Distribution Planning to Optimize Profits in the Motion Picture Industry

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Abstract

We consider the distribution planning problem for the motion picture industry. In this problem, the distributor chooses the location of theaters where the movie will be screened, while the exhibitor chooses the duration of play at the particular theater. We model the distributor's location selection problem and the exhibitor's duration selection problem as integer programming based optimization models. A critical parameter for these models is the box office revenue forecast for individual theaters. To estimate this parameter, we develop a procedure to calculate the box office revenues at the theater-level as a function of movie attributes and theater characteristics. We tested our methods on realistic box office data and show that it has the potential to improve average distributor profits by 5.5%, or around $2.2 million per movie. We also develop some insights into why our methods outperform existing practice, which are crucial to their successful practical implementation.

1. INTRODUCTION

The motion picture industry is an important sector of the U.S. economy. Movie releases at theaters generated approximately $9.5 billion in revenue in 2004, representing an 80% increase since the beginning of the previous decade. In spite of this significant increase, the Motion Picture Association of America (MPAA) reports that only 19 of the 483 movies released in 2004 generated profit higher than $50 million (http://www.mpaa.org). The average cost of a studio-released movie was nearly $98 million in 2003, up from $50.3 million in 1994. The major component of this cost is the development and production budget, which averaged around $63.6 million in 2004, followed by marketing ($34.4 million), and distribution costs. Depending on the distribution strategy chosen by the movie distributor (e.g. platforming, wide, or saturation release), distribution costs can be up to $9 million averaging around $3.74 million per movie in 2004. While distribution represents a smaller portion of a movie's total investment in comparison to development, production, and marketing costs, effective distribution is critical to box office success and ultimate financial return from a movie (Reardon, 1992; Thomas, 1998).

Distribution of a motion picture is handled by its distributor, who forms an important link in the motion picture industry supply chain (Figure 1). Examples of domestic distributors in the motion picture industry include Buena Vista (Walt Disney's distribution arm), Columbia (Sony's distribution arm), DreamWorks, and Warner Brothers. The distributor secures rights from the producer, undertakes marketing (including advertising in television, local and national media), and develops a distribution plan that specifies how many and which theaters to screen a new motion picture or movie. This is done in conjunction with exhibitors, who typically own the theaters. Some examples of domestic exhibitors include AMC, Regal, AVCO, General Cinema, and Mann. The exhibitor looks at the portfolio of movies that are going to be released, decides whether to choose the movie or not at a particular theater and then specifies how long a movie can be shown in each theater. In making these decisions, both the distributor and exhibitor rely on box office revenue forecasts for a given movie. For simplicity, we will refer to these activities of the distributor and exhibitor as distribution planning.

INSERT FIGURE 1 ABOUT HERE

Distribution planning in the motion picture industry is made under a complex environment. On the supplier side, there are multiple major and smaller independent distributors releasing more than 400 movies in a year. On the retailer side, there are major exhibition chains, smaller chains, and independent theaters at more than 7000 locations in the United States and Canada. Forecasting box offices revenues for a given duration is challenging as movies are experiential products (Hirschmann and Holbrook, 1982) and, consequently, it is difficult to forecast their audience appeal until they are already available in the theaters. This is because audience appeal varies widely depending on movie attributes such as genre, star presence, special effects, MPAA ratings, critical reviews, etc. Even veterans in the industry have not been very successful in forecasting box office revenues (Table 1). While there has been some academic research on aggregate box office forecasting, disaggregate theater-level forecasting is even harder due to dissimilarity in location specific characteristics of the theater such as amenities and demographics (i.e., median income, age, population density, etc.). This causes significant differences in revenues for the same movie across different markets (Table 2). All these factors conspire to make the distribution planning in the motion picture industry an extremely challenging problem.

INSERT TABLE 1 AND TABLE 2 ABOUT HERE

Despite the practical relevance and complexity of this problem, we have found nothing in the academic or managerial literature that describes how to undertake effective distribution planning to optimize profits in the motion picture industry. This paper presents a methodology for resolving this issue. Specifically, we model the distributor's location selection problem and the exhibitor's duration selection problem as integer programming based optimization models. A key parameter required to operationalize our models is the theater-level box office revenue forecast of the movie. To estimate this parameter, we developed an empirical technique that provides box office revenue estimate over time for a new motion picture at a selected theater. We test our methods on actual industry data and show that our approach offers the potential to significantly improve box office profits for new movies.

This paper is organized as follows. In Section 2, we review the literature relating to the development of our methodology. In Section 3, we formulate the distributor's location selection problem. We develop heuristics to solve this problem and also construct upper bounds to evaluate the quality of these heuristics. In Section 4, we construct the exhibitor's duration selection problem. In Section 5, we develop an empirical method to estimate the theater-level box office revenue for a given movie. We present computational results from this method and from the distributors and exhibitors problems in Section 6. We use these results to draw some insight into how distribution planning could be improved in the motion picture industry. In Section 7, we test our methods on realistic box office data and show that it has the potential to significantly improve current industry practice. In the concluding section, we summarize our work and present future research directions.

2. LITERATURE REVIEW

There is extensive literature on motion pictures in the popular press and in the telecommunications, film, and television areas (Bart, 2000; Vogel, 2001; Hayes and Bing, 2004). However much of this is descriptive in nature and relies heavily on anecdotal industry knowledge. There is a limited, but emerging, stream of academic research in the motion picture industry. Areas include product diffusion (Neelamegham and Chintagunta, 1999; Elberse and Eliashberg, 2003), seasonal release patterns (Krider and Weinberg, 1998 and Einav, 2002), ancillary markets (Lehmann and Weinberg, 2000) and contract design and competition (de Vany and Walls, 1996).

To the best of our knowledge, we have not encountered any previous work on the distributor's location selection problem. There are though, a set of papers that discuss problems related to motion picture exhibition. Davis (2000) examines the spatial competition in retail markets with special attention to movie theaters. Chisholm and Norman (2002) analyze the relationship between spatial competition and demand, and evaluate their models using data from the motion picture exhibition industry. Swami et al. (1999) study screen management at the local theater-level using approaches from the machine scheduling literature. An additional paper (Swami et al., 2001) examines this problem using a Markov Decision Process model. However, these papers do not explicitly consider the impact of time varying shares in profits offered by the distributor. In addition, they do not estimate weekly sales at the theater-level and, thus, do not directly consider the impact of movie attributes and theater characteristics.

There has been a significant stream of research on *aggregate* box office forecasting of new motion pictures. Many previous studies attempt to explain aggregate box office success as a function of movie attributes, such as budget, star power, MPAA rating, release timing and

Academy Award nominations and winners (Litman and Ahn, 1998, Wyatt, 1994,). Recent work focuses on whether the presence of a major distributor influences box office revenues (Sochay, 1994), the role of advertising and critics on box office success (Zufryden, 1996, Eliashberg and Shugan, 1997 and Zuckerman and Kim, 2003). Sawhney and Eliashberg (1996) develop a parsimonious aggregate forecasting model and test it on realistic data. However, none of this work considers *disaggregated* theater-level box office revenue forecasts for a new movie, which is a crucial input for both the distributors and exhibitors problem.

This paper makes the following contributions. First, we develop a method to calculate detailed disaggregated theater-level box office sales forecasts, based on both movie attributes and theater characteristics. Second, unlike the work discussed above, we directly consider the distributor's location selection problem. Correct selection of theater location is essential to ultimate box office success of a movie and we develop an optimization model to make this choice. Third, we develop the exhibitor's duration selection problem that optimally determines how long a given movie will be shown at any theater, by explicitly considering movie attributes and theater characteristics through the theater-level box office forecasts. This model also allows for the share in profits that are offered by the distributor to vary with time in a manner required by contractual specifications. Fourth, we test this model extensively on realistic data and show that it has the potential to significantly improve existing industry practice.

3. THE DISTRIBUTOR'S LOCATION SELECTION PROBLEM

Consider a distributor who has to choose which theaters to show a new movie at to maximize profits. To provide a precise statement of this problem, we consider *n* possible theaters and let *j,*

$j' \in N = (1, \ldots, n)$ index the set of theaters. These movie theaters are located in r regions indexed by $r \in P = (1, \ldots, p)$.

Define the variables:

$$W_j = \begin{cases} 1 & \text{if theater } j \text{ is chosen to screen a given movie;} \\ 0 & \text{otherwise.} \end{cases}$$

We are given:

$K^{(MAX)}$: maximum number of theaters required across all regions

$K_r^{(MIN)}$: minimum number of theaters required in region r

$$L_{jr} = \begin{cases} 1 & \text{if theater } j \text{ is located in region } r; \\ 0 & \text{otherwise.} \end{cases}$$

$$b_{jj'} = \begin{cases} 1 & \text{if theater } j \text{ competes with theater } j'; \\ 0 & \text{otherwise.} \end{cases}$$

Let π_{ijt} define the box office revenue forecast when movie i is shown at theater j in week t, where $i \in M = (1, \ldots, m)$ indexes the set of movies and $t \in Q = (1, \ldots, q)$ indexes the set of time periods. Let $t_{ij} \in Q$ denote duration chosen by the exhibitor to show movie i at theater j. Then, the distributor's expected box office profit during this period is given by $\theta_{ij}^D = \sum_{t=1}^{t_{ij}} s_{ijt}^D \pi_{ijt} - c_{ij}^D$ where c_{ij}^D is the distribution cost of movie i to theater j, and constant s_{ijt}^D represents the portion of revenues allocated to the distributor for movie i at theater j on week t. This factor depends on the duration chosen by the exhibitor and the nature of the contractual agreement between the exhibitor and distributor. Note that two key parameters required for the computation of θ_{ij}^D are

t_{ij} and π_{ijt}. In Section 5, we develop an optimization model to calculate t_{ij}, while π_{ijt} is calculated by the method described in Section 6.

The Distributor's Location Selection Problem (DLSP) can be represented by the following binary integer program:

$$\text{(DLSP)} \qquad V_i^D = Max\sum_{j=1}^{n}\theta_{ij}^D W_j \qquad (1)$$

$$\text{Such that:} \qquad \sum_{r=1}^{p}\sum_{j=1}^{n} L_{jr}W_j \le K^{(MAX)} \qquad (2)$$

$$\sum_{j=1}^{n} L_{jr}W_j \ge K_r^{(MIN)}, \quad \forall r \qquad (3)$$

$$W_j + b_{jj'}W_{j'} \le 1, \qquad \forall j, j' \neq j \qquad (4)$$

$$W_j \in \{0,1\}, \qquad \forall j \qquad (5)$$

Objective function (1) is chosen to maximize the distributor's total estimated box office profits for movie i by the appropriate choice of theaters. Constraint (2) ensures that the total number of theaters selected to screen a movie does not exceed $K^{(MAX)}$. In practice, this parameter is defined by the type of distribution strategy (i.e. platforming, wide or saturation release) chosen for the particular movie. Constraints (3) guarantee that a set minimum number of theaters are picked for each region. Constraints (4) ensure that the distributor does not pick competing theaters. This is important because distributors often choose multiple exhibitors to show a movie. Consequently, to prevent dilution of sales at the selected theater, exhibitors require that competing theaters within the vicinity are not picked. Finally, 0-1 integrality of the variables is imposed by constraints (5).

Proposition 1: The DLSP is NP-Complete.

Proof: The maximum weighted independent set problem can be derived as a special instance of the DLSP by setting the coefficients $L_{jr}, \forall j, r$ to zero. Since it is known that the weighted independent problem is NP-Complete (Garey and Johnson, 2000), the reduction establishes that DLSP is NP-Complete. ■

In light of Proposition 1, it is unlikely that we could solve large, realistic problems to optimality. In particular, we found in our computational analysis that when the number of theaters is large (over 1800 theaters) and when each theater has many competing theaters (averaging over seven per theater), we could not find solutions using leading commercial software tools such as the XPRESS and CPLEX solvers in GAMS (Brooke et al. 1992). Consequently, we elected to develop heuristics to solve such instances of this problem and also present upper bounds to evaluate the quality of these heuristics.

3.1 Upper Bounds

To develop upper bounds on the DLSP, one could relax one or more of Constraints (2) through (4) by introducing Lagrange multipliers and solve the resulting sub-problem optimally. Then this sub-problem can be optimized over the multipliers to provide a tight upper bound. However, the upper bound from any such relaxation would be no smaller than a simple linear programming relaxation of the DLSP, in which we relax Constraint (5) by allowing $W_j \in [0,1], \ \forall j$. This is because relaxations of the DLSP involving Constraints (2) through (4) have the integrality property (Geoffrion, 1974) as established by the following proposition.

Proposition 2: Relaxations of the DLSP involving Constraints (2) through (4) possess the integrality property.

Proof: We can represent the DLSP in a generalized matrix form as $\underset{x}{Max}\{fx \mid Ax \le b, Cx \le d, x \in X\}$, where A, b, C, d and f are the appropriate matrices, $Ax \le b$ represents the set of constraints we keep, $Cx \le d$ represents the set of constraints we relax and $x \in X$ represents the integrality constraints. Let $Co\{x \, \varepsilon X \mid Cx \le d\}$ represent the convex hull formed by the constraints we relax. Since in the DLSP, $K_r^{(MIN)}, K^{(MAX)} \in \boldsymbol{N}^+, \ \forall r$ and $L_{jr}, b_{jj'} \in \{0,1\}, \ \forall r, j, j'$ note that $Co\{x \in X \mid Cx \le d\} = \{x \mid Cx \le d\}$ for any relaxation involving Constraints (2), (3) and (4). It follows from Geoffrion (1974) that any relaxation of the DLSP involving these constraints has the integrality property. ■

In light of Proposition 2, we generate an upper bound for the DLSP by solving its linear programming relaxation with $W_j \in [0,1], \ \forall j$.

3.2 Heuristics

In general, the solution provided by the upper bounds may not be feasible for the DLSP due to the violation of the integrality constraints (5). To achieve this feasibility, we develop the following heuristics.

1. The Myopic Heuristic

In the myopic heuristic, we select the theaters by first ignoring Constraints (4) and optimally solving the resulting problem. Then, we develop an interchange procedure to ensure that Constraints (4) are satisfied. This heuristic is formalized by the following steps:

Step 1: In each region, select $K_r^{(MIN)}$ theaters in descending order of θ_{ij}^D . This satisfies Constraints (3). Remove the selected theaters from consideration. Sort all of the remaining theaters in descending order of θ_{ij}^D and pick additional $\left(K^{MAX} - \sum_{r=1}^{p} K_r^{(MIN)} \right)$ theaters. Thus, Constraint (2) is binding.

Step 2: We consider the theaters selected by Step 1 and look for those theaters that violate Constraints (4). We first remove the violating theater with the lowest θ_{ij}^D and find a replacement with the highest available θ_{ij}^D . This procedure is repeated until all violating theaters across all regions are eliminated.

2. The Profit/Competition Heuristic.

This heuristic selects the most profitable theaters with the least number of competitors and consists of the following steps:

Step 1: Calculate $\theta_{ij}^D / \sum_{j' \neq j} b_{jj'}$ for each theater.

Step 2: For each region, select the theater with highest ratio, and in the case of a tie select the theater with the lowest number of competing theaters. Once this theater is selected, remove all the competing theaters from consideration. If, at this point, Constraint (3) is satisfied, go to next region. If Constraint (3) is not satisfied, pick the ratio with the next highest value and repeat this procedure. Continue until Constraint (3) is satisfied. If this still does not lead to a feasible solution, restart this procedure with the theater with the next highest ratio and continue until this constraint is satisfied. Repeat this step for every region.

Step 3: Remove all theaters selected in Step 2 from consideration.

Step 4: Consider the theaters that have not been removed in Steps 2 or 3 and choose the remaining $K^{MAX} - \sum_{r=1}^{p} K_r^{(MIN)}$ theaters in decreasing order of $\theta_{ij}^{D} / \sum_{j' \neq j} b_{jj'}$. After each selection, remove the competing theaters associated with the chosen theater.

Note that this heuristic satisfies constraints (3) and (4) in step 2, and constraint (2) in step 4. In Section 6, we test the performance of both these heuristics and the upper bound across a variety of data sets.

4. THE EXHIBITOR'S DURATION SELECTION PROBLEM

While the distributor chooses which theaters to show the movie at, this choice is dependant on the expected profits at a given theater, which, in turn, partially depends on t_{ij} , the duration for which movie *i* is shown in theater *j*. The duration of play is selected by the exhibitor who owns this theater and makes this decision based on optimizing individual theater profits by choosing from a set of movies that could be released during that period.

To determine t_{ij} for the exhibitor, we consider *m* possible movies. These movies are released on week R_i ($R_i \in Q$) of a theatrical season. To select the movies, we define the variables:

$$X_{it} = \begin{cases} 1 \text{ if movie i is selected for week t;} \\ 0 \text{ otherwise.} \end{cases}$$

$$Y_i = \begin{cases} 1 \text{ if movie i is acquired for showing starting on its release week,} \\ 0 \text{ otherwise.} \end{cases}$$

We are given:

F_i : acquisition cost of movie i ($1,000s),

B_t : number of available screens in theater on week t,

$P^{(MIN)}$: minimum number of weeks in play length.

We define the exhibitor's expected box office profit for movie i at theater j in week t as $\theta_{ijt}^{E} = \pi_{ijt}(1 - s_{ijt}^{D}) - c_{ijt}^{E}$, where c_{ijt}^{E} denotes the exhibitor's cost for screening movie i at theater j during week t. The Exhibitor's Duration Selection Problem (EDSP) can be represented by the following binary integer program:

(EDSP)

$$Z_j^E = \quad Max \sum_{t=1}^{q} \sum_{i=1}^{m} \theta_{ijt}^{E} X_{it} - \sum_{i=1}^{m} F_i Y_i \tag{6}$$

Such that:

$$X_{it} \le Y_i, \forall i, t \tag{7}$$

$$\sum_{i=1}^{m} X_{it} \le B_t, \forall t \tag{8}$$

$$P^{(MIN)} Y_i \le \sum_{t=R_i}^{q} X_{it}, \forall i \tag{9}$$

$$X_{i,t+1} \le X_{it}, \quad i \in M' = \{i | R_i \le t\}, \quad \forall t \tag{10}$$

$$X_{it}, Y_i \in \{0,1\}, \quad \forall i, t \tag{11}$$

Objective function (6) is chosen to maximize the exhibitor's expected box office profits at theater j by the appropriate choice of movies and their duration of play. Constraints (7) ensure that a movie is not played unless it is purchased by the theater. Constraint (8) enforces that the number of movies that can be played for a given week does not exceed the number of screens available during that week. Constraints (9) ensure that, if a movie is selected for play, it is kept on the screen for a predetermined period of time represented by the minimum play length described in the distributor-exhibitor agreement. Constraints (10) guarantee that a movie is only played in consecutive weeks. Finally, 0-1 integrality of the variables is enforced by Constraints (11).

Let $\{X_{it}^{*}, Y_{i}^{*} \mid \forall i, t\}$ denote the optimal solution to the EDSP. Then, the distributor sets $t_{ij} = \sum_{t=R_i}^{q} X_{it}^{*}$. We were able to solve large instances of the EDSP to optimality using the XPRESS solver in GAMS. We present insights from these solutions in Section 6.

5. FORECASTING THEATER-LEVEL BOX OFFICE REVENUES

In this section, we develop a method to estimate π_{ijt}, the box office revenue forecast for movie i at theater j in week t, $\forall i, j, t$. Estimating this parameter is critical for several reasons. First, when we tabulate total box office revenue across theaters over time (i.e. $\sum_{j=1}^{n} \pi_{ijt}, \forall t)$, we get the adoption pattern for movie i. This pattern provides crucial guidance for several strategic decisions that the distributor needs to make such as determining the distribution strategy (i.e., platforming, wide, or saturation release). This, in turn, is used in determining the maximum number of theaters across regions (i.e. $K^{(MAX)}$), the minimum play length at any theater (i.e., $P^{(MIN)}$), and, finally, negotiating the revenue sharing contract during and after this minimum play length (i.e., s_{ijt}^{D}). Second, π_{ijt} is a key parameter in the DLSP, which determines which theaters the movie will be screened and, ultimately, its box office success. Third, π_{ijt} is an important parameter for the EDSP, which determines if and how long a movie will be shown at a given theater.

However, estimating π_{ijt} is challenging for several reasons. First, it is difficult to understand which movie attributes and theater characteristics affect theater-level box office revenue and how they do so. Second, it is challenging to estimate how this complex relationship between theater characteristics, movie attributes, and box office revenues changes over time. Finally, this estimation is hard, as forecasting box office revenues requires an understanding of the individual moviegoer's decision process to see a given movie and an abstraction of this process in the estimation method.

Typical industry practice to forecast box office revenue is to compare a new movie to recently released movies that are similar in *one* movie attribute and employ multiple, separate comparisons to study the effect of different movie attributes on box office revenues. While this procedure is simple and provides flexibility to incorporate subjective expertise, it is not very accurate. This is because this approach does not explicitly consider any of the above discussed aspects that make estimating π_{ijt} an immensely challenging problem. There are box office estimation models in the academic literature that incorporate some of these aspects. Most of these models run multiple regressions directly on box office revenue as a function of certain sets of movie attributes (Litman and Ahn, 1998). A major limitation of this method is that it only provides point estimates for box office revenues by assuming an unrestricted horizon for exhibiting the movie. In addition, this approach does not consider significant variations in box office revenues across time periods and differences in adoption patterns across movies. Sawhney and Eliashberg (1996) developed a parsimonious model to forecast a movie's box office success as a function of time. They employ an innovative method that incorporates an individual moviegoers decision process to adopt (or see) a given movie and also consider the impact of different movie adoption patterns. However, the objective of this model is to estimate box office revenue at the *national* or aggregate level, and this approach cannot be used to provide *local* or disaggregate location specific theater-level estimates.

To overcome the described challenges inherent in estimating π_{ijt}, we develop a four-step method. These steps are outlined in Figure 2. In Step 1, we extend the Sawhney and Eliashberg (1996) model to include location specific theater-level characteristics. We estimate the parameters of this model using nonlinear regression on a historical database of realized box office revenues for selected movies. In Step 2, we use multiple regression models to link the estimated model parameters to movie attributes and theater characteristics corresponding to this historical database and also consider any potential interactions between these variables. This step provides us with a function that estimates model parameters given a set of movie attributes and theater specific characteristics. In Step 3, we use this function to estimate the model parameters for a new movie given its attributes and the location specific characteristics for the theater under consideration. In Step 4, we use these estimates of the model parameters in the model of Step 1 to estimate the box office revenue for the new movie at a given theater across time. Below, we describe each of these steps in detail.

INSERT FIGURE 2 ABOUT HERE

Step 1: Incorporation of Theater-level Characteristics

In this step, we extend the Sawhney-Eliashberg (1996) model to include location specific theater-level characteristics. This forecasting model assumes that an individual movie patron's adoption process to watch a movie depends on two independent sub-processes: the decision to see a movie in theater j, followed by the decision to visit theater j. These processes are modeled as stochastic processes with stationary parameters λ_j representing the time-to-decide parameter and γ_j representing the time-to-act parameter. Then, the expected time to decide becomes $(1/\lambda_j)$ and the expected time to act becomes $(1/\gamma_j)$. Although it is plausible that the time-to-decide process is mainly influenced by movie attributes, we believe that the availability of the chosen movie in a theater that is acceptable to the patron affects the time-to-decide parameter. Once the individual has decided to watch a movie, the next decision is where to watch it, which is again influenced by theater characteristics.

Following the approach of Sawhney and Eliashberg (1996), the expected cumulative number of adopters at theater j by time τ then can be expressed as:

$$E[N_j(\tau)] = N_j P_j(\tau) = \frac{N_j}{\lambda_j - \gamma_j}\left[\left(\lambda_j - \gamma_j\right) + \gamma_j e^{-\lambda_j \tau} - \lambda_j e^{-\gamma_j \tau}\right] \tag{16}$$

where $N_j(\tau)$ is the distribution of the cumulative number of adopters at theater j by time τ approximated using binomial distribution, N_j is the maximum potential market size in the vicinity of theater j, and $P_j(\tau)$ is the cumulative density function for the event that an individual decides to see the new movie in theater j and acts on the decision by time τ. We estimate parameters N_j, λ_j, γ_j by using nonlinear regression to fit Equation (16) to historical data of aggregate box office revenues over time across a range of movies.

Step 2: Calibration of Regression Model Parameters from Historical Box Office Information

The second step in the estimation process is to connect the estimated value of the three parameters, $\hat{N}_j, \hat{\lambda}_j, \hat{\gamma}_j$ from the previous step to movie attributes, theater characteristics, and their possible interactions. To do this, we rely on multiple regressions based on the historical box office information used in Step 1.

Let $\boldsymbol{Z} = (Z_1, \ldots, Z_A)$ define the movie attribute vector. To incorporate theater characteristics and location specific information at the theater-level, let $\boldsymbol{S_j} = (S_{1j}, \ldots, S_{Cj})$ define the vector of theater and demographic characteristics for theater j. Then, the multiple regression equations to estimate the regression coefficients are:

$$\hat{N}_j = \alpha_N + \sum_{a=1}^{A} \beta_{N_a} Z_a + \sum_{c=1}^{C} \delta_{N_c} S_{cj} + \sum_{a'\in\Omega'} \sum_{c'\in\Lambda'} \omega_{N_{a'c'}} Z_{a'} S_{c'} + \varepsilon_N \tag{17}$$

$$\hat{\lambda}_j = \alpha_\lambda + \sum_{a=1}^{A} \beta_{\lambda_a} Z_a + \sum_{c=1}^{C} \delta_{\lambda_c} S_{cj} + \sum_{a'\in\Omega'} \sum_{c'\in\Lambda'} \omega_{\lambda_{a'c'}} Z_{a'} S_{c'} + \varepsilon_\lambda \tag{18}$$

$$\hat{\gamma}_j = \alpha_\gamma + \sum_{a=1}^{A} \beta_{\gamma_a} Z_a + \sum_{c=1}^{C} \delta_{\gamma_c} S_{cj} + \sum_{a'\in\Omega'} \sum_{c'\in\Lambda'} \omega_{\gamma_{a'c'}} Z_{a'} S_{c'} + \varepsilon_\gamma \tag{19}$$

Here, α_N, α_λ and α_γ denote the population intercepts and β_{N_a} , β_{λ_a} , and β_{γ_a} ($a \in \Omega = (1,\ldots,A)$), and δ_{N_c}, δ_{λ_c},and δ_{γ_c} ($c \in \Lambda = (1,\ldots,C)$) represent the regression coefficients associated with vectors **Z** and ***S***, respectively. In addition, the interaction terms associated with regression coefficients are $\omega_{N_{a'c'}}$, $\omega_{\lambda_{a'c'}}$ and $\omega_{\gamma_{a'c'}}$, where $a' \in \Omega' \subseteq \Omega$ and $c' \in \Lambda' \subseteq \Lambda$. Finally, ε_N, ε_λ, and ε_γ are independent and identically distributed random error terms.

Step 3: Estimation of Model Parameters for a New Movie

Let $\tilde{N}_{ij}, \tilde{\lambda}_{ij}, \tilde{\gamma}_{ij}$ represent the model parameters for a new movie i in theater j. Once the regression coefficients of Equations (17) through (19) are calculated, we use the known attributes $Z_i = (Z_{1i}, \ldots Z_{Ai})$ of new movie i and individual theater characteristics across all theaters to estimate parameters $\tilde{N}_{ij}, \tilde{\lambda}_{ij}$ and $\tilde{\gamma}_{ij}$ as:

$$\widetilde{N}_{ij} = \alpha_N + \sum_{a=1}^{A} \beta_{N_a} Z_{ai} + \sum_{c=1}^{C} \delta_{N_c} S_{cj} + \sum_{a'\in\Omega'} \sum_{c'\in\Lambda'} \omega_{N_{a'c'}} Z_{a'} S_{c'} \tag{20}$$

$$\widetilde{\lambda}_{ij} = \alpha_\lambda + \sum_{a=1}^{A} \beta_{\lambda_a} Z_{ai} + \sum_{c=1}^{C} \delta_{\lambda_c} S_{cj} + \sum_{a'\in\Omega'} \sum_{c'\in\Lambda'} \omega_{\lambda_{a'c'}} Z_{a'} S_{c'} \tag{21}$$

$$\widetilde{\gamma}_{ij} = \alpha_\gamma + \sum_{a=1}^{A} \beta_{\gamma_a} Z_{ai} + \sum_{c=1}^{C} \delta_{\gamma_c} S_{cj} + \sum_{a'\in\Omega'} \sum_{c'\in\Lambda'} \omega_{\gamma_{a'c'}} Z_{a'} S_{c'} \tag{22}$$

Step 4: Estimation of Theater-level Box Office Revenues for a New Movie

In this step, we use the estimates of $\widetilde{N}_{ij}, \widetilde{\lambda}_{ij}$ and $\widetilde{\gamma}_{ij}$ from (20) through (22) in (16), to estimate $E[\widetilde{N}_{ij}(\tau)]$, the expected cumulative number of adopters for new movie *i* at theater *j* until time τ as:

$$E[\widetilde{N}_{ij}(\tau)] = \frac{\widetilde{N}_{ij}}{\widetilde{\lambda}_{ij} - \widetilde{\gamma}_{ij}} \left[\left(\widetilde{\lambda}_{ij} - \widetilde{\gamma}_{ij} \right) + \widetilde{\gamma}_{ij} e^{-\widetilde{\lambda}_{ij}\tau} - \widetilde{\lambda}_{ij} e^{-\widetilde{\gamma}_{ij}\tau} \right] \tag{23}$$

Let ϕ_j be the ticket price at theater *j* and $t = \tau_2 - \tau_1$ be the time interval under consideration. Then, we calculate π_{ijt} as:

$$\pi_{ijt} = (E[\widetilde{N}_{ij}(\tau_2) - E[\widetilde{N}_{ij}(\tau_1))\, \phi_j \tag{24}$$

In the next section, we conduct computational experiments to test the accuracy of this method and to provide computational results for the DLSP and the EDSP.

6. COMPUTATIONAL STUDY

The financial box office data required for the computational study was purchased from Nielsen Entertainment Data Incorporated (EDI) located in Beverly Hills, California. We selected theaters located within the continental United States and purchased data of weekly box office revenues for all movies played at a given theater between May 22, 2000 and May 25, 2001.[1] The time period was chosen to completely cover an entire major release period during summer. In addition to purchasing financial data, we built two separate databases to collect information regarding motion picture attributes and theater characteristics corresponding to the movies and theaters in this period. These were created using Microsoft Access 2000. The final sample consisted of 149 movies and 97 theaters after removing theaters with incomplete financial information and movies released prior to May 22, 2000, or having less then three weeks of playtime, or both.

6.1. Results for Estimation Method

We summarize our results corresponding to the sequence of steps in the estimation method outlined in Section 5.

Step 1: Incorporation of Theater-level Characteristics

We use nonlinear regression to approximate the model parameters: the maximum potential market size (N_j), time-to-decide (λ_j), and time-to-act (γ_j) for a given movie. To execute this regression, we used the Levenberg-Marquardt algorithm (Bates and Watts, 1988) of the NLIN procedure of SAS, a commercially available statistical software (SAS, 2003). Results of the parameter approximation for selected pairs of movies and theaters are presented in Table 3.

INSERT TABLE 3 ABOUT HERE

From Table 3, we make two important observations. First, note that the magnitude of estimated box office revenue for the same movie can change significantly across theaters. For example, the estimated box office revenue for the movie WOMEN varied from \$31,400 to \$171,535. Thus, including location specific theater characteristics in the box office estimation

[1] To check whether the sample represents a typical year of movie releases, we tested the difference between the proportions of movies released over time and across genres in the previous and following years. We could accept the null hypothesis at the 95% confidence level that the two sample population proportions are equal in each class.

procedure is an important aspect in optimizing box office profits. Second, even when estimated box office revenues were similar, these could have been derived from very dissimilar parameter estimates and adoption patterns. For instance, the estimated box office revenue for movie LIESBTH at theater WYNN was \$22,607, while this estimate for the movie VELN at theater FENW was \$22,691. However, the estimate for LIESBTH at theater WYNN was based on $N = 22.994$, $\lambda = 26.168$, and $\gamma = 0.341$, while the box office estimate for VELN at theater FENW was based on $N = 23.049$, $\lambda = 2.043$, and $\gamma = 2.43$). This meant that LIESBTH followed an exponential shaped adoption pattern, whereas VELN's adoption pattern was consistent with the shape of the Erlang-2 distribution. These differences in adoption patterns have very different implications on the distribution strategy for these movies and, ultimately for parameter $K^{(MAX)}$.

Since these parameters vary significantly across movies and theaters and lead to different adoption patterns, it is critical that they are estimated by incorporating the impact of both movie attributes and theater characteristics. However, we found that we could not develop generalizations or simple rules to determine how these aspects affected these parameters. To overcome this, we resorted to multiple regressions, described in the next step.

Step 2: Calibration of Regression Model Parameters from Historical Box Office Information

To run the multiple regression connecting the parameters with movie characteristics and theater attributes, we collected 1,218 parameter triplets ($\hat{N}_j, \hat{\lambda}_j, \hat{\gamma}_j$) from the nonlinear regression of Step 1. We divided this in to two sets. The first set is the calibration sample with 609 triplets used to calibrate the multiple regression coefficients. The second was the holdout sample with the remaining 609 triplets and was used to test the validity of the regression results in Steps 3 and 4. We employed the IML procedure, a multiple regression routine in SAS to run the regressions.

The regression results showed that homoscedasticity (equal variance) was violated, so, to correct this problem, we transformed the dependent variables to their natural logarithm. The regression results are summarized in Table 4.

INSERT TABLE 4 ABOUT HERE

The regression results show that movie attributes and theater characteristics are good predictors of the maximum box office revenues at the theater-level ($R_N^2 = 0.53$). In addition, this regression in general possesses more predictive power for the time-to-act parameter (γ) than for the time-to-decide parameter (λ) ($R_\lambda^2 = 0.15, R_\gamma^2 = 0.27$). These results also provide interesting insight into which movie attributes and theater characteristics affect N, λ and γ.

Significant movie attributes that affect box office revenues include production budget, critic reviews, genre, and release date. As expected, higher budget and positive critics' reviews add to the box office success. Certain genres, specifically animation and fantasy, influenced box office revenues, since fantasy and sci-fi movies usually cater to specialized crowds. These results were similar to those of Litman and Ahn (1998). We also found that spring release dates adversely impacted box office revenues, possibly because of spring time travel and the restart of outdoor activities. Significant theater characteristics affecting box office revenues included increased presence of competing theaters, amount of discount on the ticket price, median age, population density and geographical location specified in one of seven, broadly classified regions in the United States. We found that the presence of competing theaters positively impacted box office revenues at a particular theater. While this result seems counter intuitive, this could be due to clustering effect (Chisholms and Norman, 2002), in which the collective presence of stores offering the same or similar services increases customer demand. As expected, the size of the ticket discount was negatively correlated with the box office revenues and explained why distributors often request to limit the number of discounted tickets. We also found that median age was negatively correlated to box office revenue, while increased population density positively influenced box office revenue. Finally, the geographical location of a theater had a significant impact on box office revenue.

The time-to-decide parameter λ was influenced by movie attributes and theater characteristics. Several movie related attributes, such as fantasy and animation genres, star presence, and movies heavy in special effects were positively correlated with λ. This is because these attributes pull audiences into theaters earlier, reducing the expected time to decide. On the other hand, restrictive MPAA rates and winter release timing were negatively correlated with λ, since these attributes dampen interest and, thus, increase the expected time to decide. Significant theater characteristics included adult prices and number of screens. As anticipated, the magnitude of the discount on ticket prices was positively correlated with λ, as this reduced the time to act. In addition, increasing number of screens was negatively correlated with λ. This is because increasing the number of screens typically increased the time to act presumably due to the perception that when there are more screens, the duration of movies would be longer and this could also reduce the chances of movies being sold out in subsequent weeks.

Movie attributes such as positive critiques, wider MPAA ratings and winter or spring opening dates were negatively correlated with γ. This is because all these attributes increase the expected time to act. Conversely, genre (animation and fantasy) and sequels were positively correlated with γ, since they typically catered to special audiences whose expected time to act is smaller. The only significant theater characteristic was the number of competing theaters within a five-mile radius. As expected, this was positively correlated with γ since the previously described clustering effect could reduce the expected time to act.

Finally, we list the results on the interaction terms relating location related variables with movie related ones. None of the interaction terms proved to be significant predictors of box office revenue in our sample, but could be significant in other samples.

Steps 3 and 4: Estimation of Model Parameters and Theater-level Box Office Revenues for a New Movie

We define a new movie as a movie shown in theaters in the holdout part of our sample. To validate the multiple regression results, we estimated model parameters $\tilde{N}_{ij}, \tilde{\lambda}_{ij}, \tilde{\gamma}_{ij}$, calculated box office revenue estimates for the holdout sample, and compared those to the actual achieved box office revenue. Table 4 shows model parameter and box office revenue estimates for several movie-theater pairs.

INSERT TABLE 4 ABOUT HERE

To better assess the performance of the box office estimation procedure, we also developed a benchmark model. This was based on a multiple regression model directly running box office revenue against the complete set of movie attributes that was used for our model *without* including any theater characteristics. This benchmark model itself was an enhancement on industry practice that was based upon choosing *one* movie attribute per simple regression run and employing multiple separate regressions to study the effect of different movie attributes on box office result (Tannenbaum, 2001). We observed heteroscedasticity (unequal variance) in the error terms; therefore, we transformed the actual box office measure to its logarithm. The predictive power of the benchmark model was significantly weaker than that of our model ($R^2_{benchmark} = 0.20, \mathrm{p} < 0.0001$). In addition, we observed that critic ratings were positively correlated with box office revenue estimates, while this estimate was negatively correlated to spring season release dates. Table 4 also shows forecasting results for the benchmark model.

Comparing our estimation model to the benchmark models, we found that our model reduced the average absolute forecast error of the benchmark model from 64.26% to 43.00%, or by around 33%. We next use the estimates of box office revenues from our model to test the DLSP and the EDSP.

6.2 Results for the Distributor's Location Selection Problem

The parameters required for DLSP, such as the portion of revenues allocated to the distributor, distribution costs, maximum number of theaters, and minimum number of theaters sought per region, were set based on specific movie level information. In addition, π_{ijt}, the expected theater-level box office revenue of a given movie during a given week (calculated using the procedure outlined in Section 5) and t_{ij}, the duration of play (calculated from the solution to EDSP), were key inputs to the DLSP.

We used XPRESS, a mixed-integer programming solver in GAMS to solve the DLSP. This generated optimal solutions for instances of the DLSP, when, on average, each theater had less than seven competing theaters. The average time for each run was around 53 seconds. However, we found that when each theater had more than seven competing theaters on average, GAMS could not solve the DLSP. This provided the motivation for developing the heuristics (i.e. lower bounds) to address this problem and upper bounds to evaluate the quality of the heuristics. To derive the upper bound, we solved the DLSP as a linear program using XPRESS in which W_j is relaxed to be continuous between 0 and 1. A specialized Microsoft VisualStudio.Net program was written to calculate the lower bounds using the heuristics of Section 3.1.

To examine how our models perform in larger problems, we used the reference data to construct larger problems with n theaters and m movies, where n = 1,000 and 3,000 and, m = 50, 150, and 300. We first analyzed the historical box office data and defined the probability distributions for the significant variables in the multiple regressions defined by (17) through (19). We then ran Monte Carlo simulations on these variables to generate expected theater-level revenues for these larger problems for the required choice of n and m. However, we observed that some of these problem instances generated an excessive amount of data without providing additional insights. Therefore, after careful consideration, we elected to analyze 1000 theater, 50 movie instances of the DLSP and the EDSP.

Table 6 summarizes some of our salient results from our computational tests. In this table, a row represents the solution technique used. These included the optimal solutions generated by GAMS, the upper bound generated by the LP relaxation in GAMS, and the lower bounds based on the myopic and greedy heuristics. Columns in the table represent the problem size of the DLSP represented by the number of theaters, movies, and the average number of competing theaters per theater, which we refer to as the competition density. The numbers in the body of the table describe the percentage gap of the given technique from a reference point, if that technique was successful in generating a solution for the given problem. Since the DLSP problems were solved to optimality at a competition density level of six competitors per theater, this was used as a reference point. However, GAMS was unable to generate optimal solutions due to resource limitation for problem with higher competition densities. Consequently, for the remaining problem, the upper bound was used as this reference.

INSERT TABLE 6 ABOUT HERE

Our computational results have been quite encouraging. From Table 6, observe that when we use the myopic heuristic for a competition density level of 15 competitors per theater, the average gap from the upper bound was 9.3%, while the corresponding gap with the greedy heuristic was 10.1%. As the competition density level decreased, the performance of both heuristics improved. For instance, the average gaps for the 6 competitors per theater problem reduced to 1.48% and 0.92% for the myopic and greedy heuristics, respectively. We also wanted to better understand the circumstances under which the percentage gaps change. This could provide us with insights to improve the upper bound and the heuristics. We observe from our analysis of the problem at 15 competitors per theater-level that these gaps were uniformly higher when the number of available theaters for selection as provided by the EDSP solutions was higher. Conversely, the gaps were significantly lower when the number of available theaters was lower. It is important to note that these gaps were reduced because the upper bound became tighter. These results show that the heuristics perform well across a range of data and there is scope to improve the upper bounds.

To test how sensitive the *value* of the heuristics were to estimates of theater-level box office revenues, we scaled π_{ijt} by *(1-x)* and *(1+ x)* where x = 0.1, 0.2 and 0.3. Note that our scaling procedure resulted in 6 additional data sets for the 1000 theater, 50 movie problem set at a density level of 15 competitors per theater. Table 7 summarizes the average gaps for the myopic and the greedy heuristics. These results show that average gaps for the myopic heuristic ranged from 8.94% to 10.01%, while the corresponding gaps for the greedy heuristic ranged from 9.42% to 12.04%. These results show that the heuristics were not significantly sensitive to estimation errors in π_{ijt} and, thus, provide a reliable basis to address this problem.

INSERT TABLE 7 ABOUT HERE

To see how sensitive the *solution*s of the heuristics were to the estimates of π_{ijt}, we compared the optimal theater locations selected across the heuristics for the scaled values of π_{ijt}. We found that, while the total number of locations and the composition of those locations in terms of theater types were stable, the actual locations proposed by the different solutions varied with differences of scale in π_{ijt}. For examples, about 41.7% of the optimal theater locations recommended by the DLSP solution changed for movies that were selected by less than 400 theaters to play across all EDSP solutions. These changes in actual theater selection underline the importance of pairing the appropriate theater with the given new movie. Therefore, the distributor should consider carefully the theater location as a crucial element in determining the optimal distribution planning decision.

We wanted to better understand the effects of minimum play length (i.e. $P^{(MIN)}$) on the solution of the DLSP. $P^{(MIN)}$ is an important parameter in the EDSP, therefore influencing the optimal selections for the DLSP. Distributors consider this parameter vital toward achieving the desired exposure, which in turn will affect the financial potential of a movie. Consequently, they go to great length to ensure that the agreed-upon screen is allocated to the particular movie for the requested period of time. Exhibitors, on the other hand, aim to decrease the commitment period to gain more freedom in shifting movies among their screens. We tested our procedures with $P^{(MIN)}$ set to 3 weeks as the base case, and 2 and 4 weeks as alternative settings for the 1000 theater, 50 movie, 15 competitors per theater problem. The sensitivity analysis of this parameter provided several interesting insights.

We compared the solutions with the alternative values of $P^{(MIN)}$ to the base case and found that a major shift occurs in the optimal theater locations when the minimum play length requirement was reduced to two weeks. At first glance, the change in the number of theaters and the actual selection were minimal, but a more detailed analysis of the proposed set of theaters for the base case revealed a significantly different set of theaters to target. For example, the total number of theaters selected by DLSP showed a very modest increase from 360 to 364 theaters, but more than 50% of the theaters recommended for the base case were replaced for the shortened commitment period. The extent of this change is surprising and goes unrecognized by distributors. The new solution selected more mini- and multi- type theaters in moderately priced locations in contrast with the original solution's heavy dominance by multi- and mega- theaters in highly priced areas. When $P^{(MIN)}= 4$, the total number of theaters decreased by 30, as the movie had to be shown for a longer duration, but, here again, more than 33% of those differed from the base case. This shows that the minimum play length requirement strongly influences the optimal theater selection and distributors should carefully examine the effect of changing $P^{(MIN)}$ on theater choice before agreeing to change it on a movie-by-movie basis. The DLSP provides a structured and robust basis for conducting this assessment.

6.3. Results for the Exhibitor's Duration Selection Problem

The parameters required for the EDSP, such as the portion of revenues allocated to the exhibitor, movie release schedule, number of screens at each theater location, minimum play length and fixed movie acquisition costs, were set based upon specific movie and theater-level information. The expected theater-level box office revenue of a given movie during a given week (i.e., π_{ijt}) was calculated using the procedure outlined in Section 5. The optimal solution to the EDSP provides the exhibitors the optimal schedule of movies and the duration of play for each movie (i.e., t_{ij}), which is used in the DLSP.

We used XPRESS in GAMS, to solve the EDSP for 1000 individual theaters using 9 different parameter settings. GAMS generated solutions to the EDSP by either solving the integer program to optimality in 37 seconds on average or achieving a relative gap of 0.9% or less on average by employing an embedded branch-and-bound heuristic.

To test how sensitive the *value* of optimal solutions to the EDSP were to estimates of theater-level box office revenues, we solved the EDSP for the six additional data sets generated in

Section 6.2 by scaling π_{ijt} by *(1-x)* and *(1+ x)* where x = 0.1, 0.2 and 0.3 for the 1000 theater, 50 movie problem set at 15 competitors per theater density level problem. Note that this required that we solve 6000 EDSP problems each with 50 movies. We observed that the range of variation in optimal solutions to these problems was smaller than the corresponding variation in π_{ijt}.

Next, we examined how the corresponding *solutions* changed in terms of the number and types of theaters selecting a certain movie, and how this scaling affected the duration of play in any particular theater. Here, we found that the number of theaters selecting a given movie was stable over the wide range of π_{ijt}. On average, 90% of the theaters stayed with their original movie selection across all movies and levels of variation in π_{ijt}. The 10% of theaters that did modify their programming decision typically belonged to the same type of theaters and movies were changed to reduce the fixed costs associated with acquiring a movie. In addition, we found that certain theater types were preferred by each movie and that the ratio of each type for a movie did not shift due to variation in the profit coefficient. Finally, if a movie was selected at a theater, its duration of play did not change significantly from the corresponding value when the profit coefficient was not varied. These findings show the stability of the EDSP solutions in face of uncertain profit forecasts. This stability can be reassuring, but can also be misleading to the movie distributor. It is reassuring because the distributor can select the number of theaters prescribed by its general distribution plan using reasonably accurate theater-level revenue forecasts over time. On the other hand, the stability might mislead the distributor into thinking that one has to only consider the same set of theaters across *different* movies. However, due to changes in movie attributes, the set of theaters may vary widely across movies. For instance, across the 50 movies, we found that on average only 43% of the theaters were common. This result reinforces the importance of including theater characteristics and movie attributes when determining the distribution plan.

To understand the effects of the minimum play length requirement (i.e., $P^{(MIN)}$) on the EDSP, we ran the EDSP on the 1000 theater, 50 movie, 15 competitors per theater problem with $P^{(MIN)}$ set to 3 weeks as the base case, and 2 and 4 weeks as alternatives. The results are summarized in Table 8. These results show that when the minimum play length is decreased from the base case to two weeks, exhibitors tended to program nearly twice as many movies for their screens. Even movies that received no screen time for the base case received exposure on the screen for $P^{(MIN)} = 2$. But, in order to accommodate more movies on the same number of screens, certain movies had to be selected by fewer theaters and the average duration of play was reduced from 5 weeks for the base case to 2.96 weeks. However, across all theaters, there was a 13.3% increase in expected box office profit for the exhibitors after deducting higher acquisition costs associated with booking more movies. Single- and mini-type theaters generated up to 15% higher box office profit, while multi- and mega-types saw up to 30% increases. Only 1.1% of the theaters experienced a decrease in their expected box office profit, and this decrease was no more than 7%. In addition, all the decreases occurred at single screen theaters because movies on average were shown for shorter durations and more movies were shown, resulting in greater fixed acquisition costs. Conversely, when the duration of play increased from the base case to $P^{(MIN)} = 4$, fewer movies were shown for a longer duration and this reduced both exhibitor and distributor profits.

INSERT TABLE 8 ABOUT HERE

The results in Table 8 also show that an increase in the number of movies selected directly increases movie distributor profits, because more movies have the chance to reach a greater proportion of audiences in theaters. This is desirable for distributors since these movies start generating income earlier than waiting for other ancillary distribution channels like pay-per-view or DVDs. Finally, when $P^{(MIN)} = 2$, around 86% of the movies generated better expected box office profits for the distributor producing an additional 17% profits. Therefore, distributors should consider shifting to the two-week minimum for this group of movies. The remaining movies were selected by a fewer theaters or played for a shorter duration, or both. For these movies, the distributors should consider adjusting the leasing agreement terms or insist on longer commitment periods. The EDSP provides a scientific and structured basis for both exhibitors and distributors to identify these categories of movies and enable them to increase overall box office profits.

7. APPLICATION

We have compared the methods in this paper to the theater selection decisions made by motion picture distributors by using the database developed in Section 6.1 for the 1000 theater, 50 movie problem with the number of average competing theaters set to the highest level of 15 competitors per theater. We first solved the EDSP for each of the individual theaters to determine the duration of play across all the movies. We then ran the myopic and greedy heuristic to solve the DLSP for each movie and picked the best solution.

Next, we constructed a distribution plan for a given movie replicating the procedure that distributors would have used in practice, based on extensive discussions with several leading distributors. In this procedure, distributors first ranked theaters in decreasing order of historical revenues across all movies. Next, they picked the highest ranked theaters in each region while ensuring that the minimum number of theaters and competition constraints where met in each region. Finally, they also made sure that the total number of theaters across regions did not exceed the maximum number of theaters required, which was set based upon the distribution strategy for each movie.

We wanted to compare our method with the distributor's procedure. To ensure that the quality of the theater-level box office forecast and the duration of play did not affect this comparison, theater rankings were developed using theater-level revenue forecasts using our estimation procedure, while the optimal duration of play for a given movie at each theater was determined by the EDSP. Comparing our method with the distributor's procedure, we found that theaters chosen by our method were 46% different on average than those selected by the distributor. In addition, had our method been implemented, this would have increased the average box office profit by $ 4,487 per theater, or an average of around $2.2 million per movie. This translates to a 5.5% increase in expected distributor's box office profit. In addition, individual percentage and absolute revenue improvements for some movies were as high as 10% and $4 million respectively

It is important to note that these numbers underestimate true gains. In practice the distributor's method would have performed worse than these results indicate without the advantage of a more accurate theater-level forecast from our estimation procedure and without using the optimal duration of play for each movie from the EDSP. We believe that our method outperforms the distributor's procedure because its allocation of theaters is based not just on historical sales volumes across all movies, but matching the attributes of a given new movie with the characteristics of the theater under consideration.

8. CONCLUSIONS

Our goal in this paper is to expose the reader to an intellectually engaging problem context laden with opportunities for research that can have a high impact on profits in the motion picture industry. The following conclusions can be drawn from this research.

- There are significant differences in revenue across the same movie in different theaters due to dissimilarity in location specific characteristics at the theater such as amenities and demographics (i.e., median income, age, population density, etc.). Therefore, it is important to develop detailed, disaggregate theater-level box office forecasts and to use this forecast to determine the distribution plan on which theaters and how long to show a given movie.
- Estimating effective theater-level revenue forecasts is challenging as it requires an understanding of which movie attributes and theater characteristics will affect revenues, how they do so, and how this changes over time. In addition, one needs to also understand how the decision process of individual moviegoers to see a given movie affects theater-level forecasts and how this can be incorporated in the forecasting process. The estimation procedure developed in this paper incorporates these aspects in determining detailed theater-level revenue forecasts. This procedure reduces average forecast error by over 33% compared to benchmark models based on industry practice. These forecasts are also critical in optimally determining where and how long to show a movie.
- Determining the duration of play of a movie at a theater to optimize profits is difficult, as the exhibitor has a fixed number of screens and several movies that could be played during a period. These movies generally have differences in release dates, fixed

acquisition costs, minimum play length requirements, and expected profits over time. The EDSP can be effectively used to address this problem. We found that the solution of this model is robust to variation in theater-level revenue forecasts. In addition, it was beneficial for both distributors and exhibitors to have a shorter play length for a wide range of movies. This model provided a structured and objective basis from which to identify the type of movies that could benefit from reduced minimum play length.

- Given theater-level revenue forecasts over time and optimal play lengths, the distributor faces the problem of determining at which theaters to show a given movie at in order to optimize profits. This problem is complicated because a minimum number of theaters has to be selected in each region and because the distributor needs to ensure that competing theaters are not selected. The DLSP provides an effective basis to approach this problem. We also found that this model was robust to variations in the theater-level revenue forecast and provides a basis to understanding the impact of changes in minimum play length on theater choice. In addition, the DLSP outperformed the method used by the distributors to select theaters and has the potential to increase average distributor profits by 5.5%, or around $2.2 million per movie.

This paper provides several avenues for future research. First, refinements could be developed to further improve the accuracy of the theater-level box office revenue forecasting procedure. Second, improvements could be made on the heuristics to increase the profits from the DLSP. Third, the DLSP could be extended to include the case when the distributor releases multiple movies during the same period. In this case, the problem would be to choose the optimal theater locations to exhibit these movies while considering the competitive interactions between these movies. This, in turn, would require significant modification to the structure and solution methodology of the DLSP, and changes to the theater-level box office revenue estimation procedure. Finally, the approach developed in this paper in which we determine the best locations to show a movie by estimating profit as a function of movie (or product) attributes and theater (or location) characteristics can be applied in a variety of service industry settings. For instance, one could use this idea to choose the best locations for concerts in the music industry, to determine the optimal location of specialty boutiques in the retail industry, and to pick the locations of resorts and restaurants in the hospitality industry. The modifications required to apply our model in these contexts could be a promising area for new research.

In conclusion, we believe that the methods presented in this paper provide a useful framework to analyze and improve distribution planning in the motion picture industry.

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Table 1. Examples Of Missed Box Office Forecasts

Title	NRG Estimate[1] ($M)	Actual Box Office ($M)	Relative Percentage Error [2]
X-Men	29.5	54.5	-45.9%
The Mummy Returns	50	70.1	-28.7%
Charlie's Angels	28	40.1	-30.2%
The Perfect Storm	21.5	41.3	-47.9%
Chicken Run	9	17.5	-48.6%
The Patriot	25	22.4	+11.6%
The Story of Us	18	9.7	+8.6%
Fight Club	14	11	+27.3%
Titan A. E.	12.5	9.4	+33%
Star Wars -Episode I.	150	105.7	+41.9%

(1) National Research Group's estimate of the opening weekend box office.

(2) Relative percentage error = (NRG Estimate – Actual BO) / Actual BO.

Source: IMDB and Variety.

Table 2. Box Office Revenues At Selected Markets

Markets*	The Matrix Reloaded	Star Wars: Episode II
Boston	$38,371	$35,829
Cleveland	$21,105	$21,559
Dallas	$31,207	$35,385
Denver	$31,423	$28,663
New York	$36,968	$53,499
Pittsburgh	$23,193	$18,683
San Francisco	$60,336	$60,169
St. Louis	$22,813	$23,410
Tampa	$25,043	$25,763

* Movies are shown in the same number of theaters in each market and revenues reported are Friday to Sunday averages over duration of play.

Data Source: Nielsen EDI's Box Office Sample.

Table 3. Results Of The Nonlinear Regression

Theater type	Theater	Film	T (wks)	Actual BO ($1,000)	N	λ	γ	Type of Pattern	Predicted BO ($1,000)	Absolute	MSE
Multi	WYNN	LIESBTH	12	23.009	22.994	26.168	0.341	Exponential	22.607	1.75%	1.172
Multi	WYNN	PROOF	5	4.798	5.712	0.659	0.659	Erlang-2	4.802	0.09%	0.387
Multi	WYNN	WOMEN	11	32.248	32.318	2.946	0.335	Gen. Gamma	31.400	2.63%	1.773
Multi	FENW	VALEN	3	22.64	23.049	2.043	2.043	Erlang-2	22.691	0.23%	0.097
Multi	FENW	CHARLIE	9	255.337	254.914	7.011	0.573	Gen. Gamma	253.314	0.79%	6.193
Multi	FENW	CHICKEN	10	151.711	154.752	59.795	0.391	Exponential	151.615	0.06%	2.723
Multi	KTLA	CHICKEN	7	48.535	53.029	98.852	0.379	Exponential	49.283	1.54%	1.744
Multi	KTLA	WOMEN	7	59.411	64.666	3.559	0.369	Gen. Gamma	59.221	0.32%	0.988
Multi	KTLA	FOCKER	11	76.058	90.369	16.231	0.183	Exponential	78.126	2.72%	3.762
Mega	ALIS	CHICKEN	7	131.56	134.969	5.662	0.550	Gen. Gamma	131.783	0.17%	1.846
Mega	ALIS	WOMEN	8	171.214	175.080	5.031	0.501	Gen. Gamma	171.535	0.19%	3.506
Mega	ALIS	LIESBTH	9	146.5	146.282	45.437	0.487	Exponential	144.430	1.41%	4.836
Mega	HADL	CHARLIE	8	50.418	51.558	11.083	0.565	Exponential	50.965	1.09%	1.903
Mega	HADL	CHICKEN	6	37.457	41.732	10.523	0.385	Exponential	37.429	0.07%	0.522
Mega	HADL	WOMEN	11	56.044	56.213	6.311	0.364	Gen. Gamma	55.126	1.64%	1.980
Mini	FAUL	HEADOV	3	1.626	1.91082	1.1953	1.195	Erlang-2	1.668	2.58%	0.122
Mini	FAUL	SHAFT	5	6.581	7.35186	6.5085	0.476	Gen. Gamma	6.616	0.53%	0.086
Mini	FAUL	DISKID	4	10.375	13.2302	4.5361	0.412	Gen. Gamma	10.434	0.56%	0.189
Mini	HANF	CHARLIE	10	26.963	27.7511	89.985	0.399	Exponential	27.235	1.01%	0.779
Mini	HANF	SPYKIDS	9	24.654	25.2809	3.6908	0.399	Gen. Gamma	24.497	0.64%	0.889
Mini	HANF	FOCKER	11	30.19	37.8096	17.694	0.151	Exponential	30.537	1.15%	1.166
Single	GRAH	NUTTY	4	23.089	24.6119	10.631	0.715	Exponential	23.100	0.05%	0.027
Single	GRAH	DRSEUSS	5	41.8	44.2847	14.218	0.518	Exponential	40.841	2.30%	2.170
Single	CMEO	MISSCON	7	36.838	38.4956	36.113	0.406	Exponential	36.231	1.65%	1.154
Average errors										**1.05%**	**1.667**

Table 4. Multiple Regression Results

	Dependent Variables (Standard Errors In Parentheses)			
	Multiple Regression Model			Benchmark
	Log N	**Log λ**	**Log γ**	**Log Actual**
Intercept	3.584* (0.281)	2.374* (0.397)	-0.831* (0.193)	3.000* (0.200)
Theater Related Variables				
Rockies Region	-0.282** (0.144)	0.010 (0.203)	0.161 (0.099)	
North Central Region	-0.584* (0.119)	-0.091 (0.168)	0.064 (0.082)	
South Central Region	-0.231** (0.113)	0.156 (0.160)	0.108 (0.078)	
Midwest Region	-0.173 (0.147)	0.127 (0.207)	0.137 (0.100)	
North Eastern Region	-0.440* (0.088)	-0.025 (0.124)	0.092 (0.060)	
South Eastern Region	-0.291* (0.105)	0.059 (0.147)	0.062 (0.72)	
Median Age	-0.015*** (0.008)	-0.007 (0.011)	0.008 (0.005)	
Percentage Of Singles	0.009 (0.005)	0.005 (0.008)	0.004 (0.004)	
Population Density (1,000s)	0.012* (0.004)	-0.011** (0.005)	-0.003 (0.003)	
Median Household Income ($1,000s)	0.001 (0.002)	-0.001 (0.003)	0.001 (0.001)	
Adult Ticket Price ($)	0.269* (0.057)	0.169** (0.080)	0.043 (0.039)	
Ticket Discount ($)	-0.143*** (0.085)	0.019 (0.120)	0.022 (0.058)	
Stadium Seating	0.364* (0.065)	0.032 (0.092)	0.010 (0.045)	
Mini Type (2-7)	-0.172 (0.265)	-0.780** (0.375)	-0.067 (0.182)	
Multi Type (8-15)	-0.148 (0.264)	-0.799** (0.373)	0.086 (0.181)	
Mega Type (16+)	-0.071 (0.275)	-0.737 (0.389)	0.192 (0.189)	
Numbers Of Neighboring Theaters (5 Miles)	0.015** (0.007)	-0.003 (0.009)	0.012** (0.005)	
Numbers Of Neighboring Theaters (10 Miles)	-0.002 (0.009)	-0.018 (0.013)	-0.008 (0.006)	
Numbers Of Neighboring Theaters (15 Miles)	-0.004 (0.004)	0.013** (0.005)	0.001 (0.003)	
Movie Related Variables				
Runtime (Min)	0.000 (0.002)	0.004 (0.003)	-0.002 (0.002)	-0.002 (0.003)
Production Budget ($M)	0.008* (0.001)	-0.001 (0.002)	-0.001 (0.001)	0.004** (0.002)
MPAA Rating	0.053 (0.050)	-0.136*** (0.07)	-0.087* (0.034)	0.049 (0.065)
Critique's Rating	0.305*	0.031	-0.148*	0.330*

	Dependent Variables (Standard Errors In Parentheses)			
	Multiple Regression Model			Benchmark
	Log N	Log λ	Log γ	Log Actual
	(0.033)	(0.046)	(0.022)	(0.044)
Special Effects	0.063 (0.086)	0.276** (0.122)	0.046 (0.060)	0.119 (0.113)
Star Presence	0.042 (0.077)	0.211*** (0.109)	0.003 (0.053)	0.030 (0.101)
Sequel	0.192 (0.171)	-0.120 (0.242)	0.154 (0.118)	0.165 (0.224)
Animation Genre	-0.325*** (0.173)	0.555** (0.244)	0.397* (0.119)	-0.328 (0.223)
Comedy Genre	(0.097 (0.082)	0.215 (0.115)	-0.063 (0.056)	0.157 (0.106)
Drama Genre	-0.113 (0.108)	0.176 (0.156)	0.054 (0.075)	-0.051 (0.142)
Horror Genre	0.024 (0.127)	0.280 (0.180)	0.063 (0.088)	0.072 (0.181)
Fantasy Genre	-0.360* (0.128)	0.374** (0.180)	0.263* (0.088)	-0.227 (0.168)
Holiday Period Opening	0.050 (0.079)	-0.499* (0.111)	-0.067 (0.054)	0.167 (0.103)
Winter Period Opening	0.051 (0.095)	-0.220 (0.135)	-0.198* (0.065)	-0.046 (0.125)
Spring Period Opening	-0.229** (0.098)	-0.231 (0.139)	-0.050 (0.066)	-0.253*** (0.133)
Fall Period Opening	-0.094 (0.094)	-0.145 (0.133)	-0.190* (0.065)	0.015 (0.123)
Interaction Terms				
Median Age * MPPA Rating	-0.003 (0.006)	-0.002 (0.008)	-0.006 (0.004)	
% Of Singles * MPAA Rating	-0.006 0.004)	-0.003 (0.005)	-0.003 (0.002)	
Median Household * Rating	0.001 (0.001)	0.001 (0.002)	-0.001 (0.001)	
Population * Rating	-0.002 0.002)	0.005 (0.004)	0.003 (0.002)	
Star Presence * Median Age	0.013 (0.010)	-0.002 (0.015)	-0.001 (0.007)	
Star Presence * % Of Singles	-0.007 0.005)	0.003 (0.008)	-0.003 (0.004)	
Model R^2	0.5337	0.1510	0.2719	0.2004
Adjusted R^2	0.4998	0.0893	0.2190	0.1774
F Value	15.74	2.45	5.14	8.70
Pr > F	<0.0001	<0.0001	<0.0001	<0.0001
N	605	605	605	607

* Statistically significant at 1%. ** Statistically significant at 5%. *** Statistically significant at 10%.

Table 5. Results for New Movies and The Benchmark Models

					Estimation Model						Benchmark Model		
Theater	**Type**	**Movie**	**T**	**Actual BO**	**N_SREM**	**lam_SREM**	**gam_SREM**	**Predicted BO**	**%SREM**	**Absolute error**	**Predicted**	**%bench**	**Absolute**
ALIS	Mega	ARTWR	4	34.132	49.159	22.592	0.645	45.326	32.80%	11.194	11.991	64.869%	22.141
CENT	Multi	ARTWR	5	66.058	72.232	25.783	0.994	71.710	8.56%	5.652	11.991	81.848%	54.067
INDE	Multi	ARTWR	5	9.554	13.217	6.928	0.600	12.498	30.81%	2.944	11.991	25.506%	2.437
MONM	Multi	ARTWR	5	33.947	24.966	7.463	0.608	23.663	30.29%	10.284	11.991	64.678%	21.956
CIGR	Multi	BIGMMA	10	54.802	14.353	6.481	0.469	14.210	74.07%	40.592	16.218	70.406%	38.584
COPL	Multi	BIGMMA	7	118.312	36.917	11.437	0.548	36.081	69.50%	82.231	16.218	86.292%	102.094
COUN	Mega	BIGMMA	9	64.614	35.705	26.140	0.475	35.197	45.53%	29.417	16.218	74.900%	48.396
EHIL	Multi	BIGMMA	7	33.978	28.519	8.480	0.475	27.429	19.28%	6.549	16.218	52.268%	17.760
HANF	Mini	BIGMMA	7	33.677	23.806	7.889	0.480	22.928	31.92%	10.749	16.218	51.842%	17.459
CHST	Mini	DRSEUSS	5	20.92	37.146	8.432	0.494	33.807	61.60%	12.887	32.335	54.563%	11.415
CIGR	Multi	DRSEUSS	4	20.277	27.908	4.992	0.482	23.419	15.50%	3.142	32.335	59.465%	12.058
FACT	Multi	DRSEUSS	5	81.875	57.539	9.889	0.500	52.561	35.80%	29.314	32.335	60.507%	49.540
GASL	Mini	DRSEUSS	8	63.872	28.538	5.600	0.488	27.908	56.31%	35.964	32.335	49.376%	31.537
ROYA	Mega	DRSEUSS	11	126.958	63.955	14.362	0.506	63.701	49.83%	63.257	32.335	74.531%	94.623
CENT	Multi	RUGRT	5	53.143	88.667	18.133	0.847	87.319	64.31%	34.176	22.290	58.056%	30.853
CHAP	Multi	RUGRT	8	19.757	19.181	4.086	0.524	18.848	4.60%	0.909	22.290	12.821%	2.533
FAIR	Mega	RUGRT	5	69.845	51.900	11.228	0.512	47.686	31.73%	22.159	22.290	68.086%	47.555
NTOW	Multi	RUGRT	5	13.176	9.763	3.795	0.524	8.938	32.16%	4.238	22.290	69.172%	9.114
PKWA	Multi	RUGRT	5	11.391	21.271	3.614	0.512	19.351	69.88%	7.960	22.290	95.681%	10.899
CHAP	Multi	TITANS	4	35.156	26.547	4.426	0.318	18.524	47.31%	16.632	22.290	36.597%	12.866
FAIR	Mega	TITANS	10	109.805	71.831	12.163	0.310	68.519	37.60%	41.286	22.290	79.700%	87.515
HARW	Mini	TITANS	5	10.537	26.956	5.671	0.318	21.127	100.50%	10.590	22.290	111.541%	11.753
NPAR	Multi	TITANS	12	60.46	34.547	4.057	0.307	33.603	44.42%	26.857	22.290	63.133%	38.170
WEST	Multi	TITANS	6	26.983	26.301	4.258	0.318	22.079	18.18%	4.904	22.290	17.392%	4.693
CAEL	Mini	WOMEN	5	11.924	22.271	3.768	0.429	19.334	62.15%	7.410	25.640	115.026%	13.716
CONE	Mini	WOMEN	9	42.044	21.210	3.380	0.414	20.629	50.93%	21.415	25.640	39.017%	16.404
EWAL	Multi	WOMEN	9	375.016	257.807	12.653	0.638	256.937	31.49%	118.079	25.640	93.163%	349.376
MALC	Multi	WOMEN	17	67.064	28.193	3.281	0.409	28.162	58.01%	38.902	25.640	61.768%	41.424
NPAR	Multi	WOMEN	9	60.801	45.710	3.485	0.404	44.352	27.05%	16.449	25.640	57.830%	35.161
SBEA	Mega	WOMEN	7	114.77	63.092	10.726	0.424	59.722	47.96%	55.048	25.640	77.660%	89.130
								AVERAGE	43.00%	25.706		64.26%	44.174

Table 6. Average Percentage Gaps From Tightest Bound Across 50 Problems For The DLSP

Average (Minimum/ Maximum)	**Problem Size (Theaters/ Movies/ Competition Density)**		
	1000/ 50/ 6	**1000/ 50/ 11**	**1000/ 50/ 15**
Optimal Solution: Gams	*	N/A	N/A
Upper Bound: Linear Programming Relaxation	4.68%	*	*
Lower Bounds:			
Myopic Heuristic	1.48%	7.5%	9.3%
Greedy Heuristic	0.92%	8.1%	10.1%

Table 7. Average Percentage Gaps From Upper Bound with Scaled π_{Ijt}.

Scale Factor	-30%	-20%	-10%	0%	10%	20%	30%
Myopic Heuristic	9.27	8.94	10.01	9.31	9.72	9.07	9.56
Greedy Heuristic	10.01	9.63	10.36	10.10	10.26	9.42	12.04

Table 8. Results with Varying Duration of Play or The Exhibitors Duration Selection Problem (EDSP)

	Minimum Play Length P^{MIN} = 2 weeks	Minimum Play Length P^{MIN} = 3 weeks	Minimum Play Length P^{MIN} = 4 weeks
Average Duration of Play (weeks)	2.96	4.46	7.14
Average Number of Movies Selected	19.27	12.82	8.50
Exhibitor Profits ($1000)	214,215.6	187,905.5	156,600.8
Distributor Profits ($1000)	527,124.7	446,196.9	345,317.2

Figure 1. The Motion Picture Industry Supply Chain

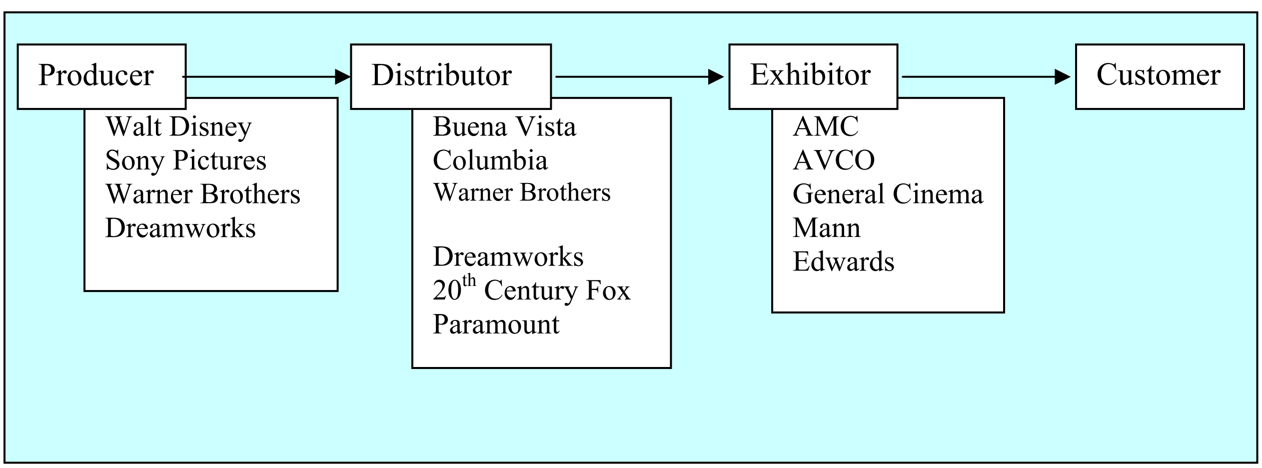

Figure 2. Methodology For Estimation Of π_{ijt}: Theater-level Box Office Revenues For A New Movie