Essays on Learning and Investor Behavior

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by

Juhani Linnainmaa

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The dissertation of Juhani Linmainmaa is approved.

Kevin McCardle

Hanno Lustig

Antonio Bernardo

Mark Grinblatt, Committee Chair

University of California, Los Angeles
2006
to my wife Sari
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Vita

1978 Born, Helsinki, Finland.

2001 B.A. and MS.c. (Econ), Helsinki School of Economics, Finland

2002–2006 Teaching Assistant
UCLA Anderson School of Management, Los Angeles, CA

2005 Allstate Dissertation Fellowship
UCLA Anderson School of Management, Los Angeles, CA

Presentations

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The three chapters of this dissertation examine learning and investor behavior. The first chapter shows that parameter uncertainty about the return/labor income process can lead to limited stock market participation. The key result is that the resulting rational nonparticipation is empirically indistinguishable from irrational nonparticipation as well as from nonparticipation generated by non-standard preferences such as ambiguity aversion. The second chapter studies whether uncertainty about day trading profits may have lead to the rise in the number of day traders in the late 1990s. I use unique Finnish data to examine this question empirically after developing a theoretical learning and portfolio choice model. I find individual day traders’ exit and trade size decisions to be consistent with the learning hypothesis—and inconsistent with the common explanation of stubborn beliefs. The third chapter is an empirical re-examination of the determinants individual investors’ behavior. The key idea is that limit orders have mechanistic features that affect inferences about investor behavior. I show that limit order use is an important component of the disposition effect, contrarian behavior, coordinated trading, and the tendency to trade attention-grabbing stocks. Limit orders also bias inferences about individuals’ stock-picking abilities.
CHAPTER 1

Learning and Stock Market Participation

1.1 Introduction

Stock market participants have historically been a minority.\(^1\) Despite the exceptional historical equity premium, many individuals stay out of the market. This is a puzzle because everyone should participate if the risk premium is even slightly positive and there are no frictions or incomplete markets (Arrow 1965). Many empirical determinants of participation are known. For example, education matters: 50% of individuals with a college degree own stocks while this rate is only 20% for those without a degree (Hong, Kubik, and Stein 2004). Wealth is the strongest determinant of participation: the participation rate increases from 3% to 55% from the first to the fifth wealth quintile. However, the non-participants are not only those who have nothing to invest: Mankiw and Zeldes (1991) find that even individuals with more than $100,000 in liquid assets have a participation rate of only 47.7%. The limited participation puzzle is our inability to understand why so many individuals choose to stay out of the market.

We propose a novel mechanism that generates non-participation in a perfectly rational setting. We first describe this mechanism with an example. Suppose an

---

\(^1\)Only the latest Survey of Consumer Finances from 2001 finds that, for the first time in the US history, stockholders have become a majority with a 51.9% participation rate. The participation rate was 31.7% in 1989. The SCF participation rates include direct and retirement account holdings of stocks and stock mutual funds.
agent works in an industry sensitive to the macroeconomic conditions. If the economy stays healthy, the agent retains her job. However, the agent might lose her job in an economic downturn. If this happens, the agent’s wage covaries positively with the dividends: if the economy recovers, firms pay higher dividends and the agent is rehired. However, if the economy remains weak, firms pay low dividends and the agent receives no labor income. The unemployed agent would hedge against the risk of not finding a new job by shorting the market.

Suppose now that the agent cannot open a margin account in this unemployed state. This inability to hedge has two consequences. First, the direct effect is that the agent stays out of the market after losing her job, generating a welfare loss relative to the “hedging allowed” case. Second, this potential future welfare loss is important at an earlier date for the employed agent. If the agent invested in the market today, an economic downturn would have important repercussions: the agent would not only be unemployed and restricted from hedging but she would also have high marginal utility because she invested. By staying out of the market today, the agent hedges against this risk.

This paper explores this mechanism and its consequences. We first demonstrate this feedback effect from future portfolio constraints with a stylized life-cycle model where an agent learns about the risk premium over time. Because the agent learns about the risk premium, the agent faces poor investment opportunities after bad realizations. (For example, the agent revises her beliefs downwards after a low realized return.) We show that the agent may stay out of the market despite a large risk premium.\footnote{Learning is central to this non-participation mechanism. For example, it would not be surprising to find that an agent with currently poor investment opportunities stays out of the market. However, our finding is more surprising: the feedback from future trading restrictions can be so strong that an agent reduces her holdings to zero despite a high risk premium.} We also introduce implied risk-aversion as a
measure of how much learning and portfolio constraints skew investor behavior. For example, an agent who stays out of the market despite a high risk premium appears infinitely risk-averse to an outsider who ignores the role of learning.

We then consider an equilibrium model where heterogeneous agents resolve uncertainty about the covariance between their nonfinancial income and dividends. Specifically, we consider the case where one of the agents becomes unemployed after a low dividend. We use this model to address two questions. First, does the unemployment risk together with market incompleteness generate a hedging demand that keeps the agent out of the market at an earlier date? Second, what are the consequences of this type of non-participation on the size of the risk premium? The latter question is important because the limited participation puzzle is intimately connected to the size of the equity premium.³

We demonstrate two results. The first result is that the risk of binding constraints alone may induce an agent to stay out of the market at date zero: the agent would participate today if she could hedge in the unemployed state. The second result is that the risk premium is high relative to the unconstrained economy—i.e., the same economy but without portfolio constraints—when the agent is currently out of the market, but

1. would buy a positive amount of the asset if the short-sale constraints were lifted and

2. is close to being indifferent between participating and not participating in

³For example, Hong, Kubik, and Stein (2004) motivate their analysis by suggesting that an understanding of what drives participation can shed light on the equity premium puzzle of Mehra and Prescott (1985). Yet, most studies that examine why some individuals do not participate sidestep this issue. It is not obvious what the effect on the risk premium should be. For example, if the short-sale constraints let the participating agents to hold only the entire market (instead of holding more), they only need to be compensated with a smaller risk premium.
This risk premium result holds also for agents with log-preferences. Hence, an agent may stay out at date 0 even though her labor income is constant, her preferences would generally lead to myopic behavior, and the risk premium is relatively high. This suggests that an empirical analysis of the determinants of non-participation may be difficult when agents hedge against the future risks of not being able to trade all the assets as smoothly as classical models presume.

The rest of the paper is organized as follows. The next section discusses related research. Section 1.2 solves a tractable life-cycle model that illustrates the feedback mechanism. Section 1.3 formulates a heterogeneous-agents equilibrium model with non-participation. Section 1.4 concludes.

1.1.1 Relation to Prior Research

1.1.1.1 Parameter Uncertainty and Learning

Many recent studies have examined parameter uncertainty (or ambiguity) and learning. For example, Brennan (1998) assumes that agents learn about the expected return in a Merton (1969) setup; Xia’s (2001) agents learn about the predictive ability of an observable state variable; Brennan and Xia (2001) show that uncertainty about dividend growth may contribute towards an explanation to the equity premium puzzle; Epstein and Miao (2003) solve an equilibrium model where agents have different prior views about the economy; and Pástor and Veronesi (2005a, 2005b) show that uncertainty about future profitability

---

4Williams (1977), Detemple (1986), and Gennotte (1986) are early contributions to asset pricing and portfolio choice under parameter uncertainty. Baron (1974) and McCardle and Winkler (1989) are portfolio choice models with learning where agents display risk-preference. Bayesian learning in a portfolio choice setup can be viewed as a particular definition of the Merton (1973) ICAPM state-variable.
may explain both IPO waves and high Nasdaq valuations in the 1990s.

This paper is closely related to the studies on parameter uncertainty. The life-cycle model of Section 1.2 is a discrete-time analogue of the Brennan (1998) model where an agent accounts for the estimation uncertainty in the expected return. The pivotal difference that generates our non-participation result is that our agent faces trading frictions.

1.1.1.2 Labor Income, Illiquidity, and Trading Constraints

Human capital is an important component of individuals’ wealth: labor income accounts for 75% of consumption (Santos and Veronesi 2005). Many studies have studied the role of human capital in asset pricing, beginning with Mayers (1972). For example, Santos and Veronesi (2005) let agents derive income from two sources, dividends and wage. Their key assumption is that the total income (the sum of wages and dividends) grows over time while the wage share depends on economic conditions. This assumption generates return predictability and the growth/value effect. Lettau and Ludvigson (2004) examine the link between wealth (including human capital) and consumption and find that only permanent wealth changes affect consumer spending. Malloy, Moskowitz, and Vissing-Jørgensen (2005a) find asset pricing success by using firing/hiring data to measure persistent labor income shocks. These studies emphasize the potential role of labor income for explaining individuals’ consumption choices and asset prices. This paper’s equilibrium model is related because we let one agent face the possibility of an unemployment. The key difference to the extant studies is our focus on how the possibility of future unemployment and trading frictions generate non-participation already at earlier periods.

Numerous studies have considered the general effects of asset illiquidity and
non-tradability. For example, Longstaff (2001) shows that an agent who must accumulate or unwind positions over a period of time can behave as if she faced endogenous borrowing and short-sale constraints. Liu, Longstaff, and Pan (2003) show that an agent facing jump risk is less willing to take levered or short positions. Longstaff (2005) considers a two-asset, heterogeneous agents model where one of the assets is traded initially but then enters a blackout period. He finds that this non-tradability can significantly skew agents’ portfolio choices and that liquidity is an important component of an asset’s equilibrium value. Pástor and Stambaugh (2003) find empirically that assets more sensitive to liquidity command a premium over low-sensitivity stocks. The common theme in this literature is that agents endowed with an illiquid asset act more cautiously than they would if the markets were complete and frictionless. In this paper, the portfolio constraints and learning skew the agents’ behavior.

1.1.1.3 The Limited Participation Puzzle

The limited participation puzzle has attracted attention for two reasons. The first line of research examines why so many individuals choose to stay out of the market despite the exceptional historical equity premium. For example, Vissing-Jørgensen (2002b) suggests that the decision to stay out of the market may be optimal for about half of the non-participating individuals even if the fixed costs are relatively modest. However, the estimated costs are too high for the other half for these costs alone to be a reasonable explanation to the limited participation puzzle. Theoretical studies by Dow and Werlang (1992), Ang, Bekaert, and Liu (2005), Epstein and Schneider (2005), and Cao, Wang, and Zhang (2005)

\footnote{For example, suppose an individual has only $5,000 in liquid assets. If the annual stock market participation cost—e.g., brokerage fees and information costs—is, say, $50, the risk-return tradeoff from the market may not be good enough to induce the agent to participate.}
introduce non-standard utility functions to generate non-participation. Dow and Werlang (1992) (a static model), Epstein and Schneider (2005), and Cao, Wang, and Zhang (2005) (dynamic models) rely on ambiguity aversion. The latter papers are related to the present study because the agents in the paper learn over time. Ang, Bekaert, and Liu (2005) generate non-participation by assuming that investors are disappointment averse.

The second line of participation research begins with the idea that investor heterogeneity may generate a higher theoretical equity premium. It is possible that if some investors are shut out of the market, their consumption processes “contaminate” aggregate consumption data. This could lead to falsely reject consumption-based asset pricing models. For example, the agents in Basak and Cuoco (1998) face frictions that shut them out of the market. In equilibrium, these agents’ consumption processes do not covary with aggregate consumption. These studies argue that data on stockholders’ consumption alone may fare better in asset pricing because the non-stockholders do not price the assets. Vissing-Jørgensen (2002a) estimates the bond and stock return Euler equations separately for market participants and non-participants and finds support for this idea. The problem with these studies is that even if they find success, they do not address the question of why non-stockholders are not pricing the assets. Cochrane (2005, pp. 61) concludes his survey of the extant literature as follows:

For example, Mankiw (1986) and others suggest that particular type of heterogeneity in individuals’ marginal rate of substitution could result into higher premium but conclude that the agents still come close to complete risk-sharing even by trading just one asset in a frictionless market. Constantinides and Duffie (1996, pp. 221) note that these negative results are largely due to the assumption that each agent’s labor income share is a stationary process. The paper shows how to match any historical equity premium with time-additive power utility and idiosyncratic income risk. Cochrane (2005, pp. 57) cautions that the Constantinides and Duffie solution may still require unreasonable level of variation in each individuals’ consumption growth: “Can it be true that if aggregate consumption growth is 2%, the typical person you meet either has +73% or −63% consumption growth?”
“Must we use microdata? While initially appealing, however, it’s not clear that the stockholder/nonstockholder distinction is vital. Are people who hold no stocks really not “marginal?” The costs of joining the stock market are trivial... Thus, people who do not invest at all choose not to do so in the face of trivial fixed costs.”

This paper addresses Cochrane’s critique: the agents in our model face no transaction costs but choose to stay out of the market in equilibrium because of the possibility that the short-sale constraints bind in the future. Our paper also complements earlier studies by simultaneously considering both the causes (future learning) and consequences (risk premium) of non-participation.

1.2 An Example of the Feedback Effect: A Life-Cycle Model

This section solves a tractable life-cycle model where a short-sale constrained agent learns about the risk premium over time. We use this model to demonstrate the feedback mechanism before turning to a more realistic equilibrium model in Section 1.3. We discuss two features of this model before detailing our assumptions and solving the model.

First, the agent in our model knows that the true risk premium is constant but is uncertain about its precise value in the beginning. The agent considers the possibility that the risk premium may turn out to be negative. If this happens, the asset becomes effectively useless to the agent because of the short-sale constraints. The assumption that the risk premium can become negative is a shorthand way of modeling subjective investment opportunity sets. For example, an agent who faces permanent labor income shocks (e.g., she may be hired, fired, or tenured)
wants to short the market when her wage covaries sufficiently with the market returns even when the risk premium is restricted to strictly positive values.

Second, the stock price follows a binomial tree with constant up- and down-tick parameters. This means that the agent might optimally take a short position in the asset and that the assumption of short-sale constraints has repercussions. Note that if the return distribution were unbounded from above, the agent would endogenously refrain from any short positions.\(^7\) (The continuous-time equivalent would be either jump-risk or assets that are not always tradable.) Note that our assumption is not very restrictive for two reasons. First, a binomial model converges to a diffusion process as the number of horizons increases and the length of each period shortens. Second, our emphasis is on what happens when the agent faces exogenous constraints on top of the endogenous constraints and not on the exact nature of this additional constraint.

1.2.1 Setup

We make the following assumptions:

- A single agent lives for \(T + 1\) periods, indexed from 0, \ldots, \(T\).

- The agent maximizes power utility over date \(T\) wealth,

\[
U_T(W_T) = \frac{W_T^{1-\gamma} - 1}{1 - \gamma}.
\] (1.1)

- There is a single risky stock and a risk-free asset. These assets are traded each period. The stock price follows a binomial tree (Cox, Ross, and

\(^7\)The agent’s utility function also needs to satisfy the Inada conditions—namely that \(\lim_{W \to 0} U'(W) = \infty\).
Rubinstein 1979, Liu and Neis 2002):

\[ S_{t+1} = \begin{cases} S_t(R_f + u) & \text{with probability } p \\ S_t(R_f - d) & \text{with probability } 1 - p \end{cases} \quad (1.2) \]

where \( ud > 0 \) to rule out arbitrage. The risk-free asset returns \( R_f \) each period for sure.

- The agent decides how much to invest in the risky asset at the beginning of each period after observing the previous period’s realized return.
- The agent cannot short the stock, \( \theta_t \geq 0 \), where \( \theta_t \) is the number of shares.
- The agent knows all the parameters of the economy precisely except for the probability \( p \). The agent has a Beta-distributed prior about \( p \) and updates her beliefs as a Bayesian at the beginning of each period.

The wealth dynamics from these assumptions are

\[ W_t = W_{t-1}(R_f + f_{t-1}\tilde{\epsilon}), \quad (1.3) \]

where \( \tilde{\epsilon} = \begin{cases} u & \text{with probability } p \\ -d & \text{with probability } 1 - p \end{cases} \)

\[ f_{t-1} = \frac{\theta_{t-1}S_{t-1}}{W_{t-1}} \quad \text{(fraction of wealth in stock).} \]

The agent’s problem can be broken into two steps: an inference problem in which the agent updates her estimate of \( p \) and an optimization problem in which the agent chooses the optimal investment given the current wealth and the estimate of \( p \) (Gennotte 1986, Brennan and Xia 2001). We consider first the inference problem.
1.2.2 The Agent’s Inference Problem

The agent has a conjugate prior distribution \( \text{Beta}(\alpha_0, \beta_0) \) about \( p \) at date 0. The assumption about a binomial stock price process and a Beta-distributed prior makes the inference problem particularly tractable. The date \( t \) posterior after observing \( N_t \) positive stock price movements is \( \text{Beta}(\alpha_0 + N_t, \beta_0 + t - N_t) \)-distributed (see, e.g., DeGroot (1970, pp. 160)). The mean and variance of the posterior distribution are given by

\[
E_t(p) = \frac{\alpha_0 + N_t}{\alpha_0 + \beta_0 + t}, \quad \text{var}_t(p) = \frac{(\alpha_0 + N_t)(\beta_0 + t - N_t)}{(\alpha_0 + \beta_0 + t)^2(\alpha_0 + \beta_0 + t + 1)}. \quad (1.4)
\]

We let \( \alpha_t \equiv \alpha_0 + N_t \) and \( \beta_t \equiv \beta_0 + t - N_t \) to denote the date \( t \) belief parameters. The intuition for updating is simple: the parameters of the Beta-distribution keep track of the number of the stock price’s up- and downticks. For example, if the agent starts with parameters \((1, 2)\), the parameters become \((2, 2)\) after an uptick and \((1, 3)\) after a downtick. Figure 1.1 illustrates how the agent’s wealth and beliefs evolve in this problem.
1.2.3 The Agent’s Optimization Problem

We solve the agent’s optimization problem with dynamic programming. The agent receives utility $V_T(W_T) = \frac{W_T^{1-\gamma} - 1}{1-\gamma}$ in the last period. (We assume that $\gamma \neq 1$.)

We begin with the conjecture that the date $t+1$ Bellman equation has the form

$$V_{t+1}(W_{t+1}, (\alpha_{t+1}, \beta_{t+1})) = \frac{W_{t+1}^{1-\gamma}}{1-\gamma} k_{t+1}(\alpha_{t+1}, \beta_{t+1})$$

and then later show that it satisfies this form. With this conjecture, the date $t$ Bellman equation solves

$$V_t(W_t, (\alpha_t, \beta_t)) = \max_{f_t \geq 0} \left\{ E_t [V_{t+1}(W_t(R_f + f_t\epsilon), (\alpha_{t+1}, \beta_{t+1}))] \right\}$$

$$= \max_{f_t \geq 0} \left\{ \frac{W_t^{1-\gamma}}{1-\gamma} \left[ \frac{\alpha_t}{\alpha_t + \beta_t} (R_f + f_t u)^{1-\gamma} k_{t+1}(\alpha_{t+1} + 1, \beta_{t+1}) + \frac{\beta_t}{\alpha_t + \beta_t} (R_f - f_t d)^{1-\gamma} k_{t+1}(\alpha_{t+1}, \beta_{t+1} + 1) \right] \right\}.$$  (1.6)

We let $k_{t+1}^u \equiv k_{t+1}(\alpha_{t+1} + 1, \beta_{t+1})$ and $k_{t+1}^d \equiv k_{t+1}(\alpha_{t+1}, \beta_{t+1} + 1)$ to simplify the notation.

The optimal investment from the first-order condition is

$$f_t^* = \begin{cases} \frac{(\alpha_t u k_{t+1}^u)^{1/2} - (\beta_t d k_{t+1}^d)^{1/2}}{(\alpha_t u k_{t+1}^u)\frac{1}{2} d + (\beta_t d k_{t+1}^d)\frac{1}{2} u} R_f & \text{if } \alpha_t u k_{t+1}^u > \beta_t d k_{t+1}^d \\ 0 & \text{otherwise.} \end{cases}$$  (1.7)

The following proposition gives the functional form of the coefficient $k_t(\alpha_t, \beta_t)$:

**Theorem 1.** The coefficient $k_t(\alpha_t, \beta_t)$ that satisfies the Bellman equation (Eq.
1.6) is given recursively by

\[
\begin{align*}
  k_t(\alpha_t, \beta_t) &= \begin{cases} 
  (u + d)^{1-\gamma} \left( (\alpha_t k_{t+1} u)^{\frac{1}{\gamma}} d + (\beta_t k_{t+1} d)^{\frac{1}{\gamma}} u \right)^{\frac{1}{\gamma}} R_f^{1-\gamma} & \text{if } \alpha_t u k_{t+1} > \beta_t d k_{t+1} \\
  \frac{\alpha_t k_{t+1}^u + \beta_t k_{t+1}^d}{\alpha_t + \beta_t} R_f^{1-\gamma} & \text{otherwise}
  \end{cases} \\
  k_T(\alpha_T, \beta_T) &= 1.
\end{align*}
\]

(1.8) (1.9)

Proof of Proposition 1. This can be proven by substituting the optimal investment (Eq. 1.7) into the Bellman equation (Eq. 1.6). The functional form of \( k_t(\alpha_t, \beta_t) \) given in the proposition satisfies the resulting equation.

We note two issues before characterizing the agent’s behavior in the model. First, the optimal investment depends only on current wealth and beliefs and not on the sequence of outcomes. Second, for comparison and for future reference, we can infer the behavior of a ‘no parameter uncertainty’ agent from the solution. If the agent knows the parameter \( p \) precisely—or does not update her beliefs over time—the coefficient \( k_t(\alpha_t, \beta_t) \) becomes a function of time alone. It follows (from Eq. 1.7) that the date \( t \) optimal investment of the ‘no parameter uncertainty’ agent is

\[
  f_t^* = \begin{cases} 
  \frac{(p u)^{\frac{1}{\gamma}} - ((1 - p) d)^{\frac{1}{\gamma}}}{(p u)^{\frac{1}{\gamma}} d + ((1 - p) d)^{\frac{1}{\gamma}} u} R_f & \text{if } p u > (1 - p) d \\
  0 & \text{otherwise}.
  \end{cases}
\]

(1.10)

The agent invests a positive amount if the risk premium is positive. Otherwise the agent would short the stock.
1.2.4 Characterizing Optimal Behavior

We now characterize the agent’s optimal behavior in the model. All proofs are in Appendix 1.5. We begin with the key result that an agent may stay out of the market even when the risk premium is strictly positive:

**Theorem 2.** The optimal investment is decreasing in the variance of the prior distribution if $\gamma > 1$ and increasing in the variance if $\gamma < 1$.

**Corollary 1.** $\exists \delta > 0$ such that an agent with $\gamma > 1$ does not invest when $E_0(\tilde{e}) \leq \delta$ and an agent with $\gamma < 1$ invests a strictly positive amount when $E_0(\tilde{e}) \geq -\delta$.

These results distinguish our life-cycle model from classical models where an agent invests a positive amount if the risk premium is positive. A mildly risk-averse agent with $\gamma < 1$ prefers uncertainty: the investment is increasing in the variance of the prior. This is the same as saying that the agent wants to take an actuarially unfair gamble. The agent’s willingness to pay to learn does not generate this behavior—note that the agent observes realized returns even without an investment. The reason is that a $\gamma < 1$ agent “weights” positive outcomes more than negative outcomes. Starting from a situation with a negative risk premium, the agent knows that the risk premium may turn out to be positive. The agent maximizes her wealth in the states with good investment opportunities by investing today.

An agent with $\gamma > 1$, on the other hand, needs to be compensated for the extra source of risk brought on by parameter uncertainty. This generates a non-participation region: faced with enough uncertainty, an agent with $\gamma > 1$ allocates everything in the risk-free asset even if her prior about the risk premium is strictly positive. This difference to the $\gamma < 1$ case arises because this more risk-averse
agent weights negative outcomes more than positive outcomes. Starting from a situation with a positive risk premium, the agent knows that the risk premium may turn out to be negative. The agent maximizes her wealth in the states with poor investment opportunities by staying out today.\(^8\)

It is straightforward to show that if the short-sale constraints are lifted, the agent behaves similar to the Brennan (1998) agent. The future learning still matters—e.g., an agent with risk-aversion above \(\gamma > 1\) always invests proportionally less than what she would invest in absence of parameter uncertainty—but the investment is always strictly positive if the risk premium is positive.\(^9\) The non-participation result requires that the short-sale constraints bind in some future states—i.e., that the expected risk premium can turn negative. As noted earlier, this assumption is a shorthand way of modeling subjective investment opportunity sets. For example, an agent may invest in an MBA degree and receive labor income out of this degree for the rest of her life.

The following proposition determines what beliefs an agent who knows “nothing” about the expected return must have about \(p\) to invest in the asset:

**Theorem 3.** An agent with maximally dispersed prior who faces two periods of

\(^8\)The interpretation of differences in weighting is intuitive. From the form of the value function, the solution to the model is similar identical to the no-learning case except that the agent “weights” different outcomes using \(k_t\)s as the weights. To see how these weights change, consider the last period of investment (time \(T - 1\)). First, if an agent with \(\gamma > 1\) does not invest, \(k_{T-1}(\alpha_{T-1}, \beta_{T-1}) = 1\), and if the agent invests, \(k_{T-1}(\alpha_{T-1}, \beta_{T-1}) < 1\). Hence, the agent weights poor future opportunities more, or put differently, the indirect utility is convex in some regions. Second, if an agent with \(\gamma < 1\) does not invest, \(k_{T-1}(\alpha_{T-1}, \beta_{T-1}) = 1\), and if the agent invests, \(k_{T-1}(\alpha_{T-1}, \beta_{T-1}) > 1\). Hence, the agent weights good future opportunities more—the indirect utility function is more concave in some regions.

\(^9\)The non-participation result does not depend on the assumption that there is no intermediate consumption. The solution is nearly identical with time-separable power utility; the distinction is that each period the agent first consumes some fraction of her wealth and then allocates the rest between the assets.
investment \((T = 2)\) invests if and only if

\[
E_0(p) \geq \frac{1}{1 + \frac{u}{d} \left( \frac{u + d}{d} \right)^{1 - \gamma}}.
\]

(1.11)

This proposition assumes that the agent has a completely non-informative prior—i.e., the variance of the prior is maximized by fixing \(E_0(p)\) and letting \(\alpha, \beta \to 0\)—and gives the boundary for the uptick probability that guarantees a positive investment. Note that because the optimal investment is decreasing in the variance of the prior for \(\gamma > 1\)-agents, this proposition gives the upper-bound of what these agents require of \(E_0(p)\) when their priors became completely uninformative. We can use this result show that the non-participation region can be substantial even in a three-period setting. For example, suppose that the risk premium is symmetric around zero \((u = d)\). Then, while an agent with \(\gamma = 2\) may require that the probability of an uptick is \(E_0(p) \geq 0.67\), an agent with \(\gamma = 5\) may stay out of the market until \(E_0(p) \geq 0.94!\) If \(u = d = 20\%\), these boundaries correspond to (expected) risk premia of 6.7\% and 17.6\%, respectively.

Suppose that there is an outsider who observes the agent’s behavior, ignores parameter uncertainty, and backs out what the agent’s behavior implies about her risk-aversion. The following proposition shows this implied risk-aversion has an intuitive form:

**Theorem 4.** An outsider who sets \(p = E_0(p)\) infers the agent’s risk-aversion as being

\[
\hat{\gamma} = \gamma \frac{\log \left( \frac{\alpha u}{\beta d} \right)}{\log \left( \frac{\alpha u}{\beta d} \right) + \log \left( \frac{k^u}{k^d} \right)} \quad \text{when} \ \alpha u > \beta d k^d \ \text{and} \ \alpha u > \beta d.
\]

(1.12)
This implied risk-aversion is strictly higher than the true risk-aversion $\gamma$ if $\gamma > 1$ and strictly less if $\gamma < 1$.

This measure quantifies the impact of parameter uncertainty and short-sale restrictions on portfolio choice. We know from Proposition 2 that an agent with $\gamma > 1$ always appears strictly more risk-averse than she really is to an outsider who ignores parameter uncertainty. At the limit, an agent who stays out of the market when $E_t[\tilde{\epsilon}] > 0$ appears infinitely risk-averse.

1.2.5 Examples

Figure 1.2 illustrates the results by plotting the optimal investment for an agent with a risk-aversion of $\gamma = 2$ and an investment horizon of $T = \{10, 50\}$ periods. The optimal investment is drawn as a function of the parameters of the prior distribution, $(\alpha, \beta)$. The non-participation region is the white area between the $45^\circ$ line and the filled area. Note that a Merton (1969) or Brennan (1998) agent would enter the market for all parameters to the right of the diagonal. In contrast, our agent stays out of the market for a wide range of parameters because of the risk of binding short-sale constraints.

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The figure also plots the implied risk-aversion for the $T = 10$ period case. The implied risk-aversion increases sharply when the agent moves towards the non-participation region or when the variance of the prior increases. Note that the implied risk-aversion is significantly above the true value of $\gamma = 2$ for all parameters in the figure.\(^{10}\) These life-cycle model results show that a short-

\(^{10}\)These plots are for an investor with moderate risk-aversion, $\gamma = 2$, and the results are more dramatic for more risk-averse agents. Many studies find relative risk-aversions above two. For example, Nielsen and Vissing-Jørgensen (2005) get an estimate around 5 from data on labor incomes and educational choices; Halek and Eisenhauer (2001) obtain a distribution of estimates with a median of 0.89 and a mean of 3.4 with insurance data; Bliss and Panigirtzoglou
An agent with power-utility over terminal date $T$ wealth trades a risky stock. The stock price follows a binomial process: $S_{t+1} = S_t(R_f + u)$ with probability $p$ and $S_{t+1} = S_t(R_f - d)$ with probability $1 - p$. The agent has a Beta-distributed prior about $p$ and updates her beliefs as a Bayesian. This figure sets $R_f = 1$, $u = d = 0.2$.

Panels A and B show optimal investments for an agent with risk-aversion $\gamma = 2$ when there are $T = 10$ or $T = 50$ periods of investment. The optimal investment is drawn as a function of the parameters of the prior distribution, $(\alpha, \beta)$. The 45° line is the fair-gamble threshold; i.e., when $\alpha = \beta$, the agent’s prior about the excess return is zero. The white area between the diagonal and the filled region is the non-participation region where the agent does not invest despite a positive risk premium. Panel C shows the implied risk-aversion for the $\gamma = 2$, $T = 10$ case. The implied risk-aversion (from Eq. 1.12) is the solution to an inference problem: how risk-averse does the agent appear to an outsider who ignores parameter uncertainty. The $z$-axis is truncated for implied risk-aversions above ten.

Sale constrained investor with limited information may optimally stay out of the market even if the (perceived) risk premium is sizable.

We have focused on the possibility that a short-sale constrained agent may stay completely out of the market because of the feedback from the short-sale constraints. However, more generally, the possibility of binding constraints always

(2004) derive (“representative agent”) mean estimates between 2 and 8 from the FTSE100 and S&P 500 option prices; Brennan and Xia (2001) suggest that a relative risk-aversion as high as 15 may be reasonable on theoretical grounds.
Figure 1.2: Optimal Investment and Implied Risk-Aversion. (cont’d)

reduces the optimal date 0 portfolio holdings of a $\gamma > 1$ agent. For example, suppose that a $\gamma = 2$ agent with an initial wealth of $10,000 is offered two double-or-nothing gambles and that the agent has a prior Beta(0.23, 0.2) about the probability of winning ($E_0(p) = 0.535$). The optimal date 0 investment is very sensitive to learning and constraints:

- **No learning, no short-sale constraints.** The agent takes $p = 0.535$ as
a fixed parameter and invests $349.

- **Learning, no short-sale constraints.** The agent invests $289.

- **Learning, short-sale constraints.** The agent stays out.

If we now fix the mean and decrease the variance of the prior by moving to Beta(0.575, 0.5)-distribution, the second scenario investment increases to $296 and the optimal investment under short-sale constraints becomes positive but is still only $162. We now turn to an equilibrium model that dispenses some of our unrealistic assumptions and shows that the same non-participation mechanism arises with permanent labor income shocks.

### 1.3 An Equilibrium Model with Non-Participation

This section solves an heterogeneous-agents equilibrium model where one of the agents may lose her job at a later date. The purpose of this model is two-fold. First, we show that the uncertainty about *future* labor income can significantly skew *today’s* decisions when the agent is restricted from hedging with the risky asset. This generates the same type of non-participation as observed in Section 1.2’s life-cycle model: the agent stays out only because of the *risk* of binding constraints. Second, we examine what consequences this type of non-participation has on the risk premium. For example, the first intuition would be that an introduction of short-sale constraints (if they matter at all) would reduce the risk premium: because the remaining agents only need to hold the entire market and not more, they require smaller compensation for risk.\textsuperscript{11} However, we show that

\textsuperscript{11}For example, Cao, Wang, and Zhang (2005) generate non-participation with ambiguity aversion and find that the risk premium in the full economy is always higher than what it is in the limited participation economy.
non-participation from the feedback effect can lead to a higher risk premium.

1.3.1 The Economy

We assume the following:

- There are two agents, indexed $i \in \{A, B\}$, who live for three periods, $t = 0, 1, 2$. Trading takes place at dates 0 and 1.
- The agents maximize power utility over date 2 wealth,
  \[ U^i = \frac{W^i_2^{1-\gamma} - 1}{1 - \gamma}. \] (1.13)
  The agents have the same risk-aversion parameter.
- There is a single risky asset in unit net supply. This asset pays a high dividend $D_h$ with probability $p$ and a low dividend $D_l$ with probability $1 - p$ at dates 1 and 2, where $D_h > D_l$.
- A risk-free technology with a gross-return of $R$ is in perfectly elastic supply.
- The agents are initially endowed shares $\bar{\theta}^i$ and consumption good $\bar{X}^i$.
- There are short-sale restrictions on the risky asset, $\theta^i_t \geq 0$, where $\theta^i_t$ is the agent $i$’s tradable asset holdings at date $t$.
- The agents are endowed with a non-tradable asset (e.g., human capital) that pays income (e.g., wage) at dates 1 and 2. If the date 1 dividend is high, agent $i$ receives a payoff of $y^{i,h}_1$ at date 1. If the dividend is low, the agent receives $y^{i,l}_1$. 21
the agent is endowed with $\theta^i, X^i$.

\[D_{h}, y_{i,h}^{i,h} \quad D_{l}, y_{i,l}^{i,l} \]
\[D_{h}, y_{2,h}^{i,h} \quad D_{l}, y_{2,l}^{i,l} \]
\[D_{h}, y_{2,h}^{i,h} \quad D_{l}, y_{2,l}^{i,l} \]

Date 0 Date 1 Date 2

Figure 1.3: The Timeline for Agent $i$ in the Equilibrium Model

- The date 2 income is \textit{contingent} on the date 1 dividend. If the date 1 dividend is high, the date 2 income is $y_{2,h}^{i,h}$ or $y_{2,l}^{i,l}$ and if the dividend is low, the date 2 income is $y_{2,l}^{i,l}$ or $y_{2,l}^{i,l}$.

The last assumption lets agents resolve uncertainty about their future income at date 1. A natural interpretation for this date 1 signal is the risk of losing a job due to a macroeconomic shock. The key insight captured by the model is that permanent labor income shocks are positively correlated with macroeconomic shocks. We assume two states and perfect correlation between the assets for tractability.\textsuperscript{12}

Figure 1.3 shows the timeline of the events for Agent $i$. The agent starts at date 0 with an endowment of shares and the consumption good. She decides how much to hold of the risky asset and puts the rest into the risk-free asset. The agent receives dividend and non-tradable asset income at date 1. She also learns the values of the date 2 incomes and then makes the date 1 investment decisions. Finally, the agent receives date 2 payoffs and consumes her terminal wealth.

\textsuperscript{12}The assumption that labor income and dividend streams are perfectly correlated is a very particular assumption. This does, however, capture the idea that labor income shocks are affected by market conditions: when an agent’s labor income stream covaries positively with the dividends, the agent is effectively already invested in the market by default, creating a hedging demand.
We proceed as follows in the remainder of this section. First, we solve the equilibrium prices in an economy that does not have short-sale constraints. Second, we compute the equilibrium prices for a particular type of a short-sale constrained economy (“a non-participation economy”) and give conditions under which these prices constitute equilibrium. Third, we show that the risk premium in the constrained economy is higher than in an unconstrained economy (i.e., an otherwise identical economy but without short-sale constraints) in particular when one of the agents is close to being indifferent between participating and not participating.

1.3.2 Equilibrium without Short-Sale Constraints

We first solve the date 1 problem and then move backwards to the date 0 problem. Note that the agent $i$’s wealth constraints bind with equality:

- **Date 0:** \[ \bar{\theta}^i P_0 + \bar{X}^i = \theta^i_0 P_0 + X^i_0 \]
- **Date 1:** \[ \theta^i_0 (P_1 + \bar{D}_i) + X^i_0 R + y^i_1 = \theta^i_1 P_1 + X^i_1 \]
- **Date 2:** \[ \theta^i_1 D_2 + X^i_1 R + y^i_2 = W^i_2 \]

where $X^i_t$ is the amount borrowed or lent at the rate $R$. After substituting out $X^i_t$, we have

\[ W^i_2 = W^i_1 R + \theta^i_1 (D_2 - P_1 R) + y^i_2, \tag{1.14} \]

where \[ W^i_1 \equiv (\bar{X}^i + \bar{\theta}^i P_0) R + \theta^i_0 (P_1 + D_1 - P_0 R) + y^i_1. \]
1.3.2.1 Date 1 Decisions and Prices

Agent $i$’s date 1 Bellman equation in state $s = \{h, l\}$ solves

$$V_{1,s}(W^i_1) = \max_{\theta^i_1} \left\{ E_1 \left[ \frac{(W^i_1 R + \theta^i_1 (\tilde{D}_2 - P_1 R) + \tilde{y}^i_{2,s})^{1-\gamma}}{1 - \gamma} \right] \right\}. \quad (1.15)$$

The optimal demand from the first-order condition is\(^{13}\)

$$\theta^*_1(W^i_1) = \frac{[p(D_h - P^*_1 R)]^{-\gamma}(W^i_1 R + \tilde{y}^i_{2,s}) - [(1 - p)(P^*_1 R - D_l)]^{-\gamma}(W^i_1 R + \tilde{y}^i_{2,s})}{[p(D_h - P^*_1 R)]^{-\gamma}(D_h - P^*_1 R) + [(1 - p)(P^*_1 R - D_l)]^{-\gamma}(P^*_1 R - D_l)}. \quad (1.16)$$

The date 2 equilibrium price results from summing the agents’ first order conditions and using the market-clearing condition, $\sum_i \tilde{\theta}^i = \sum_i \theta^*_i = \sum_i \theta^i_1 = 1$:

$$P^*_1 = \frac{1}{R} \frac{p(\omega^h_{1,s})^{-\gamma} D_h + (1 - p)(\omega^l_{1,s})^{-\gamma} D_l}{p(\omega^h_{1,s})^{-\gamma} + (1 - p)(\omega^l_{1,s})^{-\gamma}}, \quad (1.17)$$

where $\omega^s_{1,s} \equiv R^2 \sum_i \tilde{X}^i + R \left( D_1 + \sum_i \tilde{y}^i_{1,s} \right) + \sum_i \tilde{y}^i_{2,s} + D_{s'}$.

The price is a weighted average of date 2 dividends where the weights are functions of total wealth in the two states and their probabilities. The initial distribution of allocations does not matter because both agents’ non-tradable asset income is perfectly correlated with dividends: the agents can use the tradable asset to hedge perfectly against the income risk.

\(^{13}\)Substituting the optimal demand back into Eq. 1.15, the value function becomes

$$V_{1,s}(W^i_1) = \frac{[(D_h - D_l)W^i_{1,s} + c^i,s]^{1-\gamma}}{1 - \gamma} k_s. \quad (See \ Eq. 1.18 \ for \ c^i,s \ and \ k_s.)$$

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1.3.2.2 Date 0 Decisions and Price

Agent $i$’s date 0 Bellman equation solves

$$V_i^0(\bar{X}, \bar{\theta}) = \max_{\theta_0} \left\{ E_0 \left[ \tilde{W}_1 \right] \right\},$$

where $\tilde{W}_1 \equiv (D_h - D_l)(\bar{X} + \bar{\theta} P_0)R^2 + \theta_0(\bar{P}_1 + \bar{D}_1 - P_0 R) + \bar{y}^1 R + \bar{c}^i$,

$$c_{i,s} \equiv y_{i,s}^1 (P_{i,s} R - D_l) + y_{i,s}^2 (D_h - P_{i,s} R),$$

$$k^s \equiv \left( p^\gamma (P^s R - D_l) \right)^{1-\frac{1}{\gamma}} + (1 - p)^{\gamma} (D_h - P^s R)^{1-\frac{1}{\gamma}} \gamma. \quad (1.18)$$

The optimal demands can be solved from the first-order conditions. The date 0 equilibrium price follows from summing the agents’ first order conditions and using the market-clearing condition:

$$P^s_0 = \frac{1}{R} \frac{p k^h \omega_{0,h} (P^h - D_h) + (1 - p) k^l \omega_{0,l} (P^l + D_l)}{p k^h \omega_{0,h} + (1 - p) k^l \omega_{0,l}}, \quad (1.19)$$

where $\omega_{0,s} = (D_h - D_l) \left( R^2 \sum_i X_i + R (P^s + D_s + \sum_i y_{i,s}^1) \right) + \sum_i c_{i,s}.$

The equilibrium price is a weighted average of date 1 dividends and the prices in the two states. The weights are functions of total date 1 wealth in the two states and their probabilities. The following proposition summarizes these results:

**Theorem 5.** The equilibrium prices in the unconstrained economy are given by Eqs. 1.17 (the date 1 prices) and 1.19 (the date 0 price).

Note that the prices in the unconstrained economy do not depend on how the initial allocation is distributed between the two agents because the markets are effectively complete. Hence, the prices in the economy would be the same if there was only a representative agent who received all the endowments.
1.3.3 Equilibrium with Short-Sale Constraints

The non-tradable asset income in the economy can generate negative asset demands. For example, suppose that Agent A’s income covaries positively with dividends after a low date 1 dividend but that Agent B’s income is constant. Agent A would then short the asset after a low dividend if the covariance were sufficiently high. The introduction of short-sale constraints has two effects. First, the direct consequence is that Agent A increases her date 1 holdings after a low date 1 dividend from negative to nothing. Second, the restriction on the agent’s ability to hedge at date 1 may induce Agent A to reduce her date 0 holdings. This effect may be significant enough to let Agent B hold the whole supply while Agent A exits the market. We now construct this equilibrium.

1.3.3.1 Equilibrium Prices in a Non-Participation Economy

The equilibrium prices cannot, in general, be solved in closed-form when there are short-sale constraints.\textsuperscript{14} We focus on an exception where Agent A (i) has no initial endowment of the tradable asset, (\textit{ii}) optimally decides not to hold any asset at date 0 or at date 1 after a low dividend, and (\textit{iii}) has a holding between 0 and 1 at date 1 after a high dividend. (We later give the conditions to verify the optimality.) We also add the following assumptions for the sake of tractability:

1. The risk-free asset yields $R = 1$.

2. Agent A receives non-tradable asset income only at date 2.

3. Agent B does not receive non-tradable asset income.

\textsuperscript{14}The difficulty is that if the date 1 constraints bind for one agent (i.e., the marginal utility at zero holdings is negative), the market-clearing conditions together with the first-order conditions for the remaining agents are not (generally) enough to solve for equilibrium prices.
4. The agents are not endowed any consumption good, $\bar{X}^A = \bar{X}^B = 0$.

(Henceforth, we omit agent and date identifiers from $y$ because there is no ambiguity; we write Agent B’s date 2 income as $y_{s'}^s$ for $s, s' \in \{h, l\}$.) The following pricing formulas follow directly from Proposition 5.

**Corollary 2.** Non-participation equilibrium is equilibrium where Agent A has zero demand for the risky asset at date 0 and at date 1 after a low dividend. Both agents have strictly positive demands at date 1 after a high dividend. The non-participation equilibrium prices are

\[
P^h_1 = \frac{p(2D_h + y_h^h)^{-\gamma}D_h + (1 - p)(D_h + D_l + y_h^l)^{-\gamma}D_l}{p(2D_h + y_h^h)^{-\gamma} + (1 - p)(D_h + D_l + y_h^l)^{-\gamma}}, \quad (1.20)
\]
\[
P^l_1 = \frac{p(D_h + D_l)^{-\gamma}D_h + (1 - p)(2D_l)^{-\gamma}D_l}{p(D_h + D_l)^{-\gamma} + (1 - p)(2D_l)^{-\gamma}}, \quad (1.21)
\]
\[
P_0 = \frac{pk^h(P^h_1 + D_h)^{1-\gamma} + (1 - p)k^l(P^l_1 + D_l)^{1-\gamma}}{pk^h(P^h_1 + D_h)^{-\gamma} + (1 - p)k^l(P^l_1 + D_l)^{-\gamma}}, \quad (1.22)
\]

where $k^s = \left( p^{\frac{1}{\gamma}}(D_h - P^s_1)^{\frac{1}{\gamma}-1} + (1 - p)^{\frac{1}{\gamma}}(P^s_1 - D_l)^{\frac{1}{\gamma}-1} \right)^{\gamma}$.

Note that the equilibrium price after a high dividend is the same as it is in the unconstrained economy. Also, the distribution of Agent A’s non-tradable asset income after a low dividend does not affect any of the prices. We now give the conditions on $\{y_h^l, y_h^h, y_l^h, y_h^l\}$ that guarantee that the prices in Eqs. 1.20, 1.21, and 1.22 constitute equilibrium. We also show that there is such an equilibrium. The proof is in the appendix.

**Theorem 6.** The prices in Corollary 2 are the market-clearing prices if

- Conditions 1A and 1B (Agent A’s optimal holding between zero and one
after a high date 1 dividend):

\[
\frac{y_h^l}{y_h^h} \leq \frac{2D_h}{D_h + D_l}, \quad (1A)
\]
\[
y_h^l - y_h^h \leq D_h - D_l. \quad (1B)
\]

- **Condition 2** (Agent A’s optimal holding zero after a low date 1 dividend):

\[
\frac{y_l^h}{y_l^l} \geq \frac{D_h + D_l}{2D_l}. \quad (1.24)
\]

- **Condition 3** (Agent A’s optimal holding zero at date 0):

\[
\left[ \frac{p(2D_h + y_h^h)^{-\gamma}y_h^h + (1 - p)(D_h + D_l + y_h^h)^{-\gamma}y_l^h}{p(2D_h + y_l^h)^{-\gamma}2D_h + (1 - p)(D_h + D_l + y_l^h)^{-\gamma}(D_h + D_l)} \right]^{-\gamma} \leq \frac{p(y_l^h)^{-\gamma} + (1 - p)(y_l^l)^{-\gamma}}{p(D_h + D_l)^{-\gamma} + (1 - p)(2D_l)^{-\gamma}}. \quad (1.25)
\]

This system of inequalities has multiple solutions \( \{y_h^h, y_h^l, y_l^h, y_l^l\} \) for any \( D_h > D_l \).

Conditions 1A, 1B, and 2 are intuitive. Conditions 1A and 1B state that Agent A’s income cannot have a too high or too low cash-flow covariance with the tradable asset after a high date 1 dividend. Otherwise, the agent would want to short or hold more than the entire supply of the asset, respectively. Condition 2 gives the boundary for the ratio of Agent A’s income after a low dividend that ensures that the agent does not want to hold any of the asset—note that this condition mirrors Condition 1A.\(^{15}\)

\(^{15}\)Note that there is a minor openness consideration with Conditions 1A, 1B, and 2 in Proposition 6. These conditions must be satisfied with strict inequality for equilibrium to hold for sure. Note that if this is not the case—i.e., if one of the agents is indifferent between participating and not—the agent is precisely at the kink of her indirect utility function at date 1. Then, the date 0 zero analysis of how an infinitesimal increase in the date 0 holdings affects the
The expression in Condition 3 is less obvious. It states that Agent A must derive higher expected utility from staying out of the market than from buying an infinitesimal amount of the asset at the equilibrium price. Note that the LHS of Condition 3 is decreasing in $y^h_l$ and $y^l_l$, and the RHS is decreasing in $y^h_l$ and $y^l_l$. Hence, this condition says that the date 2 income following a low date 1 dividend must be small relative to the high-state income. If an agent’s income is low after a low dividend, an agent ending up in this state has high marginal utility. Hence, the intuition for the condition is that the marginal utility in the upstate must be sufficiently lower than the marginal utility in the downstate. If this is the case, the agent wants to hedge against the risk of ending up in the downstate by staying out of the market at date 0. (Note that Condition 3 is always satisfied for sufficiently small $y^h_l$ and $y^l_l$. In addition, agents with log-preferences can meet all the conditions. The reason why these agents do not behave myopically is that the date 1 market incompleteness creates a kink in these agents’ indirect utilities.\(^{16}\)

1.3.3.2 Example

Figure 1.4 shows feasible parameters for the non-tradable asset income after a low date 1 dividend, \{y^h_l, y^l_l\}, that generate non-participation equilibria. The figure assumes that both agents have log-preferences and that Agent A receives constant income after a high date 1 dividend. (See the figure text for the parameters of the example economy.) The x-axis is the value of low-state income and y-axis is the value of the high-state income. Note that the cash-flow covariance between Agent A’s non-tradable asset and the tradable asset is positive when $y^h_l > y^l_l$; agent’s utility is invalid. We write these conditions with non-strict inequality to remind that the equalities denote the agents’ indifference points.

\(^{16}\)In related research, Cochrane, Longstaff, and Santa-Clara (2005) and Longstaff (2005) analyze “Two Lucas (1978) Trees” models and show that market-clearing and black-out (i.e., non-tradability) periods also cause log-utility investors to behave non-myopically.
Figure 1.4: Non-Participation Equilibrium.
This figure shows feasible parameters for non-tradable asset income \( \{y^h, y^l\} \) that generate the non-participation equilibrium of Proposition 2. The following parameters are fixed: \( D_h = 1.1, \ D_l = 1, \ \gamma = 1, \ p = 0.5, \ y^h = y^l = 0.1 \) (i.e., the agents have log-preferences). The \( x \)-axis is the low-state income after a low date 1 dividend, \( y^l \). The \( y \)-axis is the high-state income after a low date 1 dividend, \( y^h \). The colored grid indicates the set of parameters that generate the non-participation economy. This region is divided into two sections to indicate what type of a position Agent A would take in an unconstrained economy. The darker region denotes cases where an unconstrained Agent A takes a short position in the risky asset at date 0. The lighter region denotes parameters where an unconstrained Agent A takes a long position in the risky asset at date 0.

hence, by keeping the \( x \)-axis value fixed and increasing the value on the \( y \)-axis, the tradable asset becomes increasingly worse to the agent.\(^{17}\)

The non-participation equilibrium is divided into two areas to indicate what type of date 0 position Agent A would take in an unconstrained economy (i.e., an otherwise identical economy but without short-sale constraints). The darker area represents cases where Agent A takes a short position at date 0 in the unconstrained economy. The lighter area consists of more interesting equilibria where Agent A’s strictly positive date zero holding disappears when the trading

\(^{17}\)Note that the \( y \)-axis in the figure is truncated at 0.5—the set of equilibria continues beyond the boundaries of the figure.
constraints are introduced. Here, the agent would take a long position if it were not for the possibility of binding short-sale constraints.\(^{18}\)

### 1.3.4 Risk Premium in the Non-Participation Economy

We now discuss how the date 0 risk premium changes when we move from the unconstrained economy to the short-sale constrained economy. The proof of the following proposition is in the appendix:

**Theorem 7.** The expected date 0 payoff is always higher in the constrained economy than in an otherwise identical economy but without short-sale constraints ("the unconstrained economy"). The risk premium is higher in the constrained economy when

1. Agent A is close to participating at date 1 after a low dividend \( \frac{y^h}{y^l} \approx \frac{D_h + D_l}{2D_l} \); see Condition 2 of Proposition 6)

2. and Agent A’s date 2 nonfinancial income in the upstate is small relative to the dividends \( y^h, y^l \ll D_h, D_l \).

These are sufficient conditions.

The second part of the proposition gives sufficient conditions for the date 0 price to be strictly lower in the constrained economy. Because the expected payoff in the constrained economy is always higher than the expected payoff in

\[^{18}\]Agent A takes a long position at date 0 in the unconstrained economy if

\[
\frac{p(2D_h + y^h)^{1-\gamma} + (1-p)(D_h + D_l + y^l)^{1-\gamma}}{p(2D_h + y^h)^{-\gamma}(2D_h) + (1-p)(D_h + D_l + y^l)^{-\gamma}(D_h + D_l)} \\
\leq \frac{p(D_h + D_l + y^l)^{1-\gamma} + (1-p)(2D_l + y^l)^{1-\gamma}}{p(D_h + D_l + y^l)^{-\gamma}(D_h + D_l) + (1-p)(2D_l + y^l)^{-\gamma}(2D_l)}, \quad \text{(Condition 3')} \quad (1.26)
\]
the unconstrained economy, less strict conditions in practice guarantee that the
difference in risk premia between the constrained and unconstrained economies
is positive. The necessary condition is that the date 0 price cannot change \textit{too much} in response to the introduction of constraints to reverse the effect of the
higher expected payoff.

However, these stricter conditions have an interesting and important implica-
tion: the risk premium in the constrained economy is particularly high (relative
to the unconstrained economy) when it is most puzzling that Agent A stays out—i.e., when Agent A is indifferent between participating and not participat-
ing at date 1. Moreover, these necessary conditions turn out to be the same
that guarantee that Agent A buys a strictly positive amount of the asset in the
unconstrained economy.\footnote{To see, this suppose that \(y_l^h = \alpha(D_h + D_l)\) where \(\alpha > 0\) and let Condition 2 first bind
with equality, \(y_l^l = \frac{2D_l}{D_h + D_l} y_l^h\). Then, taking the limit \(y_l^h, y_l^l \to 0\) of the LHS of Condition 3',
the condition becomes \(1 \leq 1 + \alpha\). Because this holds with strict inequality, we can choose
\(y_l^l < \frac{2D_l}{D_h + D_l} y_l^h\) and get equilibrium. This shows that Agent A would take a long position in the
risky asset at date 0 if it were not for the future short-sale constraints.}

Figure 1.5 uses the same parameters as the example economy of Figure 1.4 and
draws the difference in the risk premia between the constrained and unconstrained
economies. By Corollary 2, the risk premium in the constrained economy does
not depend on the income ratio after a low dividend. Hence, in the figure, the
risk premium in the constrained economy is always 0.06\% because everything
but \(\{y_l^h, y_l^l\}\) is fixed. However, the risk premium in the unconstrained economy
depends on these values. In particular, note that unconstrained-economy risk
premium is decreasing in both \(y_l^l\) and \(y_l^h\). Hence, moving towards the region
where the agent would participate if it were not for the short-sale constraints
(i.e., towards higher values of \(y_l^h\)), the risk premium increases. Note that in this
case the area with positive (relative) risk premium almost coincides with the
This figure shows the relative risk premia in equilibria where Agent A does not hold any risky asset at date 0 or after a low dividend at date 1 ("non-participation equilibrium"). The relative risk premium is defined as the difference between the risk premium in the constrained economy and the risk premium in otherwise identical but unconstrained economy, $r_{\text{const}} - r_{\text{unconst}}$. The change in the shading from dark to light indicates where the risk premium difference turns positive. The following parameters are fixed: $D_h = 1.1$, $D_l = 1$, $\gamma = 1$, $p = 0.5$, $y_h^l = y_h^l = 0.1$ (i.e., the agents have log-preferences).

The $x$-axis is the low-state income after a low date 1 dividend, $y_l$. The $y$-axis is the high-state income after a low date 1 dividend, $y_h$. The equity-premium in the short-sale constrained economy is constant, 0.06%, because it does not depend on parameters $\{y_h^l, y_l^l\}$.

Although this does not always need to be the case, Proposition 7 ensures that if we move far enough into the correct direction without breaking equilibrium, the risk premium spread is eventually positive.

1.3.5 Implications of the Model

Our model generates non-participation by assuming that individuals may face binding trading restrictions in the future. A natural interpretation for the date 1 signal is the risk of losing a job due to a macroeconomic shock. Poor economic
growth induces firms to cut workforce and dividends. If an agent loses her job because of an economic downturn (i.e., a low date 1 dividend), she faces a positive covariance between her wage and dividends; a subsequent turnaround in the economy makes it more probably that the agent is hired and dividends increase. However, a further weakness in the economy means that dividends stay low and the agent is likely to remain jobless. In this context, the short-sale constraint assumption states that the jobless agent at date 1 finds it impossible to open a margin account and short the market to hedge against the risk of not finding a new job. If there is no initial downturn at date 1 (i.e., a high date 1 dividend), the agent retains her job.\footnote{An alternative set of assumptions that generate same type of prediction involves the housing market. The housing market is illiquid and positively correlated with the stock market. The extant research has recognized the importance of homeownership on asset allocation (Cauley, Pavlov, and Schwartz 2005, Yao and Zhang 2005). Homeownership acts in the same way as human capital in the context of our model. An agent may not want to invest in the stock market because the home is already effectively such an investment.}

There is empirical support to the idea that learning about labor income (and not just the contemporaneous covariance with the stock market) may affect market participation and asset prices. First, Malloy, Moskowitz, and Vissing-Jørgensen (2005b) find that stockholders’ (i.e., excluding non-participants) long-run consumption risk does particularly well in pricing asset returns. Malloy, Moskowitz, and Vissing-Jørgensen (2005a) find asset pricing success by using firing/hiring data to measure persistent labor income shocks, consistent with our interpretation of the model’s date 1 signal. Second, Guvenen (2005) considers a model where individuals enter the labor market with a prior belief about their income profiles and estimates that individuals can forecast (only) 40 percent of variation in income rates at time zero. This suggests that uncertainty about future labor income is a very real source of risk to most individuals. Finally, Vissing-Jørgensen (2002b) finds that a higher volatility of nonfinancial income

\[20\]
has a negative impact on the probability of market participation. This is consistent with our model where it is the uncertainty about future labor income, not the current covariance between labor income and stock returns that generates non-participation. For example, in our non-participation equilibrium, the date 0 one-period correlation between labor income and the asset return is trivially zero.

The most interesting feature of our model is not that it generates non-participation but what it implies about the risk-sharing and risk premium in the economy. The standard motivation for the limited participation puzzle is the question why many individuals choose to stay out despite very high historical equity premium. An implication of our model is that the risk premium is relatively high in the constrained economy precisely when it is most puzzling that some agents decide to stay out. For example, an agent with log-preferences would take a long position in the risky asset if it were not for the risk of facing binding constraints in the future. The practical implication of this result is important: an econometrician who ignores uncertainty, frictions, and the permanent shocks to investors’ investment opportunities may encounter difficulties in explaining not only limited participation but also the risk premium.

1.4 Conclusions

This paper describes an intuitive mechanism that keeps some individuals out of the market even when there are no participation costs and when the current equity premium is high: uncertainty about investment opportunities and learning together with the possibility of binding trading restrictions. The life-cycle model illustrates this mechanism. Suppose that an agent has a prior about the risk premium and revises her beliefs after each new observation. A consequence of this
setup is that the future changes in investment opportunities are positively correlated with realized returns. (For example, the agent revises her beliefs downwards after observing a low return.) Thus, a short-sale constrained agent is unable to profit from her refined information in those future states where she has learned that her investment opportunities are poor. Moreover, because of the positive correlation between realizations and expectations, these states are precisely the ones where the agent’s marginal utility is high. This creates a feedback to date zero decisions: because the agent knows that the constraints may become binding in the future, she requires a higher risk premium at date 0.

We generalize this idea with an equilibrium model where agents resolve uncertainty about the covariance between a non-tradable asset (“human capital”) and a risky asset (“stock”). This setup creates a similar feedback from future short-sale constraints: the possibility of being constrained in the future may be enough to induce the agent to leave the market at date 0. This feedback can thus create situation where the equity premium is high yet an agent with no non-financial income risk today stays out of the market. Our model has the interesting implication that the equity premium is relatively high precisely when it is most puzzling that some agents choose to stay out—i.e., when the agent is currently out of the market, but

1. would buy a positive amount of the asset if the short-sale constraints were lifted and

2. is close to being indifferent between participating and not participating in the future.

The result that agents may stay out of the market because of uninsurable shocks in the future is potentially important in explaining some negative results in
the participation literature. In our model, the risk of a high covariance generates non-participation. The forward-looking expectation effect—i.e., agents stay out of the market before their labor income covaries positively with the market—may make it difficult to detect the role of labor income in microdata. For example, our model’s mechanism can explain why Vissing-Jørgensen (2002b, pp. 33) concludes:

“...the consumption growth of non-stockholders covaries substantially less with the stock return than the consumption growth of stockholders...This indicates that the primary reason for nonparticipation is not that nonstockholders are faced with nonfinancial income which is highly correlated with stock market returns.”

Our results suggest that learning and labor income shocks driven by macroeconomic conditions may simultaneously generate non-participation, a sizable equity premium, and an insignificant contemporaneous correlation between the stock return and the income of those who do not participate.

1.5 Proofs

We first prove Propositions 2, 3, and 4 for the case when \( T = 2 \) and when \( \beta d - u < \alpha u < (\beta + 1)d \) holds for the prior distribution.\(^{21}\) (We omit subscripts; e.g., \( \alpha \) denotes \( \alpha_0 \).) Eq. 1.7 shows that under this assumption learning matters: the agent invests at time 1 only if the date 0 outcome is positive. (If \( \alpha u < \beta d - u \), the agent never invests at date 0; if \( \alpha u > (\beta + 1)d \), the agent always invests at date 0. Propositions 2 and 4 also hold for the latter case.)

\(^{21}\)Proposition 2 and Corollary 1 can be proven for the general \( T \) period problem with an induction argument while the proof of Proposition 4 remains the same. The boundary in Proposition 3 is specific to the three period model.
The optimal date 0 investment (if any) in this case is

\[ f_0^* = \frac{\alpha^{\frac{1}{\gamma}}(u + d)^{\frac{1}{\gamma} - 1} \left( ((\alpha + 1)u)^{\frac{1}{\gamma}d} + (\beta d)^{\frac{1}{\gamma}u} \right) - (rd)^{\frac{1}{\gamma}}(ud)^{\frac{1}{\gamma}}(\alpha + \beta + 1)^{\frac{1}{\gamma}}}{(u + d)^{\frac{1}{\gamma} - 1} \left( ((\alpha + 1)u)^{\frac{1}{\gamma}d} + (\beta d)^{\frac{1}{\gamma}u} \right) d - (rd)^{\frac{1}{\gamma}}(ud)^{\frac{1}{\gamma}}(\alpha + \beta + 1)^{\frac{1}{\gamma}}u} \] \tag{1.27}

where we use Eq. 1.7 and Proposition 1. We define \( r = \frac{\beta}{\alpha} \) in the second line. With this substitution, the variance of the prior is decreasing in \( \alpha \) while the mean stays constant.\(^{22}\)

**Proofs of Proposition 2 and Corollary 1.** The optimal date 0 investment in Eq. 1.27 can be written as

\[ f_0^* = \frac{g(\alpha) - h(\alpha)}{g(\alpha)d - h(\alpha)u}. \tag{1.28} \]

Hence, the optimal investment is increasing in \( \alpha \) iff

\[ g'(\alpha)h(\alpha) > g(\alpha)h'(\alpha) \tag{1.29} \]

which is a condition about the relative concavity functions \( g \) and \( h \). This inequality reduces to

\[ \left[ \frac{\alpha dr}{(\alpha + 1)u} \right]^{1 - \frac{1}{\gamma}} < 0. \tag{1.30} \]

The fraction inside brackets is always less than unity by the assumption that \( \beta d - u < \alpha u \) (see above). Hence, the inequality is satisfied iff \( \gamma > 1 \). This shows that the optimal investment is increasing in the variance of the prior if an agent is more risk-averse than a log-utility investor. The proof for the \( \gamma < 1 \) agent is

\:^{22}\text{We have, after a substitution, } E_t(p) = \frac{1}{1 + r} \text{ and } \text{var}_t(p) = \frac{r}{(1 + r)^2} \frac{1}{\alpha (1 + r) + 1}. \text{ For future reference, note that } \lim_{\alpha \to 0} \text{var}_t(p) = \frac{r}{(1 + r)^2}.\]
We now prove the market participation result (Corollary 1). Suppose that
\( E_0(\tilde{\epsilon}) = \frac{\alpha}{\alpha + \beta} u - \frac{\beta}{\alpha + \beta} d = 0, \) i.e. the prior about the stock’s risk premium is zero. The condition for a positive investment from Eq. 1.27 becomes:
\[
\frac{((\alpha + 1)u)^{\frac{1}{\gamma}} d + (\beta d)^{\frac{1}{\gamma}} u}{((\alpha + 1)ud + \beta ud)^{\frac{1}{\gamma}}} \geq (u + d)^{1 - \frac{1}{\gamma}}.
\]
(1.31)

Defining \( c = \frac{\beta d}{(\alpha + 1)u} \), the LHS of this inequality can be written as
\[
L(c) = \frac{d + c^{\frac{1}{\gamma}} u}{(d + cu)^{\frac{1}{\gamma}}}.
\]
(1.32)

First, we note that \( L(1) = (u + d)^{1 - \frac{1}{\gamma}} \). Second, we observe that \( L'(c) > 0 \) if \( \gamma > 1 \) and \( L'(c) < 0 \) if \( \gamma < 1 \). Third, note that \( c < 1 \) by assumption. These imply that if \( \gamma > 1 \), the LHS in Eq. 1.31 is strictly less than the quantity on the RHS, violating the inequality. Hence, an investor more risk-averse than a log-utility investor strictly prefers not investing when the risk premium is zero. By contrast, an agent with \( \gamma < 1 \) makes a strictly positive investment in the same situation. Because the optimal investment is a continuous in all the parameters, an agent with \( \gamma > 1 \) does not invest even when \( E_0(\tilde{\epsilon}) = \delta \) with \( \delta > 0 \) and vice versa.

Proof of Proposition 3. The condition for a positive investment from Eq. 1.27 is:
\[
\frac{((\alpha + 1)u)^{\frac{1}{\gamma}} d + (\beta d)^{\frac{1}{\gamma}} u}{((\alpha + 1)ud + \beta ud)^{\frac{1}{\gamma}}} \geq (u + d)^{1 - \frac{1}{\gamma}} \left( \frac{rd}{u} \right)^{\frac{1}{\gamma}}.
\]
(1.33)

We let the variance of the prior to tend to its maximum \((\alpha \rightarrow 0)\), \( \frac{r}{(1 + r)^2} \). The
condition for positive investment becomes

\[ r \leq \frac{u}{d} \left( \frac{u + d}{d} \right)^{1-\gamma}. \] (1.34)

We get the boundary by writing \( r \) in terms of the mean of the prior, \( r = \frac{1}{E_0(p)} - 1 \). This boundary is

\[ E_0(p) \geq \frac{1}{1 + \frac{u}{d} \left( \frac{u + d}{d} \right)^{1-\gamma}}. \] (1.35)

(The equality is reached at the limit \( \alpha \to 0 \).)

**Proof of Proposition 4.** We get the expression for the implied risk-aversion by first setting \( p = \frac{\alpha}{\alpha + \beta} \) and \( \gamma = \hat{\gamma} \) in the equation for optimal investment for the ‘no parameter uncertainty’ agent (Eq. 1.10). Eq. 1.12 follows from equating this with the optimal investment of the ‘parameter uncertainty’ agent (Eq. 1.7) and solving for \( \hat{\gamma} \).

**Proof of Proposition 6.** Conditions 1A, 1B, and 2 follow after some algebra from evaluating both agents’ first-order conditions at \( \theta_1 = 0 \) after a high dividend and from evaluating Agent A’s first-order condition at \( \theta_1 = 0 \) after a low dividend.

To get Condition 3, first write down Agent A’s date 0 problem:

\[
V_0^A(X^A, \bar{\theta}^A) = \max_{\theta_0} \left\{ p \left( \frac{(D_h - D_l)(\theta_0(P_h^1 + D_h - P_0)) + c^{A,h})^{1-\gamma} \right) k^h \right. \\
+ (1 - p) \left( \frac{\theta_0(P_l^1 + D_l - P_0) + y^h)}{1 - \gamma} \right)^{1-\gamma} \right. \\
+ \left. (1 - p) \left( \frac{(\theta_0(P_l^1 + D_l - P_0) + y^h)}{1 - \gamma} \right) \right\}. \] (1.36)

(See Equation 1.18 for the values of \( k^h \) and \( c^{A,h} \).) This problem takes into account
the postulated form of non-participation equilibrium: the agent invests a positive amount after a high dividend (the indirect utility on the first line) but stays out of the market after a low dividend (the indirect utility on the second line). The indirect marginal utility condition is then

\[ pk_h(D_h - D_l)(P_{1h} + D_h - P_0)c^{A,h-\gamma} + (1 - p)(P_{1l} + D_l - P_0)(py_h^{l-\gamma} + (1 - p)y_l^{l-\gamma}) \leq 0. \] (1.37)

Condition 3 follows from substituting in the equilibrium prices from Corollary 2.

We prove the existence of a solution by constructing one. First, choose \( y_h^l = k_1(D_h + D_l) \) where \( k_1 < 1 \) and let Condition 2 to bind with equality to get \( y_l^l = 2kD_l \). Next, choose \( y_h^l = D_h + D_l \) and let Condition 1A to bind with equality to get \( y_h^h = 2D_h \). Now, conditions 1A and 2 are satisfied by assumption. Condition 1B is satisfied exactly and the LHS in Condition 3 is equal to one. The RHS is strictly greater than one by the assumptions about \( y_l^l \) and \( y_h^h \). It follows from the strict inequality in Condition 3 and the continuity of all conditions that the income process parameters can be perturbed while retaining equilibrium. \( \square \)

**Proof of Proposition 7.** We first show that the expected date 0 payoff is higher in the constrained economy. First, note that the date 1 price after a high dividend is the same in both economies. Second, the unconstrained and constrained date 1 prices after a low dividend can be written as

\[ P_{1,uc}^l = \frac{(D_h + D_l + y_h^l)^{-\gamma}D_h + (2D_l + y_l^l)^{-\gamma}D_l}{(D_h + D_l + y_h^l)^{-\gamma} + (2D_l + y_l^l)^{-\gamma}}, \] (1.38)

\[ P_{1,c}^l = \frac{(D_h + D_l)^{-\gamma}D_h + (2D_l)^{-\gamma}D_l}{(D_h + D_l)^{-\gamma} + (2D_l)^{-\gamma}}. \] (1.39)
Let us now define function $\lambda(x)$ as

$$\lambda(x) = \frac{(D_h + D_l + x)^{-\gamma}D_h + (2D_l + kx)^{-\gamma}D_l}{(D_h + D_l + x)^{-\gamma} + (2D_l + kx)^{-\gamma}}$$

(1.40)

and note that $\lambda(0) = P_{1,c}^l$ and $\lambda(y_h^l) = P_{1,uc}^l$ for a proper choice of $k$. Differentiating, the condition $\frac{d}{dx}\lambda(x) \geq 0$ can be written as

$$k \geq \frac{2D_l}{D_h + D_l}.$$  

(1.41)

This is the Condition 2 of Proposition 6 and hence, satisfied in equilibrium. This shows that the date 1 price after a low dividend is decreasing in nonfinancial income. Hence, $P_{1,c}^l \geq P_{1,uc}^l$, and because the state-probabilities are the same in the constrained and unconstrained economies, the expected payoff is higher in the constrained economy.

The constrained economy risk premium is higher than the risk premium in the unconstrained economy if

$$\frac{E[\text{Constrained Payoff}]}{E[\text{Unconstrained Payoff}]} \geq \frac{P_{0,c}}{P_{0,uc}}.$$  

(1.42)

Because the expected constrained payoff is always higher than the expected unconstrained payoff, this condition is a requirement that the date 0 price cannot change “too much” to compensate for this increase. We prove a slightly stronger condition by examining when the date 0 price is lower in the constrained economy.
The unconstrained and constrained date 0 prices can be written as

\[ P_{0,uc} = \frac{pA_1 + (1-p)B_1}{pA_2 + (1-p)B_2}, \]

\[ P_{0,c} = \frac{pwA_1 + (1-p)B'_1}{pwA_2 + (1-p)B'_2}, \]

where

\[ A_1 = p(2D_h + y_h^h)^{-\gamma}2D_h + (1-p)(D_h + D_l + y_h^l)^{-\gamma}(D_h + D_l), \]

\[ A_2 = p(2D_h + y_h^h)^{-\gamma} + (1-p)(D_h + D_l + y_h^l)^{-\gamma}, \]

\[ B_1 = p(D_h + D_l + y_h^h)^{-\gamma}(D_h + D_l) + (1-p)(2D_l + y_l^l)^{-\gamma}(2D_l), \]

\[ B_2 = p(D_h + D_l + y_h^l)^{-\gamma} + (1-p)(2D_l + y_l^l)^{-\gamma}, \]

\[ B'_1 = p(D_h + D_l)^{1-\gamma} + (1-p)(2D_l)^{1-\gamma}, \]

\[ B'_2 = p(D_h + D_l)^{-\gamma} + (1-p)(2D_l)^{-\gamma}, \]

\[ w = \frac{p(2D_h + y_h^h)^{1-\gamma} + (1-p)(D_h + D_l + y_h^l)^{1-\gamma}}{p(2D_h + y_h^h)^{-\gamma}2D_h + (1-p)(D_h + D_l + y_h^l)^{-\gamma}(D_h + D_l)}. \]

First, note that for \( y_h^h, y_l^l \geq 0, w \geq 1, \) and when \( y_h^h, y_l^l \to 0, w \to 1. \)

Second, similar to our approach above, let us define function \( \lambda(x) \) as

\[ \lambda(x, w) = \frac{pwA_1 + (1-p)\{p(D_h + D_l + x)^{-\gamma}(D_h + D_l) + (1-p)(2D_l + kx)^{-\gamma}(2D_l)\}}{pwA_2 + (1-p)\{p(D_h + D_l + x)^{-\gamma} + (1-p)(2D_l + kx)^{-\gamma}\}} \]

(1.45)

Note that \( \lambda(0, w) = P_{0,c} \) and that for a proper choice of \( k, \lambda(y_h^h, 1) = P_{0,uc} . \) The

\[ \text{This is a condition about the relative size of the dividends and nonfinancial income. Alternatively, holding } y_h^h \text{ and } y_l^l \text{ fixed, } w \to 1 \text{ when } D_h, D_l \to \infty. \]
condition \( \frac{\partial}{\partial x} \lambda(x, w) \leq 0 \) can be written after some algebra as

\[
w p^2 (D_h + D_l + x)^{\gamma-1}(2D_h + y^h_h)^{-\gamma} \\
+ (1 - p)k(2D_l + kx)^{\gamma-1}\left\{2p(2D_h + y^h_h)^{-\gamma} + (1 - p)(D_h + D_l + y^h_h)^{-\gamma}\right\} \\
\geq (1 - p)^2 (D_h + D_l + x)^{\gamma-1}(2D_l + kx)^{\gamma-1}(2D_l - k(D_h + D_l)). \quad (1.46)
\]

Note that if \( k = \frac{2D_l}{D_h + D_l} \), this inequality is satisfied strictly. Hence, the constrained date 0 price is strictly lower than the unconstrained price if (i) Agent A’s non-financial income \( \{y^h_h, y^l_h\} \) is small relative to the dividends (i.e., \( w \approx 1 \) when \( y^h_h, y^l_h \to 0 \) or, equivalently, \( D_h, D_l \to \infty \)) and (ii) Agent A is close to being indifferent between participating and not participating after a low dividend at date 1 (i.e., \( \frac{y^h_h}{y^l_l} = \frac{2D_l}{D_h + D_l} \)). Because \( \lambda(x, w) \) is continuous in all the parameters of the economy, the risk premium is strictly higher also when \( \frac{y^h_h}{y^l_l} > \frac{2D_l}{D_h + D_l}, y^h_h, y^l_h > 0 \) and \( D_h, D_l < \infty. \)
“Three years of falling markets have shaken out the inexperienced investors who signed up to online accounts without any real understanding of the markets.”

—Financial Times, “Day Trading: Those still trading online have kept their day jobs and learnt how to hedge” (February 22, 2003)

2.1 Introduction

Most economic models assume that investors are capable of very complicated computations. For example, Merton’s (1969) life-cycle model and its extensions assume that investors can solve Bellman equations while rational expectations equilibrium (REE) models assume that agents can solve fixed-point problems. In practice, individuals’ ability to carry out these computations may be limited (Hirshleifer 2001) but probably not zero. A broad literature in macroeconomics assumes that investors do not know everything about the economy but they learn about the parameters over time. A typical question posed in this adaptive learning literature is the following: suppose agents postulate a functional form for the prices in a rational expectations model; does the economy converge to equilibri-
rium if the agents revise their estimates each period based on new observations?\footnote{See Evans and Honkapohja (2001) for a textbook treatment.}

Similarly, Grossman and Stiglitz (1980) motivate their REE model as a statistical equilibrium where the economy converges because of learning.

Despite many theoretical arguments for why learning is crucial, empirical work on the issue is nonexistent.\footnote{Few recent papers examine changes in investor behavior over time. For example, Barber, Odean, and Strahilevitz (2004) find that individuals prefer to repurchase stocks they previously sold for a gain rather than stocks they previously sold for a loss. Feng and Seasholes (2005) find that trading experience reduces individual investors’ disposition effect (Shefrin and Statman 1985). Barber, Lee, Liu, and Odean (2004) study Taiwanese day traders and find a positive relation between performance and subsequent day trading activity. Nicolosi, Peng, and Zhu (2004) find that individuals’ trading experience improves portfolio performance. However, these papers do not examine whether these changes in investor behavior reflect rational learning.}

This is not surprising: it is difficult to examine how an investor changes her behavior in response to new information because \((i)\) the investor’s prior beliefs are inherently unobservable and \((ii)\) we often do not even know what the investor observes. For example, if the investor actively trades in the stock market, she could eventually learn that her stock picking skills are not good enough to justify the high transaction costs. She might then stop and transfer her assets to an index fund. However, it would empirically be very difficult to detect such rational learning because the feedback from stock-picking is slow and noisy.

We address the important question of learning by studying the decisions of \emph{individual day traders}. We argue that these investors are ideal for a study of learning for several reasons:

1. \textit{Dispersed Beliefs}. It is very difficult for an individual to learn about her own day trading skills without actually day trading.\footnote{Even if an individual can access delayed quotes, “paper trading” may not allow the individual to learn perfectly the profitability of day trading. First, actual trades would affect the market in a way that cannot be inferred from delayed quotes. Second, day traders’ reliance}
may have a reason to entertain the possibility that day trading might be profitable: for example, a neighbor’s positive experiences may cast enough doubt so that the individual wants to find out for herself. Dispersed beliefs are also ideal for a study of learning: new data influence beliefs (and behavior) more when investors’ priors are uninformative.

2. **Immediate feedback.** A day trader tries to buy and sell stocks during a single trading day at profit. The profit or loss from each day trade provides the investor with immediate and precise feedback. This distinguishes day trading from general stock-picking where feedback is often slow and noisy.

3. **Sequential observations.** A day trader makes sequential decisions and each of these decisions is observable. For example, if the day trader learns from experience, outcomes drive changes in day trade sizes and determine when the day trader quits.

The first argument about dispersed priors is important. For example, the question why some individuals day trade is puzzling by itself because most of these day traders lose money (Barber, Lee, Liu, and Odean 2004, Linnainmaa 2005a). A casual explanation is that day traders have stubborn beliefs about their own abilities. This explanation argues that day traders day trade, lose money, and

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on their ability to use limit orders to earn a profit (Linnainmaa 2005a) creates an additional problem: delayed quotes do not reveal the direction of the order-flow. Limit orders also suffer from the same problem as market orders in that they would affect the market in a way that cannot be inferred from delayed quotes. Hence, even an investor who initially engages in paper trading still retains “residual uncertainty” that may warrant experimentation with real money. (We thank Deborah Lucas for making this point.)

4 Other studies that examine day traders and other short-term trades include Harris and Schultz (1998), Garvey and Murphy (2001), Jordan and Diltz (2002), and Seasholes and Wu (2004).

5 We use term “stubborn beliefs” to describe an investor whose subjective prior about the day trading return distribution is degenerate. Such an investor ignores all feedback and always acts as if the subjective prior is the truth. We avoid using term “overconfidence” because
continue to day trade. However, an alternative possibility is that individuals may just be uncertain about the profitability of day trading. If this is true, investors may day trade just to learn: the potential gains are too high to be offset by the small costs of learning—time and transaction costs. (Section 2.2’s model makes this argument precise.)

This paper tests whether individuals learn from experience by comparing two explanations for the existence of day traders. These competing hypotheses—rational learning versus stubborn beliefs—make very different predictions about day traders’ behavior. If individuals day trade because of stubborn beliefs, (i) they quit only after running out of money and (ii) their trade sizes respond to prior outcomes only because of the outcomes’ wealth effects. In contrast, if individuals day trade because of uncertainty and learning, (i) they quit after updating their beliefs sufficiently downwards and (ii) their trade sizes change as their beliefs evolve. For example, a positive experience leads an investor to revise her beliefs upwards, and hence, to increase her trade size. Moreover, individuals who believe that day trading is unprofitable—but entertain the possibility that it could be profitable—may place small exploratory bets to learn about the profitability. 7

We test the learning hypothesis using a unique data set from Finland. This

6 The idea that some individuals do not perfectly understand financial markets is not novel. For example, Brennan (1995, pp. 61) contrasts an individual investor against the representative agent: “The representative investor is assumed to understand the economy and the process determining asset prices; the individual investor frequently does not.”

7 We follow Lefèvre (1923, pp. 128): “It is simple arithmetic to prove that it is a wise thing to have the big bet down only when you win, and when you lose to lose only a small exploratory bet, as it were.”
data set contains all transactions and positions of all individual day traders in Finland. Many characteristics of these investors support the possibility that they are uncertain about the profitability of day trading. For example, many have surprisingly limited investment experience before starting to day trade: one of every ten day traders owned or traded stocks for the very first time less than three months before the first day trade. Furthermore, the majority of day traders day trade only once or twice. This may reflect learning: investors are pessimistic about the profitability of day trading to begin with and only need a few observations to confirm their beliefs.

Our results show that investors learn from experience. Day traders quit after experiencing losses and adjust their trade sizes in response to outcomes even after controlling for the outcomes’ wealth effects. Day traders also experiment with very small trades that are without speculative motives. These results show that agents update their beliefs in the correct direction in response to new information but they do not address the rationality of the updating rules. We study whether investors accurately process new information by estimating the belief-updating rules from the data. We find—consistent with the biased self-attribution arguments of Gervais and Odean (2001) and Daniel, Hirshleifer, and Subrahmanyam (1998)—that individuals update beliefs asymmetrically, placing over three times more weight on a positive outcome than on a negative outcome. We also use the learning model to back out investors’ priors about their day trading abilities and find that most individuals (57.9%) did not believe in their ability to make money by day trading at the time they day traded for the first time.

We conclude that it is uncertainty and learning—not stubborn beliefs about one’s own abilities—that explains why (some) investors day trade. This intuitive result is important: many behavioral finance studies assume that behavioral bi-
ases persist over time. However, if investors learn from experience—as almost all non-behavioral studies assume either directly or by placing high demands on the agents’ cognitive abilities—an individual’s welfare-reducing behavioral biases will dissipate over the life-cycle.

The rest of the paper is organized as follows. Section 2.2 illustrates the effects of parameter uncertainty and learning using a three-period model. Section 2.3 discusses the data set and Section 2.4 presents the empirical results. Section 2.5 concludes.

2.2 Model

This section presents a tractable life-cycle model to demonstrate how parameter uncertainty and learning affect optimal investment. (We give a multiperiod extension of the model in Appendix 2.6.2. This model extends the life-cycle model of Linnainmaa (2005b) by adding transaction costs and by dropping the assumption that the agent observes outcomes even without an investment.) We also compare the model’s predictions against the predictions of the “stubborn beliefs” hypothesis. Section 2.4.6 estimates the multiperiod version of the model from the data on individual day traders.

2.2.1 Assumptions

We make the following assumptions:

- An agent lives for three dates, $t = 0, 1, 2$, and maximizes power utility over date 2 wealth,

\[
U(w_2) = \frac{w_2^{1-\gamma} - 1}{1 - \gamma}.
\]  

(2.1)
• The agent can day trade an amount \( x_t \geq 0 \) at dates 0 and 1.

• The day trade’s outcome has a binomial distribution:
  
  – The day trade is a success with probability \( p \) and pays off \((1 + \delta)x\)
  
  – The day trade is a failure with probability \( 1 - p \) and pays off \((1 - \delta)x\).

  where \( 0 < \delta \leq 1 \).

• The agent must *always* pay a transaction cost \( c > 0 \) to day trade. The agent can pay the cost without day trading (i.e., by choosing \( x = 0 \)) if he only wants to observe the outcome.\(^8\)

• There is a risk-free asset that pays no interest.

• The agent has initial wealth \( w_0 \gg c \).

• The agent does not know parameter \( p \) (the probability of a success) but has a prior about it:
  
  – The date 0 prior beliefs are \( \text{Beta}(\alpha_0, \beta_0) \) distributed.
  
  – If the agent day trades—or pays the transaction cost and chooses \( x = 0 \)—he observes the outcome and updates his beliefs as a Bayesian.
  
  – If he does not pay the cost, he does not get to observe the outcome.

The agent solves his problem in two steps: first, an inference problem in which the agent updates his estimate of \( p \) and second, an optimization problem in which the agent chooses the optimal investment given the current wealth and

---

\(^8\)The transaction cost may consist of items such as the spread and brokerage fee. An agent paying the cost and choosing \( x = 0 \) is effectively placing a very small trade just to learn from experience. For example, investors in actual markets may wish to *try out* a trading strategy with small trades. Our assumption about infinitesimal trades captures this intuition.
the estimate of $p$ (Gennotte 1986, Brennan and Xia 2001). We consider first the inference problem.

### 2.2.2 The Agent’s Inference Problem

The agent has a conjugate prior distribution $\text{Beta}(\alpha_0, \beta_0)$ about $p$ at date 0. The assumption about a binomial stock price process and a Beta-distributed prior makes the inference problem particularly tractable. The date $t$ posterior after observing $N_t$ positive stock price movements is $\text{Beta}(\alpha_0 + N_t, \beta_0 + t - N_t)$-distributed (see, e.g., DeGroot (1970, pp. 160)). The mean of the posterior distribution at date $t$ is

$$E_t(p) = \frac{\alpha_0 + N_t}{\alpha_0 + \beta_0 + t}. \quad (2.2)$$

We let $\alpha_t \equiv \alpha_0 + N_t$ and $\beta_t \equiv \beta_0 + t - N_t$ to denote the date $t$ belief parameters. The intuition for this updating is simple: the parameters of the Beta-distribution keep track of the number of good and bad observations. For example, if the agent starts with parameters $(1, 2)$, the parameters are $(2, 2)$ after a good outcome and $(1, 3)$ after a bad outcome. Figure 2.1 illustrates the agent’s date 0 problem.
2.2.3 The Agent’s Optimization Problem

We first solve the agent’s date 1 problem and then move backwards and solve the date 0 problem. Because the agent has to pay $c$ to learn, we must compare agent’s utilities under three different policies at each step:

- **Invest**: the agent pays the cost $c$ and day trades a strictly positive amount, $x_t > 0$.
- **Pay the Cost**: the agent pays the cost $c$ but only to observe the outcome, $x_t = 0$.
- **Do Nothing**: the agent does not pay the cost and does not observe the outcome.

### 2.2.3.1 The Date 1 Problem

We first compute the agent’s indirect utilities under different policies and then characterize the agent’s optimal behavior by comparing the utilities from following these policies. The agent’s date 1 Bellman equation solves the following problem under the *invest* policy:

$$
V_{1}^{\text{invest}}(w_1, (\alpha_1, \beta_1)) = \max_{x_1 \geq 0} \left\{ \frac{\alpha_1}{\alpha_1 + \beta_1} \left( w_1 + \delta x_1 - c \right)^{1-\gamma} + \frac{\beta_1}{\alpha_1 + \beta_1} \left( w_1 - \delta x_1 - c \right)^{1-\gamma} \right\}.
$$

The optimal investment from the first order condition is

$$
x_{1}^{*}(w_1, (\alpha_1, \beta_1)) = \frac{1}{\delta} \frac{\alpha_1}{\alpha_1 + \beta_1} (w_1 - c).
$$
We rewrite the agent’s value function as

\[ V_{\text{invest}}^1(w_1, (\alpha_1, \beta_1)) = \begin{cases} \frac{(w_1 - c)^{1-\gamma}}{1-\gamma}k_1(\alpha_1, \beta_1) & \text{if } x_1^*(w_1, (\alpha_1, \beta_1)) > 0 \\ -\infty & \text{otherwise} \end{cases} \] (2.5)

where

\[ k_1(\alpha_1, \beta_1) = \frac{2^{1-\gamma}}{\alpha_1 + \beta_1} \left( \alpha_1^{\frac{1}{2}} + \beta_1^{\frac{1}{2}} \right)^{\gamma}. \] (2.6)

We make two observations about \( k_1(\alpha_1, \beta_1) \). First, \( k_1(\alpha_1, \beta_1) \geq 1 \) if \( \gamma < 1 \) and \( k_1(\alpha_1, \beta_1) \geq 1 \) if \( \gamma > 1 \). Second, \( \frac{\partial}{\partial \alpha_1}k_1(\alpha_1, \beta_1) > 0 \) if \( \gamma < 1 \) and \( \frac{\partial}{\partial \alpha_1}k_1(\alpha_1, \beta_1) > 0 \) if \( \gamma > 1 \).

If the agent chooses the *do nothing* policy, the indirect utility is

\[ V_{\text{do nothing}}^1(w_1, (\alpha_1, \beta_1)) = \frac{w_1^{1-\gamma}}{1-\gamma}. \] (2.7)

Note that it is never optimal for the agent to just pay the cost at date 1 so we ignore this policy. The optimal date 1 rule is utility-maximizing:

\[ V_1(w_1, (\alpha_1, \beta_1)) = \arg \max_{\{\text{invest, do nothing}\}} \left\{ V_{\text{invest}}^1, V_{\text{do nothing}}^1 \right\}. \] (2.8)

Hence, the agent day trades if (i) \( \alpha_1 > \beta_1 \) and (ii) \( c \leq \left( 1 - k_1(\alpha_1, \beta_1)^{-\frac{1}{1-\gamma}} \right) w_1 \). (The second condition follows from Eq. 2.8 after substitution.) This condition says that the cost \( c \) cannot be too high or the agent will not invest. Let us denote the RHS of this condition by \( \bar{c}(w_1, (\alpha_1, \beta_1)) \)—this is the upper bound for the cost—and note that \( \frac{\partial}{\partial w_1} \bar{c} > 0 \) and \( \frac{\partial}{\partial \alpha_1} \bar{c} > 0 \). We can write the date 1 value function as

\[ V_1(w_1, (\alpha_1, \beta_1)) = \frac{(w_1 - b_1 + c)^{1-\gamma}}{1-\gamma}k_1(\alpha_1, \beta_1) \] (2.9)

where \( b_1 = 0 \) and \( k_1(\alpha_1, \beta_1) = 1 \) if the agent does not day trade at date 1.
2.2.3.2 The Date 0 Problem

The optimal behavior at date 0 can be solved in the same way as the date 1 behavior. We compute the indirect utilities of the three policies separately and then let the agent choose the one that yields the highest utility.

First, the agent can choose to day trade at date 0. Note that this can only happen if the agent may believe tomorrow that day trading is profitable: i.e., the necessary condition is that $\alpha_0 + 1 > \beta_0$. If this is not the case, the agent would never day trade tomorrow. Hence, we temporarily assume that the agent would want to day trade tomorrow at least after a positive outcome today. The agent’s date 0 Bellman equation then solves

$$V_{invest}^0(w_0, (\alpha_0, \beta_0)) = \max_{x_0 \geq 0} \left\{ \frac{\alpha_0}{\alpha_0 + \beta_0} V_1(w_0 + \delta x_0 - c, (\alpha_0 + 1, \beta_0)) + \frac{\beta_0}{\alpha_0 + \beta_0} V_1(w_0 - \delta x_0 - c, (\alpha_0, \beta_0 + 1)) \right\}. \quad (2.10)$$

The optimal investment from the first-order condition is

$$x_0^*(w_0, (\alpha_0, \beta_0)) = \frac{1}{\delta} \left[ \frac{(\alpha k^+)^{\frac{1}{\gamma}} - (\beta k^-)^{\frac{1}{\gamma}}}{(\alpha k^+)^{\frac{1}{\gamma}} + (\beta k^-)^{\frac{1}{\gamma}}} w_0 - \frac{(\alpha k^+)^{\frac{1}{\gamma}} b^- - (\beta k^-)^{\frac{1}{\gamma}} b^+}{(\alpha k^+)^{\frac{1}{\gamma}} + (\beta k^-)^{\frac{1}{\gamma}}} c \right]. \quad (2.11)$$

where $k^+$ and $b^+$ are the coefficients of the date 1 value function (Eq. 2.9) after a positive outcome today and $k^-$ and $b^-$ are the coefficients after a negative outcome. (For example, if the agent invests in both states tomorrow, $b^+ = b^- = 1$, but if the agent only invests in the upstate, $b^+ = 1$ and $b^- = 0.$) The value
function can be rewritten as

\[
V_{0}^{\text{inv}}(w_0, (\alpha_0, \beta_0)) = \frac{(w_1 - b_0 - c)^{1-\gamma}}{1-\gamma} k_0(\alpha_0, \beta_0)
\] (2.12)

where \( b_0 = \begin{cases} 
2 & \text{if invest in both date 1 states} \\
\frac{3}{2} & \text{if invest only after a good outcome today,} \\
\frac{2^{1-\gamma}}{\alpha_0 + \beta_0} \left[ (\alpha_0 k_2(\alpha_0 + 1, \beta_0))^{\frac{1}{\gamma}} + (\beta_0 k_2(\alpha_0, \beta_0 + 1))^{\frac{1}{\gamma}} \right]^\gamma & \text{both states} \\
\frac{2^{1-\gamma}}{\alpha_0 + \beta_0} \left[ (\alpha_0 k_2(\alpha_0 + 1, \beta_0))^{\frac{1}{\gamma}} + \beta_0^{\frac{1}{\gamma}} \right]^\gamma & \text{upstate only.} 
\end{cases}
\]

Second, if the agent pays the cost, he derives indirect utility

\[
V_{0}^{\text{pay the cost}}(w_0, (\alpha_0, \beta_0)) = \frac{\alpha_0}{\alpha_0 + \beta_0} V_1(w_0 - c, (\alpha_0 + 1, \beta_0)) + \frac{\beta_0}{\alpha_0 + \beta_0} V_1(w_0 - c, (\alpha_0, \beta_0 + 1))
\] (2.13)

which is the tomorrow’s expected indirect utility after paying the cost today.

Third, the indirect utility of doing nothing at date 0 is \( V_{0}^{\text{do nothing}}(w_0, (\alpha_0, \beta_0)) = \frac{(w_0 - b_0 - c)^{1-\gamma}}{1-\gamma} \). The optimal date 0 rule is utility-maximizing:

\[
V_0(w_0, (\alpha_0, \beta_0)) = \arg\max_{\{\text{invest, pay the cost, do nothing}\}} \{ V_0^{\text{invest}}, V_0^{\text{pay the cost}}, V_0^{\text{do nothing}} \}.
\] (2.14)

### 2.2.4 The Agent’s Optimal Behavior

We characterize the agent’s optimal behavior with three results before turning to examples. The proofs are in the appendix.

**Theorem 8.** The agent always pays at least the trading cost at date 0 if the prior
is sufficiently dispersed and the trading cost $c$ is small.

This result says that the agent always pays the cost if he is sufficiently unsure of himself and the cost is not “too high”. This an intuitive result: the agent is willing to pay a small cost because the potential gains—i.e., if day trading turns out to be profitable—are very large.

**Theorem 9.** The optimal investment is increasing in the number of positive outcomes, $\alpha$.

This result says that, ceteris paribus, an agent with higher expectations about the profitability day trades more.

**Theorem 10.** The optimal investment is decreasing in the agent’s risk aversion, $\gamma$.

This result shows that an agent with low risk-aversion is willing to day trade for a wider range of parameter values relative to an otherwise identical but more risk-averse agent.

### 2.2.5 Regions of Optimal Behavior

Figure 2.2 illustrates the agent’s optimal behavior in the model. The agent has an initial wealth of $w_1 = 10,000$ and a risk-aversion of either $\gamma = 2$ or $\gamma = 0.5$, and has to pay $c = 20$ to day trade. The three possible choices are drawn as functions of the parameters of the prior distribution. Note that the area below the diagonal (where $\alpha_0 > \beta_0$) represents beliefs that day trading is profitable.

It is useful to consider where the agent gets his prior before discussing the results. A natural influence for the initial beliefs is the media and the word-of-mouth information sharing among neighbors (Hong, Kubik, and Stein 2004).
Figure 2.2: Optimal Behavior under Parameter Uncertainty.

This figure shows the optimal behavior of an agent who is uncertain about the profitability of day trading and maximizes power utility over terminal wealth. The agent has an initial wealth of \( w_0 = \$10,000 \), a risk-aversion of \( \gamma = 2 \) (Panel A) or \( \gamma = 0.5 \) (Panel B) and can day trade amount \( x \geq 0 \) at dates 0 and 1 at a cost \( c = \$20 \). The day trade is a success with a probability \( p \) and pays off \( (1 + \delta)x \); it is a failure with a probability \( 1 - p \) and pays off \( (1 - \delta)x \). The investor has a Beta\((\alpha_0, \beta_0)\) distributed prior about \( p \). If the agent pays the cost, he observes the outcome and updates his beliefs as a Bayesian. This figure plots the three possible actions that the agent can take at date 0: do nothing, pay the cost, or day trade (“invest”). The regions are plotted as functions of parameters of the prior distribution. An increase in \( \alpha \) while keeping \( \beta \) fixed means that the investor has a more positive view of the profitability. An increase in both \( \alpha \) and \( \beta \) for a fixed \( \frac{\alpha}{\beta} \) lowers the variance of the prior. An agent in the region above (below) the 45° line believes that day trading is unprofitable (profitable).
For example, suppose the agent has three neighbors known for their active day trading. If two of them are now hiding from debtors while one is driving a new luxury car, the agent may pick Beta(1, 2) as a reasonable description of his beliefs (“one good, two bad observations”). However, the agent may be more uncertain than this because his neighbors’ experiences may not be very informative of his own day trading skills. If so, the agent beliefs might be better described by Beta(0.1, 0.2) distribution. The agent has the same level of expectations as before but more uncertainty.⁹

The figure reveals how the ability to learn affects behavior. We discuss three observations: (i) agents are usually willing to pay the cost and day trade when they believe day trading is even marginally profitable; (ii) agents are often willing to pay the cost to learn even when they believe day trading is probably unprofitable; and (iii) the less risk-averse agent is willing to day trade and pay the cost for a wider range of parameters. First, an agent who currently believes that day trading is profitable usually pays the cost and day trades, \( x_0^* > 0 \). The only exceptions are the cases where the agent’s prior about \( p \) is very close to \( \frac{1}{2} \) and the agent is very confident about his view. In this case, the cost of day trading makes the activity unattractive. (These cases lie just below the diagonal outside the upper-right corner of Figure 2.2; not shown.) Put differently, even if the agent believes he could earn marginally positive day trading returns before brokerage fees, brokerage fees reverse the situation.

Second, agents are often willing to place exploratory bets (i.e., they pay the cost to learn) even when they believe that day trading is unprofitable. (This is

---

⁹The following two facts about Beta-distribution are useful. First, the ratio \( \frac{\alpha}{\beta} \) determines the mean. Second, when this ratio is fixed and \( \alpha \) and \( \beta \) increase, the variance decreases. Hence, in our example, the agent’s belief about the level of profitability is unchanged when he moves from parameters \((\alpha_0, \beta_0) = (1, 2)\) to \((\alpha_0, \beta_0) = (0.1, 0.2)\) but the dispersion increases. (Pratt, Raiffa, and Schlaifer (1995) discuss at length how to parameterize prior distributions.)
the shaded area above the diagonal in the figure.) The requirement is that the agent is sufficiently uncertain about his beliefs; i.e., when $\alpha_0$ and $\beta_0$ tend closer to zero. Third, the level of risk-aversion significantly affects optimal decisions. The Panel B’s agent day trades (or at least pays the cost) for a wider range of parameters than the Panel A’s agent.

The model illustrates that parameter uncertainty can potentially explain why some investors day trade. We expect day traders to be a specific part of the population: those who initially believe that day trading is profitable or those who are just sufficiently uncertain. Moreover, ceteris paribus, an investor with lower risk-aversion is more likely to become a day trader. This tractable Bayesian model leads to an important conclusion: even in a world where everyone believes that day traders lose money, some investors become day traders.

2.2.5.1 A Numerical Example

How does the agent’s behavior change over time because of learning? Figure 2.3 addresses this question with two examples using a five-period version of the model (see Appendix 2.6.2). The parameters of this example are the same as before: an initial wealth of $10,000, a cost of $c = 20, and a risk-aversion of $\gamma = 2$. The agent in the first example starts with a prior $(\alpha_0, \beta_0) = (1, 2)$, i.e., with a belief that day trading is unprofitable while the agent in the second examples starts with a more optimistic prior, $(\alpha_0, \beta_0) = (2, 1)$.

The first example shows how the agent’s behavior is affected by what may happen: the agent is willing to pay the cost because day trading may turn out to be profitable. In fact, he requires a sequence of two positive outcomes before day trading more than the minimum amount (i.e., he then pays the cost and invests $930). If the outcome is again positive, the investment increases to $1,862 for
Figure 2.3: Day Trader’s Optimal Behavior under Parameter Uncertainty.
This figure illustrates optimal behavior of an agent who is uncertain about the profitability of day trading and maximizes power utility over terminal wealth. The agent has an initial wealth of $w_0 = 10,000$, a risk-aversion of $\gamma = 2$ and can day trade amount $x \geq 0$ at dates 1, 2, 3, and 4 at a cost $c = 20$. The day trade is a success with a probability $p$ and pays off $(1 + \delta)x$; it is a failure with a probability $1 - p$ and pays off $(1 - \delta)x$. There is also a risk-free asset than pays no interest. The investor has a Beta($\alpha_0, \beta_0$) distributed prior about $p$. If the agent pays the cost, he observes the outcome and updates his beliefs as a Bayesian. The agent’s prior in the first example is Beta(2, 1)-distributed and it is Beta(1, 2) in the second example. Each node in this figure shows the agent’s optimal decision, his beliefs and wealth, and how much he invests (the cost is denoted by “+$20”). Terminal nodes ($t = 5$) are not shown.

the last period before the agent consumes his terminal wealth. However, a more probably scenario ex ante (given the agent’s prior) is that the agent observes a negative outcome already at date 0 or at date 1. If this happens, the agent quits. Note that the agent already rationally believes before date 0 that he will have quit after date 1 with probability $\frac{2}{3} + \frac{1}{3} \times \frac{1}{2} = \frac{5}{6}$ and after date 2 with probability $\frac{9}{10}$. 61
Yet, the agent is willing to pay the small cost to learn because of the possibility that day trading may turn out to be profitable.

The second example illustrates how parameter uncertainty affects behavior even when the agent starts with a high prior. The agent’s trade size increases after positive outcomes and falls after negative outcomes. This is not surprising, but it is in strong contrast with a scenario where the agent does not learn. For example, suppose that the agent took the probability of a successful day trade \( \left( \frac{2}{3} \right) \) as a fixed parameter. A quick computation shows that the agent would always
invests a constant fraction (17.2%) of his after-trading costs wealth. Hence, the initial investment would be higher, $1,702 (versus $1,243 with learning). Second, his next period investment would be either $1,984 or $1400, depending on the first period’s outcome—the upstate investment is less than what our agent invests, and the downstate investment is more.

These two examples illustrate how the evolution of beliefs generates dynamics in trade sizes and exit decisions, and how the predictions of a stubborn agent’s behavior are very different from the behavior of agent who learns over time. We now make this comparison more precise.

2.2.6 Learning versus Stubborn Beliefs

We now show formally that the competing hypothesis of stubborn beliefs predicts that day traders’ behavior is “constant” over time. Investors with stubborn beliefs day trade because they believe they can make money by day trading. They do not revise their beliefs over time and, consequently, changes in wealth and horizon alone influence investment behavior. Suppose that an agent has a degenerate subjective distribution $E_t(p) = \hat{p}$—i.e., that the agent’s belief about the profitability of day trading is fixed. This type of an agent day trades only if $\hat{p} > \frac{1}{2}$. If the agent does believe that day trading is profitable, he always day trades a constant amount of after-trading costs wealth:

$$x_t^* = \frac{\hat{p}^\gamma - (1 - \hat{p})^\frac{1}{\gamma}}{\hat{p}^\gamma + (1 - \hat{p})^\frac{1}{\gamma}} (w_t - (T - t + 1) c) \quad (2.15)$$

where $T$ is the index of the last trading day. Note that the adjustment for trading costs takes into account all the costs the agent will pay over the life-cycle. This result shows that the ratio of two consecutive trade sizes is independent of the
level of beliefs and that the investor exits only when \( w_t < (T - t + 1) c \).

This result is in strong contrast to the learning hypothesis: outcomes do not affect the exit decision nor do they influence trade sizes after controlling for wealth changes. Moreover, only investors who learn over time place exploratory, minimum size day trades. Agents with stubborn beliefs either day trade strictly positive amount or they do not day trade at all. We now empirically test these predictions about trade sizes, exit decisions, and exploratory day trades.

### 2.3 Data and Sample

This section describes the institutional setting of the Finnish market and our data sets. We also construct the sample used in our tests, define day traders, and compare day traders to other investors in the same market.

#### 2.3.1 Helsinki Exchanges

Trading on the Helsinki Exchanges (HEX) is divided into sessions. Each trading day starts at 10:10 am with an opening call. Orders that are not executed at the opening remain on the book and form the basis for the continuous trading session. This trading session takes place between 10:30 am and 5:30 pm in a fully automated limit order book, the automated trading and information system (HETI). After-hours trading (5:30 – 5:45 pm) takes place after the continuous trading session and again the next morning (9:30 – 10:00 am) before the next opening call. (Two changes to the trading schedule were made during the sample period. On August 31, 2000, the regular trading session was extended to 6:00 pm and the after-hours session was moved to match this change. On April 10, 2001, an evening session that extended trading hours to 9:00 pm was introduced.)
The HEX trading system displays the five best price levels of the book to the market participants. The public can view this book in a market-by-price form while financial institutions receive market-by-order feed. Simple rules govern trading on the limit order book. There are no designated market makers or specialists; the market is completely order-driven. An investor trades by submitting limit orders. The minimum tick size is €0.01. An investor who wants immediate execution must place the order at the best price level on the opposite side of the book—consistent with the market convention, we call these orders *market orders*. An investor who wants to buy or sell more shares than what is currently outstanding at the best price level must “walk up or down the book” by submitting separate orders for each price level. If a limit order executes against a smaller order, the unfilled portion stays on the book as a new order. Time and price priority between limit orders is enforced. For example, if an investor submits a buy order at a price level that already has other buy orders outstanding, all the old orders must execute before the new order.

The total market value of the 158 companies in the Helsinki Exchanges was €383.14 billion in the middle of the sample period (May 2000). The average realized log-spread during the sample period was 0.43% for the 30 largest stocks and as low as 0.13% for Nokia, the most actively traded company. The most popular online broker charged a fixed fee of €8.25 and 0.2% of the trade value towards the end of the sample period. There was also a flat 28% capital gains tax during the sample period.

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10A market-by-price book displays the five levels on both sides of the market but only indicates the total number of shares outstanding at each price level. A market-by-order book shows each order separately and also shows which broker/dealer submitted each order.
2.3.2 Data

Our data are (i) the complete trading records and holdings information of all Finnish investors and (ii) the transaction data for all the stocks listed on the HEX.

1. The Finnish Central Securities Depository registry (FCSD) provided us the investor holdings and trading records for the period from January 1995 to November 2002. Each record includes a date-stamp, price, volume, stock code, a code that identifies the investor type—a domestic institution, a domestic household, or a foreigner—and other demographic information. We classify investors as individuals or institutions for this study. Grinblatt and Keloharju (2000) give the full details of this data set.

2. The transactions data are derived from the supervisory files from the HEX for a period from January 1995 to October 23, 2001. Each entry in this data set is a single trade. Each entry contains a unique trade identifier, the price, volume, and brokerage firm identities for both the buyer and the seller.

2.3.3 Matching the Data Sets

We match the investor trading records against the transactions data to obtain information on at what time each trade is completed. We describe the matching procedure here. Each trade record in the transaction data contains all the same information as the investor trading records except the investor identity. We use common elements to link the data sets.

There is no ambiguity in matching two types of trades: trades with unique price-volume combinations and non-unique trades that must originate from the
same investor.\footnote{We say that a trade has a \textit{unique} price-volume combination if, for example, there is only one trade (in one stock-day) with a price of €82 and a volume of 1,200 shares. A trade is non-unique if, say, three trades have the same price-volume combination. In this example, “all must originate from the same investor” would mean that a single investor in the investor data set is the buyer or the seller in all the three trades.} We call these trades \textit{uniquely matched trades}. There is no one-to-one link between the data sets for the remaining trades. However, it is often possible to determine almost exactly when a trade took place. For example, even if there are two trades with a price/volume combination of 40.1 euros/200 shares, the trades may have occurred almost simultaneously. We follow this idea and compute the lower and upper bound for trade time (i.e., upper and lower timestamps) for each non-unique entry in the investor trading records. Each investor trading record now contains either the exact time of the trade or at least bounds for the time of the trade.

\subsection*{2.3.4 Sample and Demographics}

A \textit{day trade} is a purchase and a sale of the same stock (in any order) on the same day.\footnote{An important question is whether this \textit{ex post} identification of day trades is valid. In particular, it is possible that investors always set target prices when they purchase shares and sometimes these targets are breached during the same-day, generating “day trades”. Linnaimaa (2005a) studies the same data as we do and tests—and strongly rejects—this hypothesis. For example, over 1/3 of the day trades result in losses but under the target price scenario there should be none.} If the investor buys and sells the same number of shares, we say that the day trade is \textit{complete}. If the amounts differ, we say that the day trade is \textit{partial}. A \textit{day trader} is an investor who day trades at least once. There are 1,055,505 individual investors in the FCSD registry with at least some holdings or trades during the sample period. A total of 22,529 investors day trade at least once. Most day traders (52.6\%) day trade only once or twice. (There \textit{are} very active day traders as well: 6.0\% of individuals day trade at least 50 times, and the...
Figure 2.4: Day Trading Activity in Finland, 1997–2002.
This figure plots the level of the Helsinki Exchanges market index (the Finnish stock market; the value at the end of January 1997 is set to 100) and the number of Finnish individual investors that day trade each month. A day trade is a purchase and a sale of the same stock (in any order) on the same day. A total of 22,529 individuals completed at least one day trade between January 1995 and November 2002.

most active day trader in the sample day trades 1,715 times.) We now discuss the overall day trading phenomenon and then compare individual day traders against the rest of the individual investor population.

Figure 2.4 shows how day trading activity varied between January 1997 and November 2002. It plots the number of investors who completed at least one day trade each month. The figure shows that although the day trading activity reached its peak in 2000, it did not die off: over 1,500 investors day traded each month towards the end of the sample.\(^\text{13}\)

Table 2.1 compares demographics of day traders against all other Finnish investors. We highlight several regularities. First, day traders are mostly male (81% vs. 54%) and on average 10 years younger than other investors. Second, day traders trade more even after excluding trades directly related to day trading. The

\(^{13}\text{Although not shown in the figure, there is a constant in- and out-flow of day traders each month. For example, 357 of the investors who day traded in November 2002 day traded for the first time.}\)

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median day trader completes 56 trades whereas this number is only three for the rest of the investor population. Third, the demographics suggest that the average day trader is less risk-averse than what the other investors are. For example, the stocks that day traders purchase have an average beta of 1.58 compared to the 1.42 for the others (t-value for the difference is −46.1). Day traders’ activity in trading warrants (15% versus 2%) supports the same conclusion: the typical day trader is less risk-averse than what the typical investor in the rest of the population is. Fourth and finally, many day traders had very limited amount of investment experience before they began to day trade. Panel B in Table 2.1 shows that 483 (2.1%) day traders had never owned or traded stocks before their first day trade. Moreover, 3,311 (14.7%) day traders had less than six months of experience.

Many of these demographics are consistent with the learning hypothesis. First, most day traders day trade only a few times. This is not what one would expect if stubborn beliefs were the main reason for day trading. Second, the evidence about lower risk-aversion is consistent with the model of Section 2.2: a mildly risk-averse investor day trades for a wide range of priors. Third, some day traders’ limited experience is also inconsistent with stubborn beliefs. If stubborn beliefs were the main reason for day trading, we would expect only heavy traders to turn into day traders. We argue that if there are any investors with “parameter uncertainty” about the profitability of day trading, they are most likely those with the least exposure to the market.14

14Many day traders are active traders as well, consistent with these investors’ being possibly overconfident about their stock-picking abilities (Barber and Odean 2001). However, this evidence against the learning hypothesis is not conclusive for several reasons. First, many day traders are very inactive day traders. Second, overconfidence in stock-picking skills does not imply that these active traders have perfectly stubborn beliefs about the profitability of day trading as well. Finally, many of the “other trades” may in fact be indirectly related to day trading: Linnainmaa (2005a) shows that individual day traders often abort their day trades
Table 2.1: Day Trader and Investor Population Demographics

A day trader is an individual investor who buys and sells the same stock (in any order) on the same day at least once during the sample period from January 1995 to November 2002. Panel A reports the demographics for all day traders and for the rest of the investor population. Traded Warrants is the fraction of investors that traded warrants during the sample period. Average Beta is the average beta of all purchases. (“Day trades” are ignored when computing Number of Trades, Average Trade Size, Traded Warrants, and Average Beta.) Day trade is complete (partial) if the sale and purchase volumes are equal (unequal). Panel B reports how much investment experience day traders had at the time of the first day trade. This experience is measured from the day the investor first acquired stocks. Investors who already owned stocks at the beginning of the sample are included into the “over a year” category.

Panel A: Day Trader and Investor Population Demographics

<table>
<thead>
<tr>
<th></th>
<th>Day Traders</th>
<th>Remaining Population</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>Birth Year</td>
<td>1958.3</td>
<td>1960</td>
</tr>
<tr>
<td>Proportion Male</td>
<td>81.3%</td>
<td>54.0%</td>
</tr>
<tr>
<td>Number of Trades</td>
<td>99.9</td>
<td>56</td>
</tr>
<tr>
<td>Average Trade Size</td>
<td>€11,953</td>
<td>€5,037</td>
</tr>
<tr>
<td>Traded Warrants</td>
<td>15.1%</td>
<td></td>
</tr>
<tr>
<td>Average Beta</td>
<td>1.58</td>
<td>1.59</td>
</tr>
<tr>
<td>Number of Day Trades</td>
<td>Complete</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>Partial</td>
<td>9.3</td>
</tr>
<tr>
<td>N</td>
<td>22,529</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Day Traders’ Investment Experience before the First Day Trade

<table>
<thead>
<tr>
<th>Experience</th>
<th>Fraction of Day Traders</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>2.1%</td>
</tr>
<tr>
<td>Less than a Month</td>
<td>3.2%</td>
</tr>
<tr>
<td>One to Three Months</td>
<td>4.4%</td>
</tr>
<tr>
<td>Three to Six Months</td>
<td>5.0%</td>
</tr>
<tr>
<td>Six Months to a Year</td>
<td>7.8%</td>
</tr>
<tr>
<td>Over a Year</td>
<td>77.5%</td>
</tr>
</tbody>
</table>

when the stock price moves against them, leaving only an observation of a purchase in the data.
2.4 Empirical Tests

We test the learning hypothesis by studying individual day traders’ two choices: the day trade size and the decision to quit day trading. We test following hypotheses:

**Hypothesis 1.** *Day traders stop day trading after negative outcomes, controlling for the outcomes’ wealth effects.*

**Hypothesis 2.** *Individuals who day trade in small quantities relative to their wealth are more likely to quit day trading.*

**Hypothesis 3.** *Day traders increase their trade sizes after positive outcomes and decrease them after negative outcomes, controlling for the outcomes’ wealth effects.*

These predictions follow the learning hypothesis, formalized in Section 2.2 as a Bayesian life-cycle model. For example, Hypotheses 1 and 3 test the most straightforward predictions of the learning model: investors revise their beliefs over time and decrease their trade sizes or quit after poor outcomes. Hypothesis 2 uses the idea that trade size is informative of the parameters of the prior distribution: the optimal trade size depends on the investor’s wealth and beliefs. Hence, an investor who commits only a small proportion of wealth to day trading is closer, ceteris paribus, to quitting than an investor who places a large day trade (see Proposition 9).

The stubborn beliefs hypothesis is the alternative hypothesis in our tests. This hypothesis predicts that only the outcomes’ wealth effects matter. A possible reaction to this choice is that this is a straw-man hypothesis: surely investors learn from their experience. However, this is precisely our point: numerous studies assume that investors are, e.g., permanently overconfident, or that their learning is biased in a way that ultimately leads to perfect overconfidence (Gervais and
Odean 2001). Hence, our null hypothesis of perfectly stubborn beliefs is a natural starting point for studying whether investors learn from experience. Section 2.4.6 addresses directly the optimality of individuals’ belief-updating rules.

2.4.1 Methodology

We use data on individual day traders’ sequential day trades in our tests. Each entry contains information about a single day trade. We also link investors’ day trades so that we also have all the information for both the previous and following day trades (if any).15

2.4.1.1 Variable Definitions and Description of Controls

Definitions. We compute day trading profits as

$$\Pi = \min(V_b, V_s) \times (p_s - p_b)$$  \hspace{1cm} (2.16)$$

where $V_b$ and $V_s$ are the number of shares purchased and sold, and $p_b$ and $p_s$ are the average purchase and sale prices, respectively. We define Loss as a dummy variable that is set to one if the day trade loses money (i.e., $p_s > p_b$). We use the Loss dummy to test the impact of outcomes; by ignoring the size of the gain or loss, we can control for the outcomes’ wealth effects. We define Proportion of Earlier Losses as the number of losing day trades divided by the total

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15 Each entry contains the following information: the number of shares purchased and sold; the average purchase, sale, and same-day closing prices; the intraday sequence of individuals trades (i.e., whether the investors first purchased shares and later sold them, or vice versa; this is blank when the upper and lower time-stamps in Section 2.3.3 overlap and create ambiguity about the sequence); the number of trading days from the same investor’s previous day trade; the gross profit or loss for each previous day trade; the portfolio value at yesterday’s close; the number of earlier trades and day trades; and the number of shares the investor owned yesterday along with the amount of unrealized capital gains or losses computed with the FIFO principle.
number of day trades. (For example, if the agent has taken losses in 60% of his
day trades before this one, we set the variable equal to 0.6.) We measure day
trade size as

\[
\text{trade size} = \min(V_b, V_s) \times \text{yesterday’s closing price.}
\]  

(2.17)

We use this definition instead of the actual value of the trade (i.e., \(V_b \times p_b + V_s \times p_s\))
to avoid correlation between the day trade’s performance and the trade size.\textsuperscript{16}

**Controls.** We control for the day trading outcomes’ wealth effects to distinguish learning from stubborn beliefs. These controls are necessary because even a stubborn beliefs-agent would respond to wealth changes. We include two log-portfolio values: the value of the investor’s portfolio at yesterday’s close (“current portfolio”) and the value of the investor’s portfolio before the first day trade (“initial portfolio”). The trade size test—that examines changes in the sizes of consecutive day trades—replaces the initial portfolio value with the value of the investor’s portfolio before the previous day trade. These variables keep track of how much wealthier or poorer the investor is relative to an earlier date. We also include an additional wealth control into the exit regression: the fraction of money the investor has made or lost in day trading. We compute this as the sum of day trading profits (or losses; Eq. 2.16) divided by the value of the initial portfolio.

\textsuperscript{16}To illustrate the problem that might otherwise arise, suppose that (i) an individual’s day trade outcomes are independently and identically distributed and that (ii) the investor always buys shares worth the same amount of money and sells them later the same day. Then, a “good” day trade would have a higher trade size than a “bad” day trade (because \(p_s > p_b\) in the actual trade value computation, \(V \times (p_b + p_s)\)). Such an investor’s good performance would predict a lower trade size next period because of the i.i.d. assumption. Our method ensures that this does not affect our results. Moreover, as discussed in Section 2.4.5.4, the results are nearly identical even if we use the actual trade size. This shows that the mechanistic effect described here is empirically of only second-order importance.
We control for the possibility that some day trades are tax motivated. The capital gains tax in Finland creates an incentive for investors to realize losses if they have already realized gains (Grinblatt and Keloharju 2004). Some day trades may not speculative because of this incentive; rather, an investor sells and purchases the shares back almost simultaneously to realize a loss. (This is illegal but the law is difficult to enforce.) We control for tax-loss trading by adding monthly dummies, an ownership dummy, and a *Capital Loss* dummy.\(^{17}\) We also include the following unreported control variables in the exit and trade size regressions: 93 monthly dummies, 6 dummy variables to define the day trade type\(^{18}\), the number of stocks in the investor’s portfolio, “no portfolio” dummies for the current and the first portfolio, a dummy for the first day trade, a dummy variable for a missing FIFO price.

2.4.2 Regression Specifications

2.4.2.1 Specification of the Exit Regression

We estimate logistic regression to study whether individuals are more likely to quit day trading after observing poor outcomes. We set the dependent variable to one for each investor’s last day trade; otherwise, the variable is set to zero. Some day traders probably continued to day trade after the end of our sample.

\(^{17}\)The results are almost unchanged if these controls are omitted. Moreover, the results are very similar if we exclude all day trades where the investor previously owns shares in the stock that is day traded. This suggests that even though some day trades may be tax motivated, such day trades do not affect our analysis.

\(^{18}\)We use the same categorization as Linnainmaa (2005a): a day trade is classified based on whether it is complete (i.e., \(V_b = V_s\)) or partial (i.e., \(V_b \neq V_s\)) and whether the investor initiated the day trade with a purchase or a sale. An additional category represents day trades where the investor leaves open an overnight short position. We also include a dummy variable set to one if the day trade type is unknown (i.e., the upper and lower time stamps overlap for the investor’s trades).
However, because it is impossible to distinguish a true exit from a pause in day trading at the end of the sample, we always use this definition. This treatment is conservative: we add noise to the data if some of our exits are only pauses. However, our monthly dummies help to control for the deterministic increase in the unconditional exit probability. (Section 2.4.5.4 shows that the results are robust to this specification.)

We include three main explanatory variables to test the predictions of the learning hypothesis specific to quitting: (i) the loss from the current day trade (a dummy variable), (ii) the proportion of earlier losses, and (iii) the log-trade size. We also include the control variables described in Section 2.4.1.1. We estimate the exit regression for two samples. First, we estimate the regression by including all 22,529 day traders. However, there is a concern that some very active day traders may drive the results. We address this possibility by estimating a second regression, this time only with those individuals who day trade at most 10 times. This second regression introduces a selection bias on purpose: the results are conditional on the investor quitting relatively soon after starting to day trade. For example, such investors may have more pessimistic expectations to begin with.

2.4.2.2 Specification of the Trade Size Regression

We estimate an OLS regression to study whether investors adjust their trade sizes in response to earlier outcomes. The log-difference between the sizes of two consecutive day trades is the regression’s dependent variable. We use the trade size definition in Eq. 2.17 to avoid correlation between the trade sizes and performance.

We include similar explanatory variables into the trade size regression as we
used in the exit regression. The distinction between these two regressions is that we now lag most of our observations by one day trade. Hence, the key learning variable is now the loss dummy from the previous day trade. We also include the proportion of earlier losses before the previous day trade. We also include the control variables described in Section 2.4.1.1 and estimate the regression again separately for all day traders and for those who day trade at most 10 times.

2.4.3 Results on Day Traders’ Decision to Quit Day Trading

The exit regression estimates in Table 2.2 show that negative outcomes induce day traders to quit day trading. This conclusion is supported by all three of our key learning variables. First, an investor is significantly more likely to quit after a negative outcome than after a positive outcome. The coefficient estimate of 0.38 ($t$-value is 21.5) for the entire sample shows that the odds-ratio for quitting is 1.47 times larger after a negative outcome. This strongly supports the hypothesis that day traders revise their beliefs depending on what type of outcomes they observe. Second, an investor with a higher proportion of bad outcomes in the past is more likely to exit today. For example, a switch from “no bad outcomes” to “all bad outcomes” increases the quitting odds-ratio by 1.21 times. This result is intuitive: ceteris paribus, an investor who has had more bad experiences has lower expectations about the profitability of day trading. This effect is—both economically and statistically—somewhat stronger in the second regression that only includes “less than ten times” day traders.
Table 2.2: The Impact of Losses on the Decision to Quit Day Trading

This table estimates a logistic regression to examine when day traders quit day trading. Each observation is a single day trade, defined as a purchase and a sale of the same stock on the same day. The dependent variable is set to one for each investor’s last day trade. Current Loss is a dummy variable set to one if the average sale price for the day trade is less than the average purchase price. Proportion of Earlier Losses is the number of losses from the earlier day trades divided by the total number of earlier day trades. Trade Size is the minimum of purchase and sale volumes multiplied by the yesterday’s closing price. Current and First Portfolio Values are the values of the investor’s portfolio at yesterday’s close and the day before the first day trade. Cumulative Profits is the sum of day trading profits from all earlier day trades. Number of 3Mos Day Trades is the number of times the investor day traded during the previous three months. The number of trades is defined similarly. Ownership is a dummy variable set to one if the investor already owned shares in the stock that is being day traded. Unrealized Capital Loss is a dummy variable set to one if the FIFO price is higher than the yesterday’s closing price. The second column estimates the regression only for those individuals who day trade at most 10 times. (The regression also includes unreported control variables; see text.)
Table 2.2: The Impact of Losses on the Decision to Quit Day Trading. (cont’d)

<table>
<thead>
<tr>
<th>Learning Variables</th>
<th>Coeff.</th>
<th>t-value</th>
<th>Coeff.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Loss</td>
<td>0.38</td>
<td>21.5</td>
<td>0.32</td>
<td>14.0</td>
</tr>
<tr>
<td>Proportion of Earlier Losses</td>
<td>0.19</td>
<td>5.8</td>
<td>0.27</td>
<td>7.1</td>
</tr>
<tr>
<td>( \ln(\text{Trade Size}) )</td>
<td>-0.23</td>
<td>-29.0</td>
<td>-0.15</td>
<td>-15.3</td>
</tr>
<tr>
<td>Wealth Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln(\text{Current Portfolio Value}) )</td>
<td>0.07</td>
<td>7.2</td>
<td>0.02</td>
<td>1.2</td>
</tr>
<tr>
<td>( \ln(\text{First Portfolio Value}) )</td>
<td>-0.02</td>
<td>-2.8</td>
<td>0.01</td>
<td>1.0</td>
</tr>
<tr>
<td>Cum. Profits / 1st Portfolio Value</td>
<td>0.12</td>
<td>5.3</td>
<td>0.22</td>
<td>6.2</td>
</tr>
<tr>
<td>Other Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \ln(\text{Number of 3MOs Day Trades}) )</td>
<td>-0.60</td>
<td>-64.9</td>
<td>0.24</td>
<td>8.3</td>
</tr>
<tr>
<td>( \ln(\text{Number of 3MOs Trades}) )</td>
<td>-0.72</td>
<td>-75.5</td>
<td>-0.48</td>
<td>-37.8</td>
</tr>
<tr>
<td>Ownership</td>
<td>0.35</td>
<td>7.0</td>
<td>0.30</td>
<td>4.9</td>
</tr>
<tr>
<td>Own. * Unreal. Capital Loss</td>
<td>-0.06</td>
<td>-2.7</td>
<td>-0.02</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

\[ N = 281,350 \]  \[ \text{Day Traders with } N_{dt} \leq 10 \]

Third, investors with smaller day trades are more likely to quit. This finding is consistent with the hypothesis that a small day trade size implies “poor” beliefs. For example, ceteris paribus, if we double the trade size, the quitting odds-ratio is 1.17 times smaller. On the other hand, if one investor makes a day trade worth $10,000 while the other (otherwise identical) only day trades $1,000, our results indicate that the odds of the latter quitting are 1.7 times higher.

There is no support for the hypothesis that investors quit only after they have lost enough money. Note that the current portfolio log-value has a positive sign while the first portfolio log-value has a negative sign. This indicates that a positive change in the portfolio value is associated with a higher exit probability. Similarly, large cumulative day trading profits (this is the “cumulative profits / first portfolio value” variable in the regression) predicts higher exit probability.
However, both the economic and statistical significance of these results is weak.\textsuperscript{19} We also note that the regression estimates are fairly similar for both samples.\textsuperscript{20} This suggests that the presence of very active day traders is not a concern for our results. Hence, our exit regression results are consistent with the learning hypothesis: day trading seems to be often motivated simply by uncertainty about the profitability of day trading, not by stubborn beliefs.

### 2.4.4 Results on Day Traders’ Choice of Day Trade Sizes

The trade size regression results in Table 2.3 also strongly support the hypothesis that day traders revise their beliefs over time. A day trader decreases his day trade size by 5.9\% after a loss compared to the change after a gain. (The impact of negative outcomes is −3.0\% for the day traders in the second column.) Note that this figure is conditional on the investor continuing day trading: we are “missing” those observations where the investor drops the day trade size to zero.\textsuperscript{21}

A higher proportion of earlier losses predicts a modest increase in the trade size. Note that this variable’s value is negative correlated with the size of the

\begin{itemize}
  \item Our results also debunk one alternative, behavioral explanation to why investors day trade. It is possible that investors derive gambling or horse race-type of entertainment from day trading and that they have made an initial allocation to a “day trading fund or mental account”. If so, investors day trade until they have exhausted their allocation. Our estimate for cumulative profits suggests that this is not the case, at least for most day traders. Note that it is likely that some investors derive entertainment from such activity (Grinblatt and Keloharju 2006); our conclusion is simply that this sensation-seeking explanation does not appear to describe the motivations of the majority of day traders.

  \item The only significant differences are between “the number of earlier (day) trades”. This is not surprising. Because there is a small number of extremely active day traders but a large number of casual day traders, a high “past trades” value catches this skewness in the distribution.

  \item We also examine day traders’ unconditional trade size changes. The average change in the trade size is −0.9\% for the whole sample and −1.6\% for the day traders in the second column. This indicates that there is a drift down in trade sizes, consistent with the idea that most day traders learn that they cannot make money by day trading. Collectively, our results show that investors revise their beliefs downwards, decrease their trade sizes, and eventually quit.
\end{itemize}
Table 2.3: The Impact of Losses on the Day Trade Size

This table estimates a regression where the dependent variable is the log-difference between the size of the current day trade and the size of the previous day trade. The size of the day trade is defined as the minimum of purchase and sale volumes multiplied by the stock price at the previous close. *Previous Loss* is a dummy variable set to one if the average sale price of the previous day trade is below the average purchase price and *Proportion of Earlier Losses* is the number of losses from the earlier day trades divided by the total number of day trades (this excludes also the previous day trade’s outcome). *Current and Previous Portfolio Values* are the values of the investor’s portfolio at yesterday’s close and the day before the previous day trade. *Number of 3Mos Day Trades* is the number of times the investor day traded during the previous three months. The number of trades is defined similarly. *Ownership* is a dummy variable set to one if the investor already owned shares in the stock that is being day traded. *Unrealized Capital Loss* is a dummy variable set to one if the FIFO price is higher than the yesterday’s closing price. *Days from the Previous Day Trade* is the number of trading days from the Previous Day Trade. The second column estimates the regression only for those individuals who day trade at most 10 times. (The regression also includes unreported control variables; see text.)

<table>
<thead>
<tr>
<th>Learning Variables</th>
<th>All Day Traders</th>
<th>Day Traders with (N_{dt} \leq 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous Loss</td>
<td>−0.059</td>
<td>−0.030</td>
</tr>
<tr>
<td>Proportion of Earlier Loss</td>
<td>0.028</td>
<td>0.035</td>
</tr>
<tr>
<td>Wealth Controls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\ln(\text{Current Portfolio Value}))</td>
<td>0.022</td>
<td>0.041</td>
</tr>
<tr>
<td>(\ln(\text{Previous Portfolio Value}))</td>
<td>−0.022</td>
<td>−0.042</td>
</tr>
<tr>
<td>Other Controls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\ln(\text{Number of 3MOs Day Trades}))</td>
<td>−0.015</td>
<td>−0.050</td>
</tr>
<tr>
<td>(\ln(\text{Number of 3MOs Trades}))</td>
<td>0.008</td>
<td>0.024</td>
</tr>
<tr>
<td>Ownership</td>
<td>0.023</td>
<td>0.041</td>
</tr>
<tr>
<td>Own. * Unreal. Capital Loss</td>
<td>−0.062</td>
<td>−0.077</td>
</tr>
<tr>
<td>(\ln(\text{Days from Prev. Day Trade}))</td>
<td>−0.013</td>
<td>−0.023</td>
</tr>
<tr>
<td>(N)</td>
<td>256,940</td>
<td>25,452</td>
</tr>
<tr>
<td>Adjusted (R^2)</td>
<td>7.6%</td>
<td>7.8%</td>
</tr>
</tbody>
</table>

previous day trade because of the way it is lagged: the *previous* day trade \((t - 1)\) is relatively small when there are more poor *earlier* observations \((\tau < t - 1)\). The
estimate shows that, conditional on keeping on day trading after poor observations, day traders increase their trade sizes. However, the economic significance of this result is rather small: the increase in the relative trade size is predicted to be only 2.8% if the investor switches from “no bad outcomes” to “all bad outcomes”.

The results in Table 2.3 show that the outcomes’ wealth effects also matter. Investors increase their trade sizes if their investment wealth today is higher than what it was at the time of the previous day trade. For example, the point estimate in the first regression predicts that day traders increase their day trade sizes by 2.2% of the percentage change in the portfolio value. For example, if the portfolio value has increased by 20% from the previous day trade, the relative increase in the trade size is only 0.5%. Hence, this wealth effect is economically modest. These results support the hypothesis that individuals learn from experience: individuals revise their beliefs over time and change their trade sizes more than what they would if they ignored feedback.

2.4.5 Robustness Tests

We now show that the conclusions drawn from our exit and trade size regressions are robust to alternative specifications and further tests. First, we argue that some of investors’ initial day trades appear to be exploratory (Section 2.4.5.1). Second, we show that investors avoid very risky strategies when day trading for the first time (Section 2.4.5.2). Third, we demonstrate that the link between an investor’s trade size and exit decisions leaves a very particular mark in the data: there is a positive drift in the average trade sizes of those investors who continue day trading (Section 2.4.5.3). Fourth, we describe additional robustness checks of model specifications and variable definitions (Section 2.4.5.4). These analyses
support the learning hypothesis and are difficult to explain by the stubborn beliefs hypothesis.

2.4.5.1 Exploratory Day Trades

Our parameter uncertainty model predicts that an investor who \textit{does not} believe that day trading is profitable may still place a small exploratory day trade to learn about this profitability. We now argue that some trades in the data do not appear to be speculative because of their small size.

Many investors day trading for the first time day trade in very small quantities. For example, 5.5\% of the first day trades are less than €500 in size (Eq. 2.17) while this fraction is only 1.0\% for all the other day trades. Day trades this small are suspicious because of the effect of trading costs. The cheapest online broker at the time of the sample charged a commission of €8.25 per trade and 0.2\% of the trade value. Let us ignore the proportional commission and consider only the role of the fixed cost. If a trade of €500 is speculative, the investor must expect to predict correctly a price movement of approx. 3.3\% (2*8.25/500) to break even! The median required price movement for these small initial day trades is 6.3\%. This is an enormous daily price change and seems to contradict the assumption that these small day trades are speculative. Moreover, there is a second problem with the speculation argument. Even if we assume that these small day trades are speculative, their sizes are curious. If the investor expects to predict correctly a significant price change, he should be willing to trade more aggressively to exploit the opportunity. The only plausible explanation seems to be that these small day trades are exploratory, not speculative.
2.4.5.2 Initial Day Trades and Risk-Taking

The learning hypothesis suggests that many of the investors day trading for the first time are likely those who only day trade to learn more. Such investors want to limit the riskiness of their trades. We examine this possibility by studying whether investors avoid a particular type of a day trade: a day trade where the investor takes a short-position with the intent of covering the short later the same day at profit. (We call this the “short” strategy.) This strategy is riskier than an alternative sequence where an investor first buys shares with intent of selling them back later (the “long” strategy) for at least two reasons. First, the potential downside is unlimited because the stock price can make an arbitrarily large jump upwards. Second, the investor has to cover the short even if the liquidity in the stock dries up—i.e., if the spreads widen significantly or even disappear.

We test the hypothesis about the riskiness of initial day trades as follows. First, we drop day trades where the investor owns shares in the stock that is being day traded. Second, we classify investors into two groups: investors who are day trading for the first time and those who have already day traded at least once before. Third, we compute the number of “short” and “long” day trades for both groups for each day in the sample. We then drop days with none or only one day trade from either group. Next, we compute the proportion of “short” day trades for each day-group and take the difference between the two groups. The resulting daily time-series of differences has 752 observations. This time-series measures in a very controlled setting whether investors who are day trading for the first time prefer the safer “long” strategy.

The time-series average difference between the “first” and “other” groups is $-12.2\%$ (the s.e. is 0.7\%). The investors who are day trading for the first time employ the riskier strategy in 13.2\% of the cases; hence, more experienced day
traders are approximately twice as likely to short. This result shows that investors day trading for the first time have a distaste for the riskier strategy, consistent with the hypothesis that investors learn from experience. Investors who have day traded before are less likely be day trading just to learn about the profitability of day trading.

2.4.5.3 Trade Sizes and the Decision to Exit

If investors day trading in smaller quantities are closer to quitting (Proposition 9), the cross-sectional average trade size will drift upwards. (‘‘The cross-sectional average trade size’’ is the average day trade size of those investors who are day trading for the first time, for the second time, and so forth.) The intuition is that those who keep on day trading believe that day trading is profitable and day trade in larger quantities while those who quit drop out from the left tail of the trade size distribution. Hence, as more time passes, only those who believe that day trading is profitability remain. Note that this is a very strong prediction: we know from Section 2.4.4 (see footnote 21) that, unconditionally, the typical day trader decreases his trade size from one day trade to the next. This suggests that we should, if anything, find a negative drift in cross-sectional average day trade sizes.

The data strongly supports the link between trade sizes and the exit decision: the cross-sectional average trade size drifts up from €13,174 to €23,096 from the first day trade to the tenth day trade. All the intermediate changes (e.g., from the first day trade to the second day trade) are positive. Furthermore, outliers do not drive this dramatic increase: the median trade size increases from €4,250 to €10,750 with all intermediate changes positive. (The Mann-Whitney z-value for the difference between the tenth and the first day trade is 32.2.) These results
show that investors who keep on day trading trade in larger quantities than those who drop out. Those who drop out have low expectations about the profitability and trade in small quantities because both the exit and trade size decisions are determined by beliefs.

2.4.5.4 Additional Robustness Results

We conclude our discussion of the empirical evidence by describing several robustness checks of our variable definitions and model specifications. First, the exit regression results are similar if the model is estimated as a linear probability model and not as a logistic model. Similarly, if we recode the dependent variable in the trade size regression as an increase / decrease dummy variable and estimate the model as a logistic regression, the results are qualitatively the same.

Second, we find that the results are nearly unchanged if the trading profit measure is changed to include a brokerage fee and the residual position (i.e., \( V_b - V_s \)) is taken into account by marking it to the market at the closing price. Similarly, the results are nearly unchanged if we define trade size as the true value of the trade, or if we replace the \( \min \)-function in Eq. 2.17 by the sum of purchase and sale volumes.

Third, we examine whether there are any problems in our methodology of identifying when day traders quit. (Our exit regression in Section 2.4.3 included all day trades from each investor and defined each investor’s last day trade as the exit. We argued that this is a conservative treatment and also included monthly dummies as control variables.) We address this concern by leaving out investors who complete any day trades during the last six months of our data set. This restriction guards against the possibility that some of the “late” day traders are just taking a brief pause from day trading and have not really quit. We find
slightly stronger results for this subsample. (We report the whole sample results in Section 2.4.3 because our arbitrary six-month rule may create a selection bias: we are probably cutting out day traders who have discovered that they can make money by day trading.)

Fourth, we estimate the exit regression separately for several subsamples to show that the results are not created by, e.g., nonlinearities in the wealth controls or by heterogeneity across investors day trading for the $i$th time versus those day trading for the first time. We partition all day trades into subsamples in two steps. In the first step, all day trades (with $2 \leq i \leq 10$, where $i$ is the number of the current day trade) are assigned into bins based on the number of the day trade, $i$. In the second step, we assign each bin’s observations into deciles based on the investor’s cumulative day trading profits. By construction, investors in each of the resulting 90 subsamples have lost or gained an almost equal amount of money over the same number of day trades. We then estimate the exit regression for each of these subsamples.\textsuperscript{22} We use “the proportion of losses” to test the learning hypothesis. We define this variables as the number of losing day trades (including the current day trade) divided by the total number of day trades.\textsuperscript{23} The coefficient estimate for this variable is positive in 70% of the subsamples and has an average of 0.397 (the standard error for this average estimate is 0.094). This result shows that outcomes matter beyond their wealth effects, consistent with the learning hypothesis. It also suggests that omitted controls do not explain Table 2.2’s exit regression results.

\textsuperscript{22}We modify the regression of Table 2.2 to omit some variables that would cause multicollinearity and reduce the number of control variables because of smaller sample sizes. The details are available upon request.

\textsuperscript{23}For example, we set this variable to $3/5$ for an investor who has lost money in three out of five day trades.
2.4.6 Direct Estimation of the Learning Model

The empirical results on day traders’ exit and trade size decisions indicate that investors update their beliefs in the correct direction: a bad outcome leads an investor to revise his or her beliefs downwards and vice versa. These results, however, cannot and should not be interpreted as evidence that day traders are Bayesians. This section estimates the learning model of Section 2.2 to study two questions:

- Do individuals react asymmetrically to good and bad outcomes?
- What type of priors did individuals have about the profitability of day trading at the time of the first day trade?

A Bayesian investor in the learning model updates his or her beliefs from \((\alpha_0, \beta_0)\) to \((\alpha_0 + 1, \beta_0)\) after a good outcome and to \((\alpha_0, \beta_0 + 1)\) after a bad outcome. We relax this updating rule by defining general updating parameters \(\kappa_\alpha\) and \(\kappa_\beta\): an investor with a prior \((\alpha_0, \beta_0)\) updates to \((\alpha_0 + \kappa_\alpha, \beta_0)\) after a good outcome and to \((\alpha_0, \beta_0 + \kappa_\beta)\) after a bad outcome. We estimate \(\kappa_\alpha\) and \(\kappa_\beta\) by fitting the learning model to day traders’ observed behavior.

2.4.6.1 Methodology

The learning model does not have an analytical solution and cannot be inverted directly for estimates of the learning parameters. Our approach is to first discretize parameters \((\kappa_\alpha, \kappa_\beta, \alpha_0, \beta_0)\) and then to solve the model for all possible combinations. We proceed as follows. For each \((\kappa_\alpha, \kappa_\beta)\) pair and for each day trader \(i\), we pick the prior \((\alpha_0^i, \beta_0^i)\) that minimizes a distance measure between actual choices in the data and the choices predicted by the model. (Henceforth,
we omit superscript $i$.) Let $x_t^a$ denote the actual trade size (including transaction cost $c$) and let $x_t^\rho$ denote the trade size predicted by a prior distribution $\rho \equiv (\alpha_0, \beta_0)$. Then, define $y_t$ as the change in the (actual or predicted) trade size as

$$y_t = \begin{cases} \frac{x_t}{x_{t-1}} - 1 & \text{if } x_{t-1} > 0 \\ 0 & \text{if } x_{t-1} = 0 \end{cases}$$

Then, we define the distance between the actual and predicted trade sizes (for $t > 1$) as

$$e^\rho_t = \begin{cases} 1 & \text{if } (x_t^a > 0 \text{ and } x_t^\rho = 0) \text{ or } (x_t^a = 0 \text{ and } x_t^\rho > 0) \\ y_t^a - y_t^\rho & \text{otherwise} \end{cases}$$

(2.18)

This distance is the difference between actual and predicted trade size changes with a penalty for observations where an agent does not participate even though the prior predicts participation or vice versa.\textsuperscript{24} For fixed updating parameters $(\kappa_\alpha, \kappa_\beta)$, we pick for each individual $i$ the optimal prior $\rho^i*$ minimizes the sum of squared distances over the first $K$ day trades,

$$\rho^i* = \arg \min_{\rho} \sum_{t=2}^{K} (e^\rho_t)^2.$$ 

(2.19)

\textsuperscript{24}We use the changes in trade sizes to construct the distance measure for two reasons. First, when the change in the trade size is defined as a proportion, we do not need to specify or identify the scaling parameter $\delta$ (see, e.g., Eq. 2.11). Second, the task for an algorithm is now to match only \textit{changes} in trade sizes without having to pay any attention to getting the \textit{levels} right. This is the proper objective when drawing conclusions about how individuals update their beliefs.
Our point estimates for the updating parameters are those that generate the smallest total sum of squared residuals in a sample of $M$ individuals,

$$(\kappa^*_\alpha, \kappa^*_\beta) = \arg \min_{(\kappa_\alpha, \kappa_\beta)} \sum_{m=1}^{M} \rho^*_m.$$ 

We resample the original sample of investors $Q$ times to get a bootstrapped estimate of the sampling distribution for $(\kappa^*_\alpha, \kappa^*_\beta)$. This estimation procedure also gives us the implied distribution of day traders’ priors—conditional on the choice of $(\kappa_\alpha, \kappa_\beta)$—as a by-product.

We empirically implement this approach as follows. First, we create a grid of 900 priors by letting both $\alpha_0$ and $\beta_0$ take on 30 different values between 0.5 and 20. Second, we fix $\kappa_\alpha = 1$ and let $\kappa_\beta$ take on 100 different values from 0 to 2. The reason for fixing $\kappa_\alpha = 1$ is that we cannot simultaneously identify priors and updating parameters; for example, observe that updating parameters $(\kappa_\alpha, \kappa_\beta)$ and a prior $(\alpha_0, \beta_0)$ are equivalent to updating parameters $(\frac{\kappa_\alpha}{2}, \frac{\kappa_\beta}{2})$ and a prior $(\frac{\alpha_0}{2}, \frac{\beta_0}{2})$. This constrained estimation allows us to test whether individuals update their beliefs symmetrically—i.e., whether $\kappa_\beta = \kappa_\alpha$—as should be the case if investors process new information as Bayesians.\(^{25}\) We fix the following parameters in the model: the horizon is set to $T = 10$, each agent’s risk aversion is set to $\gamma = 2$, and the transaction cost is set to $c = €20$.\(^{26}\) We define a day trade as a “loss” if the average sale price is less than the average purchase price and as a “gain” otherwise. We use each individual’s actual outcomes to generate

\(^{25}\)An alternative restriction for identification would be, e.g., $\alpha_0 + \beta_0 = K$, where $K$ is a fixed number.

\(^{26}\)The homogeneous risk aversion assumption is nearly innocuous because the objective function focuses on trade size changes and not at the trade size levels. We could also estimate risk-aversion separately for each individual but this would make computations more cumbersome (“the curse of dimensionality”) and would also impose additional identification problems.
trade size predictions for each day trade. We evaluate the fit of the model for each individual \( i \) using information on their first five day trades \((K = 5 \text{ in Eq. } 2.19)\).

We define the sample in this section as follows. We include investors who day traded at least three times, had at least some stockholdings at the time of the first day trade, and initially held no shares of the stock they day traded in their first day trade. (The first requirement “overidentifies” each agent’s prior; the second requirement provides an estimate of each agent’s initial wealth; the last requirement guards against investors day trading for tax reasons.) These restrictions lead to a sample of 3,410 day traders. We compute the initial wealth of each investor by assuming that stockholdings represent one-fifth of investors’ total wealth.\(^{27}\) We resample the initial sample \( Q = 20,000 \) times, letting the bootstrap sample size to be equal to the original sample size.

### 2.4.6.2 Results

Figure 2.5 shows the bootstrapped sampling distribution for \( \kappa_\beta \). The results show that \( \kappa_\beta \) is significantly less than 1 in our estimation: 99.47% of \( \kappa_\beta \) estimates lie between 0.26 and 0.28, with the mean and median being 0.294 and 0.28, respectively. This result indicates that individuals downweight bad outcomes relative to good outcomes—a Bayesian agent, as discussed in Section 2.2, would update symmetrically with \( \kappa_\alpha = \kappa_\beta = 1 \). This finding is consistent with the notion of biased self-attribution in the behavioral decision-making literature. For example, Daniel, Hirshleifer, and Subrahmanyam (1998) model this feature of individuals’ updating process to generate under- and overreaction in stock prices. Similarly, Gervais and Odean (2001) study the consequences of non-Bayesian up-

\(^{27}\)This assumption about investors’ equity shares is based on Campbell (2006, figure 3).
A sample of 3,410 individual day traders is used to estimate how day traders update their beliefs after gains and losses. The empirical estimation of the Section 2.2 model relaxes the assumption about Bayesian updating rules, allowing $\kappa_{\alpha}$ and $\kappa_{\beta}$ to differ from 1. The updating parameter $\kappa_{\alpha}$ after gains is constrained to 1 (for identification; see text for details) and $\kappa_{\beta}$ is estimated from the data. The value of $\kappa_{\beta}$ is allowed to vary from 0 to 2. Each individual’s prior is chosen to minimize a distance measure (Eq. 2.18) between the actual and predicted behavior of each day trader over his or her first five day trades. The original sample is resampled 20,000 times, and for each new sample $\kappa_{\beta}$ is chosen to minimize the sum of distance measures across all individuals in the sample.

dating rules: what happens if individuals overweight good outcomes relative to bad outcomes.\textsuperscript{28} Our estimate $\kappa_{\beta} \ll \kappa_{\alpha}$ has the same interpretation: although individuals learn from outcomes, the way they process new information is some-

\textsuperscript{28}It is possible that the empirical discrepancy between $\kappa_{\alpha}$ and $\kappa_{\beta}$ in the estimation is driven by rational factors. First, this result could be a consequence of the fact that we model the outcome of a day trade as a binomial variable. It is conceivable that if the true return distribution is skewed, the discretization of the outcome into “loss” and “profit” categories leads to an estimate of $\kappa_{\beta} \neq \kappa_{\alpha}$ even though agents really are Bayesians. Second, it is possible that agents learn about more than just the mean after observing new outcomes. An agent in a more general setup could be allowed to learn simultaneously about the variance of the day trading return distribution. Again, an estimate $\kappa_{\beta} < \kappa_{\alpha}$ might follow from neglecting this type of a channel. \textit{Ex ante}, it seems unlikely that this type of considerations could be driving the result. For example, intraday returns are very symmetric (Campbell, Lo, and MacKinlay 1997, p. 17), suggesting that skewness is an unlikely explanation.
what suboptimal relative to the Bayesian base case of $\kappa_\alpha = \kappa_\beta = 1$.

It is important to note that we are implicitly imposing a restriction that all agents update their beliefs in the same way. Other studies, such as Nicolosi, Peng, and Zhu (2004), have found heterogeneity in learning behavior. Consistent with this of type of a finding, the sampling distribution in Figure 2.5 displays “anomalous” roughness in the density close to $\kappa_\beta = 1$. This suggests the possibility that some subset of investors do, in fact, learn more optimally than the (vast) majority. (An interesting direction for subsequent research would be to partition investors by demographics, such as gender and age, and to investigate directly heterogeneity in learning.)

Figure 2.6 backs out individual day traders’ priors under the assumption that agents update by $\kappa_\alpha = 1$ and $\kappa_\beta = 0.28$ rules.\(^2\)\(^9\) The diagonal in the figure is drawn to represent the set of before-transaction cost break-even points. The results show two interesting regularities. First, most individuals do not believe that they can make money by day trading at the time of their first day trades: 57.9% of priors lie above the diagonal where $E_0(p) < \frac{1}{2}$. This finding is consistent with the theoretical considerations about (general) speculators’ beliefs in Mahani and Bernhardt (2005). Furthermore, this type of result strongly supports our learning hypothesis: many individuals’ initially day trade simply because day trading might be profitable, not because they stubbornly believe that it is. (In terms of the discussion about optimal regions of behavior in Section 2.2.5, Figure 2.6 indicates that most individuals initially reside in the “pay the cost” region;

\(^2\)\(^9\)We omit observations where the prior is not uniquely identified—i.e., when an agent’s observed behavior is equally well described by priors $(\alpha_0, \beta_0)$ and $(\alpha_0', \beta_0')$. If a representative prior is randomly chosen for these investors, the resulting graph is almost identical to the one shown here. For example, the mean belief about $p$ based on the observations included in the figure is 0.457 whereas this mean becomes 0.467 when the remaining observations are included by randomizing across possible priors.
Figure 2.6: Implied Distribution of Individual Day Traders’ Priors.

A sample of 3,410 individual day traders is used to infer day traders’ priors at the time of the first day trade by calibrating the Section 2.2 learning model. The calibration assumes that investors update by $\kappa_\alpha = 1$ and $\kappa_\beta = 0.28$ rules (see the density for $\kappa_\beta$ in Figure 2.5). Each individual’s prior is chosen to minimize a distance measure (Eq. 2.18) between the actual and predicted behavior over the first five day trades. The priors are restricted to a grid consisting of 10,000 points where $\alpha_0$ and $\beta_0$ range from 0.1 to 10; each agent’s risk-aversion is constrained to $\gamma = 2$; and the horizon is fixed to $T = 10$. Each circle in the figure represents the parameters of one individual’s prior distribution.

it is instructive to juxtapose Figures 2.6 and 2.2.) Second, individuals’ priors are clustered close to $(\alpha_0 = 0, \beta_0 = 0)$, indicating that, relatively speaking, many individuals were initially very uncertain about the profitability of day trading. (There are, however, many investors who do have very tight priors (i.e., high $\alpha_0 + \beta_0$) and who strongly believe in their own abilities ($\alpha_0 \gg \beta_0$).)

2.5 Conclusions

This paper studies whether individual investors are capable of learning from experience by examining the complete trading records of all Finnish (individual)
day traders. This is an important question because many economic models either assume hyperrational agents or, at the very least, assume that agents adjust their behavior over time to improve their well-being (i.e., adaptive learning). We circumvent the problem that beliefs (and the changes in beliefs) are inherently unobservable by studying individual day traders. These investors are ideal for studying learning for several reasons: first, they may have widely dispersed beliefs about the profitability; second, they receive immediate (and observable) feedback from their day trades; and finally, observations are sequential—it is easy to observe how the behavior changes from one day trade to the next. We compare two competing hypotheses:

- **The learning hypothesis.** Individuals do not know whether they can make money by day trading but have prior beliefs about their abilities. Some individuals—i.e., those with very dispersed priors, low risk-aversion, or optimistic priors—day trade to learn about the profitability, adjusting trade sizes or quitting depending on realized outcomes.

  An investor with pessimistic (but uncertain) beliefs about the profitability of day trading may want to day trade because the downside of being right is significantly smaller than the upside of being wrong. For example, the cost of a small round-trip trade was approx. €17 during the sample period. Hence, day trading is relatively cheap, and because of this, an investor wants to make sure that day trading really is unprofitable before ignoring it. Even in a world where *everyone* believes that day traders lose money, some investors become day traders.

- **The stubborn beliefs hypothesis.** Day traders are investors who believe in their abilities to make money by day trading. These investors have degenerate subjective prior distributions about their own abilities and ad-
just their behavior only because of the outcomes’ wealth effects. (This hypothesis is a common assumption in behavioral finance: investors are permanently overconfident about their own abilities.)

Many facts about day traders are consistent with the hypothesis that it is learning, not stubborn beliefs, that explains why investors day trade. For example, many individuals have very limited investment experience before starting to day trade; most day traders day trade only a few times; and many initial day trades do not seem to have a speculative motive. Our main evidence comes from the trade size and exit decisions: day traders are more likely to quit after negative outcomes and they respond to outcomes by increasing and decreasing their trade sizes. We control for the outcomes’ wealth effects in both cases: day traders do not reduce trade sizes or quit simply because their wealth is smaller. Collectively, our evidence suggests that only parameter uncertainty and learning, not stubborn beliefs, can explain the type of behavior observed in the data. The direct estimation of the learning model from the data shows that individuals’ are not perfect Bayesians: they overweight good outcomes relative to bad outcomes by a factor of three. We back out individuals’ implied priors and find that most individuals did not believe in their ability to make money by day trading at the time of their first day trade.

Our positive results about investors’ abilities to learn from experience are important. For example, many behavioral finance studies suggest that investors exhibit behavioral biases with costly consequences. However, as long as individu-

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30 We cannot rule out the possibility that some day traders day trade because they have stubborn beliefs about their abilities. However, given the promptness of the feedback from day trading, this does not seem plausible for explaining the motivations of the majority of day traders. It is possible that innately poor quality of feedback from stock picking prohibits regular investors from learning (Odean 1998), but such an argument does not apply to day trading. Our results suggest that in most cases, the reason for day trading is uncertainty, not stubborn beliefs.
als learn from experience, these biases should not be too costly: it seems plausible that precisely those biases that cause significant welfare losses are those that the investor most easily observes. For example, an agent who reduces her performance by 10% each year is more likely to learn from experience than an agent who experiences a performance gap of only 1%. Even if individuals are not as hyperrational as some studies assume, it appears unlikely they could completely ignore feedback from any meaningful welfare losses.

2.6 Appendix

2.6.1 Proofs

Proof of Proposition 8. Assume that the agent wants to day trade at date 1 if and only if the date 0 outcome is positive: i.e., suppose that \( \beta_0 - 1 < \alpha_0 < \beta_0 + 1 \). The indirect utility of the pay the cost policy is

\[
V^\text{pay the cost}_0(w_0, (\alpha_0, \beta_0)) = \frac{\alpha_0}{\alpha_0 + \beta_0} \frac{(w_0 - 2 \ast c)^{1-\gamma}}{1 - \gamma} k_1(\alpha_0 + 1, \beta_0) + \frac{\beta_0}{\alpha_0 + \beta_0} \frac{(w_0 - c)^{1-\gamma}}{1 - \gamma}. \tag{2.20}
\]

Let \( r_0 \equiv \frac{\beta_0}{\alpha_0} \), substitute \( \beta_0 \equiv \alpha_0 r_0 \) into the value function, take the limit \( \alpha \to 0 \) (the agent’s prior is completely uninformative at the limit), and evaluate the expression at \( c = 0 \) to get

\[
\lim_{\alpha_0 \to 0} V^\text{pay the cost}_0(w_0, (\alpha_0, r_0)) \bigg|_{c=0} = \frac{w_0^{1-\gamma}}{1 - \gamma} \left[ \frac{2^{1-\gamma} + r_0}{1 + r_0} \right].
\]

\(^{31}\)If the lower bound is violated, it is always optimal to do nothing. If the upper bound is violated, it is always be optimal to invest.
Suppose that $\gamma > 1$ (an identical argument applies for $\gamma < 1$). Then,

$$V_{\text{pay the cost}} > V_{\text{nothing}} \iff 2^{1-\gamma} < 1.$$ 

This inequality holds because $\gamma > 1$. The continuity of $V_{\text{pay the cost}}(w_1, (\alpha_0, r_0))$ with respect to $\alpha_0$ and $c$ together with the strict inequality implies that “pay the cost” yields higher indirect utility than the “do nothing” policy even when $c$ is strictly positive and the variance of the prior is not completely uninformative.

Proof of Propositions 9 and 10. These propositions follow directly from differentiating the optimal investment (Eq. 2.11) with respect to $\alpha$ and $\gamma$, respectively. We only need to use the earlier result that $\frac{\partial}{\partial \alpha_1} k_1(\alpha_1, \beta_1) > 0$ if $\gamma < 1$ and $\frac{\partial}{\partial \alpha_1} k_1(\alpha_1, \beta_1) > 0$ if $\gamma > 1$. 

2.6.2 Extended Model

This section derives an approximate solution to a $T$-period version of the model of Section 2.2. The market incompleteness (i.e., short-sale restrictions) means that we can only express the optimal behavior recursively. We approximate the indirect utility of the “pay the cost” policy to obtain simple recursion formulas. Assume that the value functions tomorrow can be written as $V_{t+1}^+ = \left(\frac{w_{t+1} - b^+ c}{1-\gamma}\right)^{1-\gamma} k^+$ and $V_{t+1}^- = \left(\frac{w_{t+1} - b^- c}{1-\gamma}\right)^{1-\gamma} k^-$ where superscripts $+$ and $-$ indicate the sign of the current outcome. (We later show that it satisfies this form.) The actual value of
the pay the cost policy is then

\[ V_t^{\text{pay}}(w_t, (\alpha, \beta)) = \frac{\alpha (w_t - c - b^+ c)^{1-\gamma}}{\alpha + \beta} k^+ + \frac{\beta (w_t - c - b^- c)^{1-\gamma}}{\alpha + \beta} k^- \] (2.21)

However, if \( c \) is small relative to \( w_t \), the following approximation is good:\(^{32}\)

\[ V_t^{\text{pay}}(w_t, (\alpha, \beta)) \approx \frac{(w_t - \frac{b_1 + b_2 + 2}{2} c)^{1-\gamma}}{1 - \gamma} \frac{\alpha k^+ + \beta k^-}{\alpha + \beta}. \] (2.22)

Then, the value functions conditional of three possible policies have the following recursive forms:

\[
V_t^{\text{invest}}(w_t, (\alpha, \beta)) = \frac{(w_t - \frac{b_1 + b_2 + 2}{2} c)^{1-\gamma}}{1 - \gamma} k_{\text{invest}},
\]
\[
V_t^{\text{pay}}(w_t, (\alpha, \beta)) \approx \frac{(w_t - \frac{b_1 + b_2 + 2}{2} c)^{1-\gamma}}{1 - \gamma} \frac{\alpha k^+ + \beta k^-}{\alpha + \beta} = \frac{(w_t - \frac{b_1 + b_2 + 2}{2} c)^{1-\gamma}}{1 - \gamma} k_{\text{pay}},
\]
\[
V_t^{\text{nothing}}(w_t, (\alpha, \beta)) = \frac{w_t^{1-\gamma}}{1 - \gamma}.
\]

We set \( V_t^{\text{invest}} \) equal to \(-\infty\) if the optimal investment \( x_t^* < 0 \).\(^{33}\) Note that all the value functions corresponding to different policies are now of the form

\(^{32}\)We can, for example, make the following computation to check how good the approximation is. The actual numbers in Figure 2.3 are computed without this approximation. If the approximation is used instead, the largest absolute error in the optimal investment is $33.

\(^{33}\)The optimal investment is given by

\[
x_t^* = \frac{(\alpha k^+)^{\frac{1}{\gamma}} - (\beta k^-)^{\frac{1}{\gamma}}}{(\alpha k^+)^{\frac{1}{\gamma}} + (\beta k^-)^{\frac{1}{\gamma}}} (w - (b^- + 1)c) + \frac{(\beta k^-)^{\frac{1}{\gamma}}}{(\alpha k^+)^{\frac{1}{\gamma}} + (\beta k^-)^{\frac{1}{\gamma}}} (b^+ - b^- c) \] (2.23)
\[ V = \frac{(w-bc)^{1-\gamma}}{1-\gamma} k \] for some \( b \) and \( k \), satisfying our assumption above. The strategy for solving the problem is as follows. Starting at date \( T-1 \), compute the indirect utilities of all policies and choose the utility-maximizing. Next, write the indirect utility in each “node” as \( \frac{(w-bc)^{1-\gamma}}{1-\gamma} k \) and record \( b \) and \( k \). Repeat these steps for dates \( T-2, T-3, \ldots, 1 \) to get the complete optimal policy.\(^{34}\)

## 2.6.3 Simulated Regressions

We use the extended model of Appendix 2.6.2 to generate simulated day trader data. We show that the exit and trade size results from this data are similar to our empirical results in Tables 2.2 and 2.3. We use the following parameters for our simulations: \( T = 15 \), \( c = 20 \), \( \gamma = 2 \), and \( p = 0.45 \). We compute optimal day trading behavior for 2,500 day traders and record their decisions.\(^{35}\)

Each day trader’s characteristics are generated as follows: (i) the initial wealth, \( w_i \), is uniformly distributed between $5,000 and $50,000; (ii) the mean of the prior distribution, \( \mu_i \), is Beta distributed with parameters (3, 3); and (iii) the variance of the prior distribution is distributed \( \sigma^2_i \sim U[\frac{1}{3}(1-\mu_i)\mu_i, (1-\mu_i)\mu_i] \).

The term \( \frac{1-p}{p} \) in the mean ensures that \( E[\mu_i] = p \); i.e., the average investor in the population has no misconceptions about how profitable day trading is. The distributional assumption for the variance determines that each individual’s variance is between 33% and 100% of the maximum possible variance. (We avoid

\(^{34}\)A problem with this approach is that—because of the lump sum trading cost \( c \)—wealth is a state-variable that affects optimal decisions. A solution to this problem is to discretize this additional state variable and to compute the optimal policy in each node for \( N \) different levels of wealth. For example, we can assume that an investor’s wealth always lies between \( W_1 \) and \( W_N \) (with \( W_N > W_1 \)). Then, we can create an exponential wealth grid \( \{W_1, W_2, \ldots, W_N\} \) using \( W_i = W_1 - 1 + (W_N - W_1 + 1)^{\frac{i-1}{N-1}} \).

\(^{35}\)If the investor’s optimal decision at date \( t = 0 \) is to do nothing, we create a new investor. Similar to our treatment of the actual data, we also include investors who do not quit before the end of sample (i.e., we retain investors who still day trade at date \( t = T \)). We use the approximation described in Appendix 2.6.2 to compute the solution.
Table 2.4: Exit and Trade Size Regressions with Simulated Data

This table estimates the exit and trade size regressions with simulated data and compares the results to the estimates from the actual data (from Tables 2.2 and 2.2). We create the simulated data from the extended portfolio choice model of Appendix 2.6.2 for 2,500 day traders. The following parameters are fixed: \( T = 15, c = 20, \gamma = 2, \) and \( p = 0.45. \) Each investor’s initial beliefs and wealth are randomized: wealth is uniformly distributed between $5,000 and $50,000, the mean of the prior distribution drawn from a Beta\((3,3,1-p^2)\)-distribution, and the variance of the prior is drawn from \( U\left[\frac{1}{3}(1-\mu_i)\mu_i, (1-\mu_i)\mu_i\right] \) (see text). We first simulate outcomes using the actual probability \( p \) and then solve for each day trader’s optimal behavior. The simulated sample has the following characteristics: the average day trader in the sample day trades 8.01 times (23.5% of them day trade once or twice); the average mean of the prior distribution is 0.453; and the average cumulative profit is \(-$1,656 \) (max = $324,501, min = \(-$17,123).)

Panel A: Exit Regression

<table>
<thead>
<tr>
<th></th>
<th>Simulated Coeff.</th>
<th>Simulated t-value</th>
<th>Real Data Coeff.</th>
<th>Real Data t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning Variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Loss</td>
<td>3.53</td>
<td>38.2</td>
<td>0.38</td>
<td>21.5</td>
</tr>
<tr>
<td>Proportion of Earlier Losses</td>
<td>25.76</td>
<td>40.2</td>
<td>0.19</td>
<td>5.8</td>
</tr>
<tr>
<td>ln(Trade Size)</td>
<td>-0.20</td>
<td>-9.3</td>
<td>-0.23</td>
<td>-29.0</td>
</tr>
<tr>
<td>Wealth Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Current Portfolio Value)</td>
<td>5.31</td>
<td>22.7</td>
<td>0.07</td>
<td>7.2</td>
</tr>
<tr>
<td>ln(First Portfolio Value)</td>
<td>-6.53</td>
<td>-26.0</td>
<td>-0.02</td>
<td>-2.8</td>
</tr>
<tr>
<td>Other Controls</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Number of Earlier Day Trades)</td>
<td>8.91</td>
<td>40.8</td>
<td>-0.60</td>
<td>-64.9</td>
</tr>
<tr>
<td>( N )</td>
<td></td>
<td></td>
<td>20,015</td>
<td>281,350</td>
</tr>
<tr>
<td>Nagelkerke ( R^2 )</td>
<td></td>
<td></td>
<td>61.4%</td>
<td>41.1%</td>
</tr>
</tbody>
</table>

creating investors who have almost degenerate beliefs.)

Table 2.4 summarizes the characteristics of our simulated data and shows the results from the exit and trade size regressions. We include the corresponding results from Tables 2.2 and 2.3 for comparison. The simulated results are similar to the empirical results: the key learning variables have the same signs in both data sets. (The magnitudes are different because the model is not calibrated to
Table 2.4: Exit and Trade Size Regressions with Simulated Data. (cont’d)

Panel B: Trade Size Regression

<table>
<thead>
<tr>
<th></th>
<th>Simulated Coeff.</th>
<th>t-value</th>
<th>Real Data Coeff.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Learning Variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Previous Loss</td>
<td>−2.954</td>
<td>−104.1</td>
<td>−0.059</td>
<td>−21.6</td>
</tr>
<tr>
<td>Proportion of Earlier Losses</td>
<td>0.255</td>
<td>6.0</td>
<td>0.028</td>
<td>4.4</td>
</tr>
<tr>
<td><strong>Wealth Controls</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Current Portfolio Value)</td>
<td>−3.243</td>
<td>−23.4</td>
<td>0.022</td>
<td>12.5</td>
</tr>
<tr>
<td>ln(Previous Portfolio Value)</td>
<td>3.314</td>
<td>23.7</td>
<td>−0.022</td>
<td>−12.9</td>
</tr>
<tr>
<td><strong>Other Controls</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Number of Prior Day Trades)</td>
<td>−0.021</td>
<td>−0.8</td>
<td>−0.015</td>
<td>−11.5</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>17,514</td>
<td></td>
<td>256,940</td>
<td></td>
</tr>
<tr>
<td><strong>Adjusted R²</strong></td>
<td>48.6%</td>
<td></td>
<td>7.6%</td>
<td></td>
</tr>
</tbody>
</table>

the data.) These simulations show that a fully-rational model can generate data that has same features as our real life day trader data. The simulated data has an attractive feature: 60% of the day traders have a prior \( E_0[p] < 0.5 \); i.e., they already initially believe that that day traders lose money.
CHAPTER 3

The Limit Order Effect

3.1 Introduction

Numerous studies find that individual investors’ behavior is very sensitive to news and short-term returns. For example, Odean (1998) and Grinblatt and Keloharju (2000) find that individuals sell when prices rise and buy when prices fall. Hirshleifer, Myers, Myers, and Teoh (2003) show that individuals trade against earnings surprises, selling companies that release better-than-expected earnings and vice versa. Barber and Odean (2002) and Seasholes and Wu (2005) report that individuals trade stocks that grab their attention—i.e., stocks that release news or experience significant price movements. Moreover, many studies find that individuals trade systematically in the wrong direction.\(^1\) Collectively, the evidence suggests that individuals monitor the market closely but systematically misinterpret new information, losing to other investors. This description of individual investors is puzzling. First, we would expect individuals to be trend-followers, not contrarians: uninformed investors’ beliefs are more sensitive to news, inducing them to trade in the direction of the news (Brennan and Cao 1997).\(^2\) Second, individuals trade against earnings surprises, selling companies that release better-than-expected earnings and vice versa. Barber and Odean (2002) and Seasholes and Wu (2005) report that individual investors are more sensitive to news, inducing them to trade in the direction of the news (Brennan and Cao 1997).\(^2\) Second,

\(^{1}\)For example, Odean (1999) concludes: “What is more certain is that these [individual] investors do have useful information which they are somehow misinterpreting.”

\(^{2}\)Suppose that there are two investors who initially have the same level of beliefs about the stock’s intrinsic value but one (the uninformed) is more uncertain about the value than the other (the informed). Now, if the company releases negative news, the uninformed investor revises her beliefs downwards more. Hence, after the announcement, the uninformed investor
it seems inexplicable how uninformed investors could systematically lose money in their trades (Fama 1970).

![Buy Orders](Image)

Buy Orders | Sell Orders | Price
---|---|---
Time 0 (Pre-Event)

![Time 1 (Post-Event)](Image)

Buy Orders | market orders | Sell Orders | SO | Price
---|---|---|---|---
Time 1 (Post-Event)

Figure 3.1: An Example of Limit Orders Triggered by News

This paper shows that individual investors’ use of limit orders can explain why individuals seem to misinterpret new information and systematically trade in the “wrong direction”. The following example illustrates how limit orders affect inferences about investor behavior (see Figure 3.1). Suppose there is no disagreement about a stock’s fair value \( V_0 \) so that investors trade only for liquidity reasons. Those willing to wait place limit orders, hoping that an impatient investor trades against them.

Now, suppose that the company unexpectedly must be selling shares to the informed investor until their beliefs converge. See Brennan and Cao (1997) and Brennan, Cao, Strong, and Xu (2005) for a detailed analysis.

3We develop a limit order model in Appendix 3.5 to formalize this example. We use the model to show that the limit order characteristics discussed here—for example, that more limit orders execute when there is a shock to the fundamentals—are equilibrium characteristics and not ad hoc intuition.

4Even when there may be privately informed investors in the market, an investor trading for liquidity reasons is likely to prefer a limit order to a market order. See, e.g., Glosten (1994) and Handa and Schwartz (1996) for reasons why uninformed traders should use limit orders. Kyle (1985)-type of models with multiple informed agents, such as Holden and Subrahmanyam (1992), address the market order side. They predict that any private information is rapidly revealed through informed agents’ aggressive competition. Bloomfield, O’Hara, and Saar (2005) is one of the dissidents in the limit versus market order choice literature. The paper finds that informed traders use more limit orders than liquidity traders in a laboratory experiment. However, even if investors with private information prefer limit orders, investors competing to gain from a release of public information must employ market orders.
grades its earnings guidance. Those who observe this announcement in real-time react by submitting market buy orders, triggering sell limit orders that are not withdrawn in time. All the sell limit orders in the book up to the new valuation $V_1$ execute and lose money.\(^5\) Thus, the arrival of news creates an appearance that (many) liquidity traders react to the news and lose money because of their poor decisions.

This paper uses unique data set that combines investor trading records with limit order data to examine how significantly this mechanism—**the limit order effect**—contributes to inferences about investor behavior and performance. We focus on the following:

1. **Disposition Effect.** Shefrin and Statman (1985) label investors’ preference to realize gains too soon and losses too late “the disposition effect”. Odean (1998), Shapira and Venezia (2001), Grinblatt and Keloharju (2001), Feng and Seasholes (2005), Dhar and Zhu (2006) and others have shown that an individual is more likely to sell a holding with paper gains than with paper losses. Limit order use may create an appearance that investors prefer to sell stocks with capital gains. For example, suppose an investor just bought ten stocks and needs to sell one of them. She may place a good-till-canceled sell limit order for each stock 10% above the current market price. When one of the orders executes, the stock that is sold will be the one with the highest capital gain while the unsold stocks must have unrealized returns of $r_i < 10\%$.\(^6\)

\(^5\)This is the adverse selection risk, first described by Bagehot (1971) and formalized by Copeland and Galai (1983), Glosten and Milgrom (1985), and others. Note also that those investors who had submitted buy limit orders now need to pay more than before for their shares.

\(^6\)The fact that the return distributions of the unsold stocks are truncated from above gen-
2. **Contrarian Behavior.** Heath, Huddart, and Lang (1999), Nofsinger and Sias (1999), Choe, Kho, and Stulz (1999), Grinblatt and Keloharju (2000), Grinblatt and Keloharju (2001), Richards (2004), and others have found that individuals follow contrarian trading strategies. Limit orders are always *contrarian* orders because limit orders execute only when the stock price moves against the order.\(^7\)

3. **Coordinated Trading.** Barber, Odean, and Zhu (2003) and Kumar and Lee (2006) find that individual investors are often on the same side of the market on the same day in the same stock. Limit orders may intensify coordination: if there is new information, a single market order trader may empty one side of the limit order book, creating an appearance that all limit order traders herd on the same side.

4. **Attention-Grabbing Behavior.** Barber and Odean (2002) and Seasholes and Wu (2005) find that individuals trade attention-grabbing stocks—i.e., stocks that release news or experience significant price movements. Limit order traders may appear to concentrate their trades in high attention stocks because more limit orders trigger when something happens in the market. This mechanism is particularly relevant when realized returns proxy for “attention”.

\[^7\]Similar to this argument, Grinblatt and Keloharju (2001, p. 590) observe that “the impact of past returns on the buy versus sell decision is complicated by equilibrium constraints. For example, not all investors can be contrarians if all buys are sells and vice versa.” It is possible that all *active* investors follow momentum strategies, forcing *passive* investors into contrarian trades.
5. Underperformance of Attention-Grabbing Trades. Barber and Odean (2002) argue that attention-grabbing behavior may be costly. They document that individuals’ trades in the high-attention stocks perform poorly: the stocks sold outperform the stocks purchased. Limit orders may be the cause: if it is new information that creates “attention”, conditioning on “attention” is equivalent to conditioning on realizations of the adverse selection risk.

6. Misinterpreting New Information. Hirshleifer, Myers, Myers, and Teoh (2003) find that ”individuals... tend to make contrarian trades in opposition to the direction of earnings surprises.” These contrarian and losing trades may be limit orders. An arrival of new information induces market order traders to cut through the limit order book, leaving limit order traders (passively) on the wrong side of the market.

7. Negative Stock Picking Skills. Barber and Odean (1999), Odean (1999), Grinblatt and Keloharju (2000), Barber, Lee, Liu, and Odean (2005), Grinblatt and Keloharju (2006), and others compare the returns of “buy” and “sell” portfolios to measure trading performance. These papers find that individuals display negative stock picking skills, shifting towards future losers and away from future winners. The adverse selection risk must affect long-term performance when information is long-lived. An informed investor hides in the order-flow (Kyle 1985), slowly trading against the uninformed investors. The uninformed investors must be on the wrong side of the market at the time the information is revealed.8

8Harris (2003) makes a similar observation: “Uninformed traders do not lose because they systematically want to trade on the wrong side. Even if they flip a coin to decide on which side to trade, uninformed traders tend to lose. If private signals are symmetric, and the average loss to the informed investors is $\alpha\%$, a buy-sell performance analysis “biases” results downwards by $2\alpha\%$ because investors face adverse selection with both purchases and sales.”
Table 3.1: Quantifying the Limit Order Effect

This table reports what proportion various behavioral patterns and performance results are attributable to limit order use. The numbers in the “attributable” column are computed as follows. “Disposition effect” (Section 3.3.1) is the average reduction in the capital gain coefficient of a sell versus hold logistic regression when a market order indicator variable is switched on. “Contrarian behavior” (Section 3.3.2) is the average reduction in the date \( t \) return coefficient of a buy versus sell logistic regression when the market order indicator variable is switched on. We report averages of \( \bar{r}_t \) and \( s_t \) of Table 3.4. “Coordinated Trading” (Section 3.3.3) is the reduction in the Lakonishok, Shleifer, and Vishny (1992) measure of herding, adjusted for observations where market and limit order traders herd on the opposite sides of the market. “Attention-Grabbing Behavior” (Section 3.3.4) is the reduction in the squared deviations around the mean for buy-sell imbalances when moving from limit order-initiated trades to market order-initiated trades. A positive number indicates that the effect is smaller for market orders than it is for limit orders. “Underperformance of Attention-Grabbing Trades” (Section 3.3.4) measures investors’ performance in high-attention stocks. “Negative Stock-Picking Skills” (Section 3.3.5) is a buy versus sell performance analysis. “Misinterpreting New Information” (Section 3.3.6) measures trading gains around scheduled and unscheduled earnings announcements.
Table 3.1 summarizes our key results. We find that limit orders contribute significantly to the behavioral results. The percentage contributions of limit orders range from a low of 27.2% (long-term contrarian behavior) to a high of 94.1% (attention-grabbing behavior). Limit order use is an even more important determinant of investor performance. It is the passive limit order traders who appear to possess negative stock picking skills, who seem to misinterpret new information, and who lose money when trading high-attention stocks. Market order traders, in contrast, earn positive returns when reacting the new information and when trading high-attention stocks. The limit order effect may be a simple yet powerful explanation for many findings about individual investors’ behavior.
3.2 Data

This section discusses the rules of the Helsinki Exchanges and the Finnish market during the sample period from September 1998 to October 2001. We also discuss the contents of our data set, categorize order types, and explain how we match trading records with the limit order data.

3.2.1 Helsinki Exchanges

Trading on the Helsinki Exchanges (HEX) is divided into sessions. Each trading day starts at 10:10 am with an opening call. Orders that are not executed at the opening remain on the book and form the basis for the continuous trading session. This trading session takes place between 10:30 am and 5:30 pm in a fully automated limit order book, the automated trading and information system (HETI). After-hours trading (5:30 – 5:45 pm) takes place after the continuous trading session and again the next morning (9:30 – 10:00 am) before the next opening call. (Two changes to the trading schedule were made during the sample period. On August 31, 2000, the regular trading session was extended to 6:00 pm...
and the after-hours session was moved to match this change. On April 10, 2001, an evening session that extended trading hours to 9:00 pm was introduced.

The HEX trading system displays the five best price levels of the book to the market participants on both sides. The public can view this book in a market-by-price form while financial institutions receive market-by-order feed. Simple rules govern trading on the limit order book. There are no designated market makers or specialists; the market is completely order-driven. An investor trades by submitting limit orders. The minimum tick size is EUR 0.01. An investor who wants immediate execution must place the order at the best price level on the opposite side of the book. An investor who wants to buy or sell more shares than what is currently outstanding at the best price level must “walk up or down the book” by submitting separate orders for each price level. If a limit order executes against a smaller order, the unfilled portion stays on the book as a new order. Time and price priority between limit orders is enforced. For example, if an investor submits a buy order at a price level that already has other buy orders outstanding, all the old orders must execute before the new order.

The total market value of the 158 companies on the Helsinki Exchanges was EUR 383 billion in the middle of the sample period (May 2000). We report several sample statistics for the 30 most actively traded stocks for future reference:

- A total of 14.2 million trades took place in these stocks. The most active stock is Nokia with 2.7 million trades.

- These stocks have an average realized log-spread of 0.44%. (Nokia’s average spread is the lowest, 0.13%.)

9 A market-by-price book displays the five levels on both sides of the market but only indicates the total number of shares outstanding at each price level. A market-by-order book shows each order separately and also shows which broker/dealer submitted each order.
Three out of ten trades originate from households. Households’ participation—as measured by the proportion of household trades—ranges from a low of 15% to a high of 72% in these 30 stocks.

### 3.2.2 Investor Trading Records and Limit Order Data

We use the following data sets in this study:

1. **The complete trading records and holdings information of all Finnish investors.** The Finnish Central Securities Depository registry (FCSD) provided us these data for the period from January 1995 to November 2002. Each trade record includes a date-stamp, a stock identifier, and the price, volume, and direction of the trade. Each record also identifies the investor type—a domestic institution, a domestic household, or a foreigner—and gives other demographic information. We classify all investors as either individuals or institutions (including foreigners) for this study. Grinblatt and Keloharju (2000) give the full details of this data set.

2. **The limit order data for all HEX stocks.** These data are the supervisory files from the HEX from September 18, 1998 to October 23, 2001. Each entry is a single order entered into the trading system, containing a unique order identifier, date- and time-stamps, a session code, a code for the brokerage firm submitting the order, a trade type indicator (i.e., upstairs/downstairs/odd-lot), and the price, volume, and direction of the order. All entries also contain a set of codes for tracking the life of an order—an order can expire, be partly or completely filled, or modified. We use these data to reconstruct the limit order book for each second of every trading day for all the stocks. Data before July 10, 2001 is missing the time-stamp that identifies when
an unfilled order is withdrawn.

3.2.2.1 Matching the Data Sets

We match the investor trading records against the limit order data using executed trades to obtain information on, e.g., what type of orders different investors use and when trades take place. Each trade record in the limit order data contains all the same information as the investor trading records except the investor identity. We use common elements to link the data sets.

There is no ambiguity in matching two types of trades: trades with unique price-volume combinations and non-unique trades that must originate from the same investor. We call these trades uniquely matched trades and we use only these trades in most of our analysis. There is no one-to-one link between the data sets for the remaining trades. We match these trades in two steps, using brokerage firm identities to improve the match. In the first step, we use the uniquely matched trades to determine each investor’s broker. In the second step, we match the non-unique trades so that, when possible, an investor’s trade ends up originating from the preferred broker. The matching of the non-unique trades can generate errors. However, a matching error where investors A and B switch places only matters if one is an institution and the other is a household. Moreover, these errors make all our tests more conservative: because this process adds noise, the true differences in trading behavior are even stronger than what we find in our tests.

---

10We say that a trade has a unique price-volume combination if, for example, there is only one trade (in one stock-day) with a price of EUR 82 and a volume of 1,200 shares. A trade is non-unique if, say, three trades have the same price-volume combination. In this example, “all must originate from the same investor” would mean that a single investor in the investor data set is the buyer or the seller in all the three trades.
3.2.2.2 Order Classification

We use the following categorization to classify every order entered into the trading system:

- **Market order.** An order placed at the best price level on the opposite side of the book to get immediate execution.\(^{11}\)

- **Inside the Spread Limit Order.** An order placed inside the current bid-ask spread.

- **At the Spread Limit Order.** An order placed at the best bid (for a buy order) or at the best ask (for a sell order).

- **Outside the Spread Limit Order.** An order placed outside the current bid-ask spread.

- **Pre-Open Limit Order.** A limit order entered into the system before the continuous trading session begins (the limit order book is not viewable to any market participants at this time). This category includes *stale limit orders*; these are unfilled orders carried over from the previous trading day.

Figure 3.2 shows an example of how a buy order is classified when best bid is currently $58 and the best ask is $63. For example, an order with a price between $58.01 and $62.99 is classified as an inside-the-spread order.

---

\(^{11}\)All orders are technically limit orders; however, many brokers use the term *market order* for these active orders. We adopt the same terminology because there is no risk of confusion.
Table 3.2: Individuals’ and Institutions’ Use of Market and Limit Orders

This table shows how institutions and individuals use market and limit orders. The sample consists of all uniquely matched trades in the 30 most actively traded stocks on the Helsinki Exchanges between September 18, 1998 to October 23, 2001. This table reports the frequencies of different order types for executed trades. Market Order is an order that is placed on the opposite side of the book so that it executes immediately. Each limit order is classified as follows: (i) inside is an order placed inside the current bid-ask spread; (ii) at is an order placed inside the bid-ask spread; (iii) outside is an order placed outside the current bid-ask spread; (iv) pre-open is an order entered into the system before the trading day beings; (v) stale is an order carried over from the previous trading day. Stale limit orders are not (double-)counted as pre-open orders. Note that (1) “Inside” + · · · + “Stale” = “Limit Order” and (2) “Limit Order” + “Market Order” = 100%. N is the number of trades.

<table>
<thead>
<tr>
<th>Order Type</th>
<th>Households</th>
<th>Institutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limit Order</td>
<td>56.1%</td>
<td>48.8%</td>
</tr>
<tr>
<td>Inside</td>
<td>20.2%</td>
<td>25.9%</td>
</tr>
<tr>
<td>At</td>
<td>7.1%</td>
<td>11.1%</td>
</tr>
<tr>
<td>Outside</td>
<td>16.2%</td>
<td>7.9%</td>
</tr>
<tr>
<td>Pre-Open</td>
<td>9.8%</td>
<td>3.6%</td>
</tr>
<tr>
<td>Stale</td>
<td>2.6%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Market Order</td>
<td>43.9%</td>
<td>51.2%</td>
</tr>
<tr>
<td>N</td>
<td>1,333,793</td>
<td>4,700,231</td>
</tr>
</tbody>
</table>

Table 3.2 shows summary statistics about how individuals and institutions use different types of orders in our data set. It shows that individual investors (55.7%) employ more limit orders than institutions (47.0%). Moreover, there are significant differences in the composition of the limit orders. Individuals’ limit orders are more price sensitive: 12.8% of all orders (or 23.0% of limit orders) are placed outside the spread or carried over from the previous trading day. For institutions, these fractions are 3.7% and 7.9%, respectively. Hence, the mechanistic features of limit orders are more important for individuals’ behavior and performance.
3.3 Empirical Tests of the Limit Order Effect

3.3.1 Disposition Effect

3.3.1.1 Methodology

We use a logistic regression to study the choice an investor faces making a sell versus hold decision. We construct a sample out of all investor-days when the investor sells at least one of his or her holdings. We generate separate observations for all sold and unsold holdings for each investor-day. The dependent variable is set to one if the investor sells shares of stock $i$ ("sell") and to zero otherwise ("hold").

We measure the disposition effect with a $Gain$ indicator variable that is set to one if the sale price is higher than the FIFO price. (We use the same-day close as the benchmark price for unsold positions.) We also include an indicator variable for market order-initiated trades and its interaction with the capital gain variable. If the actual sale is completed with a market order, all the “hold” observations are also flagged as market order-initiated trades.\textsuperscript{12}

We estimate the logistic regression separately for each stock with more than 5,000 trades by individual investors (78 stocks) in the sample to account for heterogeneity across stocks—i.e., investors might be more likely to keep stock X instead of stock Y because of some omitted variable—and to equally-weight the

\textsuperscript{12}A logistic regression approach, unlike the alternative PGR/PLR (proportion of gains/losses realized) measure, allows us to control for two important determinants of the buy versus keep decision. First, we include the first five powers of the length of the holding period—defined as the number of days from the previous purchase of shares divided by 100—to control for the negative relation between the length of the holding period and the likelihood of a sale (Feng and Seasholes 2005). Second, we control for differences in the unconditional probability of observing an actual sale by including the number of stocks actually sold divided by the number of stocks in the portfolio as an explanatory variable. (For example, if an investor has only one stock in his or her portfolio, an actual sale would always be observed.)
sample across stocks. The latter is a particularly important concern in a market with a small numbers of actively traded stocks. We use the Fama and MacBeth (1973) method and examine average coefficients across individual regressions. We estimate the stock-specific regressions in four samples for the sake of robustness:

1. **Full Sample**: includes all sell versus hold observations

2. **Position Size-Constrained Sample**: only includes such unsold holdings where the euro value of the position is as large as the euro value of the actual sale(s).\textsuperscript{13}

3. **Small Trades**: includes sales in the first trade size quintile. All the observations are ranked by the euro value of the actual sale.

4. **Large Trades**: includes sales in the fifth trade size quintile.

\textsuperscript{13}For example, if an investor sells EUR 10,000 worth of shares by selling two stocks, we include only such unsold positions where the position value is \( \geq \frac{\text{EUR 10,000}}{2} = \text{EUR 5,000}. \)
Table 3.3: A Logistic Regression Analysis of the Disposition Effect

This table reports estimates of a disposition effect logistic regression. The sample consists of “sell” and “hold” observations—a hold observation is generated when an investor sells another stock from the portfolio. All observations must satisfy the following requirements: (1) The investor must have both winners and losers in his portfolio to ensure that there is a choice between realizing a loss or gain, (2) The investor must not sell all his holdings, (3) An unsold holding is included only if the investor does not purchase more shares of that stock on the same day, and (4) The observation must have a valid FIFO price. The dependent variable is set to one if the stock is sold and to zero if the stock is kept. The explanatory variables are: a capital gain indicator variable that is set to one if the stock is sold at a price higher than the FIFO price, a market order indicator variable and its interaction with the capital gain indicator variable, the first five powers of the length of the hold period, and the proportion of stocks sold from the portfolio on the day of the trade. The regression is estimated separately for all 78 stocks with at least 5,000 trades from individual investors during the sample period from September 23, 1998 through October 23, 2001. This table reports the average coefficient estimates for the capital gain indicator variable and the interaction term. $\%$-Chg is the average change in the gain coefficient when the market order indicator variable is switched on:

$$\%\text{-Chg} = \max \left( \min \left( \frac{\hat{b}_{\text{mkt, gain}}}{\hat{b}_{\text{gain}}}, 1 \right), -1 \right)$$

where $\hat{b}_{\text{gain}} > 0$  

where $\hat{b}_{\text{gain}}$ is the base coefficient estimate and $\hat{b}_{\text{mkt, gain}}$ is the interaction term. The results are reported for four samples: a sample with all observations (“Full Sample”), a sample that excludes unsold positions that are smaller than the actual value of sales (“Position-Size Constrained Sample”), a sample that contains trades in the smallest trade size quintile (“Small Trades Sample”), and a sample that contains large trades (“Large Trades Sample”). $f_{\text{neg}}$ is the proportion of negative coefficient estimates.
Table 3.3: A Logistic Regression Analysis of the Disposition Effect. (cont’d)

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Gain</th>
<th>Mkt * Gain</th>
<th>%-Chg</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full Sample</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.294</td>
<td>−0.503</td>
<td>−41.4%</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.041</td>
<td>0.021</td>
<td>2.1%</td>
</tr>
<tr>
<td>Median</td>
<td>1.355</td>
<td>−0.508</td>
<td>−38.0%</td>
</tr>
<tr>
<td>$f_{neg}$</td>
<td>0.0%</td>
<td>98.7%</td>
<td></td>
</tr>
<tr>
<td><strong>Position-Size Constrained Sample</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.847</td>
<td>−0.372</td>
<td>−47.1%</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.036</td>
<td>0.021</td>
<td>2.7%</td>
</tr>
<tr>
<td>Median</td>
<td>0.868</td>
<td>−0.370</td>
<td>−45.0%</td>
</tr>
<tr>
<td>$f_{neg}$</td>
<td>1.3%</td>
<td>98.7%</td>
<td></td>
</tr>
<tr>
<td><strong>Small Trades Sample</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.164</td>
<td>−0.538</td>
<td>−45.8%</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.049</td>
<td>0.037</td>
<td>3.3%</td>
</tr>
<tr>
<td>Median</td>
<td>1.217</td>
<td>−0.518</td>
<td>−44.8%</td>
</tr>
<tr>
<td>$f_{neg}$</td>
<td>2.6%</td>
<td>96.2%</td>
<td></td>
</tr>
<tr>
<td><strong>Large Trades Sample</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.163</td>
<td>−0.436</td>
<td>−38.7%</td>
</tr>
<tr>
<td>s.e.</td>
<td>0.053</td>
<td>0.033</td>
<td>3.0%</td>
</tr>
<tr>
<td>Median</td>
<td>1.263</td>
<td>−0.412</td>
<td>−33.2%</td>
</tr>
<tr>
<td>$f_{neg}$</td>
<td>1.3%</td>
<td>96.2%</td>
<td></td>
</tr>
</tbody>
</table>

### 3.3.1.2 Results

Table 3.3 shows that investors display a strong preference for selling stocks with capital gains. The average capital gain-coefficient estimate across individual stock regressions is 1.294 in the full sample. The disposition effect is significantly weaker when the sale is executed with a market order instead of a limit order in all but one regression.\(^{14}\) The average coefficient estimate for the interaction term is $−0.503.\(^{15}\) This change is economically very significant: the average percentage change in the capital gain coefficient is $−41.4\%$ (column $%\text{-Chg}$). Hence, almost

\(^{14}\)The only positive coefficient estimate is statistically insignificant.

\(^{15}\)If an individual has the same number of stocks with capital gains and losses in her portfolio, these estimates mean that the probability of realizing a gain drops from 0.785 to 0.688 when the market order indicator variable is switched on.
two-fifths of the disposition effect is specific to limit orders.

The results are similar in the other samples. The overall level of the disposition effect is lower and the percentage contribution of limit orders greater in the position-size constrained sample. The trade size-specific subsample results show that although limit orders are a somewhat more important determinant with small trades—45.8% (small trades) versus 38.7% (large trades)—the results show unambiguously that effect is not only limited to these trades.

3.3.2 Contrarian Behavior

3.3.2.1 Methodology

This section modifies Grinblatt and Keloharju’s (2001) approach to study how lagged returns affect buy versus sell decision, and whether this relation is different for limit and market orders. We estimate a logistic regression in a sample that contains individual investors’ all purchases and sales. The dependent variable is set to one for sales and to zero for purchases. We include 24 return variables on the right-hand side: eight are market returns over the previous three months\(^{16}\) and the rest are stock-specific excess returns. We include positive and negative stock-specific returns as separate variables. Finally, we include a market order indicator variable and its interactions with all the 24 return variables.

We estimate the logistic regressions for each of the 78 stocks with more than 5,000 trades from individual investors and compute cross-sectional coefficient estimates to analyze the role of limit orders.

\(^{16}\)The return intervals we use are the same as those used by Grinblatt and Keloharju (2001) except that we leave out past returns that are more than three months old. (Grinblatt and Keloharju (2001) find insignificant coefficient estimates at these longer lags.) The return intervals used to explain day \(t\) trading behavior are: \(t, t - 1, t - 2, t - 3, t - 4, [t - 19, t - 5], [t - 39, t - 20],\) and \([t - 60, t - 40].\)
Table 3.4: Limit Orders and Individual Investors’ Contrarian Trading Strategies

This table estimates a logistic regression of individual investors’ buy versus sell decision. The dependent variable is set to one for sales and to zero for purchases. The explanatory variables are: 8 lagged market returns, 16 stock-specific excess returns, a market order indicator variable, and the interactions between the market order indicator variable and all the return variables. The returns are decomposed into positive and negative components: $r_{[t_1,t_2]} = \max(r_{[t_1,t_2]}, 0)$ and $r_{[t_1,t_2]} = \min(r_{[t_1,t_2]}, 0)$. The logistic regression is estimated separately for those 78 stocks that had at least 5,000 trades from individual investors during the sample period from September 23, 1998 through October 23, 2001. This table reports the average coefficient estimates for past returns (Base Coefficients) and the interaction terms between past returns and the market order indicator variable ($Mkt-Order \times r_t$). The percentage fraction of negative estimates is reported in column $f_{neg}$. Column $Mkt f_{neg}$ reports the fraction of estimates where the sum of the base and interaction term estimates is negative. $%-Chg$ is the average change in the return coefficient when the Market indicator variable is set to one. (The change is bounded between $-100\%$ and $100\%$; see Eq. 3.1.) The regression is estimated in three samples: a sample with all observations (“Full Sample”), the smallest trade size quintile sample (“Small Trades Sample”), and a large trades sample (“Large Trades Sample”).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Base Coefficients</th>
<th>Mkt-Order * $r_t$</th>
<th>Mkt</th>
<th>$f_{neg}$</th>
<th>$%-Chg$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>s.e.</td>
<td>$f_{neg}$</td>
<td>Mean</td>
<td>s.e.</td>
</tr>
<tr>
<td>$r_0$</td>
<td>16.19</td>
<td>1.22</td>
<td>0</td>
<td>-19.62</td>
<td>1.34</td>
</tr>
<tr>
<td>$r_{-1}$</td>
<td>5.56</td>
<td>0.77</td>
<td>17</td>
<td>-5.46</td>
<td>0.74</td>
</tr>
<tr>
<td>$r_{-2}$</td>
<td>4.01</td>
<td>0.59</td>
<td>21</td>
<td>-2.66</td>
<td>0.55</td>
</tr>
<tr>
<td>$r_{-3}$</td>
<td>3.89</td>
<td>0.45</td>
<td>12</td>
<td>-2.90</td>
<td>0.40</td>
</tr>
<tr>
<td>$r_{-4}$</td>
<td>2.99</td>
<td>0.47</td>
<td>23</td>
<td>-2.02</td>
<td>0.47</td>
</tr>
<tr>
<td>$r_{[-19, -5]}$</td>
<td>1.81</td>
<td>0.30</td>
<td>19</td>
<td>-0.33</td>
<td>0.18</td>
</tr>
<tr>
<td>$r_{[-39, -20]}$</td>
<td>1.23</td>
<td>0.17</td>
<td>15</td>
<td>-0.49</td>
<td>0.15</td>
</tr>
<tr>
<td>$r_{[-60, -40]}$</td>
<td>0.60</td>
<td>0.17</td>
<td>33</td>
<td>-0.43</td>
<td>0.18</td>
</tr>
<tr>
<td>$\Xi_0$</td>
<td>18.48</td>
<td>1.21</td>
<td>0</td>
<td>-19.90</td>
<td>1.54</td>
</tr>
<tr>
<td>$\Xi_{-1}$</td>
<td>8.35</td>
<td>0.75</td>
<td>4</td>
<td>-6.76</td>
<td>0.74</td>
</tr>
<tr>
<td>$\Xi_{-2}$</td>
<td>4.61</td>
<td>0.51</td>
<td>12</td>
<td>-2.13</td>
<td>0.52</td>
</tr>
<tr>
<td>$\Xi_{-3}$</td>
<td>4.24</td>
<td>0.56</td>
<td>21</td>
<td>-1.77</td>
<td>0.58</td>
</tr>
<tr>
<td>$\Xi_{-4}$</td>
<td>3.56</td>
<td>0.43</td>
<td>17</td>
<td>-1.78</td>
<td>0.43</td>
</tr>
<tr>
<td>$\Xi_{[-19, -5]}$</td>
<td>1.94</td>
<td>0.22</td>
<td>19</td>
<td>-0.77</td>
<td>0.19</td>
</tr>
<tr>
<td>$\Xi_{[-39, -20]}$</td>
<td>1.14</td>
<td>0.14</td>
<td>21</td>
<td>-0.24</td>
<td>0.14</td>
</tr>
<tr>
<td>$\Xi_{[-60, -40]}$</td>
<td>0.92</td>
<td>0.14</td>
<td>19</td>
<td>-0.35</td>
<td>0.15</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th></th>
<th>Small Trades Sample</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Base Coefficients</td>
<td>Mkt-Order * $r_t$</td>
<td>Mkt $r_t$</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>s.e.</td>
<td>$f_{neg}$</td>
<td>Mean</td>
</tr>
<tr>
<td>$r_0$</td>
<td>18.53</td>
<td>1.58</td>
<td>1</td>
<td>21.43</td>
</tr>
<tr>
<td>$r_{-1}$</td>
<td>7.29</td>
<td>1.28</td>
<td>15</td>
<td>4.16</td>
</tr>
<tr>
<td>$r_{-2}$</td>
<td>5.02</td>
<td>1.18</td>
<td>27</td>
<td>-2.91</td>
</tr>
<tr>
<td>$r_{-3}$</td>
<td>2.84</td>
<td>0.97</td>
<td>37</td>
<td>-1.07</td>
</tr>
<tr>
<td>$r_{-4}$</td>
<td>4.72</td>
<td>1.36</td>
<td>31</td>
<td>-1.72</td>
</tr>
<tr>
<td>$r_{[-19,-5]}$</td>
<td>2.56</td>
<td>0.65</td>
<td>32</td>
<td>-0.12</td>
</tr>
<tr>
<td>$r_{[-39,-20]}$</td>
<td>1.83</td>
<td>0.43</td>
<td>31</td>
<td>0.58</td>
</tr>
<tr>
<td>$r_{[-60,-40]}$</td>
<td>1.28</td>
<td>0.62</td>
<td>45</td>
<td>-0.54</td>
</tr>
<tr>
<td>$\Sigma_0$</td>
<td>18.82</td>
<td>1.44</td>
<td>3</td>
<td>-19.91</td>
</tr>
<tr>
<td>$\Sigma_{-1}$</td>
<td>11.06</td>
<td>1.40</td>
<td>8</td>
<td>-9.85</td>
</tr>
<tr>
<td>$\Sigma_{-2}$</td>
<td>5.39</td>
<td>0.91</td>
<td>22</td>
<td>-0.38</td>
</tr>
<tr>
<td>$\Sigma_{-3}$</td>
<td>5.48</td>
<td>1.01</td>
<td>24</td>
<td>-0.03</td>
</tr>
<tr>
<td>$\Sigma_{-4}$</td>
<td>3.16</td>
<td>0.94</td>
<td>35</td>
<td>-0.60</td>
</tr>
<tr>
<td>$\Sigma_{[19,5]}$</td>
<td>2.92</td>
<td>0.41</td>
<td>26</td>
<td>-0.49</td>
</tr>
<tr>
<td>$\Sigma_{[39,20]}$</td>
<td>2.40</td>
<td>0.51</td>
<td>21</td>
<td>-0.54</td>
</tr>
<tr>
<td>$\Sigma_{[60,40]}$</td>
<td>1.50</td>
<td>0.32</td>
<td>28</td>
<td>-0.63</td>
</tr>
<tr>
<td>Large Trades Sample</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_0$</td>
<td>16.89</td>
<td>1.41</td>
<td>1</td>
<td>-19.06</td>
</tr>
<tr>
<td>$r_{-1}$</td>
<td>5.46</td>
<td>0.93</td>
<td>28</td>
<td>-3.70</td>
</tr>
<tr>
<td>$r_{-2}$</td>
<td>4.85</td>
<td>0.76</td>
<td>22</td>
<td>-2.64</td>
</tr>
<tr>
<td>$r_{-3}$</td>
<td>4.78</td>
<td>0.66</td>
<td>17</td>
<td>-2.78</td>
</tr>
<tr>
<td>$r_{-4}$</td>
<td>3.68</td>
<td>0.62</td>
<td>29</td>
<td>-1.38</td>
</tr>
<tr>
<td>$r_{[-19,-5]}$</td>
<td>2.09</td>
<td>0.35</td>
<td>24</td>
<td>0.02</td>
</tr>
<tr>
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<td>1.53</td>
<td>0.24</td>
<td>23</td>
<td>-0.47</td>
</tr>
<tr>
<td>$r_{[-60,-40]}$</td>
<td>0.76</td>
<td>0.24</td>
<td>35</td>
<td>-0.42</td>
</tr>
<tr>
<td>$\Sigma_0$</td>
<td>20.36</td>
<td>1.34</td>
<td>1</td>
<td>-21.06</td>
</tr>
<tr>
<td>$\Sigma_{-1}$</td>
<td>8.60</td>
<td>0.87</td>
<td>8</td>
<td>-7.01</td>
</tr>
<tr>
<td>$\Sigma_{-2}$</td>
<td>4.74</td>
<td>0.69</td>
<td>17</td>
<td>-2.64</td>
</tr>
<tr>
<td>$\Sigma_{-3}$</td>
<td>4.52</td>
<td>0.71</td>
<td>23</td>
<td>-1.32</td>
</tr>
<tr>
<td>$\Sigma_{-4}$</td>
<td>3.44</td>
<td>0.55</td>
<td>29</td>
<td>-0.95</td>
</tr>
<tr>
<td>$\Sigma_{[19,5]}$</td>
<td>2.15</td>
<td>0.27</td>
<td>19</td>
<td>-0.74</td>
</tr>
<tr>
<td>$\Sigma_{[39,20]}$</td>
<td>1.14</td>
<td>0.20</td>
<td>21</td>
<td>-0.29</td>
</tr>
<tr>
<td>$\Sigma_{[60,40]}$</td>
<td>0.99</td>
<td>0.19</td>
<td>24</td>
<td>-0.45</td>
</tr>
</tbody>
</table>

### 3.3.2.2 Results

Table 3.4 reports the average coefficient estimates for the stock-specific excess return variables. The full sample replicates the Grinblatt and Keloharju (2001) result: individual investors employ contrarian trading strategies. For example, in
almost all regressions, an investor is more likely to sell shares when the same-day return is positive, and to buy when the return is negative (column $f_{neg}$). The proportion of negative coefficient estimates increases as the horizon lengthens: two-thirds of the positive return component estimates are positive at the longest horizon.

Limit orders contribute significantly to individuals’ contrarian behavior: market order traders are significantly more momentum-oriented than limit order traders. The market order * return interaction is negative for all positive same-day returns and for 95% of the negative returns. The sum of the base coefficient and the interaction term—a hypothetical coefficient in a market orders-only sample—is negative 83% of the time for positive returns and 53% of the time for negative returns (column $Mkt \ f_{neg}$). Moreover, even when the coefficient does not become negative, it often falls significantly: the average reduction is 97% for positive returns and 78% for negative returns (column $\%$-Chg). The percentage reductions in the previous day’s coefficients are 74% (positive returns) and 60% (negative returns) and the reductions are statistically significant at almost all lags. The difference between market and limit order traders is economically large at all horizons. The average reduction is still between one-fourth and one-third for the three longest horizon returns. The trade size-specific estimates show that small or large trades do not exclusively drive the results.
3.3.3 Coordinated Trading

3.3.3.1 Methodology

We study coordinated trading using the Lakonishok, Shleifer, and Vishny (1992) (LSV) herding measure. This measure is defined for a stock/time interval as

\[ H(i) = \left| \frac{B(i)}{B(i) + S(i)} - p \right| - AF(i) \]  

(3.2)

where \( B(i) \) is the number of purchases during the interval, \( S(i) \) is the number of sales, and \( p \) is the aggregate purchases-to-sales ratio for the interval. \( AF(i) \) is the expectation of the first term, computed under the assumption that \( B(i) \) has a binomial distribution with \( B(i) + S(i) \) draws and a success probability of \( p \). We follow Dorn, Huberman, and Sengmueller (2005) and compute the LSV measure at daily, weekly, and monthly frequencies, conditional on order type. We measure differences in herding between market and limit order traders with the difference \( H(i)_{\text{lim}} - H(i)_{\text{mkt}} \). We restrict the sample to trades where the investor held shares a day before the trade. This controls for the spurious coordination generated by short-sale constraints (Wylie 2005).

We define an indicator variable \( \text{Diff} \) as

\[ \text{Diff}_i = \begin{cases} 1 & \text{if } \left( \frac{B(i)}{B(i) + S(i)} - p \right) \left( \frac{B(i)_{\text{lim}}}{(B(i)_{\text{lim}} + S(i)_{\text{lim}})} - p \right) < 0 \\ 0 & \text{otherwise} \end{cases} \]  

(3.3)

to measure how often limit and market order traders are on the opposite sides of the market in a stock relative to the unconditional probability \( p \). We use \( \text{Diff} \) to
define an adjusted limit versus market difference in the LSV measure:

$$\text{Adjusted Lim} - \text{Mkt} = \begin{cases} 
H(i)_{\text{lim}} - H(i)_{\text{mkt}} & \text{if Diff}_i = 0 \\
H(i)_{\text{lim}} & \text{if Diff}_i = 1 
\end{cases}$$

(3.4)

This adjusted difference measures how much of the coordination among limit order traders is passive: if market and limit order traders are on the opposite sides, $H(i)_{\text{mkt}}$ is ignored and the difference is set to $H(i)_{\text{lim}}$.

### 3.3.3.2 Results

Table 3.5 shows that market order traders are less coordinated than limit order traders. The daily difference in the LSV-measure is 2.87%. This is comparable with the difference of 2.30% reported by Dorn, Huberman, and Sengmueller (2005). The differences in the degree of herding persist at the longer horizons: the weekly and monthly differences are 2.80% and 2.06%, respectively. These differences, however, hide the fact that market and limit order traders often herd on the opposite sides of the market. For example, this is the case in 44.5% of the daily observations. Hence, the adjusted measure that represents differences in active herding is significantly higher, 5.51% at the daily horizon. This translates to a percentage difference of 52.1% in the LSV measure between the two trader types. Market and limit order traders fall on the different sides of the market also at longer horizons. This, the adjusted differences and the role of limit orders are also greater than the raw differences. The percentage differences in active herding are 41.5% and 36.0% at weekly and monthly horizons, respectively.
Table 3.5: Limit Order Use and Coordination among Individual Investors

This table reports the average LSV (Lakonishok, Shleifer, and Vishny 1992) herding measures computed from all trades by Finnish individual investors between September 23, 1998 and October 23, 2001. The sample is restricted to observations where the investor held shares of the stock a day before the trade (see text). The herding measure is estimated at daily, weekly, and monthly frequencies. *All Trades* reports the herding measure for a sample consisting of both market and limit order trades; *Market*-column reports herding measures computed only from market order-initiated trades; and *Limit*-column reports measures computed from limit order-initiated trades. *Lim – Mkt* is the pairwise difference between the herding measures. *Adj. Lim – Mkt* adjusts the difference for days when market and limit order investors herd on the opposite sides of the market relative to the unconditional buy-probability $p$ in the LSV measure: if the investors herd on the different sides of the market, the measure is set equal to $H(i)_{lim}$ and if the investors are on the same side, the measure is the actual difference, $H(i)_{lim} - H(i)_{mkt}$. *Diff* is the proportion of observations where limit and market order investors herd on the opposite sides of the market relative to the unconditional probability.

<table>
<thead>
<tr>
<th></th>
<th>All Trades</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Market</td>
<td>Limit</td>
<td>$\text{Lim} - \text{Mkt}$</td>
<td>$\text{Adj. Lim} - \text{Mkt}$</td>
<td>Diff</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>8.20%</td>
<td>7.71%</td>
<td>10.58%</td>
<td>2.87%</td>
<td>5.51%</td>
</tr>
<tr>
<td></td>
<td>s.e.</td>
<td>0.06%</td>
<td>0.07%</td>
<td>0.07%</td>
<td>0.10%</td>
<td>0.09%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>6.77%</td>
<td>6.13%</td>
<td>10.00%</td>
<td>2.81%</td>
<td>5.15%</td>
</tr>
<tr>
<td>Weekly</td>
<td>Mean</td>
<td>9.02%</td>
<td>8.03%</td>
<td>10.83%</td>
<td>2.80%</td>
<td>4.49%</td>
</tr>
<tr>
<td></td>
<td>s.e.</td>
<td>0.10%</td>
<td>0.11%</td>
<td>0.12%</td>
<td>0.14%</td>
<td>0.13%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>7.36%</td>
<td>6.59%</td>
<td>9.61%</td>
<td>2.62%</td>
<td>3.99%</td>
</tr>
<tr>
<td>Monthly</td>
<td>Mean</td>
<td>9.18%</td>
<td>8.36%</td>
<td>10.42%</td>
<td>2.06%</td>
<td>3.75%</td>
</tr>
<tr>
<td></td>
<td>s.e.</td>
<td>0.17%</td>
<td>0.18%</td>
<td>0.18%</td>
<td>0.20%</td>
<td>0.20%</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>7.65%</td>
<td>6.67%</td>
<td>9.01%</td>
<td>1.75%</td>
<td>3.26%</td>
</tr>
</tbody>
</table>

### 3.3.4 Attention-Grabbing Behavior

#### 3.3.4.1 Methodology

We modify the Barber and Odean (2002) methodology to measure how much limit order use contributes to the finding that individuals trade stocks that grab their attention. We also re-examine Barber and Odean’s evidence that suggests
that the high-attention trades exhibit poor subsequent performance. We proceed as follows. First, we sort stocks into deciles each day by three different measures of “attention”:

1. The close-to-close returns for the same trading day.
2. The close-to-close returns for the previous trading day.
3. The abnormal trading volume for the same trading day, computed as

\[ V_i^* = \frac{V_t}{\sum_{i=1}^{126} V_{t-i}} \] \hspace{1cm} (3.5)

where \( V_t \) is the number of shares traded.

(We analyze the 78 stocks that have at least 5,000 trades from individual investors during the sample period.) Second, for each decile-day, we compute buy-sell imbalances as

\[ \text{Buy-Sell Imbalance} = \frac{\# \text{Purchases} - \# \text{Sales}}{\# \text{Purchases} + \# \text{Sales}}. \] \hspace{1cm} (3.6)

We compute imbalances separately for market and limit orders and also by using a value-weighted version of Eq. 3.6. Finally, we compute time-series averages and Newey-West \( k = 8 \) adjusted standard errors across trading days for each order type/decile. We compute squared deviations around the mean

\[ SS = \sum_{i=1}^{10} (BS_i - \overline{BS})^2 \text{ where } BS_i = \text{buy-sell imbalance in decile } i. \] \hspace{1cm} (3.7)

to quantify how responsive market and limit order traders are to the return and volume sorts.

We measure performance of attention-grabbing trades by replicating the methodology of Barber and Odean (2002). We compute the average one-month returns
Figure 3.3: Limit Orders and Attention-Grabbing Behavior.
This figure shows buy-sell imbalances for Finnish individual investors’ limit and market orders for deciles formed by the same-day return, yesterday’s return, and same-day abnormal volume. Abnormal volume is computed by dividing the same-day trading volume by the average trading volume over the previous six months. The sample consists of trades by individuals in the 78 with at least 5,000 trades from individual investors during the sample period from September 18, 1998 to October 23, 2001. These stocks are assigned into return/volume deciles each day. The equally-weighted buy-sell imbalances in Panels A through C are computed separately for each day/decile/order type as the difference between the number of purchases and sales divided by the total number of trades. The imbalances in Panels D through F weight observations by the euro value of trades. The plots are drawn by computing time-series averages of the daily observations. The standard errors are Newey-West adjusted with $k = 8$ lags.

for purchases and sales separately for each order type/decile/day. We use the time-series difference between the returns on purchases and sales to study performance on “high” and “low” attention days.
Table 3.6: The Performance of Attention-Grabbing Trades

This table reports the performance of individual investors' market and limit order trades for deciles based on yesterday’s return and same-day abnormal volume. Abnormal volume is computed by dividing the same-day trading volume by the average trading volume over the previous six months. The sample consists of trades by individuals in the 78 with at least 5,000 trades from individual investors during the sample period from September 18, 1998 to October 23, 2001. The stocks are assigned into return/volume deciles each day. The equally-weighted performance in Panel A is computed by weighting each trade equally; Panel B weights trades by the euro value of each trade. The average return from the date $t$ closing price to the closing price one month later ($t+20$) is computed separately for purchases and sales for each day/decile/order type. This table reports the time-series averages and standard errors (Newey-West adjusted with $k = 8$ lags; reported in italics) of the daily differences between the buy and sell portfolio returns. Lines $\text{Mkt}$ and $\text{Lim}$ report the performance of market and limit order-initiated trades. $M-L$ is the pairwise difference between market and limit order-initiated trades.

**Panel A: Equally-Weighted Performance**

<table>
<thead>
<tr>
<th>Return/Abnormal Volume Decile</th>
<th>Lo</th>
<th>2</th>
<th>3</th>
<th>8</th>
<th>9</th>
<th>Hi</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sort by Yesterday’s Return</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{Lim}$</td>
<td>0.26</td>
<td>0.28</td>
<td>0.22</td>
<td>-0.11</td>
<td>-0.14</td>
<td>-0.37</td>
</tr>
<tr>
<td></td>
<td>0.29</td>
<td>0.27</td>
<td>0.29</td>
<td>0.29</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>$\text{Mkt}$</td>
<td>0.40</td>
<td>0.78</td>
<td>0.82</td>
<td>0.23</td>
<td>0.33</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>0.26</td>
<td>0.31</td>
<td>0.29</td>
<td>0.23</td>
<td>0.24</td>
</tr>
<tr>
<td>$\text{M-L}$</td>
<td>0.14</td>
<td>0.50</td>
<td>0.59</td>
<td>0.34</td>
<td>0.45</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>0.28</td>
<td>0.32</td>
<td>0.27</td>
<td>0.31</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Sort by Same-Day Abnormal Volume</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{Lim}$</td>
<td>0.30</td>
<td>-0.04</td>
<td>-0.57</td>
<td>0.39</td>
<td>0.50</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>0.34</td>
<td>0.24</td>
<td>0.24</td>
<td>0.30</td>
<td>0.26</td>
</tr>
<tr>
<td>$\text{Mkt}$</td>
<td>-0.36</td>
<td>0.01</td>
<td>0.32</td>
<td>0.30</td>
<td>0.73</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>0.33</td>
<td>0.27</td>
<td>0.25</td>
<td>0.26</td>
<td>0.27</td>
</tr>
<tr>
<td>$\text{M-L}$</td>
<td>-0.60</td>
<td>0.05</td>
<td>0.90</td>
<td>-0.08</td>
<td>0.22</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>0.65</td>
<td>0.43</td>
<td>0.32</td>
<td>0.23</td>
<td>0.28</td>
<td>0.33</td>
</tr>
</tbody>
</table>

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Table 3.6: The Performance of Attention-Grabbing Trades. (cont’d)

Panel B: Value-Weighted Performance

<table>
<thead>
<tr>
<th></th>
<th>Sort by Yesterday’s Return</th>
<th>Sort by Same-Day Abnormal Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lim</strong></td>
<td>0.26 0.16 0.40 0.06 0.01 -0.39</td>
<td>0.23 0.20 -0.44 0.48 0.43 0.08</td>
</tr>
<tr>
<td></td>
<td>0.32 0.31 0.35 0.38 0.30 0.27</td>
<td>0.38 0.39 0.29 0.25 0.34 0.30</td>
</tr>
<tr>
<td><strong>Mkt</strong></td>
<td>0.31 0.66 0.65 0.37 0.33 0.71</td>
<td>-0.42 0.29 0.52 0.28 0.86 0.84</td>
</tr>
<tr>
<td></td>
<td>0.25 0.23 0.33 0.30 0.27 0.24</td>
<td>0.38 0.35 0.31 0.24 0.29 0.28</td>
</tr>
<tr>
<td><strong>M-L</strong></td>
<td>0.04 0.50 0.24 0.31 0.30 1.10</td>
<td>0.63 0.10 0.97 -0.20 0.42 0.79</td>
</tr>
<tr>
<td></td>
<td>0.33 0.28 0.37 0.35 0.28 0.32</td>
<td>0.65 0.48 0.33 0.22 0.28 0.35</td>
</tr>
</tbody>
</table>

3.3.4.2 Results on Behavior

Figure 3.3 shows that limit order traders’ order imbalances respond mechanically to the return and volume sorts. The effect is the strongest when the decile sort is based on the same-day returns. The imbalance decreases from a high of 44.2% (in \(D_1\)) to a low of −39.0% (in \(D_{10}\)).\(^{17}\) Although the market order traders also respond to the same-day return sort, the resulting imbalances are modest compared to the limit order imbalances. The percentage reduction in the squared deviations (Eq. 3.7) when moving from limit to market order traders is 94.1% with the same-day return sort using the equally-weighted imbalance measure. Hence, most of individual investors’ attention-grabbing behavior in our sample comes from the triggering of individuals’ (passive) limit orders.

\(^{17}\)The results are nearly identical in a sample limited to trades where an individual submits the limit order and an institution triggers the order with a market order. Thus, our results show that institutions actively sell to and buy from households. Cohen, Gompers, and Vuolteenaho (2002) find a similar result.
Market and limit order traders respond very differently also to the yesterday’s return and abnormal same-day volume sorts. This is not surprising: because volume, news, and returns are all interrelated and individual stock volatility is highly autocorrelated, all of these sorts are linked to the mechanistic nature of limit orders. For example, although both market and limit order traders are net buyers of stocks that have fallen the day before, (i) market order traders’ imbalances are less extreme and (ii) market order traders are net buyers in the stocks in the highest deciles while limit order traders are net sellers. The percentage reduction in the squared deviations measure is 66.9% in the yesterday’s return sort.

Market and limit order traders behave in almost diametrically opposite ways in the same-day abnormal volume sort; e.g., limit order traders are net buyers of thinly traded stocks while market order traders are net sellers of these issues. Market order traders respond as strongly or even stronger to the volume sort than limit order traders: the change in the equally-weighted measure is 19.9%.\(^{19}\)

### 3.3.4.3 Results on Performance

Table 3.6 reports the performance of limit and market order traders for deciles formed by yesterday’s returns and the same-day abnormal trading volume. The results show that limit orders perform poorly in the highest return and abnormal volume deciles.\(^{20}\) The difference in the performance between limit and market

---

\(^{18}\)He and Wang (1995), for example, is a rational model with this feature.

\(^{19}\)The changes in the sum of squared deviations are similar with the value-weighted imbalance measure except for the volume sort. These changes are \(-94.7\%\) (same-day return), \(-64.8\%\) (yesterday’s return), and \(-42.2\%\) (abnormal volume).

\(^{20}\)The asymmetry of the return results may originate from trading constraints. Even if private signals are symmetrically distributed around the current market value, investors may not be able to trade on the (at least small) negative signals because of short-sale constraints (Saar 2001).
order traders is statistically significant in both equally- and value-weighted samples. For example, market order purchases outperform market order sales in the highest yesterday’s return decile by 0.69% whereas this spread is −0.37% for limit orders.

These results suggest that investors’ poor performance in the “high attention” stocks arises not from poor cognitive heuristics but from limit order traders’ passivity. Limit orders suffer losses when something significant happens in the market—i.e., when “attention” is high. Even though market order traders’ buy-sell imbalances also respond to return and volume sorts, this behavior does not entail costs. In fact, the results show that market order traders’ trading decisions in the high-attention stocks are very good.

### 3.3.5 Individual Investors’ Stock Picking Skills

#### 3.3.5.1 Methodology

We examine how limit order use affects a performance analysis that compares the performance of buy and sell portfolios. We compute the average $K$-day cumulative return for all purchases and sales on date $t$ as

$$R_{t,t+K} = \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \ln P_{s_i,t+K} - \ln P_{s_i,t} \right)$$

where $s_i$ is the stock that is traded and $N_t$ is the number of purchases/sales. We compute returns up to six months ($K = 130$). Second, we compute the difference between the average buy and sell returns for each day. We repeat these computations for market and limit orders, and also differentiate between different

---

The asymmetry of the volume result is similar to the high-volume premium documented by Gervais, Kaniel, and Milgeldrin (2001).
types of limit orders (see Section 3.2.2.2). We measure stock picking skills using
the time-series averages of the buy versus sell differences and use the pairwise
differences between the order types to compute how much limit orders influence
the performance results.

3.3.5.2 Results

Figure 3.4 shows that different order types vary not only in the stock price move-
ments that precede the trade\(^{21}\) \((t < 0)\) but also in the subsequent performance of
the trades \((t > 0)\). Outside-the-spread and pre-open limit orders—the more price
sensitive order types—display poor long-term performance. For example, after
three months \((t = 60)\), the average difference between purchases and sales is sig-
nificantly negative \((-1.8\%)\) for outside-the-spread limit orders. These results are
consistent with the possibility that investors with long-lived private information
trade against the uninformed liquidity traders.

Individuals’ market orders do not show any evidence of systematically poor
timing. In fact, the buy-sell difference is significantly positive at a 95\% confi-
dence level up to one-month after the trade. This suggests that some segment
of individual investors may possess useful private information (Coval, Hirshleifer,
and Shumway 2005). However, this superior performance turns statistically in-
significant at longer horizons and even modestly negative towards the end of the
return window. At the end of the six-month window, the average cumulative buy
versus sell difference for market order-initiated trades is \(-0.4\%)\) with a Newey-
West adjusted 95\% confidence interval \([-2.0\%, 1.2\%]\). The difference between

\(^{21}\)The pre-trade differences supplement the analysis in Section 3.3.2: more price sensitive
orders require larger price movements to execute. For example, the cumulative return difference
between outside-the-spread limit buy and sell orders is \(-6.4\%). This is the average drop in stock
price that is required to trigger a buy order relative to a sell order.
Figure 3.4: Limit Orders and Performance Analysis with Buy-Sell Portfolios. This figure shows the performance of individual investors’ purchase decisions compared to their sale decisions by constructing “buy” and “sell” portfolios for each day. The cumulative performance from the same-day close up to six months (130 trading days) is computed for both portfolios each day. The returns are also plotted two weeks backwards. A positive value (in the $t > 0$-region) indicates that the stocks bought outperform the stocks sold. Panel A plots the “buy minus sell” difference conditional on the order type. Panel B plots the difference between market orders and (all) limit orders together with a Newey-West ($k = 8$) adjusted 95% confidence interval for this second-order difference. The sample period in Panel A is July 10, 2000—October 23, 2001 and the period in Panel B is September 23, 1998—October 23, 2001.

market and limit order-initiated trades, however, remains both economically and statistically significant up to six months (Figure 3.4, Panel B).\textsuperscript{22}

\textsuperscript{22}The drifts in Figure 3.4 may arise from two sources that do not contradict market effi-
Table 3.7: Performance Analysis with Buy–Sell Portfolios

This table reports the performance of individual investors’ purchase decisions compared to their sale decisions by constructing “buy” and “sell” portfolios for each day from September 23, 1998 through October 23, 2001. This table reports the difference in the cumulative performance between the two portfolios from the same-day close up to six months (130 trading days). A positive value indicates that the stocks purchased outperform the stocks sold. Lines Mkt and Lim report the average performance of market and limit order-initiated trades. Line M-L is the pairwise difference between market and limit order-initiated trades. Neway-West ($k = 8$) adjusted standard errors are reported in italics. The buy-sell performance is computed for three samples: a sample with all observations (“Full Sample”), a sample that contains the trades in the smallest trade size quintile (“Small Trades Sample”), and a largest trade size quintile sample (“Large Trades Sample”).

<table>
<thead>
<tr>
<th># of Trading Days after Trade</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>60</th>
<th>130</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full Sample</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mkt</td>
<td>0.10%</td>
<td>0.28%</td>
<td>0.35%</td>
<td>0.47%</td>
<td>0.48%</td>
<td>−0.37%</td>
</tr>
<tr>
<td></td>
<td>0.03%</td>
<td>0.12%</td>
<td>0.20%</td>
<td>0.28%</td>
<td>0.58%</td>
<td>0.89%</td>
</tr>
<tr>
<td>Lim</td>
<td>−0.14%</td>
<td>0.05%</td>
<td>0.10%</td>
<td>−0.09%</td>
<td>−1.12%</td>
<td>−1.97%</td>
</tr>
<tr>
<td></td>
<td>0.03%</td>
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<td>0.57%</td>
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<tr>
<td>M-L</td>
<td>0.25%</td>
<td>0.23%</td>
<td>0.25%</td>
<td>0.56%</td>
<td>1.60%</td>
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<tr>
<td></td>
<td>0.04%</td>
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<tr>
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<td></td>
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<td></td>
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<tr>
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<td>0.60%</td>
<td>0.65%</td>
<td>−0.70%</td>
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<tr>
<td></td>
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<td>0.19%</td>
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<td>0.41%</td>
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<tr>
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<tr>
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<td>0.71%</td>
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<tr>
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<tr>
<td><strong>Large Trades Sample</strong></td>
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<td></td>
<td></td>
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<tr>
<td>Mkt</td>
<td>0.11%</td>
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<td>0.45%</td>
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<tr>
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<tr>
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<td>0.15%</td>
<td>0.21%</td>
<td>0.44%</td>
<td>0.58%</td>
</tr>
</tbody>
</table>

ciency. First, the drifts can arise from Kyle (1985) type of slow diffusion of information where price changes do not predict future price changes. Then, the drift can only be observed ex
3.3.6 Misinterpreting New Information

This section uses data on earnings announcements released during regular trading hours to examine how limit and market order traders respond to new information. Earnings announcements are ideal for this purpose: they endow some market participants (i.e., those who monitor the market closely) a transient trading opportunity. An investor who reacts to the information before her competitors can gain from all the limit orders between the current and post-announcement values. An earnings announcement also renders all pre-announcement orders stale.

We study trading around two types of earnings announcements: pre-scheduled announcements (i.e., the company has pre-announced the date and time) and unscheduled announcements (i.e., the company complies with (the equivalent) of SEC’s Form 8-K disclosure requirements).

We use data on all earnings announcements during the sample period. The resulting sample consists of 586 pre-scheduled and 117 unscheduled earnings announcements. Each observation contains the date and time of the announcement.

---

The market might endogenously shut down just before a pre-scheduled announcement because all limit orders become stale in response to the news release. There are several reasons why this may not happen in practice. First, it is possible that market participants with orders in the book have a precise signal about the announcement and try to gain from a liquidity-driven shock or they hope that some investors misinterpret the announcement. Second, it may be that market participants with orders in the book do not know that the announcement is to be released (e.g., because of high monitoring costs) or fail to withdraw their orders in time.

The time-stamp is rounded downwards to the nearest minute. For example, if the time-stamp reads 12:03pm, the exact time of the announcement must be \( t \in [12:03:00, 12:03:59] \). An announcement is usually released both in English and in Finnish. We use the time-stamp from the announcement that arrives first. Note that we do not classify announcements as positive or negative surprises or exclude announcements that cause no price movements.
Table 3.8: Returns on Market Order and Stale Limit Order-Initiated Trades around Earnings Announcements

This table reports trading gains for individuals’ stale limit orders and market orders executed around earnings announcements. The sample consists of 586 pre-scheduled earnings announcements (Panel A) and 117 unscheduled earnings announcements (Panel B) released during the regular trading hours on the Helsinki Exchanges between September 18, 1998 to October 23, 2001. A stale limit order is an order entered into the book before the release of an announcement. Before contains trades executed before the announcement, during contains trades executed during the first five minutes after the announcement, and after contains trades executed after these five minutes. The average trading gains are first computed for each interval/announcement with at least two trades. This table reports the means and standard errors of the first-stage averages. N is the number of announcements and # of Trades is the total number of trades.

3.3.6.1 Methodology

We compute average trading gains for all individuals’ executed orders around each announcement. Trading gains are defined as

\[ r_{i,s,t} = \begin{cases} 
\ln(c_{s,t+k}) - \ln(p_{i,s,t}) & \text{if a buy order} \\
\ln(p_{i,s,t}) - \ln(c_{s,t+k}) & \text{if a sell order}
\end{cases} \]  \hspace{1cm} (3.9)

where \( p_{i,s,t} \) = the trade price

\( c_{s,t+k} \) = the closing price in stock \( s \) on date \( t + k \).

We compute trading gains at three horizons: the same-day close, one week later, and two weeks later. We examine two-minute time intervals around the announcements as well as before, during, and after windows. Before contains all the same day trades executed before the announcement, during contains trades executed during the first five minutes after the announcement, and after contains all the same day trades executed after these five minutes. We first compute average trading gains for each interval (or window)/announcement and then get cross-sectional estimates by averaging across announcements.
Figure 3.5: Trading Gains for Individuals’ Market Orders and Stale Limit Orders around Earnings Announcements.

This figure shows trading gains for individuals’ stale limit orders (Panel A) and market orders (Panel B) around earnings announcements. A stale limit order is an order entered into the book before the release of an announcement. The sample consists of all 586 pre-scheduled earnings announcements released during the regular trading hours on the Helsinki Exchanges between September 18, 1998 to October 23, 2001. The average trading gain to the same-day close is first computed (Eq. 3.9) separately for each investor type/order type/announcement/two-minutes interval with at least two trades. This figure shows the means and 95% confidence intervals of the first-stage averages for each time interval.
Table 3.8: Returns on Market Order and Stale Limit Order-Initiated Trades around Earnings Announcements. (cont’d)

Panel A: Scheduled Earnings Announcements

<table>
<thead>
<tr>
<th>Trading Gain Horizon</th>
<th>Stale Limit Orders</th>
<th>Market Orders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td># of Trades</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>Same-Day</td>
<td>One-Week</td>
</tr>
<tr>
<td>Before</td>
<td>8,776</td>
<td>379</td>
</tr>
<tr>
<td>During</td>
<td>2,516</td>
<td>207</td>
</tr>
<tr>
<td>After</td>
<td>3,103</td>
<td>368</td>
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</table>

Panel B: Unscheduled Earnings Announcements

<table>
<thead>
<tr>
<th></th>
<th>Stale Limit Orders</th>
<th>Market Orders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td># of Trades</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>Same-Day</td>
<td>One-Week</td>
</tr>
<tr>
<td>Before</td>
<td>3,471</td>
<td>85</td>
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<tr>
<td>During</td>
<td>1,339</td>
<td>70</td>
</tr>
<tr>
<td>After</td>
<td>633</td>
<td>65</td>
</tr>
</tbody>
</table>

3.3.6.2 Results

Figure 3.5 shows that stale limit orders triggered after an announcement perform very poorly. The same-day and one-week trading gains are significantly negative.
for orders executed during the first eight minutes after the announcement.\textsuperscript{25} For example, the cross-sectional average same-day return on orders executed during the $[0, 120s]$ interval is $-2.5\%$ (with a $t$-value of $-2.5$). Panel B shows that individuals lose only on their limit orders: the market orders have positive trading gains up to ten minutes after the announcement.

Table 3.8 shows that the performance of stale limit orders worsens with the horizon. For example, the average one-week return for the limit orders in the first interval is $-4.7\%$. This increase may arise from the post-earnings announcement drift. However, note that the standard errors also increase with the horizon—the statistical significance of the losses is almost unchanged. Hence, a strategy designed to exploit this effect would be very risky.\textsuperscript{26} Stale limit orders lose even more when the earnings announcement is unexpected. For example, the one-week loss for the stale limit orders triggered in the during-window is $2.92\%$ for the scheduled announcements and $8.99\%$ for the unscheduled announcements. Most of these losses accrue the day of the announcement. Market order traders’ gains are statistically significantly positive immediately after unscheduled earnings announcements. These results sharply contrast the view that individuals systematically misinterpret new information: the investors who do lose, lose because of their passivity—active investors execute good trades. The mechanism that generates the stale limit order losses is important: it would not be possible

\textsuperscript{25}This persistence in losses may be an artifact of aggregation. For example, suppose that we pool data on three earnings announcements and that the market reacts fully to each announcement as soon as it is observed (i.e., a thinly traded stock may not be monitored continuously). If the market reacts in Stock A between times 0 and 1, in Stock B between times 1 and 2, and in Stock C between times 2 and 3, the aggregated data shows a reaction between times 0 and 3.

\textsuperscript{26}For example, an investor could use the data from the first minute after an announcement to observe the direction of the order-flow and then mimic this behavior. However, this strategy is linked to the PEAD because the first minute reaction is probably a good proxy for the direction of the earnings surprise. Hence, this strategy would only capture the well-known result (Ball and Brown 1968) that prices continue to drift after earnings announcements.
back out and reverse these individuals’ trading strategies to make money. These investors’ poor timing mirrors the great timing of other investors.

3.3.6.3 Demographics of Stale Limit Order Investors

Who loses money with stale limit orders? Individuals with stale limit orders in the book during the first five minutes after the announcements are very different from the market order investors during the same period. First, the average age of stale limit order investors is 50.2 whereas it is 43.1 years for the market order traders. Second, stale limit order traders have less investment experience, as measured by the number of earlier trades: 179.9 versus 591.8 trades. (The distributions are highly positively skewed.) Third, stale limit order traders live in rural areas relative to the market order traders. The average urban zip code index for stale limit order investors is 19.4% while it is 25.1% for market order trades. (We compute this index for zip code \( i \) by dividing the number of investors living in zip code \( i \) by the number of investors in the most densely populated zip code.)

These differences in demographics suggest that stale limit order investors may face higher monitoring costs. They are less active traders who may have decided that the cost of acquiring relevant information outweighs the expected loss from this type of adverse selection risk.

\[27\] All these differences are statistically significant. The pair-wise age difference is \(-7.38\) years with a standard error of 3.41 years. The pair-wise difference in the number of trades is 434.52 with a standard error of 83.89 trades. Finally, the pair-wise difference in the zip code index is 5.18% with a standard error of 1.83%. These pair-wise differences are computed from 162 earnings announcements. We compute all the demographics as cross-sectional averages of announcement-specific averages.

\[28\] The relation between order use and experience is more general. For example, pre-open/stale limit orders account for 25.4% of all limit orders for investors in the lowest experience decile. (We define experience as the number of earlier trades.) This proportion is only 6.7% for investors in the highest experience decile. This shift in order strategies may help to understand why some studies have found a significant relation between experience and trading behavior—Seasholes and Wu (2005) find that investors with more experience display lower disposition effect—and
3.4 Conclusions

This paper analyzes how limit orders alter inferences about investor behavior. Because limit orders are mechanically contrarian and exposed to the adverse selection risk, limit orders

- are more likely to execute when there is an information event
- generate losses when there is an information event
- create an appearance that the investor placing the order is reacting to news

For example, a limit order investor appears to exhibit negative market timing when an informed investor triggers the order. A study that does not account for the fact that limit order investors exhibit “passive reaction” to news runs the risk of confounding cause and effect.

We find that limit order use is an important determinant of numerous behavioral patterns: the disposition effect (47.1%), contrarian behavior (27.2%–87.9%), coordinated trading (36.0%–52.1%), and attention-grabbing behavior (66.9% with yesterday’s return sort). More importantly, we find that limit order use reverses inferences about investors’ stock-picking skills. The extant literature argues that behavioral biases are important because of these biases appear to be costly: individuals’ heuristics lead to systematic underperformance. Our results suggest that this underperformance is an artifact of individual investors being uninformed and passive: they lose money when new information is released because they cannot withdraw their limit orders in time, and they tend to lose in the long-run because investors with long-lived private information trade against between experience and investor performance—Nicolosi, Peng, and Zhu (2004) find that that more experienced investors perform better than less experienced investors.
them. This performance result is important: individual investors do not possess negative market timing skills; they lose because they are uninformed.

This paper does not try to address two important questions. First, we have not analyzed how optimal individual investors’ order use is. Although uninformed investors must lose to the informed investors (see footnote 8), suboptimal order use may increase these losses. This issue is complicated by the fact that we do not observe investors’ information sets and objectives. For example, if an investor is forced to sell something, she will not care about the adverse selection risk: the objective is to maximize the sale price. Second, we do not investigate the possibility that investors use limit orders to exhibit behavioral biases. For example, an individual who wants to trade attention-grabbing stocks could use stale limit orders. Although this type of strategies seem unattractive because of the adverse selection risk, we cannot rule out the possibility. Hence, our estimates of the limit order effect in Table 3.1 should be treated cautiously.

Although we use Finnish data because of its unique attributes, there is a good reason to believe that the limit order effect is also important in the US markets. First, limit orders are widely used also in the US markets. For example, the Securities Exchange Act Release (September 6, 1996) states that “limit orders accounted for 50% of [the NYSE] customer trades of 100-500 shares and 66% of customer trades of 600-1000 shares.” Lo, MacKinlay, and Zhang (2002) report that limit orders account for 45% of the total order flow on the NYSE. Moreover, many limit orders in the US are good-till-canceled orders: Bae, Jang, and Park (2003) report a fraction of 14.4% with the TORQ data. In comparison, this proportion is only 2.6% for individuals and 0.3% for institutions in the Finnish data. Hence, stale limit orders appear to be significantly more common in the US markets than what they are in our data set. Third, although we lack
direct US evidence, we do know that the results are not solely Finland-specific. For example, Dorn, Huberman, and Sengmueller (2005) show that limit orders cause spurious coordination among individuals using data from a German broker. Richards (2004, pp. 31) refers to the earlier version of this paper and concludes: “...the similarity between the [Korea Stock Exchange] data and the Finnish evidence...suggests that greater use of limit orders by households may be a fairly widespread phenomenon. It is therefore likely that order-submission effects are a substantial cause of the finding that domestic individual investors in Asian equity markets appear to be contrarian investors.” There are good reasons to believe that the limit order effect is an important determinant of individual investors’ behavior and performance also in the US markets.

3.5 A Simple Model of a Limit Order-Driven Market

This section formulates a tractable and stylized equilibrium model of a limit order-driven market and uses it to demonstrate the mechanical aspects of limit orders (the limit order effect). Our approach is motivated by the models of Glosten (1994) and Handa and Schwartz (1996).\(^\text{29}\) The spread in our model arises endogenously from the risk of adverse selection—there are no order-processing or inventory effects. The motivation for the use of limit orders in our model is that the order may trigger because of a liquidity-shock.

3.5.1 Setup

We assume the following:

There are three dates, $t = 0, 1, 2$. There is a single stock traded in an order-driven market with a current intrinsic value $V_0$ (known to everyone). The date 2 value is $V_2$, unknown at date 0.

The market consists of many uninformed and risk-neutral agents who want purchase or sell a single share for liquidity reasons.

The investor can submit a limit order at date 0 at price $L$, or wait until date 2 and submit a market order. If the investor submits a market order at date 2, the execution price is $V_0 \pm s$, where $s$ is the half-spread. If the investor’s limit order does not execute, the investor always submits a date 2 market order.

The investors maximize expected profits: $E[\tilde{V}_2 - P]$ for a buyer and $E[P - \tilde{V}_2]$ for a seller, where $P$ is the execution price.

At date 1, a large institutional investor enters the market. This investor trades on a private signal with probability $\pi$ and is otherwise trading for liquidity reasons. The two possible outcomes are:

- If there is a signal, the date 2 intrinsic value is drawn from a uniform distribution $U(V_0 - \delta_I, V_0 + \delta_I)$, where $\delta_I > 0$ is an exogenous parameter.

- If there is no signal, a liquidity-shock temporarily pushes the stock price to a level drawn from $U(V_0 - \delta_U, V_0 + \delta_U)$, where $\delta_U > 0$ is an exogenous parameter. The date 2 intrinsic value is the same as the date 0 value, $V_2 = V_0$.

\[\text{Note that this model does not explicitly consider how market orders and limit orders interact; instead, we use “the outsider” as the source of both information and liquidity-driven shocks. We do this for tractability: our stylized model emphasis the information/liquidity shock tradeoff faced by a limit order trader while shutting out unnecessary complications.}\]
Agents can submit limit orders  A signal arrives with prob. \( \pi \)  Agents submit market orders if necessary

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Date 1</th>
<th>Date 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Intrinsic Value ( V_0 )]</td>
<td></td>
<td>[Intrinsic Value ( \bar{V}_2 )]</td>
</tr>
</tbody>
</table>

Figure 3.6: The Timeline of the Events in the Limit Order Model

Figure 3.6 illustrates the sequence of the model’s events.

### 3.5.2 Optimal Behavior, the Choice of Limit Price, and Equilibrium

We analyze the problem by focusing on a single investor who needs to buy one share. The investor can do two things: wait until date 2 and submit a market order or submit a limit order at date 0. If the investor decides to wait until date 2, the expected loss is equal to the half-spread. Now, suppose that the investor places a limit order. There are four possible outcomes:

1. **There is a signal but the order does not execute.** The expected terminal value is \( E_0[V_2 | V_2 > L] = \frac{V_0 + \delta_U + L}{2} \). The investor pays the spread \( s \) to buy the share with a market order at date 2. The probability of this event is \( \pi \frac{V_0 + \delta_U + L}{2\delta_U} \).

2. **There is a signal and the order executes.** The expected terminal value is \( E_0[V_2 | V_2 \leq L] = \frac{V_0 - \delta_U + L}{2} \) and the profit \( \frac{V_0 - \delta_U - L}{2} \). The probability of this event is \( \pi \frac{L - V_0 + \delta_U}{2\delta_U} \).

3. **There is no signal and the order does not execute.** The terminal value is \( V_0 \) and the investor pays the spread \( s \) to buy the share at date 2. The probability of this event is \( (1 - \pi) \frac{V_0 + \delta_U - L}{2\delta_U} \).

4. **There is no signal and the order executes.** The terminal value is \( V_0 \) and the profit \( V_0 - L \). The probability of this event is \( (1 - \pi) \frac{L - V_0 + \delta_U}{2\delta_U} \).
The investor’s optimal limit price maximizes the expected profit

\[ L^* = \arg\max_L \left\{ \frac{-\pi(L - V_0 + \delta_I)^2}{4\delta_I} + \frac{\pi L - (V_0 - \delta_I)(-s)}{2\delta_I} \right\} \]

\[ = \frac{\pi\delta_U(V_0 + s - \delta_I) + 2(1 - \pi)\delta_I(V_0 - \frac{\delta_U - s}{2})}{\pi\delta_U + 2(1 - \pi)\delta_I}. \]  

The agent prefers a limit order to a market order when the expected return at \( L^* \) is higher than \(-s\) (the expected loss from a market order):

\[ -s \leq \frac{-\pi(V_0 + \delta_I - L^*)^2}{4\delta_I} + \frac{(1 - \pi)(L^* - \delta_I)(-s)}{2\delta_I} \]

\[ = \frac{-\pi(V_0 + \delta_I - L^*)^2\delta_U + 2(1 - \pi)(L^* - V_0 + \delta_U)\delta_I(V_0 - L)}{2\pi(L^* - V_0 + \delta_I)\delta_U + 2(1 - \pi)(L^* - V_0 + \delta_U)\delta_I}. \]  

We now use two aspects of equilibrium in an order-driven market:

- The market is order-driven so the spread must arise from investors limit orders (i.e., \( V_0 - L^* = s \)).

- All investors in the model are homogeneous—except that some want to buy and others want to sell—so that at equilibrium, investors must be indifferent between submitting limit and market orders (i.e., Eq. 3.12 holds with equality).

The equilibrium spread from substituting \( L^* \) from Eq. 3.11 into Eq. 3.12 is

\[ s^* = \frac{-(1 - \pi)\delta_I^2\delta_U + \sqrt{E}}{\pi\delta_U + (1 - \pi)\delta_I}. \]  

\[ E \equiv \pi(1 - \pi)\delta_I^2\delta_U(\delta_I - \delta_U)(\pi\delta_U + (1 - \pi)\delta_I)((1 - \pi)\delta_I + (1 + \pi)\delta_U) \]
A necessary condition for the existence of equilibrium \((s^* > 0)\) is that \(\delta_I > \delta_U\).\(^{31}\)

We suppose that parameters \(\{\pi, \delta_I, \delta_U\}\) constitute such equilibrium.

### 3.5.3 Implications

1. **More limit orders execute when there is an information event.** Suppose that there is a buy limit order at price \(L_b > V_0 - \delta_U\) and a sell limit order at price \(L_s < V_0 + \delta_U\) so that both a signal and a liquidity shock can trigger the orders. (Whether \(L_b\) and \(L_s\) are equilibrium values or arbitrary is inconsequential for our argument.) The probability of a limit order executing when there is no signal is 
   \[
   \frac{2\delta_U - (L_s - L_b)}{2\delta_U}
   \]
   and when there is a signal,
   \[
   \frac{2\delta_I - (L_s - L_b)}{2\delta_I}
   \]
   Hence, more orders execute when there is a signal if \(\delta_I > \delta_U\), which is a necessary condition for equilibrium.

2. **Limit orders placed farther from the fair value are more likely to trigger when there is an information event.** For example, a buy limit order placed in the interval \([V_0 - \delta_I, V_0 - \delta_U)\) in our model executes only when there is an information event. The ratio of information versus liquidity-driven execution probabilities increases in the distance from the spread.

3. **Investors using limit orders trade in the “wrong” direction in response to news.** Limit order traders trade in the wrong direction when there is an information event because only orders overshot by the intrinsic value execute. Limit orders triggered due to an information event on average lose 
   \[-\frac{1}{2}(\delta_I - s^*)\]
   in equilibrium.

\(^{31}\)The sufficient condition can be written as

\[
\pi(\delta_I - \delta_U)(\pi\delta_U + 2(1 - \pi)\delta_I)((1 - \pi)\delta_I + (1 + \pi)\delta_U) > (1 - \pi)\delta_I^2\delta_U
\]

which is a cubic equation in \(\delta_I, \delta_U, \) and \(\pi\). Note that this condition is satisfied when \(\delta_I > \delta_U\) and \(\pi\) tends close to 1; i.e., when the risk of adverse selection is sufficiently high.
4. *Investors (on average) gain less than the half-spread from using limit orders.*

This is the risk of adverse selection. The first and fourth terms in Eq. 3.10 are the two opposing effects: (1) if the limit order executes because of a liquidity shock, it earns the half-spread but (2) the losses to the informed investor offset some of these gains. In our risk-neutral model, these losses offset all the gains (because Eq. 3.12 holds with equality).


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