

Designing Supply Contracts: Contract Type and Information Asymmetry

Charles J. Corbett, Deming Zhou, Christopher S. Tang

The Anderson School of Management, University of California, Los Angeles, 110 Westwood Plaza, Los Angeles, California 90095-1481
{charles.corbett@anderson.ucla.edu, deming.zhou@anderson.ucla.edu, christopher.tang@anderson.ucla.edu}

This paper studies the value to a supplier of obtaining better information about a buyer's cost structure, and of being able to offer more general contracts. We use the bilateral monopoly setting to analyze six scenarios: three increasingly general contracts (wholesale-pricing schemes, two-part linear schemes, and two-part nonlinear schemes), each under full and incomplete information about the buyer's cost structure. We allow both sides to refuse to trade by explicitly including reservation profit levels for both; for the supplier, this is implemented through a cutoff policy. We derive the supplier's optimal contracts and profits for all six scenarios and examine the value of information and of more general contracts. Our key findings are as follows: First, the value of information is higher under two-part contracts; second, the value of offering two-part contracts is higher under full information; and third, the proportion of buyers the supplier will choose to exclude can be substantial.

Key words: supply chains; contracting; pricing; asymmetric information

History: Accepted by Teck H. Ho, special issue editor; received September 2001. This paper was with the authors 15 months for 1 revision.

1. Introduction

Consider a monopolist supplier who is about to enter a new geographic market, modifying an existing product to meet local needs. For instance, a European maker of a novel household appliance would need to adapt its product to 110 volts for export to the United States. The supplier needs to find a local retailer through which to sell the product. The classic double-marginalization problem occurs: If both parties charge a markup, final retail price will be higher and total demand will be lower than in the vertically integrated case. It is well known that two-part contracts, under which the supplier sells the product at its marginal cost and charges a fixed side payment, can coordinate the channel.

This changes when the retailer has private information about her internal variable costs, e.g., processing and handling costs. The economics and supply-chain management literatures analyze a variety of contract types our supplier could use; in general, he can no longer achieve the first-best outcome. We examine three common types, each a special case of the next: the basic wholesale-pricing scheme, a two-part linear scheme with fixed wholesale price and side payment, and a two-part nonlinear scheme with wholesale price and side payment depending on quantity purchased. Under full information, the two-part linear scheme is sufficient to coordinate the channel; under asymmetric information, even two-part nonlinear schemes cannot do so. Some practical questions remain unanswered. What is the value to the

supplier of obtaining information about the buyer's costs? How much extra profits can the supplier extract by offering more general contracts? If a supplier currently offers a simple wholesale price under asymmetric information, should he focus on getting better information or on offering more general contracts?

Typically, the retailer (or agent) has an outside opportunity, giving her a reservation profit level—for instance, the profit she could obtain from allocating the shelf space to another supplier's product. If the supplier's contract is not sufficiently attractive, the buyer will refuse to trade. The supplier's reservation profit level is usually ignored. Not giving the supplier an outside opportunity is a rather strong assumption that confers substantial power to the buyer. In this paper, we allow the supplier to have a reservation profit level, too. In the example above, this reservation profit level could be the product adaptation costs. If the supplier cannot earn enough profits from this new market to cover these adaptation costs, he will not enter the market at all. Alternatively, the supplier's reservation profit level could arise from the fixed portion of the account management costs associated with dealing with a new buyer, as in Chen et al. (2001).

The notion of excluding undesirable types of agent is in itself not new. Baron and Myerson (1982) allow the regulator to shut down an inefficient monopolist, resulting in the same type of cutoff policy as we study below. Moorthy (1984) shows that it can be optimal for a firm to exclude certain customer types in

designing product lines. Here, we allow the supplier to refuse to trade with buyers with costs above a certain “cutoff level.” Ha (2001) confirms that such policies are indeed optimal (in a more general setting).

Our analysis yields several interesting observations. First, it is often optimal for the supplier to exclude a substantial portion of buyers, up to 25% or more in our numerical experiments. Ignoring the supplier’s reservation profit level is, hence, a strong assumption. Second, the value of information about the buyer’s costs can be linked to the variance of the supplier’s prior distribution. Third, the value of information is greater under two-part contracts than under one-part contracts, as they allow greater freedom to exploit that information. Fourth, the value of being able to offer two-part contracts rather than one-part contracts is greater under full information than under asymmetric information. Together, this gives the important finding that better information and two-part contracts are strategic complements: Achieving one increases the incremental value of the other. Last, in our numerical experiments, the cost of being restricted to two-part linear rather than nonlinear contracts was very small. Some of these results can be verified analytically only for the traditional case when the supplier must trade or when it is optimal for the supplier not to exclude any buyers, but the numerical experiments suggest that our results carry over to the case without that assumption.

In §2, we review pertinent literature. Section 3 introduces the modeling framework. The buyer’s optimization problem is solved in §4. The supplier’s problem under full and under asymmetric information is solved in §§5 and 6, respectively, for all three contract types. Our key results follow in §7, which compares profits and cutoff policies in various scenarios. Numerical experiments are presented in §8, and §9 offers some areas for future research. Some proofs and additional comments are provided in a separate document.

2. Literature Review

Double marginalization in bilateral monopoly is discussed in, among many others, Tirole (1988). It is well known that, under full information, the supplier can achieve first-best using two-part contracts. Corbett and Tang (1999) assume the buyer has private information about her variable costs; the current paper goes further by allowing both sides to refuse to trade. Ha (2001) studies optimal contracts in a more general setting than we do, including nonlinear stochastic demand, at the expense of gaining less insight into the value of information and of more general contracts. Several other contracting issues have been studied in the recent supply-chain management literature. Weng

(1995) studies quantity discounts for achieving coordination, and Chen et al. (2001) extend this to multiple customers and more general cost structures. Tsay et al. (1999) provide a review of other work in this area.

The economics literature on double marginalization is extensive. Dobbs and Hill (1993) discuss contracting solutions when the buyer has private information about demand. Rochet (1985) studies bilateral monopoly with uncertainty and general two-sided information asymmetry. Bresnahan and Reiss (1985) show how the ratio of the profit margins of manufacturer and retailer under simple wholesale pricing with full information depends on the convexity of the demand function. Gal-Or (1991a) shows that when the buyer has private information about demand and retail costs, the supplier generally cannot achieve the vertically integrated solution. Gal-Or (1991b) studies the situation with two suppliers, but complete information, and finds that equilibrium is sometimes achieved with linear pricing and sometimes with a franchise fee contract. Irmen (1997) shows that using percentage markups leads to higher downstream profits and lower final price than the traditional absolute markups. We add to this literature by comparing the value of information with the value of more general contracts.

A related problem is that of regulating a monopolist under asymmetric information. Baron and Myerson (1982) allow the regulator to shut down inefficient firms, and show that a cutoff policy that excludes all firms with costs higher than a critical level c^* is optimal. Lewis and Sappington (1988a) show that information asymmetry about demand has different effects on the behavior of the optimal policy than information asymmetry about cost; Lewis and Sappington (1988b) show that, under information asymmetry about both demand and cost, the firm is given less flexibility to set prices than when only one of these two parameters is privately known. Armstrong (1999) extends the latter work to include the regulator’s shutdown option more explicitly. Maggi and Rodríguez-Clare (1995) allow the agent’s reservation utility to depend on his type, while Rochet and Stole (2002) consider agents with random reservation utilities. Jullien (2000) deals with the regulator’s shutdown option by transforming the case where some firm types are excluded into a more tractable full-participation model.

3. The Modeling Framework

We consider a single supplier and single buyer who resells the supplier’s product to a final market. The quantity q demanded per period in the final market is given by the demand function $q = a - bp$, where $a > 0$

and $b > 0$ are known parameters. The buyer selects either q or p ; the other is then immediately determined. It is possible to obtain the optimal contracts for polynomial demand functions $q = (a - bp)^n$, but then we can no longer characterize the value of information and the value of more general contracts; therefore, we focus on the linear demand case throughout. The supplier's variable costs are s ; the buyer's internal variable costs (excluding the cost of the supplier's part) are c . In general, the supplier does not know c ; we assume the supplier holds a prior distribution $F(c)$ with continuous density $f(c)$, defined on $[\underline{c}, \bar{c}]$, where $0 \leq \underline{c} \leq \bar{c} \leq \infty$. The supplier has a reservation profit level Π_s^- and will refuse to trade with any buyer if that would lead to profits below Π_s^- . We will see that the supplier's profits are always decreasing in c , so he can follow a cutoff policy: The supplier will not trade if $c > \beta$ for some optimal cutoff point $\beta \in [\underline{c}, \bar{c}]$. Proposition 4(a) in Ha (2001, p. 54) shows that this type of policy is indeed optimal. To ensure nonnegative demand, we assume throughout that $a - b(s + \beta) \geq 0$. All parameters except c are common knowledge.

We consider three types of contract, each a special case of the next. First is a one-part linear contract w , in which the supplier can only specify a constant wholesale price w . Second is a two-part linear contract (w, L) in which the supplier offers a per-period lump-sum side payment L to the buyer, but where w and L are independent of q . $L > 0$ can be interpreted as a slotting fee, common among large retailers; $L < 0$ corresponds to a franchise fee, common when the supplier has a strong brand. The most general is the two-part nonlinear contract $\{w(q), L(q)\}$, where wholesale price and side payment depend on q . Forcing the supplier to use a more restrictive contract essentially limits his bargaining power and his ability to coordinate the system.

The sequence of events is as follows. Depending on the case, the supplier knows c or only holds a prior $F(c)$. He then offers a (menu of) contract(s); the buyer chooses an order quantity q based on her internal cost c and pays the supplier $w(q)q - L(q)$. We study three types of contract under full and asymmetric information, leading to six cases. In Cases F1, F2, and F3, the supplier has full information about c and offers a one-part linear contract w , a two-part linear contract (w, L) , and a two-part nonlinear menu of contracts $\{w(q), L(q)\}$, respectively. Cases A1, A2, and A3 are analogous, except now the supplier does not know c . For any revealed choice of q , the supplier can infer the buyer's true cost c , so by the revelation principle (Baron and Myerson 1982), we can reformulate this equivalently as a menu of contracts $\{w(c), L(c)\}$. A buyer announcing \hat{c} then chooses a contract $(w(\hat{c}), L(\hat{c}))$.

Clearly, Case A3 is the most general, so it is sufficient to formulate the supplier's problem allowing two-part nonlinear contracts under asymmetric information first, and then analyze the more constrained types of contract and the full information cases as special cases.

$$\mathcal{P}_{A3} \max_{w(c), L(c), \beta} \int_{\underline{c}}^{\beta} \Pi_s(c, q(c)) f(c) dc + \int_{\beta}^{\bar{c}} \Pi_s^- f(c) dc \quad (1)$$

$$\text{s.t. ICb: } q(c) = \arg \max_q \Pi_b(c, q) \quad \forall \underline{c} \leq c \leq \beta \quad (2)$$

$$\text{IRb: } \Pi_b(c, q(c)) := (p(q(c)) - w(c) - c)q(c) + L(c) \geq \Pi_b^- \quad \forall \underline{c} \leq c \leq \beta \quad (3)$$

$$\Pi_s(c, q(c)) := (w(c) - s)q(c) - L(c) \geq \Pi_s^- \quad \forall \underline{c} \leq c \leq \beta \quad (4)$$

$$\underline{c} \leq \beta \leq \bar{c} \quad (5)$$

The supplier's expected net profits in (1) depend on the quantity q ordered by the buyer through the definitional constraint (4), which depends on the buyer's internal costs c , unknown to the supplier. Under two-part contracts, the supplier also offers a side payment $L(c)$. Condition (2) is the buyer's incentive-compatibility constraint: Presented with any menu of contracts, a buyer with costs c will choose q to maximize her profits. With $p(q)$ the retail price, inequality (3) represents the buyer's individual rationality constraint IRb: Her net profits must exceed her reservation profit level Π_b^- . The objective function (1) executes the supplier's cutoff policy: If the supplier stands to earn less than Π_s^- by trading with a specific buyer, he will refuse to do so. The supplier's actual profits are decreasing in c in our cases. If $\Pi_s(c, q(c)) \geq \Pi_s^-$ holds for $c = \beta$, it will also hold for any $c \leq \beta$.

4. The Buyer's Optimization Problem

The buyer's optimization problem for a given contract is well known; we briefly review it for completeness. In Case F1, the supplier offers a wholesale price w . The buyer solves problem \mathcal{B}_{F1} , maximizing $\Pi_b := (p(q) - w - c)q$ over q . Since $q = a - bp$, the solution to this is

$$p^* = \frac{a + b(w + c)}{2b}, \quad q^* = \frac{1}{2}(a - b(w + c)). \quad (6)$$

In Case A1, the buyer solves the same problem, so $\mathcal{B}_{A1} \equiv \mathcal{B}_{F1}$. Under a two-part linear contract, the supplier also offers a side payment L , but as L is independent of q , it will not affect the buyer's order quantity. Hence, the buyer still selects p^* and q^* as in (6); i.e.,

$\mathcal{B}_{A2} \equiv \mathcal{B}_{F2} \cong \mathcal{B}_{F1}$. Cases F2 and F3 are equivalent, as discussed below, so we ignore F3 here.

In Case A3, the supplier offers a menu $\{w(c), L(c)\}$, and the buyer chooses which cost parameter \hat{c} to announce. Once she has announced \hat{c} , wholesale price $w(\hat{c})$ and side payment $L(\hat{c})$ are fixed, so the buyer will choose order quantity $q^*(w(\hat{c}))$ and set final market price $p^*(w(\hat{c}))$, where p^* and q^* are as given in (6). The buyer solves \mathcal{B}_{A3} , maximizing $\Pi_b(c, \hat{c}) := (p^*(w(\hat{c})) - w(\hat{c}) - c)q^*(w(\hat{c})) + L(\hat{c})$ over \hat{c} . The revelation principle states that there is an optimal contract under which the buyer will announce $\hat{c} = c$. $\Pi_b(c, \hat{c})$ is concave in \hat{c} , so requiring that the first-order condition for \mathcal{B}_{A3} be solved at $\hat{c} = c$ gives the buyer's incentive compatibility constraint (ICb) as $L'(c) = \frac{1}{2}(a - b(w(c) + c))w'(c) \forall c \leq c \leq \beta$.

5. Optimal Contracts Under Full Information

Propositions 1 and 2 summarize the supplier's optimal contracts under full information for Cases F1 and F2, respectively. Additional comments are given in a separate document.

PROPOSITION 1. *In Case F1, the supplier's optimal contract and profits are as follows. When $\underline{c} \leq c \leq \alpha$, then $w_{F1}^* = a/2b + \frac{1}{2}(s - c)$ and $\Pi_{s,F1}^* = (1/8b)(a - b(s + c))^2$, and when $\alpha \leq c \leq \beta_{F1}^*$, then $w_{F1}^* = a/b - 2\sqrt{\Pi_b^-}/b - c$ and $\Pi_{s,F1}^* = (a - b(s + c))\sqrt{\Pi_b^-}/b - 2\Pi_b^-$, where $\Pi_s^- \leq 2\Pi_b^- \Rightarrow \alpha = a/b - s - 4\sqrt{\Pi_b^-}/b$ and $\beta_{F1}^* = a/b - s - 2\sqrt{\Pi_b^-}/b - \Pi_s^-/\sqrt{b\Pi_b^-}$, and $\Pi_s^- > 2\Pi_b^- \Rightarrow \alpha = \beta_{F1}^* = a/b - s - 2\sqrt{2\Pi_s^-}/b$. w_{F1}^* is decreasing in c , increasing in a and (weakly) in s ; $\Pi_{s,F1}^*$ is increasing in a and in Π_s^- and decreasing in b, s, c , and Π_b^- ; β_{F1}^* is (weakly) decreasing in Π_b^- and Π_s^- , increasing in a , and decreasing in s . Also, if $\Pi_s^- = 2\Pi_b^-$, $\alpha = \beta_{F1}^*$ and the two cases coincide.*

PROPOSITION 2. *In Cases F2 and F3, the supplier's optimal contract is as follows. When $\underline{c} \leq c \leq \beta_{F2}^*$, then $w_{F2}^* = s$, $L_{F2}^* = \Pi_b^- - (1/4b)(a - b(s + c))^2$, and $\Pi_{s,F2}^* = -\Pi_b^- + (1/4b)(a - b(s + c))^2$, where $\beta_{F2}^* = (1/b)[a - 2\sqrt{b(\Pi_b^- + \Pi_s^-)}]$. β_{F2}^* is (weakly) decreasing in b, s, Π_b^- , and Π_s^- , and increasing in a . $\Pi_{s,F2}^*$ is increasing in a and Π_s^- and decreasing in b, s, c , and Π_b^- .*

The link between β_{F2}^* and s and Π_b^- corresponds to Proposition 1(c) in Ha (2001, p. 47). Note that the supplier may choose to exclude buyers ($\beta_{F2}^* < \bar{c}$) regardless of whether Π_s^- is positive or negative.

6. Optimal Contracts Under Asymmetric Information

In Case A1, the supplier's problem is as in \mathcal{S}_{A3} , with the additional constraints that $L = 0$ and $w(c) = w \forall c \leq c \leq \beta$. We can obtain an implicit solution:

PROPOSITION 3. *In Case A1, the supplier's optimal contract can be found as follows. Find all pairs (w, β) that*

satisfy the following equations:

$$\begin{aligned} & \frac{1}{2}(a - b(2w - s))\frac{F(\beta)}{f(\beta)} - \frac{1}{2}b\frac{E[c | c \leq \beta]}{f(\beta)} \\ & = \frac{1}{2}(w - s)(a - b(w + \beta)) - \Pi_s^-; \end{aligned} \quad (7)$$

$$\frac{1}{4b}(a - b(w + \beta))^2 = \Pi_b^-. \quad (8)$$

Compute $E[\Pi_s(w, \beta)]$ for each such pair (w, β) and for $(w, \beta) = (a/2b + \frac{1}{2}(s - E[c]), \bar{c})$ to find the optimal (w_{A1}^, β_{A1}^*) . If $\underline{c} \leq \beta \leq \bar{c}$, w_{A1}^* increases whenever β_{A1}^* decreases and v.v., by (8). If $\beta_{A1}^* = \bar{c}$, the optimal solution is $w_{A1}^* = a/2b + \frac{1}{2}(s - E[c])$ and $E[\Pi_{s,A1}] = (1/8b)(a - b(s + E[c]))^2$.*

The supplier can lower the cutoff point β_{A1}^* but, by (8), he will then charge a higher wholesale price w_{A1}^* . Without the cutoff point, the wholesale price w_{A1}^* and supplier's expected profits $E[\Pi_{s,A1}]$ decrease in $E[c]$.

For Case A2, we start with \mathcal{S}_{A3} , now restricted to $L(c) = L$ and $w(c) = w \forall c \leq c \leq \beta$. Define the projection of x on the interval $[l, u]$ by $x \perp [l, u] := \max\{l, \min\{x, u\}\}$.

PROPOSITION 4. *In Case A2, the supplier's optimal contract and resulting profits are $w_{A2}^* = s + \beta - E[c | c \leq \beta]$, $L_{A2}^* = \Pi_b^- - (1/4b)(a - b(s + 2\beta - E[c | c \leq \beta]))^2$, and $E[\Pi_{s,A2}] = (\frac{1}{2}(\beta - E[c | c \leq \beta])(a - b(s + \beta)) + (1/4b)((a - b(s + 2\beta - E[c | c \leq \beta]))^2 - \Pi_b^- - \Pi_s^-)F(\beta) + \Pi_s^-$, where the optimal cutoff point is given by $\beta_{A2}^* = \beta_0 \perp [c, \bar{c}]$ where β_0 solves*

$$\begin{aligned} & \frac{1}{4b}(a - b(s + E[c | c \leq \beta])) \\ & = \frac{1}{2}\frac{F(\beta)}{f(\beta)} + \frac{\Pi_b^- + \Pi_s^-}{a - b(s + 2\beta - E[c | c \leq \beta])}. \end{aligned} \quad (9)$$

The left-hand side of (9) decreases in β and the right-hand side increases, so β_{A2}^* is decreasing in $\Pi_b^- + \Pi_s^-$. w_{A2}^* now increases with β_{A2}^* , in contrast to Case A1 where w_{A1}^* and β_{A1}^* move in opposite directions. When $\beta_0 \geq \bar{c}$ so that $\beta_{A2}^* = \bar{c}$, the solution simplifies to $w_{A2}^* = s + \bar{c} - E[c]$, $L_{A2}^* = \Pi_b^- - (1/4b)(a - b(s + 2\bar{c} - E[c]))^2$, and $E[\Pi_{s,A2}] = \frac{1}{2}(\bar{c} - E[c])(a - b(s + \bar{c}) + (1/4b)(a - b(s + 2\bar{c} - E[c]))^2 - \Pi_b^-)$. In the degenerate case where $\underline{c} = \bar{c}$, Case A2 collapses to F2.

For Case A3, we make the usual assumption that $F(c)$ has a decreasing reverse hazard rate, i.e., that $F(c)/f(c)$ is increasing in c . Without this assumption, tractable results are impossible to obtain; see Ha (2001) for a theoretical treatment of the more general case. Using optimal control with variable endpoint conditions, we find:

PROPOSITION 5. *In Case A3, the supplier's optimal contract satisfies, $\forall \underline{c} \leq c \leq \beta_{A3}^*$: $w_{A3}^*(c) = s + F(c)/f(c)$,*

$dL_{A3}^*/dc = \frac{1}{2}(a - b(w_{A3}^*(c) + c))(dw_{A3}^*(c)/dc)$ and $L_{A3}^*(\beta_{A3}^*) = -\Pi_s^-$, where the optimal cutoff point is given by $\beta_{A3}^* = \beta_0 \perp [\underline{c}, \bar{c}]$, where β_0 solves

$$\beta + \frac{F(\beta)}{f(\beta)} = \frac{a - 2\sqrt{b(\Pi_b^- + \Pi_s^-)}}{b} - s. \quad (10)$$

β_{A3}^* is decreasing in s , Π_b^- , and Π_s^- , and increasing in a ; $w_{A3}^*(c)$ is increasing in c .

The effects of Π_b^- and s on β_{A3}^* confirm Proposition 4(b) in Ha (2001, p. 54).

7. Comparison of the Six Cases

We can now answer our original questions about the value of information and of more general contracts. Although we omitted derivations of the buyer’s profits and the joint profits for brevity, we do include those perspectives in some of the comparisons below.

7.1. Comparing Cutoff Points

PROPOSITION 6. *The cutoff points satisfy the following inequalities: $\beta_{F3}^* = \beta_{F2}^* \geq \beta_{A3}^* \geq \beta_{A2}^*$ and $\beta_{F3}^* = \beta_{F2}^* \geq \beta_{F1}^* \geq \beta_{A1}^*$.*

The proof is in the Appendix. Proposition 6 extends Proposition 4(c) in Ha (2001, p. 54), which states that $\beta_{F2}^* \geq \beta_{A3}^*$, to cover all other cases we consider here. Because β_{F2}^* is the highest, any buyer who should be excluded in Case F2 should also be excluded in all other cases. The numerical experiments show that we cannot rank the cutoff points any further, as β_{F1} and β_{A1} are greater than β_{A2} and β_{A3} in some cases and smaller in others. The practical implication of Proposition 6 is that, without a change in contract type, the supplier should exclude more buyers when he does not know their cost.

7.2. The Value of Information

How much can the supplier gain by learning the buyer’s costs, without changing the type of contract? Introduce the difference operator $\Delta_{mn}\Pi_k := E_c[\Pi_{k,m}(w_m^*, L_m^*)] - E_c[\Pi_{k,n}(w_n^*, L_n^*)]$, where $m, n \in \{F1, F2, A1, A2, A3\}$ are the cases being compared and $k \in \{s, b, j\}$ indicates the supplier’s, the buyer’s, and the joint profits. We can obtain closed-form expressions when $\beta_{A1}^* = \beta_{F1}^* = \bar{c}$; for other cases, we resort to numerical experiments. Most literature to date does not allow cutoff policies. Whenever we assume $\beta^* = \bar{c}$, this is equivalent to the case where no cutoff policy is allowed, so the propositions below are in fact more general than the traditional case without cutoff policy. Recall from Proposition 6 that $\beta_{A1}^* = \bar{c}$ implies $\beta_{F1}^* = \bar{c}$.

PROPOSITION 7. *If $\beta_{A1}^* = \bar{c}$, or when no cutoff policy is followed, then $\Delta_{F1,A1}\Pi_s = (b/8)\text{Var}(c) \geq 0$, $\Delta_{F1,A1}\Pi_b = -(3b/16)\text{Var}(c) \leq 0$, and $\Delta_{F1,A1}\Pi_j = -(b/16)\text{Var}(c) \leq 0$.*

The proof is straightforward. Proposition 7 implies that the value of full information to the supplier increases with the uncertainty about the buyer’s cost, i.e., with $\text{Var}(c)$, and with the price sensitivity of demand, i.e., with b . Not surprisingly, the buyer’s profits decrease if the supplier gets full information. More surprisingly, joint profits also decrease. Under the rigidity of a one-part contract, full information does not facilitate a more efficient outcome, but only helps the supplier extract more profits from the buyer.

Next, recall that $\beta_{A2}^* = \bar{c}$ implies that $\beta_{F2}^* = \bar{c}$. We can state, again without proof:

PROPOSITION 8. *If $\beta_{A2}^* = \bar{c}$, or when no cutoff policy is followed, then $\Delta_{F2,A2}\Pi_s = (b/4)\text{Var}(c) + \frac{1}{2}(\bar{c} - E[c]) \cdot (a - b(s + \bar{c})) \geq 0$, $\Delta_{F2,A2}\Pi_b = -(b/4)\text{Var}(c) - \frac{1}{2}(\bar{c} - E[c])(a - b(s + \bar{c})) + (b/4)(\bar{c} - E[c])^2 \leq 0$, and $\Delta_{F2,A2}\Pi_j = (b/4)(\bar{c} - E[c])^2 \geq 0$.*

The value of information still depends on price sensitivity b and on $\text{Var}(c)$, but now also on the worst-case deviation from the expected value $E[c]$. This is because the supplier now offers a side payment to make IRb binding at \bar{c} ; he cannot do so in Cases F1 and A1. We already knew that the buyer’s profits decrease going from A2 to F2. However, joint profits now increase if the supplier has full information, and this effect increases with price sensitivity b and with the gap between \bar{c} and $E[c]$. Combining Propositions 7 and 8, we get:

PROPOSITION 9. *If $\beta_{A1}^* = \beta_{A2}^* = \bar{c}$, or when no cutoff policy is followed, then $\Delta_{F2,A2}\Pi_s \geq \Delta_{F1,A1}\Pi_s$, and $\Delta_{F2,A2}\Pi_j \geq \Delta_{F1,A1}\Pi_j$.*

PROOF. The statement follows directly from $\bar{c} \geq E[c]$ and $a - b(s + \bar{c}) \geq 0$, which is true by assumption (to ensure nonnegative quantities). □

When no buyers are excluded, the value of information to the supplier (and to the system) is greater with two-part contracts. Allowing $\beta < \bar{c}$, the expressions are no longer tractable, but our numerical findings are consistent with Proposition 9.

7.3. The Value of More General Contracts

How much can the supplier gain from offering more general contracts? As above, we focus part of our analysis on the special case with $\beta_{A1}^* = \beta_{F1}^* = \bar{c}$.

PROPOSITION 10. $\Delta_{F2,F1}\Pi_s \geq -\Pi_b^- + (a - b(s + c))^2 / (8b) \geq 0$, where the first inequality is tight when $\beta_{F1}^* = \beta_{F2}^* = \bar{c}$ or when no cutoff policy is followed. In those cases, $\Delta_{F2,F1}\Pi_b = \Pi_b^- - (a - b(s + c))^2 / 16b \leq 0$ and $\Delta_{F2,F1}\Pi_j = (a - b(s + c))^2 / 16b \geq 0$.

The value of offering two-part instead of one-part contracts under full information decreases with Π_b^- and with price sensitivity b . A partial explanation can be found by taking b very small; i.e., demand is insensitive to retail price p . In this case, w has almost

no effect on price or quantity, so the supplier must rely on the side payment L to achieve a higher profit. Therefore, two-part contracts are more valuable when b is very small.

PROPOSITION 11. *If $\beta_{A1}^* = \beta_{A2}^* = \bar{c}$, or when no cutoff policy is followed, then*

$$\begin{aligned} \Delta_{A2, A1} \Pi_s &= -\Pi_b^- + (1/8b)(a - b(s + 2\bar{c} - E[c]))^2 \geq 0, \\ \Delta_{A2, A1} \Pi_b &= \Pi_b^- - (1/16b)(a - b(s + 2\bar{c} - E[c]))^2 \\ &\quad + (1/4)(\bar{c} - E[c])(a - b(s + 2\bar{c} - E[c])), \end{aligned}$$

and

$$\begin{aligned} \Delta_{A2, A1} \Pi_j &= (1/16b)(a - b(s + 2\bar{c} - E[c]))^2 \\ &\quad + (1/4)(\bar{c} - E[c])(a - b(s + 2\bar{c} - E[c])) \geq 0. \end{aligned}$$

The value of more general contracts under asymmetric information decreases with Π_b^- and with price sensitivity b . The next result follows immediately:

PROPOSITION 12. *If $\beta_{F1}^* = \beta_{F2}^* = \beta_{A1}^* = \beta_{A2}^* = \bar{c}$, or if no cutoff policy is followed, then $\Delta_{F2, F1} \Pi_s \geq \Delta_{A2, A1} \Pi_s$, and $\Delta_{F2, F1} \Pi_j \geq \Delta_{A2, A1} \Pi_j$.*

This implies that the value to the supplier (and to the system) of more general contracts is greater under full information than under asymmetric information, when all optimal cutoff points are equal to \bar{c} . The numerical experiments below for the general case are again consistent with this. Together, Propositions 9 and 12 give us the key insight that information and two-part contracts are strategic complements. The practical significance of this is immediate: Although obtaining better information and offering two-part

contracts are both ways for the supplier to increase his profits, they are not substitutes. The incremental value of either approach increases after achieving the other. One might still ask where to start: get better information or offer two-part contracts? This means comparing $\Delta_{F1, A1} \Pi_s$ with $\Delta_{A2, A1} \Pi_s$. Using the expressions in (and under the conditions of) Propositions 7 and 11, we see that $\Delta_{F1, A1} \Pi_s > \Delta_{A2, A1} \Pi_s$ is more likely to hold as $\text{Var}(c)$ increases: The greater the uncertainty about c , the greater the value of information relative to the value of more general contracts.

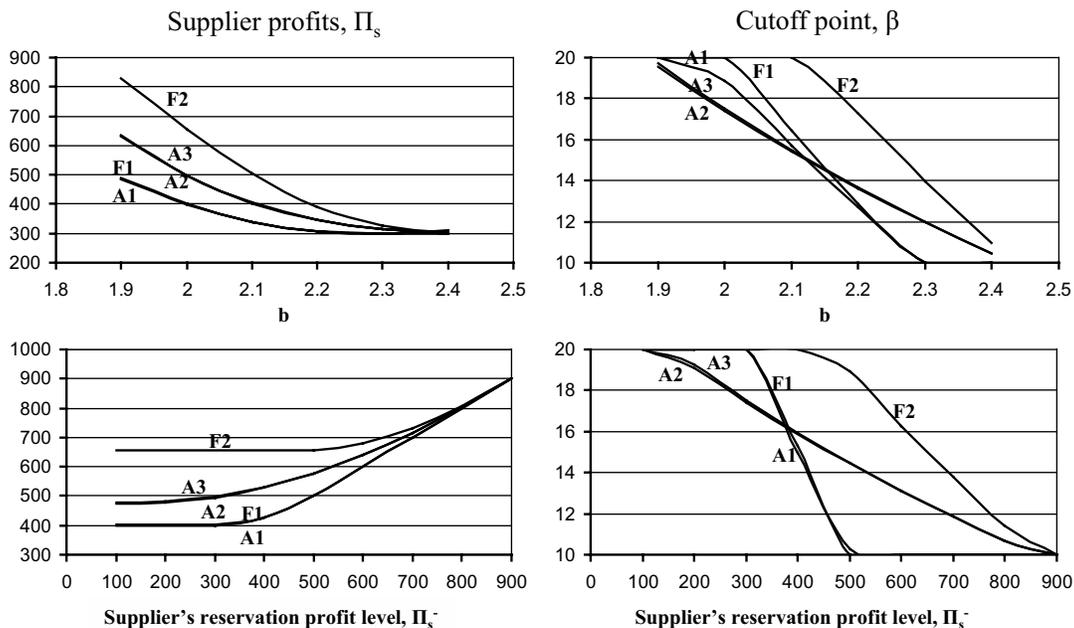
8. Numerical Experiments

We used numerical experiments to illustrate the preceding propositions without the no-exclusion assumption under various prior distributions. We set $q = a - bp = 200 - 2p$, $s = 45$, $\Pi_b^- = 150$, and $\Pi_s^- = 300$. Figure 1 shows how profits and cutoff points change with b and Π_s^- when c is uniformly distributed on $[\underline{c}, \bar{c}] = [10, 20]$. The charts for the cutoff points verify that we cannot rank them further than in Proposition 6. We also experimented with quadratic demand $q = (a - bp)^2$; the results on value of information and of more general contracts are highly similar.

8.1. Sensitivity Analysis with Respect to Prior Distribution

We performed experiments with uniform, normal, beta, and exponential prior distributions. With the normal, we can examine the effect of variance with a mean-preserving spread. With the beta, we can change the mean without changing the variance. The exponential allows us to examine the effect of simultaneous changes of mean and variance.

Figure 1 Sensitivity Analysis of Expected Profits and Cutoff Points



We truncate all distributions to $[\underline{c}, \bar{c}]$ to ensure nonnegative demand; i.e., $a - b(s + \bar{c}) \geq 0$. For the normal distributions, we define the truncated density as $f_T(c) = f(c)/(F(\bar{c}) - F(\underline{c}))$, where f and F are the original normal density and distribution. Beta distributions can have $f(0) = f(1) = 0$. To prevent division by zero in $F(c)/f(c)$, we transform the domain of the beta prior from $[0, 1]$ to $[\underline{c} - 2, \bar{c} + 2] = [8, 22]$ and truncate the resulting distribution to $[\underline{c}, \bar{c}]$ as for the normal. Similarly, we shift the exponential distribution from $[0, \infty)$ to $[\underline{c}, \infty)$ and then truncate

the tail to $[\underline{c}, \bar{c}]$. As the variance of a normal or exponential distribution goes to ∞ , that distribution truncated to $[\underline{c}, \bar{c}]$ converges to the uniform, so the variance of our truncated distributions cannot exceed the variance of $U[\underline{c}, \bar{c}]$, which is 8.33. N1, N2, and N3 are normal distributions with increasing variance, U the limiting uniform distribution, B1 and B2 beta distributions with increasing mean, and E1 and E2 exponential distributions with increasing mean and variance. The exact parameters are shown in Table 1.

Table 1 Effect of Mean and Variance on Supplier Profits

Effect of variance on supplier profits, using mean-preserving spread under truncated normal prior distribution, with limiting uniform distribution												
Case:	N ₁			N ₂			N ₃			U		
Before	$\mu = 15, \sigma = 2$			$\mu = 15, \sigma = 3.23$			$\mu = 15, \sigma = 9$			[10, 20]		
After	Mean = 15, Variance = 3.65			Mean = 15, Variance = 6.01			Mean = 15, Variance = 7.90			Mean = 15, Variance = 8.33		
	$E\Pi_s$	β	R									
F1	400.91	20.00	0.00	401.50	20.00	0.00	402.00	20.00	0.00	402.08	20.00	0.00
F2	651.82	20.00	0.00	653.00	20.00	0.00	654.00	20.00	0.00	654.17	20.00	0.00
A1	398.61	18.42	0.04	397.66	18.67	0.08	396.98	18.83	0.11	396.87	18.86	0.11
A2	532.92	17.12	0.14	512.02	17.33	0.20	497.45	17.39	0.25	495.16	17.39	0.26
A3	534.03	17.19	0.13	513.29	17.41	0.19	499.08	17.50	0.24	496.88	17.50	0.25

Effect of mean on supplier profits, keeping variance constant, under truncated beta distributions						
Case:	B1			B2		
Before	$\alpha = 2, \beta = 3$			$\alpha = 3, \beta = 2$		
After	Mean = 14.00, Variance = 5.98			Mean = 16.00, Variance = 5.98		
	$E\Pi_s$	β	R	$E\Pi_s$	β	R
F1	421.65	20.00	0.00	381.84	20.00	0.00
F2	693.30	20.00	0.00	613.67	20.00	0.00
A1	417.75	18.12	0.06	378.14	19.27	0.08
A2	543.47	16.25	0.20	481.23	18.62	0.17
A3	545.32	16.36	0.19	482.14	18.67	0.16

Effect of mean and variance on supplier profits, under truncated exponential distribution, with limiting uniform distribution									
Case:	E1			E2			U		
Before	$\theta = 3.75$			$\theta = 8.20$			$\theta \rightarrow \infty$		
After	Mean = 13.00, Variance = 6.04			Mean = 14.01, Variance = 7.75			Mean = 15.00, Variance = 8.33		
	$E\Pi_s$	β	R	$E\Pi_s$	β	R	$E\Pi_s$	β	R
F1	442.44	20.00	0.00	422.02	20.00	0.00	402.08	20.00	0.00
F2	734.88	20.00	0.00	694.04	20.00	0.00	654.17	20.00	0.00
A1	438.34	17.50	0.07	416.94	18.21	0.10	396.87	18.86	0.11
A2	581.06	14.74	0.23	532.87	15.90	0.27	495.16	17.39	0.26
A3	583.57	14.90	0.22	535.07	16.05	0.26	496.88	17.50	0.25

Note. All distributions are truncated to $[\underline{c}, \bar{c}] = [10, 20]$, as described in the text, so the exponential distribution tends to the uniform distribution as $\theta \rightarrow \infty$. R is the probability that supplier and customer will not trade; i.e., $R = 1 - F(\beta)$. "Before" refers to the parameters used to generate the distribution before truncation; "after" refers to the mean and variance of the resulting distribution after truncation.

Table 2 Value of Information

	N1	N2	N3	B1	B2	E1	E2	U
$E\Pi_{s,F1} - E\Pi_{s,A1}$	2.31	3.84	5.02	3.89	3.70	4.10	5.08	5.21
$(E\Pi_{s,F1} - E\Pi_{s,A1})/E\Pi_{s,F1}$ (%)	0.58	0.96	1.25	0.92	0.97	0.93	1.20	1.30
$E\Pi_{s,F2} - E\Pi_{s,A2}$	118.91	140.98	156.55	149.83	132.44	153.82	161.17	159.01
$(E\Pi_{s,F2} - E\Pi_{s,A2})/E\Pi_{s,F2}$ (%)	18.24	21.59	23.94	21.61	21.58	20.93	23.22	24.31
$E\Pi_{s,F3} - E\Pi_{s,A3}$	117.80	139.71	154.92	147.97	131.53	151.32	158.97	157.29
$(E\Pi_{s,F3} - E\Pi_{s,A3})/E\Pi_{s,F3}$ (%)	18.07	21.39	23.69	21.34	21.43	20.59	22.90	24.04

In Table 1, increasing variance significantly reduces $E[\Pi_{s,A2}]$ and $E[\Pi_{s,A3}]$, but has little effect on $E[\Pi_{s,A1}]$. Cutoff points increase with variance, but the proportion of buyers excluded increases too. Increasing the mean without changing the variance also reduces profits; cutoff points increase, but now the proportion of buyers excluded goes both ways. Increasing both mean and variance substantially reduces profits.

8.2. Information, General Contracts, and Cutoff Policy

Table 2 confirms Proposition 9: The value of information is greater under two-part than under one-part linear contracts. Table 3 confirms Proposition 12: The value of offering two-part rather than one-part linear contracts is greater under full information. Many suppliers, in practice, find themselves in Case A1, using the simplest contract and with incomplete information about the buyer’s cost. Should the supplier offer more general contracts or seek better information? Tables 2 and 3 show that $\Delta_{A2,A1}\Pi_s$ is far greater than $\Delta_{F1,A1}\Pi_s$; i.e., in our sample, offering a two-part contract under asymmetric information is better than a wholesale price scheme under full information. Two-part contracts and better information are still complements in our numerical experiments.

Table 4 shows that the value to the supplier of using cutoff policies is greatest in Case A2, which is consistent with the fact that β_{A2}^* is always lower than β_{A3}^* . As the mean and/or the variance increases, the value of the cutoff policy in Case A2 decreases, because the optimal cutoff point increases with the mean and the variance in our sample. Requiring $\beta = \bar{c}$ reduces profits by between 2.23% and 16.19% in Case A2. In Case A3, the restriction to $\beta = \bar{c}$ reduces

profits by between 1.93% and 9.11%. In Case A1, the cost of requiring $\beta = \bar{c}$ is surprisingly small, less than 1.92% in our sample. In Table 1, the proportion of buyers excluded varies widely, but sometimes reached 25%. This suggests that ignoring the supplier’s ability to exclude unprofitable buyers is a rather severe restriction, adding further justification to the cutoff policies studied here. In our experiments with quadratic demand functions the proportion often reached 45%, suggesting that convex decreasing demand aggravates the unattractiveness of high-cost buyers.

8.3. Comparing Cutoff Points and Contracts

The data are consistent with our results in Proposition 6: $\beta_{F2}^* \geq \beta_{A3}^* \geq \beta_{A2}^*$, and $\beta_{F2}^* \geq \beta_{F1}^* \geq \beta_{A1}^*$. Figure 1 suggests that β_{A3} and β_{A2} are less sensitive than β_{A1} to changes in parameters such as b and Π_s^- . Profits in Cases A2 and A3 are very close. In our experiments, $\Delta_{A3,A2}\Pi_s$ was always below 0.40%. Our experiments with quadratic demand yielded comparably small differences. With nonlinear stochastic demand, Ha (2001) finds that the supplier’s profits in Case A2 are between 0.9% and 14.7% lower than in Case A3. Together, this suggests that for nonlinear contracts to be valuable, one must have stochastic or nonlinear demand, and that it may be the stochasticity rather than the nonlinearity of demand that makes nonlinear contracts valuable, though this needs to be studied further.

9. Summary and Conclusions

We have examined the value to a supplier of offering more general contracts and the value of obtaining better information about the buyer’s cost structure.

Table 3 Value of More General Contracts

	N1	N2	N3	B1	B2	E1	E2	U
$E\Pi_{s,F2} - E\Pi_{s,F1}$	250.91	251.50	252.00	271.65	231.84	292.44	272.02	252.08
$(E\Pi_{s,F2} - E\Pi_{s,F1})/E\Pi_{s,F2}$ (%)	38.49	38.51	38.53	39.18	37.78	39.79	39.19	38.54
$E\Pi_{s,A2} - E\Pi_{s,A1}$	134.31	114.36	100.47	125.72	103.09	142.72	115.93	98.29
$(E\Pi_{s,A2} - E\Pi_{s,A1})/E\Pi_{s,A2}$ (%)	25.20	22.34	20.20	23.13	21.42	24.56	21.76	19.85
$E\Pi_{s,A3} - E\Pi_{s,A2}$	1.11	1.27	1.63	1.85	0.91	2.51	2.20	1.72
$(E\Pi_{s,A3} - E\Pi_{s,A2})/E\Pi_{s,A3}$ (%)	0.21	0.25	0.33	0.34	0.19	0.43	0.41	0.35

Table 4 Value of Cutoff Policy

	N1	N2	N3	B1	B2	E1	E2	U
$E\Pi_{s,A1}(\beta_{A1}^*) - E\Pi_{s,A1}(\bar{c})$	3.99	3.05	2.36	5.54	1.12	8.43	4.79	2.25
$(E\Pi_{s,A1}(\beta_{A1}^*) - E\Pi_{s,A1}(\bar{c}))/E\Pi_{s,A1}(\beta_{A1}^*)$ (%)	1.00	0.77	0.60	1.33	0.30	1.92	1.15	0.57
$E\Pi_{s,A2}(\beta_{A2}^*) - E\Pi_{s,A2}(\bar{c})$	57.92	37.02	22.45	63.00	10.71	94.08	52.42	20.16
$(E\Pi_{s,A2}(\beta_{A2}^*) - E\Pi_{s,A2}(\bar{c}))/E\Pi_{s,A2}(\beta_{A2}^*)$ (%)	10.87	7.23	4.51	11.59	2.23	16.19	9.84	4.07
$E\Pi_{s,A3}(\beta_{A3}^*) - E\Pi_{s,A3}(\bar{c})$	19.41	28.70	19.57	37.60	9.30	53.16	41.51	17.71
$(E\Pi_{s,A3}(\beta_{A3}^*) - E\Pi_{s,A3}(\bar{c}))/E\Pi_{s,A3}(\beta_{A3}^*)$ (%)	3.63	5.59	3.92	6.89	1.93	9.11	7.76	3.56

We have explicitly included a reservation profit level for the supplier, often ignored in the contracting literature. Several interesting findings emerge. First, the value to the supplier of information about the buyer’s retail costs can be linked to the variance of the prior distribution. Second, information and two-part contracts are strategic complements: The value of information is greater under two-part contracts than under one-part contracts, and the value of being able to offer two-part contracts rather than one-part contracts is greater under full information than under asymmetric information. Some of these results are proven analytically; others can only be proven for the traditional case without cutoff policy, but are numerically supported. Finally, the proportion of buyers excluded in the optimal contract varied widely, but was often 25% or more, indicating that forcing the supplier to deal with all buyer types is a strong assumption. We were unable to characterize precisely when the supplier will choose not to exclude any buyers, but view this as an interesting question for further study.

The contributions of this paper are to critically examine the role of reservation profit levels for both parties, of asymmetric information, and of contract type, and to give insight into how different contracts behave under various prior distributions. It would be interesting to examine these same phenomena in a competitive environment or in a multiperiod context. Most of the optimal contracts are still similar in structure when using a nonlinear demand function of the form $q = (a - bp)^n$. However, the supplier’s expected profits in Case A2 become untractable, which makes analytical comparisons between cases impossible too. Information asymmetry about demand function parameters can be handled analogously, but again leads to problems with the supplier’s expected profits. A full analysis of nonlinear demand and of information asymmetry about demand parameters is left for future work, as are other mechanisms such as the quantity fix contract included in Ha (2001).

Appendix

PROOF OF PROPOSITION 6. Comparing the expressions in Propositions 1 and 2 yields $\beta_{F2} \geq \beta_{F1}$. Next, we show

$\beta_{A3} \geq \beta_{A2}$. The equations defining β_{A2} and β_{A3} are, respectively,

$$\frac{1}{2}(\beta_{A2} - E[c | c \leq \beta_{A2}])(a - b(s + 2\beta_{A2} - E[c | c \leq \beta_{A2}])) + \frac{1}{4b}(a - b(s + 2\beta_{A2} - E[c | c \leq \beta_{A2}]))^2 - \frac{1}{2} \frac{F(\beta_{A2})}{f(\beta_{A2})} \cdot (a - b(s + 2\beta_{A2} - E[c | c \leq \beta_{A2}])) = \Pi_s^- + \Pi_b^-$$

and

$$\frac{1}{4b} \left(a - b \left(s + \frac{F(\beta_{A3})}{f(\beta_{A3})} + \beta_{A3} \right) \right)^2 = \Pi_s^- + \Pi_b^-.$$

Setting the two expressions equal, we get

$$\left(a - b \left(s + \beta_{A2} + \frac{F(\beta_{A2})}{f(\beta_{A2})} \right) \right)^2 - b^2 \left(\beta_{A2} - E[c | c \leq \beta_{A2}] - \frac{F(\beta_{A2})}{f(\beta_{A2})} \right)^2 = \left(a - b \left(s + \frac{F(\beta_{A3})}{f(\beta_{A3})} + \beta_{A3} \right) \right)^2,$$

which implies

$$\left(a - b \left(s + \beta_{A2} + \frac{F(\beta_{A2})}{f(\beta_{A2})} \right) \right)^2 \geq \left(a - b \left(s + \frac{F(\beta_{A3})}{f(\beta_{A3})} + \beta_{A3} \right) \right)^2$$

or $\beta_{A2} + \frac{F(\beta_{A2})}{f(\beta_{A2})} \leq \beta_{A3} + \frac{F(\beta_{A3})}{f(\beta_{A3})}$.

Because we assumed that $F(c)/f(c)$ is increasing in c , this implies that $\beta_{A3} \geq \beta_{A2}$, as desired.

The statement that $\beta_{F2} \geq \beta_{A3}$ is trivially true from the expressions for β_{F2} and β_{A3} and the fact that $F(c)/f(c) \geq 0$. To show that $\beta_{F1} \geq \beta_{A1}$, we derive a contradiction. Assume $\beta_{F1} < \beta_{A1}$. Then, for $\beta_{F1} < c < \beta_{A1}$, the supplier’s profit $\Pi_{s,A1}(c)$ in Case A1 is greater than Π_s^- . By contrast, in Case F1, $\Pi_{s,F1}(c) = \Pi_s^-$, so that the supplier is worse off under complete information than under incomplete information, which is impossible, because in Case F1, the supplier can always at least replicate the optimal contract from Case A1. □

References

Armstrong, M. 1999. Optimal regulation with unknown demand and cost functions. *J. Econom. Theory* 84 195–215.
 Baron, D. B., R. B. Myerson. 1982. Regulating a monopolist with unknown costs. *Econometrica* 50(4) 911–930.
 Bresnahan, T. F., P. C. Reiss. 1985. Dealer and manufacturer margins. *RAND J. Econom.* 16(2) 253–268.

- Chen, F., A. Federgruen, Y.-S. Zheng. 2001. Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management Sci.* **47**(5) 693–708.
- Corbett, C. J., C. S. Tang. 1999. Designing supply contracts: Contract type and information asymmetry. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative Models for Supply Chain Management*. Kluwer, Norwell, PA.
- Dobbs, I. A., M. B. Hill. 1993. Pricing solutions to the bilateral monopoly problem under uncertainty. *Southern Econom. J.* **60**(2) 479–489.
- Gal-Or, E. 1991a. Vertical restraints with incomplete information. *J. Indust. Econom.* **39** 503–516.
- Gal-Or, E. 1991b. Duopolistic vertical restraints. *Eur. Econom. Rev.* **35** 1237–1253.
- Ha, A. Y. 2001. Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation. *Naval Res. Logist.* **48** 41–64.
- Irmen, A. 1997. Mark-up pricing and bilateral monopoly. *Econom. Lett.* **54** 179–184.
- Jullien, B. 2000. Participation constraints in adverse selection models. *J. Econom. Theory* **93** 1–47.
- Lewis, T. R., D. B. M. Sappington. 1988a. Regulating a monopolist with unknown demand. *Amer. Econom. Rev.* **78**(5) 986–998.
- Lewis, T. R., D. B. M. Sappington. 1988b. Regulating a monopolist with unknown demand and cost functions. *RAND J. Econom.* **19**(3) 438–457.
- Maggi, G., A. Rodríguez-Clare. 1995. On countervailing incentives. *J. Econom. Theory* **66** 238–263.
- Moorthy, K. S. 1984. Market segmentation, self-selection, and product life design. *Marketing Sci.* **3**(4) 288–307.
- Rochet, J.-C. 1985. Bilateral monopoly with imperfect information. *J. Econom. Theory* **36** 214–236.
- Rochet, J.-C., L. A. Stole. 2002. Nonlinear pricing with random participation. *Rev. Econom. Stud.* **69** 277–311.
- Tirole, J. 1988. *The Theory of Industrial Organization*. The MIT Press, Cambridge, MA.
- Tsay, A. A., S. Nahmias, N. Agrawal. 1999. Modeling supply chain contracts: A review. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Norwell, PA.
- Weng, Z. K. 1995. Channel coordination and quantity discounts. *Management Sci.* **41** 1509–1522.