Modeling the Impact of an Outcome-Oriented Reimbursement Policy on Clinic, Patients, and Pharmaceutical Firms

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This paper is dedicated to the memory of Dr. Anthony P. Moore, M.D. (May 6, 1949–May 10, 1998). He has been an inspirational character to many individuals throughout the world. He is deeply missed by his family, friends, colleagues, patients, his Executive MBA classmates, instructors, and administrators at UCLA.

Tackling the steep increase in drug costs is an especially important issue among many health care providers and insurers. To entice the clinics to become more cost efficient, the U.S. federal government, as well as many HMOs, have developed various cost containment initiatives recently. However, the impact of these initiatives on the patients’ well-being, the clinic’s profitability, and the pharmaceutical firm’s profitability has not been formally analyzed. In this paper we develop a mathematical model that is intended to examine the impact of a reimbursement policy for drug usage. Despite the simplistic structure of our model, the analysis enhances our understanding of the joint impact of the reimbursement policy on the patients, the clinic, and the pharmaceutical firm. Thus, our analysis can provide valuable information for evaluating the effectiveness of implementing such a reimbursement policy. In addition, we utilize the data gathered from a clinic to help support the assumptions and results of the underlying model.

(Health Care; Mathematical Model; Management)

1. Introduction

According to a recent study in Health Care Financial Review (1996), there is a major change in the cost of providing the necessary health care services. In 1967, inpatient services comprised 91.6% ($3.9 billion) while outpatient services accounted for 8.4% ($0.4 billion). However, in 1994, inpatient services accounted for 78% ($114.2 billion) while outpatient services accounted for 22% ($32.2 billion). The increase in total health care costs as well as the trend toward more outpatient services, along with the anticipated health cost needed to support a large increase in the elderly population, has created major concerns for the government to develop various cost containment initiatives, including limited-choice managed care, limited-choice prescription, and various reimbursement policies. The impact of these different initiatives on the many parties involved (patients, clinics and hospitals, pharmaceuticals and biotech firms) is a central issue for health policy makers and industry business leaders.
While managed-care providers can deploy various containment initiatives to reduce costs, the impact on the quality of the service provided needs to be carefully assessed. There are several survey studies of this particular issue. Donelan et al. (1996) examined the experience of 4000 patients associated with access, cost, and quality of health services in the United States, Canada, and Germany, and found that U.S. patients have more problems in getting necessary treatment under the limited-choice managed-care program than under the fee-for-service plan. Curtis and Rubenfeld (1997) found that Medicare patients in managed care received fewer end-of-life procedures than did those in fee-for-service plans. In addition, Blazer (1996) argues that reducing reimbursement for psychiatric disorders in late life gives the impression that the costs of mental health services for psychiatric disorders in older persons are lower than those for other age groups, and therefore underestimates the need for mental health services necessary to treat psychiatric disorders in the elderly.

Cost containment initiatives can have significant impact on clinics or hospitals as health service providers. Historically, cost containment initiatives were primarily designed to encourage hospitals to improve their efficiency (cf. Fox 1990), and relatively little attention has been paid to understanding the managerial behavior at clinics or hospitals. The work of Reid and Coburn (1996) was one of the first that examined the behavior of the nursing facility administrators before and after the introduction of Maine’s nursing home prospective payment system. They found that there was a decline in the proportion of Medicaid patient days in the three years following the implementation of the payment system, and that most nursing home administrators would maintain a high occupancy rate and retain a stable proportion of heavy care Medicaid patients.

Various cost containment efforts deal with the steep increase in drug costs. According to a recent survey conducted by a health care consulting firm (see the report in the Los Angeles Times, January 26, 1999), employers, on average, spent 13.8% more on prescription drugs for their employees in 1998 than in the previous year. The rising drug costs have also caused higher premiums and out-of-pocket charges for patients. At the same time, both the government and health care insurers face the challenge of tackling this trend of rising drug costs. For example, many HMOs have limited their coverage list of expensive prescription drugs to encourage their patients to switch to cheaper generic alternatives, and Medicare has been considering more restrictive policies for drug reimbursement. Therefore, these cost containment initiatives have significant impact on the profitability of pharmaceutical firms. For example, Amgen’s stock price dropped by 12% on August 12, 1997 when the Federal Health Care Finance Administration (FHCFa) proposed changing the reimbursement policy of Epo-gen, a drug for boosting red blood cell counts for patients with chronic kidney disease.1 Many investors were concerned about how the new reimbursement policy would affect Amgen’s profit.

To date, most of the empirical research studies have focused on only one of the involved parties, i.e., patients only, hospitals or clinics only, or pharmaceutical firms only. We feel that analytical models that can study the joint impact of a cost containment initiative on all involved parties are of vital importance. Such models are useful in generating valuable insights into how different initiatives could affect different parties. Many what-if questions can be addressed and different scenarios or alternate options can be quickly assessed before the implementation of any particular program choice. Application of operations research modeling techniques have been abundant in addressing many health care delivery issues. Pierskalla and Brailer (1994) have reviewed hundreds of research works in recent years which have focused on health care delivery, and we believe that operations research models will continue to serve as valuable tools for generating useful information and insights for health care policy makers in addressing the emerging issues of designing and evaluating possible cost containment initiatives.

Motivated by the Amgen problem described earlier,

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we have developed a model that captures the underlying structure of the reimbursement policy proposed by the Federal Health Care Finance Administration for the reimbursement of Epogen in 1997. The policy is one under which Medicare will not reimburse clinics or hospitals for the fees associated with Epogen treatment in any given month for those patients whose average red blood cell counts exceed a certain threshold during the last 90-day period. This policy is therefore an outcome oriented reimbursement which provides incentives for caregivers to offer appropriate patient care with effective use of resources. Throughout this paper, we shall refer to this outcome oriented reimbursement policy as the threshold policy. The proposed threshold policy has motivated us to examine the following questions: How would the threshold policy affect the clinic’s prescription policy on the drug usage? How would it affect the clinic’s profitability? How would it affect the patients’ well-being? How would it affect the pharmaceutical firm’s profitability? To answer these questions, we use our model to examine the impact of the threshold policy on the patient, the clinic, and the pharmaceutical firm. The objective is to analyze a simple mathematical model so as to formalize our understanding of how the threshold policy can affect all involved parties.

This paper is organized as follows. In §2, we first present a model for analyzing the interaction between the patient’s well-being and the prescription policy of a drug. Then we determine an optimal prescription policy that maximizes the expected profit for the clinic. We further analyze the data gathered from a local clinic to help support the assumptions and results derived from our model. Section 3 examines the impact of the reimbursement policy on the clinic’s prescription policy, the patients’ well-being, and the clinic’s profitability, as well as the pharmaceutical firm’s revenue. Section 4 examines two possible extensions. This paper ends with a discussion in §5.

2. Model Formulation and Analysis
Consider a clinic that purchases a drug from a pharmaceutical firm and prescribes this drug to patients. The patients receive the drug treatments from the clinic on a regular basis over an extended period of time. After providing a drug treatment to a patient, the clinic files a claim with the patient’s insurer (e.g. Medicare) for reimbursement that covers the cost of the drug, various operating costs, and a profit margin. The insurer has developed an outcome oriented threshold reimbursement scheme that is intended to discourage unnecessary treatment or excessive prescription. Under this scheme, the clinic’s claim will be approved only when the patient’s well-being is below a certain threshold value. However, the clinic is liable to pay the pharmaceutical firm for the drug regardless of the claim, so that the clinic bears the entire risk of not receiving the reimbursement. To analyze how this reimbursement policy will affect all involved parties, we begin by modeling how the reimbursement policy affects the behavior of the clinic.

2.1. Prescription Policy and Patients’ Well-Being
We first develop a model that examines the relationship between the clinic’s prescription policy and the patient’s well-being. Here, the response of the drug treatment and the patient’s well-being are measured primarily using a single score. For example, Epogen, a drug product manufactured by Amgen, is commonly used to treat anemia in patients with kidney disease. Epogen is prescribed by clinics to help hemodialysis patients to increase their hematocrit (Hct) level. Hematocrit is the measurement of the ratio of red blood cell count relative to all other fluids in the blood, has a range of normal values varying with age, and can be determined by spinning a capillary tube full of blood in a centrifuge and measuring the ratio. Low hematocrit level can cause severe illness and, possibly, death in hemodialysis patients. In this case, the hematocrit level is the score being monitored during the course of the drug treatment.

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3 See van Kemenade (1998) for a report that explores the possibilities and desirability of outcome oriented reimbursement and Mulcahy (1991) for a description of a sophisticated new medical payment system by Blue Cross and Blue Shield of Minnesota that directly links hospital reimbursement to patient outcomes.
Let $X_0$ be the patient’s initial score, representing the patient’s initial condition before treatment. At the end of each period $t - 1, t = 1, 2, \ldots$, the clinic measures the patient’s score $X_{t-1}$. Upon measuring the score $X_{t-1}$, the clinic determines the dosage (i.e., the amount of the drug to be prescribed), denoted by $D_t$, to the patient at the beginning of period $t$, and the patient’s score $X_t$ is then measured at the end of period $t$. Therefore, the sequence of events is as follows:

$$X_0, D_1, X_1, D_2, X_2, \ldots, X_{t-1}, D_t, X_t, \ldots$$

We use a linear relationship to model the response curve of the dosage on the score:

$$X_t = X_0 + \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_t + \alpha D_t. \quad (2.1)$$

For any $i$, $\varepsilon_i$ is an i.i.d. $N(0, \sigma^2)$ random variable that represents some random fluctuation added to the underlying patient’s score in period $i$. These fluctuations are cumulative over the course of drug treatment, and are normally caused by the patient’s health condition, interaction of other drugs taken by the patient, and the patient’s diet regimen, etc. The parameter $\alpha > 0$ represents the (known) response rate of the drug per unit dosage on the score.

We next model the prescription policy adopted by the clinic. Such a policy should be simple to implement in practice, and have the desirable effect of keeping the patient’s score at some target level so as to minimize the variability in the score. For example, hematocrit variability has been demonstrated to be associated with increased mortality risk in hemodialysis patients. Specifically, we assume that the clinic adopts the target prescription policy that can be described as follows: Based on the patient’s score $X_{t-1}$ at the end of period $t - 1$, the clinic would specify the subsequent dosage $D_t$ that would adjust the patient’s score to the target level $T$ (in expectation) at the end of period $t$. (Based on our discussion with the director of a dialysis clinic, the $T$-policy captures the characteristics of the actual prescription policy for Epogen being used.) We shall refer to this prescription policy as the $T$-policy.

We now analyze the characteristics of the dosage and the score under the $T$-policy. Consider the case when $t = 1$. From (2.1),

$$X_1 = X_0 + \varepsilon_1 + \alpha D_1,$$

To bring the score to the target level $T$ in expectation by the end of period 1; i.e., $E(X_1|X_0) = T$, the initial dosage $D_1$ is then given by

$$D_1 = \frac{T - X_0}{\alpha}. \quad (2.2)$$

Here, we also use the notation $d$ to denote this initial dosage. We can assume that $X_0 < T$, i.e., patients with initial scores above $T$ are considered to be well, and do not require the treatment.

Observe from (2.1) that $X_{t-1} = X_0 + \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_{t-1} + \alpha D_{t-1}$. Thus, we have:

$$X_t = X_{t-1} + \varepsilon_t + \alpha(D_t - D_{t-1}), \quad t \geq 2. \quad (2.3)$$

Under the $T$-policy, the clinic prescribes the dosage $D_t$ so as to adjust the score $X_t$ to the target level $T$ (in expectation) by the end of period $t$; i.e., $E(X_t|X_{t-1}) = T$. Using (2.3) above, the dosage $D_t$ can be expressed as:

$$D_t = D_{t-1} + \left(\frac{T - X_{t-1}}{\alpha}\right), \quad t \geq 2. \quad (2.4)$$

Observe that the prescribed dosage $D_t$ is adjusted upward or downward from the previous dosage amount $D_{t-1}$ according to whether $X_{t-1}$ is below or above the target level $T$. Note that we allow $D_t$, given in (2.4) to be negative, which is negligible as long as $\sigma^2$ is small compared with $D_{t-1}$. This approximation simplifies our subsequent analysis for generating useful insights for understanding the impact of the reimbursement scheme.\(^4\)

It follows from (2.3) and (2.4) that $X_t = T + \varepsilon_t$ for all $t$. Hence, $X_t$ is an i.i.d. $N(T, \sigma^2)$ random variable, and is independent of $D_t$. It can be shown from (2.4) that $E(D_t) = d$ and $\text{Var}(D_t - D_{t-1}) = \sigma^2/\alpha^2$ for all $t$. In summary, we have shown that:

**Proposition 1.** Under the $T$-policy (a) $X_t$ is an i.i.d. $N(T, \sigma^2)$ random variable; (b) $E(D_t) = d$ and $\text{Var}(D_t)$,\(^4\)

Denardo and Tang (1992) and Tang (1990) consider a similar production control policy for controlling inventory in a stochastic manufacturing system. They show that a similar assumption can serve as a good approximation.
\(- D_{t-1} = \sigma^2 / \alpha^2\); and (c) \(X_t\) and \(D_t\) are independent for all \(t \geq 1\).

2.2. Optimal Prescription Policy

Given the prescription policy defined in (2.4), we now define the threshold reimbursement policy and determine the clinic’s optimal prescription policy. Consider the case in which the clinic prescribes dosage \(D_t\) to a patient at the beginning of period \(t\). The clinic has to pay the drug manufacturer in the amount of \(c D_t\) at the beginning of period \(t\), where \(c\) is the cost per unit dosage. After the treatment, the clinic files a claim with the patient’s insurer in the amount of \(pD_t\), where \(p\) (\(p > c > 0\)) is the rate per unit dosage. For simplicity, we ignore any other direct costs associated with administering the treatment, but one can easily include these direct costs into \(c\) as well. Under the threshold reimbursement policy, the clinic’s claim will be approved and the clinic will receive \(pD_t\) if the score of the patient at the end of the period \(t\), \(X_t\), is less than or equal to some threshold level \(K\). Otherwise, the clinic’s claim will be denied. Figure 1 depicts the transactions between the manufacturer (pharmaceutical firm), the clinic, and the insurer.

In general, there exist certain clinical guidelines for specifying target level. Let us assume that the target \(T\) must lie within some specified range, \(L \leq T \leq U\). For instance, Amgen has suggested that the desirable Hct level for patients on dialysis should range between 30% to 36%. Within this range \([L, U]\), the clinic can select its optimal target \(T^*\) based on the patient’s initial condition so as to maximize its long-run average expected profit.

Let \(R_t\) denote the profit of the clinic for a patient at period \(t\). Then it is easy to see that

\[
E(R_t | D_t) = pD_t p(X_t \leq K) - cD_t = D_t (p p P(X_t \leq K) - c).
\]

Following from Proposition 1 that \(D_t\) and \(X_t\) are independent, we have:

\[
E(R_t) = E(D_t p p P(X_t \leq K) - c) = \left( \frac{T - X_0}{\alpha} \right) (p P(X_t \leq K) - c). \tag{2.5}
\]

By denoting \(X_0 = \mu\), the clinic’s problem is to determine the optimal target \(T^*\) so as to maximize the expected profit. Specifically, the clinic’s problem can be written as:

\[
\max_{L \leq T \leq U} E(R_t) = \max_T \left( \frac{T - \mu}{\alpha} \right) (p P(X_t \leq K) - c), \tag{2.6}
\]

where \(X_t\) is an \(N(T, \sigma^2)\) random variable from Proposition 1. Once \(T^*\) is determined, the clinic can apply (2.4) to determine the corresponding optimal prescription policy \(D_t\) at period \(t\). For this reason, it is
sufficient for us to focus on the characteristics of the optimal target \( T^* \).

The problem formulation of (2.6) represents a basic tradeoff associated with the target \( T \). To elaborate, let us recall from Proposition 1 that \( X_t \) is an i.i.d. \( N(\mu, \sigma^2) \) random variable. In this case, if \( T \) is high, the term \((T - \mu) / \alpha\) is high, and yet the probability of the clinic’s claim getting approved, i.e., \( P[X_t \leq K] \), is low. On the other hand, if \( T \) is low, then we have the opposite effect. Therefore, this problem is similar in nature to the classic newsvendor problem. As we shall see below, the optimal \( T^* \) possesses a structure that is similar to the fractile solution for the newsvendor problem.

Let

\[
f(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad F(y) = \int_{-\infty}^{y} f(t) \, dt
\]

denote the p.d.f. and cumulative distribution of a standard \( N(0, 1) \) random variable.

**Proposition 2.** The optimal target is given by

\[
T^* = \max\{L, \min (K - s^* \sigma, U)\}
\]

in which \( s^* \) is the unique solution to \( h(y) = 0 \) where

\[
h(y) = pF(y) + p\left(y - \frac{K - \mu}{\sigma}\right)f(y) - c. \tag{2.7}
\]

Furthermore,

\[
s^* \in \left[\frac{(K - \mu) - \sqrt{(K - \mu)^2 + 8\sigma^2}}{2\sigma}, \frac{(K - \mu) + \sqrt{(K - \mu)^2 + 8\sigma^2}}{2\sigma}\right].
\]

**Proof.** Consider first the unconstrained optimization problem. From (2.6),

\[
T^*_u = \arg\max_T \left( \frac{T - \mu}{\alpha} \right) \left( pF\left( \frac{K - T}{\sigma} \right) - c \right).
\]

Let \( t = T - \mu \) and \( k = K - \mu \). Then, \( T^*_u \) can be expressed as

\[
T^*_u = \mu + t^*,
\]

where \( t^* \) maximizes the function \( g(t) \) and

\[
g(t) = ptF\left( \frac{k - t}{\sigma} \right) - ct. \tag{2.8}
\]

Direct differentiation of (2.8) gives

\[
g'(t) = pF\left( \frac{k - t}{\sigma} \right) - \frac{\sigma}{\sigma} f\left( \frac{k - t}{\sigma} \right) - c.
\]

Let \( y = (k - t)/\sigma \) and

\[
h(y) = g'(y - y\sigma) = pF(y) + p\left(y - \frac{k}{\sigma}\right)f(y) - c.
\]

Then,

\[
h'(y) = pF(y) + \left(y - \frac{k}{\sigma}\right)f(y) + pf(y).
\]

Because \( f'(y) = -yf(y) \), we have

\[
h'(y) = pF(y) \left(2 + \left(y - \frac{k}{\sigma}\right)(-y)\right)
\]

\[
= -pF(y)\left(y^2 - (k/\sigma)y - 2\right)
\]

\[
= -pF(y)\left(y - \frac{k - \sqrt{k^2 + 8\sigma^2}}{2\sigma}\right)
\]

\[
\times \left(\frac{y - \frac{k + \sqrt{k^2 + 8\sigma^2}}{2\sigma}}{2\sigma}\right).
\]

Because \( f(y) > 0 \) for all \( y \),

\[
\begin{cases}
  h'(y) < 0, & y < \frac{k - \sqrt{k^2 + 8\sigma^2}}{2\sigma}, \\
  h'(y) \geq 0, & \frac{k - \sqrt{k^2 + 8\sigma^2}}{2\sigma} \leq y \leq \frac{k + \sqrt{k^2 + 8\sigma^2}}{2\sigma}, \\
  h'(y) < 0, & y > \frac{k + \sqrt{k^2 + 8\sigma^2}}{2\sigma}.
\end{cases} \tag{2.9}
\]

Because \( f(\cdot) \) has a finite mean, \( yf(y) \to 0 \) as \( y \to -\infty \) and as \( y \to \infty \). Therefore, \( h(-\infty) = -c \) and \( h(\infty) = p - c \). From the first and third inequalities in (2.9), \( h(y) < -c < 0 \) when \( y < (k - \sqrt{k^2 + 8\sigma^2})/2\sigma \), and \( h(y) > p - c > 0 \) when \( y > (k + \sqrt{k^2 + 8\sigma^2})/2\sigma \), respectively. Furthermore, the second inequality in (2.9) shows that \( h(y) \) is increasing for \( k \)
− \sqrt{k^2 + 8\sigma^2}/2\sigma < y < (k + \sqrt{k^2 + 8\sigma^2})/2\sigma. This implies that h(y) must have changed its sign from −ve to +ve at some unique point \( s^* \in \{(k - \sqrt{k^2 + 8\sigma^2})/2\sigma, (k + \sqrt{k^2 + 8\sigma^2})/2\sigma\} \), such that

\[
\begin{align*}
\{ & h(y) < 0, \quad y < s^*, \\
& h(s^*) = 0, \\
& h(y) > 0, \quad y > s^*.
\end{align*}
\]

(2.10)

Because \( h(y) = g'(k - y\sigma) \), it follows from (2.10) that

\[
\begin{align*}
\{ & g'(t) < 0, \quad t > k - s^*\sigma, \\
& g'(k - s^*\sigma) = 0, \\
& g'(t) > 0, \quad t < k - s^*\sigma.
\end{align*}
\]

(2.11)

In other words, the function \( g(t) \) is unimodal, and is maximized at \( t^* = k - s^*\sigma \). This implies that

\[
T^*_s = \mu + t^* = \mu + k - s^*\sigma = K - s^*\sigma
\]

is the unique optimal target for the unconstrained problem.

For the constrained problem where \( L \leq T \leq U \), it follows from the fact \( g(t) \) is unimodal that \( T^* = T^*_s \) if \( L \leq T^*_s \leq U \), \( T^* = L \) if \( T^*_s < L \), and \( T^* = U \) if \( T^*_s > U \). This result can be expressed in a compact form as provided in Proposition 2. □

Proposition 2 suggests that \( T^* \) is set at \( s^* \) multiplied by standard deviations of the random fluctuation of the score below the reimbursement threshold \( K \), as long as \( T^* \) is within the specified range \( [L, U] \). Notice that \( s^* \) is akin to the “safety factor” that is well known in the inventory theory literature. However, \( s^* \) can be negative here. From the proof of Proposition 2, we can deduce that \( s^* \geq 0 \) is equivalent to \( h(0) \leq 0 \), which can be simplified as

\[
\frac{1}{\pi} \leq \frac{c}{p} + \frac{K - \mu}{\sigma \sqrt{\pi}}.
\]

(2.12)

Assuming that \( K > \mu \), it is easy to check from (2.12) that \( s^* \geq 0 \) if \( (p - c)/c \leq 100\% \); i.e., when the clinic’s profit margin is below 100%. Therefore, it is more likely to have \( s^* \geq 0 \), which implies that \( T^* \leq K \). Observe from (2.10) that \( s^* \) can easily be computed using a simple bisection method. Furthermore, we will use (2.10) to analyze how the optimal \( T^* \) would change with respect to some of the model parameters in the next section.

2.3. An Alternate Model

Equation (2.4) indicates that the prescribed dosage \( D_t \) is adjusted upward or downward from the previous dosage amount \( D_{t-1} \) according to whether \( X_{t-1} \) is below or above the target level \( T \). Since \( E(D_{t-1}) = d \) for all \( t \), one can consider a simpler prescription scheme where the prescribed dosage \( D_t \) is adjusted upward or downward from the fixed “target” dosage \( d \) (rather than previous dosage amount \( D_{t-1} \)) according to whether \( X_{t-1} \) is below or above the target level \( T \), i.e.,

\[
D_t = d + \left( \frac{T - X_{t-1}}{\alpha} \right).
\]

For tractable analysis in this case, however, we also need to modify the model of the relationship between the score and dosage as given in (2.3) by

\[
X_t = X_{t-1} + \epsilon_t + \alpha(D_t - d), \quad t \geq 2.
\]

Then it can easily be shown that the results in Proposition 1 remain valid in this alternate model except that \( D_t \) is now an i.i.d. \( \mathcal{N}(d, \sigma^2/\alpha^2) \) random variable. Furthermore, the clinic’s optimization problem for \( T^* \) is identical to the one before, so that all results in Proposition 2 as well as in subsequent sections of this paper remain valid.

2.4. Empirical Data Analysis

Before we analyze the impact of different factors on the optimal prescription policy, we shall perform some exploratory data analysis that enables us to examine the reasonableness of our basic model as described in § 2.1. We obtained a data set from a local clinic administering Epogen to patients with chronic kidney disease. The data set contains 1,012 patient records over a 4-month period in 1997. Each patient record consists of the patient’s weekly Epogen dosage and the patient’s hematocrit (Hct) level at the end of each month over the 4-month period. Because the patients’ Hct levels (score), \( X_t \), were measured and given only at the end of each month, we used the average weekly dosage during that month as the patients’ average monthly dosage, \( D_{t,i} \), in our analysis.

Amgen has suggested the following prescription guidelines for administering Epogen: Starting dosage is 50–100 units/kg based on patient’s weight, with
maintenance dosage to keep the patient’s Hct level between a target range of 30% to 36% for all patients. Dosage adjustment is required when the patient’s Hct level is below 30% (increase dosage) or above 36% (decrease dosage). This local clinic follows the above prescription guidelines and uses discretion to determine the actual dosage according to the patient’s health conditions. Therefore, their prescription policy is akin to our prescription policy as described in (2.2) and (2.4), whereas a desirable target range of 30% to 36%, rather than a fixed target, is used with dosage adjustments when the patient’s scores fall outside the target range. However, the dosage adjustments might not be linear in general. Nevertheless, we believe that our linear adjustment rule with a fixed target, with its simplicity for tractable analysis, can be used as an approximation of the actual practice here.

We first computed the overall average Hct level, \( E(X) = 33.2 \), and the average dosage, \( E(D) = 12,707 \), over the 4-month period. We next calculated the correlation coefficient between \( E(X) \) and \( E(D) \), which was found to be equal to \(-0.381\). This suggests that a higher (lower) average dosage is usually prescribed to patients with lower (higher) average Hct score or with worse (better) initial condition. The negative correlation between the dosage and the score is consistent with our prescription policy as described in (2.2) and (2.4).

In our model, we assume that different patients have different initial scores and different target scores. However, the scores of all patients have the same random fluctuation \( \epsilon_t \), where \( \epsilon_t \) is an i.i.d. \( N(0, \sigma^2) \) random variable. To examine the reasonableness of this assumption, we segregated the data records into separate blocks according to the patients’ average Hct level, and then analyzed each block of patient records individually. Specifically, for each block of patient records, we computed the standard deviation of the following measures: the scores \( (\sigma(X_t)) \), the dosage \( (\sigma(D_t)) \), and the difference between dosage in two consecutive periods \( (\sigma(D_t - D_{t-1})) \). We also computed the ratio \( \sigma(X_t) / \sigma(D_t - D_{t-1}) \), and the results were summarized in Table 1. In Table 1, we also indicated the number of patients within each block.\(^5\)

Observe from Table 1 that \( \sigma(X_t) \), \( \sigma(D_t) \), and \( \sigma(D_t - D_{t-1}) \) appear to be fairly constant, except for the high value of \( \sigma(D_t) \) for the block that has \( E(X) \) between 28.0–28.9. Furthermore, we studied the dis-

\(^5\) In our analysis, we ignored the records with \( E(X) \) below 26 or above 39, as there are only a few data points scattered over these ranges for any meaningful analysis.

<table>
<thead>
<tr>
<th>Average ( E(X) )</th>
<th># Patients</th>
<th>( \sigma(X_t) )</th>
<th>( \sigma(D_t) )</th>
<th>( \sigma(D_t - D_{t-1}) )</th>
<th>( \sigma(X_t)/\sigma(D_t - D_{t-1}) = \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.0–26.9</td>
<td>12</td>
<td>2.83</td>
<td>10911</td>
<td>6174</td>
<td>0.000458</td>
</tr>
<tr>
<td>27.0–27.9</td>
<td>14</td>
<td>2.36</td>
<td>8559</td>
<td>5763</td>
<td>0.000410</td>
</tr>
<tr>
<td>28.0–28.9</td>
<td>26</td>
<td>2.18</td>
<td>14305</td>
<td>3873</td>
<td>0.000563</td>
</tr>
<tr>
<td>29.0–29.9</td>
<td>48</td>
<td>2.37</td>
<td>10521</td>
<td>4807</td>
<td>0.000493</td>
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<tr>
<td>30.0–30.9</td>
<td>66</td>
<td>2.25</td>
<td>9789</td>
<td>4762</td>
<td>0.000472</td>
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<tr>
<td>31.0–31.9</td>
<td>87</td>
<td>2.30</td>
<td>7462</td>
<td>4047</td>
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<td>4334</td>
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<td>158</td>
<td>2.08</td>
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<td>7226</td>
<td>4092</td>
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<td>3811</td>
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<tr>
<td>average</td>
<td>2.29</td>
<td>8826</td>
<td>4391</td>
<td></td>
<td></td>
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</table>

Total \# of patients = 915
tribution of \( X_t \) and found it to be approximately normal. This supports our model assumption that \( \epsilon_t \), the random fluctuation of the score, is normally distributed. Also, observe from Table 1 that the ratio \( \sigma(X_i)/\sigma(D_i - D_{i-1}) \) appears to be fairly constant and falls within the range of 0.0004–0.0006. From Proposition 1, it is easy to observe that \( \sigma(X_i)/\sigma(D_i - D_{i-1}) = \alpha \). This estimated range for \( \alpha \), the response rate of Epogen on the Hct level, is consistent with the actual response rate given to the clinic. Therefore, our data analysis corroborates Proposition 1.

3. Operational Characteristics

We now utilize Proposition 2 to examine the impact of different factors on the clinic’s optimal prescription policy \( T^* \), the clinic’s expected profit \( E(R_t) \), the patient’s well-being measured in terms of the probability that the score is above a certain level \( P(X_t \geq l) \), and the drug manufacturer’s expected revenue given by \( cd = \frac{c[(T^* - \mu)/\alpha]}{\mu e^{(T^*/\mu)}} \).

3.1. Impact of Patient’s Initial Condition \( \mu \)

For a patient with a higher initial score \( \mu \) (better initial health condition), we were able to show that the optimal target \( T^* \) will be higher. With a higher target, the clinic runs a higher risk of a claim being denied. However, the higher potential profit (due to higher dosage) associated with the higher target outweighs the increased risk for the clinic as the initial score increases. Specifically, we show that:

**Proposition 3.** \( T^* \) increases as \( \mu \) increases.

**Proof.** It follows from (2.7) that \( \frac{\partial h(y)}{\partial \mu} = \frac{pf(y)}{\sigma} > 0 \). This implies the function \( h(y) \) increases as \( \mu \) increases. Combining this observation with (2.10), it is easy to see that \( s^* \) decreases as \( \mu \) increases. Therefore, \( T^*_\mu = K - s^*\sigma \) is increasing in \( \mu \). Since \( T^* = \max \{L, \min (T^*_\mu, U)\} \), \( T^* \) increases as \( \mu \) increases. □

We now present some numerical results that further illustrate how the initial patient score \( \mu \) affects the optimal target \( T^* \) as well as the probability that a claim will be approved in terms of \( P(X_t \leq K) \). In addition, we also analyze how the initial patient score \( \mu \) would affect the patient’s well-being in terms of the probability that the score will exceed a certain “minimum acceptable level” \( l \); i.e., \( P(X_t \geq l) \). In our numerical examples, we set \( \alpha = 1, K = 100, p = 1, c = 0.7, \sigma = 5, L = 90, U = 100 \), and the minimum acceptable
level $l = 85$. Figure 2 shows the optimal $T^*$ for different values of $\mu$ ranging from 70 to 90. Observe from Figure 2 that as $\mu$ decreases, the optimal target $T^*$ decreases, which implies that the well-being deteriorates for patients with lower initial score under the proposed reimbursement policy. However, as $T^*$ decreases, the probability of getting a claim approved increases.

Figure 3 shows that under the optimal target policy, the optimal expected profit per period $E(R_i)$ for a patient decreases as the patient’s initial score $\mu$ increases. This implies that patients with higher initial scores will generate lower profits for the clinic due to lower dosage, as the average dosage for the patient is equal to $(T^* - \mu) / \alpha$.

3.2. Impact of Relative Treatment Cost to Revenue $c/p$

As the relative cost to revenue of the drug treatment increases, the clinic needs to reduce the risk of a claim being denied. As a result, the clinic will become more conservative and set a lower target $T^*$. This result is highlighted in Proposition 4:

**Proposition 4.** $T^*$ decreases as $c/p$ increases.

**Proof.** Define

$$\tilde{h}(y) = \frac{h(y)}{p} = F(y) + \left( y - \frac{K - \mu}{\sigma} \right) f(y) - \frac{c}{p}.$$

It follows from Proposition 2 that $s^*$ is the unique solution to $\tilde{h}(y) = 0$. For any fixed $y$, $\tilde{h}(y)$ decreases as $c/p$ increases. Therefore, it follows from (2.10) that $s^*$ increases as $c/p$ increases. Thus, $T_u^*$ (and hence $T^*$) decreases as $c/p$ increases.

We illustrate how this cost to revenue ratio affects the optimal target $T^*$ in Figure 4. We set $\mu = 90$ and set all other model parameters to the same values as in the previous examples. By fixing $p = 1$, it suffices to vary $c$ from 0 to 0.99 so as to analyze the impact of $c/p$. Figure 4 summarizes the overall impact of $c$ on $T^*$, $P[X_i \leq K]$, and $P[X_i \geq l]$.

Observe that $T^*$ drops quite sharply as the ratio of $c/p$ approaches one. This suggests that the clinic would greatly reduce the prescription target for its patients as its profit margin gets very small. As the ratio of $c/p$ approaches one, the probability of getting a claim approved is very high, but the patient’s well-being deteriorates as the probability of maintaining the score above the minimum acceptable level decreases sharply.

3.3. Impact of Random Fluctuation $\sigma$

We now examine the impact of the random fluctuation of the patient’s score $\sigma$.

**Proposition 5.** $T^*$ is decreasing in $\sigma$ when $s^* \geq 1$.

**Proof.** Using the fact that $h(s^*) = 0$ and that $f'(s) = -sf(s)$, we can differentiate (2.7) with respect to $\sigma$ to obtain

$$pf(s^*) \frac{ds^*}{d\sigma} + p \left( \frac{ds^*}{d\sigma} + \frac{K - \mu}{\sigma^2} \right) f(s^*)$$

$$- p \left( s^* - \frac{K - \mu}{\sigma} \right) s^* f(s^*) \frac{ds^*}{d\sigma} = 0,$$

which simplifies to

$$\frac{ds^*}{d\sigma} = - \frac{(K - \mu)/\sigma^2}{2 - s^*(s^* - (K - \mu)/\sigma)}.$$  \(3.1\)

Since $T_u^* = K - s^*\sigma$, we have $dT_u^*/d\sigma = -s^* - \sigma(ds^*/d\sigma)$. By substituting (3.1) and $T_u^* = K - s^*\sigma$ and using our assumption that the target $T_u^*$ is greater than the initial score $X_o = \mu$, we deduce that

$$\frac{dT_u^*}{d\sigma} = -s^* + (1 - s^*)(T_u^* - \mu)/\sigma \leq 0$$
when \( s^* \leq 1 \). Therefore, \( T_u^* \) (and hence \( T^* \)) is decreasing in \( \sigma \) when \( s^* \leq 1 \). □

Note from (2.10) that \( s^* \geq 1 \) is equivalent to \( h(1) \leq 0 \), which can be written as

\[
F(1) + \left( 1 - \frac{K - \mu}{\sigma} \right) f(1) = 0.841 \\
+ 0.242 \left( 1 - \frac{K - \mu}{\sigma} \right) \leq c / p.
\]
Therefore, Proposition 5 shows that when the profit margin is small \((c/p\) is high) and/or when the reimbursement threshold is much higher than the patient’s initial condition \((K - \mu)/\sigma < 1\), \(T^*\) is decreasing in \(\sigma\).

We now illustrate the impact of \(\sigma\). In this example, we kept the previous values of other model parameters with \(\mu = 90\) and \(c = 0.7\), while we varied \(\sigma\) from 0 to 10. Figure 5 summarizes the impact of \(\sigma\) on the optimal target, patient’s well-being and the probability of reimbursement, and Figure 6 shows how \(\sigma\) affects the profit of the clinic as well as the revenue of the drug manufacturer.

Observe that \(T^*\) is decreasing in \(\sigma\) in Figure 5 (notice that \(s^* > 1\) for all \(\sigma\) between 0 and 10 in this example, so Proposition 5 applies here). Thus, a higher fluctuation of the patient’s score will prompt the clinic to set a lower target \(T^*\) in order to reduce the risk of being denied reimbursement. This lower target, in turn, leads to a lower average dosage for the patient. Also, observe from Figure 5 that the probability of reimbursement decreases and the patient’s well-being \((P(X_t > 1))\) deteriorates as \(\sigma\) increases.

Figure 6 shows that the average profit for the clinic increases (due to higher average dosage and reimbursement probability) as \(\sigma\) decreases. Furthermore, the revenue of the drug manufacturer also increases (due to higher average dosage) as \(\sigma\) decreases. These results provide a very interesting implication. Reducing the random fluctuation \(\sigma\) is beneficial to all parties involved: increased well-being for the patient, higher profit for the clinic, and higher revenue for the drug manufacturer. In many instances, one can reduce the random fluctuation \(\sigma\) by certain diet and exercise programs.

### 3.4. Impact of Reimbursement Threshold \(K\)

We can expect that a higher reimbursement threshold \(K\) should result in a higher target set by the clinic. The following proposition shows that this relationship is generally true.

**Proposition 6.** \(T^*\) is increasing in \(K\) when \(s^* \geq 0\).

**Proof.** Using the fact that \(h(s^*) = 0\) and that \(f'(s) = -sf(s)\), we can differentiate (2.7) with respect to \(K\) to obtain

\[
pf(s^*)\frac{ds^*}{dK} + p\left(\frac{ds^*}{dK} - \frac{1}{\sigma}\right)f(s^*)
- p\left(s^* - \frac{K - \mu}{\sigma}\right)s^*f(s^*)\frac{ds^*}{dK} = 0,
\]
which simplifies to
\[
\frac{ds^*}{dK} = \frac{1/\sigma}{2 - s^*(s^* - (K - \mu)/\sigma)}. \tag{3.2}
\]

Because \( T_u^* = K - s^*\sigma \), we have \( (dT_u^*/dK) = 1 - \sigma(ds^*/dK) \). By substituting (3.2) and \( T_u^* = K - s^*\sigma \), and using our assumption that the target \( T_u^* \) is greater than the initial score \( X_o = \mu \), we deduce that
\[
\frac{dT_u^*}{dK} = 1 - \frac{1}{2 + s^*((T_u^* - u)/\sigma)} \geq 0
\]
when \( s^* \geq 0 \). Therefore, \( T_u^* \) (and hence \( T^* \)) is increasing in \( K \) when \( s^* \geq 0 \). \( \square \)

Equation (2.12) provides the condition under which \( s^* \geq 0 \). Thus, Proposition 6 shows that when the clinic’s profit margin is below 100% (i.e., \( (p - c)/c \leq 100\% \)), \( T^* \) is increasing in \( K \), and this condition is true for most dialysis clinics administering Epogen.

The next set of numerical results studies how the threshold policy \( K \) can affect the clinic’s behavior as well as its profit. In these examples, all parameters remain the same as before except we set \( \mu = 90 \), \( c = 0.7 \) and \( \sigma = 5 \). Figure 7 illustrates how the threshold \( K \) can affect the optimal target \( T^* \), the patient’s well-being (i.e., \( P(X_t \geq l) \)), and the probability of a claim being approved.

Observe from Figure 7 that as the threshold \( K \) increases, the optimal target \( T^* \) increases, resulting in a larger average dosage and an increased well-being for the patient. Furthermore, observe that as the threshold \( K \) increases, the probability of reimbursement increases even when the target \( T^* \) increases.

From the clinic’s perspective, it is also important to understand how the threshold policy would affect its overall profit. Because the clinic has patients with different health conditions, we can estimate the potential impact as follows. For patients with initial score \( \mu \), we can calculate their average profit per period. Then, the average profit per period for the clinic can be computed once the patient profile of the distribution of the initial scores is given. (For our purpose here, we ignore the situation in which different insurers might have different reimbursement schedules for their patients so that the target policy only depends on the patient’s initial score here.) To illustrate this, consider the numerical examples in § 3.1 and assume that the initial scores of the patients in the clinics are uniformly distributed at integer values between 70 and 90. The
average profit per period per patient in the clinic was computed for different values of $K$, and these are summarized in Figure 8. Observe that the average profit decreases almost linearly as the reimbursement threshold $K$ decreases.

We further study how the threshold policy $K$ can affect the revenue of the drug manufacturer for a given patient profile of the clinic. Because the expected revenue for a patient with initial score $x$ is given by $cd = c(T^* - x)/\alpha$, we computed the average revenue for the manufacturer for the given patient profile for our numerical example here and also summarized the results in Figure 8. Observe that as $K$ decreases, the manufacturer’s average revenue decreases almost linearly due to the reduced dosage prescribed by the clinic when lower reimbursement threshold $K$ is imposed. In summary, the results in Figures 7 and 8 allow us to jointly evaluate the impact of the threshold value $K$ on the patient’s well-being, as well as the clinic’s profitability and the drug manufacturer’s revenue.

4. Extensions

In this section, we consider two extensions of our model to deal with some additional issues that have not been addressed in our basic model presented in § 2. The first one considers a more general prescription policy with smoothing to reduce the fluctuation in dosage amount from period to period. The second extension considers a more general situation in which the reimbursement policy is based on the average score over an extended period of time instead of a single period.

4.1. Prescription Policy with Smoothing

The target prescription policy described in § 2 can lead to a significant change in dosage between subsequent periods, which could cause ill effects on the patients. Therefore, the clinic needs to adopt a more general prescription policy by introducing some smoothing factor to dampen the change in dosage levels. Consider the following general prescription policy with smoothing that is given by $D_t = d_t$, and

$$D_t = D_{t-1} + \theta \left( \frac{T - X_{t-1}}{\alpha} \right), \quad t \geq 2,$$

where $\theta$ is a smoothing factor and $0 < \theta \leq 1$. Observe that when $\theta = 1$, this reduces to the basic target prescription policy given by (2.4). Here, the factor $\theta < 1$ plays the role of smoothing the dosage fluctuation over time by adjusting the dosage in each period only.
partially to the deviation of the patient’s score from the target level \( T \). The following proposition summarizes the operating characteristics of the dosage and the patient’s score over time when the clinic adopts this more general target prescription policy with smoothing factor \( \theta \), denoted by the \((T, \theta)\)-policy:

**Proposition 7.** Under the \((T, \theta)\)-policy,

(a) \( X_t \) and \( D_t \) are independent for \( t \geq 1 \);
(b) \( E(X_t) = T \) as \( t \to \infty \);
(c) \( \text{Var}(X_t) = \sigma^2/(1 - (1 - \theta)^2) \) as \( t \to \infty \);
(d) \( E(D_t) = d \) as \( t \to \infty \);
(e) \( \text{Var}(D_t - D_{t-1}) = (\theta^2/(1 - (1 - \theta)^2))(\sigma^2/\alpha^2) \) as \( t \to \infty \).

**Proof.** (a) Substitute (4.1) into (2.3) and obtain, for \( t \geq 2 \),

\[
X_t = X_{t-1} + \theta(T - X_{t-1}) + \epsilon_t = \theta T + (1 - \theta)X_{t-1} + \epsilon_t. \tag{4.2}
\]

Using (4.1) recursively, we obtain

\[
D_t - D_{t-1} = (D_t - D_{t-2}) - \theta \left( \frac{X_{t-1} - X_{t-2}}{\alpha} \right).
\]

Substitute (2.3) for \( X_{t-1} \) into the above equation and after simplification, we obtain

\[
D_t = D_{t-1} + (1 - \theta)(D_{t-1} - D_{t-2}) + \frac{\theta}{\alpha} \epsilon_{t-1}. \tag{4.3}
\]

From the definition, \( D_1 \) and \( X_1 \) are independent. Suppose \( D_{t-1} \) and \( X_{t-1} \) are independent. Then, by using (4.2) and (4.3), we can deduce that \( D_t \) and \( X_t \) are independent from the fact that \( \epsilon_{t-1} \) and \( \epsilon_t \) are independent. By induction, we prove (a).

(b) It follows from (4.2) that \( E(X_t) = \theta T + (1 - \theta)E(X_{t-1}) \), from which we can deduce the result.

(c) It also follows from (4.2) that \( \text{Var}(X_t|X_{t-1}) = \sigma^2 \) and that \( \text{Var}(X_t) = \sigma^2 + (1 - \theta)^2 \text{Var}(X_{t-1}) \).

Therefore, \( \text{Var}(X_t) = \sigma^2/[1 - (1 - \theta)^2] \) as \( t \to \infty \).

(d) Since \( E(X_t) = T \) as \( t \to \infty \), it follows from (4.1) that \( E(D_t) = d \) as \( t \to \infty \).

(e) It follows from (4.1) that \( \text{Var}(D_t - D_{t-1}) = \theta^2/(\alpha^2) \text{Var}(X_{t-1}) \). The result follows from part (c). \( \square \)

Observe from (4.2) that the target prescription policy with smoothing adjusts the patient’s score partially back to its target \( T \) (in expectation) when \( \theta < 1 \), and fully when \( \theta = 1 \). Proposition 7 shows that in either case, the expected patient’s score is \( T \) and the average patient’s dosage is \( d \) in the long run. Furthermore, observe that \( 1/(1 - (1 - \theta)^2) \) is decreasing and convex in \( \theta \) while \( \theta^2/(1 - (1 - \theta)^2) = \theta/(2 - \theta) \) is increasing and convex in \( \theta \). Therefore, \( \text{Var}(X_t) \) is convex and decreasing in \( \theta \), while \( \text{Var}(D_t - D_{t-1}) \) is convex and increasing in \( \theta \) as \( t \to \infty \). This presents an interesting tradeoff for choosing the smoothing factor \( \theta \). A prescription policy with large \( \theta \) attempts to quickly adjust the patient score back to the target level \( T \) at the expense of a higher fluctuation between successive dosage levels. On the other hand, a prescription policy with small \( \theta \) reduces the fluctuation in dosage, but results in a higher fluctuation of the patient score around its target \( T \).

Suppose we select a smoothing factor \( \theta \) that yields an acceptable variance of the dosage \( D_t \). Then, for any given value of \( \theta \), we can apply Proposition 7 to formulate the clinic’s problem so as to determine the optimal target \( T^\ast \) that maximizes the expected profit \( E(R_t) \). In this case, the clinic’s problem can be written as:

\[
\max_{\text{subject to } \mu \leq \alpha} \frac{T - \mu}{\alpha} \left( \text{Pr}(X_t \leq K) - c \right), \tag{4.4}
\]

where \( X_t \) is a \( N(T, \sigma^2) \) random variable and \( \sigma^2 = \sigma^2/[1 - (1 - \theta)^2] \).

Therefore, all previous results stated in Propositions 2 through 7 remain valid. To see that, simply replace \( \sigma \) with \( \hat{\sigma} \) as defined above. Since a higher value of \( \theta \) results in smaller \( \hat{\sigma} \), we can apply Proposition 5 to show that the optimal target \( T^\ast \) is increasing in \( \theta \) when \( s^\ast \geq 1 \). In particular, when the profit margin is sufficiently small (i.e., \( c/p \) is large), more smoothing

\[\text{Suppose the clinic imposes a bound, denoted by } B, \text{ on the variance between successive dosage } (D_t - D_{t-1}). \text{ Then we would need to choose the optimal smoothing factor } \theta^\ast \text{ and the optimal target } T^\ast \text{ jointly. To do so, one can utilize the closed form expression for the variance of } (D_t - D_{t-1}) \text{ to develop an additional constraint: } (\theta^2/(1 - (1 - \theta)^2))/(\sigma^2/\alpha^2) \leq B. \text{ By imposing this additional constraint to the clinic’s problem, one can determine the optimal smoothing factor } \theta^\ast \text{ and the optimal target } T^\ast \text{ jointly. However, the analysis of the joint optimal smoothing factor } \theta^\ast \text{ and the optimal target } T^\ast \text{ is complex and is beyond the scope of this paper. We omit the details.} \]
(i.e., a lower value of $\theta$) results in a lower optimal target $T^*$, which presents a tradeoff between the fluctuation in dosage and the patient’s well-being (measured in terms of $T^*$).

Figure 9 illustrates the impact of $\theta$ on the corresponding optimal target $T^*$, the probability of a claim being approved and the probability that a patient’s score is above 85. In this numerical example, all parameters remain the same as before except we set $\alpha = 1$, $K = 100$, $p = 1$, $c = 0.7$, $\sigma = 5$, and $\mu = 90$.

Our results here are consistent with the previous one on the impact of $\sigma$. For example, since $\hat{\sigma}$ is decreasing in $\theta$, the optimal target $T^*$ here is increasing in $\theta$. In other words, a high value of the smoothing factor $\theta$ results in a higher target $T^*$.

### 4.2. Other Reimbursement Policies

We now study a more general reimbursement policy under which the prescribed dosage at period $t$ is reimbursed only if the average patient score over $n$ periods, denoted by $X^n_t$, does not exceed the threshold level $K$.§ Since $X_t = T + \epsilon_t$, $X^n_t = \frac{X_t + X_{t-1} + \ldots + X_{t-n+1}}{n}$

$$= T + \frac{\epsilon_t}{n} + \frac{\epsilon_{t-1}}{n} + \ldots + \frac{\epsilon_{t-n+1}}{n} = T + \epsilon^n_t.$$  

Note that $X^n_t$ is an $N(T, \sigma^2/n)$ random variable; however, $X^n_i$ and $X^n_j$ are no longer independent if $|i - j| < n$. This implies that the variance of the patient’s score reduces when the reimbursement policy depends on the average score over multiple periods. In this case, we can select the optimal $T^*$ as before, replacing $\sigma^2$ by $\sigma^2/n$ accordingly. Thus, this reimbursement policy has the same effect as reducing $\sigma$ in the original model; see §3.3.

### 5. Discussion

We developed an analytical model to better understand how an outcome oriented reimbursement policy can affect the patients, the clinic, and the drug manufacturer. Using a linear response function and a target

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7 For instance, the policy issued by Health Care Financing Administration in August 1997, known as Hematocrit Measurement Audit, allowed for Epogen payments based on a 3-month rolling average Hct measurement. However, for various reasons this policy was changed in March 1998 so that the reimbursement policy now depends only on the most recent Hct measurement.
prescription policy, we derived the optimal target within some constrained range (as imposed by clinical guidelines) that maximizes the clinic’s average profit. We further analyzed how the patient’s initial condition, the cost parameters, the fluctuation of the patient’s score, and the reimbursement threshold would affect the optimal target as well as the clinic’s profit and the drug manufacturer’s revenue.

Our model captures the clinic’s risk of being denied reimbursement if the patient’s score after the treatment exceeds the predetermined reimbursement threshold. Therefore, the model can provide the clinic with important information about how the different factors would change its selection of the optimal target prescription policy. In particular, we show in § 3 that such factors as patients with worse initial conditions, lower profit margin for the drug treatment, higher fluctuation of the patient score (under certain conditions as given in Proposition 5), or a lower reimbursement threshold would all result in a lower target being set by the clinic. We further use several numerical examples to help quantify the associated impact of these factors on the patient, the clinic, and the drug manufacturer. Our results suggest that all of the above factors change the optimal target and the associated operating characteristics of the model substantially.

The results in § 3.4 suggest that the clinic lowers their prescription target as the reimbursement threshold decreases. This seems intuitive, as it is the clinic who is paying for the drug cost, but assumes the risk of no reimbursement. However, a lower target implies lower drug usage, and therefore the reimbursement policy also adversely affects the revenue of the drug manufacturer. So, one interesting question is how the drug manufacturer can react to the insurer’s reimbursement policy $K$ so as to improve its revenue.

There are various ways that the drug manufacturer can provide incentives for the clinic to increase the target (and hence, the dosage) for the patients within the clinical guidelines. One approach is for the drug manufacturer to simply lower the purchase cost $c$ to the clinic. Another approach is to consider some risk-sharing scheme whereby the manufacturer offers some partial reimbursement $r$ ($r < c$) to the clinic for each unit dosage when the treatment is denied reimbursement. However, such a risk-sharing scheme can be perceived as a disguised version of a “rebate” policy, which is considered illegal by the health insurance agencies. Yet another possible approach is to reduce the random fluctuations of the patient’s score $\sigma$, as suggested by the result in Proposition 5. For example, the drug manufacturer can develop and distribute promotional programs/brochures to increase the awareness of the patients about the importance of diet and exercise programs which in many instances help to reduce the fluctuations. Our model can be used to provide useful information for evaluating the associated costs and potential benefits for each of these approaches.

While our model captures the essence of a specific problem, there are other issues that deserve attention for future research. First, we consider the case in which a patient’s well-being is captured by a single score. A more general situation might require the patient’s well-being be measured by multiple scores (such as electrolytes, blood pressure, blood sugar level, etc.). In such a case, one needs to extend our single-dimension model to a multidimension model. Second, we assume that the response rate of the drug $\alpha$ is linear and independent of the patient’s condition. A nonlinear function or a response rate $\alpha(X_t)$ depending on the patient’s score might be needed to model other applications. Third, we assume that the fluctuation of the score $\sigma$ is stationary and is known to the clinic. However, in reality, it is very difficult to estimate $\sigma$. This is because the fluctuation of the score can be caused by many factors including the patient’s diet program, health condition, drug interactions, etc. In any event, our model does provide a better understanding of the impact of $\sigma$ on the clinic as well as the patients. Finally, there are other types of reimbursement policy to be examined as well.

Health care policy is a difficult and complex subject. The important issue of how to control the escalating costs of providing health care while ensuring the overall quality of health care services provided to the patients.

8 For this situation, we can simply replace $p$ with $p - r$ and $c$ with $c - r$ in our model. Furthermore, we can easily derive from our previous results that both $T^*$ and $E(D_t)$ are increasing in $r$. We omit the details here.
remains a challenging task for government, industry business leaders, and academicians to tackle. It is our hope that this paper will generate additional interest for operations researchers to apply their skills to address the many important emerging issues arising from the cost containment efforts in the health care industry.9

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