}}{dw^E} < 0$ for all w^E. In addition, when $w^E = \frac{a}{b}$, $\Pi_R^{(E,H)} - \Pi_R^{(E,E)} = -\beta_R f_R < 0$ from (28). Therefore, there exists a single point $\tau^E < \frac{a}{b}$ such that $\Pi_R^{(E,E)}$ and $\Pi_R^{(E,H)}$ cross exactly once at $w^E = \tau^E$, where τ^E is given in (17). □

Proof of Proposition 3. Notice from (17) that $\tau^E = \frac{a}{2b}$ when $f_R = \underline{f}_R^E$. We consider the following three intervals of f_R: (i) $f_R \geq f_R^a$, (ii) $\underline{f}_R^E < f_R < f_R^a$, and (iii) $0 \leq f_R \leq \underline{f}_R^E$.

(i) If $f_R \geq f_R^a$, $\tau^E \leq 0$, hence $w^{(E,*)} = \frac{a}{2b}$ and $\Pi_M^{(E,*)} = \frac{a^2}{8b}$.

(ii) If $\underline{f}_R^E < f_R < f_R^a$, $0 < \tau^E < \frac{a}{2b}$. So, $\Pi_M^{(E,E)} = \frac{a^2}{8b}$ is attained at $w^E = \frac{a}{2b}$ and $\Pi_M^{(E,H)} = \frac{4ab_R\theta_R(1-\beta_R)}{b_R - b\theta_R}\left\{\sqrt{\frac{bf_R}{A}} - \frac{2bf_R}{a(b_R - b\theta_R)}\right\}$ is attained at $w^E = \tau^E$. To determine $\Pi_M^{(E,*)} = \max\{\Pi_M^{(E,E)}, \Pi_M^{(E,H)}\}$, we compare $\Pi_M^{(E,E)}$ and $\Pi_M^{(E,H)}$ for $f_R \in (\underline{f}_R^E, f_R^a)$. When $f_R = \underline{f}_R^E$ (i.e. $\tau^E = \frac{a}{2b}$), both $\Pi_M^{(E,E)}$ and $\Pi_M^{(E,H)}$ are attained at $w^E = \frac{a}{2b}$, yielding

$$\Pi_M^{(E,E)} - \Pi_M^{(E,H)} = -\frac{a^2\beta_R(b_R - b\theta_R)^2}{8bA} < 0. \tag{29}$$

When $f_R = f_R^a$, $\tau^E = 0$, hence $\Pi_M^{(E,E)} > \Pi_M^{(E,H)}$. In addition, $\frac{\partial \Pi_M^{(E,H)}}{\partial f_R} < 0$ for $f_R > \underline{f}_R^E$. Therefore, there exists $\bar{f}_R^E \in (\underline{f}_R^E, f_R^a)$ such that if $\bar{f}_R^E \leq f_R < f_R^a$, $w^{(E,*)} = \frac{a}{2b}$ and $\Pi_M^{(E,*)} = \Pi_M^{(E,E)}$, and if $\underline{f}_R^E < f_R \leq \bar{f}_R^E$, $w^{(E,*)} = \tau^E$ and $\Pi_M^{(E,*)} = \Pi_M^{(E,H)}$. We obtain the closed form of $\bar{f}_R^E$ by solving $\Pi_M^{(E,H)} = \Pi_M^{(E,E)}$ for f_R.

(iii) If $0 \leq f_R \leq \underline{f}_R^E$, $\frac{a}{2b} \leq \tau^E \leq \frac{a}{b}$. So, $\Pi_M^{(E,E)}$ is attained at $w^E = \tau^E$ and $\Pi_M^{(E,H)}$ is attained at $w^E = \frac{a}{2b}$. Since the optimal $\Pi_M^{(E,E)}$ attained at the boundary $w^E = \tau^E$ is smaller than that at the interior solution $w^E = \frac{a}{2b}$, by (29), we get $w^{(E,*)} = \frac{a}{2b}$ and $\Pi_M^{(E,*)} = \Pi_M^{(E,H)} = \frac{a^2 b_R\theta_R(1-\beta_R)}{2A}$. □

Proof of Corollary 2. $k^{(E,*)}$ follows immediately from Proposition 3. By substituting $w^{(E,*)}$ into the results in Proposition 1, we obtain $p^{(E,*)}, r_R^{(E,*)}$ and $\Pi_R^{(E,*)}$. □

Remarks on Proposition 3 and Corollary 2: The structural properties of $w^{(E,*)}, r_R^{(E,*)}$ and $\Pi_R^{(E,*)}$ with respect to f_R are straightforward from observations of their closed forms. $\Pi_M^{(E,*)}$ is decreasing and concave in $f_R \in (\underline{f}_R^E, \bar{f}_R^E)$ because $\frac{\partial}{\partial f_R}\Pi_M^{(E,*)} = 0$ at $f_R = \underline{f}_R^E$ and $\frac{\partial^2}{\partial f_R^2}\Pi_M^{(E,*)} < 0$. $p^{(E,*)}$ is decreasing and convex in $f_R \in (\underline{f}_R^E, \bar{f}_R^E)$ by the assumption that $p^E \in \left(0, \frac{a}{b}\right)$.

Proof of Proposition 4. The proof follows the same approach as presented in the proof of Proposition 1, and hence is omitted. □

Proof of Proposition 5. The unique threshold τ^H is obtained by solving $\Pi_R^{(H,E)} = \Pi_R^{(H,H)}$ for w^H, so that $\Pi_R^{(H,E)} \geq \Pi_R^{(H,H)}$ if and only if $w^H \geq \tau^H$. □

Proposition 6 *Suppose that the manufacturer offers rebates (i.e., $j = H$) and that a unique equilibrium exists in subgame 2. Then there exist two thresholds $\underline{f}_R^H$ and $\bar{f}_R^H$ such that the manufacturer's optimal wholesale price $w^{(H,*)}$, manufacturer rebate value $r_M^{(H,*)}$, and the resulting profit $\Pi_M^{(H,*)}$ in stage 2 satisfy:*

$$w^{(H,*)} = \begin{cases} \frac{a\theta_M\{4b_M-\beta_M(3b_M+b\theta_M\}}{B_1} & if\ f_R \geq \bar{f}_R^H \\ \frac{a}{b} - \frac{2}{b_R-b\theta_R}\sqrt{\frac{Af_R}{b}}\left\{1 - \frac{\beta_M\theta_R b_R(1-\beta_R)(b_M-b\theta_M)}{\theta_M(1-\beta_M)A}\right\} & if\ \underline{f}_R^H < f_R \leq \bar{f}_R^H \\ \frac{a\theta_M B_3}{bB_2} & otherwise \end{cases}$$

$$r_M^{(H,*)} = \begin{cases} \frac{a(b_M-b\theta_M)}{B_1} & if\ f_R \geq \bar{f}_R^H \\ \frac{2\theta_R b_R(1-\beta_R)(b_M-b\theta_M)}{\theta_M b_M(1-\beta_M)(b_R-b\theta_R)}\sqrt{\frac{bf_R}{A}} & if\ \underline{f}_R^H < f_R \leq \bar{f}_R^H \\ \frac{a\theta_R b_R(1-\beta_R)(b_M-b\theta_M)}{B_2} & otherwise \end{cases}$$

$$\Pi_M^{(H,*)} = \begin{cases} \frac{a^2\theta_M b_M(1-\beta_M)}{B_1} - \beta_M f_M & if\ f_R \geq \bar{f}_R^H \\ \frac{4ab_R\theta_R(1-\beta_R)}{b_R-b\theta_R}\left\{\sqrt{\frac{bf_R}{A}} - \frac{2bf_R}{a(b_R-b\theta_R)}\right\} + B_4 f_R - \beta_M f_M & if\ \underline{f}_R^H < f_R \leq \bar{f}_R^H \\ \frac{a^2\theta_R b_R\theta_M b_M(1-\beta_R)(1-\beta_M)}{B_2} - \beta_M f_M & otherwise. \end{cases}$$

where, the positive intermediate terms B_1 through B_6 (which depend on parameters other than f_R and f_M) are defined as:

$B_1 = 8\theta_M b_M b - \beta_M(b_M^2 + 6b\theta_M b_M + b^2\theta_M^2)$,

$B_2 = \beta_M\beta_R\left\{b\theta_R\theta_M\left(2b_M b_R + b\theta_M b_R + 2b\theta_R b_M\right) + b_M^2 b_R\theta_R + 2\theta_M b_M b_R^2\right\} - 2\theta_M b_M\beta_R\left(b_R + b\theta_R\right)^2 + \theta_R b_R B_1$,

$B_3 = \beta_M\beta_R\left\{b_M\left(b^2\theta_R^2 + b\theta_R b_R + b_R^2\right) + \theta_M b^2\theta_R b_R\right\} - b\theta_R b_R\beta_M\left(3b_M + b\theta_M\right) + b_M A$,

$B_4 = \frac{4b\beta_M b_R^2\theta_R^2(1-\beta_R)^2(b_M-b\theta_M)^2}{Ab_M\theta_M(1-\beta_M)(b_R-b\theta_R)^2}$,

$B_5 = \frac{\theta_R\beta_M(1-\beta_R)(b_M-b\theta_M)}{(1-\beta_M)\theta_M\{2b\theta_R-\beta_R(b_R+b\theta_R)\}}$, and

$B_6 = b_M\left\{6b\theta_R b_R - \beta_R\left(b_R^2 + 2b^2\theta_R^2 + 3b\theta_R b_R\right)\right\} - b_M\beta_M\left\{b\theta_R\left(b_R - b\theta_R\beta_R\right) + A\right\} - b^2\theta_M\theta_R b_R\beta_M\left(1-\beta_R\right)$.

Proof of Proposition 6. We follow the procedure similar to the proof of Proposition 3. Let $(w^{(H,E)}, r_M^{(H,E)})$ denote a pair of (w^H, r_M^H) that maximizes $\Pi_M(w^H, r_M^H, p^{(H,E)})$. Suppose the fol-

lowing inequality holds:

$$1 < \frac{b_M}{b\theta_M} < \left(\frac{4}{\beta_M} - 3\right) + \sqrt{\left(\frac{4}{\beta_M} - 3\right)^2 - 1}.$$

Then, $\Pi_M(w^H, r_M^H, p^{(H,E)})$ is jointly concave with respect to (w^H, r_M^H). From the first-order conditions, we obtain

$$(w^{(H,E)}, r_M^{(H,E)}) = \left(\frac{a\theta_M\{4b_M - \beta_M(3b_M + b\theta_M)\}}{B_1}, \frac{a(b_M - b\theta_M)}{B_1}\right). \tag{30}$$

Similarly, let $(w^{(H,H)}, r_M^{(H,H)})$ denote a pair of (w^H, r_M^H) that maximizes $\Pi_M(w^H, r_M^H, p^{(H,H)}, r_R^{(H,H)})$. Assuming the joint concavity of Π_M, we obtain

$$(w^{(H,H)}, r_M^{(H,H)}) = \left(\frac{a\theta_M B_3}{bB_2}, \frac{a\theta_R b_R(1-\beta_R)(b_M - b\theta_M)}{B_2}\right). \tag{31}$$

From (30) and (31), we get

$$w^{(H,H)} - w^{(H,E)} = \frac{a\theta_M b_M \beta_M \beta_R (1-\beta_M)(b_M - b\theta_M)(b_M + b\theta_M)(b_R - b\theta_R)^2}{bB_1B_2} > 0, \tag{32}$$

$$r_M^{(H,H)} - r_M^{(H,E)} = \frac{2a\theta_M b_M \beta_R (1-\beta_M)(b_M - b\theta_M)(b_R - b\theta_R)^2}{B_1B_2} > 0, \text{ and} \tag{33}$$

$$\frac{w^{(H,H)} - w^{(H,E)}}{r_M^{(H,H)} - r_M^{(H,E)}} = \frac{\beta_M(b_M + b\theta_M)}{2b} < \frac{\beta_M b_M}{b} = \frac{\partial \tau^H}{\partial r_M^H}. \tag{34}$$

By Proposition 5, if $w^{(H,E)} \geq \tau^H(r_M^{(H,E)})$, then the interior solution $(w^{(H,E)}, r_M^{(H,E)})$ belongs to the set $T^{(H,E)}$, so this solution is optimal to the problem $\max_{(w^H, r_M^H) \in T^{(H,E)}} \Pi_M(w^H, r_M^H, p^{(H,E)})$ $(\equiv \Pi_M^{(H,E)})$. Otherwise, the optimal solution is obtained at the boundary $w^H = \tau^H(r_M^H)$ because Π_M is jointly concave with respect to (w^H, r_M^H). Similarly, if $w^{(H,H)} \leq \tau^H(r_M^{(H,H)})$, then the optimal solution to the problem $\max_{(w^H, r_M^H) \in T^{(H,H)}} \Pi_M(w^H, r_M^H, p^{(H,H)}, r_R^{(H,H)})$ $(\equiv \Pi_M^{(H,H)})$ is $(w^{(H,H)}, r_M^{(H,H)})$; otherwise, it satisfies $w^H = \tau^H(r_M^H)$.

To determine $\Pi_M^{(H,*)} \equiv \max\{\Pi_M^{(H,E)}, \Pi_M^{(H,H)}\}$, we define f_R^b, $\underline{f}_R^H$ and f_R^c such that $\tau^H(\frac{a}{b\theta_M}) = 0$ at $f_R = f_R^b$, $\tau^H(r_M^{(H,H)}) = w^{(H,H)}$ at $f_R = \underline{f}_R^H$, and $\tau^H(r_M^{(H,E)}) = w^{(H,E)}$ at $f_R = f_R^c$. Combining the fact that τ^H is decreasing in f_R with (32)-(34), we get $f_R^b > \underline{f}_R^H > f_R^c$. We now consider four cases depending on the value of f_R: (i) $f_R \geq f_R^b$, (ii) $\underline{f}_R^H < f_R < f_R^b$, (iii) $f_R^c \leq f_R \leq \underline{f}_R^H$, and (iv) $0 < f_R < f_R^c$.

(i) If $f_R \geq f_R^b$, all feasible (w^H, r_M^H) belong to $T^{(H,E)}$. Since $w^{(H,E)} > 0 > \tau^H(r_M^{(H,E)})$, $(w^{(H,*)}, r_M^{(H,*)}) = (w^{(H,E)}, r_M^{(H,E)})$ and $\Pi_M^{(H,*)} = \Pi_M^{(H,E)}$.

(ii) If $\underline{f}_R^H < f_R < f_R^b$, $w^{(H,H)} > \tau^H(r_M^{(H,H)})$ and $w^{(H,E)} > \tau^H(r_M^{(H,E)})$. Thus, $\Pi_M^{(H,E)}$ is attained at

$(w^{(H,E)}, r_M^{(H,E)})$ and $\Pi_M^{(H,H)}$ is attained at the boundary $w^H = \tau^H(r_M^H)$. Substituting $w^H = \tau^H(r_M^H)$ into $\Pi_M(w^H, r_M^H, p^{(H,H)}, r_R^{(H,H)})$, we obtain the optimal value of r_M^H, denoted by $r_M^{(H,H)'}$, from the first-order condition $\frac{\partial \Pi_M}{\partial r_M^H} = 0$ (because $\frac{\partial^2 \Pi_M}{\partial (r_M^H)^2} < 0$), and retrieve the optimal value of w^H,denoted by $w^{(H,H)'}$, from $w^H = \tau^H(r_M^H)$:

$$\begin{aligned} r_M^{(H,H)'} &= \frac{2\theta_R b_R(1-\beta_R)(b_M - b\theta_M)}{\theta_M b_M(1-\beta_M)(b_R - b\theta_R)}\sqrt{\frac{bf_R}{A}}, \text{ and} \\ w^{(H,H)'} &= \frac{a}{b} - \frac{2}{b_R - b\theta_R}\sqrt{\frac{Af_R}{b}}\left\{1 - \frac{\beta_M\theta_R b_R(1-\beta_R)(b_M - b\theta_M)}{\theta_M(1-\beta_M)A}\right\}. \end{aligned}$$

The resulting profit $\Pi_M^{(H,H)}$ is $\frac{4ab_R\theta_R(1-\beta_R)}{b_R - b\theta_R}\left\{\sqrt{\frac{bf_R}{A}} - \frac{2bf_R}{a(b_R - b\theta_R)}\right\} + B_4 f_R - \beta_M f_M$. To determine $\Pi_M^{(H,*)} = \max\left\{\Pi_M^{(H,E)}, \Pi_M^{(H,H)}\right\}$, we compare $\Pi_M^{(H,E)}$ and $\Pi_M^{(H,H)}$ for $f_R \in (\underline{f}_R^H, f_R^b)$. When $f_R = \underline{f}_R^H$, $\Pi_M^{(H,E)}$ is attained at $(w^{(H,E)}, r_M^{(H,E)})$ and $\Pi_M^{(H,H)}$ is attained at $(w^{(H,H)}, r_M^{(H,H)})$, yielding

$$\Pi_M^{(H,E)} - \Pi_M^{(H,H)} = -\frac{2a^2\theta_M^2 b_M^2 \beta_R(1-\beta_M)^2(b_R - b\theta_R)^2}{B_1 B_2} < 0. \tag{35}$$

When $f_R = f_R^b$, $\tau^H = 0$, hence $\Pi_M^{(H,E)} > \Pi_M^{(H,H)}$. In addition, $\frac{\partial \Pi_M^{(H,H)}}{\partial f_R} < 0$ for $f_R > \underline{f}_R^H$. Therefore, there exists $\bar{f}_R^H \in (\underline{f}_R^H, f_R^b)$ such that if $\bar{f}_R^H \le f_R < f_R^b$, $(w^{(H,*)}, r_M^{(H,*)}) = (w^{(H,E)}, r_M^{(H,E)})$ and $\Pi_M^{(H,*)} = \Pi_M^{(H,E)}$, and if $f_R^b < f_R \le \bar{f}_R^H$, $(w^{(H,*)}, r_M^{(H,*)}) = (w^{(H,H)'}, r_M^{(H,H)'})$ and $\Pi_M^{(H,*)} = \Pi_M^{(H,H)}$.

(iii) If $f_R^c \le f_R \le \underline{f}_R^H$, $w^{(H,H)} \le \tau^H(r_M^{(H,H)})$ and $w^{(H,E)} \ge \tau^H(r_M^{(H,E)})$. So, $\Pi_M^{(H,E)}$ is attained at $(w^{(H,E)}, r_M^{(H,E)})$ and $\Pi_M^{(H,H)}$ is attained at $(w^{(H,H)}, r_M^{(H,H)})$. By (35), $(w^{(H,*)}, r_M^{(H,*)}) = (w^{(H,H)}, r_M^{(H,H)})$ and $\Pi_M^{(H,*)} = \Pi_M^{(H,H)}$.

(iv) If $0 < f_R < f_R^c$, $w^{(H,H)} < \tau^H(r_M^{(H,H)})$ and $w^{(H,E)} < \tau^H(r_M^{(H,E)})$. So, $\Pi_M^{(H,E)}$ is attained at the boundary $w^H = \tau^H(r_M^H)$ and $\Pi_M^{(H,H)}$ is attained at $(w^{(H,H)}, r_M^{(H,H)})$. Since the optimal $\Pi_M^{(H,E)}$ attained at the boundary $w^H = \tau^H(r_M^H)$ is smaller than that at the interior solution $(w^{(H,E)}, r_M^{(H,E)})$, by (35), $(w^{(H,*)}, r_M^{(H,*)}) = (w^{(H,H)}, r_M^{(H,H)})$ and $\Pi_M^{(H,*)} = \Pi_M^{(H,H)}$. □

Corollary 3 *Suppose the manufacturer offers rebates (i.e., $j = H$). Given the manufacturer's optimal wholesale price $w^{(H,*)}$ and manufacturer rebate value $r_M^{(H,*)}$, the retailer's best response and the resulting profit are:*

$$k^{(H,*)} = \begin{cases} E \\ H \\ H \end{cases} ; p^{(H,*)} = \begin{cases} \frac{a\theta_M\{6b_M - \beta_M(5b_M + b\theta_M)\}}{B_1} & if\ f_R \ge \bar{f}_R^H \\ \frac{a}{b} - \frac{2b_R\{2b\theta_R - \beta_R(b_R + b\theta_R)\}(1-B_5)}{b_R - b\theta_R}\sqrt{\frac{f_R}{Ab}} & if\ \underline{f}_R^H < f_R \le \bar{f}_R^H \\ \frac{a\theta_M B_6}{bB_2} & otherwise \end{cases}$$

$$r_R^{(H,*)} = \begin{cases} 0 \\ 2\sqrt{\frac{bf_R}{A}} \\ \frac{ab_M\theta_M(1-\beta_M)(b_R - b\theta_R)}{B_2} \end{cases} ; \Pi_R^{(H,*)} = \begin{cases} \frac{4a^2 b\theta_M^2 b_M^2(1-\beta_M)^2}{B_1^2} & if\ f_R \ge \bar{f}_R^H \\ \frac{Af_R}{(b_R - b\theta_R)^2} & if\ \underline{f}_R^H < f_R \le \bar{f}_R^H \\ \frac{a^2 b_M^2\theta_M^2 b_R\theta_R(1-\beta_R)(1-\beta_M)^2 A}{B_2^2} - \beta_R f_R & otherwise. \end{cases}$$

Proof of Corollary 3. $k^{(H,*)}$ follows immediately from Proposition 6. By substituting $(w^{(H,*)}, r_M^{(H,*)})$ into the results in Proposition 4, we obtain $p^{(H,*)}, r_R^{(H,*)}$ and $\Pi_R^{(H,*)}$. $\square$

Remarks on Proposition 6 and Corollary 3

1. In order for a unique equilibrium to exist, the following conditions must be satisfied:

(i) $\Pi_M(w^H, r_M^H, p^{(H,E)})$ and $\Pi_M(w^H, r_M^H, p^{(H,H)}, r_R^{(H,H)})$ are jointly concave with respect to (w^H, r_M^H);

(ii) $w^{(H,*)} \in (0, \frac{a}{b})$ and $p^{(H,*)} \in (0, \frac{a}{b})$;

(iii) $r_M^{(H,*)} > 0$ and $r_R^{(H,*)} > 0$.

Due to the complexity of these conditions, we omit the details here. As discussed in subgame 1, these conditions are satisfied in a wide range of parameter values.

2. The structural properties of $r_M^{(H,*)}, r_R^{(H,*)}$ and $\Pi_R^{(H,*)}$ with respect to f_R are straightforward from observations of their closed forms. For $f_R \in (\underline{f}_R^H, \bar{f}_R^H)$,

(i) $\Pi_M^{(H,*)}$ is decreasing and concave in f_R because $\frac{\partial}{\partial f_R}\Pi_M^{(H,*)} = 0$ at $f_R = \underline{f}_R^H$ and $\frac{\partial^2}{\partial f_R^2}\Pi_M^{(H,*)} < 0$.

(ii) $p^{(H,*)}$ is decreasing and convex in f_R by the assumption that $p^H \in \left(0, \frac{a}{b}\right)$, which implies

$$1 - \frac{\theta_R \beta_M (1-\beta_R)(b_M - b\theta_M)}{\theta_M (1-\beta_M)\{2b\theta_R - \beta_R (b_R + b\theta_R)\}} > 0. \tag{36}$$

(iii) $w^{(H,*)}$ is decreasing and convex in f_R if and only if $1 - \frac{\theta_R b_R \beta_M (1-\beta_R)(b_M - b\theta_M)}{\theta_M (1-\beta_M) A} > 0$. By (36), this condition holds if $\frac{A}{b_R} - \{2b\theta_R - \beta_R (b_R + b\theta_R)\} = \frac{b\theta_R}{b_R}(2b_R - \beta_R(b_R + b\theta_R)) > 0$. The result follows because $b_R > b\theta_R$.

Proof of Lemma 1. At $f_R = \underline{f}_R^E$, $\tau^E = \frac{a}{2b}$. At $f_R = \underline{f}_R^H$, $\tau^H(r_M^{(H,H)}) = \tau^E + \frac{\beta_M b_M}{b} r_M^{(H,H)} = w^{(H,H)}$, hence $\tau^E = w^{(H,H)} - \frac{\beta_M b_M}{b} r_M^{(H,H)}$. From (17), τ^E is decreasing in f_R. Therefore, $\underline{f}_R^E < \underline{f}_R^H$ if and only if

$$\frac{a}{2b} - \left(w^{(H,H)} - \frac{\beta_M b_M}{b} r_M^{(H,H)}\right) > 0. \tag{37}$$

Substituting $\left(w^{(H,H)}, r_M^{(H,H)}\right)$ in (31) into (37), we get after simplification: $\frac{a\theta_R b_R \beta_M (1-\beta_R)(b_M - b\theta_M)^2}{2bB_2} > 0$. $\square$

Proof of Theorem 1 We present the proofs of Parts (a), (c) and (e) regarding the threshold $f_M^*(f_R)$. The same method applies to Parts (b) and (d) regarding the threshold $f_R^*(f_M)$.

(a) From Proposition 6, $\Pi_M^{(H,*)}$ is linearly decreasing in f_M with slope β_M. Therefore, for the existence of $f_M^*(f_R)$ at which $\Pi_M^{(H,*)} = \Pi_M^{(E,*)}$, it suffices to show that when $f_M = 0$, $\Pi_M^{(H,*)} > \Pi_M^{(E,*)}$ for all f_R. If we define $\Pi_M^H \equiv \Pi_M^{(H,*)} + \beta_M f_M$, then $f_M^*(f_R) = \frac{1}{\beta_M}\left(\Pi_M^H - \Pi_M^{(E,*)}\right)$. From Propositions 3 and 6, we know: (i) in I_2, Π_M^H is constant and $\Pi_M^{(E,*)}$ is decreasing in f_R; and (ii) Π_M^H in I_4 is greater than Π_M^H in I_5, while $\Pi_M^{(E,*)}$ is the same in both I_4 and I_5. From (i), if $\Pi_M^H > \Pi_M^{(E,*)}$ in I_1, then $\Pi_M^H > \Pi_M^{(E,*)}$ in I_2. From (ii), if $\Pi_M^H > \Pi_M^{(E,*)}$ in I_5, then $\Pi_M^H > \Pi_M^{(E,*)}$ in I_4. Therefore,

it suffices to show that $\Pi_M^H > \Pi_M^{(E,*)}$ in I_1, I_3 and I_5. From Propositions 3 and 6, we obtain after simplification:

$$\Pi_M^H - \Pi_M^{(E,*)} = \begin{cases} \frac{a^2\theta_R^2 b_R^2 \beta_M (1-\beta_R)^2 (b_M - b\theta_M)^2}{2AB_2} & if\ f_R \in I_1 \\ \frac{4bb_R^2\theta_R^2 \beta_M (1-\beta_R)^2 (b_M - b\theta_M)^2}{Ab_M\theta_M (b_R - b\theta_R)^2 (1-\beta_M)} f_R & if\ f_R \in I_3 \\ \frac{a^2 \beta_M (b_M - b\theta_M)^2}{8bB_1} & if\ f_R \in I_5 \end{cases} . \tag{38}$$

The existence of $f_M^*(f_R)$ follows from the fact that (38) is positive in every interval.

(c) From (38), $f_M^* \left(= \frac{1}{\beta_M}\left(\Pi_M^H - \Pi_M^{(E,*)}\right)\right)$ is constant with respect to f_R in $I_1 \cup I_5$, and f_M^* is linear to f_R in I_3. From Propositions 3 and 6, $\frac{\partial f_M^*}{\partial f_R} = -\frac{1}{\beta_M}\frac{\partial \Pi_M^{(E,*)}}{\partial f_R}$ for $f_R \in I_2$. Since $\Pi_M^{(E,*)}$ is concave and decreasing in $f_R \in I_2$, f_M^* is convex and increasing in $f_R \in I_2$. Similarly, $\frac{\partial f_M^*}{\partial f_R} = \frac{1}{\beta_M}\frac{\partial \Pi_M^H}{\partial f_R}$ for $f_R \in I_4$, hence f_M^* is concave and decreasing in $f_R \in I_4$.

(e) Substituting $\beta_M(b_M - b\theta_M) = 0$ into (38), $\Pi_M^H - \Pi_M^{(E,*)} = 0$, hence $f_M^*(f_R) = 0$ for all f_R. From Part (d), it follows that $f_R^*(f_M) = \bar{f}_R^E = \bar{f}_R^H$ for all f_M. $\square$