The Implications of Customer Purchasing Behavior and In-store Display Formats

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Abstract

Consider a retailer announces both the regular price and the post-season clearance price at the beginning of the selling season. Throughout the season, customers arrive in accord with a Poisson process. In this paper we analyze the impact of two types of customer purchasing behavior and two common in-store display formats on the retailer's optimal expected profit and optimal order quantity. We consider the case when all customers are either myopic (purchase immediately upon arrival) or strategic (either purchase at the regular price upon arrival or attempt to purchase at the clearance price after the season ends). In addition, we consider the case when the retailer would display either all available units or one unit at a time on the sales floor. When all customers have identical valuation, we show that, in equilibrium, each strategic customer's purchasing decision is based on a threshold policy that depends on the inventory level at the time of arrival. We prove analytically that the retailer would obtain a higher expected profit and would order more when the customers are myopic. Also, we show analytically that the retailer would earn a higher expected profit and would order more under the display one unit format when the customers are strategic. We illustrate numerically the penalty when the retailer mistakenly assumes that the strategic customers are myopic. We extend our analysis to the case in which customers belong to multiple classes, each of which has a class-specific valuation, and also to the case in which the post-season clearance price depends on the actual end-of-season inventory level.

Keyword: Retailing, Purchasing Behavior, In-Store Display Formats, Ordering Decision.

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1 Introduction

Consider a retailer who sells a fashion product with uncertain customer arrivals over a single selling season. For any given selling price p_h during the season and salvage value s, one can formulate the problem as the newsvendor problem and obtain the optimal order quantity and the corresponding optimal expected profit. The elegant newsvendor solution is based on an assumption that the customers are 'myopic' in the sense that they will purchase the product immediately upon arrival. However, the myopic assumption becomes questionable when the retailer deploys different pricing strategies.

To obtain a clearance price that is higher than the salvage value s, retailers have developed different dynamic pricing mechanisms since the late 1980s. When selling seasonal goods, a common form of dynamic pricing strategy is the markdown pricing strategy. Fisher et al. (1994) reported that 26% of fashion goods are sold at markdown prices. As retailers offer different markdown pricing mechanisms, customers would take the future price into consideration when making their purchasing decisions. As such, customers are becoming more 'strategic' in the sense that they might wait for a sale instead of purchasing the product immediately upon arrival. This type of strategic purchasing behavior has been reported in Kadet (2004) and McWilliams (2004).

As customers become strategic, the newsvendor solution no longer holds. This new shift in customer purchasing behavior has motivated us to develop a model for examining the impact of strategic purchasing behavior on the retailer's optimal order quantity and optimal expected profit. As an initial attempt to study this issue, we shall focus our analysis on the case in which the retailer adopts a simple form of markdown pricing mechanism that can be described as follows. At the beginning of the season, the retailer orders $Q \geq 1$ units and announces that the product will be sold at the reduced price $p_l < p_h$ if it is not sold at the regular price p_h by the end of the season.[1] A customer can either purchase the product (if available) during the season at price p_h or attempt to purchase the product at the reduced price p_l after the season ends. Suppose there are k units available at the end of the season and suppose there are n customers who decided to wait for the end-of-season sale. Then each customer will get 1 unit if $n \leq k$ and each of the n customers has an equal probability of $\frac{k}{n}$ for getting 1 unit when $n > k$. This rationing policy mimics the situation when all markdown items are sold on a first-come-first-serve basis. A more general version of this markdown pricing mechanism has been adopted by the Filene's Basement

[1] This is equivalent to the case in which the price is dropped from p_h to p_l within the season.

store in Boston since 1908. At the Filene's Basement store, the "automatic markdown plan" is pre-announced: most unsold items after 2, 4 and 6 weeks will be sold at 25%, 50% and 75% off the regular price, respectively. Filene's Basement will donate all the unsold items after 2 months to the charity. The reader is referred to Bell and Starr (1998) for more details.

In addition to markdown pricing mechanisms, in-store display format has a direct impact on the strategic purchasing behavior as well. This is because different in-store display formats can create different impressions about the actual inventory level at the store. As an initial attempt to analyze the impact of in-store display formats on the retailer's optimal order quantity and optimal expected profit when the customers are strategic, we consider two basic in-store display formats under which the retailer would either display all units or display one unit at a time on the sales floor.[2] The 'Display All' format has been adopted by many fashion retailers such as Filene's Basement and Benetton, while the 'Display One' format has been adopted by various high-end stores such as the Bally handbag stores in Taiwan and the Hour Glass watch stores in Singapore and Hong-Kong. Both in-store display formats have different implications. The Display All format allows a retailer to utilize the available space more effectively by maximizing the sales floor space. Also, it provides each arriving customer perfect information about the actual inventory level available for sale at the time of arrival, which has direct impact on the customers' strategic purchasing decisions. The Display One format allows a retailer to use the limited sales space to display an assortment of different designs instead of multiple units of the same design. By displaying one unit at a time, it creates an impression of scarcity, which would urge interested customers to purchase the product immediately upon arrival.

When customers are either myopic or strategic and when the retailer adopts either the Display All or the Display One format, we are interested in examining the following questions for any given values of p_h and p_l:

1. When the customers are strategic, how would different display formats affect their optimal purchasing behavior?

2. When the customers are strategic, how would different display formats affect the retailer's optimal expected profit and optimal ordering decision?

[2]There are other in-store display formats including the case in which the retailer, say, Zara, displays only a few items on the sales floor, and the case in which the retailer display the products in different strategic locations such as front of the aisle, entrance to the store, etc. The reader is referred to Ghemawat and Nueno (2003) for details.

3. Suppose the retailer incorrectly assumes that the strategic customers are myopic. How would this incorrect assumption affect the retailer's optimal expected profit and optimal ordering decision for each display format?

To answer these questions, we develop a model that incorporates stochastic customer arrivals and rational purchasing behavior. In our base model, we analyze the case in which customers with identical valuation arrive to the store in accord with a Poisson process. We first show that each customer's optimal purchasing decision is based on a threshold that depends on the inventory level at the time of arrival. This result implies that, when the customers are strategic, the total demand for the product during the season would depend on the number of customers arrived during the season and their actual arrival times. In contrast, when the customers are myopic, the total demand depends only on the number of customers arrived during the season. Therefore, the total demand associated with the strategic customer case is more uncertain than that of the myopic customer case. Hence, one would speculate that the retailer's optimal order quantity would be higher when the customers are strategic. However, when customers are strategic, some customers pay p_h and others pay $p_l < p_h$. Hence, the effective price would be lower than p_h and one would conjecture that the retailer's optimal order quantity would be lower when customers are strategic. In light of these two opposite views, the net impact of strategic purchasing behavior on the retailer's optimal order quantity is not obvious to us.

To investigate how customer's purchasing behavior affects retailer's optimal expected profit and optimal order quantity, we determine the retailer's expected profit associated with both display formats when the customers are myopic and strategic. We show that: (a) The retailer will order more and enjoy a higher expected profit when the customers are myopic instead of strategic; and (b) The retailer will order more and enjoy a higher profit under the Display One format than that of the Display All format when the customers are strategic. In addition, we show analytically that the retailer will over-order and will obtain a lower expected profit when the retailer incorrectly assumes that the strategic customers are myopic.[3] These results imply that the customer's purchasing behavior (myopic or strategic) and the in-store display formats (Display All or Display One) have significant impacts on the retailer's optimal order quantity and optimal expected profit. We also extend our analysis to the case in which customers belong to different classes, each of which has a class-specific valuation. We obtain similar analytical results when the retailer adopts the Display

[3]This analytical result is consistent with the numerical results obtained by Aviv and Pazgal (2005) and Levin et al. (2005).

One format. Furthermore, we extend our analysis to the case in which the post-season clearance price depends on the actual end-of-season inventory level.

2 Literature Review

Our paper is related to two groups of recent papers that analyze dynamic pricing issues for the case when customers are strategic.[4] In the first group, all customers are assumed to be present at the beginning of the selling season. Besides the earlier economic models developed by Stokey (1979), Besanko and Winston (1990), and Harris and Raviv (1981), we review three recent papers that are based on the assumption that all customers are present at the beginning of the selling season. Elmaghraby, Gulcu and Keshkinocak (2004) examine a situation in which the retailer pre-announces the price markdown schedule. The customers may demand multiple units of the product and can choose the number of units to purchase at each price drop. By determining the rational purchase behavior of each customer, they compare the retailer's expected profit associated with different pre-announced markdown mechanisms. Levin et al. (2005) present a stochastic dynamic game formulation for the dynamic pricing problem. They prove the existence of a unique subgame perfect equilibrium dynamic pricing policy and they obtain monotonicity results for two special cases: (a) when customers are myopic; and (b) when customers are strategic but do not need to compete for the items since the inventory level is sufficient to satisfy all customers. Liu and van Ryzin (2005) study a situation when the retailer commits to a pre-announced markdown price schedule. By assuming that all customers are present simultaneously at the beginning of the selling season, they determine the optimal ordering decision and they develop conditions under which it is optimal for the retailer to create shortages by understocking products.

In the second group, customers arrive at different times throughout the selling season. Su (2005) examines a situation in which customers may belong to either the high-valuation segment or the low-valuation segment, and may either be strategic or myopic. When customers arrive continuously in a deterministic manner throughout the season, he determines the optimal dynamic pricing policy over time as well as the optimal ordering decision for the retailer. Elmaghraby et al. (2005) analyze a situation in which the retailer sells 1 unit under two operating regimes. Under

[4] The reader is referred to an article by Weatherford and Bodily (1992), a comprehensive review by Elmaghraby and Keskinocak (2003), and a book by Talluri and van Ryzin (2004) for more in-depth discussion about dynamic pricing.

the reservation regime, a buyer can either purchase the product at the regular price or reserve the product at the post-season clearance price. If the buyer reserves the product and if it remains unsold at the end of the season, he is obligated to purchase the product at the clearance price. Under the no reservation regime, a buyer can either purchase the product at the regular price or he enters a lottery to purchase the product at the clearance price if the product remains unsold. In the presence of Poisson customer arrivals, they show that the retailer can always obtain a higher expected profit under the reservation regime when there is a single class of customers with identical valuation. However, when there are multiple classes of customers with class-specific valuations, they establish conditions under which the reservation regime dominates the no reservation regime. Aviv and Pazgal (2005) study two pricing strategies: inventory contingent discounting strategy and announced fixed-discount strategy. In the first strategy, the retailer would only announce the clearance price after the actual end-of-season inventory level is realized. In the second strategy, the retailer would announce the clearance price at the beginning of the season. They assume that each arriving customer only knows the initial order quantity Q, but does not know the actual inventory level at the time of arrival. This assumption enables them to show that it is optimal for customers to purchase according to individual thresholds that depend on the individual valuations and arrival times. This assumption also enables them to develop a subgame perfect Nash equilibrium for the game between the retailer and the customers.

While our model is based on a pre-announced markdown pricing scheme, the focus is different from the aforementioned papers in the following ways. First, most of the aforementioned papers focused on the retailer's optimal pricing policy, while our focus is on the retailer's optimal expected profit and optimal order quantity. Second, most of the papers are based on the assumption that all customers are present at the beginning of the season or customer arrivals are deterministic, while our paper considers stochastic customer arrivals as in Aviv and Pazgal (2005) and Elmagrahby et al. (2005). Third, Elmagraghby et al. (2005) examines the retailer's optimal profit associated with two operating regimes for the case when there is only 1 unit to sell, while we consider the case when the retailer has Q units available for sale at the beginning of the season. Also, we determine the retailer's optimal profit and optimal order quantity for the case when all customers are either myopic or strategic and when the retailer adopts either the Display All or Display One format. Fourth, unlike the assumption considered in Aviv and Pazgal (2005), we assume that each arriving customer knows the actual inventory level at the time of arrival, which is probably more reasonable in a traditional retailing environment. Also, we consider different in-store display formats: Display All and Display One. Under both display formats, we show that it is optimal for each customer to purchase according to his threshold that depends on his valuation, his arrival time, and the actual inventory level upon the time of arrival. By taking the customer's strategic purchasing behavior into consideration, we are able to express the retailer's optimal expected profit and optimal order quantity in implicit functional forms for both display formats. These implicit functional forms enable us to compare the retailer's optimal expected profits and optimal order quantities associated with different scenarios analytically.

Our paper is organized as follows. By assuming all customers belong to a single class with identical valuation, Section 3 examines the base models in which all customers can be either myopic or strategic and the retailer adopts either the Display All or the Display One format. When the customers are strategic, we establish the optimal purchasing rule and determine the retailer's expected profit under both display formats. In Section 4, we compare the retailer's optimal expected profits and optimal order quantities for each of the base models, and we show analytically that the optimal order quantity is higher when the customers are myopic and the optimal order quantity is higher under the Display One format when the customers are strategic. We consider two extensions in Section 5. In the first extension, we extend our analysis to the case in which the customers belong to multiple classes, each of which has a class-specific valuation. Under the Display One format, we determine the optimal purchasing rule and the retailer's expected profit when the customers are strategic. We compare the retailer's optimal profits and optimal order quantities for the cases when customers are either myopic or strategic. In the second extension, we consider a situation in which the post-season clearance price depends on the end-of-season inventory level. Section 6 ends our paper with some concluding remarks.

3 The Base Model

Consider a retailer who orders and sells $Q \geq 1$ units of a single product with unit cost c over a selling season that spans over $[0, T]$. At the beginning of the selling season, the retailer announces both the price p_h at which the product will be sold during the selling season and the post-season clearance price p_l for the unsold items, where $c < p_l < p_h$. The retailer will obtain a salvage value $s < c$ for each unit that remains unsold after the post-season clearance. In this paper, we consider two display formats: Display all and Display one. Under the Display All format, all available units are displayed on the sales floor at all times, and hence, each arriving customer has perfect information regarding the inventory level at the time of arrival. Under the Display One format, the retailer displays only 1 item on the sales floor, and keeps other available units in the storeroom. Once the display item is sold, the retailer will display a new item retrieved from the storeroom. We assume that each arriving customer thinks that the retailer has only 1 item (if available) for sale at the time of arrival.[5]

During the season, customers arrive in accord with a Poisson process with rate λ, where λ remains constant throughout the entire season. Upon arrival, each customer can either purchase one unit (if available) during the season at p_h or wait and then attempt to purchase at the reduced price p_l after the season ends. When the season ends, each customer who waited will get one unit at the reduced price p_l if the leftover inventory exceeds the number of interested customers. Otherwise, the retailer will ration out the leftover inventory to these interested customers with equal probability.[6]

In our base model, we assume that the market is comprised of a single class of customers with identical valuation v.[7] To ensure each customer might purchase the product during the season, we assume that $v > p_h > p_l$. We also assume that the parameter values v, p_h, p_l and λ are common knowledge. In addition, each customer knows his arrival time t. The base model with a single class customers enables us to understand the underlying structure of the model and to generate specific insights. In a later section, we shall extend our analysis of the Display One format to the case in which the customers belong to multiple classes, each of which has a class-specific valuation. In preparation, let $B(t)$ and $A(t)$ be the number of customers who arrive 'before' and 'after' t, respectively. Notice that $B(t)$ and $A(t)$ are independent Poisson random variables with parameters λt and $\lambda(T-t)$, respectively.

[5] This assumption is reasonable when each customer is only interested in purchasing one unit, when the customer does not ask the retailer about the actual inventory level, when the retailer does not know the actual inventory level, or when the retailer does not reveal the actual inventory level to the customers.

[6] This rationing policy mimics the case when the clearance items are sold on a first-come-first-serve basis.

[7] Our model can be easily extended to the case when there are two classes of customers. Class i customers have identical valuation v_i, for $i = 0, 1$, where $v_1 > p_h > v_0 > p_l$, so that all arriving customers of Class 0 will always attempt to purchase the item at the reduced price p_l after the season ends.

3.1 Myopic Customers

When the customers are myopic and when $v > p_h$, each arriving customer during the season will attempt to purchase the item at p_h regardless of the display format adopted by the retailer. Hence, the effective demand for the product is equal to $B(T)$. In this case, regardless of the display format adopted by the retailer, the retailer's expected profit is identical to the expected profit function associated with the newsvendor problem. Thus, when the customers are myopic, the retailer's expected profit for any order quantity Q can be written as:

$$\begin{aligned}\Pi_r^M(Q) &= \mathrm{E}\{p_h \min\{Q, B(T)\} + s[Q - B(T)]^+\} - cQ \\ &= (p_h - c)Q - (p_h - s)E[Q - B(T)]^+. \end{aligned} \tag{3.1}$$

3.2 Strategic Customers Under the Display All Format

We now determine the retailer's expected profit function associated with the Display All format for the case when the customers are strategic. To begin, let us examine the customer's strategic purchasing behavior.

3.2.1 Optimal Strategic Purchasing Rule Under the Display All Format

When the retailer adopts the Display All (DA) format, each arriving customer knows the actual number of units available for sale upon arrival. To examine how this knowledge affects a strategic customer's purchasing decision, let us consider a customer who arrives at time t and observes k units available for sale, where $1 \leq k \leq Q$. He will enjoy a surplus $v - p_h$ if he purchases the item at p_h. Alternatively, he can wait and attempt to purchase the item at the reduced price p_l after the season ends.[8] If he attempts to purchase the item at the reduced price p_l, his expected surplus is equal to $(v - p_l)H(k, t)$, where the term $H(k, t)$ represents the expected probability of getting the item at the reduced price after the season ends. By comparing the expected surpluses associated with these two purchase options, we can establish the following DA threshold purchasing rule: For any customer who arrives at time t and observes k units available for sale, he should: (a) purchase

[8] We assume that each customer who decides to wait will return to the store at time T. It is easy to check that our model can be extended to the case when a fixed proportion $0 < q \leq 1$ of customers who decide to wait will eventually return to the store at time T. For simplicity, we assume that $q = 1$ in our model.

one unit at p_h if $t \le t^*(k)$; and (b) attempt to purchase one unit at p_l after the season ends if $t > t^*(k)$, where the threshold $t^*(k) = \max\{0, t(k)\}$ and $t(k)$ satisfies:[9]

$$H(k, t(k)) = \frac{v - p_h}{v - p_l}, \tag{3.2}$$

and $t(Q) < t(Q-1) < \cdots < t(1)$.

In general, the expected probability $H(k,t)$ associated with the DA threshold purchasing rule is a complex function because it depends on the customer arrival pattern throughout the entire season. However, the expected probability $H(k,t)$ can be established easily for the case when $t = t(k)$. When $t = t(k)$, the DA threshold purchasing rule implies that, in order for a customer to observe k items available at time $t(k)$, no customers arrived before $t(k)$ would wait and all customers who arrive after $t(k)$ would wait. Therefore, all k units available at time $t(k)$ will still be available for sale at the reduced price p_l after the season ends. Under our rationing policy, the customer arriving at time $t(k)$ who decided to wait will get the item at the reduced price p_l with probability 1 when $A(t(k)) \le k-1$ and $\frac{k}{A(t(k))+1}$ when $A(t(k)) \ge k$. Combine this observation with the fact that $A(t(k))$ is a Poisson random variable with parameter $\lambda(T - t(k))$, we can express the term $H(k, t(k))$ as:

$$\begin{aligned} H(k, t(k)) &= \sum_{n=0}^{k-1} \text{Prob}(A(t(k)) = n) + \sum_{n=k}^{\infty} \text{Prob}(A(t(k)) = n) \frac{k}{n+1}, \\ &= \sum_{n=0}^{k-1} \frac{[\lambda(T - t(k))]^n}{n!} e^{-\lambda(T-t(k))} + \sum_{n=k}^{\infty} \frac{[\lambda(T - t(k))]^n}{n!} e^{-\lambda(T-t(k))} \frac{k}{n+1}. \end{aligned} \tag{3.3}$$

By examining (3.3) and (3.2), we can prove the following Lemma:

Lemma 1 *The threshold $t(k)$ that satisfies (3.2) is unique. Also, the threshold $t(k)$ has the following properties:*

1. $t(Q) < t(Q-1) < \cdots < t(k+1) < t(k) < \cdots < t(1) < T$.
2. *The threshold $t(k)$ is increasing in λ, v and p_l, and decreasing in p_h.*

Proof: All proofs are given in the Appendix.

It follows from Lemma 1 that the threshold $t(k)$ is strictly decreasing in k and the fact that $t^*(k) = \max\{0, t(k)\}$, it is easy to show that:

[9]This construct generalizes the analysis presented in Elmaghraby et al. (2005) for the case when $Q = 1$.

Proposition 1 *There exists a positive integer θ that satisfies: $\theta = argmin\ \{t(j) \le 0 : j = 1, 2, \cdots\}$. Moreover, the threshold $t^*(k) = \max\{0, t(k)\}$ has the following properties:*

1. *If $Q < \theta$, then $0 < t^*(Q) < t^*(Q-1) < \cdots < t^*(1) < T$.*

2. *If $Q \ge \theta$, then $t^*(Q) = t^*(Q-1) = \cdots = t^*(\theta) = 0 < t^*(\theta - 1) < \cdots < t^*(1) < T$.*

3. *The threshold $t^*(k)$ is increasing in λ, v and p_l, and decreasing in p_h.*

Proposition 1 implies that the thresholds $t^*(k)$ are decreasing in k and that $t^*(Q) = 0$ when the initial order quantity $Q \ge \theta$. When Q is sufficiently large so that $t^*(Q) = 0$, each customer arriving at time $t > 0 = t^*(Q)$ will observe Q units available and will attempt to purchase the product at the reduced price p_l under the DA threshold purchasing rule. This implication is intuitive because, when the initial order quantity is sufficiently large, the expected probability of getting the product at the reduced price after the seasons ends is high; i.e., $H(Q, t)$ is high. Hence, there is no incentive for any arriving customer to purchase the product at the regular price p_h. This result is consistent with that obtained by Liu and van Ryzin (2005) under the assumption that all customers are present at the beginning of the season.

By applying Proposition 1, we can compare the expected surplus of an arriving customer for the case when he follows the DA threshold purchasing rule and the case when he deviates from the DA threshold purchasing rule. This comparison enables us to prove that:

Proposition 2 *There is a Nash equilibrium in which all arriving customers follow the DA threshold purchasing rule.*

3.2.2 Expected Payoffs Under the Display All Format

We now determine the retailer's expected profit when all arriving customers follow the DA threshold purchasing rule. Notice that the retailer's profit depends on the purchasing decisions made by the customers who arrive during different time intervals $(t^*(j), t^*(i)]$ for $1 \le i < j \le Q + 1$, where $t^*(Q + 1) \equiv 0$; hence, the computation of the retailer's expected payoff is not straightforward. However, it can be computed in a recursive manner. In preparation, for $1 \le i < j \le Q + 1$, let:

$$f(j, i) \quad = \quad \text{the retailer's expected revenue to be obtained from } t^*(j) \text{ to } T$$

$$
\begin{aligned}
& \quad \text{when } i \text{ units are available for sale at time } t^*(j), \text{ and} \\
g(i) \quad = & \quad \text{the retailer's expected revenue to be obtained from } t^*(i) \text{ to } T \\
& \quad \text{when } i \text{ units are available for sale at time } t^*(i).
\end{aligned}
$$

Since the retailer has Q units available for sale at time $t^*(Q+1) \equiv 0$, the function $f(Q+1, Q)$ corresponds to the retailer's expected revenue over the entire season. Hence, for any order quantity Q, the retailer's expected profit can be expressed as:

$$\Pi_r^{DA}(Q) = f(Q+1, Q) - cQ. \tag{3.4}$$

To determine the retailer's expected profit $\Pi_r^{DA}(Q)$ for any given Q, it suffices to focus on the function $f(j,i)$ for $1 \leq i < j \leq Q+1$. To begin, let $N(j,i)$ be the number of customers who arrive within the time window $(t^*(j), t^*(i)]$. Considering three mutually exclusive and exhaustive events associated with $N(j,i)$ yields:

Proposition 3 *For $1 \leq i < j \leq Q+1$, the recursive function $f(j,i)$ and the function $g(i)$ satisfy:*[10]

1. $f(j,i) = g(i)Prob(N(j,i) = 0) + ip_h \sum_{k=i}^{\infty} Prob(N(j,i) = k) + \sum_{k=1}^{i-1}(kp_h + f(i, i-k))Prob(N(j,i) = k)$.
2. $g(i) = ip_l - (p_l - s)\sum_{k=0}^{i-1}(i-k)Prob(N(i,0) = k)$.

Since $N(j,i)$ is a Poisson random variable with parameter $\lambda(t^*(i) - t^*(j))$, we can determine the functions $f(j,i)$ and $g(i)$, and hence, the retailer's expected profit $\Pi_r^{DA}(Q)$ given in (3.4).[11]

3.3 Strategic Customers Under the Display One Format

We now examine the case when the retailer orders Q units at the beginning of the selling season, displays only 1 item on the sales floor, and keeps the rest in the storeroom. The Display One format operates as follows: once the display item is sold, the retailer will display a new item retrieved from the storeroom. We now determine the strategic customer's purchasing rule and the retailer's expected profit under the Display One format.

[10] We define $\sum_{k=1}^{0} \equiv 0$.

[11] To compute customers' expected surplus Π_c^{DA} under the Display All Policy, we can use the same approach by defining similar recursive functions $f(j,i)$ and $g(i)$. We omit the details.

3.3.1 Optimal Strategic Purchasing Rule Under the Display One Format

Under the Display One format, a customer who arrives at time t will observe $k = 1$ unit available for sale. He will enjoy a surplus $v - p_h$ if he purchases the item at p_h. Alternatively, he can wait and attempt to purchase the item at p_l after the season ends. Since he believes that there is only $k = 1$ unit available, each arriving customer would behave in accord with the case when $Q = 1$ under the Display All format. As such, the Display One format is a special case of the Display All format when $Q = 1$. Hence, we can use the same approach to prove that all customers will follow the DO threshold purchasing rule in equilibrium, where the DO threshold purchasing rule is defined as follows: (a) purchase the item at p_h if $t \leq t'$; and (b) attempt to purchase at p_l after the season ends if $t > t'$, where $t' = t^*(1) = max\{0, t(1)\}$ and $t(1)$ satisfies (3.2).

3.3.2 Expected Payoffs Under the Display One Format

Under the DO purchasing rule, all customers arriving before t' (denoted by $B(t')$) would attempt to purchase the product at p_h and all customers arriving after t' (denoted by $A(t')$) would attempt to purchase the product at the reduced price p_l after the season ends. Therefore, for any order quantity Q, the retailer can generate three revenue streams from selling $\min\{Q, B(t')\}$ items at p_h, selling $\min\{[Q - B(t')]^+, A(t')\}$ items at p_l, and disposing of $[[Q - B(t')]^+ - A(t')]^+$ items at salvage value s. Therefore, for any order quantity Q, the retailer's expected profit associated with the Display One format can be expressed as:[12]

$$\begin{aligned} \Pi_r^{DO}(Q) &= \mathrm{E}\left\{p_h \min\{Q, B(t')\} + p_l \min\{[Q - B(t')]^+, A(t')\} + s[Q - B(t') - A(t')]^+\right\} - cQ \\ &= (p_h - c)Q - (p_h - p_l)\mathrm{E}[Q - B(t')]^+ - (p_l - s)\,\mathrm{E}[Q - B(T)]^+, \end{aligned} \tag{3.5}$$

where $B(T) = A(t') + B(t')$ corresponds to the total number of customers arriving within the selling season.

[12]We can compute the customers' expected surplus under the Display One format by observing that each of the $\min\{(Q, B(t'))\}$ customers will obtain a surplus $(v - p_h)$ and each of the $\min\{([Q - B(t')]^+, A(t'))\}$ customers will obtain a surplus $(v - p_l)$. We omit the details.

4 Comparisons

We now compare the retailer's expected profits and the retailer's optimal order quantities when the customers are either myopic or strategic and when the retailer adopts either the Display All or the Display One format.

4.1 Comparison of the Retailer's Expected Profits

First, let us compare the retailer's expected profit when customers are myopic and the retailer's expected profit associated with the case of strategic customers under the Display One format as presented in Section 3.3. When customers are myopic, all arriving customers (i.e., $B(T)$) will attempt to purchase the item at p_h regardless of the display format. However, under the Display One format, only those customers arriving before t' (i.e., $B(t')$) will attempt to purchase the item at p_h when they are strategic. Since $B(T)$ and $B(t')$ are Poisson random variables with parameters λT and $\lambda t'$, respectively; and since $t' = t^*(1) < T$, $B(T)$ is stochastically larger than $B(t')$, and hence, $E[Q - B(T)]^+ \leq E[Q - B(t')]^+$. Combine this observation with the retailer's expected profits given in equations (3.1) and (3.5), we have shown that $\Pi_r^M(Q) \geq \Pi_r^{DO}(Q)$ for any given order quantity Q. Next, comparing the retailer's expected profits associated with the Display One and the Display All formats yields:

Proposition 4 $\Pi_r^M(Q) \geq \Pi_r^{DO}(Q) \geq \Pi_r^{DA}(Q)$.

For any order quantity Q, Proposition 4 indicates that the retailer will gain more when the customers are myopic. Moreover, when customers are strategic, displaying the items one at a time instead of displaying all items will enable the retailer to obtain a higher expected profit. It follows from Proposition 4, it is easy to see that: $\max_Q \ \Pi_r^M(Q) \geq \max_Q \ \Pi_r^{DO}(Q) \geq \max_Q \ \Pi_r^{DA}(Q)$. Hence, the retailer can obtain a higher optimal expected profit under the Display One format when the customers are strategic.

4.2 Comparison of Retailer's Optimal Order Quantities

We now compare the retailer's optimal order quantities when customers are myopic and when customers behave strategically under the two display formats. In preparation, we determine the optimal order quantity for each case. First, when customers are myopic, the retailer's expected profit function $\Pi_r^M(Q)$ given in (3.1) has an identical structure as in the newsvendor problem, and hence, the optimal order quantity Q^M is the smallest integer that satisfies:

$$F(Q) \geq \frac{p_h - c}{p_h - s}, \tag{4.1}$$

where $F(\cdot)$ is the cumulative distribution function of $B(T)$, a Poisson random variable with parameter λT.

Second, when the customers are strategic and when the retailer adopts the Display One format, it is easy to check from (3.5) that the retailer's profit function $\Pi_r^{DO}(Q)$ is a concave function of Q. By considering the first order condition, the retailer's optimal order quantity Q^{DO} is the smallest integer that satisfies:

$$(p_h - p_l)G(Q) + (p_l - s)F(Q) \geq p_h - c, \tag{4.2}$$

where $G(\cdot)$ is the cumulative distribution function of $B(t')$, a Poisson random variable with parameter $\lambda t'$.

Third, when customers are strategic and when the retailer adopts the Display All format, we need to consider two separate cases: $Q < \theta$ and $Q \geq \theta$. Let $Q_1 = \arg\max\{\Pi_r^{DA}(Q) : Q \text{ integer and } Q < \theta\}$, and $Q_2 = \arg\max\{\Pi_r^{DA}(Q) : Q \text{ integer and } Q \geq \theta\}$. Therefore, under the Display All format, the retailer's optimal order quantity is:

$$Q^{DA} = \begin{cases} Q_1, & \text{if } \Pi_r^{DA}(Q_1) > \Pi_r^{DA}(Q_2); \\ Q_2, & \text{otherwise.} \end{cases} \tag{4.3}$$

Comparing the optimal order quantities given in (4.1), (4.2) and (4.3) yields:

Proposition 5 $Q^M \geq Q^{DO} \geq Q^{DA}$.

Proposition 5 implies that the retailer will tend to overstock if he thinks the customers are myopic while they are indeed strategic. In addition, since $\Pi_r^{DO}(Q^{DO}) \geq \Pi_r^{DO}(Q^M)$ and $\Pi_r^{DA}(Q^{DA}) \geq \Pi_r^{DA}(Q^M)$, the retailer's expected profit will suffer if he incorrectly assumes that the strategic customers are myopic. To illustrate numerically about the penalty associated with this incorrect assumption, we set: $p_h = 100, p_l = 35, T = 8, \lambda = 1, c = 25, s = 10$ and we vary v from 115 to 190. As shown Figure 1, the retailer's relative profit loss under the Display All format

($\frac{\Pi_r^{DA}(Q^{DA})-\Pi_r^{DA}(Q^M)}{\Pi_r^{DA}(Q^{DA})}$) is increasing in the customer's valuation, ranging from 60% to 87%. However, the relative profit loss under the Display One format ($\frac{\Pi_r^{DO}(Q^{DO})-\Pi_r^{DO}(Q^M)}{\Pi_r^{DO}(Q^{DO})}$) is decreasing in the customer's valuation, ranging from 1% to 10%. Therefore, it is important for the retailer to gain a clearer understanding about customer's purchasing behavior when making ordering decision for products with short selling seasons. In addition, when customers are strategic, Proposition 5 suggests that the retailer can obtain a higher expected profit by ordering more when he adopts the Display One format instead of the Display All format. This result is verified numerically in Figure 2. In summary, customers' strategic purchasing behavior and retailer's display format can have significant impacts on the retailer's order quantity and expected profit.

Insert Figures 1 and 2 about here.

5 Extensions

5.1 Extension 1: Multiple Classes of Customers

In the base case, all arriving customers have identical valuation v. We now extend our analysis to the case in which there are n classes of customers, each of which has a class-specific valuation v_i with probability α_i, where $i = 1, 2, \cdots, n$, and $\sum_{i=1}^n \alpha_i = 1$. Without loss of generality, we assume that $p_l < p_h < v_1 < v_2 < \cdots < v_n$. Since the customer arrival process is Poisson, the arrival processes for these n classes of customers are independent Poisson processes with rates $\alpha_i\lambda$, where $i = 1, 2, \cdots, n$. As before, we let $B_i(t)$ and $A_i(t)$ be the number of customers of class i who arrive 'before' and 'after' t; respectively.

First, when the customers are myopic, each arriving customer of class i will purchase at p_h immediately upon arrival because $v_i > p_h$. As such, regardless of the display format, the retailer's expected profit remains the same as stated in (3.1). Next, when the customers are strategic and when the retailer adopts the Display All format, the exact analysis for the customers' purchasing behavior becomes intractable. For this reason, we shall restrict our attention to the case when the retailer adopts the Display One format.

We now extend the DO threshold purchasing rule to the case of multiple classes of customers. A class i customer is said to follow the DOM threshold purchasing rule if, given his arrival time t, he (a) purchases the item at p_h if $t \leq t'_i$; and (b) wait and attempt to purchase the item at

p_l after the season ends if $t > t'_i$, where $t'_1 < t'_2 < \cdots < t'_n$. First, let us consider a customer of class n with valuation v_n arriving at time t'_n. If this customer purchases the product at time t'_n, he will receive a surplus $v_n - p_h$. Alternatively, he can wait and attempt to purchase along with $\sum_{i=1}^n A_i(t'_i)$ customers (i.e., all arriving customers of class $i \geq 1$ who arrive after t'_i). In this case, this class n customer will get an expected surplus $E(\frac{1}{\sum_{i=1}^n A_i(t'_i)+1})(v_n - p_l)$. At the break-even point, t'_n must satisfy the following equation:

$$v_n - p_h \quad = \quad E(\frac{1}{\sum_{i=1}^n A_i(t'_i) + 1})(v_n - p_l). \tag{5.1}$$

Notice that $\sum_{i=1}^n A_i(t'_i)$ is a Poisson random variable with parameter $\lambda(T - \sum_{i=1}^n \alpha_i t'_i)$. Therefore (5.1) can be simplified as:

$$\frac{1 - e^{-\lambda(T - \sum_{i=1}^n \alpha_i t'_i)}}{\lambda(T - \sum_{i=1}^n \alpha_i t'_i)} \quad = \quad \frac{v_n - p_h}{v_n - p_l}. \tag{5.2}$$

Next, let us consider a customer of class $j = 1, 2, \cdots, n-1$ with valuation v_j arriving at time t'_j. He will enjoy a surplus $v_j - p_h$ if he purchases the product at time t'_j. Alternatively, he can wait and attempt to purchase after the season ends. However, since $t'_j < t'_{j+1} < \cdots < t'_n$, any customer of class k, $k \geq j+1$, arriving between t'_j and t'_k will purchase the product at p_h under the DOM rule. Because the retailer displays one item at a time, this customer of class j thinks that he will get nothing by postponing his purchase if at least 1 customer of class k, $k \geq j+1$, arrives between t'_j and t'_k. If no such arrivals occur, this customer of class j will attempt to purchase the item along with $\sum_{i=1}^n A_i(t'_i)$ customers after the season ends. By considering the probability of having no customers of class k arriving between t'_j and t'_k for $k = j+1, j+2, \cdots, n$, the threshold t'_j, $j = 1, 2, \cdots, n-1$, must satisfy:

$$v_j - p_h \quad = \quad \Pi_{k=j+1}^n e^{-\alpha_k \lambda(t'_k - t'_j)} \cdot E(\frac{1}{\sum_{i=1}^n A_i(t'_i) + 1})(v_j - p_l). \tag{5.3}$$

By considering (5.3), it is easy to check that:

$$t'_{j+1} - t'_j \quad = \quad \frac{1}{\lambda \sum_{k=j+1}^n \alpha_k} \cdot ln\{\frac{(v_{j+1} - p_h)(v_j - p_l)}{(v_j - p_h)(v_{j+1} - p_l)}\} > 0 \quad \text{for } j = 1, 2, \cdots, n-1. \tag{5.4}$$

The last inequality results from the fact that $v_j < v_{j+1}$. By applying (5.4) repeatedly, we can express t'_j in terms of t'_n for $j = 1, 2, \cdots, n-1$. By substituting these values for t'_j into (5.2), we can determine t'_n and then we can compute the values for t'_j, $j = 1, 2, \cdots, n-1$.[13] Notice that t'_j is a

[13] In the event when t'_j lies outside the range $[0, T]$, one can always set $t'_j = 0$ when $t'_j < 0$ and set $t'_j = T$ when $t'_j > T$ for $j = 1, \cdots, n$.

complex function that depends on α_k and v_k for $k = 1, 2, \cdots, n$. In any event, we have established the following property of t'_k for $1 \le k \le n$:

Proposition 6 $0 \le t'_1 < t'_2 < \cdots < t'_n \le T$. *For* $j = 1, 2, \cdots, n-1$, $t'_{j+1} - t'_j$ *is decreasing in* λ, v_j *and* p_l, *and increasing in* v_{j+1} *and* p_h.

By examining the expected payoff when a customer deviates from the DOM threshold purchasing rule, we can show the following Proposition.

Proposition 7 *There is a Nash equilibrium in which all arriving customers follow the DOM threshold purchasing rule.*

When each arriving customer follows the DOM threshold purchasing rule, all of the $B_i(t'_i)$ customers of class i will attempt to purchase the item at p_h (if available) upon arrival, and all of the $A_i(t'_i)$ customers will attempt to purchase at p_l after the season ends, where $i = 1, 2, \cdots, n$. Following the similar argument as presented in Section 3.3.2, the retailer's expected profit for any order quantity $Q \ge 1$ is:

$$\Pi_r^{DOM}(Q) \quad = \quad (p_h - c)Q - (p_h - p_l)\mathrm{E}[Q - \sum_{i=1}^{n} B_i(t'_i)]^+ - (p_l - s)\ \mathrm{E}[Q - B(T)]^+. \qquad (5.5)$$

Since the retailer's expected profit Π_r^{DOM} given in (5.5) is a concave function of Q, the optimal order quantity Q^{DOM} is the smallest integer that satisfies:

$$(p_h - p_l)\hat{G}(Q) + (p_l - s)F(Q) \ge p_h - c, \qquad (5.6)$$

where $\hat{G}(\cdot)$ is the cumulative distribution function of $\sum_{i=1}^{n} B_i(t'_i)$, a Poisson random variable with parameter $\lambda \sum_{i=1}^{n} \alpha_i t'_i$.

We now compare the retailer's optimal profits and optimal order quantities associated with the case of multiple customer classes when customers are myopic or strategic. By using the same argument as presented in Section 4.1 that the random variable $B(T)$ is stochastically larger than the random variable $\sum_{i=1}^{n} B_i(t'_i)$, we can prove the following Proposition:

Proposition 8 *Under the Display One format, for any given order quantity* Q, *the retailer would gain a higher expected profit when the customers are myopic; i.e.,* $\Pi_r^M(Q) \ge \Pi_r^{DOM}(Q)$. *The optimal order quantity is higher when the customers are myopic; i.e.,* $Q^M \ge Q^{DOM}$.

Next, we compare the retailer's expected profit associated with the case when the retailer operates in a homogeneous market with a single class of strategic customers with identical valuation v to the case when the retailer operates in a heterogeneous market with n classes of strategic customers, each of which has a class-specific valuation v_i, for $i = 1, 2, \cdots, n$. To establish a meaningful comparison, we shall consider the case when $v = \sum_{i=1}^{n} \alpha_i v_i$ and when all other parameters (p_h, p_l, λ) remain the same for both markets. Because the effective demand for the product during the selling season depends on n different thresholds t'_j, the effective demand appears to be more uncertain when operating in a heterogeneous market instead of a homogeneous market. This observation would lead one to conjecture that the retailer would obtain a lower expected profit when operating in a heterogeneous market. To examine this issue, we compare the retailer's expected profits given in (3.5) and (5.5) by considering the case when $v = \sum_{i=1}^{n} \alpha_i v_i$, getting:

Proposition 9 *When $v = \sum_{i=1}^{n} \alpha_i v_i$, the weighted average of the thresholds associated with the case of multiple classes of customers is higher than the threshold associated with the single class case; i.e., $\sum_{i=1}^{n} \alpha_i t'_i > t'$. In addition, for any order quantity Q, the retailer would obtain a higher expected profit when operating in a heterogeneous market; i.e., $\Pi_r^{DOM}(Q) \geq \Pi_r^{DO}(Q)$. Furthermore, it is optimal for the retailer to order more when operating in a heterogeneous market; i.e., $Q^{DOM} \geq Q^{DO}$.*

Proposition 9 presents a counter-intuitive result in which the retailer would obtain a higher expected profit when operating in a heterogeneous market. To understand the underlying reason, observe that the weighted average of the thresholds associated with the case of multiple classes of customers is higher than the threshold associated with the single class case. This implies that, when operating in a heterogeneous market, the retailer has a longer aggregate time window (i.e., $\sum_{i=1}^{n} \alpha_i t'_i$) within which each arriving customer will purchase the product at p_h. This explains why the retailer would obtain a higher expected profit in a heterogeneous market.

Finally, let us consider the case in which the retailer incorrectly assumes that a heterogeneous market is homogeneous. Specifically, the retailer assumes that the market is comprised of a single class of customers with identical valuation v, while the actual market consists of multiple classes of customers with class specific valuation v_i, for $i = 1, 2, \cdots, n$. This situation could occur when the retailer aggregates the customer's valuation into a single class so that $\sum_{i=1}^{n} \alpha_i v_i = v$. We now evaluate the relative profit loss $\frac{\Pi_r^{DOM}(Q^{DOM}) - \Pi_r^{DOM}(Q^{DO})}{\Pi_r^{DOM}(Q^{DOM})}$ associated with this incorrect assumption. To do so, we use the same parameter values as given in Section 4.2. For the homogeneous market, we set $v = 117$; however, for the heterogeneous market, we set $n = 2$ and $\alpha_1 = \alpha_2 = 0.5$. To ensure that $\sum_{i=1}^{2} \alpha_i v_i = v = 117$, we set $v_2 = 117 + \Delta > v_1 = 117 - \Delta$ and we vary Δ from 1 to 16, where Δ captures the heterogeneity of customer valuations. As shown in Figure 3, the relative profit loss increases as Δ increases, and hence, it is important for the retailer to obtain a clearer understanding about the heterogeneity of customer valuations.

Insert Figure 3 about here.

5.2 Extension 2: Inventory Dependent Clearance Price

In the base model, the post-season clearance price p_l is pre-committed at the beginning of the season and is independent of the end-of-season inventory level I, where $0 \le I \le Q$. Using this markdown pricing mechanism, the retailer is unable to set the clearance price as a response to the actual end-of-season inventory level I. In this section, we extend our analysis to the case in which the retailer would announce that the post-season clearance price will follow a specific function $p(I)$, and that $p(I)$ is a decreasing function of I. In this case, the customer knows the functional form of $p(\cdot)$ but would not know the actual post-season clearance price until the end-of-season inventory level I is realized.[14]

Besides adopting the Display All format, let us consider the situation in which the retailer announces the regular price p_h and the post-season clearance price plan $p(\cdot)$ at the beginning of the season, where $p_h > p(I)$ for $0 \le I \le Q$.[15] Suppose a customer arrives at time t and observes k units available for sale, where $1 \le k \le Q$. Then he will enjoy a surplus $v - p_h$ if he purchases the item at p_h. Alternatively, he can wait and attempt to purchase the item at the reduced price $p(I)$ after the season ends. If he decides to wait and if all customers who arrive after t would also wait, then $I = k$. In this case, he will get an expected surplus $(v - p(k))H(k, t)$, where the term $H(k, t)$ represents the expected probability of getting the item at the reduced price after the season ends.

[14] Based on our discussion with a subsidiary of a high-end handbag retailer in Taiwan, we learned that it is common for their Europe headquarter to pre-announce to the subsidiaries regarding the specific functional form of $p(\cdot)$ at the beginning of the season so that the subsidiaries in different countries can coordinate their markdown prices efficiently. While we are not aware of a situation in which the retailer would pre-announce the markdown price function $p(\cdot)$ to their customers, we think this analysis can provide additional insights regarding a different post-season clearance pricing mechanism.

[15] Since this price markdown plan depends on the end-of-season inventory level I, the customers would need to know the actual inventory level at all times. As such, the Display One format would not be appropriate for this case.

By comparing the expected surpluses associated with these two purchase options, we can establish the following Inventory Dependent Clearance (IDC) threshold purchasing rule: For any customer who arrives at time t and observes k units available for sale, he should: (a) purchase one unit at p_h if $t \leq \tilde{t}(k)$; and (b) attempt to purchase one unit at $p(I)$ after the season ends if $t > \tilde{t}(k)$, where the threshold $\tilde{t}(k) = \max\{0, t(k)\}$ and $t(k)$ satisfies the following equation:

$$H(k, t(k)) = \frac{v - p_h}{v - p(k)}, \tag{5.7}$$

where $H(k, t(k))$ is given in (3.3).

By replacing p_l with $p(k)$, it is easy to show that Lemma 1, Proposition 1 and Proposition 2 continue to hold. Specifically, Propositions 1 and 2 become:

Proposition 10 *There exists a positive integer $\tilde{\theta}$ that satisfies: $\tilde{\theta} = argmin\ \{t(j) \leq 0 : j = 1, 2, \cdots\}$. Moreover, the threshold $\tilde{t}(k) = \max\{0, t(k)\}$ has the following properties:*

1. *If $Q < \tilde{\theta}$, then $0 < \tilde{t}(Q) < \tilde{t}(Q-1) < \cdots < \tilde{t}(1) < T$.*
2. *If $Q \geq \tilde{\theta}$, then $\tilde{t}(Q) = \tilde{t}(Q-1) = \cdots = \tilde{t}(\tilde{\theta}) = 0 < \tilde{t}(\tilde{\theta}-1) < \cdots < \tilde{t}(1) < T$.*
3. *The threshold $\tilde{t}(k)$ is increasing in λ and v and $p(k)$, and is decreasing in p_h.*

Furthermore, there exists an equilibrium in which all arriving customers will follow the IDC threshold purchasing rule.

Next, let us compare the threshold $t^*(k)$ associated with the base case as presented in Section 3.2.1 and the threshold $\tilde{t}(k)$ associated with the Inventory Dependent Clearance price case. For the purpose of comparison, let us consider the case in which p_l is bounded between $p(Q)$ and $p(1)$ so that there exists a δ so that $p(1) \geq p(2) \geq \cdots \geq p(\delta) > p_l \geq p(\delta+1) \geq \cdots \geq p(Q)$. In this case, we can prove the following result:

Proposition 11 *$\tilde{t}(k) > t^*(k)$ for $1 \leq k \leq \delta$ and $\tilde{t}(k) \leq t^*(k)$ for $\delta + 1 \leq k \leq Q$.*

The result stated in Proposition 11 is intuitive. Consider the case when there are fewer items left; i.e., when k is small so that $1 \leq k \leq \delta$. In this case, $p(k) > p_l$, and hence, each customer who observes k units available for sale is more eager to purchase the item at p_h under the IDC threshold purchasing rule. This explains why $\tilde{t}(k) > t^*(k)$ when $k \leq \delta$. We can use a similar approach to explain the result $\tilde{t}(k) \leq t^*(k)$ when $k > \delta$.

Finally, let us compute the retailer's expected profit when all arriving customers follow the IDC threshold purchasing rule. In this case, we can use the same approach as presented in Section 3.2.2 to determine the retailer's expected profit. To do so, we first re-define $f(j,i)$ and $g(i)$ in Section 3.2.2 as $\tilde{f}(j,i)$ and $\tilde{g}(i)$ by replacing the threshold $t^*(k)$ with $\tilde{t}(k)$ and by replacing p_l with $p(i)$ for certain value of i. Second, we can redefine $N(j,i)$ as a Poisson random variable with parameter $\lambda(\tilde{t}(i) - \tilde{t}(j))$. Then we can show that the retailer's expected profit can be expressed as:

$$\tilde{\Pi}_r(Q) = \tilde{f}(Q+1, Q) - cQ. \tag{5.8}$$

To compare the retailer's optimal order quantity and optimal expected profit under the pre-committed clearance price in the base model with the inventory dependent clearance price in this section, we consider the same numerical example as in Section 4.2, except we set $p_l = 50$ in the pre-committed clearance price case. For any order quantity Q, we set the inventory dependent price function $p(I)$ as $40 = p(Q) = \cdots = p(Q/2) < p(Q/2+1) = \cdots = p(1) = 60$. As we can see from Figure 4, under both clearance pricing mechanisms, when the customer's valuation v is below a certain threshold, it is optimal for the retailer to order a large quantity so that all arriving customers will wait for post-season clearance sale. When the customer's valuation v is sufficiently high, the retailer's optimal order quantity increases with valuation under both clearance pricing mechanisms. Furthermore, as shown in Figure 4, it is optimal for the retailer to order more when the clearance price p_l is pre-committed at the beginning of the season. By ordering more, the retailer may be able to obtain a higher optimal profit when the clearance price p_l is pre-committed. This result is illustrated numerically in Figure 5. Therefore, having the flexibility to set the clearance price at the end of the season will actually reduce the retailer's expected profit when the customers are strategic. This result is consistent with the numerical result presented in Aviv and Pazgal (2005).

Insert Figures 4 and 5 about here.

6 Conclusions

In this paper, we have examined the retailer's optimal order quantity and the retailer's optimal expected profit under four scenarios: the customers are either myopic or strategic, and the retailer adopts either the Display All or the Display One format. To obtain tractable results, we have developed a model based on a situation in which the retailer announces both the regular price and the post-season clearance price at the beginning of the selling season and the customers arrive in accord with a Poisson process. When the customers have identical valuation, we have shown that, in equilibrium, each strategic customer should behave according to a threshold policy that depends on inventory level at the time of arrival. We have proved that the retailer would obtain a higher profit and would order more when the customers are myopic and that the retailer would earn a higher profit and would order more under the Display One format when the customers are strategic. We have illustrated numerically the penalty associated with the case when the retailer mistakenly assumes that the strategic customers are myopic. We have extended our analysis to the case in which customers belong to multiple classes and the retailer adopts the Display One format. Furthermore, we have extended our analysis to the case in which the post-season clearance price depends on the actual end-of-season inventory level.

Some of our assumptions in the paper can be relaxed. First, we have assumed a customer's valuation is fixed throughout the entire season. However, as discussed in Aviv and Pazgal (2005), the customer's valuation can be time-dependent: high in the beginning and decline over time. It is easy to check that our results continue to hold when all the customer have the same time-declining valuation function: $v(t) = Ve^{-\alpha t}$, where V is the same base valuation and α is the declining rate. Second, we have assumed that customers are risk-neutral in our paper. Liu and van Ryzin (2005) consider the retailer's optimal stocking decisions when the customers are risk neutral or risk-averse and they show that the retailer will behave differently in these two cases. Assuming the same power utility function $u(x) = x^r$ as in Liu and van Ryzin (2005), our analysis still holds.

Our paper has certain limitation in terms of the same valuation for all the customers under the Display All format. Since the exact analysis for customers' strategic purchasing behavior is intractable when customers have different valuations, a different approach is needed for our future research.

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7 Appendix: Proof

Proof of Lemma 1: In order to show that $t(k)$ that satisfies (3.2) is unique, we need to show that the equation $\hat{H}(k,t) = \frac{v-p_h}{v-p_l}$ has a unique solution, where $\hat{H}(k,t)$ is an auxiliary function associated with $H(k,t(k))$. Specifically,

$$\hat{H}(k,t) = \sum_{n=0}^{k-1} \frac{[\lambda(T-t)]^n}{n!} e^{-\lambda(T-t)} + \sum_{n=k}^{\infty} \frac{[\lambda(T-t)]^n}{n!} e^{-\lambda(T-t)} \frac{k}{n+1}. \quad (7.1)$$

By observing the fact that $\hat{H}(k,t) = 1$ when $t = T$ and $\hat{H}(k,t)$ is strictly increasing in t for any $k \geq 1$, we can conclude that there exists an unique $t(k)$ that satisfies (3.2).

Next, we prove $t(k+1) < t(k)$ by contradiction. Suppose $t(k+1) \geq t(k)$. By considering (3.2), (3.3), and the fact that $A(t(k+1))$ and $A(t(k))$ are Poisson random variables with parameters $\lambda(T - t(k+1))$ and $\lambda(T - t(k))$, respectively, we can show that:

$$\begin{aligned} \frac{v-p_h}{v-p_l} = H(k,t(k)) &= \sum_{n=0}^{k-1} \mathrm{Prob}(A(t(k)) = n) + \sum_{n=k}^{\infty} \mathrm{Prob}(A(t(k)) = n)\frac{k}{n+1}, \\ &< \sum_{n=0}^{k} \mathrm{Prob}(A(t(k+1)) = n) + \sum_{n=k+1}^{\infty} \mathrm{Prob}(A(t(k+1)) = n)\frac{k+1}{n+1} \\ &= H(k+1, t(k+1)) = \frac{v-p_h}{v-p_l}. \end{aligned} \quad (7.2)$$

This leads to a contradiction. Hence, we must have $t(k+1) < t(k)$.

Finally, we prove $t(1) < T$ by contradiction. Suppose $t(1) \geq T$. By considering (3.2) and (3.3), we have $1 > \frac{v-p_h}{v-p_l} = H(1, t(1)) = \hat{H}(1, t(1)) \geq \hat{H}(1, T) = 1$. This leads to a contradiction. Finally, to prove the third statement, we apply the implicit function theorem by taking the derivative of the equation (3.2) with respect to λ, v, p_h and p_l. We omit the details. $\square$

Proof of Proposition 1: To start, it follows from Lemma 1 that the threshold $t(k)$ is strictly decreasing in k and $t(1) < T$, there must exist a θ so that θ is the smallest integer that has $t(\theta) \leq 0$. By considering the definition of θ and the definition of $t^*(k)$, we can apply Lemma 1 to prove the remainder of the Proposition. $\square$.

Proof of Proposition 2: We prove our result by contradiction. Suppose not. Then there must exist a customer who arrives at time t, observes k units available for sale, and obtains a higher surplus by deviating from the DA threshold purchasing rule, while all other arriving customers follow the rule. To aim for a contradiction, we now show this customer cannot get a higher surplus by deviating from the DA threshold purchasing rule. Let us consider the following cases:

1. When $Q < \theta$. Since $Q < \theta$, we have $0 < t(k) = t^*(k) < T$ by Proposition 1. We now consider two scenarios:

   (a) When $t < t^*(k)$. Under the DA threshold purchasing rule, this customer would receive a surplus $(v - p_h)$ by purchasing the item at p_h. However, he deviates from the rule by attempting to purchase the product at p_l after the season ends. By doing so, he receives a surplus $(v - p_l)H(k, t)$, where $H(k, t)$ is the expected probability of getting one item at p_l after the season ends. To aim for a contradiction, it suffices to show that $H(k, t) \leq \frac{v-p_h}{v-p_l}$. The exact expression for $H(k, t)$ is quite complex because it depends on the specific customer arrival pattern after t. In preparation, let N_0 be the number of customers arriving between t and $t(k)$ and let N_i be the number of customers arriving between $t(k-i+1)$ and $t(k-i)$ for $i = 1, 2, \cdots, k-1$, where the N_i's are independent Poisson random variables. Let us make the following observations. Consider the case when $N_0 = 0$ (i.e., no customers arrive between t and $t(k)$), then it is easy to see that $H(k, t) = H(k, t(k))$. Next, consider the case when $N_0 = 1, N_1 = 0$. In this case, the customer who arrives between t and $t(k)$ will purchase one item under the DA threshold purchasing rule, and hence, there are $k - 1$ items left at time $t(k)$. However, since $N_1 = 0$ (i.e., no customers arrive between $t(k)$ and $t(k-1)$), it is easy to see that $H(k, t) = H(k-1, t(k-1))$. By using the same argument, we can enumerate certain events associated with the random variables N_i's so that in each event, we have

$H(k,t) = H(k-j, t(k-j))$ for some $j \in \{0, 1, 2, \cdots, k-1\}$. By considering the probability of the occurrence of these events, it can be shown that:

$$H(k,t) = P_0 H(k, t(k)) + P_1 H(k-1, t(k-1)) + \cdots + P_{k-1} H(1, t(1)),$$

and that $P_0 + P_1 + \cdots + P_{k-1} < 1$. It follows from (3.2) that $H(i, t(i)) = \frac{v-p_h}{v-p_l}$ for $i = k, k-1, \cdots, 1$, we can conclude that $H(k,t) < \frac{v-p_h}{v-p_l}$. This leads to a contradiction.

(b) When $t > t^*(k)$. Under the DA threshold purchasing rule, this customer would receive a surplus $(v - p_l)H(k,t)$ by attempting to purchase at p_l after the season ends. However, he obtains a surplus $(v - p_h)$ instead by purchasing the item at p_h. To aim for a contradiction, it suffices to show that $(v - p_l)H(k,t) \geq (v - p_h)$. Under the DA threshold purchasing rule, all customers arriving after $t(k)$ would wait. Hence, had the customer who arrives at time t attempted to purchase the item at p_l after the season ends, he would have received a surplus:

$$(v - p_l)H(k,t) = (v - p_l)\{\sum_{n=0}^{k-1} \text{Prob}(N + A(t) = n) + \sum_{n=k}^{\infty} \text{Prob}(N + A(t) = n)\frac{k}{n+1}\},$$

where N represents the number of customers arriving within the interval $(t(k), t)$, and $A(t)$ represents the number of customers arriving within the interval $(t, T]$. Since N and $A(t)$ are independent Poisson random variables, the distribution of $N + A(t)$ is identical to the distribution of $A(t(k))$, where $A(t(k))$ corresponds to the number of customers arriving within the interval $(t(k), T]$. It follows from (3.2), we have: $(v - p_l)H(k,t) = (v - p_l)H(k, t(k)) = v - p_h$. This leads to a contradiction.

2. When $Q \geq \theta$. Since $Q \geq \theta$, Proposition 1 implies that $t(Q) < 0$ and $t^*(Q) = 0$. In this case, since all the other customers follow the DA threshold purchasing rule by waiting, this deviating customer who arrives at time $t > 0$ would observe Q items available. Had he waited and attempted to purchase the item after the season ends, he would have received an expected surplus $H(Q, 0)(v - p_l) = \hat{H}(Q, 0)(v - p_l) > \hat{H}(Q, t(Q))(v - p_l) = v - p_h$, where the inequality follows by the fact that $\hat{H}(k,t)$ is a strictly increasing function of t. Therefore, this customer cannot gain a higher surplus by deviating from the DA threshold purchasing rule.

By combining the above cases, we can conclude that any customer who deviates from the DA threshold purchasing rule cannot obtain a higher surplus. This completes the proof. □

Proof of Proposition 3: To show the recursive formula for $f(j, i)$ in the first statement, we consider the following three events associated with $N(j, i)$:

1. When $N(j,i) = 0$; i.e., when no customers arrive within the time window $(t^*(j), t^*(i)]$. Since there are i items available for sale at time $t^*(j)$, the retailer will still have i items available at time $t^*(i)$. Hence, $f(j,i) = g(i)$.

2. When $N(j,i) \geq i$. Since $i < j$ and since $t^*(j) \leq t^*(i) \leq t^*(k)$, for $1 \leq k \leq i$ (Proposition 1), each customer arriving at time $t \in (t^*(j), t^*(i)]$ has $t \leq t^*(k)$ for $1 \leq k \leq i$. Hence, each of these arriving customers will attempt to purchase at p_h under the DA purchasing rule. Since $N(j,i) \geq i$, $f(j,i) = ip_h$.

3. When $N(j,i) = k$, where $k = 1, \cdots, i-1$. Using the same argument as before, it is easy to show that the retailer will receive kp_h from these k arriving customers and will have $i-k$ remaining units available at time $t^*(i)$. In this case, $f(j,i) = kp_h + f(i, i-k)$.

Combining the payoffs associated with the above three cases, we have proved the recursive formula for $f(j,i)$.

To prove the second statement, it remains to determine the function $g(i)$ when there are i units available at time $t^*(i)$. Under the DA threshold purchasing rule, when i units are avaliable at time $t^*(i)$, all customers arriving after time $t^*(i)$ will attempt to purchase the item at the reduced price p_l after the season ends. Based on the number of arrivals after time $t^*(i)$; i.e, $A(t^*(i))$, the retailer will sell $\min\{i, A(t^*(i))\}$ units at p_l and dispose of each of the $[i - A(t^*(i))]^+$ leftover units at salvage value s. Noting that $A(t^*(i)) = N(i,0)$ as $t^*(0) = T$, the function $g(i)$ can be expressed as:

$$\begin{aligned} g(i) &= p_l \, \mathrm{E}\{\min\{i, A(t^*(i))\}\} + s \, \mathrm{E}\{[i - A(t^*(i))]^+\} = ip_l - (p_l - s) \, \mathrm{E}\{[i - A(t^*(i))]^+\} \\ &= ip_l - (p_l - s) \sum_{k=0}^{i-1} (i-k) \, \mathrm{Prob}(N(i,0) = k). \end{aligned}$$

This completes the proof. □

Proof of Proposition 4: It remains to show $\Pi_r^{DO}(Q) \geq \Pi_r^{DA}(Q)$. We need to consider two cases: $Q \geq \theta$ and $Q < \theta$. When $Q \geq \theta$, Proposition 1 implies that $t^*(Q) = 0$. Since $t^*(Q+1) = t^*(Q) = 0$, $N(Q+1, Q) = 0$. Hence, it is easy to check from Proposition 3 that $f(Q+1, Q) = g(Q)$ and $g(Q) = Qp_l - (p_l - s)E[Q - B(T)]^+$, because $A(t^*(Q)) = A(0) = B(T)$. Therefore, the retailer's expected profit given in (3.4) can be expressed as:

$$\Pi_r^{DA}(Q) = (p_l - c)Q - (p_l - s)E[Q - B(T)]^+. \tag{7.3}$$

Compare the above equation with the retailer's expected profit under the Display One format given in (3.5), it is easy to show that $\Pi_r^{DO}(Q) \geq \Pi_r^{DA}(Q)$ when $Q \geq \theta$.

Let us consider the case when $Q < \theta$. In this case, Proposition 1 implies that $0 < t^*(Q) < \cdots < t^*(1) = t' < T$. To compare the retailer's expected profits associated with these two display formats, let us consider the following three mutually exclusive and exhaustive events:

1. When $B(t') \geq Q$. Under the DO threshold purchasing rule, all $B(t')$ customers will attempt to purchase the item at p_h, and hence, the retailer's profit is equal to $(p_h - c)Q$. Under the DA threshold purchasing rule, not all $B(t')$ customers will attempt to purchase the item at p_h because each arriving customer's purchasing decision depends on the actual inventory level upon arrival. As such, the retailer will sell k items at price p_h and sell the rest of the $Q - k$ items at price p_l, where $0 \leq k \leq Q$ and k depends on the arrival times of those $B(t')$ customers. In this case, the retailer's profit is equal to $(kp_h + (Q - k)p_l) - cQ$. Hence, the retailer would obtain a lower profit under the Display All format.

2. When $B(t') = m < Q$ and $B(T) \geq Q$. Under the DO purchasing rule, $B(t') = m$ customers will attempt to purchase the item at p_h, and $B(T) - B(t')$ customers will attempt to purchase the item at the reduced price p_l after the season ends. Since $B(t') = m < Q$ and $B(T) \geq Q$, the retailer's profit is equal to $(mp_h + (Q - m)p_l) - cQ$. Under the DA threshold purchasing rule, we can use the same argument as before to show that the retailer's profit is equal to $(kp_h + (Q - k)p_l) - cQ$, where $0 \leq k \leq m$. Hence, the retailer would obtain a lower profit under the Display All format.

3. When $B(t') = m < Q$ and $B(T) = n < Q$. Under the DO threshold purchasing rule, $B(t') = m$ customers will attempt to purchase the item at p_h, and $B(T) - B(t') = n - m$ customers will attempt to purchase the item at the reduced price p_l after the season ends. Since $n < Q$, there will be $Q - n$ units left after the post-season clearance. As such, the retailer's profit is equal to $(mp_h + (n - m)p_l + (Q - n)s) - cQ$. Under the Display All format, we can use the same argument as before to show that the retailer's profit is equal to $(kp_h + (n - k)p_l + (Q - n)s) - cQ$, where $0 \leq k \leq m$. Hence, the retailer would obtain a lower profit under the Display All format.

Since the retailer obtains a lower profit under the Display All format in all three cases as stated above, we can conclude immediately that $\Pi_r^{DO}(Q) \geq \Pi_r^{DA}(Q)$ when $Q < \theta$. This completes the proof. □

Proof of Proposition 5: We prove $Q^M \geq Q^{DO}$ first. Since $B(T)$ and $B(t')$ are Poisson random variables with parameters λT and $\lambda t'$ respectively, $B(T)$ is stochastically larger than $B(t')$. Thus, $G(x) \geq F(x)$ for $x \geq 0$, where $F(\cdot)$ and $G(\cdot)$ are the cumulative distribution functions associated with the random variables $B(T)$ and $B(t')$, respectively. Since $G(x) \geq F(x)$, $(p_h - p_l)G(Q^M) + (p_l - s)F(Q^M) \geq (p_h - s)F(Q^M) \geq p_h - c$, where the last inequality follows from (4.1). In this case, we have shown that Q^M satisfies (4.2). Since Q^{DO} is the smallest integer that satisfies (4.2), we can conclude that $Q^M \geq Q^{DO}$.

We now prove $Q^{DO} \geq Q^{DA}$. When $Q \geq \theta$, the retailer's expected profit $\Pi_r^{DA}(Q)$ is given in (7.3). By considering the first order condition, the retailer's optimal order quantity Q_1 is the smallest integer value that satisfies $F(Q) \geq \frac{p_l - c}{p_l - s}$. Next, when $Q < \theta$, the retailer's expected profit $\Pi_r^{DA}(Q)$ is given in (3.4). In this case, we can determine the optimal order quantity Q_2 by evaluating the function $\Pi_r^{DA}(Q)$ for $Q \in [1, \theta - 1]$ and obtain the optimal expected profit $\Pi_r^{DA}(Q_2)$. By the definition of Q^{DA} in (4.3), it suffices to show that $Q^{DO} \geq Q_1$ since $Q_1 \geq \theta > Q_2$. Since Q_1 is the smallest integer that satisfies $F(Q) \geq \frac{p_l - c}{p_l - s}$, it suffices to show that $F(Q^{DO}) \geq \frac{p_l - c}{p_l - s}$. We prove this by contradiction. Suppose $F(Q^{DO}) < \frac{p_l - c}{p_l - s}$. Then $(p_h - p_l)G(Q^{DO}) + (p_l - s)F(Q^{DO}) < (p_h - p_l)G(Q^{DO}) + (p_l - c) < p_h - c$, which violates the definition of Q^{DO} given in (4.2). Therefore, we can conclude that $F(Q^{DO}) \geq \frac{p_l - c}{p_l - s}$ and that $Q^{DO} \geq Q_1$. This completes the proof. □

Proof of Proposition 7: It follows by the similar arguments as in the proof of Proposition 2. We omit the details. □

Proof of Proposition 9: Notice t' satisfies $\frac{1 - e^{-\lambda(T - t')}}{\lambda(T - t')} = \frac{v - p_h}{v - p_l}$. By considering equation (5.2) and the fact that $v_n > v$, we can conclude immediately that $\sum_{i=1}^{n} \alpha_i t_i' > t'$ and $\sum_{i=1}^{n} B_i(t_i')$ is stochastically larger than $B(t')$. Therefore, $\Pi_r^{DOM}(Q) \geq \Pi_r^{DO}(Q)$ and $Q^{DOM} \geq Q^{DO}$ follow by considering equations (3.5), (5.5), (4.2) and (5.6). We omit the details. □

Proof of Proposition 10: We can use the same approach as the proof of Propositions 1 and 2 to prove our result. We omit the details. □

Proof of Proposition 11: Since $p(k) > p_l$ for $1 \leq k \leq \delta$, we have $\frac{v - p_h}{v - p(k)} > \frac{v - p_h}{v - p_l}$. We can apply Lemma 1 and Proposition 10 to show that $\tilde{t}(k) > t^*(k)$ for $1 \leq k \leq \delta$. We can complete the proof by using a similar argument. We omit the details. □

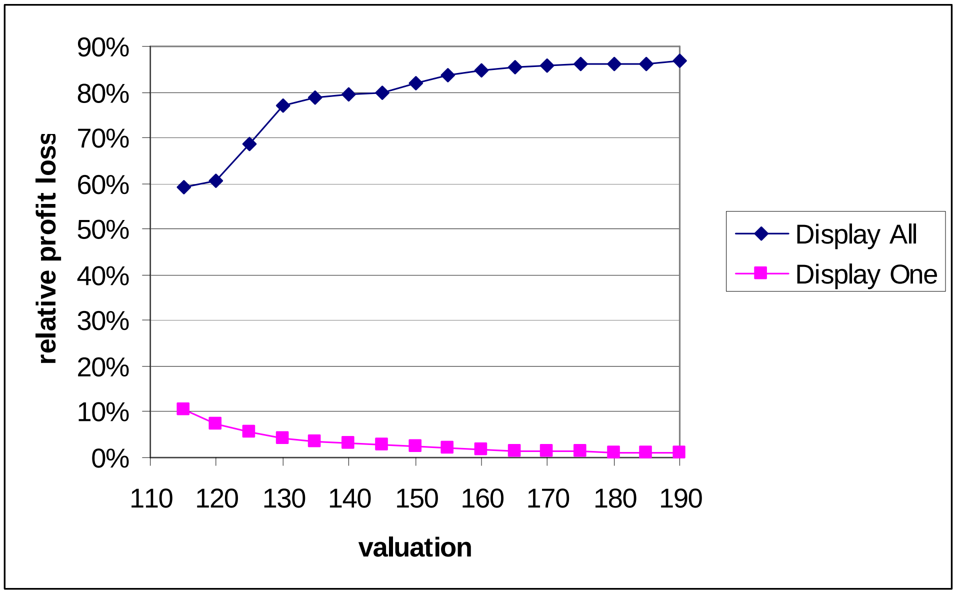

Figure 1: Impact of valuation on the retailer's relative profit loss by mistakenly assuming strategic customers as myopic

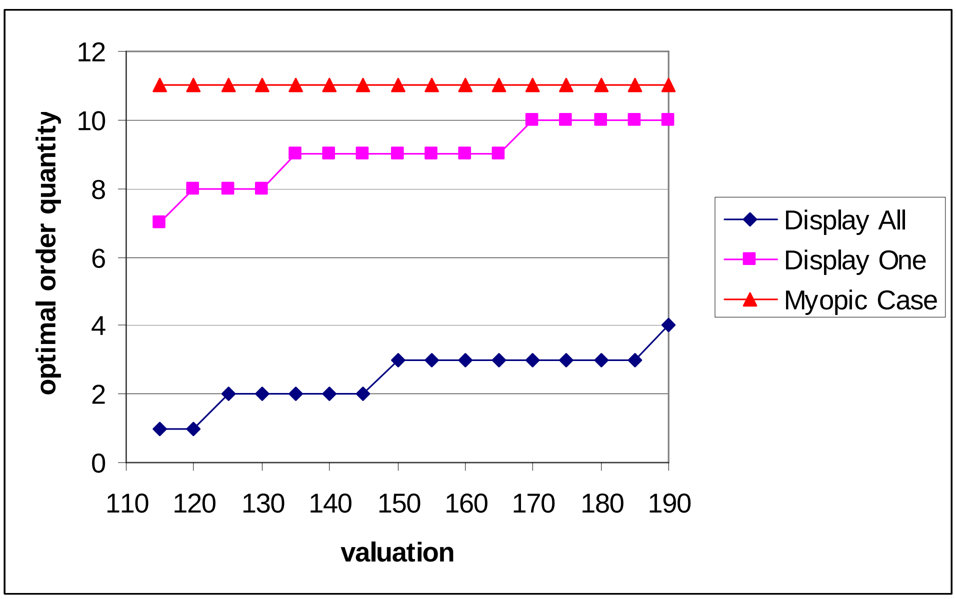

Figure 2: Impact of valuation on the retailer's optimal order quantity

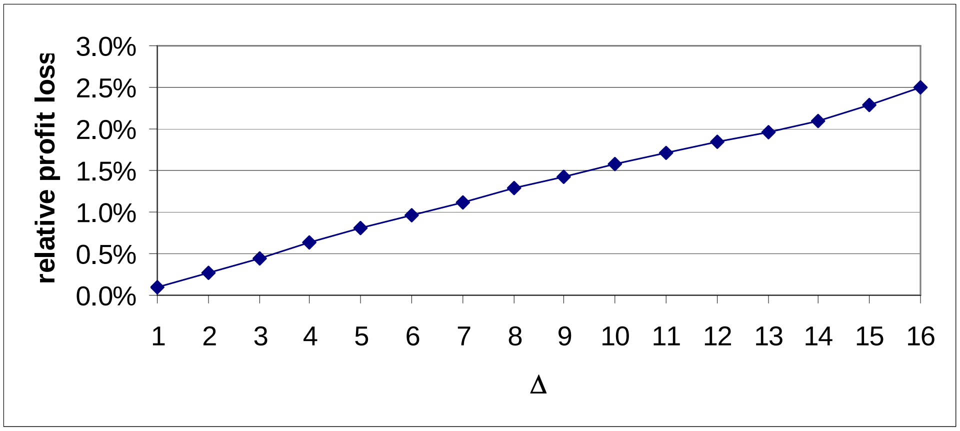

Figure 3: Impact of customer valuation heterogeneity on the retailer's relative profit loss by mistakenly assuming heterogeneous market as homogeneous

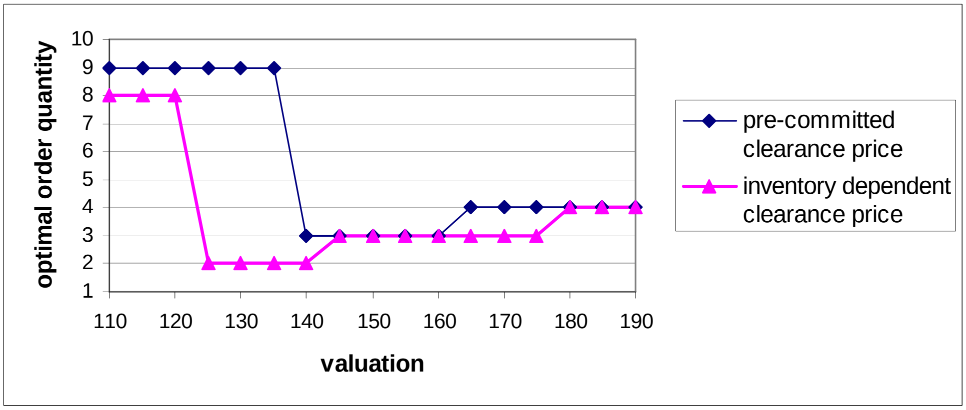

Figure 4: Impact of valuation on the retailer's optimal order quantities under pre-committed and inventory dependent clearance prices

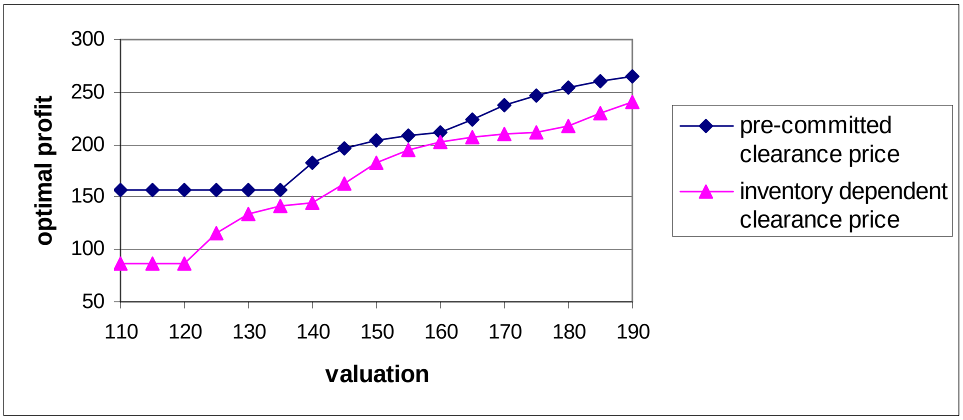

Figure 5: Impact of valuation on the retailer's optimal profits under pre-committed and inventory dependent clearance prices