Three Essays on Valuation

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy
in Management

by

Bruno Miguel Calisto Miranda

2006
The dissertation of Bruno Miguel Calisto Miranda is approved.

Antonio E. Bernardo

Bhagwan Chowdhry

Harold L. Cole

Mark J. Garmaise, Committee Chair

University of California, Los Angeles

2006
To my parents

João and Celeste Miranda

whose love, support and encouragement made an everlasting impression on my life
# Contents

1 Overvaluation Distortion ................................................................. 1

1.1 Introduction .................................................................................. 1

1.2 Equity Financing ............................................................................. 7

1.2.1 First Best .................................................................................. 10

1.2.2 Optimal Mechanism ................................................................. 11

1.3 Equity and Debt Financing ............................................................ 17

1.3.1 First Best .................................................................................. 18

1.3.2 Optimal Mechanism ................................................................. 19

1.4 Public Corporations ...................................................................... 26

1.4.1 Old Debt .................................................................................. 31

1.5 Bankruptcy Costs .......................................................................... 33

iv
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5.1</td>
<td>First Best</td>
<td>34</td>
</tr>
<tr>
<td>1.5.2</td>
<td>Incentive Constraint</td>
<td>35</td>
</tr>
<tr>
<td>1.6</td>
<td>Conclusion</td>
<td>38</td>
</tr>
<tr>
<td>1.7</td>
<td>Appendix A: Optimal Mechanism under Equity Financing</td>
<td>40</td>
</tr>
<tr>
<td>1.8</td>
<td>Appendix B: Optimal Mechanism with Equity and Debt</td>
<td>44</td>
</tr>
<tr>
<td>1.9</td>
<td>Appendix C: Bankruptcy Costs</td>
<td>47</td>
</tr>
<tr>
<td>1.10</td>
<td>References</td>
<td>54</td>
</tr>
<tr>
<td>2</td>
<td>Libor Interest Rates</td>
<td>57</td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>57</td>
</tr>
<tr>
<td>2.2</td>
<td>Model</td>
<td>66</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Private Agents</td>
<td>67</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Government</td>
<td>71</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Financial Intermediaries</td>
<td>73</td>
</tr>
<tr>
<td>2.2.4</td>
<td>Definition of Equilibrium</td>
<td>79</td>
</tr>
<tr>
<td>2.3</td>
<td>Equilibrium</td>
<td>80</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Portfolio Problem of a Financial Intermediary</td>
<td>80</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Externality behind the demand for cash</td>
<td>85</td>
</tr>
</tbody>
</table>
3 Do the pecking-order’s predictions follow from its premises? 116

3.1 Introduction ......................................................... 116

3.2 Economic Environment ........................................... 124

   3.2.1 Technology and Timing .................................... 124

   3.2.2 Managerial Objectives .................................... 130

   3.2.3 The Signaling Game ....................................... 132

3.3 Equilibrium ....................................................... 138

   3.3.1 The Possibility of Costless Separation .................. 139

   3.3.2 Low Type Policies ........................................ 142
# List of Figures

1.1 Timing of events in period $t$ ..................................................... 9

1.2 Overvaluation Distortion ......................................................... 15

1.3 Optimal Mechanism under Equity Financing .............................. 17

1.4 Overvaluation Incentives ......................................................... 21

1.5 Share Repurchases in Public Corporations ................................ 29

1.6 Value Destruction under debt holders’ control ........................... 34

1.7 Equity Issuance and Debt Exposure ........................................ 37

1.8 Stock Price and Market Value of Equity .................................... 38

1.9 Overvaluation Incentives with Value Destruction ....................... 49

2.1 A simple example ................................................................. 62

2.2 Preferences and Technology .................................................. 71
2.3 Balance Sheet ................................................................. 73
2.4 Timing of Events in Financial Sector ................................. 75
2.5 Portfolio Allocation ....................................................... 82
2.6 Money Demand ............................................................. 84
2.7 Effective Spread .......................................................... 86
2.8 Equilibrium ................................................................. 93
2.9 Effect of $\delta$ on Interbank rate ($b = 0$) ......................... 95
2.10 Effect of Inflation ........................................................ 97
2.11 Market Spreads 3 Month (Weekly Observations) ............. 101
3.1 Higher capital investment as positive signal ..................... 155
3.2 Higher debt as positive signal ...................................... 157
3.3 Equity value function .................................................... 169
3.4 Optimal capital allocations .......................................... 170
3.5 Optimal financing policies - high value of $\theta$ ............... 171
3.6 Optimal financing policies - low value of $\theta$ ................. 172
3.7 Percentage of equity sold to new shareholders ................. 173
3.8 Evolution of leverage ratio ........................................... 174
## List of Tables

3.1 Parameter Choices ......................................................... 175

3.2 Summary Statistics from Simulated Firms ............................ 176

3.3 Leverage Regressions ....................................................... 178
ACKNOWLEDGMENTS

For support, excellent comments and suggestions, I would like to thank Andy Atkeson, Tony Bernardo, Ariel Burnstein, Stephen Cauley, Bhagwan Chowdhry, Hal Cole, Mark Garmaise, Gary Hansen, Christian Hellwig, Christopher Hennessy, Francis Longstaff, Raghu Rau, Richard Roll, Eduardo Schwartz, Walter Torous, Selale Tuzel and my fellow Ph.D. students. Special thanks to Christopher Hennessy and Dmitry Livdan for their collaboration in chapter 3 of my dissertation.

I gratefully acknowledge financial support from Fundação para a Ciência e Tecnologia (FCT) and European Social Fund (ESF) under the third Community Support Framework (CSF), Anderson School of Management and Harold Price Center for Entrepreneurial Studies.
December 2, 1975
Born, Lisbon, Portugal

1998
B.A., Economics
Portuguese Catholic University
Lisbon, Portugal

1997-2001
Assistant Lecturer in Economics
Portuguese Catholic University
Lisbon, Portugal

2001-2006
Teaching Assistant in Finance
Anderson School of Management at UCLA
Los Angeles, California

2006
M.S., Mathematics
Department of Mathematics at UCLA
Los Angeles, California
ABSTRACT OF THE DISSERTATION

Three Essays on Valuation

by

Bruno Miguel Calisto Miranda

Doctor of Philosophy in Management

University of California, Los Angeles, 2006

Professor Mark J. Garmaise, Chair

The first essay studies how firms optimally design mechanisms to transmit information to outside investors in order to obtain capital for growth opportunities. The value of issued securities depends on information about future cash flows that outside investors are unable to verify. Firms claim to have better projects in order to receive cheaper financing. This creates an incentive for the firm to overinvest. This overvaluation distortion is mitigated when the firm pays low dividends, the level of collateral is high, and the firm’s equity ownership is sufficiently dispersed to allow appropriately designed share repurchase programs.

The second essay presents a dynamic general equilibrium model that disentangles
interbank interest rates from interest rates on government bonds. Firstly, I explicitly identify the externality in the financial sector that explains the use of cash in the portfolios of financial intermediaries. It is shown that the positive spread between interbank rates and treasury rates is the leading variable that drives the money market. Secondly, general equilibrium implications are analyzed. I identify the role played by interbank rates that prevents them from reflecting a pricing kernel at which the real economy is willing to transfer cash flows over time, thereby compromising the use of Libor interest rates in valuation. Additionally, an increase in expected inflation produces an increase in real interest rates and an economic slowdown.

The third essay examines the effect of asymmetric information on the evolution of corporate investment and financing using an empirically testable dynamic structural model. We do so by embedding a privately informed manager in a neoclassical investment framework with costly default. When private information is bad, the manager overinvests (relative to first-best), has negative leverage, and uses dividend cuts or equity flotations to fill any financing gaps. When private information is good, debt provides a positive signal. However, in many states the good type issues equity despite having access to default-free debt. In addition, the good type may overinvest. These predictions contradict the pecking-order folk wisdom.
Chapter 1

Overvaluation Distortion

1.1 Introduction

This paper studies how firms optimally design mechanisms to transmit information to outside investors in order to obtain capital for growth opportunities. The value of issued securities depends on information about future cash flows that outside investors are unable to verify. Firms claim to have better projects in order to receive cheaper financing. However, these projects optimally require more investment since they are more productive. This creates an incentive for the firm to overinvest. This overvaluation distortion is mitigated when the firm pays low dividends, the level of collateral is high, and the firm’s equity ownership is sufficiently dispersed to allow appropriately designed share repurchase programs.
Firms frequently rely on outside investors to finance growth opportunities. Firm insiders typically have private information about the quality of these opportunities and attempt to transmit this information with detailed business plans and prospectuses or direct communication with the investor community (such as conversations with analysts and investors). Outside investors, however, recognize that insiders have an incentive to overstate the quality of growth opportunities in order to minimize the ownership stake that must be sold to secure funding. This paper shows how this informational friction leads firms to overinvest to credibly transmit information to outside investors and also examines efficient mechanisms for mitigating this overinvestment.

To examine these issues, I develop a model of a risk-neutral firm with assets-in-place and a growth opportunity. The optimal amount of capital to allocate to the growth opportunity depends on its productivity, which is unknown to outside investors. The only assumption about the growth opportunity is that the marginal product of capital is decreasing. The firm is assumed to have insufficient internal funds to finance the project optimally and designs a mechanism through which it credibly transmits information to outside investors. The mechanism identifies its investment policy and the securities being issued to finance its investment. By the Revelation Principle, I can restrict attention to policies that are a function of the true productivity of the growth opportunity.

My main result is that credible information transmission requires the firm to overinvest in the growth opportunity. The intuition is straightforward. Since the firm has an incentive to overstate the quality of the growth opportunity to secure funding with
the lowest possible dilution of insider ownership, there must be a cost associated with overstating quality in the firm’s investment policy. In the underinvestment region, however, the firm would benefit from overstating quality both because it (i) reduces dilution and (ii) moves its investment policy closer to first-best; thus, the firm cannot credibly transmit information in the underinvestment region. Conversely, in the overinvestment region, the firm does incur a cost of overstating quality - committing to a less efficient investment policy. The overinvestment result is in stark contrast to the seminal paper of Myers and Majluf (1984) which shows informational asymmetries between the firm and the market may lead the firm to underinvest in positive NPV projects. My results differ from Myers and Majluf because they assume asymmetric information about the value of assets in place whereas I assume asymmetric information about the quality of the firm’s growth opportunity. Although both assumptions are plausible, my assumption fits better when evaluating the financing policy of young, entrepreneurial firms or, more generally, firms with significant growth opportunities such as, for example, the Telecom sector late 90s.

The financial structure and payout policy of the firm must then be designed to mitigate the overvaluation distortion on its investment policy. In this context, firms with insufficient financial slack should not pay dividends. Dividends increase the amount the firm needs to raise from external sources, thereby increasing the marginal incentives to overstate its productivity in order to obtain cheaper financing. As a consequence, the overinvestment problem is even more severe, reducing the returns to the firm’s sharehold-
ers. Fama and French (2001) identify low profitability and strong growth opportunities as typical characteristics of firms that do not pay dividends. When a firm has enough earnings to finance its growth opportunities, the overvaluation distortion disappears since the firm does not have to raise capital from outside investors. For firms with sufficient financial slack, the overvaluation distortion does not prevent them from paying dividends. In fact, DeAngelo, DeAngelo and Skinner (2004) find that dividend payments by industrial firms are concentrated among well-established firms with high earnings, such as Exxon Mobil or General Electric.

Payout policy through share repurchases, instead of dividends, constitutes an additional tool for the firm to mitigate the overvaluation distortion. The firm has an incentive to repurchase shares at a low price in order to increase the value of the firm’s remaining shares. Thus, when a firm issues debt and repurchases shares, there is an incentive to overvalue debt by overstating the quality of the growth opportunity but there is also a countervailing incentive to undervalue equity in order to repurchase its shares at a low price. Consequently, share repurchases mitigate the overvaluation distortion. In fact, a combination of debt issuance with a share repurchase implements the first-best investment policy in my model. The analysis shows that optimal share repurchase policy is given by the ratio between the marginal impact on the value of debt from overstating quality and the marginal impact on the value of equity from overstating quality. Since a firm having a growth opportunity with higher quality borrows relatively more, the marginal incentive to overvalue debt is relatively higher than the marginal incentive to undervalue
equity. Thus, the model predicts a positive price reaction upon the announcement of a share repurchase and the announcement effect is positively related to the proportion of shares repurchased. This fact is documented in, for example, Ikenberry, Lakonishok and Vermaelen (1995), and Grullon and Michaely (2002). Moreover, although firms with better projects achieve a higher stock price, the model makes the interesting prediction that the promised yield on debt is nondecreasing in the quality of the project. This is due to the fact that firms with high quality growth opportunities use more leverage to finance investment and also use more leverage to repurchase more shares, thereby decreasing the firm’s bond rating.

The use of share repurchases is not as effective in firms with concentrated equity ownership. For example, in the extreme case of a single shareholder, it is irrelevant if the shareholder sells one share at a low price since it is that same shareholder that will fully appropriate the gain on the stock price of the remaining shares. The ability to mitigate the overvaluation distortion with share repurchases provides a novel motivation for firms to go public.

My analysis also shows the importance of collateral - the value of assets in place - for financial policy. Since the value of collateral does not depend on the productivity of the firm’s growth opportunities, a firm with more collateral has weaker incentives to overstate its productivity in order to obtain cheaper financing. As a result, the firm delivers a better return to its shareholders because the overvaluation distortion on its investment policy is reduced. Thus, firms should start by issuing fully collateralized debt.
This paper provides a unified framework linking two approaches that have shaped our understanding of corporate finance. On the one hand, it is often assumed that managers have an excessive taste for running large firms, as opposed to simply profitable ones (see, e.g., Jensen (1986, 1993), Stulz (1990), and Zwiebel (1996)). The preference for “empire building” implies overinvestment and is traditionally justified as being a reduced form for greater perquisite consumption and reputation that comes from running a large business. In my paper, however, overinvestment is derived endogenously in a simple model with asymmetric information but without assuming empire building preferences. The overinvestment result is at odds with the underinvestment result obtained in the signaling literature. In the signaling approach, financial decisions by firms convey information to outside investors about the real prospects of the firm (e.g., Myers and Majluf (1984), and Miller and Rock (1985)). Subsequent to Brennan and Kraus (1987), which address situations where the amount of investment is fixed, Constantinides and Grundy (1989) obtain the result that if a fully revealing equilibrium exists then the firm has an efficient investment policy. The analysis developed in this paper provides a link between these two approaches by establishing a relation between overinvestment and the incentives that firms have to overvalue securities raised to finance investment.

The remainder of the paper is organized as follows. Section (1.2) formalizes the notion of overvaluation distortion in a simplified version of the model. Section (1.3)
extends the model to equity and debt financing. Section (1.4) considers the case of public corporations. In particular, debt repurchases are a less powerful financial tool to mitigate the overvaluation destruction than share repurchases. A debt repurchase implies a transfer of value from shareholders to old debt holders that still didn’t tender their debt, due to higher seniority on future cash flows. Section (1.5) analyzes the situation where debt holders’ control of the company, after it defaults on its debt, partially destroys cash flows. This may be a result of premature liquidation of assets, or due to the intangibility of assets that are under shareholders’ control, like customers lists or intellectual property. Equity issuances are associated with smaller stock price reactions. Firms issue equity because of more severe financial restrictions due to cash flow’s destruction under debt holders’ control, thereby associated with smaller stock price reactions. Section (1.6) concludes and gives direction for further research.

### 1.2 Equity Financing

The following model formalizes the notion of overvaluation distortion. My main results require only two important assumptions. First, the value of the security being issued depends positively on the quality of the firm’s growth opportunity in order to create incentives to overstate quality. Second, the marginal productivity of capital is decreasing in order to prevent the firm from always wishing to overstate the quality of the growth opportunity. To simplify the exposition, however, I will formalize the notion of overval-
uation distortion in a model in which security being issued is equity and the production function of the firm has constant elasticity with respect to investment.

Consider a firm that can invest equity financed capital $k$ at time $t$ and obtain observable earnings $\theta \epsilon k^\alpha$ at time $t + 1$. The parameter $\theta$ represents the quality of the growth opportunity and is known precisely by the firm’s insiders at time $t$. Outside investors, however, only know that $\theta$ is drawn from a distribution $G(\theta)$ on the support $[\theta_L, \theta_H]$, where $\theta_L > 0$. The parameter $\epsilon$ is a random variable realized at time $t + 1$ and is drawn from a distribution $H(\epsilon)$ on the support $[\epsilon_L, \epsilon_H]$, with mean $\tau$ and $\epsilon_L > 0$. There is no asymmetric information about the random variable $\epsilon$. The random variables $\theta$ and $\epsilon$ are independent with probability density functions, $g(\theta)$ and $h(\epsilon)$, respectively. The parameter $\alpha$ is in the interval $(0, 1)$ so that the production function exhibits decreasing marginal returns.

The firm has retained earnings of $E$ but these earnings are insufficient to finance optimally the growth opportunity even if it has the lowest possible productivity, i.e., $\alpha \theta_L \tau E^{\alpha - 1} \geq R$, where $R$ is the interest rate. This assumption plays no important role in the analysis but simply ensures that the firm needs external finance. All agents are risk neutral.

Figure 2.4 describes the timing of events. At the beginning of time $t$, the firm has current earnings $E$ which are publicly observed by all market participants. The firm then privately observes the productivity parameter $\theta$ and then decides (i) the dividend payout
\( p, (ii) \) the investment policy \( k, \) and \( (iii) \) the equity issuance \( s, \) where \( s \) represents the percentage of ownership sold to outside investors.\(^1\) These three decisions are observable by outside investors. At time \( t + 1 \) earnings are realized, the firm is liquidated, and the proceeds are distributed to shareholders.

\[
\begin{array}{ccc}
\text{Earnings publicly observed} & \theta \text{ privately observed by firm} & \text{Dividend payout, investment decisions and equity issuance}
\end{array}
\]

Figure 1.1: Timing of events in period \( t \)

The firm is operated in the best interest of its current shareholders thus there are no agency problems between the firm’s management and its current shareholders. The payoff to shareholders comes in the form of dividends and the market value of their shares. Let \( MV_{t-} \) denote the market value of equity prior to any corporate decisions made at time \( t \) and let \( MV_{t+} = R^{-1}\theta \pi k^\alpha \) denote the market value of equity after all corporate decisions made at time \( t. \) The firm’s objective is to maximize the return on the firm’s shares:\(^2\)

\[\text{By definition, } s = \frac{\# \text{ (new shares)}}{\# \text{ (old shares)} + \# \text{ (new shares)}}.\]

\(^2\)Maximization the firm’s stock return is the same as being operated in the best interest of its current shareholders because the firm does not repurchase any of its shares. Later in the paper, when I allow share repurchases, maximization of the return on firm’s stock creates incentives to buy back shares from current shareholders at an undervalued price.

\(^1\)By definition, \( s = \frac{\# \text{ (new shares)}}{\# \text{ (old shares)} + \# \text{ (new shares)}}.\)
\[
\max_{\{p(\theta), k(\theta) \geq 0, s(\theta) \leq 1\}} MV_{t-} = \int_{\theta_L}^{\theta_H} g(\theta) \left[ p(\theta) + (1 - s(\theta))R^{-1} \theta \epsilon_k(\theta) \alpha \right] d\theta. \tag{1.1}
\]

On the other hand, their investment \( k \) and dividend payout \( p \) are financed by current earnings and equity issuance:\(^3\)

\[
k + p = E + sE_\theta [MV_{t+}; p, s, k]. \tag{1.2}
\]

Keeping investment and dividends constant, it is in the best interest of current shareholders that each share be overvalued. Overvalued stock allows them to issue less shares and, as a consequence, sell less ownership of future earnings.

### 1.2.1 First Best

The first best is the situation where investment is efficiently allocated: optimal investment decisions \( k^*(\theta) \) are determined by \( \alpha \theta \epsilon_k(\theta) \alpha^{-1} = R \). It is the outcome obtained when \( \theta \) is observed by outside investors.

Under the assumption that \( \theta \) is publicly observed, budget equation (1.2) can immediately be used to rewrite objective function (1.1) as

\[
\max_{\{p(\theta), k(\theta) \geq 0, s(\theta) \leq 1\}} MV_{t-} = \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \theta \epsilon_k(\theta) \alpha - k(\theta) + E \right] d\theta.
\]

\(^3E_\theta \) denotes expectation with respect to quality \( \theta \). The mechanism through which rational expectations from outsiders are formed will be specified in what follows. Equation (1.2) assumes that outside investors have no bargaining power. The presence of several potential outside investors provides justification for this assumption, since they would compete for any rent at their disposal.
Thus, it is in the interest of current shareholders to choose investment $k^*(\theta)$. As in Modigliani-Miller, dividend policy becomes irrelevant. Ownership sold is then given by

$$s^*(\theta; p(\theta)) = \frac{k^*(\theta) + p(\theta) - E}{R^{-1}\theta\tau k^*(\theta)^\alpha}.$$

### 1.2.2 Optimal Mechanism

A mechanism identifies investment policy, payout policy and financial policy to finance investment and payout policy. By the revelation principle, one can restrict attention to mechanisms $(p(\theta), k(\theta), s(\theta))$ in which the true type is truthfully reported.\(^4\) Let $\hat{\theta}$ denote the announced type. The optimal mechanism is defined in problem $(P1)$:

\[
(P1) : \max_{\{p(\theta), k(\theta) \geq 0, s(\theta) \leq 1\}} MV_{L-} = \int_{\theta_L}^{\theta_H} g(\theta) \left[ p(\theta) + (1 - s(\theta))R^{-1}\theta\tau k(\theta)^\alpha \right] d\theta \tag{1.3}
\]

s.t.

\[
k(\hat{\theta}) + p(\hat{\theta}) = E + s(\hat{\theta})R^{-1}\theta\tau k(\hat{\theta})^\alpha \quad \forall \hat{\theta}, \tag{1.4}
\]

\[
p(\theta) + (1 - s(\theta))R^{-1}\theta\tau k(\theta)^\alpha \geq p(\hat{\theta}) + (1 - s(\hat{\theta}))R^{-1}\theta\tau k(\hat{\theta})^\alpha \quad \forall \theta \forall \hat{\theta}. \tag{1.5}
\]

Equation (1.4) is the budget equation for announced type $\hat{\theta}$: Investment and dividends are financed through current earnings and equity issuance. Equation (1.5) is the

---

\(^4\)Suppose a mechanism $(p(\mu), k(\mu), s(\mu))$ in which message $\mu(\theta)$ is sent to outside investors. Then $(p(\mu(\theta)), k(\mu(\theta)), s(\mu(\theta)))$ is a mechanism, in which productivity parameter $\theta$ is truthfully reported, that provides the same payoff to current shareholders and outside investors.
incentive compatibility constraint: Type $\theta$ doesn’t have strict incentives to announce a different type $\hat{\theta}$.

Problem (P1) makes the standard assumption in mechanism design that the firm can commit to a capital allocation scheme. When a share is bought, the outside investor knows which assets are being bought by the firm, although the productivity of those assets is not known. By presenting a business plan or prospectus when trying to obtain funding from outside investors, the firm commits to the level of investment and payout defined in its business plan or prospectus. It is the same commitment device we observe in any form of secured financing, as in, for example, a typical mortgage: the bank is willing to finance only after seeing that the house has been bought.\(^5\)

Substituting budget equation (1.4) in objective function (1.3) and incentive compatibility constraint (1.5), we can rewrite problem (P1) as problem (P2):

\[
(P2) : \max_{\{p(\theta), k(\theta) \geq 0, s(\theta) \leq 1\}} MV_{1-} = \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \theta \tau k(\theta)^\alpha - k(\theta) + E \right] d\theta \quad (1.6) \\
\text{s.t.} \\
k(\hat{\theta}) + p(\hat{\theta}) = E + s(\hat{\theta}) R^{-1} \hat{\theta} \tau k(\hat{\theta})^\alpha \quad \forall \hat{\theta}, (1.7) \\
R^{-1} \theta \tau k(\theta)^\alpha - k(\theta) \geq s(\hat{\theta}) R^{-1} (\hat{\theta} - \theta) \tau k(\hat{\theta})^\alpha + R^{-1} \theta \tau k(\hat{\theta})^\alpha - k(\hat{\theta}) \forall \theta \forall \hat{\theta}. (1.8)
\]

\(^5\)An alternative (unmodelled) justification could be based on reputational concerns: If the firm intends to borrow from outside investors often in the future, the firm knows that it won’t be able to provide information if outside investors anticipate revisions in its capital allocation after obtained financing.
The first term on the right side of incentive compatibility constraint (1.8) represents potential gains to current shareholders from having their equity incorrectly priced. 6 By overstating the quality of the growth opportunity, the firm can finance the same level of investment and dividends with smaller dilution of insider ownership.

It suffices to impose incentive compatibility constraint (1.8) locally, because global restrictions will then be a consequence. 7 From (1.8), type $\theta$’s marginal gain from announcing a slightly different type $\hat{\theta}$ is given by

$$
\frac{ds(\hat{\theta})}{d\hat{\theta}} \left[ R^{-1}(\hat{\theta} - \theta)\bar{c}(k(\hat{\theta}))^\alpha \right] + s(\hat{\theta}) \left[ R^{-1}\bar{c}(k(\hat{\theta}))^\alpha + R^{-1}(\hat{\theta} - \theta)\bar{c}\alpha(k(\hat{\theta}))^{\alpha-1}\frac{dk(\hat{\theta})}{d\hat{\theta}} \right]
$$

$$
+ \left[ R^{-1}\bar{c}\alpha(k(\hat{\theta}))^{\alpha-1} - 1 \right] \frac{dk(\hat{\theta})}{d\hat{\theta}}.
$$

Thus, incentive compatibility gives

$$
s(\theta)\bar{c}(k(\theta))^\alpha = [R - \theta\bar{c}\alpha(k(\theta))^{\alpha-1}]\frac{dk(\theta)}{d\theta} \quad \forall \theta. \quad (1.9)
$$

The left side of equation (1.9) is the marginal gain from issuing overvalued equity ownership $s(\theta)$ by reporting a higher type. It represents free cash that can be used to

---

6To obtain (1.8), note that incentive constraint (1.5) becomes

$$s(\theta)R^{-1}\theta\bar{c}k(\theta)^\alpha - k(\theta) + E + (1 - s(\theta))R^{-1}\theta\bar{c}k(\theta)^\alpha \geq s(\hat{\theta})R^{-1}\hat{\theta}\bar{c}k(\hat{\theta})^\alpha - k(\hat{\theta}) + E + (1 - s(\hat{\theta}))R^{-1}\hat{\theta}\bar{c}k(\hat{\theta})^\alpha,$$

or

$$R^{-1}\theta\bar{c}k(\theta)^\alpha - k(\theta) + E \geq s(\hat{\theta})R^{-1}(\hat{\theta} - \theta)\bar{c}k(\hat{\theta})^\alpha + R^{-1}\theta\bar{c}k(\theta)^\alpha - k(\hat{\theta}) + E.$$

---

7See Appendix A for a justification.
finance investment or dividends to current shareholders. The right side is the marginal cost from incurring levels of investment associated with a higher type. The optimal amount of capital to allocate to the project depends on productivity, which is unknown to outside investors. Optimal investment $k^*$, where the marginal return on investment equals the cost of capital $R$, is illustrated in figure 1.2 by the path connecting points $F_1$ and $F_2$. The firm would ideally invest on the path connecting points $F_1$ and $F_2$. However, the firm may not be able to transmit information about its productivity with an efficient investment policy, because the value of securities depends positively on the productivity of the growth opportunity. The firm would have a marginal incentive to overstate its productivity since the firm would be able to overvalue its securities, thereby selling less ownership of future cash flows, in a situation where the additional investment still returns its cost of capital. With underinvestment, the firm would have two motives to overstate its productivity: the additional capital raised creates value inside the firm because the marginal return on investment is higher than the cost of capital, and the firm can sell less ownership of future cash flows by overvaluing securities issued. Thus, we obtain an investment policy characterized by overinvestment. On the path connecting points $O_1$ and $O_2$, the firm does not have an incentive to overstate its productivity because the marginal destruction in value inside the firm exactly compensates the marginal incentive to overvalue its securities. Moreover, as figure 1.2 suggests, the higher the overinvestment, the higher the incentives to overvalue securities because the amount raised through outside financing is bigger, thereby exacerbating even more the overinvestment.
We have overinvestment as an essential feature of the optimal mechanism. The distortion is a consequence of investment being financed with assets whose value increases with the productivity of the firm.

Equation (1.9) allows us to rewrite our mechanism design problem in its final form:\(^8\)

\[
(P3) : \max_{\{p(\theta),k(\theta)\geq 0\}} MV = \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \theta \tau k(\theta)^\alpha - k(\theta) + E \right] d\theta \\
\text{s.t.} \\
p(\theta) + k(\theta) - \frac{E}{\theta} = \left[ 1 - R^{-1} \theta \tau \alpha (k(\theta))^{\alpha-1} \right] \frac{dk(\theta)}{d\theta} 
\]

\(^8\)Substituting equation (1.7) into equation (1.9), we get

\[
\frac{p(\theta) + k(\theta) - E}{R^{-1} \theta \tau \alpha (k(\theta))^{\alpha-1}} = \frac{R - \theta \tau \alpha (k(\theta))^{\alpha-1}}{\tau (k(\theta))^{\alpha}} \frac{dk(\theta)}{d\theta}.
\]

Equation (1.11) follows. Once problem (P3) is solved, \(s(\theta)\) can be obtained from equation (1.7).
Proposition 1.2.1 gives the closed-form solution to problem (P3).

**Proposition 1.2.1** At the optimum, firms don’t pay dividends and there is no distortion for the least productive type \( \theta_L \). Thus, \( \theta_L \alpha k_L^{\alpha-1} = R \), that is, \( k_L = k_L^* = \left( \frac{\theta_L \alpha R}{\epsilon} \right)^{\frac{1}{\alpha-1}}. \)

There is overinvestment for the remaining types.

A firm investing equity financed capital \( k \) has type

\[
\theta(k) = \frac{k - E}{R^{-1} \epsilon((k)^\alpha - (k_L^*)^\alpha) + \theta_L^{-1}[k_L^* - E]].
\]  

(1.12)

The market value at the beginning of period \( t \) is

\[
MV_0 = \frac{E \theta(\theta)}{\theta_L} [R^{-1} \theta_L \epsilon(k_L^*)^\alpha - k_L^*] + \frac{E \theta(\theta)}{\theta_L} E.
\]  

(1.13)

**Proof.** See Appendix A. ♦

It is not optimal to pay dividends since the firm would had to borrow even more from outside investors. As a consequence, the overvaluation distortion on the firm’s investment policy would had to be even higher to compensate the increase in the marginal incentive to overvalue the firm’s shares.

Figure 1.3 presents a graphical description of the optimal mechanism. For a given productivity, the market value of equity is higher than what it would be in the first best investment policy, because the firm raises more capital. The overinvestment, which is
The solid lines correspond to the optimal mechanism. The dashes and stars represent the first best solution. Ownership sold \( s \) for the first best solution assumes no dividends. \( P_{t+}S_{t-} \) is the market value of the old shares after the equity issuance. \( MV_{t+} = P_{t+}S_{t+} \) is the total market value of equity after the equity issuance. \( MV_{t-} = P_{t-}S_{t-} \) is the market value of the old shares before the equity issuance. \( k \) is investment and \( \theta \) the quality of the growth opportunity.

Figure 1.3: Optimal Mechanism under Equity Financing

financed with additional equity issuances, causes the firm’s stock price to be below what it would be in the first best investment policy.

### 1.3 Equity and Debt Financing

When a firm can issue equity and debt to finance investment opportunities, we naturally ask how can we mitigate the overvaluation distortion by choosing the appropriate capital structure.

Consider a firm issuing equity and debt to finance investment opportunities.\(^9\) Denote

\(^9\)This section assumes no dividends to simplify the exposition. Section (1.2) showed that dividends are not a useful financial tool to mitigate the overvaluation distortion on the firm’s investment policy.
the debt to be paid at $t+1$ for a firm of type $\theta$ by $d(\theta)$. The objective of the firm is to maximize the return on its shares:

$$\max_{\{d(\theta), k(\theta) \geq 0; 0 \leq s(\theta) \leq 1\}} MV_{t} = \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \int_{d(\theta)\frac{d\theta}{\pi k(\theta)\alpha}}^{\infty} h(\epsilon) \left[ (1 - s(\theta))(\theta \epsilon k(\theta)^\alpha - d(\theta)) \right] d\epsilon \right] d\theta.$$  \hspace{1cm} (1.14)

On the other hand, investment $k$ is financed by current earnings, equity issuance and debt issuance:

$$k(\theta) = E + s(\theta)R^{-1} \int_{d(\theta)\frac{d\theta}{\pi k(\theta)\alpha}}^{\infty} h(\epsilon) \left[ \theta \epsilon k(\theta)^\alpha - d(\theta) \right] d\epsilon$$

$$+ R^{-1} \left[ \int_{0}^{d(\theta)\frac{d\theta}{\pi k(\theta)\alpha}} h(\epsilon) \theta \epsilon k(\theta)^\alpha d\epsilon + \int_{d(\theta)\frac{d\theta}{\pi k(\theta)\alpha}}^{\infty} h(\epsilon) d(\theta) d\epsilon \right].$$  \hspace{1cm} (1.15)

### 1.3.1 First Best

Assuming that productivity $\theta$ is publicly observed, we can substitute (1.15) into objective function (1.14) and rewrite the objective as

$$\max_{\{d(\theta), k(\theta) \geq 0; 0 \leq s(\theta) \leq 1\}} MV_{t} = \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \int_{d(\theta)\frac{d\theta}{\pi k(\theta)\alpha}}^{\infty} h(\epsilon) \left[ \theta \epsilon k(\theta)^\alpha - d(\theta) \right] d\epsilon - k(\theta) + E \right.$$

$$\left. + R^{-1} \left[ \int_{0}^{d(\theta)\frac{d\theta}{\pi k(\theta)\alpha}} h(\epsilon) \theta \epsilon k(\theta)^\alpha d\epsilon + \int_{d(\theta)\frac{d\theta}{\pi k(\theta)\alpha}}^{\infty} h(\epsilon) d(\theta) d\epsilon \right] \right] d\theta$$

$$= \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \theta \epsilon k(\theta)^\alpha - k(\theta) + E \right] d\theta.$$  

Thus, it is in the interest of current shareholders to choose the first best investment $k^*(\theta)$. The precise financial plan to finance investment is irrelevant.
1.3.2 Optimal Mechanism

Problem \((P1)\) defines the optimal mechanism. Equation (1.17) is the budget equation for announced type \(\hat{\theta}\): Investment is financed through current earnings, equity issuance and debt issuance. Equation (1.18) is the incentive compatibility constraint: Type \(\theta\) doesn’t have strict incentives to announce a different type \(\hat{\theta}\).

\[
(P1) : \max_{\{d(\theta), k(\theta) \geq 0; 0 \leq s(\theta) \leq 1\}} \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \int_{d(\theta)}^{\infty} h(\epsilon) \left[ (1 - s(\theta)) (\theta \epsilon k(\theta)^\alpha - d(\theta)) \right] d\epsilon \right] d\theta
\]

\[
\text{s.t.} \quad k(\hat{\theta}) = E + s(\hat{\theta}) R^{-1} \int_{d(\hat{\theta})}^{\infty} h(\epsilon) \left[ \hat{\theta} \epsilon k(\hat{\theta})^\alpha - d(\hat{\theta}) \right] d\epsilon + R^{-1} \left[ \int_{0}^{d(\hat{\theta})} h(\epsilon) \hat{\theta} \epsilon k(\hat{\theta})^\alpha d\epsilon + \int_{d(\hat{\theta})}^{\infty} h(\epsilon) d(\hat{\theta}) d\epsilon \right] \forall \hat{\theta},
\]

\[
R^{-1} \int_{d(\hat{\theta})}^{\infty} h(\epsilon) \left[ (1 - s(\hat{\theta})) (\theta \epsilon k(\hat{\theta})^\alpha - d(\hat{\theta})) \right] d\epsilon \geq \geq R^{-1} \int_{d(\hat{\theta})}^{\infty} h(\epsilon) \left[ (1 - s(\hat{\theta})) (\theta \epsilon k(\hat{\theta})^\alpha - d(\hat{\theta})) \right] d\epsilon \forall \theta \forall \hat{\theta}.
\]

Substituting budget equation (1.17) in objective function (1.16) and incentive compatibility constraint (1.18), we can rewrite problem (P1) as problem (P2):
(P2) \text{max}_{\{d(\theta), k(\theta) \geq 0 ; 0 \leq s(\theta) \leq 1 \}} MV_L = \int_{0}^{\theta_H} g(\theta) \left[ R^{-1} \theta \epsilon_k(\theta)^\alpha - k(\theta) + E \right] d\theta \quad (1.19)

\text{s.t.}

\begin{align*}
k(\hat{\theta}) &= E + s(\hat{\theta}) R^{-1} \int_{0}^{\hat{\theta}} \frac{d(\hat{\theta})}{\hat{\theta} \epsilon_k(\hat{\theta})^\alpha} h(\epsilon) \left[ \hat{\theta} \epsilon_k(\hat{\theta})^\alpha - d(\hat{\theta}) \right] d\epsilon \\
&\quad + R^{-1} \left[ \int_{0}^{\hat{\theta}} \frac{d(\hat{\theta})}{\hat{\theta} \epsilon_k(\hat{\theta})^\alpha} h(\epsilon) \hat{\theta} \epsilon_k(\hat{\theta})^\alpha d\epsilon + \int_{\hat{\theta}}^{\infty} \frac{d(\hat{\theta})}{\hat{\theta} \epsilon_k(\hat{\theta})^\alpha} h(\epsilon) d(\hat{\theta}) d\epsilon \right] \forall \hat{\theta}, \\
R^{-1} \theta \epsilon_k(\theta)^\alpha - k(\theta) + E &\geq R^{-1} \theta \epsilon_k(\hat{\theta})^\alpha - k(\hat{\theta}) + E \\
&\quad + s(\hat{\theta}) R^{-1} \int_{0}^{\hat{\theta}} \frac{d(\hat{\theta})}{\hat{\theta} \epsilon_k(\hat{\theta})^\alpha} h(\epsilon) \left[ (\hat{\theta} - \theta) \epsilon_k(\hat{\theta})^\alpha \right] d\epsilon \\
&\quad + R^{-1} \left[ \int_{0}^{\hat{\theta}} \frac{d(\hat{\theta})}{\hat{\theta} \epsilon_k(\hat{\theta})^\alpha} \left[ \hat{\theta} \epsilon_k(\hat{\theta})^\alpha - d(\hat{\theta}) \right] d\epsilon \right. \\
&\quad \left. + R^{-1} \int_{\hat{\theta}}^{\infty} \frac{d(\hat{\theta})}{\hat{\theta} \epsilon_k(\hat{\theta})^\alpha} h(\epsilon) \left[ d(\hat{\theta}) - \theta \epsilon_k(\hat{\theta})^\alpha \right] d\epsilon \right. \\
&\quad \left. + R^{-1} \int_{0}^{\hat{\theta}} \frac{d(\hat{\theta})}{\hat{\theta} \epsilon_k(\hat{\theta})^\alpha} h(\epsilon) (\hat{\theta} - \theta) \epsilon_k(\hat{\theta})^\alpha d\epsilon \right] \forall \theta \forall \hat{\theta}.
\end{align*}

The last four terms in incentive compatibility constraint (1.21) represent potential gains to increase the firm’s stock return from obtaining financing with securities incorrectly priced, as illustrated in figure 1.4. Since the value of equity and debt used to finance investment is nondecreasing in productivity, there are incentives to report a higher type.

Terms (1) and (2) represent the gains from overvalued equity: (1) gives the artificial
$\omega(\theta)$: Critical value below which firm defaults on debt $d(\theta)$.

$\epsilon \theta k(\theta)\alpha$: Total value of a type $\theta$ firm at $t+1$.

Figure 1.4: Overvaluation Incentives

increase in cash flows to equity holders in states in which the firm does not default on its debt, and (2) gives the artificial increase in cash flows to shareholders from the reduction in states in which the firm is unable to meet its debt obligations. Terms (3) and (4) represent the gains from overvalued debt: (3) gives the artificial gain from the reduction of states in which the firm has to default, and (4) gives the artificial increase in cash flows to debt holders in states in which the firm still defaults on its debt.

In order to simplify notation and to focus on leading variables, I will formalize our mechanism using debt exposure $\omega(\theta) \equiv \frac{d(\theta)}{\theta k(\theta)\alpha}$, instead of promised debt $d(\theta)$. $\omega(\theta)$ is the critical value of $\epsilon$ below which a firm of type $\theta$ will be unable to repay all its debt. Since $H$ is the distribution function of $\epsilon$, $H(\omega(\theta))$ is the likelihood of default for a type $\theta$ firm. Thus, a higher $\omega(\theta)$ can be interpreted as a deterioration of the firm’s debt rating.
To prevent type $\theta$ from announcing a slightly different type $\hat{\theta}$, incentive compatibility constraint (1.21) gives

$$\int_{0}^{\omega(\theta)} h(\epsilon)ek(\theta)^{\alpha}d\epsilon + s(\theta) \int_{\omega(\theta)}^{\infty} h(\epsilon)ek(\theta)^{\alpha}d\epsilon = [R - \theta\tau\alpha(k(\theta))^{\alpha-1}] \frac{dk(\theta)}{d\theta} \quad \forall \theta. \quad (1.22)$$

The first term on the left side of equation (1.22) is the marginal gain from issuing overvalued debt. The second term on the left side of equation (1.22) is the marginal gain from issuing overvalued equity ownership $s(\theta)$. They both represent free cash that can be used to finance investment and increase the return on the firm’s stock. The term on the right side of equation (1.22) is the marginal cost from incurring levels of investment associated with a higher type.

Equation (1.22) allows us to rewrite our mechanism design problem in its final form (P3).

$$(P3) : \max_{\{\omega(\theta),k(\theta)\geq0;0\leq s(\theta)\leq1\}} MV_{L-} = \int_{\theta_L}^{\theta_H} g(\theta) [R^{-1}\theta\tau k(\theta)^{\alpha} - k(\theta) + E] d\theta \quad (1.23)$$

s.t.

$$k(\theta) = E + s(\theta)R^{-1}\theta k(\theta)^{\alpha} \int_{\omega(\theta)}^{\infty} h(\epsilon) [\epsilon - \omega(\theta)] d\epsilon$$

$$+ R^{-1}\theta k(\theta)^{\alpha} \left[ \int_{0}^{\omega(\theta)} h(\epsilon)\epsilon d\epsilon + \int_{\omega(\theta)}^{\infty} h(\epsilon)\omega(\theta)d\epsilon \right], \quad (1.24)$$

$$k(\theta)^{\alpha} \int_{0}^{\omega(\theta)} h(\epsilon)\epsilon d\epsilon + s(\theta)k(\theta)^{\alpha} \int_{\omega(\theta)}^{\infty} h(\epsilon)d\epsilon = [R - \theta\tau\alpha(k(\theta))^{\alpha-1}] \frac{dk(\theta)}{d\theta}. \quad (1.25)$$

\(^{10}\)Note that terms (2) and (3) don’t play a role at the margin.
Define the overvaluation distortion \( \Delta(\theta) \) from a firm of type \( \theta \):

\[
\Delta(\theta) \equiv [R - \theta \alpha(k(\theta))^\alpha \theta] \frac{dk(\theta)}{d\theta}.
\] (1.26)

The financial challenge is to construct a financial plan that mitigates the overvaluation distortion on its investment policy. From budget constraint (1.24),

\[
\frac{ds(\theta)}{d\omega(\theta)} = -\frac{(1 - s(\theta)) \int_{\omega(\theta)}^{\infty} h(\epsilon)d\epsilon}{\int_{\omega(\theta)}^{\infty} h(\epsilon)[\epsilon - \omega(\theta)]d\epsilon} < 0.
\] (1.27)

For a given level of investment, the increase in revenues obtained from issuing additional debt allows the firm to decrease equity issuance, although the value of equity previously sold goes down due to higher debt repayments in the future.

From incentive compatibility constraint (1.25),

\[
\frac{d\Delta(\theta)}{d\omega(\theta)} = k(\theta)^\alpha h(\omega(\theta)))\omega(\theta) + \frac{ds(\theta)}{d\omega(\theta)} k(\theta)^\alpha \int_{\omega(\theta)}^{\infty} h(\epsilon)\epsilon d\epsilon - s(\theta) k(\theta)^\alpha h(\omega(\theta))\omega(\theta)
\]

\[
= (1 - s(\theta)) k(\theta)^\alpha h(\omega(\theta)))\omega(\theta) + \frac{ds(\theta)}{d\omega(\theta)} k(\theta)^\alpha \int_{\omega(\theta)}^{\infty} h(\epsilon)\epsilon d\epsilon.
\] (1.28)

An increase in debt exposure increases the incentives to overvalue debt and reduces the incentives to overvalue equity. A marginal increase in reported productivity to increase

\[
0 = \frac{ds(\theta)}{d\omega(\theta)} R^{-1} \theta k(\theta)^\alpha \int_{\omega(\theta)}^{\infty} h(\epsilon)[\epsilon - \omega(\theta)]d\epsilon - s(\theta) R^{-1} \theta k(\theta)^\alpha \int_{\omega(\theta)}^{\infty} h(\epsilon)d\epsilon
\]

\[
- s(\theta) R^{-1} \theta k(\theta)^\alpha h(\omega(\theta)) [\omega(\theta) - \omega(\theta)] + R^{-1} \theta k(\theta)^\alpha \left[ h(\omega(\theta))\omega(\theta) - h(\omega(\theta))\omega(\theta) + \int_{\omega(\theta)}^{\infty} h(\epsilon)d\epsilon \right]
\]

\[
= \frac{ds(\theta)}{d\omega(\theta)} R^{-1} \theta k(\theta)^\alpha \int_{\omega(\theta)}^{\infty} h(\epsilon)[\epsilon - \omega(\theta)]d\epsilon + (1 - s(\theta)) R^{-1} \theta k(\theta)^\alpha \int_{\omega(\theta)}^{\infty} h(\epsilon)d\epsilon.
\]
revenues from debt issuance is even more appealing with larger levels of debt. On the other hand, two effects reduce the incentives to overvalue equity. Incentives to overvalue equity go down because equity ownership sold to finance investment goes down, as in (1.27), and simultaneously the value of equity ownership previously sold goes down due to larger debt repayments in the future. Therefore,

\[
\frac{d \Delta(\theta)}{d \omega(\theta)} = \left\{ h(\omega(\theta)) \omega(\theta) - \frac{\int_{\omega(\theta)}^{\infty} h(\epsilon) \epsilon d\epsilon}{\int_{\omega(\theta)}^{\infty} h(\epsilon) \epsilon - \omega(\theta) d\epsilon} \right\} (1 - s(\theta)) k(\theta)^{a} \\
= \delta(\omega(\theta); h) (1 - s(\theta)) MV_{t+}(\theta),
\]

(1.29)

where

\[
\delta(\omega(\theta); h) \equiv h(\omega(\theta)) \omega(\theta) \int_{\omega(\theta)}^{\infty} h(\epsilon) \epsilon - \omega(\theta) d\epsilon - \left( \int_{\omega(\theta)}^{\infty} h(\epsilon) \epsilon d\epsilon \right) \left( \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon \right). 
\]

(1.30)

Clearly, \( \delta(0; h) < 0 \ \forall h \): A firm issuing no debt has a strictly positive marginal gain from issuing debt since it reduces the overvaluation incentives associated with equity issuance without introducing any overvaluation incentives on debt. Since the risk of defaulting on its debt is zero, the value of the debt that the firm issues does not depend on its productivity type. As a consequence, the firm is able to achieve a more efficient investment policy if it starts issuing some debt.

Now, consider the most simple type of nontrivial densities: \( h(\epsilon) = \frac{1}{(b-a)} 1_{[a,b]} \). For \( \omega(\theta) < a \) we have \( \delta(\omega(\theta); h) < 0 \): There are no overvaluation incentives on debt because there is enough cash flow tomorrow to prevent default under any realization of \( \epsilon \). There is solely the gain from reducing the overvaluation distortion from equity issuance.\(^{12}\) For

\(^{12}\)The case \( \omega(\theta) > b \) is not relevant because it would mean that the firm is issuing more debt than it

24
\( a < \omega(\theta) < b, \)

\[
\delta(\omega(\theta); h) = \frac{\omega(\theta) (b - \omega(\theta))^2}{b - a} - \frac{(b^2 - \omega(\theta)^2)(b - \omega(\theta))}{2(b - a)} \frac{(b - a)}{b - a} \\
= \frac{(b - \omega(\theta))^2}{2(b - a)^2} \left\{ \omega(\theta)(b - \omega(\theta)) - (b^2 - \omega(\theta)^2) \right\} \\
= - \frac{b(b - \omega(\theta))^2}{2(b - a)^2} < 0.
\]

The increase in overvaluation incentives on debt is always smaller than the reduction in overvaluation incentives on equity. Since any given density \( h \) is the limit of a linear combination of step functions of the above type, we obtain the following general result.

**Proposition 1.3.1** The optimal financial policy relies solely on debt.

Define \( \bar{\theta} \equiv R \left( E/ \left[ (\alpha \bar{\tau})^{1/\alpha} - \epsilon_L (\alpha \bar{\tau})^{1/\alpha} \right] \right)^{1-\alpha}. \)

For \( \theta \leq \bar{\theta} \), firms issue fully collateralized debt and have an efficient investment policy.

For \( \theta > \bar{\theta} \), we have an investment policy characterized by overinvestment.

**Proof.** See Appendix B for details. ♦

Proposition 1.3.1 clarifies the idea that the value of collateral does not come from guaranteeing repayment, but instead on having a value that is independent of the announced productivity of the firm. As a consequence, fully collateralized financing eliminates the overvaluation distortion on the investment policy of the firm. Once the firm exhausts its collateral, it will not be able to eliminate the overvaluation distortion on

---

25
its investment policy because the value of the securities being issued, equity or debt, depends positively on productivity.

The next section presents an advantage for firms to be public corporations, with shares traded in public exchanges, instead of private firms. The overvaluation distortion can be reduced through the use of share repurchases in large public companies, since those firms have strong incentives to repurchase shares at an undervalued price. It is precisely because those firms are not concerned about a specific small shareholder that they end up delivering the best possible return to every shareholder!

1.4 Public Corporations

With share repurchases, the mechanism design problem faced by the firm is again (P3), (1.23)-(1.25), but where the domain for equity ownership sold is extended to $s(\theta) \leq 1$ (instead of $0 \leq s(\theta) \leq 1$).
In order to achieve the first best investment policy $k^*(\theta)$, the overvaluation distortion on the right side of incentive constraint (1.33) should be zero. Thus, incentive constraint (1.33) says that the percentage of shares repurchased ($-s(\theta)$) should equal the ratio between the marginal gain to overvalue debt and the marginal gain to overvalue equity, 

$$s(\theta) = \frac{\int_0^{\omega(\theta)} h(\epsilon) \epsilon d\epsilon}{\int_{\omega(\theta)}^{\infty} h(\epsilon) \epsilon d\epsilon}.$$  

When debt exposure is small, $\omega(\theta) < \epsilon_L$, investment is financed with fully collateralized debt. The firm does not have incentives to overstate its productivity in order to overvalue debt since there is no probability of default. Therefore, $s(\theta) = 0$ whenever the firm is only using fully collateralized debt. If $s(\theta) > 0$, the firm would have incentives to overvalue equity by announcing higher productivity. If $s(\theta) < 0$, the firm would have incentives to understate its productivity in order to repurchase shares at an undervalued price.

As debt exposure becomes higher, $\omega(\theta) > \epsilon_L$, the firm is forced to issue risky debt. The
riskiness of debt creates incentives to overvalue debt, which have to be compensated with the incentives to undervalue equity in order to eliminate the overvaluation distortion on the investment policy of the firm. Therefore, although the firm is borrowing from external capital markets, share repurchases exist to cancel the incentives to stimulate overvaluation when issuing risky debt. A higher level of debt exposure $\omega(\theta)$ increases the incentive to overvalue debt and reduces the value of equity. As a result, share repurchases increase with the level of debt exposure to eliminate the overvaluation distortion. That is, the model predicts that share repurchases increase with the deterioration of the firm’s bond rating. Since a deterioration of the firm’s bond rating increases the incentives to overvalue debt, the firm should repurchase more shares so that the incentives to undervalue equity cancel the incentives to overvalue debt and no overvaluation distortion on its investment policy is created.

Substituting optimal repurchase policy (1.34) into budget constraint (1.32), we obtain the debt policy to finance investment and payout policy,

$$\frac{k^*(\theta) - E}{R^{-1}\theta \tilde{\epsilon} k^*(\theta)^{\alpha}} = \frac{\omega(\theta)}{E_{\epsilon}[\epsilon|\epsilon \geq \omega(\theta)]}.$$  (1.35)

For a given positive level of current earnings, the percentage of borrowing to the total value of the firm increases with the quality of the growth opportunity (that is, the left side of (1.35) is increasing in $\theta$). As a consequence, debt exposure is nondecreasing with the quality of the growth opportunity (in order to increase the right side of equation
\( P_{t+} S_{t-} \) is the market value of the old shares after the equity issuance.
\( MV_{t+} = P_{t+} S_{t+} \) is the total market value of equity after the equity issuance.
\( \theta \) is the quality of the growth opportunity and \( s \) is ownership sold.
\( \omega \) is the critical value of \( \epsilon \) below which the firm defaults on its debt.

Figure 1.5: Share Repurchases in Public Corporations

(1.35)).\(^{13}\) The promised yield on debt is then nondecreasing in firm’s type because the likelihood of default is higher. The deterioration of the firm’s bond rating follows from the relatively higher leverage that firms with better projects have to finance higher investment and share repurchases.\(^{14}\)

Proposition (1.4.1) follows.

**Proposition 1.4.1** Let \( \bar{\theta} \equiv R \left( E / \left[ (\alpha \bar{\epsilon})^{\frac{1}{1-\alpha}} - \epsilon_L (\alpha \bar{\epsilon})^{\frac{\alpha}{1-\alpha}} \right] \right)^{1-\alpha} \).

For \( \theta \leq \bar{\theta} \), optimal financial policy relies solely on issuing fully collateralized debt to finance investment.

\(^{13}\)As the level of earnings converges to zero, debt exposures of different types get closer and closer.

When current earnings are zero, debt exposure is the same for all types.

\(^{14}\)See Appendix B for an analytical proof.
For $\theta > \bar{\theta}$, optimal financial policy relies on issuing risky debt to finance investment and share repurchases. Debt exposure and equity ownership repurchased are strictly increasing in productivity.

Under this optimal financial policy, the first best investment policy is achieved.

**Proof.** See Appendix B for details. $\diamondsuit$

In order to obtain a closed-form solution, assume that $\epsilon$ has the uniform distribution $\mathcal{U}[\epsilon_L, \epsilon_H]$. Then

$$k^*(\theta) = \left( R^{-1} \alpha \epsilon \theta \right)^{\frac{1}{1-\alpha}},$$

$$\omega(\theta) = \begin{cases} 
\frac{k^*(\theta)-E}{R^{-1} \theta k^*(\theta)\alpha} & \text{if } \theta \leq \bar{\theta} \\
\frac{\epsilon_H [k^*(\theta)-E]}{2R^{-1} \theta k^*(\theta)\alpha - [k^*(\theta)-E]} & \text{if } \theta > \bar{\theta},
\end{cases}$$

$$s(\theta) = \begin{cases} 
0 & \text{if } \theta \leq \bar{\theta} \\
-\frac{\omega(\theta)^2 - \epsilon_H^2}{\epsilon_H^2 - \omega(\theta)^2} & \text{if } \theta > \bar{\theta}.
\end{cases}$$

Figure (1.5) presents the graphical description of the optimal mechanism. For a firm that runs out of collateral ($\theta > \bar{\theta}$), the market value of old shares ($P_{t+}, S_{t-}$) is higher than the market value of equity ($MV_{t+}$) because the firm repurchases shares in order to cancel the incentives to overvalue risky debt.
1.4.1 Old Debt

The importance of share repurchases follows from the fact that the firm has incentives to understate its productivity when repurchasing shares. If the firm has old debt outstanding, would the incentives to repurchase risky debt at an undervalued price be a financial tool as powerful as share repurchases? This subsection considers the situation of a firm with old debt outstanding and good investment opportunities to finance.

Denote the debt to be paid at $t + 1$ for a firm of type $\theta$ by $d(\theta)$. The objective of the firm is to maximize the return on its shares:

$$\max_{\{d(\theta), k(\theta) \ge 0, s(\theta) \le 1\}} MV_t = \int^{\theta_L}_{\theta_L} g(\theta) \left[ R^{-1} \int^{\infty}_{d(\theta)} h(\epsilon) [(1 - s(\theta))(\theta \epsilon k(\theta)^\alpha - d(\theta))] \, d\epsilon \right] d\theta.$$  

(1.36)

Suppose the firm has old debt $d_t$ senior in its balance sheet. Investment $k$ is financed by current earnings, equity issuance and debt issuance:

$$k(\theta) = \begin{cases} 
E + s(\theta) R^{-1} \int^{\infty}_{d(\theta)} h(\epsilon) [\theta \epsilon k(\theta)^\alpha - d(\theta)] \, d\epsilon \\
+ R^{-1} \left[ \int^{d(\theta)}_{d_t} h(\epsilon) [\theta \epsilon k(\theta)^\alpha - d_t] \, d\epsilon + \int_{d(\theta)}^{\infty} h(\epsilon) [d(\theta) - d_t] \, d\epsilon \right] \\
\text{if } d(\theta) \ge d_t \\
E + s(\theta) R^{-1} \int^{d(\theta)}_{d(\theta)} h(\epsilon) [\theta \epsilon k(\theta)^\alpha - d(\theta)] \, d\epsilon \\
- R^{-1} \left[ \frac{d_t - d(\theta)}{d_t} \int_{0}^{d(\theta)} h(\epsilon) [\theta \epsilon k(\theta)^\alpha - d(\theta)] \, d\epsilon + \int_{d(\theta)}^{\infty} h(\epsilon) [d_t - d(\theta)] \, d\epsilon \right] \\
\text{if } d(\theta) < d_t. 
\end{cases}$$  

(1.37)
Substituting budget equation (1.37) into objective function (1.36), the objective is to maximize

\[
MV_{t-} = \begin{cases} 
\int_{\theta_L^H}^{\theta_H} g(\theta) \left[ R^{-1} \int_{d_t}^{\infty} h(\epsilon) \theta \epsilon k(\theta)^\alpha d\epsilon - k(\theta) + E - R^{-1} \int_{d_t}^{\infty} h(\epsilon) d\epsilon d\epsilon \right] d\theta & \text{if } d(\theta) \geq d_t \\
\int_{\theta_L^H}^{\theta_H} g(\theta) \left[ R^{-1} \int_{d_t}^{\infty} h(\epsilon) \theta \epsilon k(\theta)^\alpha d\epsilon - k(\theta) + E - R^{-1} \int_{d_t}^{\infty} h(\epsilon) d\epsilon d\epsilon \\
- R^{-1} \left[ \frac{d_t - d(\theta)}{d_t} \int_{0}^{d(\theta)} h(\epsilon) \left[ \theta \epsilon k(\theta)^\alpha - d_t \right] d\epsilon \right] \right] d\theta & \text{if } d(\theta) < d_t.
\end{cases}
\]

The additional two terms that appear in (1.38), when the firm repurchases risky debt, are negative. These two terms represent a loss to current shareholders. When a firm has risky debt outstanding and repurchases part of it, debt holders that still didn’t tender their debt have a positive gain, since they gain seniority on future cash flows. There is a transfer of value from current shareholders to debt holders that could be avoid if the firm does not repurchase old debt. Therefore, repurchasing old debt does not allow current shareholders to achieve the same return on their stock as when the firm uses share repurchases.\textsuperscript{15}

\textsuperscript{15}Additionally, as in Myers (1977), the existence of old debt can cause underinvestment since part of the benefit will go to debt holders. From (1.38), optimal investment is determined by

\[
R^{-1} \int_{d_t}^{\infty} h(\epsilon) \theta \epsilon k(\theta)^\alpha d\epsilon = 1.
\]

If the old debt doesn’t have seniority, then the firm can potentially do even better by not repurchasing...
1.5 Bankruptcy Costs

The analysis developed in the previous sections assumes that, once investment is determined, total cash flows do not depend on whether the ownership of the firm is held by shareholders or debt holders. However, it is natural to consider situations where there is value destruction when debt holders take control of the firm. The destruction of value may be a result of premature liquidation of assets, in the spirit of Diamond and Dybvig (1983), or due to intangibility of assets that are strongly dependent on shareholders’ control, like customers lists or intellectual property that literally walks out of the door with its creator. This section extends the model from section (1.4) to introduce the possibility that there is value destruction in states in which the firm is unable to repay all its debt.

The objective of the firm is again to maximize the return on its shares:

\[
\max_{\{d(\theta), k(\theta) \geq 0; s(\theta) \leq 1\}} MV_{L-} = \int_{\theta_L}^{\theta_H} g(\theta) \left[ (1 - s(\theta))(R - 1) \int_{d(\theta)}^{\infty} h(\epsilon) [\theta \epsilon k(\theta)^\alpha - d(\theta)] d\epsilon \right] d\theta. 
\]

(1.39)

On the other hand, investment \( k \) is financed by current earnings, equity issuance and debt issuance:

\[
k(\theta) = E + s(\theta) R^{-1} \int_{\theta_L}^{\theta_H} h(\epsilon) \left[ \theta \epsilon k(\theta)^\alpha - d(\theta) \right] d\epsilon \\
+ R^{-1} \left[ \int_{\gamma}^{\infty} \frac{d(\theta)}{\theta k(\theta)^\alpha} h(\epsilon) \left[ \epsilon \theta k(\theta)^\alpha - \gamma (d(\theta) - \epsilon \theta k(\theta)^\alpha) \right] d\epsilon + \int_{\theta_H}^{\infty} h(\epsilon) d(\theta) d\epsilon \right]. 
\]

(1.40)
Budget equation (1.40) describes the situation in which there is a marginal cost of $\gamma$ for each dollar that the firm is unable to repay on its debt, as illustrated in figure (1.6). The destruction of cash flows in states where debt holders take control of the company reduces the market value of risky debt.

\[ \omega(\theta) \text{: Critical value below which firm defaults on debt } d(\theta). \]
\[ \frac{\gamma}{1+\gamma} \omega(\theta) \text{: Critical value below which cash flows are zero.} \]
\[ \epsilon \theta k(\theta)^{\alpha} \text{: Total cash flow for a type } \theta \text{ firm at } t + 1. \]

Figure 1.6: Value Destruction under debt holders’ control

### 1.5.1 First Best

When productivity $\theta$ is publicly observed, budget constraint (1.40) can be used to rewrite objective function (1.39) as

\[
\max_{\left\{ d(\theta), k(\theta) \geq 0; s(\theta) \leq 1 \right\}} \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \theta \epsilon k(\theta)^{\alpha} - k(\theta) + E \right. \\
- \int_{0}^{\frac{\gamma}{1+\gamma} \frac{d(\theta)}{\theta k(\theta)^{\alpha}}} h(\epsilon) \theta \epsilon k(\theta)^{\alpha} d\epsilon - \int_{\frac{\gamma}{1+\gamma} \frac{d(\theta)}{\theta k(\theta)^{\alpha}}}^{\frac{d(\theta)}{\theta k(\theta)^{\alpha}}} h(\epsilon) \gamma (d(\theta) - \epsilon \theta k(\theta)^{\alpha}) d\epsilon \bigg] d\theta.
\]
It is in the interest of current shareholders to choose the first best investment policy \( k^*(\theta) \). However, in contrast with the situation where there is no destruction in cash flows when debt holders take control, debt is issued only if there is not any risk of default.

1.5.2 Incentive Constraint

Inequality (1.41) gives the incentive constraint.

\[
(1 - s(\theta)) R^{-1} \int_{\hat{d}(\theta) / \hat{k}(\theta)}^{\infty} h(\epsilon) \left[ \theta e k(\theta)^{\alpha} - d(\theta) \right] d\epsilon \geq \geq (1 - s(\hat{\theta})) R^{-1} \int_{\hat{d}(\hat{\theta}) / \hat{k}(\hat{\theta})}^{\infty} h(\epsilon) \left[ \theta e \hat{k}(\hat{\theta})^{\alpha} - d(\hat{\theta}) \right] d\epsilon \quad \forall \theta \neq \hat{\theta}. \quad (1.41)
\]

I assume that outside investors can infer the severity of cash flows destruction under debt holders’ control, \( \gamma \), from the firm’s business plan.

Using an approach similar to the one used before, we can rewrite the incentive constraint as

\[
k(\theta)^{\alpha}(1 + \gamma) \int_{1 + \gamma \omega(\theta)}^{\omega(\theta)} h(\epsilon)\epsilon d\epsilon + s(\theta) k(\theta)^{\alpha} \int_{\omega(\theta)}^{\infty} h(\epsilon)\epsilon d\epsilon - \gamma \int_{1 + \gamma \omega(\theta)}^{\omega(\theta)} h(\epsilon)\epsilon d\epsilon \frac{d d(\theta)}{d\theta} = = \left[ R - \left( \int_{\omega(\theta)}^{\infty} h(\epsilon)\epsilon d\epsilon + \int_{1 + \gamma \omega(\theta)}^{\omega(\theta)} h(\epsilon)(1 + \gamma)\epsilon d\epsilon \right) \theta^{\alpha}(k(\theta))^{\alpha - 1} \right] \frac{d k(\theta)}{d\theta} \quad \forall \theta. \quad (1.42)
\]

The first term on the left side of equation (1.42) is the marginal gain from issuing overvalued debt. The second term on the left is the marginal gain from issuing overval-
ued equity ownership $s(\theta)$. They both represent free cash that can be used to finance investment and increase the return on the firm’s stock. The third term gives the impact from the change of debt outstanding due to the announcement of a higher type. If better types issued more debt, the announcement of an higher type increases the destruction in value due to debt holders’ control. The term on the right side of equation (1.41) gives the marginal effect on value creation inside the firm.

In order to have a marginal return on investment equal to the cost of capital, incentive constraint (1.42) gives

$$s(\theta) = \frac{\gamma \int_{\omega(\theta)}^{\omega(\theta)} h(\epsilon) d\epsilon}{k(\theta)^\alpha \int_{\omega(\theta)}^{\infty} h(\epsilon) \epsilon d\epsilon} - \frac{(1 + \gamma) \int_{\omega(\theta)}^{\omega(\theta)} h(\epsilon) \epsilon d\epsilon}{\int_{\omega(\theta)}^{\infty} h(\epsilon) \epsilon d\epsilon}.$$

Therefore, there is an additional force, relative to the previous sections, that prevents the firm from announcing a higher type. To announce a higher type, the firm needs to raise additional debt as more productive firms do, thereby increasing the destruction of future cash flows in states in which the firm is unable to repay its debt entirely. This

\[16\text{Similarly to before, substituting equation (1.43) into budget constraint (1.40), we obtain the differential equation describing the financing mechanism. The financing mechanism identified in this section of the paper with bankruptcy costs is determined by having a marginal return on investment equal to the cost of capital, that is, to have the right-hand side of equation (1.41) equal to zero. Although an equality between marginal return on investment and cost of capital determines the optimal mechanism in the previous section without bankruptcy costs, it is beyond the scope of this paper to investigate whether such equality restricts the optimal mechanism with bankruptcy costs. It is proposed as a reasonable assumption to simplify the exposition about the effects of bankruptcy costs in the main analysis of this paper.}\]
\( \gamma, \theta, s \) and \( \omega \) are marginal cost of default, productivity, equity ownership sold and debt exposure.

Figure 1.7: Equity Issuance and Debt Exposure

effect reduces the expected value of future cash flows.

Thus, the level of share repurchases decreases with the value destruction parameter \( \gamma \). For sufficiently high levels of \( \gamma \), it is optimal not to repurchase shares since the additional debt that a firm would had to issue to convince investors of higher productivity are enough to prevent the firm from doing so, as illustrated in figure 1.7.

Additionally, the firm’s stock price is decreasing in the value destruction parameter \( \gamma \), as in figure 1.8. This follows from the fact that current shareholders need to sell more ownership to outside investors in order to raise the same level of funding, due to the value destruction when debt holders take control of the company.

Thus, equity issuances increase with the value of the destruction parameter \( \gamma \), and the stock price is decreasing in \( \gamma \). Firms issuing equity are those firms whose assets are easily destroyed under debt holders’ control, due to liquidation costs or intangibility of assets managed by shareholders. This financial restriction reduces the impact the value
The dashes represent the case $\gamma = 0$. The solid line represents a situation where $\gamma > 0$. $\gamma$ is the marginal destruction for each dollar the firm defaults. $\theta$ is the productivity of the growth opportunity. $P_{t+}S_{t-}$ is the market value of the old shares after issuing securities. $MV_{t+} \equiv P_{t+}S_{t+}$ is the total market value of equity after issuing securities.

Figure 1.8: Stock Price and Market Value of Equity

of the growth opportunity for the firm, thereby a lower stock price reaction.

1.6 Conclusion

The analysis in this paper covered equity, debt, dividend, repurchases and investment policy. All the insights and predictions obtained in this paper followed from a single notion. It is the notion of overvaluation distortion, which creates a link between over-investment and the incentives that firms have to transmit information that overvalues
securities issued to finance investment opportunities.

Another application of this overvaluation distortion is in the context of Mergers and Acquisitions. Consider a firm that can develop significant improvements when acquiring existing firms. The creation of value can be a result of synergies with existing operations, or simply a better use for the assets of acquired companies. The quality of the improvement is private information. Using a straightforward reinterpretation of the model formalized in this paper, the notion of overvaluation distortion in the context of Mergers and Acquisitions follows. The investment variable $k$ would be the market value of acquisitions, and the variable $\theta$ the quality of the improvement. The optimal level of acquisitions $k^*$ depends on the firm’s improvement ability $\theta$. Whenever the value of securities being issued to finance acquisitions depends positively on the firm’s improvement ability, we obtain an overvaluation distortion on the acquisition’s policy of the firm.
1.7 Appendix A: Optimal Mechanism under Equity Financing

The Hamiltonian to problem (P3) is given by

\[ H(k, p, \lambda) = R^{-1} \theta \tau(k(\theta))^\alpha - k(\theta) + E + \lambda(\theta) \frac{p(\theta) + k(\theta) - E}{\theta - R^{-1} \alpha^2 \tau(k(\theta))^{\alpha - 1}}, \]  

(1.44)

where \( \lambda(\theta) \) is the solution to the adjoint equation

\[
\frac{d\lambda(\theta)}{d\theta} = -\nabla_k \left\{ R^{-1} \theta \tau(k(\theta))^\alpha - k(\theta) + E \right\} - \nabla_k \left\{ \frac{p(\theta) + k(\theta) - E}{\theta - R^{-1} \alpha^2 \tau(k(\theta))^{\alpha - 1}} \right\} \lambda(\theta)
\]

\[ = 1 - R^{-1} \theta \tau \alpha(k(\theta))^{\alpha - 1} - \frac{\lambda(\theta)}{\theta - R^{-1} \theta^2 \tau \alpha(k(\theta))^{\alpha - 1}} \left\{ 1 + \frac{p(\theta) + k(\theta) - E}{\theta - R^{-1} \theta^2 \tau \alpha(k(\theta))^{\alpha - 1}} \right\}, \]

(1.45)

with boundary condition \( \lambda(\theta_H) = 0 \).  

The sum of the first two terms in the left side of equation (1.45) is positive because of overinvestment. Since \( \lambda(\theta_H) = 0 \), we have \( p(\theta) < 0 \) when \( (\theta_H - \delta) < \theta < \theta_H \), for some \( \delta > 0 \). The value of \( \lambda(\theta) \) does not return to zero, as \( \theta \) decreases. Suppose it does: as \( \lambda(\theta) \) were to approach zero, the third term in equation (1.45) would vanish and the overinvestment associated with the first two terms would push the value of \( \lambda(\theta) \) downwards. Therefore, since \( \lambda(\theta) < 0 \) for \( \theta < \theta_H \), maximization of Hamiltonian (1.44) shows that it is optimal to have no dividends, \( p(\theta) = 0 \ \forall \theta \).

Incentive compatibility constraint (1.11),

\[ p(\theta) + k(\theta) - E = [\theta - \theta^2 R^{-1} \tau \alpha k(\theta)^{\alpha - 1}] \frac{dk(\theta)}{d\theta}, \]

17Following the approach of the Potryagin maximum principle, \( \lambda(\theta) = \nabla_k J(\theta, k(\theta)) \) along the optimal path \( k(\theta) \), where \( J(\theta, k) \) is the solution to the Hamilton-Jacobi-Bellman partial differential equation associated with problem (P3).
can be rewritten as
\[
\frac{d\theta(k)}{dk} = \frac{\theta(k) R - \theta(k)\tau \alpha k^{\alpha-1}}{p(k) + k - E}.
\]

Defining \( w(k) = \theta(k)^{-1} \), so that \( \frac{d\theta(k)}{dk} = -\frac{dw(k)}{dk} \), we get
\[
\frac{dw(k)}{dk} + \frac{1}{p(k) + k - E} w(k) = \frac{R^{-1} \tau \alpha k^{\alpha-1}}{p(k) + k - E}.
\]

This gives
\[
\theta(k)^{-1} = e^{-\int [p(k) + k - E]^{-1}dk} \left[ \int e^{\int [p(k) + k - E]^{-1}dk} \frac{R^{-1} \tau \alpha k^{\alpha-1}}{p(k) + k - E} \, dk + C \right].
\]

Since \( p(k) = 0 \) \( \forall \theta \),
\[
\theta(k)^{-1} = \frac{1}{k - E} \left[ \int [k - E]^{-1} \frac{R^{-1} \tau \alpha k^{\alpha-1}}{k - E} \, dk + C \right] = \frac{R^{-1} \tau k^{\alpha} + C}{k - E}.
\]

Let \( k_L \) be the investment for the lowest type, that is, \( \theta(k_L) = \theta_L \). We have
\[
\theta_L^{-1} = \frac{R^{-1} \tau (k_L)^{\alpha} + C}{k_L - E},
\]
\[
C = \theta_L^{-1} [k_L - E] - R^{-1} \tau (k_L)^{\alpha}.
\]

Thus, the type of a firm investing equity financed capital \( k \) is given by
\[
\theta(k) = \frac{k - E}{R^{-1} \tau ((k)^{\alpha} - (k_L)^{\alpha}) + \theta_L^{-1} [k_L - E]}.
\]

\[\text{Multiplying equation (1.46) by } e^{\int [p(k) + k - E]^{-1}dk},\]
\[
\left( e^{\int [p(k) + k - E]^{-1}dk} w(k) \right)' = e^{\int [p(k) + k - E]^{-1}dk} \frac{R^{-1} \tau \alpha k^{\alpha-1}}{p(\theta) + k - E}.
\]

Thus,
\[
e^{\int [p(k) + k - E]^{-1}dk} w(k) = \int e^{\int [p(k) + k - E]^{-1}dk} \frac{R^{-1} \tau \alpha k^{\alpha-1}}{p(\theta) + k - E} \, dk + C.
\]
From equation (1.47),
\[ k + [−R^{-1}θ\theta k]^{α} + [θ(R^{-1}θ k_L^{α} + θ_L^{-1}E − θ_L^{-1}k_L) − E] = 0, \]

and the market value at the beginning of period \( t \) becomes

\[
MV_{t−} = \int_{θ_L}^{θ_H} g(θ) \left[ R^{-1}θ k(θ)^α − k(θ) + E \right] dθ
\]
\[ = \int_{θ_L}^{θ_H} g(θ)[θ(R^{-1}θ k_L^{α} + θ_L^{-1}E − θ_L^{-1}k_L)] dθ
\]
\[ = E(θ)[R^{-1}θ k_L^{α} + θ_L^{-1}E − θ_L^{-1}k_L]
\]
\[ = \frac{E(θ)}{θ_L}[R^{-1}θ k_L^{α} − k_L] + E(θ)E. \]

Thus, it is optimal to have efficient investment for the lowest type. We have \( θ_L θ k_L^{α−1} = R \), that is, \( k_L = k_L^{∗} = \left( \frac{θ L ϵ σ}{R} \right)^{1−α} \). Therefore, \(^{19}\)

\[
MV_{t−} = \int_{θ_L}^{θ_H} g(θ) \left[ R^{-1}θ k^{∗}(θ)^α − k^{∗}(θ) + E \right] dθ
\]
\[ = \int_{θ_L}^{θ_H} g(θ) \left[ \theta \theta k^{∗}(θ)^α − k^{∗}(θ) + E \right] dθ
\]
\[ = \int_{θ_L}^{θ_H} g(θ) \left[ \theta \theta k^{∗}(θ)^α − k^{∗}(θ) + E \right] dθ
\]
\[ = \left( \frac{θ}{R} \right)^{1−α} (1−α)α^{α−1} E(θ) \theta E. \]

\(^{19}\)In the first best, since \( k^{∗}(θ) = \left( \frac{αθ}{R} \right)^{1−α} \),

\[
MV_{t−} = \int_{θ_L}^{θ_H} g(θ) \left[ R^{-1}θ k^{∗}(θ)^α − k^{∗}(θ) + E \right] dθ
\]
\[ = \int_{θ_L}^{θ_H} g(θ) \left[ \frac{k^{∗}(θ)}{α} − k^{∗}(θ) + E \right] dθ
\]
\[ = \left( \frac{θ}{R} \right)^{1−α} (1−α)α^{α−1} E(θ) \theta E + E. \]
Ownership sold is then given by

\[ s(k) = \frac{p(k) + k - E}{R^{-1} \theta(k) \tau k^\alpha} = \frac{R^{-1} \tau(k^\alpha - k_L^\alpha) + \theta_L^{-1} [k_L - E]}{R^{-1} \theta \tau k^\alpha} = 1 - \frac{R^{-1} \theta \tau k^\alpha - [k_L - E]}{R^{-1} \theta \tau k^\alpha} \]

\[ = 1 - \frac{R^{-1} \theta \tau \left( \frac{\theta_L \tau k^\alpha}{R} \right)^\frac{\alpha}{R} - \left[ \left( \frac{\theta_L \tau k^\alpha}{R} \right)^\frac{1}{\alpha} - E \right]}{R^{-1} \theta \tau k^\alpha} \]

\[ = 1 - \frac{(\theta_L \tau)^\frac{\alpha}{R} \left( R^{-1} \frac{\alpha}{R} \right) \left( \frac{\alpha}{R} - 1 \right) + RE}{R^{-1} \theta \tau k^\alpha} = 1 - \frac{(\theta_L \tau)^\frac{\alpha}{R} \left( R^{-1} \frac{\alpha}{R} \right) \left( \frac{\alpha}{R} - 1 \right) + RE}{\theta \tau k^\alpha}. \tag{1.48} \]

Thus, \( s \) is decreasing in \( k \), and therefore also in \( \theta \). \(^{20}\)

By definition of \( s \), \( s = \frac{S_{t+} - S_{t-}}{S_{t+}} \), where \( S \) is the number of shares, so that \( S_{t+} = \frac{S_{t-}}{1-s} \). Letting \( P_{t+} \) denote the price per share, \( sMV_{t+} = (S_{t+} - S_{t-})P_{t+} \). Thus, \( sMV_{t+} = sS_{t+}P_{t+} = \frac{S_{t-}}{1-s}S_{t+}P_{t+} \), so that \( S_{t-}P_{t+} = (1-s)MV_{t+} \).

It remains to show incentive compatibility constraint (1.8) globally,

\[ R^{-1} \theta \tau k(\theta)^{\alpha} - k(\theta) \geq s(\hat{\theta})R^{-1}(\hat{\theta} - \theta)\tau k(\hat{\theta})^{\alpha} + R^{-1} \theta \tau k(\hat{\theta})^{\alpha} - k(\hat{\theta}) \quad \forall \theta \neq \hat{\theta}, \]

or

\[ R^{-1} \theta \tau k(\theta)^{\alpha} - k(\theta) \geq R^{-1} \hat{\theta} \tau k(\hat{\theta})^{\alpha} - k(\hat{\theta}) + (1 - s(\hat{\theta}))R^{-1}(\theta - \hat{\theta})\tau k(\hat{\theta})^{\alpha} \quad \forall \theta \neq \hat{\theta}. \]

This follows immediately from equations (1.47) and (1.48). \(^{21}\)

\(^{20}\)For the first best solution, assuming no dividend payout (to be comparable), we have

\[ s(k) = \frac{p(k) + k - E}{R^{-1} \theta(k) \tau k^\alpha} = \frac{k - E}{R^{-1} \tau k^\alpha \left[ \frac{\alpha}{R} \right] k^\alpha} = \frac{\alpha [k - E]}{k} = \alpha - \frac{\alpha E}{k}. \]

We see that \( s \) increases with capital \( k \), and therefore also with \( \theta \).

\(^{21}\)Substituting equations (1.47) and (1.48), we obtain the trivial inequality

\[ \frac{\theta}{\theta_L} \left[ R^{-1} \theta_L \tau (k_L^\alpha - k_L^\alpha + E) \right] - E \geq \frac{\hat{\theta}}{\theta_L} \left[ R^{-1} \theta_L \tau (k_L^\alpha - k_L^\alpha + E) \right] - E+ \]
1.8 Appendix B: Optimal Mechanism with Equity and Debt

In order to obtain the payoff for type $\theta$ when announcing $\hat{\theta}$, rewrite budget equation (1.17) as

$$k(\hat{\theta}) = E + s(\hat{\theta})R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ \theta \hat{c}(\hat{\theta})^\alpha - d(\hat{\theta}) \right] de$$

$$+ s(\hat{\theta})R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ \hat{\theta} \hat{c}(\hat{\theta})^\alpha - d(\hat{\theta}) \right] de + s(\hat{\theta})R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ (\hat{\theta} - \theta)\hat{c}(\hat{\theta})^\alpha \right] de$$

$$+ R^{-1}\int_{0}^{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}} h(e)\hat{c}(\hat{\theta})^\alpha de + R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e)d(\hat{\theta})de.$$

Then

$$R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ (1 - s(\hat{\theta}))(\theta \hat{c}(\hat{\theta})^\alpha - d(\hat{\theta})) \right] de =$$

$$= R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ \theta \hat{c}(\hat{\theta})^\alpha - d(\hat{\theta}) \right] de - k(\hat{\theta}) + E$$

$$+ s(\hat{\theta})R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ (\hat{\theta} - \theta)\hat{c}(\hat{\theta})^\alpha \right] de + s(\hat{\theta})R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ \hat{\theta} \hat{c}(\hat{\theta})^\alpha - d(\hat{\theta}) \right] de$$

$$+ R^{-1}\int_{0}^{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}} h(e)\hat{c}(\hat{\theta})^\alpha de + R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e)d(\hat{\theta})de$$

$$= R^{-1}\theta \hat{c}(\hat{\theta})^\alpha - k(\hat{\theta}) + E$$

$$+ s(\hat{\theta})R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ (\hat{\theta} - \theta)\hat{c}(\hat{\theta})^\alpha \right] de + s(\hat{\theta})R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e) \left[ \hat{\theta} \hat{c}(\hat{\theta})^\alpha - d(\hat{\theta}) \right] de$$

$$- R^{-1}\int_{0}^{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}} h(e)\hat{c}(\hat{\theta})^\alpha de - R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e)d(\hat{\theta})de$$

$$+ R^{-1}\int_{0}^{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}} h(e)\hat{c}(\hat{\theta})^\alpha de + R^{-1}\int_{\frac{d(\hat{\theta})}{\theta_k(\hat{\theta})^\alpha}}^{\infty} h(e)d(\hat{\theta})de$$

$$+ \left[ R^{-1}\theta_L \ell(k_L^*)^\alpha - k_L^* + E \right] \left( \frac{\hat{\theta} - \hat{\theta}}{\theta_L} \right).$$
\[ R^{-1} \theta \kappa(k(\theta)) - k(\theta) + E + s(\theta) R^{-1} \int_0^\infty h(\epsilon) \left[ (\theta - \theta) e_k(\theta)^\alpha \right] d\epsilon + s(\hat{\theta}) R^{-1} \int_0^\infty \frac{d(\hat{\theta})}{d(\theta)} h(\epsilon) \left[ \hat{\theta} e_k(\theta)^\alpha - d(\hat{\theta}) \right] d\epsilon \\
\] 
\[ + R^{-1} \int_0^\infty \frac{d(\hat{\theta})}{d(\theta)} h(\epsilon) \hat{d}(\hat{\theta}) d\epsilon + R^{-1} \int_0^\infty h(\epsilon)(\hat{\theta} - \theta) e_k(\hat{\theta})^\alpha d\epsilon - R^{-1} \int_0^\infty \frac{d(\hat{\theta})}{d(\theta)} h(\epsilon) \hat{d}(\hat{\theta}) e_k(\hat{\theta})^\alpha d\epsilon \\
= R^{-1} \theta \kappa(k(\theta)^\alpha) - k(\theta) + E + s(\theta) R^{-1} \int_0^\infty h(\epsilon) \left[ (\theta - \theta) e_k(\theta)^\alpha \right] d\epsilon + s(\hat{\theta}) R^{-1} \int_0^\infty \frac{d(\hat{\theta})}{d(\theta)} h(\epsilon) \left[ \hat{\theta} e_k(\theta)^\alpha - d(\hat{\theta}) \right] d\epsilon \\
\] 
\[ + R^{-1} \int_0^\infty \frac{d(\hat{\theta})}{d(\theta)} h(\epsilon) \left[ d(\hat{\theta}) - \theta e_k(\theta)^\alpha \right] d\epsilon + R^{-1} \int_0^\infty \frac{d(\hat{\theta})}{d(\theta)} h(\epsilon) (\hat{\theta} - \theta) e_k(\hat{\theta})^\alpha d\epsilon. \]

The marginal gain from announcing a slightly different type \( \hat{\theta} \) is given by

\[ [R^{-1} \theta \kappa(k(\theta)) - 1] \frac{dk(\theta)}{d\theta} + \int_0^\infty \frac{d(\hat{\theta})}{d(\theta)} h(\epsilon) e_k(\theta)^\alpha d\epsilon + s(\theta) \int_0^\infty \frac{d(\hat{\theta})}{d(\theta)} h(\epsilon) e_k(\theta)^\alpha d\epsilon. \]

The critical value of \( \theta \) such that \( \omega(\theta) = \epsilon_L \) is \( (R^{-1} \alpha \theta \kappa) \frac{1}{1-\alpha} - E = R^{-1} \epsilon_L \theta (R^{-1} \alpha \theta \kappa) \frac{1}{1-\alpha} \), that is,

\[ \theta = \left( \frac{ER^{\frac{1}{1-\alpha}}}{(\alpha \kappa)^{\frac{1}{1-\alpha}} - \epsilon_L \theta (\alpha \kappa)^{\frac{1}{1-\alpha}}} \right)^{1-\alpha}. \]

Optimal debt policy of equation (1.35) follows from substituting repurchase policy (1.34) into budget constraint (1.32). We have

\[ \frac{k^*(\theta) - E}{R^{-1} \theta k^*(\theta)^\alpha} = \frac{\int_0^\omega(\theta) h(\epsilon) e\epsilon d\epsilon}{\int_\omega(\theta) h(\epsilon) e\epsilon d\epsilon} \int_\omega(\theta) h(\epsilon) [\epsilon - \omega(\theta)] d\epsilon + \int_0^\omega(\theta) h(\epsilon) e\epsilon d\epsilon \]

\[ = \omega(\theta) \left( \frac{\int_0^\omega(\theta) h(\epsilon) e\epsilon d\epsilon}{\int_\omega(\theta) h(\epsilon) e\epsilon d\epsilon} \right) + \omega(\theta) \int_\omega(\theta) h(\epsilon) e\epsilon d\epsilon = \omega(\theta) \int_\omega(\theta) h(\epsilon) e\epsilon d\epsilon = \frac{\omega(\theta)}{E \epsilon \epsilon \geq \omega(\theta)} \frac{1}{1-\alpha}. \]

In order see analytically that the left side of equation (1.35) is increasing in productivity \( \theta \), note that

\[ \frac{k^*(\theta) - E}{R^{-1} \theta k^*(\theta)^\alpha} = \frac{(R^{-1} \theta \kappa)^{\frac{1}{1-\alpha}} - E}{R^{-1} \theta \kappa (R^{-1} \alpha \theta \kappa)^{\frac{1}{1-\alpha}}} = \alpha^{\frac{1}{1-\alpha}} - E (R^{-1} \theta \kappa)^{\frac{1}{1-\alpha}} \frac{1}{1-\alpha}. \]

Moreover, the right side of (1.35) is increasing in the debt exposure \( \omega(\theta) \),

\[ \left( \int_\omega(\theta) h(\epsilon) e\epsilon d\epsilon \right)^2 \frac{d(\omega(\theta))}{d\omega(\theta)} = \]

45
\[
\begin{align*}
\int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon - \omega(\theta) h(\omega(\theta)) \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon + \left[ \omega(\theta) \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon \right] h(\omega(\theta)) &= \\
= \left( \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon \right) \left( \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon - \omega(\theta) h(\omega(\theta)) \right) \\
&> 0 \text{ by taking limits of step functions (picture of } \epsilon \text{'s density gives intuition).}
\end{align*}
\]

Let \( R_Y(\theta) \) denote the yield of debt issued by a firm of type \( \theta \). We have

\[
\begin{align*}
R_Y^{-1}(\theta) \cdot d(\theta) &= R^{-1} \left[ \int_{0}^{\omega(\theta)} h(\epsilon) \epsilon \theta k(\theta)^\alpha d\epsilon + \int_{\omega(\theta)}^{\infty} h(\epsilon) d(\theta) d\epsilon \right] \\
R_Y^{-1}(\theta) &= R^{-1} \frac{\int_{0}^{\omega(\theta)} h(\epsilon) d\epsilon + \int_{\omega(\theta)}^{\infty} h(\epsilon) \omega(\theta) d\epsilon}{\omega(\theta)} \\
\frac{dR_Y^{-1}(\theta)}{d\theta} &= R^{-1} \frac{-\int_{0}^{\omega(\theta)} h(\epsilon) d\epsilon d\omega(\theta)}{\omega(\theta)^2} \leq 0,
\end{align*}
\]

Assume that \( \epsilon \) has the uniform distribution \( \mathcal{U}[\epsilon_L, \epsilon_H] \). For \( \omega(\theta) \geq \epsilon_L, \frac{k(\theta) - E}{R^{-1} \theta k(\theta)^\alpha} = \frac{\epsilon_{\omega(\theta)}}{\epsilon_{\omega(\theta) + \omega(\theta)}} = \frac{2\epsilon_{\omega(\theta)}}{\epsilon_H + \omega(\theta)} \). Thus,

\[
\omega(\theta) = \frac{\epsilon_H}{2\epsilon} \frac{k(\theta) - E}{R^{-1} \theta k(\theta)^\alpha} = \frac{\epsilon_H \left[ k^*(\theta) - E \right]}{2\epsilon \left[ R^{-1} \theta k^*(\theta)^\alpha \right] - \left[ k^*(\theta) - E \right]}.
\]

Moreover, \( S_{t-} P_{t+} = (1 - s) MV_{t+} \) and \( MV_{t+} = R^{-1} \theta \epsilon k(\theta)^\alpha - k(\theta) + E \).

When the firm has old debt outstanding \( d_t \), objective function (1.36) can be rewritten as, after substituting budget equation (1.37),
In order to obtain the payoff for type \( \theta \), rewrite budget equation (1.40) as

\[
k(\hat{\theta}) = E + s(\hat{\theta})R^{-1} \int_{\theta k(\hat{\theta})}^{\infty} h(e) \left[ \hat{\theta}ek(\hat{\theta})^\alpha - d(\hat{\theta}) \right] de \\
+ s(\hat{\theta})R^{-1} \int_{\theta k(\hat{\theta})}^{\hat{\theta}k(\hat{\theta})^\alpha} h(e) \left[ \hat{\theta}ek(\hat{\theta})^\alpha - d(\hat{\theta}) \right] de + s(\hat{\theta})R^{-1} \int_{\hat{\theta}k(\hat{\theta})^\alpha}^{\infty} h(e) \left[ (\hat{\theta} - \theta)ek(\hat{\theta})^\alpha \right] de \\
+ R^{-1} \int_{\gamma \theta k(\hat{\theta})^\alpha}^{\infty} h(e) [\hat{\theta}ek(\hat{\theta})^\alpha - \gamma(\hat{\theta} - \hat{\theta})k(\hat{\theta})^\alpha] de + R^{-1} \int_{\gamma \theta k(\hat{\theta})^\alpha}^{\infty} h(e)d(\hat{\theta}).
\]

Then

\[
(1 - s(\hat{\theta}))R^{-1} \int_{\theta k(\hat{\theta})}^{\infty} h(e) \left[ \hat{\theta}ek(\hat{\theta})^\alpha - d(\hat{\theta}) \right] de =
\]
Using budget equation (1.40), we can rewrite objective function (1.39) as

\[
\max_{\{d(\theta), k(\theta) \geq 0; s(\theta) \leq 1\}} MV_L = \int_{\theta_L}^{\theta_H} g(\theta) \left[ R^{-1} \int_{\frac{d(\theta)}{\theta \hat{k}(\theta)^{\alpha}}}^{\infty} h(\epsilon) \left( \theta(\epsilon) - d(\hat{\theta}) \right) \epsilon d\epsilon - k(\hat{\theta}) + E \right. \\
+ R^{-1} \int_{\frac{d(\theta)}{\theta \hat{k}(\theta)^{\alpha}}}^{\infty} h(\epsilon) \left( \theta(\epsilon) - d(\hat{\theta}) \right) \epsilon d\epsilon + s(\hat{\theta}) R^{-1} \int_{\frac{d(\theta)}{\theta \hat{k}(\theta)^{\alpha}}}^{\infty} h(\epsilon) \left( \theta(\epsilon) - d(\hat{\theta}) - e(\hat{\theta}) \right) \epsilon d\epsilon \\
+ R^{-1} \int_{\frac{d(\theta)}{\theta \hat{k}(\theta)^{\alpha}}}^{\infty} h(\epsilon) d(\hat{\theta}) d\epsilon + R^{-1} \int_{\frac{d(\theta)}{\theta \hat{k}(\theta)^{\alpha}}}^{\infty} h(\epsilon) \left( \theta(\epsilon) - d(\hat{\theta}) - e(\hat{\theta}) \right) \epsilon d\epsilon \\
\left. - R^{-1} \int_{\frac{d(\theta)}{\theta \hat{k}(\theta)^{\alpha}}}^{\infty} h(\epsilon) \left[ e(\hat{\theta}) - gamma(\hat{\theta}) - e(\hat{\theta}) \right] \epsilon d\epsilon \right] d\theta \tag{1.49}
\]

Substituting budget equation (1.40) in incentive compatibility constraint (1.41), we can rewrite the incentive constraint as
\[
R^{-1} \int_{d(\hat{\theta})}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) \theta e k(\hat{\theta})^\alpha d\epsilon - k(\hat{\theta}) + R^{-1} \int_{\frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}}}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) \left[ \theta e k(\hat{\theta})^\alpha - \gamma \left( d(\hat{\theta}) - e \theta k(\hat{\theta})^\alpha \right) \right] d\epsilon \geq \\
R^{-1} \int_{d(\hat{\theta})}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) \theta e k(\hat{\theta})^\alpha d\epsilon - k(\hat{\theta}) + R^{-1} \int_{\frac{\gamma d(\epsilon)}{\pi e(\epsilon)^{\alpha}}}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) \left[ \theta e k(\hat{\theta})^\alpha - \gamma \left( d(\hat{\theta}) - e \theta k(\hat{\theta})^\alpha \right) \right] d\epsilon \\
+ s(\hat{\theta}) R^{-1} \int_{d(\hat{\theta})}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) \left[ (\hat{\theta} - \theta) e k(\hat{\theta})^\alpha \right] d\epsilon + s(\hat{\theta}) R^{-1} \int_{\frac{\gamma d(\epsilon)}{\pi e(\epsilon)^{\alpha}}}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) \left[ \hat{\theta} e k(\hat{\theta})^\alpha - d(\hat{\theta}) \right] d\epsilon \tag{1.50} \\
+ R^{-1} \int_{\gamma}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) \left[ \hat{\theta} e k(\hat{\theta})^\alpha - \gamma \left( d(\hat{\theta}) - e \theta k(\hat{\theta})^\alpha \right) \right] d\epsilon + R^{-1} \int_{\frac{\gamma d(\epsilon)}{\pi e(\epsilon)^{\alpha}}}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) d(\hat{\theta}) d\epsilon \\
- R^{-1} \int_{\gamma}^{\infty} \frac{d(\epsilon)}{\pi e(\epsilon)^{\alpha}} h(\epsilon) \left[ \theta e k(\hat{\theta})^\alpha - \gamma \left( d(\hat{\theta}) - e \theta k(\hat{\theta})^\alpha \right) \right] d\epsilon \tag{3} \forall \hat{\theta} \in \hat{\theta}.
\]

Terms (1), (2) and (3), in incentive compatibility constraint (1.50), represent potential gains to current shareholders from obtaining financing with securities incorrectly priced, as illustrated in figure 1.9. Terms (1) and (2) represent the gains from overvalued equity: (1) gives the artificial increase in cash flows to equity holders in states in which the firm does not default on its debt, and (2) gives the
artificial increase in cash flows to shareholders from the reduction in states in which the firm has to default. Term (3) represents the artificial increase in cash flows to debt holders from overvalued debt.

Type \( \theta \)'s marginal gain from announcing a slightly different type \( \hat{\theta} \) is given by

\[
-R^{-1} \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \frac{dd(\theta)}{d\theta} dc + R^{-1} \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e)(1 + \gamma) k(\theta)^{\alpha} dc + s(\theta) R^{-1} \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) k(\theta)^{\alpha} dc.
\]

Substituting (1.43) into budget constraint (1.40),

\[
\frac{k(\theta) - E}{R^{-1}k(\theta)^{\alpha}} = \gamma \frac{dd(\theta)}{d\theta} \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \frac{d\beta(\theta)}{d\beta} dc + \omega(\theta) \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc
\]

\[
= \gamma \frac{dd(\theta)}{d\theta} \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \frac{d\beta(\theta)}{d\beta} dc + \omega(\theta) \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc
\]

\[
= \frac{\gamma}{k(\theta)^{\alpha}} \frac{dd(\theta)}{d\theta} \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) dc \left[ \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \frac{d\beta(\theta)}{d\beta} dc \right]
\]

\[
+ \omega(\theta) \left\{ \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc \right\} - \frac{\gamma}{k(\theta)^{\alpha}} \frac{dd(\theta)}{d\theta} \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) dc + \omega(\theta) \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc
\]

\[
= \frac{\gamma}{k(\theta)^{\alpha}} \frac{dd(\theta)}{d\theta} \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) dc \left[ 1 - \omega(\theta) \frac{\int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \frac{d\beta(\theta)}{d\beta} dc}{\int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc + \omega(\theta) \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc} \right]
\]

\[
+ \omega(\theta) \left\{ \frac{\int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc \left[ \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc \right] \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) dc + \omega(\theta) \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc} \right\}
\]

\[
+ \gamma \omega(\theta) \left\{ \frac{\int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc \left[ \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc \right] \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) dc - \omega(\theta) \int_{\frac{\beta(\theta)}{R(\theta)}}^{\frac{d(\theta)}{R(\theta)}} h(e) \omega(\theta) dc \right\}
\]
\[
\begin{align*}
    &= \frac{\gamma}{k(\theta)^\alpha} \frac{dd(\theta)}{d\theta} \int_0^{\omega(\theta)} h(\epsilon)d\epsilon \left[ 1 - \frac{\omega(\theta)}{E_\epsilon[\epsilon \geq \omega(\theta)]} + \omega(\theta) \frac{\int_0^\infty \omega(\theta) h(\epsilon)d\epsilon}{E_\epsilon[\epsilon \geq \omega(\theta)]} \right] \\
    &\quad - \gamma \omega(\theta) \int_0^{\omega(\theta)} h(\epsilon)d\epsilon \left[ 1 - \frac{E_\epsilon[\epsilon \geq \omega(\theta)]}{E_\epsilon[\epsilon \geq \omega(\theta)]} \right]
\end{align*}
\]

For \( \omega(\theta) \leq \epsilon_L \), the firm issues fully collateralized debt,

\[
\frac{k(\theta) - E}{R^{-1}\theta k(\theta)^\alpha} = \omega(\theta).
\]

Assume \( \epsilon \) follows the uniform distribution \( U[\epsilon_L, \epsilon_H] \). In order to simplify the exposition, assume

\[
\gamma < \frac{\epsilon_L}{\epsilon_H - \epsilon_L}. \quad (22)
\]

Then, for \( \omega(\theta) \geq \epsilon_L \),

\[
\begin{align*}
    &= \frac{\gamma}{k(\theta)^\alpha} \frac{dd(\theta)}{d\theta} \int_0^{\omega(\theta)} h(\epsilon)d\epsilon \left[ 1 - \frac{\omega(\theta)}{E_\epsilon[\epsilon \geq \omega(\theta)]} + \omega(\theta) \frac{\int_0^\infty \omega(\theta) h(\epsilon)d\epsilon}{E_\epsilon[\epsilon \geq \omega(\theta)]} \right] \\
    &\quad - \gamma \omega(\theta) \int_0^{\omega(\theta)} h(\epsilon)d\epsilon \left[ 1 - \frac{E_\epsilon[\epsilon \geq \omega(\theta)]}{E_\epsilon[\epsilon \geq \omega(\theta)]} \right]
\end{align*}
\]

\[
\begin{align*}
    &\quad = \frac{\gamma}{k(\theta)^\alpha} \frac{dd(\theta)}{d\theta} P[\epsilon \leq \omega(\theta)] \left[ 1 - \frac{\omega(\theta)}{E_\epsilon[\epsilon \geq \omega(\theta)]} \right] + \omega(\theta) \frac{\frac{\epsilon_L}{\epsilon_H - \epsilon_L} \omega(\theta)}{E_\epsilon[\epsilon \geq \omega(\theta)]} \\
    &\quad - \gamma \omega(\theta) P[\epsilon \leq \omega(\theta)] \left[ 1 - \frac{E_\epsilon[\epsilon \leq \omega(\theta)]}{E_\epsilon[\epsilon \geq \omega(\theta)]} \right]
\end{align*}
\]

\[
\begin{align*}
    &\quad = \frac{\gamma}{k(\theta)^\alpha} \frac{dd(\theta)}{d\theta} \omega(\theta) - \epsilon_L \left[ 1 - \frac{\omega(\theta)}{\epsilon_H - \epsilon_L} \right] + \omega(\theta) \frac{\epsilon_H + \epsilon_L}{\epsilon_H - \epsilon_L} \\
    &\quad - \gamma \omega(\theta) \left[ 1 - \frac{\omega(\theta)}{\epsilon_H - \epsilon_L} \right]
\end{align*}
\]

\[
\begin{align*}
    \frac{\gamma}{k(\theta)^\alpha} \frac{dd(\theta)}{d\theta} \omega(\theta) - \epsilon_L \left[ 1 - \frac{2\omega(\theta)}{\epsilon_H + \omega(\theta)} \right] + \omega(\theta) \frac{\epsilon_H + \epsilon_L}{\epsilon_H + \omega(\theta)} - \gamma \omega(\theta) \left( \omega(\theta) - \epsilon_L \right) \left[ 1 - \frac{\omega(\theta)}{\epsilon_H + \omega(\theta)} \right]
\end{align*}
\]

\[
\begin{align*}
    \frac{\gamma}{k(\theta)^\alpha} \frac{dd(\theta)}{d\theta} \omega(\theta) - \epsilon_L &\left( \epsilon_H - \epsilon_L \right) \left( \epsilon_H + \omega(\theta) \right) + \omega(\theta) \frac{\epsilon_H + \epsilon_L}{\epsilon_H + \omega(\theta)} - \gamma \omega(\theta) \left( \omega(\theta) - \epsilon_L \right) \left( \epsilon_H - \epsilon_L \right) \frac{\epsilon_H + \epsilon_L}{\epsilon_H + \omega(\theta)} \\
\end{align*}
\]

22This assumption on \( \gamma \) is not important. It prevents states of nature in which cash flows are zero when they would be strictly positive without debt holders’ control. It will simply allow to present the exposition with functions defined by a single branch instead of two.
\[
\begin{align*}
\omega_d \omega_d \frac{d\omega}{d\theta} &= \frac{\gamma}{k(\theta)\alpha} \left\{ \frac{d\theta}{d\theta} \frac{(\omega(\theta) - \epsilon_L)(\epsilon_H - \omega(\theta))}{(\epsilon_H - \epsilon_L)(\epsilon_H + \omega(\theta))} + \omega(\theta) \frac{\epsilon_H + \epsilon_L}{\epsilon_H + \omega(\theta)} - \gamma(\theta) \frac{\omega(\theta) - \epsilon_L}{\epsilon_H + \omega(\theta)} \right\}.
\end{align*}
\]

Investment policy is determined by

\[
R = \left( \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon + \int_{\epsilon_H - \epsilon_L}^{\omega(\theta)} h(\epsilon)(1 + \gamma) d\epsilon \right) \theta \alpha k(\theta)^{\alpha - 1}
\]

\[
= \left( \frac{\epsilon_H^2 - \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} + (1 + \gamma) \frac{\omega(\theta)^2 - \epsilon_L^2}{2(\epsilon_H - \epsilon_L)} \right) \theta \alpha k(\theta)^{\alpha - 1},
\]

\[
k(\theta) = \left[ \frac{R^{-1} \theta \alpha}{1 - \alpha} \left( \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right) \right] \frac{1}{1 + \pi},
\]

\[
\frac{dk}{d\theta} = \frac{1}{1 - \alpha} \left[ R^{-1} \theta \alpha \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right] \frac{1}{1 + \pi}.
\]

Therefore, \(\omega(\theta) = \frac{d\theta}{d\theta} k(\theta)^{\alpha} \) gives

\[
\frac{d\omega}{d\theta} = \frac{dd(\theta) k(\theta)^{\alpha} - d(\theta) \left( k(\theta)^{\alpha} + \theta \alpha k(\theta)^{\alpha - 1} \frac{dk}{d\theta} \right)}{(\theta k(\theta)^{\alpha})^2},
\]

\[
\frac{dd}{d\theta} = \frac{d\omega}{d\theta} k(\theta)^{\alpha} + \omega(\theta) \left[ k(\theta)^{\alpha} + \theta \alpha k(\theta)^{\alpha - 1} \frac{dk}{d\theta} \right],
\]

\[
\frac{dd(\theta)}{d\theta} = \frac{d\omega}{d\theta} k(\theta)^{\alpha} + \omega(\theta) k(\theta)^{\alpha - 1} \frac{1}{1 - \alpha} \left[ R^{-1} \theta \alpha \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right] \frac{1}{1 + \pi}.
\]

\[
\frac{dd(\theta)}{d\theta} = \frac{d\omega}{d\theta} \left\{ \theta k(\theta)^{\alpha} + \omega(\theta)^2 \theta \alpha k(\theta)^{\alpha - 1} \frac{1}{1 - \alpha} \left[ R^{-1} \theta \alpha \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right] \frac{1}{1 + \pi} \right\}.
\]

\[
\frac{dd(\theta)}{d\theta} = \omega(\theta) k(\theta)^{\alpha} + \omega(\theta) \theta \alpha k(\theta)^{\alpha - 1} \frac{1}{1 - \alpha} \left[ R^{-1} \theta \alpha \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right] \frac{1}{1 + \pi}.
\]

\[
= \frac{R^{-1} \theta \alpha \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)}}{2(\epsilon_H - \epsilon_L)}.
\]

\[
= \frac{R^{-1} \theta \alpha \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)}}{2(\epsilon_H - \epsilon_L)}.
\]
Defining

\[
\begin{align*}
 u_1(\theta, \omega(\theta)) &= \frac{k(\theta) - E}{R^{-1} k(\theta)^{\alpha}} - \omega(\theta) \frac{\epsilon_H + \epsilon_L}{\epsilon_H + \omega(\theta)} + \gamma \omega(\theta) \frac{\omega(\theta) - \epsilon_L}{\epsilon_H + \omega(\theta)}, \\
 u_2(\theta, \omega(\theta)) &= \frac{\gamma}{k(\theta)^{\alpha}} \frac{(\omega(\theta) - \epsilon_L)(\epsilon_H - \omega(\theta))}{(\epsilon_H - \omega(\theta)).}
\end{align*}
\]

\[
\begin{align*}
 u_3(\theta, \omega(\theta)) &= \omega(\theta) k(\theta)^{\alpha} + \omega(\theta) \theta k(\theta)^{\alpha-1} R^{-1} \theta \alpha k(\theta)^{\alpha-1} \left[ R^{-1} \theta \alpha k(\theta)^{\alpha-1} \left( \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right) \right]^{\frac{1}{1-\alpha}}.
\end{align*}
\]

\[
\begin{align*}
 u_4(\theta, \omega(\theta)) &= \frac{\gamma}{k(\theta)^{\alpha}} \frac{(\omega(\theta) - \epsilon_L)(\epsilon_H - \omega(\theta))}{(\epsilon_H - \omega(\theta)).}
\end{align*}
\]

the differential equation for \(\omega(\theta)\) is given by ²³

\[
\frac{d\omega(\theta)}{d\theta} = \frac{u_1(\theta, \omega(\theta)) - u_3(\theta, \omega(\theta))}{u_4(\theta, \omega(\theta))}.
\]

Budget constraint (1.40) gives ownership sold

\[
\begin{align*}
 s(\theta) &= k(\theta) - E - R^{-1} \theta k(\theta)^{\alpha} \left[ \int_{\omega(\theta)}^{\infty} h(\epsilon) \left[ \epsilon - \gamma (\omega(\theta) - \epsilon) \right] d\epsilon + \int_{\omega(\theta)}^{\infty} h(\epsilon) \omega(\theta) d\epsilon \right] \\
 &= \frac{k(\theta) - E - R^{-1} \theta k(\theta)^{\alpha} \left[ (1 + \gamma) \frac{\omega(\theta)^2 - \epsilon_L^2}{2(\epsilon_H - \epsilon_L)} - \gamma \omega(\theta) \frac{\omega(\theta) - \epsilon_L}{\epsilon_H - \epsilon_L} + \omega(\theta) \frac{\epsilon_H - \omega(\theta)}{\epsilon_H - \epsilon_L} \right]}{R^{-1} \theta k(\theta)^{\alpha} \left( \frac{\epsilon_H^2 - \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right) - R^{-1} \theta k(\theta)^{\alpha} \omega(\theta) \frac{\epsilon_H - \omega(\theta)}{\epsilon_H - \epsilon_L}}.
\end{align*}
\]

The market value of equity becomes

\[
\begin{align*}
 MV_{t+} &= R^{-1} \theta k(\theta)^{\alpha} \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon - k(\theta) + E \\
 &\quad + (1 + \gamma) R^{-1} \theta k(\theta)^{\alpha} \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon - \gamma R^{-1} d(\theta) \int_{\omega(\theta)}^{\infty} h(\epsilon) d\epsilon \\
 &= R^{-1} \theta k(\theta)^{\alpha} \left( \frac{\epsilon_H^2 - \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right) - k(\theta) + E \\
 &\quad + (1 + \gamma) R^{-1} \theta k(\theta)^{\alpha} \omega(\theta)^2 - \epsilon_L \frac{\epsilon_H - \omega(\theta)}{\epsilon_H - \epsilon_L} - \gamma R^{-1} d(\theta) \omega(\theta) - \epsilon_L.
\end{align*}
\]

Moreover, \(S_{t+} - P_{t+} = (1 - s) MV_{t+}\).

²³Recall that \(k(\theta, \omega(\theta)) = \left[ R^{-1} \theta \alpha \left( \frac{\epsilon_H^2 - (1 + \gamma) \epsilon_L^2 + \gamma \omega(\theta)^2}{2(\epsilon_H - \epsilon_L)} \right) \right]^{\frac{1}{1-\alpha}}\).
1.10 References


Baron, David, and Adam Meirowitz; 2004; “Fully-revealing equilibria of multiple-sender signaling and screening models ”; Working paper.


54


Maskin, Eric, and Jean Tirole; 1992; “The Principal-Agent Relationship with an Informed Principal, II: Common Values”; *Econometrica* 60, 1-42.


Myers, Stewart, and Nicholas S. Majluf; 1984; “Corporate Financing and Investment Decisions when firms have information that investors do not have”; *Journal of Financial Economics* 13, 187-221.


Chapter 2

Libor Interest Rates

2.1 Introduction

This paper presents a dynamic general equilibrium model that disentangles interbank interest rates from interest rates on government bonds.

Firstly, I explicitly identify the externality in the financial sector that explains the use of cash in the portfolios of financial intermediaries. Cash provides insurance for the interest rate used to finance loans. However, when a financial intermediary decides his level of insurance, he ignores the negative effect of his decision on the remaining financial sector. It is shown that the positive spread between interbank rates and treasury rates is the leading variable that drives the money market. The spread has to increase when financial intermediaries desire less cash than the cash available in the economy. Similarly,
the spread decreases when there is less cash in the economy than desired by financial intermediaries.

Secondly, general equilibrium implications are analyzed. The externality creates an ex-ante distortion in the financial sector. As a result, borrowing interest rates have to increase relative to lending interest rates. The analysis is developed in a model where there are private investors, financial intermediaries and a government. The model is simple enough to allow close-form solutions. I identify the role played by interbank rates that prevents them from reflecting a pricing kernel at which the real economy is willing to transfer cash flows over time, thereby compromising the use of Libor interest rates in valuation. Additionally, an increase in expected inflation exacerbates the externality identified in this paper. As a consequence, expected inflation increases the equilibrium costs of financial intermediation. The result is an increase in real interest rates and an economic slowdown.

Why should we develop equilibrium models allowing the fact that interbank interest rates are different from interest rates on government bonds in financial markets?

There are two strong motives. Firstly, the use of Libor\(^1\) interest rates in financial markets is predominant. Libor interest rates are used as reference points for the construction of some of the most widely used financial contracts such as loans or swaps\(^2\).

\(^1\)“London Interbank Offered Rate”. 
\(^2\)There is a practical reason to use Libor rates, instead of the popular treasury rates, as reference points. Although prices on treasuries are available daily, they don’t typically come in the form of zero
Additionally, Libor interest rates are explicitly used in standard finance textbooks as discount rates to obtain present values of future cash flows\(^3\), and are also implicitly used in financial markets as discount rates when loans are made whose payments are contingent on future Libor interest rates. Therefore, we need to understand what drives interbank interest rates and whether or not they reflect rates at which the real economy is willing to transfer cash flows over time.

Secondly, monetary policy operates by targeting the Federal funds rate, which is an overnight interbank rate. Since the positive spread between interbank interest rates and treasury rates is always changing in financial markets (figure 2.11 will provide empirical evidence), the importance of having equilibrium models with endogeneous spreads between these two interest rates lies at the heart of an understanding of monetary policy.

In this context, this paper gives two main contributions. The first contribution is to identify the externality among financial intermediaries that explains the demand for cash in their portfolios. I show how the ex-post heterogeneity of financial intermediaries creates the externality.

By looking at the intertemporal resource constraint of the economy, economic theory was able to identify the nominal interest rate as the opportunity cost to hold cash be-

\(^3\)The analytic convenience of using Libor rates lies in the fact that, when cash flows are discounted at Libor interest rates, a bond with a variable interest rate pegged at future Libor interest rates should be sold at par.
cause the government, by issuing cash instead of a bond, saves the nominal interest rate (Friedman (1968)). To justify a demand, however, economic literature has exogenously imposed a value on cash that is not present in the remaining financial assets. Three standard monetary economies have been analyzed extensively: a cash-credit model, a money-in-the-utility-function model, and a shopping-time model (see, for example, Lucas and Stokey (1983), Chari, Christiano and Kehoe (1996) or Diamond and Rajan (2003)).

In all these frameworks, the money demand comes from equating the nominal interest rate, seen as the opportunity cost of holding cash, to the marginal gain from holding cash. The benefit to hold cash is the opportunity to consume in a cash-in-advance framework, marginal utility in a money-in-the-utility-function model, and the time saved in transactions in a shopping-time model. In money-in-the-utility-function and shopping-time models, the benefit is exogenously specified. Similarly, in a cash-in-advance model, since it is difficult to view a large percentage of transactions forcing buyers to make payments in cash and to hold the cash in advance, we end up looking at the cash-in-advance model as a significant reduced form model. The present paper explicitly identifies the incentives to hold cash in a general equilibrium economy, even in a framework where all transactions can be done without cash and double coincidence of wants is not an issue (as opposed to the search-theoretical literature pioneered by Kiyotaki and Wright (1989)).

When the nominal interest rate is identified as the cost to hold cash, it is both

---

4Other than transactions where there are incentives to keep the transaction hidden from tax authorities.
from the perspective of the economy as a whole as well as from the perspective of an individual agent. The tension, associated with the lack of nominal interest on cash, comes from the fact that there is no benefit for the economy as a whole to hold the cash. As a consequence, the benefit to hold cash cannot be understood solely from the perspective of a single agent in the economy. It can only be understood when the all general equilibrium is at work. Only after recognizing this, can the externality that creates the demand for cash be identified. This paper identifies the role of interbank rates in providing the benefit to hold cash. It shows that it is the spread between interbank rates and treasury rates, which is positive since otherwise an arbitrage exists, that drives the money market. That’s the variable that financial markets should look at to understand what is happening in the money market.

The argument can be illustrated with a simple example. Consider an economy with two financial intermediaries, $A$ and $B$, and suppose the demand for credit is $200$ dollars, as in figure 2.1. At the beginning of the day, each financial intermediary makes a loan of $100$ dollars and decides his level of cash. Borrowers are then allowed to issue checks or to make electronic transfers, from the account that the financial intermediary opened to him. The $200$ dollars are spent in the economy and subsequently deposited within the financial sector in the same day, since otherwise the owners would lose the nominal interest rate on deposits. However, the clearing across financial intermediaries is made at the end of the day, near of after the market be close. Only at the end of the day will financial intermediaries know where deposits were made. Suppose two equally likely
states of nature. In the first, $A$ gets a deposit of $150 and $B$ a deposit of $50, and in the second state $A$ gets a deposit of $50 and $B$ of $150. In the first state, $B$ has a debt of $50 dollars to $A$ associated with the financial assets underwritten by $B$ that end up being deposited at $A$. $B$ can settle the debt in cash or borrow at the overnight interbank rate.\footnote{Note that payments made through account transfers do not exist between two financial intermediaries.} Similarly, $A$ has to settle a debt of $50 dollars in the second state. Let $M_j \leq 50$, $R$, $i$ and $\kappa$ denote the cash held by intermediary $j$, the nominal interest rate on loans, on deposits and overnight interbank interest rate, respectively. The expected profit for $A$ is

$$
\pi_A(M_A|M_B) = \frac{1}{2}[M_A + 100(1 + R) + M_B + (50 - M_B)(1 + \kappa) - M_A(1 + i) - 150(1 + i)]
+ \frac{1}{2}[100(1 + R) - M_A(1 + i) - 50(1 + i) - (50 - M_A)(1 + \kappa)]
= 100(R - i) - iM_A - \frac{\kappa}{2}(50 - M_A) + \frac{\kappa}{2}(50 - M_B).
$$

Figure 2.1: A simple example
Therefore, as long as $\kappa \geq 2i$, $A$ wants to hold $50$ dollars in cash. The marginal cost is $i$, which is the cost of capital for this financial intermediary, but the marginal gain is $\kappa/2$, which is the probability of using that additional unit of cash, $1/2$, times the overnight interbank rate that he saves by financing his loans with cash instead of at the overnight interbank rate. Similarly, $B$ wants to hold $50$ dollars in cash. For the financial sector as a whole, the cost to hold $1$ dollar in cash is the nominal interest rate on deposits times the amount of cash, since cash is obtained by issuing a deposit, or equity, that asks for the nominal interest rate $i$. There is no net benefit to hold cash. Indeed,

$$\pi_A(M_A|M_B) + \pi_B(M_B|M_A) = 100(R - i) - iM_A - \frac{\kappa}{2}(50 - M_A) + \frac{\kappa}{2}(50 - M_B) + 100(R - i) - iM_B - \frac{\kappa}{2}(50 - M_B) + \frac{\kappa}{2}(50 - M_A) = 200(R - i) - i(M_A + M_B)$$

However, there is an externality in the decision of each financial intermediary. By increasing the level of cash, in order to avoid financing his loans at the overnight interbank interest rate instead of at the smaller interest rate on deposits in the second state, $A$ ignores the negative effect on $B$. $B$ is more likely to be paid in cash instead of at the overnight interbank interest rate. Similarly, $B$ ignores the negative effect on $A$ in the first state. This externality, associated with the ex-post heterogeneity of financial intermediaries, creates the demand for cash and forces $R > i$ in a zero expected profits equilibrium. If it were possible for these two financial intermediaries to commit to not use cash, or were simply not allowed to use cash, then $A$ would make a $50$ dollar loan to $B$ at the interbank interest rate in the first state and would borrow $50$ dollars
from $B$ at the interbank interest rate in the second state. Independent of the level of interbank rates, there wouldn’t be any ex-ante distortion and in equilibrium we would have $R = i$. However, due to the externality, financial intermediaries want to hold cash in their portfolios, even though they operate in an economy where all transactions could be executed without cash.

In the model to be presented, there is a continuous distribution of deposits. Thus, from the perspective of a financial intermediary, the probability of using an additional unit of cash to finance his loans decreases with the amount of cash already available in his portfolio. Therefore, a financial intermediary doesn’t want to hold cash in his portfolio to avoid financing each of his loans at the interbank rate under all states of nature. After a certain level of cash, the probability of using an additional unit of cash is small enough that does not compensate the nominal interest rate on deposits.

Moreover, the model allows that, at the end of the day, when a financial intermediary has to settle his debt with the remaining financial intermediaries, he can use government bonds in addition to cash. Because the debt is dollar denominated, cash is a clean way to settle it. A financial intermediary can decide to settle his debt, associated with securities underwritten by him, by paying it with financial assets that allow the owner to obtain cash flows in the future. However, since the outside option is to borrow at the overnight interbank interest rate, the debt holder will only accept treasuries if those cash flows are discounted at the interbank rate. Although assets such as government bonds pay positive nominal interest ex-ante, they are not perfect substitutes for cash.
There are three important assumptions for the externality to exist. The first assumption is the existence of more than one financial intermediary, since otherwise he would completely integrate the externality. The second is that, when a given financial intermediary makes a loan, he doesn’t know in advance the interest rate that he will end up using to finance the loan.\(^6\) Cash provides insurance for the interest rate used to finance the loan. The third is that, when a financial intermediary collects cash payments to liquidate debts from the remaining financial sector at the end of the day, he cannot instantaneously reinvest the cash. Note that, even in a fictional world, where markets are open 24 hours, the externality is still there as long as it takes the financial intermediary 1 minute to reinvest the cash and interest rates are compounded on a minute basis.

Finally, the second contribution of this paper is, by incorporating the above argument into general equilibrium, to provide a framework to understand the simultaneous behavior of interest rates on loans, deposits, government bonds and interbank interest rates. The dependence of the equilibrium on the degree of contemporaneous uncertainty on deposits

\(^6\)This assumption will be justified in section 2.2.3. Previous literature (Diamond and Rajan 2001) has identified demand deposits as a contract that allows financial intermediaries to finance their loans while committing their human capital to the service of investors. Additionally, as illustrated in the previous example, there is a physical impossibility in assuming that loans and deposits are made at exactly the same instant in time. When a financial intermediary makes a loan, he allows the borrower to make payments, by issuing checks or making account transfers, from the account that the financial intermediary opened to him. Those financial assets can only be deposited at the financial sector after the borrower has actually used them in purchases.
and the dependence on inflation, due to the interaction with the externality identified in this paper, is analyzed.

The paper is organized as follows. In section 2, I describe the model. Sections 3 and 4 solve and characterize the equilibrium, respectively. Section 5 concludes.

### 2.2 Model

Consider a discrete-time economy, $t \geq 0$, with a single homogeneous good. There is a continuum of measure one of agents that have the same preferences, given by

$$E_t \left( \sum_{l \geq t} \beta^{l-t} c_l \right),$$

(2.1)

where $c_t$ is consumption at date $t$, $\beta \in (0, 1)$ is a discount factor for future utility, and $E_t$ is expectations formed at date $t$.\(^7\)

Each period, the endowment of each agent is given by $\omega$ goods and, with constant probability $p$, each agent has access to a constant-returns-to-scale production technology:

$$y_{t+1} = \alpha x_t,$$

(2.2)

\(^7\)The use of a infinite horizon economy, instead of a more parsimonious two-period economy, is justified by the fact that, in a two period economy, there wouldn’t be any investment decisions in the last period, including cash holdings decisions. Since there wouldn’t be any demand for cash, the price level in the last period wouldn’t be determined. This would make the nominal interest rates from the first to the second period undetermined. Therefore, no demand for cash would be obtained even in the first period.
where \( x_t \) is investment of goods at date \( t \) and \( y_{t+1} \) is output of goods at date \( t+1 \). Agents with access to such technology can invest their own wealth and also to obtain loans from financial intermediaries. When an agent obtains credit, the financial intermediary allows him to issue checks or to make account transfers, from the account that the financial intermediary opened to him. The maximum amount that each financial intermediary is able to collect for each unit invested by an agent with access to the production technology (2.2) is \( \theta \alpha \). \(^8\)

The economy is populated by three types of agents: private agents, financial intermediaries and a government.

### 2.2.1 Private Agents

Let \( P_t \) be the nominal price of a unit of good at time \( t \), \( \tau \) the tax rate on produced goods, \( T_t \) nominal lump-sum transfers from the government, \((1 + i_t)\) and \((1 + R_t)\) the noncontingent nominal interest rate on deposits in the financial sector and loans from \( t \) to \( t+1 \), respectively.

Taking prices as given, the problem of a private agent is to choose consumption \((c_t \equiv \frac{C_t}{P_t})\), real deposits \((d_t \equiv \frac{D_t}{P_t})\), real private equity \((h_t \equiv \frac{H_t}{P_t})\) and real debt \((z_t \equiv \frac{Z_t}{P_t})\),

---

\(^8\)The borrowing constraint can be justified by the fact that, when an entrepreneur needs to raise funds from an investor, he cannot commit not to withdraw his human capital from the project, as in Hart and Moore (1994).
to maximize expected utility (2.1): \(^9\)

\[
\max E_0 \left( \sum_{t \geq 0} \beta^t c_t \right),
\]  

s.t.

\[
P_t c_t + P_t d_t + P_t h_t = \begin{cases} 
  P_t \omega + P_{t-1} d_{t-1} (1 + i_{t-1}) + T_t & \text{if nonproductive at } t - 1 \\
  P_t \omega + P_{t-1} d_{t-1} (1 + i_{t-1}) + T_t + (1 - \tau) P_t \alpha (h_{t-1} + z_{t-1}) & \text{if productive at } t - 1,
\end{cases}
\]  

\[
\frac{(1 + R_t) P_t}{P_{t+1}} z_t \leq \theta \alpha (h_t + z_t),
\]  

\[c_t, d_t, h_t, z_t \geq 0.\]  

In order to obtain the real return on equity for an agent with access to technology (2.2), note that the borrowing constraint gives\(^{10}\)

\[
z_t \leq \frac{\theta \alpha}{1 + r_{t+1} - \theta \alpha} h_t.
\]  

Thus, it is the highest \(1 + r_{t+1}\) (that is, lowest \(P_{t+1}\)) that restricts the amount of debt.

Defining \(1 + \bar{r}_t \equiv \sup \{1 + r_{t+1} : P_{t+1} \}\), the real return on equity \((1 + e_{t+1})\) is then given

\(^9\)In equilibrium, private agents will optimally choose not to hold cash from one period to the next since it doesn’t pay any nominal interest. Financial intermediaries will optimally hold the cash.

\(^{10}\)The nominal return on loans \(1 + R_t\) charged by financial intermediaries is known at time \(t\) but the real return on loans is not necessarily known since it depends on \(P_{t+1}\).
\[(1 + e_{t+1}) = (1 - \tau)\alpha(1 + \frac{\theta\alpha}{1 + \bar{r}_t - \theta\alpha}) - (1 + r_{t+1})\frac{\theta\alpha}{1 + \bar{r}_t - \theta\alpha} \]
\[= \frac{(1 - \tau)\alpha(1 + \bar{r}_t) - (1 + r_{t+1})\theta\alpha}{1 + \bar{r}_t - \theta\alpha}. \quad (2.8)\]

Letting \(v_t \geq 1\) denote the value of an unit of private equity at \(t\) for an agent with access to technology (2.2), and the real return on deposits by \(1 + s_{t+1} \equiv \frac{1 + \bar{r}_t}{P_{t+1}/P_t}\), the problems for productive and nonproductive agents give us, respectively,

\[1 = E_t[\beta(1 + s_{t+1})(pv_{t+1} + (1 - p))], \quad (2.9)\]
\[v_t = E_t[\beta(1 + e_{t+1})(pv_{t+1} + (1 - p))]. \quad (2.10)\]

A nonproductive agent can consume the good, or make a deposit, allowing future investment or consumption. Thus, the real return on deposits has to satisfy equation (2.9). On the other hand, when an agent has access to the production technology (2.2), he can invest at the return on equity, which includes the leverage effect of debt, and either reinvest or consume in the future, as in (2.10). Given risk neutrality and debt constraint (2.7), a productive agent invests all his wealth and issues debt as in (2.7), as long as

\[\frac{\partial(1 + e_{t+1})}{\partial\bar{r}_t} = \frac{-\alpha(1 + \bar{r}_t)\alpha(1 + \bar{r}_t - \theta\alpha)}{(1 + \bar{r}_t - \theta\alpha)^2} = \frac{\alpha(1 + \bar{r}_t)\alpha(1 + \bar{r}_t - \theta\alpha)}{(1 + \bar{r}_t - \theta\alpha)^2} \left[\alpha(1 - \tau) - \alpha(1 + r_{t+1})\right].\]

Note that there is no tax advantage from issuing debt. This allows us to abstract from fiscal policy considerations, although a similar analysis applies under the presence of tax shields from issuing debt.

---

\[^{11}\text{Implicitly, I assume that } \theta\alpha < 1 + \bar{r}_t \text{ to prevent issuing infinite amounts of debt, and } E_t[1 + r_{t+1}] < (1 - \tau)\alpha \text{ to allow some gain from issuing debt. Clearly, the return on equity is decreasing in } \tau \text{ and the real interest rates on loans, and increasing in } \alpha \text{ and } \theta \text{ since } \frac{\partial(1 + e_{t+1})}{\partial\bar{r}_t} = \frac{-\alpha(1 + \bar{r}_t)\alpha(1 + \bar{r}_t - \theta\alpha)}{(1 + \bar{r}_t - \theta\alpha)^2} = \frac{\alpha(1 + \bar{r}_t)\alpha(1 + \bar{r}_t - \theta\alpha)}{(1 + \bar{r}_t - \theta\alpha)^2} \left[\alpha(1 - \tau) - \alpha(1 + r_{t+1})\right].\]
Nonproductive agents are indifferent between consuming or investing.

In order to elucidate further equations (2.9) and (2.10), assume that interest rates are constant. Then equations (2.9) and (2.10) become

\[ 1 = \beta (1 + s) [pv + (1 - p)], \quad (2.11) \]
\[ v = \beta \frac{(1 - \tau - \theta)\alpha (1 + r)}{1 + r - \theta\alpha} [pv + (1 - p)]. \quad (2.12) \]

As \( v \) increases, the real interest rate on deposits decreases since a nonproductive agent is more willing to make a deposit because the value of an unit of private equity in the future is higher, as in figure 2.2. Also, when the real interest rate on debt increases, the value of an unit of equity \( v \) decreases since the level of debt that a productive agent is able to issue to exploit his investment opportunities is restricted. \(^\text{13}\)

As figure 2.2 illustrates, the spread between the real interest rate on loans \((1 + r)\) and the real interest rate on deposits \((1 + s)\) increases as the unit value of private equity \(v\) decreases. \(^\text{14}\) A change in the unit value of equity \(v\) only affects the real interest rate\(^\text{12}\) Note that Modigliani and Miller’s analysis does not apply because cash flows are not independent of financial decisions.

\(^\text{13}\)When the real interest rate is constant, \((1 + e) = \frac{(1 - \tau - \theta)\alpha (1 + r)}{1 + r - \theta\alpha}\). Thus, \(\frac{\partial (1 + e)}{\partial (1 + r)} = -\frac{\theta\alpha (1 - \tau - \theta)\alpha}{(1 + r - \theta\alpha)^2} < 0\).

An increase in the interest rate on loans decreases the return on equity since it reduces the amount of resources that can be invested issuing debt.

\(^\text{14}\)From (2.11), \((1 + s) = \frac{\beta^{-1}}{1 - pv(1 - p)}\) and \(\frac{\partial (1 + s)}{\partial v} = \frac{-p\beta^{-1}}{(1 - pv(1 - p))^2} = -p\beta(1 + s)^2\). From (2.12), \((1 + r) = \frac{\theta\alpha}{1 - p\beta(1 - \tau - \theta)\alpha - (1 - p)\beta(1 - \tau - \theta)\alpha} \) and \(\frac{\partial (1 + r)}{\partial v} = \frac{-\theta\alpha(1 - p)\beta(1 - \tau - \theta)\alpha}{(1 - p)(1 - \tau - \theta)\alpha - (1 - p)\beta(1 - \tau - \theta)\alpha} \). Thus, positive spreads increase with the interest rate on loans as long as \((1 - p)\frac{1 - \tau - \theta}{\theta\alpha^2} > p\). Letting \(\gamma\) denote the marginal real cost from making a loan, the parameter condition \((1 - p)(1 - \gamma)\frac{1 - \tau - \theta}{\theta\alpha^2} > p\) is
on deposits by its effect on expectations of nonproductive agents about the future value of private equity in the event that they become productive. Thus, the real interest rate on deposits reacts much less to changes in the value of equity than real interest rates on loans.

2.2.2 Government

The government constraint is given by

\[ M_t - M_{t-1} + P_t^B B_t - B_{t-1} = T_t + (G_t - \tau Y_t), \quad t \geq 0. \tag{2.13} \]

assumed in the remaining of this paper, implying that the effective real spread \((1 - \gamma)(1 + r) - (1 + s)\) increases with the real interest rate on loans. As \(p \downarrow 0\), the parameter condition is satisfied since the effect on the interest rate on deposits is negligible. Also, as the borrowing constraint becomes more severe, \(\theta \downarrow 0\), the parameter condition becomes satisfied because the value of equity becomes hardly affected by the interest rate on loans, since the use of debt becomes negligible.
The government taxes sales of produced goods $Y_t$ at the rate $\tau$. In order to abstract from fiscal policy considerations, I assume that government expenses are equal to $\tau$ times the amount of produced goods, $G_t = \tau Y_t$. A change in the monetary base $M_t$ is then injected into the economy through nominal lump-sum transfers $T_t$ to private agents and open market operations $(B_{t-1} - P_t^B B_t)$, where $B_t$ is the notational value of debt issued at time $t$ and due at $t + 1$, and $P_t^B$ his price at time $t$.

In order to solve for the equilibrium, the government policy has to be specified. I will assume that the government is committed, and has such a commitment technology, to maintain a constant inflation, $\Pi_t = \Pi$, $t \geq 0$, and a constant real value of debt, $\frac{B_t}{P_{t+1}} = b$, $t \geq 0$. 


table

Balance Sheet

\[
\begin{array}{c|c|c}
L_t & D_t^B & D_t^M \\
\hline
M_t & P_t^B B_t & D_t^R \\
\end{array}
\]

Figure 2.3: Balance Sheet

2.2.3 Financial Intermediaries

Financial intermediaries maximize their expected profits and free entry forces zero expected profits in equilibrium.

Each financial intermediary makes nominal loans $L_t$ at date $t$ which mature and are paid off at date $t + 1$. The noncontingent nominal interest rate on these loans is $R_t$. Similar to Diamond and Rajan (2003b), loans are so dependent on the financial intermediary’s specific skills for collection (that is, they are so illiquid) that the financial

\footnote{The fact that there are taxes and government expenses doesn’t play an important role in the main analysis of this paper. The paper includes them since they don’t ask for any additional effort in solving and understanding the model and provide an understanding to how the tax rate affects the equilibrium allocation, given the close form solution obtained in this paper. The same approach applies to $\gamma$.}

\footnote{To solve for the equilibrium, an approach similar to the one developed in this paper can be applied under alternative specifications of the government policy. The assumption that the government is committed to maintain a constant inflation is chosen because it completely illustrates the results highlighted in this paper and simultaneously simplifies the exposition because once ex-ante nominal interest rates are determined, the ex-post real interest rates are known.}

73
intermediary is unable to sell them.\textsuperscript{17} For each unit collected, the financial intermediary pays a marginal real cost of \( \gamma \).\textsuperscript{18}

These loans can be financed by deposits \( D^R_t \) with noncontingent nominal interest rate \( i_t \). Deposits are satisfied on demand. As explained by Diamond and Rajan (2001), demand deposits allow financial intermediaries to finance their loans while committing their human capital to the service of investors\textsuperscript{19}.

In order to prevent that a single financial intermediary be able to satisfy the all demand for loans or to collect the all supply of deposits, an upper bound on the amount of loans that each financial intermediary can make and an upper bound on the amount of deposits that each financial intermediary can collect are assumed\textsuperscript{20}.

In addition to loans, financial intermediaries can hold tradable securities in their

\textsuperscript{17}This assumption does not play a role in the analysis. It is done solely to simplify the exposition.

\textsuperscript{18}\( \gamma \) represents a cost in making the loan and enforcing the debt repayment.

\textsuperscript{19}To quickly recall the main point, the problem is that having funding from investors promising a certain repayment, a financial intermediary can threaten to hold back his collection skills unless investors reduce the required repayment. The way to finance lending while committing his human capital is to issue uninsured demand deposits, which, by their “first come, first served” component, cannot be negotiated down.

\textsuperscript{20}The exact upper bounds don’t have to be specified. Although the number of financial intermediaries in the market will depend on the exact upper bounds, their role is solely to prevent that one single financial intermediary be able to satisfy the all market for loans or deposits. As already mentioned in the introduction, an important assumption for the externality identified in this paper to hold is the existence of more than one financial intermediary, to guarantee that the externality is not fully integrated.
portfolios, as illustrated in figure 2.3. Given the simplifying assumption that loans are illiquid, treasuries and cash are the tradable securities that financial intermediaries can hold in their portfolios.

The timing of events in period $t$ is described in figure 2.4.

![Figure 2.4: Timing of Events in Financial Sector](image)
Financial intermediaries announcing the highest nominal interest rate on deposits then simultaneously choose their levels of cash deposits, $D^M_t$, and treasury deposits, $D^B_t$, to hold in their portfolios. Given the announced interest rates on loans, productive agents start looking for credit, among the financial intermediaries announcing the smallest interest rate on loans, as explained in section 2.2.1. Loans are then equally distributed among the financial intermediaries announcing the smallest interest rate on loans. When a private agent obtains credit, he is allowed to make payments, by issuing checks or by making electronic account transfers, from the account that a financial intermediary opened to him. Productive agents then start using their credit, buying goods to exploit their investment opportunities.

When nonproductive agents sell their goods, they end up with checks in their hands or credits in accounts that they have with specific financial intermediaries. These financial assets are necessarily deposited within the financial sector in that same day, among the financial intermediaries announcing the highest interest rate on deposits, to avoid loosing nominal interest. However, financial assets underwritten by a specific financial intermediary are not necessarily deposited at that same financial intermediary, but typically at some other financial intermediary. The fact that the interest rate used by financial intermediaries to finance their loans is not known in advance is thus justified by this feature of the financial technology.\textsuperscript{22} For the financial sector as a whole, the value

\textsuperscript{22}Additionally, as previously mentioned, deposits on demand have been identified in the literature as a way to solve enforcement problems between financial intermediaries and depositors.
of deposits made through checks or electronic transfers, $D_t^R$, is equal to the amount of credit conceded, $L_t$. But a specific financial intermediary only knows that the expectation of his deposits $D_t^R$ is the total value of deposits in the financial sector divided by the number of financial intermediaries,\footnote{Which will be equal to the amount of loans he made since loans are equally distributed.} not his idiosyncratic realization. For simplicity, assume that for a specific financial intermediary announcing the highest interest rate on deposits, his level of deposits divided by the above expected value, $\frac{D_t^R}{E[D_t^R]}$, has a uniform distribution with support $[1-\delta, 1+\delta]$, $\delta \leq 1$.

Only at the end of the day, near or after the market being close, can the clearing across financial institutions be finished. When a financial intermediary underwrites more credit than the level of deposits that he ends up collecting, he needs to settle his debt. He can either borrow at the overnight interbank rate or pay his debt using tradable securities, by paying in cash or, if the owner of the debt allows him, paying in treasuries.\footnote{Note that payments made through account transfers do not exist between two financial intermediaries. Additionally, recall that private loans are illiquid due to their dependence on the financial intermediary’s specific skills for collection intermediary, although this assumption is only to simplify the exposition.} It is assumed that financial intermediaries take the interbank interest rate as given. Similarly, if a financial intermediary collects more deposits than the amount of credit underwritten, he will present those financial assets behind the deposits to the financial intermediaries that underwrote them and ask for payment. Some financial claims will be paid in cash, treasuries or by lending at the interbank rate, depending on the specific

23 Which will be equal to the amount of loans he made since loans are equally distributed.
24 Note that payments made through account transfers do not exist between two financial intermediaries. Additionally, recall that private loans are illiquid due to their dependence on the financial intermediary’s specific skills for collection intermediary, although this assumption is only to simplify the exposition.
financial intermediary to whom the financial claim behind the deposit applies.\footnote{The incentive to choose to have cash in his portfolio in period \( t \) comes from the random realization of deposits and the subsequent clearing across financial institutions at the end of period \( t \). The random realization of deposits and clearing next period does not introduce any gain from holding cash from \( t \) to \( t + 1 \), since that cash can be obtained next period, before the realization of deposits and the clearing across financial intermediaries in period \( t + 1 \). By waiting for the next period, the financial intermediary saves the nominal interest rate on deposits. Therefore, we can restrict our attention to portfolio decisions by financial intermediaries with an horizon of only one period since there are no precautionary motives to horde cash.}

The following definition of a strategy for a financial intermediary will ignore decisions in the last subgame associated with figure 2.4, due to their simplicity. When a financial intermediary has to settle his debt, he has a strict incentive to use the available cash in his portfolio, since it doesn’t provide him any interest. When cash is no longer available, he can use treasuries, but since the outside option is to borrow at the interbank rate, the owner of the debt will only allow him to pay in treasuries if the associated cash flows are discounted at the interbank rate. Thus, when cash is no longer available, he is indifferent between paying in treasuries or borrowing at the interbank rate.\footnote{We can interpret some of the deposits as equity because the expected return is the same. Some financial intermediaries will end up having higher return on equity and some smaller, but it will not matter in the aggregate since everybody is risk neutral and the probability of being productive each period is the same for every private agent.}

\textbf{Definition 2.2.3.1} \textit{A strategy for a financial intermediary is defined as the announcement of his nominal interest rate on loans and deposits, and a level of cash and treasuries}
holdings contingent on previously announced interest rates by him and the remaining financial intermediaries.  

2.2.4 Definition of Equilibrium

Definition 2.2.4.1 An equilibrium is a sequence of prices \( \{P_t, P^B_t, R_t, i_t, \kappa_t\}_{t \geq 0} \) and allocations

\( \{c_t, x_t, M_t, B_t, D_t, L_t, N_t, \tau, T_t\}_{t \geq 0} \) such that

(i) Given prices, private agents maximize their utility (2.1) subject to budget constraints (2.4), (2.5) and (2.6);

(ii) Taking as given the interbank interest rate, the strategies of financial intermediaries for interest rate on loans, deposits, cash and treasuries holdings, constitute a symmetric subgame perfect equilibrium for the game associated with figure 2.4;

(iii) The government constraint (2.13) is satisfied. The government is committed to an inflation target \( \Pi \) and debt target \( b \);

(iv) The monetary, treasury, goods, deposits, loans and interbank markets clear:

- \( M^*_t = M^d_t \), where \( M^*_t \) is the exogenous supply of cash by the government and \( M^d_t \)

is the demand by financial intermediaries;

\footnote{The assumption of free entry will force zero expected profits and therefore dynamic contracts between financial intermediaries do not occur, which justifies the fact that the definition of equilibrium will be restricted to the horizon of only one period.}
• $B^s_t = B^d_t$, where $B^s_t$ is the exogenous supply of government bonds and $B^d_t$ is the demand by private agents and financial intermediaries;

• $P_t \omega + P_t \alpha x_{t-1} = P_t c_t + P_t g_t + P_t x_t + \gamma (1 + R_{t-1}) L_{t-1}$, where $g_t = \tau \alpha x_{t-1}$ and $x_t = \frac{X_t}{P_t}$ is real investment at time $t$;

• $D^s_t = D^d_t$, where $D^s_t$ is the supply of deposits by financial intermediaries and $D^d_t$ the demand by private agents;

• $L_t = Z_t$, where $L_t$ is the supply of loans by financial intermediaries and $Z_t$ the supply of debt by private agents;

• $N^s_t = N^d_t$, where $N^s_t$ is the supply of deposits between financial intermediaries and $N^d_t$ the demand of deposits between financial intermediaries.

2.3 Equilibrium

2.3.1 Portfolio Problem of a Financial Intermediary

Lemma 2.3.1 Symmetric subgame perfect equilibria of the game associated with figure 2.4 exist. Financial intermediaries have zero expected profits.

Proof. See Appendix A. ♦

In order to obtain the level of cash chosen by a given financial intermediary,

\[28\] For notational reasons, no superscript is used to identify a specific financial intermediary although in the remaining of this section all variables refer to a specific financial intermediary.
profit \( \pi_t = \pi_t(M_t, B_t | M'_t, B'_t) \) of a financial intermediary, at each subgame starting after interest rates being announced and entry decisions being made, is given by (2.14).\(^{29}\)

In states of nature where cash, treasuries and \( D_t^R \) are not enough to finance liabilities \( L_t \), loans will be financed by the available amount of deposits \( D_t^R \) that will earn \((1+i_t)\), all the available cash and treasuries, and the remaining \((L_t-M_t-\frac{B_t}{1+\kappa_t})-D_t^R\) at the interbank interest rate \( \kappa_t \) (see (a) in figure 2.5 and first term in expression (2.15)). For states where \( L_t - M_t - \frac{B_t}{1+\kappa_t} < D_t^R < L_t - M_t \), the difference \((L_t - D_t^R)\) is financed with all the available cash and a proportion \( w \) of treasuries such that \( wB_t = (1+\kappa_t)[(L_t-D_t^R) - M_t] \) ((b) in figure 2.5 and second term in (2.15)). When \( L_t - M_t < D_t^R < L_t \), the difference \((L_t - D_t^R)\) is financed with cash and the remaining cash \((M_t -(L_t-D_t))\) is retained ((c) in figure 2.5 and third term in (2.15)). For states of nature where \( D_t^R > L_t \), you retain the cash and treasuries, and obtain cash, treasuries or a loan at the interbank rate depending on the particular financial intermediary to whom the financial claim behind the deposit applies, having an expected return \( g(\kappa_t, M'_t) \) ((c) in figure 2.5 and fourth term in (2.15)).\(^{30}\)

\(^{29}\)It is clear that there is no incentive to hold \( M_t > \delta L_t \) since the amount \((M_t - \delta L_t)\) of cash would never be used to finance a loan but would ask for the nominal interest rate on deposits. \( M'_t \) and \( B'_t \) refer to cash and treasuries holding decisions by the remaining financial intermediaries. Note that \( D_t^M = M_t \) and \( D_t^B = P_t^B B_t \). Given the indifference between paying in treasuries or borrowing at the interbank rate, expression (2.14) assumes that the available treasuries in the portfolio of a financial intermediary are used before starting borrowing at the interbank rate.

\(^{30}\)The exact form of the function \( g(\kappa_t, M'_t) \) is not important because, when choosing the amount of cash to hold in his portfolio, a financial intermediary is unable to affect his payoff in states of nature where \( D_t^R > L_t \), which depends on the interbank interest rate \( \kappa_t \) and the levels of cash held by the
Figure 2.5: Portfolio Allocation
\[
\pi_t = \int_{L_t - \delta L_t}^{L_t - \delta L_t} \frac{1}{2\delta L_t} [-M_t(1 + i_t) - P_t^B B_t(1 + i_t) - D_t^R(1 + i_t) \\
- ((L_t - M_t - B_t) - D_t^R(1 + \kappa_t) + L_t(1 + R_t)(1 - \gamma)]dD_t^R \\
+ \int_{L_t - \delta L_t}^{L_t - \delta L_t} \frac{1}{2\delta L_t} [-M_t(1 + i_t) - P_t^B B_t(1 + i_t) \\
- D_t^R(1 + i_t) + B_t - ((L_t - D_t^R) - M_t)(1 + \kappa_t) + L_t(1 + R_t)(1 - \gamma)]dD_t^R \\
+ \int_{L_t}^{L_t + \delta L_t} \frac{1}{2\delta L_t} [-M_t(1 + i_t) - P_t^B B_t(1 + i_t) - D_t^R(1 + i_t) \\
+ B_t + (M_t - (L_t - D_t^R)) + L_t(1 + R_t)(1 - \gamma)]dD_t^R \\
+ \int_{L_t}^{L_t + \delta L_t} \frac{1}{2\delta L_t} [-M_t(1 + i_t) - P_t^B B_t(1 + i_t) - D_t^R(1 + i_t) + B_t \\
+ M_t + (1 + g(\kappa_t, M'_t))(D_t^R - L_t) + L_t(1 + R_t)(1 - \gamma)]dD_t^R \\
= -M_i + B_t(P_t^B(1 + i_t) - 1) + L_t[(1 + R_t)(1 - \gamma) - (1 + i_t)] - \frac{\kappa_t B_t}{2} \left(1 - \frac{M_t}{\delta L_t} - \frac{B_t}{(1 + \kappa_t)\delta L_t}\right) \\
+ \frac{\kappa_t \delta L_t}{4} \left(1 + \frac{\kappa_t}{\delta L_t} \frac{B_t}{\delta L_t} \right)^2 - (1 + \frac{M_t}{\delta L_t} - \frac{B_t}{\delta L_t})^2 - \frac{\kappa_t \delta L_t}{4} \left(\frac{B_t}{(1 + \kappa_t)\delta L_t} \right)^2 + \frac{g(\kappa_t, M'_t)\delta L_t}{4} \\
\text{(2.14)}
\]

Since\(^{31}\)

\[
\frac{\partial \pi_t}{\partial B_t} = -(P_t^B(1 + i_t) - 1),
\]

\(P_t^B \geq \frac{1}{1 + i_t}\) to prevent arbitrage. Therefore, in equilibrium, \(P_t^B = \frac{1}{1 + i_t}\) since otherwise no agent in the economy would hold treasuries.

We have\(^{32}\)

\[
\frac{\partial \pi_t}{\partial M_t} = -i_t + \frac{\kappa_t}{2} \left(1 - \frac{M_t}{\delta L_t}\right) \leq 0.
\]

\(^{31}\)See appendix B for a step-by-step derivation of (2.15) from (2.14), and to obtain the first order conditions.

\(^{32}\)It is clear that (2.15) is a concave function of \(M_t\).
The cost of one unit of cash is $i_t$ since, to obtain that unit of cash, the financial intermediary has to obtain a deposit.\footnote{Alternatively, it can be interpreted as issuing equity.} The benefit to hold one more unit of cash is given by the product of the probability that deposits end up being low enough such that loans cannot be financed by the available deposits and cash, $(1 - \frac{M_t}{\delta L_t})/2$, times the interbank interest rate that the financial intermediary would end up paying to finance his own loans. At the optimum, one more unit of cash would be too costly to hold since the probability of using that additional unit of cash to avoid using the interbank interest rate to finance his loans is already too small. A similar reasoning applies to an amount of cash less than the optimal one, where the probability is still high enough. Thus,

$$M_t = \begin{cases} 
\delta L_t \left(1 - \frac{2i_t}{\kappa_t}\right) & \kappa_t \geq 2i_t \\
0 & \kappa_t < 2i_t.
\end{cases} \quad (2.16)$$

When the interbank interest rate is not high relative to the nominal interest rate on deposits, $\kappa_t < 2i_t$, the financial intermediary wants $M_t = 0$, since the cost to finance liabilities when deposits end up being low is relatively small. As the interbank interest rate
rate increases, the cost to finance those loans increases. In the limit, the level of cash becomes closer and closer to full insurance $M_t = \delta L_t$, as illustrated in figure 2.6.\footnote{The fact that interbank rates need to be twice the interest rate on deposits to create a demand for cash comes from the fact that financial intermediaries are identical ex-ante. By allowing ex-ante heterogeneity among financial intermediaries, a demand for cash can be obtained for interbank rates smaller than twice the interest rate on deposits, although arbitrage prevents interbank rates to be smaller than the interest rate on deposits.}

### 2.3.2 Externality behind the demand for cash

In equilibrium, the zero expected profit condition $\pi_t(M_t) = 0$ holds. We have

$$\frac{\pi_t}{L_t} = \begin{cases} -\delta(1 - \frac{2i_t}{\kappa_t})i_t + [(1 + R_t)(1 - \gamma) - (1 + i_t)] & \text{if } \kappa_t \geq 2i_t \\ (1 + R_t)(1 - \gamma) - (1 + i_t) & \text{if } \kappa_t < 2i_t. \end{cases}$$

Thus, in equilibrium,

$$(1 + R_t)(1 - \gamma) = \begin{cases} (1 + i_t) + \delta(1 - \frac{2i_t}{\kappa_t})i_t & \text{if } \kappa_t \geq 2i_t \\ (1 + i_t) & \text{if } \kappa_t < 2i_t. \end{cases}$$

Define the effective spread $ES_t \equiv (1 + R_t)(1 - \gamma) - (1 + i_t) = +\delta(1 - \frac{2i_t}{\kappa_t})i_t$. Then $\frac{\partial ES_t}{\partial \kappa_t} = \frac{2i_t^2}{\kappa_t^2} > 0$ since an increase in the interbank interest rate increases the demand for cash because the cost to finance loans when deposits end up being low increases. Also, $\frac{\partial ES_t}{\partial i_t} = \delta(1 - \frac{4i_t}{\kappa_t})$. Thus, as illustrated in figure 2.7, the maximum spread is $\frac{\delta i_t}{8}$, when
\[ i_t = \frac{\kappa_t}{4}. \]

When \( i_t < \frac{\kappa_t}{4} \), the reduction in money demand as a result of an higher \( i_t \) does not compensate the increase in the cost of holding cash \( i_t \), and when \( i_t > \frac{\kappa_t}{4} \) the reduction in money demand is higher than the increase in the cost of holding cash \( i_t \).

The positive effective spread exists because of the negative externality among financial intermediaries when deciding how much cash to hold in their portfolios. In order to clarify this externality, suppose financial intermediaries were able to coordinate and commit not to hold cash in their portfolios, or were simply not allowed to hold cash. In states of nature where the level of deposits for a particular financial intermediary is low, he would finance some of his loans at the interbank interest rate, and when deposits are high, he would finance loans from other financial intermediaries at the interbank interest rate. Independent of the level of interbank rates, the effective spread would be zero. Sometimes a particular financial intermediary would have positive profits and sometimes negative profits, but the uncertainty in deposits would not generate any effective spread between lending and borrowing interest rates from the financial sector. However, by being allowed to hold cash, a particular financial intermediary doesn’t obtain any gain in states of nature where his deposits are high, but he can affect his payoff when his deposits

---

Figure 2.7: Effective Spread

\[ ES = \frac{\delta\kappa_t}{8} \]

\[ 0 \quad \frac{\kappa_t}{4} \quad \frac{\kappa_t}{2} \quad i_t \]
are low by financing some of his loans with cash, that only asked for the nominal interest rate on deposits, instead of at an higher interbank interest rate. What he ignores in his decision is the negative effect that financing his loans with cash has on the payoff that financial intermediaries with an high amount of deposits have. It is precisely this externality that creates the demand for cash, and thereby a positive effective spread.

### 2.3.3 Equilibrium Allocation

In the present section, four lemmas will provide intermediate results, followed by the close form solution of the equilibrium presented in theorem 2.3.1.

**Lemma 2.3.2** The equilibrium values of $v$, $(1 + s)$, $(1+r)$, $\kappa$ and $z$ are determined by the following 5 equations:

\[
1 = \beta (1 + s) [pv + (1 - p)] \tag{2.17}
\]

\[
v = \beta \frac{(1 - \tau - \theta) \alpha (1 + r)}{1 + r - \theta \alpha} [pv + (1 - p)] \tag{2.18}
\]

\[
(1 - \gamma)(1 + r) = (1 + s) + \delta (1 - \frac{2i}{\kappa}) \frac{(1 + s) \Pi - 1}{\Pi} \tag{2.19}
\]

\[
\frac{1 - p(1 - \tau - \theta) \alpha}{\theta \alpha} (1 + r) = p \omega + p [1 + \delta (1 - \frac{2i}{\kappa})] (1 + s) z + [1 + p(\frac{\Pi - 1}{\Pi})\delta (1 - \frac{2i}{\kappa})] z + \frac{pb}{1 + s} \tag{2.20}
\]

\[
\frac{T}{P} = \frac{\Pi - 1}{\Pi} \delta (1 - \frac{2i}{\kappa}) z - \frac{sb}{1 + s} \tag{2.21}
\]

They are the valuation equation of nonproductive agents, valuation equation of productive agents, zero expected profit condition of financial intermediaries, budget restriction
from productive agents and government constraint, respectively. Equations (2.17) and (2.18) will be called *valuation conditions*. Equations (2.19), (2.20) and (2.21) will be called *market conditions*.

**Proof.** By Walras law, it is not necessary to explicitly impose the budget restriction from nonproductive agents. See Appendix C for details. ♦

The *valuation conditions* imply an effective spread $ES^{val}$ that comes from preferences as seen in section 2.2.1. A reduction in the value of a unit of equity induces an increase in the effective return on loans $(1 - \gamma)(1 + r)$ higher than the increase in the interest rate on deposits $(1 + s)$ since it only affects the real interest rate on deposits by its effect on expectations of nonproductive agents about future values of equity, implying a negative relation between $(1 + r)$ and the effective spread $ES^{val}$. On the other hand, the *market conditions* imply an effective spread $ES^{mkt}$ that comes from imposing market equilibrium. The next three lemmas analyse the *market conditions*, setting the stage for the equilibrium solution presented in the subsequent theorem. Equilibrium is achieved when $ES^{val} = ES^{mkt}$.

**Lemma 2.3.3** *Market conditions imply that an increase in the real interest rate on loans induces an increase in the amount of cash that each financial intermediary holds per loan made.*
Proof. Substituting (2.19) and (2.21) into (2.20),
\[ 1 - \frac{p(1 - \tau - \theta)\alpha}{\theta\alpha} (1 + r) - p(1 - \gamma)(1 + r) - 1 \frac{\Pi^T P}{(\Pi - 1)\delta(1 - \frac{2i}{\kappa})} = p\omega + p\frac{\Pi^T P}{\Pi + \frac{1}{1 + s}}. \]
Thus,
\[ \delta(1 - \frac{2i}{\kappa}) = \left[ 1 - \frac{p(1 - \tau - \theta)\alpha}{\theta\alpha} (1 + r) - p(1 - \gamma)(1 + r) - 1 \frac{\Pi^T P}{(\Pi - 1)\delta(1 - \frac{2i}{\kappa})} \right]. \tag{2.22} \]
Therefore, \( \frac{\partial[\delta(1 - \frac{2i}{\kappa})]}{\partial(1 + r)} = \left[ 1 - \frac{p(1 - \tau - \theta)\alpha}{\theta\alpha} - p(1 - \gamma) \right] \frac{\Pi^T P}{\omega(\Pi - 1) + p\frac{\Pi^T P}{\Pi + \frac{1}{1 + s}}} \] > 0 since \( \delta(1 - \frac{2i}{\kappa}) \geq 0. \)

An increase in the real interest rate on loans \((1 + r)\) forces productive agents to reduce the amount of resources obtained from issuing debt, and consequently a reduction in the amount of credit provided by the financial sector.\textsuperscript{35} Since the real value of government transfers remains the same, the same real value of cash is injected into the economy.\textsuperscript{35}

\textsuperscript{35}This is the natural outcome to be obtained in any model of debt issuance. In the present model of debt issuance,
\[ \frac{(1 + r)}{\theta\alpha} - \frac{p(1 - \tau - \theta)\alpha}{\theta\alpha} (1 + r) - p(1 - \gamma)(1 + r) - 1 \frac{\Pi^T P}{(\Pi - 1)\delta(1 - \frac{2i}{\kappa})} = \]
\[ = p\omega + p\frac{\Pi^T P}{\Pi - 1} + \frac{pb}{1 + s}. \]

For the same level of debt, the amount of private equity, and consequently the level of investment, would have had to go up to maintain previous levels of debt when real interest rates on loans increase. However, since the increase in investment would be higher than the increase in the wealth of productive agents that comes from higher investment and higher returns on loans, productive agents are unable to sustain the previous level of debt. To see that the second term is the private return per debt, recall that the return on equity is \((1 + e) = \frac{(1 - \tau - \theta)\alpha}{1 + r - \theta\alpha} (1 + r)\) and \(h = \frac{1 + r - \theta\alpha}{\theta\alpha} z\), giving a return per debt of \(\frac{(1 - \tau - \theta)\alpha}{\theta\alpha}(1 + r)\). Thus, an increase in the real interest rate on loans induces a reduction in debt, equity since the economy is unable to take advantage of investment opportunities as before because of the reduction in debt issuance, investment and therefore consumption.
Therefore, interbank interest rates have to increase relative to interest rates on deposits, \( i/\kappa \) has to decrease, in order to induce financial intermediaries to hold an higher amount of cash per loan in their portfolios.

**Lemma 2.3.4** Market conditions imply that an increase in the real interest rate on loans induces an increase in the real interest rate on deposits if and only if

\[
\frac{T}{\overline{P}}((1-p)(1-\gamma)\Pi + p(1-\gamma) - \frac{1-p(1-\tau-\theta)}{\theta \alpha} \alpha] < (1-\gamma)p(\Pi - 1)\left[\omega + \frac{b}{1+s}\right].
\]

**Proof.** See Appendix C. ♦

An increase in \((1 + r)\) has two opposite effects in \((1 + s)\). It increases the effective return per loan but also increases the amount of cash per loan. By increasing the real return on each loan made by the financial sector, it allows the financial sector to increase the real interest rate on deposits. However, an increase in the real interest rate on loans reduces the demand for loans. Since government transfers remain the same, interbank interest rates have to increase relative to nominal interest rates on deposits in order to induce the financial sector to hold higher levels of cash per loan in their portfolios, as in lemma 2.3.3. This increases the costs of financial intermediation, thereby restricting the real interest rate on deposits that can be offered by the financial sector. The second effect should restrict but not compensate the first, that is, the weight of government transfers in the economy is such that it allows a competitive financial sector to increase the interest rate on deposits when the interest rate on loans increases, although restricting the effect
by increasing the costs of financial intermediation.\footnote{The parameter condition \(\frac{T}{f}(1-p)(1-\gamma)\Pi + p(1-\gamma) - \frac{1-p(1-\gamma)\alpha}{\gamma} \) will therefore be assumed in the remaining of this paper. Clearly, as \(\frac{T}{f}\) decreases or \(\omega\) increases, the inequality becomes satisfied, since the weight of government transfers in the economy decreases.}

**Lemma 2.3.5** Market conditions imply that an increase in the real interest rate on loans induces an increase in the effective spread \(ES^{mkt}\).

**Proof.** Since \(ES^{mkt} = \delta(1 - \frac{2\tau}{\kappa})[\frac{\Pi(1+s)-1}{\Pi}]\), the result follows from lemmas 2.3.3 and 2.3.4. ♦

An increase in the real interest rate on loans has two effects in the effective spread \(ES^{mkt}\). First, by increasing interbank interest rates relative to nominal interest rates, it induces the financial sector to hold more cash per loan made, as in lemma 2.3.3. Moreover, the increase in the real interest rate on loans allows financial intermediaries to increase the real interest rate on deposits, that is, to increase the nominal interest rate on deposits since inflation remained the same, as in lemma 2.3.4. Thus, both effects induce an increase in the effective spread \(ES^{mkt}\), since the amount of cash per loan and the cost of holding cash both increase.

The previous three lemmas described the behaviour of the market conditions (2.19), (2.20) and (2.21). The equilibrium solution presented in the next theorem can now be understood and is illustrated in figure 5. Equilibrium is achieved when \(ES^{val} = ES^{mkt}\). For interest rates on loans such that the valuation conditions ask for almost no effective \(\omega\)
spread, \((1 + r)(1 - \gamma) \approx (1 + s)\), we have \(E_{\text{val}} < E_{\text{mkt}}\) because financial intermediaries would be unable to offer the interest rate on deposits that nonproductive agents ask, due to the cost of holding cash in their portfolios, thereby unable to attract the necessary funding to finance the demand for credit by productive agents. As the real interest rate on loans increases, \(E_{\text{val}}\) increases significantly because nonproductive agents only ask for a small increase in the real interest rate on deposits since the increase in the real interest rates on loans only affects their expectations of the private value of equity in the future, as seen in section 2.2.1. However, the increase in \(E_{\text{mkt}}\) obtained in lemma 2.3.5 is relatively small compared to the increase in \(E_{\text{val}}\) for reasonable weights of government transfers in the economy since the effect comes only from the increase in the amount of cash per loan in the portfolio of financial intermediaries, motivated by the increase in the interbank interest rate relative to the nominal interest rate on deposits and due to the reduction in credit demand as a result of the increase in the real interest rate on loans.\(^37\)

The real interest rate on loans increases until \(E_{\text{val}} = E_{\text{mkt}}\). A further increase in the real interest rate on loans would again induce a significant change in \(E_{\text{val}}\) compared to the change in \(E_{\text{mkt}}\), thereby providing an overwhelming amount of funding to the financial sector when compared to the demand for credit by productive agents. The above argument is quantitatively expressed in close form by the next theorem.

\(^37\)As explained, the impact on \(E_{\text{mkt}}\) of an increase in the real interest rates on loans depends on the weight of government transfers in the economy. Looking at expression (2.37), we see that as \(\omega \uparrow \infty \) or \(\frac{T}{P} \downarrow 0\), the dependence of \(E_{\text{mkt}}\) on \((1 + r)\) disappears. The parameter condition \(\frac{\partial E_{\text{val}}}{\partial (1 + r)} = (1 - \gamma) - \frac{\theta \alpha}{[(1 - p)(1 + r - \theta \alpha)]^2} > \frac{\partial E_{\text{mkt}}}{\partial (1 + r)}\) is assumed in the remaining of this paper.
Theorem 2.3.1 The equilibrium interest rate on loans is given by $(1+r)^* = \frac{-b_1 + \sqrt{b_2 - 4b_0\gamma}}{2b_2}$, where $b_2$, $b_1$ and $b_0$ are constants.\(^{38}\) The real interest rate on deposits, real return on eq-

\(^{38}\)Explicitly,

\[
\begin{align*}
b_2 &\equiv (1-\gamma)(1-p)[\rho\omega(\Pi - 1) - (1-p)\Pi T P + \frac{p(\Pi - 1)b}{1+s}] \\
&\quad + [(1-\gamma)(1-p)\theta\alpha - \beta^{-1} + (1-\tau - \theta)\alpha p][\frac{1-p(1-\tau - \theta)\alpha}{\theta\alpha}\Pi T P - p(1-\gamma)\Pi T P] \\
&\quad + [\frac{1-p(1-\tau - \theta)\alpha}{\theta\alpha} T (1-\gamma)\Pi - p(1-\gamma)\gamma T P](1-p)\theta\alpha - [\frac{1-p(1-\tau - \theta)\alpha}{\theta\alpha} T P] \\
&\quad + p(1-\gamma)\frac{T P}{P} - \frac{T}{P}(1-\gamma)\Pi (1-p), \\
\end{align*}
\]

\[
\begin{align*}
b_1 &\equiv [- (1-\gamma)(1-p)\theta\alpha - \beta^{-1} + (1-\tau - \theta)\alpha p][\rho\omega(\Pi - 1) + p\Pi T P + \frac{p(\Pi - 1)b}{1+s} - \Pi T P] \\
&\quad + [\beta^{-1}\theta\alpha\frac{1-p(1-\tau - \theta)\alpha}{\theta\alpha} T P - p(1-\gamma)\Pi T P] + [\frac{1-p(1-\tau - \theta)\alpha}{\theta\alpha} T P + p(1-\gamma) T P] \\
&\quad - \frac{T}{P}(1-\gamma)\Pi (1-p)\theta\alpha - \frac{T}{P}(1-p), \\
\end{align*}
\]

\[
\begin{align*}
b_0 &\equiv \beta^{-1}\theta\alpha[\rho\omega(\Pi - 1) + p\Pi T P + \frac{p(\Pi - 1)b}{1+s} - \Pi T P] + \frac{T}{P}(1-p)\theta\alpha. \\
\end{align*}
\]

\(\alpha = 1.6, \theta = 0.2, \gamma = 0.0001, \tau = 0.01, \Pi = 1.05, \)
\(\delta = 0.1, p = 0.25, \beta = 0.8, \omega = 100, T/P = 0.1, b = 0.\)

Figure 2.8: Equilibrium
unity, unit value of private equity, effective spread, cash per loan, debt, equity, investment, consumption and deposits, are given by, respectively,

\[
(1 + s)^* = \frac{1 - p\beta(1 - \tau - \theta)\alpha - \frac{\theta\alpha}{(1+r)^*}}{\beta(1 - p) - \frac{p\beta(1-p)\theta\alpha}{(1+r)^*}},
\]

(2.23)

\[
(1 + e)^* = \frac{(1 - \tau - \theta)\alpha}{1 - \frac{\theta\alpha}{(1+r)^*}},
\]

(2.24)

\[
v^* = \frac{\beta(1 - \tau - \theta)\alpha(1 - p)}{1 - p\beta(1 - \tau - \theta)\alpha - \frac{\theta\alpha}{(1+r)^*}};
\]

(2.25)

\[
ES^* = (1 - \gamma)(1 + r)^* - \frac{\beta^{-1}}{1 - p} + \frac{(1 - \tau - \theta)\alpha(1 + r)^* p}{(1 - p)((1 + r)^* - \theta\alpha)},
\]

(2.26)

\[
\delta(1 - \frac{2i}{\kappa})^* = \frac{[1 - p(1 - \tau - \theta)\alpha(1 + r)^* - p(1 - \gamma)(1 + r)^* - 1]\Pi P}{p\omega(\Pi - 1) + p\Pi P + \frac{p\beta}{1 + s}}
\]

(2.27)

\[
z^* = \frac{p\omega + \frac{p\beta}{1 + s}}{1 - p(1 - \tau - \theta)\alpha(1 + r) - p(1 - \gamma)(1 + r) - 1 - p\delta(1 - \frac{2i}{\kappa})},
\]

(2.28)

\[
h^* = \frac{(1 + r)^* - \theta\alpha}{\theta\alpha} z^*,
\]

(2.29)

\[
x^* = \frac{(1 + r)^*}{\theta\alpha} z^*,
\]

(2.30)

\[
c^* = \omega + [(1 - \tau)\alpha - \gamma\theta\alpha - 1]x^*,
\]

(2.31)

\[
d^* = (1 + \delta(1 - \frac{2i}{\kappa})^*)z^* + \frac{b}{1 + s}.
\]

(2.32)

**Proof.** See appendix C. ♦

Figure 2.8 provides a numerical example.
2.4 Equilibrium Characterization

Given the close form solution obtained in theorem 2.3.1, several comparative statics can be obtained. For the purposes of the present paper, two results will be highlighted due to the interaction with the externality identified in this paper.

**Proposition 2.4.1** An increase in the contemporaneous uncertainty parameter $\delta$ produces a decrease in the interbank interest rate $\kappa$. All other equilibrium variables remain the same.

**Proof.** The statement follows immediately from the system of equations described in lemma 2.3.2. ♦

Keeping interest rates the same, the increase in the contemporaneous uncertainty parameter $\delta$ induces the financial sector to hold more cash per loan in their portfolios, due to higher uncertainty about the interest rate used to finance loans. An increase in the degree of contemporaneous uncertainty in deposits exacerbates the externality among financial intermediaries, as explained in section 2.2.3, since the probability that
the last unit of cash that a financial intermediary holds in his portfolio is used to avoid financing loans at the interbank interest rate, instead of at the nominal interest rate on deposits, increases. The excess demand for cash induces a decrease in interbank interest rates to restore equilibrium, by decreasing the costs that each particular financial intermediary has when deposits end up being low, thereby reducing the incentives of financial intermediaries to hold cash in their portfolios.

The previous proposition has important implications for the valuation practice of financial claims. In the context of proposition 2.4.1, interbank interest rates simply reflect how stable the matching in the provision of funds in the financial sector is. Interbank interest rates should be high when the matching in the provision of funds in the financial sector is very stable, in order to induce financial intermediaries to hold cash, and should be small when the matching is quite unstable, since the increase in uncertainty about the interest rate used to finance loans increases, thereby exacerbating the incentive to hold cash even when interbank interest rates are small. More generally, interbank interest rates represent a price that provides the adjustment between the demand and supply of cash. In particular, Libor interest rates are far from simply reflecting the rate at which the private economy is willing to transfer resources in time. This result compromises the use of Libor interest rates as appropriate discount rates in valuation.

Proposition 2.4.2 Assume \( b = 0 \). An increase in inflation, followed by the corresponding increase in the value of transfers due to the increase of government revenues from
the inflation tax, implies an increase in the real interest rates on deposits, treasuries and loans, an increase in the spreads of interbank rates to treasury or deposits rates, and a decrease in real debt issuance, equity, investment, consumption, production and unit value of equity.

Proof. Fixing the real interest rate on loans \((1+r)\), the amount of cash per loan remains the same from equation (2.22), 
\[
\delta(1-2i) = \frac{\left[1-p(1-r-\theta)\alpha(1+r)-p(1-\gamma)(1+r)-1\right]\Phi}{p\omega(1-\gamma)+p\Pi + \frac{p(\Pi-1)b}{1+s}}. 
\]
Then, since \((1+s) = \frac{(1-\gamma)(1+r)+\frac{\delta(1-2i)}{\Pi}}{1+\delta(1-\frac{2i}{\Pi})}\) from equation (2.19), the real interest rate on deposits decreases. Thus, there is an increase in effective market spreads. 

\(^{39}\)That is, the proportional increase in the inflation tax \(\frac{\Pi-1}{\Pi}\) equals the proportional increase in government revenues from the inflation tax. The increase in revenues represents an increase in transfers \(\frac{T}{\Pi}\) as in equation (2.21).

\(^{40}\)From equation (2.28), 
\[
z = \frac{\rho\omega + \rho b}{1-\rho(1-r-\theta)\alpha(1+r)-p(1-\gamma)(1+r)-1-p\delta(1-\frac{2i}{\Pi})},
\]
the level of debt also remains the same.
Since the revenues from the inflation tax are transferred back to private agents, the amount of debt that productive agents are able to issue and the amount of real cash at the disposal of financial intermediaries are unaffected. However, in order to keep real interest rates, the opportunity cost of holding cash for the financial sector, \( i \), has to increase. This increases the costs of financial intermediation, thereby reducing the real interest rates on deposits and an increase in market spreads. This is represented as the movement from point \( A \) to point \( B \) in figure 7. At point \( B \), nonproductive agents are not willing to provide the desired funding to the financial sector since \( (1 + s) \) is too small, forcing an increase in \( (1 + r) \) and \( (1 + s) \) from \( B \) to \( C \) to restore market equilibrium by reducing the demand for credit and attracting funding from nonproductive agents to finance credit. Thus, there is an increase in the amount of cash held per loan in the portfolios of financial intermediaries, induced by an increase in interbank interest rates relative to the nominal interest rate on deposits,\(^{41}\) a reduction in debt issuance due to the increase in real interest rates on loans, a reduction in equity because productive agents are unable to use debt to exploit their investment opportunities as before, thus a reduction in investment, consumption and the unit value of equity \( v \).

Proposition 2.4.2 shows that expected inflation distorts investment decisions even when any uncertainty typically associated with inflation is eliminated. The only way the economy has to protect against inflation is to completely eliminate the use of cash in

\[ \kappa - i = i(\frac{\kappa}{r} - 1), \]

the spread increases when both nominal interest rates and the ratio of interbank rates to nominal interest rates increase.
all portfolio decisions. However, due to the externality identified in this paper, financial intermediaries still want to hold cash in their portfolios. Expected inflation, by taxing financial intermediaries in their portfolio decisions, distorts lending and borrowing interest rates in the financial sector. This leads to an increase in real interest rates and an economic slowdown.

To finish the characterization of equilibrium, note that, under commitment, the Friedman rule holds in this economy. In the context of this model, where interest rates on deposits are different from interest rates on loans, the Friedman rule should be interpreted as zero nominal interest rates on deposits. With a zero nominal interest rate on deposits, financial intermediaries don’t incur any cost by holding cash in their portfolios. Thus, under the Friedman rule, they will hold enough cash to fully insure themselves about any realization of deposits as long as interbank interest rates are strictly positive. This means that, from equation (2.16), $M_t = \delta L_t$. Since the opportunity cost to hold cash is zero, the spread between the effective real interest rate on loans and the real interest rate

42In order to understand the implementation of the Friedman rule in this economy, let $(1 + s)^{**}$ be the real interest rate on deposits associated with zero effective spread from the valuation conditions, equations (2.17) and (2.18). Since $(1 + s) \equiv \frac{1 + i_{\Pi}}{\Pi}$, the Friedman rule is implemented by taxing, in a lump-sum way, the private agents in this economy. These taxes are such that the money supply decreases at a rate equal to the real interest rate on deposits $(1 + s)^{**}$, in order to implement a zero nominal interest rate on deposits.

43It is interesting to observe that the specific value for the interbank interest rate would be undetermined under the Friedman rule.
on deposits completely disappears, as can be seen from equation (2.19). The elimination of this effective spread allows an increase in expected consumption from the perspective of each private agent since more resources can be used to exploit investment opportunities.

2.5 Conclusion

This paper presented a dynamic general equilibrium model that disentangles interbank rates from interest rates on deposits. Firstly, an externality, among financial intermediaries satisfying deposits on demand, was identified. It was shown that it is able to solve the tension that previous models of money have faced when trying to justify the demand for an asset that doesn’t pay any interest. Additionally, it shows that the positive spread between interbank rates and treasury rates is the leading variable that drives the money market. Secondly, this paper provided an equilibrium framework to understand the simultaneous behaviour of interest rates on deposits, loans and interbank interest rates. The dependence of the equilibrium on the degree of contemporaneous uncertainty on deposits and inflation, being variables at the heart of the above externality, was analyzed. The analysis showed that interbank interest rates play a role in the economy that prevents them from reflecting a discount rate at which the real economy is willing to transfer cash flows over time. Additionally, an increase in expected inflation increases the equilibrium costs in financial intermediation, inducing an increase in real interest rates and an economic slowdown.
2.6 Empirical Spreads

![Graph of Libor 3 Month - Treasuries 3 Month with data points from 1983 to 2003. First Observation: 1/8/82, Last Observation: 1/28/2005. Weekly figures are averages of 7 calendar days ending on Wednesday of current week. H.15 Federal Reserve Board.]

Figure 2.11: Market Spreads 3 Month (Weekly Observations)

2.7 Appendix A: Proof of lemma 2.3.1

**Proof.** [Lemma 2.3.1] In order to check equilibrium requirements, let $R_t$ and $i_t$ be the announced interest rates. If financial intermediaries have strictly positive expected profits, there is always the incentive for one additional potential entrant to enter the market, announce exactly the same interest rates and redistribute the expected profits by one more financial intermediary. The eight cases associated with increasing or decreasing at least one of the two announced interest rates need to be checked.

Suppose that a financial intermediary increases his $R_t$ and preserves his $i_t$. Then
the loans that he could previously make would be distributed among the remaining financial intermediaries. Since he would still collect the same amount of deposits, the remaining financial intermediaries would increase their cash holdings in exactly the amount of the loans that he was previously making. The net effect at the end of the day would be to substitute his previous loans for cash, thereby not compensating the nominal interest rate that deposits ask.

There is no incentive to increase \( R_t \) and decrease \( i_t \) since that would correspond to get out of the financial market.

Suppose he announces the same \( R_t \) and reduces \( i_t \). Then he wouldn’t be able to collect any type of deposits (in cash, treasuries, or through checks or account transfers) and would be forced to finance his previous loans at the interbank interest rate. Since, as a result of the zero expected profit condition, the effective interest rate on loans is smaller than the interbank interest rate, such deviation is not profitable.\(^{44}\)

The remaining five cases associated with either a decrease in \( R_t \) or an increase in \( i_t \) do not constitute a feasible deviation due to the capacity upper bounds assumed for

\(^{44}\)The fact that the effective rate on loans \((1 - \gamma)(1 + R)\) is smaller that the interbank rate \(1 + \kappa\) follows from the zero expected profit condition in the financial sector as will appear in lemma 2.3.2:

\[
(1 - \gamma)(1 + r) = (1 + s) + \delta(1 - \frac{2i}{\kappa})\left[\frac{(1 + s)\Pi - 1}{\Pi}\right]
\]

Since, from section 2.2.3, \( \delta \leq 1 \) and \( \kappa \geq 2i \), we have \( 0 \leq \delta(1 - \frac{2i}{\kappa}) < 1 \). From the zero profit condition, after multiplying by \( \Pi \), it follows that \( i > (1 - \gamma)(1 + R) - (1 + i) \geq 0 \). Finally, because \( \kappa \geq 2i \), we conclude that \((1 - \gamma)(1 + R) < 1 + \kappa\).
the technology that a specific financial intermediary has to make loans and collect
deposits, which prevents him from being the only one making loans or collecting
deposits from lenders in the market.\footnote{They are $R_t \uparrow i_t \uparrow$, $\overline{R_t} \uparrow i_t \uparrow$, $R_t \downarrow i_t \uparrow$, $R_t \downarrow \overline{i_t}$ and $R_t \downarrow i_t \downarrow$.}
2.8 Appendix B: Expected profit of a Financial Intermediary

When financial intermediaries hold treasuries in their portfolios and use them to settle their debts,

\[
\pi_t = \int_{L_t-\delta L_t}^{L_t-M_t} \frac{1}{2\delta L_t} \left[ -M_t(1+i_t) - P_t^B B_t(1+i_t) - D_t^R(1+i_t) \right. \\
-((L_t - M_t - \frac{B_t}{1+\kappa_t}) - D_t^R(1+\kappa_t) + L_t(1+R_t)(1-\gamma))dD_t^R \\
+ \int_{L_t-M_t}^{L_t} \frac{1}{2\delta L_t} [-M_t(1+i_t) - P_t^B B_t(1+i_t) - D_t^R(1+i_t) + B_t + (M_t - (L_t - D_t^R))] \\
+ L_t(1+R_t)(1-\gamma)dD_t^R \\
+ \int_{L_t}^{L_t+\delta L_t} \frac{1}{2\delta L_t} [-M_t(1+i_t) - P_t^B B_t(1+i_t) - D_t^R(1+i_t) + B_t + M_t] \\
+(1+g(\kappa_t, M'_t, B'_t))(D_t^R - L_t) + L_t(1+R_t)(1-\gamma)dD_t^R \\
\] 

\[
= -M_t(1+i_t) - P_t^B B_t(1+i_t) - L_t(1+i_t) + L_t(1+R_t)(1-\gamma) \\
+ \int_{L_t-\delta L_t}^{L_t-M_t} \frac{1}{2\delta L_t} [(D_t^R - L_t + M_t + \frac{B_t}{1+\kappa_t})(1+\kappa_t)]dD_t^R \\
+ \int_{L_t-M_t}^{L_t} \frac{1}{2\delta L_t} [B_t + (D_t^R + M_t - L_t)(1+\kappa_t)]dD_t^R \\
+ \int_{L_t}^{L_t+\delta L_t} \frac{1}{2\delta L_t} [B_t + M_t - L_t + D_t^R]dD_t^R \\
+ \int_{L_t}^{L_t+\delta L_t} \frac{1}{2\delta L_t} [B_t + M_t + (D_t^R - L_t)(1+g(\kappa_t, M'_t, B'_t))]dD_t^R \\
\]
\[
\begin{align*}
&= -M_t(1 + i_t) - P_t^R B_t(1 + i_t) + L_t(1 + R_t)(1 - \gamma) - (1 + i_t) \bigg] \\
&+ \int_{L_t - \delta L_t}^{L_t - M_t} \frac{1}{2\delta L_t} \left[ \left( \frac{B_t}{1 + \kappa_t} - B_t \right) (1 + \kappa_t) \right] dD_t^R \\
&+ \int_{L_t - \delta L_t}^{L_t - M_t} \frac{1}{2\delta L_t} \left[ (D_t^R - L_t + M_t + B_t)(1 + \kappa_t) \right] dD_t^R \\
&+ \int_{L_t - \delta L_t}^{L_t - M_t} 1 \left[ B_t + (D_t^R + M_t - L_t)(1 + \kappa_t) \right] dD_t^R \\
&+ \int_{L_t}^{L_t + \delta L_t} \frac{1}{2\delta L_t} [B_t + M_t - L_t + D_t^R] dD_t^R \\
&+ \int_{L_t}^{L_t + \delta L_t} \frac{1}{2\delta L_t} [B_t + M_t + (D_t^R - L_t)(1 + g(\kappa_t, M_t', B_t'))] dD_t^R \\
&= -M_t(1 + i_t) - P_t^R B_t(1 + i_t) + L_t[(1 + R_t)(1 - \gamma) - (1 + i_t)] \\
- \frac{\kappa_t B_t}{2} (1 - \frac{M_t}{\delta L_t} - \frac{B_t}{(1 + \kappa_t)\delta L_t}) + B_t + M_t \\
+ \kappa_t \int_{L_t - \delta L_t}^{L_t - M_t} \frac{1}{2\delta L_t} [D_t^R - L_t + M_t + B_t] dD_t^R + \kappa_t \int_{L_t - \delta L_t}^{L_t - M_t} \frac{1}{2\delta L_t} [D_t^R + M_t - L_t] dD_t^R \\
+ g(\kappa_t, M_t', B_t') \int_{L_t}^{L_t + \delta L_t} \frac{1}{2\delta L_t} [D_t^R - L_t] dD_t^R \\
&= -M_t i_t - B_t(P_t^R(1 + i_t) - 1) + L_t[(1 + R_t)(1 - \gamma) - (1 + i_t)] - \frac{\kappa_t B_t}{2} \left( 1 - \frac{M_t}{\delta L_t} - \frac{B_t}{(1 + \kappa_t)\delta L_t} \right) \\
+ \frac{\kappa_t}{4\delta L_t} \left[ (\frac{\kappa_t B_t}{1 + \kappa_t} - B_t)^2 - (\delta L_t - M_t - B_t)^2 \right] - \frac{\kappa_t}{4\delta L_t} \left( \frac{B_t}{1 + \kappa_t} \right)^2 + \frac{g(\kappa_t, M_t', B_t')}{4\delta L_t} (\delta L_t)^2 \\
&= -M_t i_t - B_t(P_t^R(1 + i_t) - 1) + L_t[(1 + R_t)(1 - \gamma) - (1 + i_t)] - \frac{\kappa_t B_t}{2} \left( 1 - \frac{M_t}{\delta L_t} - \frac{B_t}{(1 + \kappa_t)\delta L_t} \right) \\
+ \frac{\kappa_t}{4} \left[ (\frac{\kappa_t}{1 + \kappa_t} B_t)^2 - (1 - \frac{M_t}{\delta L_t} - \frac{B_t}{\delta L_t})^2 \right] - \frac{\kappa_t}{4} \left( \frac{B_t}{1 + \kappa_t} \right)^2 + \frac{g(\kappa_t, M_t', B_t')}{4} (\delta L_t)^2.
\end{align*}
\]
Then

\[
\frac{\partial \pi_t}{\partial M_t} = -i_t + \frac{\kappa_t B_t}{2 \delta L_t} + \frac{\kappa_t (1 - \frac{M_t}{\delta L_t} - \frac{B_t}{\delta L_t})}{2}
\]

\[
= -i_t + \frac{\kappa_t}{2} (1 - \frac{M_t}{\delta L_t}),
\]

\[
\frac{\partial \pi_t}{\partial B_t} = -(P_t^B (1 + i_t) - 1) - \frac{\kappa_t (1 - \frac{M_t}{\delta L_t} - \frac{B_t}{\delta L_t})}{2 (1 + \kappa_t \delta L_t)} + \frac{\kappa_t B_t}{2 (1 + \kappa_t \delta L_t)}
\]

\[
+ \frac{\kappa_t}{2} \left( \frac{\kappa_t}{1 + \kappa_t} \frac{B_t}{\delta L_t} \right) \frac{\kappa_t}{1 + \kappa_t} + \frac{\kappa_t}{2} (1 - \frac{M_t}{\delta L_t} - \frac{B_t}{\delta L_t})
\]

\[
- \kappa_t \delta L_t \frac{B_t}{2 (1 + \kappa_t \delta L_t)} \frac{1}{(1 + \kappa_t \delta L_t)}
\]

\[
= -(P_t^B (1 + i_t) - 1) + \left( \frac{\kappa_t}{1 + \kappa_t} + \frac{\kappa_t^3}{2 (1 + \kappa_t)^2} - \frac{\kappa_t}{2} - \frac{\kappa_t}{2 (1 + \kappa_t)^2} \right) \frac{B_t}{\delta L_t}
\]

\[
= -(P_t^B (1 + i_t) - 1) + \left( \frac{2 \kappa_t - \kappa_t (1 + \kappa_t)}{2 (1 + \kappa_t)} + \frac{\kappa_t (\kappa_t^2 - 1)}{2 (1 + \kappa_t)^2} \right) \frac{B_t}{\delta L_t}
\]

\[
= -(P_t^B (1 + i_t) - 1) + \left( \frac{\kappa_t - \kappa_t^2}{2 (1 + \kappa_t)} + \frac{\kappa_t (\kappa_t - 1)}{2 (1 + \kappa_t)} \right) \frac{B_t}{\delta L_t}
\]

\[
= -(P_t^B (1 + i_t) - 1).
\]

Alternatively, when treasuries are not used by financial intermediaries in their port-
\[ \pi_t(M_t|M'_t) = \int_{L_t-M_t}^{L_t-M_t} \frac{1}{\delta L_t} \left[ -M_t(1+i_t) - D_t^R(1+i_t) - ((L_t - M_t) - D_t^R)(1 + \kappa_t) ight. \\
+ L_t(1+R_t)(1-\gamma)]dD_t^R \\
+ \int_{L_t-M_t}^{L_t-M_t} \frac{1}{\delta L_t} \left[ -M_t(1+i_t) - D_t^R(1+i_t) + (M_t - (L_t - D_t^R)) ight. \\
+ L_t(1+R_t)(1-\gamma)]dD_t^R \\
+ \int_{L_t}^{L_t+\delta L_t} \frac{1}{\delta L_t} \left[ -M_t(1+i_t) - D_t^R(1+i_t) + M_t + (1 + g(\kappa_t, M'_t))(D_t^R - L_t) ight. \\
+ L_t(1+R_t)(1-\gamma)]dD_t^R \\
= -M_t(1+i_t) - L_t(1+i_t) + L_t(1+R_t)(1-\gamma) \\
+ \int_{L_t-M_t}^{L_t+\delta L_t} \frac{1}{\delta L_t} \left[ M_t - (L_t - D_t^R) \right] dD_t^R \\
- \kappa_t \int_{L_t-M_t}^{L_t-M_t} \frac{1}{\delta L_t} \left[ (L_t - M_t) - D_t^R \right] dD_t^R \\
+ g(\kappa_t, M'_t) \int_{L_t}^{L_t+\delta L_t} \frac{1}{\delta L_t} \left( D_t^R - L_t \right) dD_t^R \\
= -i_t M_t + L_t[(1+R_t)(1-\gamma) - (1+i_t)] - \kappa_t \int_{L_t-M_t}^{L_t-M_t} \frac{1}{\delta L_t} \left[ (L_t - M_t) - D_t^R \right] dD_t^R \\
+ g(\kappa_t, M'_t) \int_{L_t}^{L_t+\delta L_t} \frac{1}{\delta L_t} \left( D_t^R - L_t \right) dD_t^R \\
= -i_t M_t + L_t[(1+R_t)(1-\gamma) - (1+i_t)] \\
- \frac{\kappa_t}{2\delta L_t} \frac{(\delta L_t-M_t)^2}{2} + \frac{g(\kappa_t, M'_t)\delta^2 L_t^2}{2\delta L_t} \\
= -i_t M_t + L_t[(1+R_t)(1-\gamma) - (1+i_t)] - \frac{\kappa_t\delta L_t}{4}(1 - \frac{M_t}{\delta L_t})^2 + \frac{g(\kappa_t, M'_t)\delta L_t}{4}. \\

The first term represents the cost of receiving deposits in cash and holding the cash as an asset. The second term gives the gain associated with the spread between the interest rate charged on loans, liquid of costs associated with making the loan and enforcing
repayment, and the interest rate on deposits that are used to finance those loans. The third gives the cost that the financial intermediary has in states of nature where deposits end up being too low, forcing him to finance some of his loans at the interbank interest rate $\kappa_t$. Finally, the fourth term gives the gain associated with states of nature where the amount of deposits received by the financial intermediary is higher than his loans, where some of them will end up in a loan to some other financial intermediary at the interbank interest rate $\kappa_t$ and others will be promptly paid in cash.

2.9 Appendix C: Proof of lemmas 2.3.2, 2.3.4 and theorem 2.3.1

**Proof.** [Lemma 2.3.2] Recalling that the debt issue from a private agent with access to production technology (2.2) is given by (2.7), total investment, being the sum of private equity plus the debt issue, is given by

$$x_t = h_t + z_t = h_t + \frac{\theta \alpha}{1 + \bar{r}_1 - \theta \alpha} h_t = \frac{1 + \bar{r}_1}{1 + \bar{r}_1 - \theta \alpha} h_t.$$

Thus, $z_t = \frac{\theta \alpha}{1 + \bar{r}_1 - \theta \alpha} h_t = \frac{\theta \alpha}{1 + \bar{r}_1} x_t$.

Since the government is committed to maintain a constant inflation rate $\Pi$, interest rates are independent of the current state of the economy. At time $t$, real costs associated with making loans and enforcing debt repayments are then given by

$$\gamma(1 + r_t) z_{t-1} = \gamma(1 + r_t) \frac{\theta \alpha}{1 + r_t} x_{t-1} = \gamma \theta \alpha x_{t-1}.$$  

Goods market clearing condition says that production is applied in government expenses, private consumption, investment and costs by financial intermediaries,

$$\omega + \alpha x_{t-1} = \tau \alpha x_{t-1} + c_t + x_t + \gamma \theta \alpha x_{t-1},$$  

or
\[ \omega + [(1 - \tau)\alpha - \gamma \theta \alpha] x_{t-1} = c_t + x_t. \]

Using money demand (2.16), since \( \Pi_t = \frac{P_t}{P_{t-1}}, \frac{T_t}{P_t} = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} + \frac{1}{P_t} (P_t B_t - B_{t-1}) = (\frac{\Pi - 1}{\Pi}) \delta (1 - \frac{2i}{\kappa}) z + \frac{1}{1+s} \frac{B_t}{P_t} \Pi - \frac{B_{t-1}}{P_{t-1}} = (\frac{\Pi - 1}{\Pi}) \delta (1 - \frac{2i}{\kappa}) z - \frac{s}{1+s} b \]

in order to maintain a constant inflation rate \( \Pi \).

46 Holding the real demand for cash \( \delta (1 - \frac{2i}{\kappa}) z \) constant, an higher level of inflation represents an higher real value of lump-sum transfers since the same tax base is taxed at an higher rate.

Private agents arrive at \( t \) with real financial wealth \( w_t \) that comes from endowment, returns on equity, deposits and transfers from the government:

\[
w_t = \omega + (1 + \epsilon) h_{t-1} + d_{t-1}(1 + s) + \frac{T}{P} = \omega + \left( \frac{(1 - \tau)\alpha(1 + r) - (1 + r)\theta \alpha}{1 + r - \theta \alpha} \frac{1 + r - \theta \alpha}{\theta \alpha} \right) z + \left( z + \frac{M}{P} + \frac{b}{1 + s} \right)(1 + s) + \frac{\Pi - 1}{\Pi} \delta (1 - \frac{2i}{\kappa}) z - \frac{s}{1+s} b \]

\[ = \omega + \left\{ (1 - \tau - \frac{\theta}{\theta}) (1 + r) + (1 + \delta (1 - \frac{2i}{\kappa})) (1 + s) + \frac{\Pi - 1}{\Pi} \delta (1 - \frac{2i}{\kappa}) \right\} z + \frac{b}{1 + s}. \]

\[ (2.33) \]

Nonproductive agents apply their financial wealth in consumption and deposits,

\[ c_t + d_t = (1 - p) w_t. \]

Thus, \( \omega + [(1 - \tau)\alpha - \gamma \theta \alpha - 1] \frac{1 + r}{\theta \alpha} z + (1 + \delta (1 - \frac{2i}{\kappa})) z + \frac{b}{1 + s} = (1 - p) \omega + (1 - p) \left\{ \frac{1 - \tau - \theta}{\theta} (1 + r) + (1 + \delta (1 - \frac{2i}{\kappa})) (1 + s) + \frac{\Pi - 1}{\Pi} \delta (1 - \frac{2i}{\kappa}) \right\} z + \frac{(1 - p)b}{1 + s}. \]

The budget restriction from productive agents, \( x - z = pw \), gives \( \frac{1 + r - \theta \alpha}{\theta \alpha} z = \frac{1 + r}{\theta \alpha} w_t \)\( - b \).
pω + p\frac{1-\tau-\theta}{\theta^2}(1+r)z + p(1+\delta(1-\frac{2i}{\kappa}))(1+s)z + p(\frac{\Pi-1}{\Pi})\delta(1-\frac{2i}{\kappa})z + \frac{pb}{1+s}, or

\[ \frac{1-p(1-\tau-\theta\alpha)}{\theta^\alpha}(1+r)z = pω + p(1+\delta(1-\frac{2i}{\kappa}))(1+s)z + [1+p(\frac{\Pi-1}{\Pi})\delta(1-\frac{2i}{\kappa})]z + \frac{pb}{1+s}. \]

(2.34)

Adding both budget constraints, \( c + \frac{D}{p} = (1-p)w \) and \( x - z = pw \), we get

\[ [\omega + (1-\tau)\alpha - \gamma^\theta\alpha - 1]\frac{1+r}{\theta^\alpha}z + [1+\delta(1-\frac{2i}{\kappa})]z + \frac{b}{1+s} + \frac{1+r}{\theta^\alpha}z - z = \omega + \left\{ \frac{1-\tau-\theta}{p^2}(1+r) + (1+\delta(1-\frac{2i}{\kappa}))(1+s) + \frac{\Pi-1}{\Pi}\delta(1-\frac{2i}{\kappa}) \right\}z + \frac{b}{1+s}, \]

or

\[ (1-\gamma)(1+r) = (1+\delta(1-\frac{2i}{\kappa}))(1+s) - \frac{\delta(1-\frac{2i}{\kappa})}{\Pi}. \]

(2.35)

Equation (2.35) is precisely the expected zero profit condition for the financial sector, showing that the cost of holding cash is the nominal interest rate since revenues are returned to the economy. This shows that Walras law is verified, justifying that the budget constraint from nonproductive agents can be forgotten from the system of equations described in the statement of the lemma. ♦

**Proof.** [Lemma 2.3.4] Equation (2.19) says that the real return on deposits \((1+s)\) is given by the return on loans plus the value of the cash per loan that remains after the inflation tax, divided by deposits per loan, \((1+s) = \frac{(1-\gamma)(1+r) + \delta(1-\frac{2i}{\kappa})}{1+\delta(1-\frac{2i}{\kappa})}. \)
Thus,
\[
[1 + \delta(1 - \frac{2i}{\kappa})] \frac{\partial(1 + s)}{\partial (1 + r)} = [(1 - \gamma) + \frac{1}{\Pi} \partial[\delta(1 - \frac{2i}{\kappa})][1 + \delta(1 - \frac{2i}{\kappa})] \\
-[(1 - \gamma)(1 + r) + \frac{\delta(1 - \frac{2i}{\kappa})}{\Pi} \partial[\delta(1 - \frac{2i}{\kappa})],
\]
\[
(1 + \delta(1 - \frac{2i}{\kappa})) \frac{\partial(1 + s)}{\partial (1 + r)} = (1 - \gamma) - \frac{(1 + s)\Pi - 1}{\Pi} \frac{\partial[\delta(1 - \frac{2i}{\kappa})]}{\partial (1 + r)}
\]
\[
(1 + \delta(1 - \frac{2i}{\kappa})) \frac{\partial(1 + s)}{\partial (1 + r)} = (1 - \gamma) - \frac{(1 - \gamma)(1 + r) + \frac{1}{\Pi} \partial[\delta(1 - \frac{2i}{\kappa})]}{1 + \delta(1 - \frac{2i}{\kappa})}.
\]

Therefore, from the proof of lemma (2.3.3), the real interest rate on deposits increases if and only if

\[
(1 - \gamma) - [(1 - \gamma)(1 + r) - \frac{1}{\Pi}] > 0.
\]

That is, \( (1 - \gamma)p_\omega(\Pi - 1) + p\Pi T - p[1 + s] + \frac{[1 - p(1 - \tau - \theta)\alpha]}{\alpha} (1 + r) - p(1 - \gamma)(1 + r) - 1] \Pi T > 0. \]

That is, \((1 - \gamma)p_\omega(\Pi - 1) + (1 - \gamma)p\Pi T + (1 - \gamma) \frac{[1 - p(1 - \tau - \theta)\alpha]}{\alpha} (1 + r) - p(1 - \gamma)(1 + r) - 1] \Pi T > 0. \]

That is, \((1 - \gamma)p_\omega(\Pi - 1) + (1 - \gamma)p\Pi T + (1 - \gamma) \frac{[1 - p(1 - \tau - \theta)\alpha]}{\alpha} (1 + r) - p(1 - \gamma)(1 + r) - 1] \Pi T > 0. \]

\[
(1 - \gamma) = (1 - \gamma)(1 + r) - \frac{1}{\Pi} \frac{\partial[\delta(1 - \frac{2i}{\kappa})]}{\partial (1 + r)}.
\]

\[
\text{Proof.} \ [\text{Theorem 2.3.1}] \text{ The valuation equation for productive agents, (2.18), gives}
\]
\[
v = \frac{\beta(1 - \tau - \theta)\alpha(1 - p)(1 + r)}{(1 + r - \theta\alpha) - \beta(1 - \tau - \theta)\alpha}.
\]

Using (2.17), \((1 + s) = \frac{1}{\beta(1 + r - \theta\alpha) - \beta(1 - \tau - \theta)\alpha} \]
\[
\text{This shows equations (2.23), (2.24) and (2.25). Moreover,}
\]
\[
E_{\text{S}^{\text{prof}}} = (1 - \gamma)(1 + r) - \frac{\beta^{-1} \beta(1 - \tau - \theta)\alpha(1 - p)(1 + r)}{p(1 + r - \theta\alpha) - \beta(1 - \tau - \theta)\alpha(1 + r) - p(1 - p)]} = \frac{(1 - \gamma)(1 + r) - \beta^{-1} \beta(1 - \tau - \theta)\alpha(1 + r)(1 - p)}{1 - p}.
\]
Also, substituting (2.22) back into (2.19),

\[
(1 + s) = \frac{(1 - \gamma)(1 + r) + \frac{1-p(1-r-\theta)\alpha}{\theta\alpha}(1+r)-p(1-\gamma)(1+r)-1}{p\omega(\Pi-1)+p\Pi^T + b(\Pi-1)^T}\n\]

Thus, using (2.19) again,

\[
ES_{mkt} = \frac{(1-p(1-r-\theta)\alpha)(1+r) - p(1-\gamma)(1+r) - 1}{p\omega(\Pi-1) + p\Pi^T + \frac{b(\Pi-1)^T}{1+s}} (1-\gamma)(1+r)\Pi - 1
\]

Equating (2.36) to (2.37), we obtain a quadratic equation since the third order terms cancel, giving the desired expression for \((1+r)^*\). Expression (2.27) follows from (2.22). To get the level of debt, substitute equation (2.19) into (2.20), to obtain

\[
z = \frac{p\omega + \frac{\rho b}{1+s}}{1-p(1-r-\theta)\alpha(1+r)-p(1-\gamma)(1+r)-1-p\delta(1-\frac{2}{\alpha})}. \quad \text{The remaining variables follow by substitution.}
\]

\[\diamond \]

### 2.10 References


Athey, Susan, Andrew Atkeson and Patrick Kehoe; 2003; “The Optimal Degree of Discretion in Monetary Policy ” *Working Paper Federal Reserve Bank of Minneapolis*


Chari, V.V., Lawrence Christiano and Patrick Kehoe; 1996; “Optimality of the Friedman rule in economies with distorting taxes”; Journal of Monetary Economics Vol. 37, 203-223.

Debreu, Gerard; 1987; Theory of Value, Yale University Press.


Hart, Oliver and John Moore; 1994; “A Theory of Debt based on the Inalienability
of Human Capital”; Quarterly Journal of Economics .


Chapter 3

Do the pecking-order’s predictions follow from its premises?

3.1 Introduction

We examine the effect of asymmetric information on the evolution of corporate investment and financing using an empirically testable dynamic structural model. We do so by embedding a privately informed manager in a neoclassical investment framework with costly default. When private information is bad, the manager overinvests (relative to first-best), has negative leverage, and uses dividend cuts or equity flotations to fill any financing gaps. When private information is good, debt provides a positive signal. However, in many states the good type issues equity despite having access to default-free debt.
In addition, the good type may overinvest. These predictions contradict the pecking-order folk wisdom. The effects in our model are absent from static models which fail to account for the fact that asymmetric information raises the value of future cash inflows from investment and the shadow cost of cash outflows associated with debt service. Consistent with empirical observation, the simulated firm exhibits mean-reverting leverage and a negative leverage-lagged cash flow relationship. When we mimic traditional reduced-form regressions, coefficient values are inconsistent with what is commonly imputed to pecking-order theory. In addition, “market-timing” proxies are insignificant despite the fact that managers want to exploit mispricing. These results call into question standard reduced-form regressions used to test the significance of informational asymmetry for investment and financing behavior.

As argued by Myers (1984), the two leading theories of corporate financial behavior are the trade-off and pecking-order theories. The former posits that firms choose their financial mix by equating tax benefits of debt with bankruptcy costs at the margin. The latter posits that asymmetric information causes firms to follow a simple rule of thumb for financing: internal funds should be used first, followed by debt, followed by external equity. Given the prominence of these theories, it comes as little surprise that a voluminous empirical literature has emerged comparing their ability to explain the stylized facts. For examples, see Shyam-Sunder and Myers (1999) and Fama and French (2002).

*In this paper, we ask two fundamental questions related to the validity and meaning of*
empirical tests of the pecking-order hypotheses. In an economy with asymmetric information, will firms act in the way that Myers and Majluf (1984) and Myers (1984) predict? Relatedly, will firms optimizing under asymmetric information generate regression coefficients that are consistent with those predicted by advocates of the pecking-order?

In order to address these questions, we develop a dynamic structural model embedding a privately informed manager in a neoclassical investment framework with costly default. The model is rich in that it endogenizes investment, debt, savings, equity flotations, dividends, and share repurchases. In addition to providing clear empirical predictions, our model also fills an important void in the literature. There is a long line of dynamic structural models of the trade-off theory. However, we are unaware of any dynamic structural models that capture the type of ex ante informational asymmetries emphasized by Myers and Majluf (1984), Leland and Pyle (1977) and Ross (1977). This state of affairs has not gone without notice. For example, after presenting his variant of dynamic trade-off theory, Ross (2005) argues that “The introduction of the issues raised by the presence of asymmetric information in the determination of the capital structure and the integration of these issues into the intertemporal neoclassical model are a major challenge.”

Some background will be useful in framing the results of our analysis. Empirical tests of the pecking-order have focused on the following predictions, based upon the arguments contained in the seminal paper by Myers and Majluf (1984): (P1) debt is always preferred to equity; (P2) firms use debt in order to fill any financing gaps; (P3)
firms only issue equity when the costs of distress become acute; (P4) dividends are sticky and do not adjust in order to fill financing gaps; (P5) asymmetric information induces firms to invest less than they would in an economy with symmetric information; (P6) leverage is declining in lagged profits; and (P7) firms hoard cash in order to avoid distortions associated with asymmetric information.

We begin with a standard neoclassical investment framework where the firm is endowed with a stochastic concave profit function. Each period, the firm is run by a single-period manager who observes a profit shock before outside investors. Following Myers and Majluf (1984), the manager works in the interest of risk-neutral insider-shareholders who do not buy or sell shares in the event of equity flotations or share repurchases. Under these assumptions, equity flotations represent a negative signal and share repurchases represent a positive signal, since they respectively serve to decrease and increase the percentage ownership of insiders. The manager is free to save and can borrow using a standard single-period debt contract. Financing needs are endogenous as the manager also chooses dividends and investment in the interest of insiders.

Although the manager works for a single-period, he is forward-looking and recognizes that his decisions affect net worth and equity value in the subsequent period. Thus, the endogenous evolution of net worth and the exogenous evolution of the firm’s “type” represent the underlying source of dynamics. In the model, there is an infinite sequence of signaling games played between a privately informed manager and uninformed investors. We characterize the least-cost separating (perfect Bayesian) equilibria for each level of
net worth. The equilibria of the games are properly capitalized into the equity value function by exploiting recursive features of the model.

Our main finding is that firms of either type will exhibit blatant violations of pecking-order predictions P1-P5, whereas P6 and P7 do emerge as equilibrium phenomena in our asymmetric information economy. In equilibrium, when private information is bad, the manager overinvests, has negative leverage, and uses dividend reductions or new equity issues in order to fill any financing gap. When private information is good, debt provides a positive signal. However, in many states the good type issues equity despite having access to default-free debt. In addition, the good type may overinvest.

As one would expect, the regression coefficients generated by a simulated panel of firms are inconsistent with those commonly imputed to pecking-order theory. In particular, in our simulated panel, where each type is assumed to be equally probable, debt issuance only fills 44% of the financing gap, with the remainder of the gap filled by endogenous reductions in dividends or flotations of new equity. This contradicts predictions P1-P4. The simulated model does support P6 and P7 in that firms often hoard cash and exhibit a negative leverage-lagged cash flow relationship. Also consistent with empirical observation, the simulated firms exhibit mean-reverting leverage–behavior often attributed to trade-off theory cum transactions costs.

There is a close relationship between the pecking-order story advocated by Myers and Majluf (1984) and the market-timing story of Baker and Wurgler (2002). In particular,
both stories are predicated on the notion that managers would like to issue overvalued securities in order to transfer gains to current shareholders. Using simulated data, we run regressions similar to those reported by Baker and Wurgler. In simulated data, the coefficient on their market-timing variable (external finance weighted average q) is actually insignificant. Based on this evidence, we argue that there is no reason to interpret a significant market-timing variable as indicative of managers actually attempting to time the market. In a rational economy at least, market-timing managers do not generate significant market-timing coefficients. Hennessy and Whited (2005) show that a market-timing manager is not a necessary condition for a significant market-timing variable, since the trade-off theory can generate significant market-timing regression coefficients. Taken together this evidence shows that a market-timing manager is neither necessary nor sufficient to generate significant coefficients on “market-timing” proxies.

It is worthwhile to contrast the results of our model with those emerging from static models and to pinpoint the cause of differences. A key insight provided in this paper is that static models of financing under asymmetric information are likely to overstate the case for debt and to overstate the reduction in investment because they effectively assume that the market imperfection vanishes once the first (and only) period is over. This procedure effectively imposes the restriction that the shadow value of a dollar of future internal funds (net worth) is simply a dollar. However, in a world with asymmetric information, managers recognize that future internal funds are worth more than a dollar if they allow the firm to avoid adverse selection costs. This consideration encourages firms
to invest more than the full-information first-best, since current investment generates future internal funds. The same consideration encourages saving and discourages debt.

Effectively, asymmetric information introduces concavity (pseudo-risk-aversion) into the value function of the manager, despite the fact that he is risk-neutral. Intuitively, internal funds are especially valuable in low net worth states when the firm is concerned that the flotation of large blocks of securities will send a negative signal. Considerations of efficiency demand that the investor insure the pseudo-risk-averse manager by lowering the firm’s debt commitment. However, this endogenous risk-aversion also causes the investor to view debt finance as a positive signal. Thus, there is a trade-off between efficient risk-sharing and information revelation.

The other point emphasized by our model is that the literature tends to focus too heavily on predictions regarding firm behavior when private information is good. If one is taking a theory to the data, it must be recognized that private information can be bad as well as good. With this in mind, we note that in the least-cost separating equilibrium, there is no sense in imposing costs on the low type. Therefore, when private information is bad, the firm will be allowed to issue equity on fair terms in order to build up its cash buffer-stock. In addition to precautionary motives, this fact explains why the simulated firms in our model issue more debt than predicted by the pecking-order.

At this point we provide a summary of closely related papers. The interested reader is referred to Tirole (2006) for a comprehensive survey. The pecking-order hypotheses
have been formally assessed in a number of single-period models. Noe (1988) shows that if the manager has no uncertainty regarding terminal cash flow, then P1 holds in a pooling equilibrium that satisfies standard refinements. Nachman and Noe (1994) identify necessary and sufficient conditions for debt to be the unique source of financing in a pooling equilibrium. A limitation of the Nachman and Noe framework is that it prohibits share repurchases. This is not without loss of generality. For example, Constantinides and Grundy (1990) consider a setting with variable project scale and show that share repurchases allow the firm to costlessly overcome adverse selection provided default is costless. DeMarzo and Duffie (1999) derive sufficient conditions for debt to be an optimal separating contract.

Our model is in the spirit of the signaling literature in corporate finance, which was pioneered by Ross (1977) and Leland and Pyle (1977). Although the source of deadweight loss is different, the least-cost separating equilibrium in our model has a clear analog with that constructed in the single-period model of Ambarish, John and Williams (1987). In our model, investment scale and debt are used as signals. Their model uses investment scale and taxable dividends as signals. In both models, the optimal mix of signals for the high type equates the ratio of distortion to information content across signals. Further, in both models, the low type is given an allocation that maximizes his payoff ignoring incentive constraints. In our model, the low type allocation still differs from what would emerge under symmetric information, due to endogenous precautionary saving and overinvestment incentives.
In the interest of clarity, we note that a number of recent papers have made progress in characterizing optimal contracts in dynamic settings when there is asymmetric information \textit{ex post} regarding true cash flows. For examples, see DeMarzo and Fishman (2003) and Biais et al. (2006). Our model differs fundamentally from these papers in that the timing of informational asymmetry is different. Our model is in the spirit of Myers and Majluf (1984) in that the manager enjoys superior information \textit{ex ante}, while cash flow is observable \textit{ex post}. The alternative models are in the spirit of Bolton and Scharfstein (1990), where the manager privately observes end-of-period cash flows.

The remainder of this paper is organized as follows. Section 2 presents assumptions on technology and describes the structure of the signaling game. Section 3 characterizes the least-cost separating equilibrium. In section 4, we use simulated model data to evaluate the empirical implications of informational asymmetry.

### 3.2 Economic Environment

#### 3.2.1 Technology and Timing

Time is discrete and the firm’s horizon is infinite. There is a risk-free asset paying a constant rate of interest \( r > 0 \). All agents are risk-neutral and share a common discount factor \( \beta \equiv (1+r)^{-1} \). Capital \( (k) \) decays exponentially at rate \( \delta \in [0,1] \). The following two assumptions describe the production technology and timing of information revelation.
Assumption 1. Operating profits are $\theta \varepsilon k^\alpha$ where $\alpha \in (0, 1)$. The shock $\theta$ takes values in the set $\{\theta_L, \theta_H\}$ with $0 \leq \theta_L < \theta_H$ and follows a first-order Markov process with $\pi_{ij}$ denoting the probability of $\theta_i$ conditional on lagged type $\theta_j$. The shock $\theta$ is persistent with $1 > \pi_{HH} \geq \pi_{HL} > 0$. The idiosyncratic shock $\varepsilon$ is independently and identically distributed in the set of positive real numbers $\mathbb{R}_+$. The density function $f : [\varepsilon, \infty) \to [0, 1]$ is continuous and differentiable.

Assumption 2. At the start of each period, the current $\theta$ is privately observed by the manager. Financing and investment is then determined. At the end of the period, $\varepsilon$ is revealed simultaneously to the manager and the investor. Realized output is observable, allowing the investor to infer $\theta$ at the end of the period.

Under the stated timing assumptions, the manager is “one-step-ahead” of the market. This approximates the environment facing managers. Managers know they observe some information about the firm prior to the market, but they also understand that such information will eventually become public. The model captures this economic reality in a tractable manner. The stated timing assumptions are similar to those adopted by Lucas and McDonald (1990), who also impute to the manager one-step-ahead knowledge.

The firm can raise external funds by borrowing or issuing additional shares of equity. The borrowing technology consists of a one-period debt contract, analogous to those featured in the models of Cooley and Quadrini (2001) and Hennessy and Whited (2006). The face value of debt for the type $i$ firm is denoted $b_i$. After observing this period’s
Idiosyncratic shock $\varepsilon$, *but prior to the revelation of next period’s* $\theta$, the manager decides whether to default. If the firm delivers the promised debt payment, shareholders retain ownership. In the event of default, the now symmetrically-informed lender (per Assumption 2) renegotiates with the firm. The lender has full ex post bargaining power and extracts all bilateral surplus by demanding a payment that leaves the firm indifferent between continuing or not. This set of assumptions allows us to derive endogenous default thresholds, analogous to those obtained from smooth-pasting conditions in continuous-time models. Although the manager makes his default decision prior to observing next period’s $\theta$, the persistence of $\theta$ implies that a manager with type $\theta_H$ exhibits a greater willingness to pay the firm’s debt.

The variable $w$ denotes *realized net worth*, which is the sum of capital net of depreciation plus operating profits less debt service.

$$w \equiv (1 - \delta)k + \theta\varepsilon k^\alpha - b. \quad (3.1)$$

There are two state variables in the model: the firm’s lagged type (exogenous) and its *revised net worth* ($\tilde{w}$) which is endogenous. Revised net worth is equal to realized net worth if the firm does not default. If the firm defaults, the lender extracts a payment equal to the maximum amount consistent with limited liability. This leaves a defaulting firm with a type-contingent minimal level of net worth. The set of possible values of revised net worth is denoted $\tilde{W}_j$, where $j$ indexes the manager’s type at the time the debt contract was signed.
Suppose that we are at the start of period \( t \), but that the manager has not yet observed \( \theta \) for period \( t \). The total market value of shareholder’s equity, conditional upon having drawn type \( j \) in the prior period \((t - 1)\) is denoted \( v_j : \tilde{W}_j \to \mathbb{R}_+ \). The default threshold (for net worth) for a firm that is of type \( j \) at the time of loan inception is denoted \( w^d_j \). The limited liability (endogenous default) condition can be stated as

\[
v_j(w^d_j) = 0. \tag{3.2}
\]

It is easily verified that \( v_j \) is strictly increasing in our model. Thus, equation (3.2) determines a unique default threshold for each borrower type. Further, the option value inherent in the firm implies that \( w^d_j < 0 \) for \( j \in \{L, H\} \). Intuitively, the ability to exploit future positive NPV projects ensures that firms are willing to continue even when net worth is negative. Below, we will speak of \( -w^d_j \) as representing the “going-concern” value of the firm.

In the special case where the shock \( \theta \) is i.i.d., the analysis simplifies since there is no need to keep track of the lagged type as a state variable. In particular,

\[
\theta \text{ i.i.d. } \Rightarrow v_H = v_L \equiv v \Rightarrow w^d_H = w^d_L \equiv w^d. \tag{3.3}
\]

In our model, the true \( \theta \) will be revealed to the investor each period in a separating equilibrium. However, to establish separation, it will be necessary to consider the payoffs to type-\( i \) that receives the allocation of type-\( j \) where \( j \) does not necessarily equal \( i \). Consider first incentives to default on the debt obligation. Let \( \varepsilon^d_{ij} \) denote the critical value of \( \varepsilon \) such that type-\( i \) would default given that it has received the capital stock and
debt obligation of type-\( j \). Now recall that the firm always has the option to default at the end of the period and walk out of the renegotiation with net worth \( w_{i}^{d} \). Therefore, the manager will find it optimal to default for any realization of \( \varepsilon \) such that realized net worth falls below \( w_{i}^{d} \). It follows that

\[
(1 - \delta)k_j + \theta_i \varepsilon_{ij}^d k_j^\alpha - b_j \equiv w_{i}^{d} \Rightarrow \varepsilon_{ij}^d \equiv \frac{b_j - (1 - \delta)k_j + w_{i}^{d}}{\theta_i k_j^\alpha}. \tag{3.4}
\]

Of course, \( \varepsilon_{ij}^d \leq \bar{\varepsilon} \) implies there is zero probability of default for type-\( i \) that has received the allocation of type-\( j \).

Myers (1984) states that “the modified pecking order story recognizes both asymmetric information and costs of financial distress.” The existence of default costs is supported by the empirical studies of Weiss (1990) and Andrade and Kaplan (1998), for example. To account for default costs, we assume that a fraction \( (\phi) \) of going-concern value is spent on legal fees in the event of post-default renegotiations between the firm and the lender.

We now compute the value of lender recoveries in the event of default. Recall that in the event of default the lender demands a revised payment from the firm, call it \( b_i^r \neq b_i \), that leaves the firm with revised net worth equal to \( w_{i}^{d} \). Therefore, we compute \( b_i^r \) using

\[
(1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha - b_i^r = w_{i}^{d} \Rightarrow b_i^r = (1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha - w_{i}^{d}. \tag{3.5}
\]

Recall that \(-w_{i}^{d} > 0\) represents the going-concern value of the firm. Thus, equation (3.5) tell us that the lender seizes the firm’s physical assets, all operating profits, and the going-concern value. This leaves equity with a continuation value of zero, consistent with
the absolute priority rule. The lender’s net recovery in the event of default is computed as:

\[ b_i^c + \phi w_i^d = (1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha - (1 - \phi)w_i^d. \tag{3.6} \]

In the model, the firm is also allowed to save \((b < 0)\). Myers and Majluf (1984) and Myers (1984) argue that firms should attempt to maintain financial slack in order to reduce deadweight losses associated with obtaining external funds under asymmetric information. Shyam-Sunder and Myers (1999) argue that “tax or other costs of holding excess funds” serve as a counterweight to precautionary saving incentives. To account for such costs in a tractable manner, we assume the firm incurs an up-front cost of \(\gamma b^2/2\) on deposits into a corporate savings account that offers a gross yield \(r\). Below, the variable \(\chi\) is an indicator function for \(b < 0\).

Assumption 3 summarizes the firm’s borrowing/savings technology.

**Assumption 3.** Default is endogenous. In the event of default, the lender has all ex post bargaining power. The deadweight default cost for a firm of type-\(i\) is a fraction \((\phi)\) of going-concern value \((-w_i^d)\). The gross yield on corporate saving is \(r\), with the firm incurring a cost of \(\gamma b^2/2\) up-front on saving deposits.

In the interest of brevity, let \(\Omega^{ij}\) denote the expected discounted end-of-period value of shareholders’ equity for a type-\(i\) that has taken the type-\(j\) allocation. We have:

\[ \Omega^{ij} \equiv \beta \int_{\varepsilon_{ij}^d}^{\infty} v_i[(1 - \delta)k_j + \theta_i \varepsilon k_j^\alpha - b_j]f(\varepsilon)d\varepsilon. \tag{3.7} \]
Also, let $\rho_i$ denote the market price of the debt issued by type $i$ in equilibrium.

$$\rho_i \equiv \beta \left[ b_i \int_{\varepsilon_i^u}^{\infty} f(\varepsilon) d\varepsilon + \int_{\varepsilon_i^l}^{\varepsilon_i^d} [(1 - \delta)k_i + \theta_i \varepsilon k_i^a - (1 - \phi)w_i^d] f(\varepsilon) d\varepsilon \right].$$  \hspace{1cm} (3.8)

### 3.2.2 Managerial Objectives

Each period, the firm is run in the interest of a set of single-period insider-shareholders. For simplicity, we label the insider-shareholders as “the manager.” As described in the next assumption, managerial objectives are identical to those assumed by Constantinides and Grundy (1989).

**Assumption 4.** Each period, the firm is run by a risk-neutral insider-manager who holds a fraction of outstanding shares. The manager does not buy additional shares in the event of an equity flotation and does not tender shares in the event of a share repurchase.

The assumption of risk-neutrality rules out risk exposure as a source of credible signaling, in contrast to the model of Leland and Pyle (1977), for example. The manager holds $m$ shares of stock. The total current number of shares outstanding at the start of the period, inclusive of manager shares, is $c > m$. The number of new shares issued is denoted $n$, with $n < 0$ implying that the firm is conducting a share repurchase. We define the variable $s$ as $s \equiv n/(c + n)$. Shares are issued and repurchased ex dividend.

Our manager-based modelling framework is used solely for descriptive clarity. More
generally, the same equilibrium would be obtained in an economy where managers maximize the value of the claims held by a set of dominant “current shareholders” (e.g. institutional investors) that will neither buy nor sell shares during the current period. It is worth noting that Myers and Majluf (1984) also adopt the assumption that the manager works in the interest of “passive” shareholders who do not alter their stakes even as the firm alters the number of shares outstanding.1

Dividends are denoted $d$ and are constrained to be nonnegative. Absent such a constraint, the firm could avoid the costs associated with informational asymmetries by having shareholders inject their own funds directly. Under the stated assumptions, the manager receives a fraction $m/c$ of total dividends and holds an equity stake of $m/(c+n)$ at the end of the period. Therefore, if the manager draws $\theta_i$ at the start of the period, he will choose policies to maximize

$$
\left(\frac{m}{c}\right)d + \left(\frac{m}{c+n}\right)\beta \int_\varepsilon^\infty v_i[(1-\delta)k + \theta_i\varepsilon k^\alpha - b]f(\varepsilon)d\varepsilon.
$$

(3.9)

It will be convenient to think of the manager as choosing between alternative “allocations.” An allocation $(a)$ is simply a vector $a \equiv (b, d, k, s)$. Suppose now that the manager draws $\theta_i$ at the start of the period. Using the definition of $s$, the objective function for the privately informed manager (3.9) simplifies, with the type-$i$ manager choosing the allocation that maximizes

$$
\left(\frac{m}{c}\right)d + (1-s)\beta \int_\varepsilon^\infty v_i[(1-\delta)k + \theta_i\varepsilon k^\alpha - b]f(\varepsilon)d\varepsilon.
$$

(3.10)

1See page 189 of Myers and Majluf (1984) for a discussion of this assumption.

2We have dropped a multiplicative term $(m/c)$ since it is irrelevant for incentive compatibility.
At this point it is worth noting that the vector $a$ contains all variables relevant to the informed manager’s payoff function. Of course, investor beliefs will affect the set of feasible allocations.

Assume the type-$i$ manager reveals the firm’s true type by choosing a separating allocation $(b_i, d_i, k_i, s_i)$. If the firm issues new shares ($n > 0$), the equity flotation is worth

$$s_i \left[ \beta \int_{d_i}^{\infty} v_i[(1 - \delta)k_i + \theta_i \varepsilon k_i^\alpha - b_i] f(\varepsilon) d\varepsilon \right].$$

(3.11)

It is perhaps less obvious to see that the expression in (3.11) also represents the cash outflow from a share repurchase, in which case $n$ and $s$ are both negative. To demonstrate this claim, consider a simple example. Note first that the bracketed term in (3.11) is the expected discounted value of shareholder’s equity at the end of the period. Suppose this amount is equal to 100. Suppose also that $c = 50$ and $n = -10$. After the share repurchase, each remaining share will be worth $100/(50 - 10) = 2.5$. Therefore, in order to induce ten shareholders to tender, the firm must pay $25(= 10 \times 2.5)$. Now note that $s_i \times 100 = -(10/40) \times 100 = -25$. By construction, each outsider shareholder is indifferent between tendering or not at the margin.

### 3.2.3 The Signaling Game

Although our model is dynamic, each period’s manager can be properly viewed as playing a one-shot signaling game with an investor. In each signaling game, the manager moves
first, offering an allocation $a$ to the investor. The investor then updates his beliefs and either accepts or rejects the allocation. If the investor accepts the offer, the allocation $a$ is implemented. If the offer is rejected, the firm is then constrained to finance with internal funds.

Each manager is forward-looking by construction. In particular, the value functions $(v_L, v_H)$ entering the manager’s payoff function (3.10) will be constructed to correctly capitalize the outcome of all signaling games played by future managers. The equilibrium concept is perfect Bayesian equilibrium (PBE). A PBE imposes the following requirements: the manager makes an optimal offer given the investor’s beliefs; the investor’s beliefs must be obtained through Bayesian updating wherever possible; and the investor accepts an offer only if his expected profits are weakly positive given his beliefs. We shall confine attention to the least-cost separating PBE in pure strategies. In a separating PBE, the investor will be able to perfectly identify the firm’s type upon observing the manager’s offer. The focus on separating equilibria is motivated by the empirical focus of this paper. One of the main appeals of the asymmetric information paradigm in corporate finance is its ability to explain announcement effects associated with security issuances.\(^3\) A model focused on pooling equilibria cannot explain these announcement effects, but one focused on separating equilibria can.\(^4\)

\(^3\)For example, Asquith and Mullins (1986) document a negative relation between the size of equity issues and share prices. Dann (1981) and Vermaelen (1984) document a positive reaction to share repurchases.

\(^4\)Alternatively, one can view our model as solving for the Rothschild-Stiglitz-Wilson allocation dis-
We begin by defining a broad set of technologically feasible allocations $\mathcal{A}$:

$$\mathcal{A} \equiv \{(b, d, k, s) : d \geq 0, k \geq 0, s \leq 1\}.$$

To derive the least-cost separating equilibrium we begin by solving Program L.\textsuperscript{5}

**PROGRAM L:** \(\max_{a \in \mathcal{A}} d_L + (1 - s_L)\beta \int_{\varepsilon_{LL}}^{\infty} v_L[(1 - \delta)k_L + \theta_L \varepsilon k_L^o - b_L]f(\varepsilon)d\varepsilon\)

subject to the following budget constraint

$$BC_L : d_L + k_L + \gamma b_L^2/2 - \tilde{w} \leq \beta \left[ s_L \int_{\varepsilon_{LL}}^{\infty} v_L[(1 - \delta)k_L + \theta_L \varepsilon k_L^o - b_L]f(\varepsilon)d\varepsilon + b \int_{\varepsilon_{LL}}^{\infty} f(\varepsilon)d\varepsilon \right. + \left. \int_{\tilde{z}}^{\varepsilon_{dLL}} [(1 - \delta)k_L + \theta_L \varepsilon k_L^o]f(\varepsilon)d\varepsilon \right].$$

Letting the solution to Program L be denoted \(a^*_L \equiv (b^*_L, d^*_L, k^*_L, s^*_L)\), we next solve Program H.

**PROGRAM H:** \(\max_{a \in \mathcal{A}} d_H + (1 - s_H)\beta \int_{\varepsilon_{HH}}^{\infty} v_H[(1 - \delta)k_H + \theta_H \varepsilon k_H^o - b_H]f(\varepsilon)d\varepsilon\)

subject to the budget constraint

$$BC_H : d_H + k_H + \gamma b_H^2/2 - \tilde{w} \leq \beta \left[ s_H \int_{\varepsilon_{HH}}^{\infty} v_H[(1 - \delta)k_H + \theta_H \varepsilon k_H^o - b_H]f(\varepsilon)d\varepsilon + b \int_{\varepsilon_{HH}}^{\infty} f(\varepsilon)d\varepsilon \right. + \left. \int_{\tilde{z}}^{\varepsilon_{dHH}} [(1 - \delta)k_H + \theta_H \varepsilon k_H^o]f(\varepsilon)d\varepsilon \right].$$

This program is solved for net worth levels sufficiently high such that the low type can satisfy $BC_L$. The low type gets a zero payoff if net worth is sufficiently low such that $BC_L$ cannot be met.
\[ \beta \left[ s_H \int_{\epsilon_H}^{\infty} v_H[(1-\delta)k_H + \theta_H\epsilon k_H^\alpha - b_H]f(\epsilon)d\epsilon + b \int_{\epsilon_H}^{\infty} f(\epsilon)d\epsilon \right] + \int_{\epsilon}^{\epsilon_H} [ (1-\delta)k_H + \theta_H\epsilon k_H^\alpha - (1-\phi)w_H^H]f(\epsilon)d\epsilon \]

and a no-mimic constraint

\[ NMLH : \quad d_L^* + (1-s_L^*)\beta \int_{\epsilon_L}^{\infty} v_L[(1-\delta)k_L^* + \theta_L\epsilon(k_L^*)^\alpha - b_L^*]f(\epsilon)d\epsilon \geq d_H + (1-s_H)\beta \int_{\epsilon_H}^{\infty} v_H[(1-\delta)k_H + \theta_H\epsilon k_H^\alpha - b_H]f(\epsilon)d\epsilon. \]

Notice that in solving Program L we did not impose a no-mimic constraint. Lemma 1 shows this is without loss of generality.\(^6\)

Lemma 1. Assume that \(a_H^*\) and \(a_L^*\) solve Program H and Program L respectively, and that \(s_L^* \geq 0\). Then

\[ d_H^* + (1-s_H^*)\beta \int_{\epsilon_H}^{\infty} v_H[(1-\delta)k_H^* + \theta_H\epsilon(k_H^*)^\alpha - b_H^*]f(\epsilon)d\epsilon \geq d_L^* + (1-s_L^*)\beta \int_{\epsilon_L}^{\infty} v_L[(1-\delta)k_L^* + \theta_L\epsilon(k_L^*)^\alpha - b_L^*]f(\epsilon)d\epsilon. \]

Proof. See Appendix A.

\(^6\)In solving Program L, we may confine attention to \(s_L \geq 0\) without loss of generality. To see this, note that a share repurchase by the low type can be replaced with a dividend without affecting the value of the objective function.
Let \( \hat{\theta}(a) \) denote the firm type inferred by the investor conditional upon receiving an arbitrary offer \( a \). The PBE can be supported by the following investor beliefs.

\[
\hat{\theta}(a^*_H) = \theta_H \tag{3.12}
\]

\[
\hat{\theta}(a^*_L) = \theta_L
\]

\[
\hat{\theta}(a) \in \arg \min_{\theta \in \{\theta_L, \theta_H\}} \int_{\varepsilon}^{\infty} v_\theta[(1 - \delta)k + \theta \varepsilon k^\alpha - b]f(\varepsilon)d\varepsilon + b \int_{\varepsilon}^{\infty} f(\varepsilon)d\varepsilon
\]

\[
+ \int_{\varepsilon}^{\infty} [(1 - \delta)k + \theta \varepsilon k^\alpha - (1 - \phi)w^d_{\theta}]f(\varepsilon)d\varepsilon
\]

On the equilibrium path, the beliefs in (3.12) are consistent with Bayes’ rule. Off the equilibrium path, the investor imposes “worst-case” beliefs in the sense of Brennan and Kraus (1987). In particular, the investor attaches the lowest possible valuation to any package of securities not issued in equilibrium. If the firm were to issue shares and/or debt, a worst-case belief imputes type \( \theta_L \). As another example, if the firm were to repurchase shares (and issue no debt), then a worst-case belief imputes type \( \theta_H \).

We now verify that the solutions to Programs L and H in conjunction with beliefs (3.12) constitute a PBE. First note that the beliefs are consistent with Bayes’ rule on the equilibrium path. Second, note that any offer \( a_0 \notin \{a^*_L, a^*_H\} \) that would be acceptable to the investor necessarily satisfies both \( BC_L \) and \( BC_H \), because the beliefs minimize the value of the package of securities.

We now verify that each type will choose his type-specific allocation. Consider first the low type’s incentive to offer some allocation \( a_0 \notin \{a^*_L, a^*_H\} \) that would be acceptable to the investor given the beliefs in (3.12). Since \( a_0 \) is acceptable, it must satisfy \( BC_L \).
Thus, \( a_0 \) was in the feasible set for Program L and the low type must prefer \( a^*_L \) to \( a_0 \).

This same argument shows that the low type will not make an offer that is rejected, since such an allocation is equivalent to getting zero outside funding, while zero outside funding is always acceptable to the investor. Therefore, the low type prefers the offer \( a^*_L \) to all other allocations other than \( a^*_H \). Finally, \( NM_{LH} \) ensures the low type prefers \( a^*_L \) to \( a^*_H \).

Consider next the high type’s incentive to offer some allocation \( a_0 \neq a^*_H \) that would be accepted by the investor given the beliefs in (3.12). Note that the allocation \( a_0 \) was in the feasible set when we solved Program H. To see this, note that \( BC_H \) is necessarily satisfied, since the offer is acceptable even with worst-case beliefs. Since \( a_0 \) also satisfies \( BC_L \), we know \( a_0 \) is in the feasible set for Program L. The optimality of \( a^*_L \) in Program L implies that \( NM_{LH} \) is satisfied. Thus the high type prefers \( a^*_H \) to \( a_0 \). This same argument shows that the high type will not make an offer that is rejected, since such an allocation is equivalent to getting zero outside funding, while zero outside funding is always acceptable to the investor.

The final step in the construction will be to define the equity value function recursively, using

\[
v_j(w) \equiv \pi_{Hj} \left[ d^*_H + (1 - s^*_H) \beta \int_{s^*_HH}^{\infty} v_H[(1 - \delta)k^*_H + \theta_H \varepsilon(k^*_H)^\alpha - b^*_H] f(\varepsilon) d\varepsilon \right] + (1 - \pi_{Hj}) \left[ d^*_L + (1 - s^*_L) \beta \int_{s^*_LL}^{\infty} v_L[(1 - \delta)k^*_L + \theta_L \varepsilon(k^*_L)^\alpha - b^*_L] f(\varepsilon) d\varepsilon \right]
\]

(3.13)
3.3 Equilibrium

The full-information first-best investment policy solves

$$
k_i^{FB} \in \arg\max_k \beta \int_{\varepsilon}^{\infty} [(1 - \delta)k + \theta_i \varepsilon k^\alpha] f(\varepsilon) d\varepsilon - k
\Rightarrow k_i^{FB} = \left[ \frac{\alpha \theta_i E(\varepsilon)}{r + \delta} \right]^{1/(1-\alpha)}.
$$

(3.14)

The following remark provides a description of the full-information economy. This provides a useful benchmark for assessing the relative allocative efficiency of the economy where managers have private information.

Remark. If the profit shock $\theta_i$ is public information, default is costly ($\phi > 0$), and corporate saving is costly ($\gamma > 0$), then the firm implements first-best investment ($k_i^{FB}$) using a combination of equity and default-free debt. There will be no corporate saving ($b_i \geq 0$).

Although they do not emphasize the point, one of the key premises of Myers and Majluf (1984) and Myers (1984) is the existence of costs of default. The critical role of this assumption is illustrated in the following subsection, which shows that the firm can implement first-best investment with costless signaling through debt issuance provided that default is costless. Essentially, when default is costless, the firm relies entirely upon debt to signal good information.
3.3.1 The Possibility of Costless Separation

Constantinides and Grundy (1990) adopt the assumption that default is costless. In their static model, they show that a firm with variable investment scale can costlessly implement the first-best investment policy even if the set of financing instruments is limited to equity and standard debt. In their model, the issuance of any security is a negative signal. However, equity is more sensitive to the manager’s private information than debt. First-best is achieved with the firm issuing debt in excess of the amount needed to fund the investment. The excess funds are then used to finance a share repurchase. Effectively, the negative signal content of the debt flotation is just compensated by the positive signal provided by the share repurchase. Of course, the probability of default is likely to be high under such a separating policy. However, this has no effect on total firm value under their adopted assumption of costless default.

Consider now our dynamic model. Let \( p_{i}^{FB} \) denote the net present value of investment for a firm that implements the first-best investment policy:

\[
p_{i}^{FB} \equiv \beta[(1 - \delta)k_{i}^{FB} + \theta_i E(\varepsilon)(k_{i}^{FB})^\alpha] - k_{i}^{FB}.
\] (3.15)

For a firm that can implement the full-information first-best each period using fairly-priced external financing (with no deadweight losses), the type-contingent going-concern value of the firm \((-w_{L}^{d*}, -w_{H}^{d*})\) can be found as the solution to the following system

\[-w_{H}^{d*} = \pi_{HH}[p_{H}^{FB} - \beta w_{H}^{d*}] + (1 - \pi_{HH})[p_{L}^{FB} - \beta w_{L}^{d*}]
\] (3.16)

\[-w_{L}^{d*} = \pi_{HL}[p_{H}^{FB} - \beta w_{H}^{d*}] + (1 - \pi_{HL})[p_{L}^{FB} - \beta w_{L}^{d*}].\]
Assumption 1 guarantees that the solution to the above system satisfies

\[ w_{H}^{d^*} \leq w_{L}^{d^*}. \]  

(3.17)

Finally, we note that if a firm can implement first-best investment using fairly-priced external financing with zero default cost then equity value is linear in internal resources, with

\[ v_i(\bar{w}) = v_i^{FB}(\bar{w}) \equiv \bar{w} - w_i^{d^*}. \]

This brings us to Proposition 1, which provides weak conditions on primitives such that the firm implements first-best provided that \( \phi = 0 \).

**Proposition 1.** If the idiosyncratic shock \( \varepsilon \) is bounded above by \( \bar{\varepsilon} \) and there are no costs of default, the firm achieves the full-information first-best valuation \( v_i^{FB} \) implementing first-best investment \( k_i^{FB} \) for \( i \in \{L, H\} \). If \( \bar{w} < k_{H}^{FB} \), the low type sets \( b_L = 0 \) and the high type sets \( b_H = (1 - \delta)k_{H}^{FB} + \theta_L\bar{\varepsilon}(k_{H}^{FB})^\alpha - w_i^{d^*} \). If \( \bar{w} \geq k_{H}^{FB} \), both firms finance investment using internal funds exclusively.

Proof. Fix \( k_i = k_i^{FB} \) and choose \( s_i \) to satisfy \( BC_i \). The proposed set of financing policies clearly solves Programs L and H ignoring the \( NM_{LH} \) constraint and we need only verify it is satisfied. Suppose first \( \bar{w} < k_{H}^{FB} \). Consider the above policies and set \( d_L = \max\{0, \bar{w} - k_{L}^{FB}\} \) and \( d_H = 0 \) with \( s_i \) satisfying \( BC_i \). But note that \( NM_{LH} \) is slack since

\[ d_L + (1 - s_L)\Omega_{LL} \geq (1 - s_H)\Omega_{LH} = 0. \]
Suppose next that $\tilde{w} \geq k^FB_H$. Then $NM_{LH}$ is satisfied with $s_i = b_i = 0$ and $d_i = \max\{0, \tilde{w} - k^FB_i\}$. To see this, note that each type is financing with internal funds in this case so the low type must be better off choosing $k^FB_L$. $\blacksquare$

The proof of Proposition 1 is worthy of further discussion since it runs parallel to Proposition 4 from Constantinides and Grundy (1990). Under the policies specified in our proposition, the high type is able to costlessly separate from the low type by raising the face value of its debt sufficiently high such that an imposter low type would default regardless of the realized value of $\varepsilon$. Of course, such a high debt payment increases the likelihood of default by the high type. However, the assumption that $\phi = 0$ ensures this is of no consequence. Of course, this method of separation becomes suspect once one introduces costs of default, which is one of the stated premises of Myers and Majluf (1984) and Myers (1984).

The following result shows that $\phi = 0$ is not a necessary condition for costless separation of types in our model, although the proposition does suggest that costless separation with $\phi > 0$ requires fairly strong restrictions on the underlying production technology.

**Proposition 2.** If capital has a use-life of one period ($\delta = 1$) and the low type cannot use the capital productively ($\theta_L = 0$), then the firm achieves full-information first-best valuation $v^{FB}_i$ implementing first-best investment $k^FB_i$ for $i \in \{L, H\}$ using only external equity to fill any financing gap.
Proof. Fix \( k_i = k_i^{FB} \); \( b_i = 0 \); \( s_L = 0 \); \( d_L = \tilde{w} \); \( d_H = \max\{0, \tilde{w} - k_H^{FB}\} \), with \( s_H \) determined according to \( BC_H \). Clearly, this set of policies solves Programs L and H ignoring the \( NM_{LH} \) constraint. Depending on the firm’s net worth, the \( NM_{LH} \) constraints are

\[
\tilde{w} \leq k_H^{FB} \quad \tilde{w} - w_L^{ds} \geq -(1 - s_H)w_L^{ds}
\]

\[
\tilde{w} > k_H^{FB} \quad \tilde{w} - w_L^{ds} \geq \tilde{w} - k_H^{FB} - w_L^{ds}.
\]

Both constraints are satisfied.

As a limiting case, Proposition 2 is of some interest for the analysis that follows. Under the stated assumptions, the low type places a value of zero on any physical capital. Therefore, the low type has no interest in raising funds through an equity issuance, despite the fact that it would seem to benefit from issuing overvalued equity. However, the money raised in the equity issuance is of no value to the low type since the funds go to unproductive capital.

We turn our attention in the next subsection to the nature of equilibrium when the firm cannot achieve the full-information first-best valuation.

### 3.3.2 Low Type Policies

In this subsection and the next, it is assumed that \( \phi > 0 \) which is necessary to preclude the non-distorting equilibrium described in Proposition 1. In order to express the optimality conditions compactly, we define some additional variables. It will be convenient
to compute the marginal effect of $b$ on the expected discounted end-of-period value of shareholders’ equity for a high-type (low-type) that has taken the high-type allocation. We have

$$\Omega_{b}^{HH} \equiv -\beta \int_{\epsilon_{H}^{HH}}^{\infty} v'(H)[(1 - \delta)k_H + \theta_H k_H^{\alpha} - b_H]f(\epsilon)d\epsilon$$  \hspace{1cm} (3.18)

$$\Omega_{b}^{LH} \equiv -\beta \int_{\epsilon_{L}^{HH}}^{\infty} v'(L)[(1 - \delta)k_L + \theta_L k_H^{\alpha} - b_H]f(\epsilon)d\epsilon.$$  

Similarly, we may compute the marginal effect of $k$ on the expected discounted end-of-period value of shareholders’ equity for a high-type (low-type) that has taken the high-type allocation as follows:

$$\Omega_{k}^{HH} \equiv \beta \int_{\epsilon_{H}^{HH}}^{\infty} v'(H)[(1 - \delta)k_H + \theta_H k_H^{\alpha} - b_H][1 - \delta + \alpha \theta_H k_H^{\alpha - 1}]f(\epsilon)d\epsilon$$  \hspace{1cm} (3.19)

$$\Omega_{k}^{LH} \equiv \beta \int_{\epsilon_{L}^{HH}}^{\infty} v'(L)[(1 - \delta)k_L + \theta_L k_H^{\alpha} - b_H][1 - \delta + \alpha \theta_L k_H^{\alpha - 1}]f(\epsilon)d\epsilon.$$  

We will also need to introduce the indicator function $\Phi$ denoting a firm whose equity will be worth zero if the low type is realized in the subsequent period. Of course, equity value hinges upon net worth, so we shall write $\Phi(w)$. Similarly, we shall let $\mu$ denote the multiplier on the $NM_{LH}$ constraint and write it as $\mu(w)$ to emphasize that this multiplier is also contingent upon the net worth.

In order to interpret the optimality conditions in this model, it will be useful to have a sense of the magnitude of the shadow value of internal resources. This is the subject of Lemma 2.
Lemma 2. The shadow value of internal resources on the equilibrium path is

\[ v'_L(w) = 1 + \pi_{HL}\mu(w)(\Omega^{HH} - \Omega^{LH})/\Omega^{HH} \]  
(3.20)

\[ v'_H(w) = \pi_{HH}[1 + \mu(w)(\Omega^{HH} - \Omega^{LH})/\Omega^{HH}] + (1 - \pi_{HH})[1 - \Phi(w)]. \]  
(3.21)

Proof. See Appendix A.

In order to provide some intuition for Lemma 2 we must foreshadow some results that follow. We begin first with a discussion of equation (3.20), which quantifies the shadow value of internal funds for a firm with lagged type \( \theta_L \). On the equilibrium path, a firm with lagged type \( \theta_L \) will always have strictly positive equity value in the subsequent period regardless of \( \varepsilon \) and regardless of the observed type. This is because the low type saves (\( b_L \leq 0 \)) implying that it will enter the subsequent period with \( w > 0 \) regardless of the \( \varepsilon \) shock. If the realized type in the subsequent period is \( \theta_L \), a dollar of internal funds is just worth a dollar, as the firm essentially obtains financing on fair terms. However, if the realized type in the subsequent period is \( \theta_H \), a dollar of internal funds will be worth more than a dollar if it allows the firm to avoid costs associated with asymmetric information. These effects are clearly illustrated in (3.21) as \( v'_L > 1 \) when the \( NM_{LH} \) constraint binds.

Consider next equation (3.21), which quantifies the shadow value of internal funds for a firm with lagged type \( \theta_H \). In contrast to the low type, the high type may take on debt in order to signal its type (\( b_H > 0 \)). If the firm then experiences a sufficiently low
draw of $\varepsilon$, end-of-period net worth can be negative. If the next realized type is $\theta_L$, it may be optimal to shut down. This gives rise to a variant of the Myers (1977) debt overhang problem, in that the possibility of shut-down in bad states causes the firm to place a lower value of internal funds, ceteris paribus. However, equation (3.21) tells us that the shadow value of internal funds will still exceed unity if the precautionary motive swamps the overhang effect.

Lemma 2 is of fundamental importance for our dynamic theory of investment and financing under asymmetric information. In particular, standard two-period models of the firm implicitly force $v' = 1$. This is because a firm that simply vanishes is never forced to confront informational asymmetries after the initial round of financing is obtained. Hence, in static models, the value of a dollar received at the terminal date of the firm is simply a dollar. In contrast, in a forward-looking framework, the manager recognizes that a dollar of internal funds in the future can have precautionary value.

The Lagrangian for Program $L$ is

$$L = d_L + (1 - s_L) \beta \int_{\varepsilon_{LL}}^{\infty} v_L[(1 - \delta)k_L + \theta_L \varepsilon k_{LL}^o - b_L] f(\varepsilon) d\varepsilon +$$

$$\lambda_L \{ \bar{w} - d_L - k_L - \chi \gamma b_L^2 / 2 + \beta s_L \int_{\varepsilon_{LL}}^{\infty} v_L[(1 - \delta)k_L + \theta_L \varepsilon k_{LL}^o - b_L] f(\varepsilon) d\varepsilon$$

$$+ \beta b_L \int_{\varepsilon_{LL}}^{\infty} f(\varepsilon) d\varepsilon + \beta \int_{\varepsilon_{LL}}^{\varepsilon_{LL}} [(1 - \delta)k_L + \theta_L \varepsilon k_{LL}^o - (1 - \phi) w_L^d] f(\varepsilon) d\varepsilon \}$$

$$+ \eta_L d_L + \psi_L (1 - s_L).$$

\footnote{Technically, this occurs when the low type cannot satisfy $BC_L$ even if $s_L = 1$ and $d_L = 0$. In contrast, on the non-default region, equity must have strictly positive value if $\theta_H$ is drawn.
The first-order conditions for \(d_L\) and \(s_L\) are

\[
1 - \lambda_L + \eta_L = 0 \quad (3.22)
\]
\[
(\lambda_L - 1)\Omega^{LL} - \psi_L = 0. \quad (3.23)
\]

From equations (3.22) and (3.23) it follows that \(\eta_L\Omega^{LL} = \psi_L\). This brings up two relevant scenarios. Suppose first that \(\psi_L > 0\). It follows that \(\eta_L > 0\) and \(d_L = 0\). It follows that the low type is getting a payoff of zero since \(s_L = 1\). If \(\varepsilon\) has bounded support, the constraint \(NM_{LH}\) is then satisfied by having the high type take on a sufficient amount of debt such that the low type would default in all states. If \(\varepsilon\) has unbounded support, \(\psi_L(w) > 0\) can never occur for \(w\) on the continuation region. To see this, note that the proposed equilibrium would entail the low type getting zero. But then the low type would always opt for the high type allocation unless \(s_H = 1\) and \(d_H = 0\). Of course, this implies both types get a payoff of zero which contradicts being on the continuation region. For the remainder of this subsection we confine attention to the economically interesting case where \(\psi_L = 0\).

The first-order condition pinning down \(b_L\) is

\[
\beta \left[ \int_{\varepsilon_L}^{\infty} [v'_L((1 - \delta)k_L + \theta_L\varepsilon k_L^{\alpha} - b_L) - 1] f(\varepsilon) d\varepsilon \right] = -\chi \gamma b_L + \beta \frac{\partial \varepsilon^d_{LL}}{\partial b_L} f(\varepsilon^d_{LL}) \phi w^d_L. \quad (3.24)
\]

From Lemma 2 it follows that the left side of (3.24) is positive, and strictly so provided that \(NM_{LH}\) will bind with positive probability in the subsequent period. It follows that \(b_L\) must be negative. The optimality condition for \(k_L\) takes a similar form, with

\[
\beta \left[ \int_{\varepsilon_L}^{\infty} [v'_L((1 - \delta)k_L + \theta_L\varepsilon k_L^{\alpha} - b_L)(1 - \delta + \alpha\theta_L\varepsilon k_L^{\alpha-1}) f(\varepsilon) d\varepsilon \right] = 1.
\]
The next proposition follows directly from the first-order conditions for the debt and capital of the low type.

**Proposition 3.** If the manager observes \( \theta_L \), the firm overinvests relative to first-best with \( k_L > k_L^{FB} \) if there is a positive probability of \( N M_{LH} \) binding in the subsequent period.

The low type engages in precautionary saving with

\[
b_L = -\frac{\beta}{\gamma} \left[ \int_{\epsilon}^{\infty} \left[ v'_L ((1 - \delta)k_L + \theta_L \epsilon k^a_L - b_L) - 1 \right] f(\epsilon) d\epsilon \right].
\] (3.25)

Dividends and equity issuance for the low type are contingent upon net worth with

\[
\tilde{w} < k_L - \beta b_L + \gamma b^2_L / 2 \Rightarrow d_L = 0 \text{ and } s_L > 0
\]

\[
\tilde{w} \geq k_L - \beta b_L + \gamma b^2_L / 2 \Rightarrow d_L = \tilde{w} - (k_L - \beta b_L + \gamma b^2_L / 2) \text{ and } s_L = 0.
\]

The intuition for the low type policies are as follows. The fact that the low type would benefit from security mispricing causes \( N M_{LH} \) to be the key incentive constraint. In order to discourage imitation by the low type, the least-cost separating equilibrium makes the low type as well off as possible. This is a standard feature of signaling models. However, in most signaling models, the optimal policy entails giving the low type the full-information first-best allocation. This does not hold in our dynamic model. In our model, the low type is given a “second-best” allocation which accounts for the fact that the shadow value of internal resources exceeds unity. Consequently, the low type overinvests relative to first-best and engages in precautionary saving despite the fact that
such saving entails deadweight losses. It is also worth noting that the both $b_L$ and $k_L$ are invariant to net worth ($\tilde{w}$). Effectively, the low type satisfies $BC_L$ by varying dividends and equity issuance only. Note, \textit{this is the exact opposite of the pecking-order prediction that debt (and only debt) is used to achieve budget balance.}

### 3.3.3 High Type Policies

The Lagrangian for Program H is

$$L = d_H + (1 - s_H)\beta \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1 - \delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H]f(\varepsilon) d\varepsilon$$

$$+ \lambda_H \{\bar{w} - d_H - k_H - \chi \gamma b_H^2/2 + \beta s_H \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1 - \delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H]f(\varepsilon) d\varepsilon\}$$

$$+ \beta b_H \int_{\varepsilon_{HH}^d}^{\infty} f(\varepsilon) d\varepsilon + \beta \int_{\varepsilon_{HH}^d}^{\infty} [(1 - \delta)k_H + \theta_H \varepsilon k_H^\alpha - (1 - \phi)w_H^d]f(\varepsilon) d\varepsilon\}$$

$$+ \mu \{d_L^* + (1 - s_L^*)\beta \int_{\varepsilon_{HL}^d}^{\infty} v_L[(1 - \delta)k_L^* + \theta_L \varepsilon (k_L^*)^\alpha - b_L^*]f(\varepsilon) d\varepsilon\}$$

$$- d_H - (1 - s_H)\beta \int_{\varepsilon_{HL}^d}^{\infty} v_L[(1 - \delta)k_H + \theta_L \varepsilon k_H^\alpha - b_H]f(\varepsilon) d\varepsilon\}$$

$$+ \eta_H d_H + \psi_H (1 - s_H).$$

The first-order conditions for $d_H$ and $s_H$ are

$$1 - \lambda_H - \mu + \eta_H = 0 \quad (3.26)$$

$$\lambda_H - 1\Omega_{HH}^H + \mu \Omega_{LH}^H - \psi_H = 0 \quad \Omega_{HH}^H = \Omega_{LH}^H \quad (3.27)$$

\footnote{Note, Program H is solved only if the low type achieves a strictly positive continuation value in Program L. The preceding subsection discussed the nature of equilibrium for lower net worth levels.}
Substituting (3.26) into (3.27), we obtain

\[ \eta_H \Omega^{HH} = \mu(\Omega^{HH} - \Omega^{LH}) + \psi_H. \]  

(3.28)

It is straightforward to establish \( \psi_H(w) = 0 \) on the continuation region \( (w > w^d_j) \). To see this, suppose to the contrary that \( \psi_H > 0 \). It follows from (3.28) that \( \eta_H > 0 \) and \( d_H = 0 \). Thus, the high type gets a payoff of zero. However, the low type would then also receive an allocation with a payoff of zero since the \( NM_{HL} \) constraint demands

\[ d_H + (1 - s_H)\Omega^{HH} \geq d_L + (1 - s_L)\Omega^{HL} \geq d_L + (1 - s_L)\Omega^{LL}. \]  

(3.29)

But this contradicts \( v_j(w) > 0 \). Without loss of generality we shall treat \( \psi_H = 0 \) as we solve for optimal policies on the firm’s continuation region.

Rearranging (3.28) we obtain

\[ \eta_H = \mu[(\Omega^{HH} - \Omega^{LH})/\Omega^{HH}]. \]  

(3.30)

Condition (3.30) is of fundamental importance. It tells us that whenever \( NM_{LH} \) binds, the manager would be better off if the firm could implement a rights-issue in which shareholders are paid a negative dividend. That is, when \( NM_{LH} \) binds, the manager would be better off if the firm could avoid turning to outside investors for funds. The adverse selection problem is manifest in condition (3.30), with the term in squared brackets representing the relative difference in true equity values for the two types. From this optimality condition it also follows that the high type will never pay a dividend when \( NM_{LH} \) binds since

\[ \mu > 0 \Rightarrow \eta_H > 0 \Rightarrow d_H = 0. \]  

(3.31)
The optimality condition pinning down $b_H$ is

$$\beta \left[ \int_{\varepsilon_{HH}}^{\infty} [v'_H((1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H) - 1] f(\varepsilon) d\varepsilon - \frac{\partial \varepsilon_{HH}^d}{\partial b_H} f(\varepsilon_{HH}^d) \phi w_H^d \right] + \chi b_H^2$$

$$= \left( \frac{\mu \Omega^{LH}}{\lambda_H} \right) [1 - s_H] \left[ \frac{\Omega_k^{HH}}{\Omega_k^{HH}} - \frac{\Omega_{b}^{LH}}{\Omega_{b}^{LH}} \right].$$

We now conjecture, and then verify, that the high type will choose $b_H \leq 0$ if the $NM_{LH}$ constraint is slack at the current level of net worth. Under the conjecture that $b_H \leq 0$, there is zero probability of shut-down regardless of next period’s realized type. It follows from Lemma 2 that $v'_H \geq 1$ in this case. From (3.32) it follows immediately that $\mu = 0 \Rightarrow b_H \leq 0$. Condition (3.32) leads directly to a key implication of the model, that signaling must be a concern if the firm issues a positive amount of debt. This conclusion is stated as Lemma 3.

**Lemma 3.** If the manager observes $\theta_H$ and there is a positive probability of $NM_{LH}$ binding in the subsequent period, then a necessary condition for $b_H > 0$ is that $NM_{LH}$ is binding in the current period.

The optimality condition pinning down $k_H$ is

$$1 - \beta \left[ \int_{\varepsilon_{HH}}^{\infty} [v'_H((1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H)] [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon \right] (3.33)$$

$$- \beta \left[ \int_{\varepsilon_{HH}}^{\varepsilon_{HH}^d} [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon + \frac{\partial \varepsilon_{HH}^d}{\partial k_H} f(\varepsilon_{HH}^d) \phi w_H^d \right]$$

$$= \left( \frac{\mu \Omega^{LH}}{\lambda_H} \right) [1 - s_H] \left[ \frac{\Omega_k^{HH}}{\Omega_k^{HH}} - \frac{\Omega_{k}^{LH}}{\Omega_{k}^{LH}} \right].$$

By way of contrast, the optimality condition in a full-information neoclassical economy
$1 - \beta \left[ \int_{\bar{\varepsilon}}^{\infty} [1 - \delta + \alpha \theta H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon \right] = 0.$

Anticipating the results of our numerical analysis, it is worth noting the appearance of the multiplicative term $v_H'$ in the capital optimality condition (3.33). If $v_H' > 1$, the desire to avoid adverse selection costs provides an added incentive for capital accumulation. However, the debt overhang effect in our model may also cause $v_H' < 1$, thus discouraging capital accumulation.

From (3.32) and (3.33) it also follows that

$$\mu = 0 \implies b_H = b_H^P \text{ and } k_H = k_H^P$$

where

$$b_H^P \equiv -\frac{\beta}{\gamma} \left[ \int_{\bar{\varepsilon}}^{\infty} [v_H'((1 - \delta)k_H^P + \theta H \varepsilon (k_H^P)^{1}\varepsilon - b_H^P) - 1] f(\varepsilon) d\varepsilon \right]$$

and

$$1 = \beta \left[ \int_{\bar{\varepsilon}}^{\infty} [v_H'((1 - \delta)k_H^P + \theta H \varepsilon (k_H^P)^{1}\varepsilon - b_H^P)] [1 - \delta + \alpha \theta H (k_H^P)^{1}\varepsilon] f(\varepsilon) d\varepsilon \right].$$

Proposition 4 spells out some important implications of the above optimality conditions.

**Proposition 4.** If $NM_{LH}$ is ever binding, then $\exists \bar{w}_0$ at which $NM_{LH}$ switches from binding to nonbinding. For all $\bar{w} > \bar{w}_0$, the high type overinvests relative to first-best with $k_H = k_H^P > k_H^{FB}$ and saves with $b_H = b_H^P$. For $\bar{w} > \bar{w}_0$, dividends and equity issuance are
contingent upon net worth with

\[ \tilde{w} < k_H^P - \beta b_H^P + \gamma (b_H^P)^2 / 2 \Rightarrow d_H = 0 \text{ and } s_H > 0 \]

\[ \tilde{w} \geq k_H^P - \beta b_H^P + \gamma (b_H^P)^2 / 2 \Rightarrow d_H = \tilde{w} - k_H^P - \beta b_H^P + \gamma (b_H^P)^2 / 2 \text{ and } s_H = 0. \]

Proof. See Appendix A.

Propositions 3 and 4 show that much of the folk-wisdom regarding the empirical content of asymmetric information is suspect. First, we note that the propositions show that it is possible for asymmetric information to induce both types of firms to \textit{overinvest} relative to first-best. Part of the causal mechanism behind the model’s prediction of overinvestment is that costs of adverse selection generate a precautionary motive for undertaking policies that generate \textit{future} internal funds. This mechanism is the sole source of the high type’s overinvestment incentive when \( NM_{LH} \) is slack. Anticipating results in the next subsection, signaling potentially provides an additional motive for overinvestment by the high type. We have already noted that the low type violates the pecking order’s prescription to use debt as the prime source of external funds. A second point worthy of note is that \textit{it is possible for even the high type to exhibit blatant violations of the pecking-order prescriptions in terms of financing}. In particular, when the financing gap is small and \( NM_{LH} \) is nonbinding, Proposition 4 shows that the high type simultaneously issues equity and saves.

Proposition 4 does not discuss optimal policies when \( NM_{LH} \) is binding. Discussion of the economic content of conditions \((3.32)\) and \((3.33)\) for \( \mu > 0 \) will be delayed until
the next subsection, which links the optimality conditions to single-crossing conditions common to the signaling and mechanism design literatures.

### 3.3.4 Single-Crossing Conditions

In this subsection we consider lower net worth states such that $NM_{LH}$ is binding, so that separation enters explicitly into the decision-making process of the high type. In the previous subsection it was established that $d_H = 0$ when $NM_{LH}$ binds. Let us now consider the normalized payoff $u(b, k, s; \theta_i)$ to a type-$i$ manager that takes some arbitrary allocation $(b, k, s)$ such that $d = 0$

$$u(b, k, s; \theta_i) \equiv (1 - s)\beta \int_{\epsilon_i}^\infty v_i[(1 - \delta)k + \theta_i \varepsilon k^\alpha - b]f(\varepsilon)d\varepsilon. \quad (3.34)$$

Next, we compute the total derivative of $u$, evaluated at the high type allocation. We have

$$du(b_H, k_H, s_H; \theta_i) = (1 - s_H)\Omega^{iH}_k \ast dk + (1 - s_H)\Omega^{iH}_b \ast db - \Omega^{iH} \ast ds. \quad (3.35)$$

Setting the derivative to zero, one obtains the slope of indifference curves evaluated at the high type allocation. The manager’s willingness to exchange equity ownership for additional capital is determined by

$$\frac{ds}{dk}(b_H, k_H, s_H; \theta_i) = \frac{(1 - s_H)\Omega^{iH}_k}{\Omega^{iH}}. \quad (3.36)$$
This indifference curve notation allows us to rewrite the optimality condition for the high-type capital stock (3.33) as

\[
1 - \beta \left[ \int_{\epsilon_{HH}^d}^{\infty} \left[ \int_{\epsilon_{HH}^d}^{\epsilon} \left[ v_H^d((1 - \delta)k_H + \theta_H \epsilon k_H^{\alpha - 1} - b_H)[1 - \delta + \alpha \theta_H \epsilon k_H^{\alpha - 1}]f(\epsilon) \, d\epsilon \right] \right] \, d\epsilon \right] (3.37)
\]

Due to diminishing marginal product of capital, the left-side of equation (3.37) is increasing in \( k_H \). Therefore, the optimality condition tells us that the high-type’s capital stock varies positively with the signal content of capital investment, as measured by the difference in the slope of the two type’s indifference curves in \( k-s \) space. For example, in Figure 3.1 the indifference curves are drawn under the assumption that the high type has a greater willingness to exchange equity for capital. In this case, higher capital investment provides a positive signal which encourages overinvestment relative to first-best.

Intuition suggests that four factors determine the relative slopes of the types’ indifference curves in \( k-s \) space. First, the high type generates more future cash than a low type for a given level of capital. Second, persistence in \( \theta \) implies that a high type has a high probability of being a high type in the subsequent period. For such a firm, the future internal cash generated by installed capital may be more valuable since internal funds reduce exposure to adverse selection costs. These first two effects serve to increase the slope of the high-type indifference curve. However, the low type knows his equity is less valuable than that of the high type, which increases his willingness to exchange equity for
This figure shows the indifference curves drawn under the assumption that the high type has a greater willingness to exchange equity for capital. In this case, higher capital investment provides a positive signal which encourages overinvestment relative to first-best.
capital. In addition, the low type may place a higher shadow value on a marginal dollar at the end of the period, since it necessarily realizes lower net worth when it receives the high-type allocation. The next subsection presents additional analysis of the signal content of capital investment under uniformly distributed idiosyncratic ($\varepsilon$) shocks.

The manager’s willingness to exchange equity for debt reductions is determined by

\[
\frac{ds}{db}(b_H, k_H, s_H; \theta_i) = \frac{(1 - s_H)\Omega^H_i}{\Omega^H_i}.
\]

(3.38)

Using this indifference curve relationship allows us to rewrite the debt optimality condition for the high type (3.32) as

\[
\beta \left[ \int_{\varepsilon_{HH}^L}^{\infty} [v'_H((1 - \delta)k_H + \theta_H\varepsilon k_H^\alpha - b_H) - 1]f(\varepsilon)d\varepsilon - \frac{\partial \varepsilon_{HH}^L}{\partial b_H}f(\varepsilon_{HH}^H)\phi w_H^d \right] + \gamma b_H = \left( \frac{\mu \Omega^L_H}{\lambda_H} \right) \left[ \left| \frac{ds}{db}(b_H, k_H, s_H; \theta_L) \right| - \left| \frac{ds}{db}(b_H, k_H, s_H; \theta_H) \right| \right].
\]

(3.39)

Equation (3.39) tells us that the high-type’s borrowing depends upon the signal content of debt, as measured by the difference in the slope of the indifference curves in $b$-$s$ space. Recall that when the constraint $NM_{LH}$ is slack, the high type will choose to save an amount $b_H^P < 0$ such that the left-side of (3.39) is equal to zero. Starting at this point, if the firm were to decrease its saving, the left-side of (3.39) would increase. With this in mind, consider the indifference curves in Figure 3.2. In Figure 3.2, the low type is assumed to be more willing to exchange equity for a reduction in debt. Therefore, debt provides a positive signal and the high type borrows more/saves less in order to separate from the low type. An exact treatment of the signal content of debt is provided in the following two subsections, which makes specific distributional assumptions regarding the
This figure shows the indifference curves drawn under the assumption that the low type is more willing to exchange equity for a reduction in debt. Therefore, debt provides a positive signal and the high type borrows more/saves less in order to separate from the low type.

\[ u(b, k, s; |\theta^L) = \text{const.} \]

\[ u(b, k, s; |\theta^H) = \text{const.} \]

\( \varepsilon \) shocks. Anticipating, we obtain unambiguous analytical predictions that debt provides a positive signal for both uniformly and exponentially distributed \( \varepsilon \) shocks.

Finally, we note that an efficient mix of real and financial signals equates the ratio of “distortion” to signal content at the margin. In particular, equations (3.37) and (3.39)
imply that

\[
1 - \beta \left[ \int_{\varepsilon}^{\infty} \left( v_H' \right) \ast \left[ 1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha - 1} \right] f(d\varepsilon) + \int_{\varepsilon}^{\varepsilon_H^d} \left[ 1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha - 1} \right] f(d\varepsilon) \right] \\
\frac{d\varepsilon}{d\theta_L}(b_H, k_H, s_H; \theta_L) - \frac{d\varepsilon}{d\theta_L}(b_H, k_H, s_H; \theta_H) \\
- \beta \left[ \frac{\partial \varepsilon_H^d}{\partial \varepsilon} f(\varepsilon_H^d) \phi w_H^d \right] \\
+ \frac{d\varepsilon}{d\varepsilon}(b_H, k_H, s_H; \theta_H) - \frac{d\varepsilon}{d\varepsilon}(b_H, k_H, s_H; \theta_L) \\
= \frac{\beta \left[ \int_{\varepsilon}^{\infty} \left( v_H' - 1 \right) f(d\varepsilon) - \frac{\partial \varepsilon_H^d}{\partial \varepsilon} f(\varepsilon_H^d) \phi w_H^d \right] + \chi \gamma b_H}{\left| \frac{d\varepsilon}{d\theta_L}(b_H, k_H, s_H; \theta_L) \right| - \left| \frac{d\varepsilon}{d\theta_L}(b_H, k_H, s_H; \theta_H) \right|}.
\]

### 3.3.5 Example: Uniformly Distributed Idiosyncratic Shocks

This subsection assumes \( \varepsilon \) is distributed uniformly on the interval \([\varepsilon, \overline{\varepsilon}]\), with \( \theta \) being i.i.d. We define

\[
\bar{w}_{ij} = (1 - \delta) k_j + \theta_i \varepsilon k_j^{\alpha} - b_j \\
\bar{w}_{ij} = (1 - \delta) k_j + \theta_i \overline{\varepsilon} k_j^{\alpha} - b_j.
\]

It will also be useful to note that for type-\( i \) taking the type-\( j \) allocation

\[
dw = \theta_i k_j^{\alpha} d\varepsilon \Rightarrow d\varepsilon = \frac{dw}{\theta_i k_j^{\alpha}}.
\]

Consider now the signal content of the high-type issuing an amount of debt such that \( \varepsilon < \varepsilon_H^d < \varepsilon_{LH}^d < \overline{\varepsilon} \).\(^9\) The change of variable in (3.41) allows us to rewrite the marginal

\(^9\)Similar results are obtained if one considers the issuance of safe debt by the high type. Here we confine the analysis to defaultable debt in the interest of brevity.
effect of debt on equity value, presented in equation (3.18), as follows

\[
\Omega_{ij}^b = -\beta \int_{v_{ij}}^{x} v'[ (1 - \delta) k_j + \theta \epsilon k_j^a - b_j ] (1 - \sigma - \epsilon) d\epsilon - v_{ij} d\epsilon
\]  
(3.42)

\[
= -\beta \frac{w_{ij} - w_{ij}}{v'(w)} \int_{w_{ij}}^{w_{ij}} v'(w) dw
\]

\[
= -\beta \frac{v(w_{ij})}{w_{ij} - w_{ij}}.
\]

Applying the same change of variable, it is readily verified that

\[
\Omega_{ij} = \frac{\beta}{w_{ij} - w_{ij}} \int_{w_{ij}}^{w_{ij}} v(w) dw.
\]  
(3.43)

It follows that the debt signaling term in (3.32) can be expressed as

\[
\frac{\Omega_{HH}^L - \Omega_{LH}^L}{\Omega_{HH}^L - \Omega_{LH}^L} = \frac{[v(w_{HH}) - v(w_{LH})]}{[v(w_{HH}) - v(w_{LH}) - v(w_{HH})]} - \frac{[v(w_{HH}) - v(w_{LH})]}{[v(w_{HH}) - v(w_{LH}) - v(w_{HH})]}.
\]  
(3.44)

From Lemma 2 and Proposition 4 we know \( v \) exhibits concavity as the shadow value of internal funds falls to unity for net worth sufficiently high. When the value function is concave, the numerator of the first term in (3.44) exceeds that of the second term. Next, note that the respective denominators are simply the average value of \( v \), which is strictly larger for the high type. It follows that the difference in (3.44) is positive, which implies that marginal increases in debt provide a positive signal under this subsection’s assumptions that the persistent \( \theta \) shocks are i.i.d. and the idiosyncratic \( \epsilon \) shocks are uniformly distributed.

The mathematical arguments in the preceding paragraph are closely related to the economic intuition regarding why debt issuance is a positive signal in this setting. First note that we invoked concavity of the value function \( v \). Concavity of the value function...
implies that the low type attaches a higher shadow cost to a dollar of future debt service, since the low type realizes lower net worth if it mimics the high-type. Second, the mathematical argument invoked the fact that the low type has lower equity value. Low equity value increases the low type’s willingness to part with equity in return for debt reductions. Both effects cause the low type’s \( b-s \) indifference curve to have a steeper slope.

Consider next the signal content of capital accumulation. Using the change of variable in (3.41), we obtain

\[
\frac{\Omega_{k}^{HH}}{\Omega_{k}^{LH}} - \frac{\Omega_{k}^{LH}}{\Omega_{k}^{HH}} = \left( \frac{\partial \pi_{kH}^{HH}}{\partial k_{H}^{*}} \right) \left( \frac{\partial \pi_{kH}^{LH}}{\partial k_{H}^{*}} \right) \left( \frac{\Omega_{b}^{HH}}{\Omega_{b}^{LH}} \right).
\]  (3.45)

Recall that the signal content of capital accumulation depends on the manager’s willingness to trade equity for capital. In the preceding subsection we argued that capital accumulation tends to have positive signal content since the high type has a higher marginal product of capital. This effect is captured in equation (3.45), with \( \partial \pi_{kH}^{HH} / \partial k_{H}^{*} > \partial \pi_{kH}^{LH} / \partial k_{H}^{*} \) increasing the difference between the high-type and low-type indifference curve slopes. However, it follows from (3.44) that \( \left| \Omega_{b}^{HH} / \Omega_{b}^{LH} \right| < \left| \Omega_{b}^{HH} / \Omega_{b}^{LH} \right| \), so the sign of the capital signaling expression (3.45) is ambiguous. Again, the mathematical argument runs parallel to the economic intuition. The fact that \( \left| \Omega_{b}^{HH} / \Omega_{b}^{LH} \right| < \left| \Omega_{b}^{HH} / \Omega_{b}^{LH} \right| \) was attributed to concavity of \( v \) and the fact that the low type has a lower average value of \( v \). Economically, concavity of \( v \) implies that the low type imputes a higher shadow value to a marginal dollar of future cash, such as that stemming from installed capi-
tal. In addition, the low expectation of \( v \) for the low type causes it to exhibit a greater willingness to exchange equity for capital.

### 3.3.6 Example: Exponentially Distributed Idiosyncratic Shocks

This subsection assumes \( \epsilon \) is exponentially distributed with \( f(\epsilon) \equiv \xi e^{-\xi \epsilon} \). Under this distributional assumption, we are able to obtain unambiguous results regarding the signal content of debt even allowing for correlated \( \theta \) shocks. This contrasts with the previous subsection, where we needed to assume that \( \theta \) was i.i.d. in order to obtain clear results.

In the interest of brevity, we confine attention to the signal content of debt, as the signal content of capital is impossible to sign analytically. We confine attention to the signal content of defaultable debt, although similar results are obtained when we consider the issuance of safe debt.

We begin by rewriting the expression for the marginal cost of debt (3.18) as follows

\[
\Omega_{ij}^b = \frac{-\beta}{\theta_i k_j^\alpha} \int_{\epsilon}^{\infty} f(\epsilon) \left( \frac{\partial}{\partial \epsilon} v_i ((1 - \delta)k_j + \theta_i \epsilon k_j^\alpha - b_j) \right) d\epsilon. \tag{3.46}
\]

Using integration by parts it follows that

\[
\Omega_{ij}^b = \frac{-\xi \Omega_{ij}^b}{\theta_i k_j^\alpha}. \tag{3.47}
\]

It follows directly that debt issuance is a positive signal, with

\[
\frac{\Omega_{b}^{HH}}{\Omega_{HH}} - \frac{\Omega_{b}^{LH}}{\Omega_{LH}} = \xi k_H^{-\alpha} (\theta_L^{-1} - \theta_H^{-1}) > 0. \tag{3.48}
\]
3.4 Numerical Simulation

The procedure used to solve the model numerically is presented in Appendix B. The assumed values of exogenous parameters are presented in Table 3.1. Once the model is solved, we use the wealth-contingent equilibrium policy functions \((a^*_L, a^*_H)\) to generate a panel data set for simulated firms. In particular, we draw 3000 samples consisting of 300 draws of the two profit shocks. We then use the policy functions generated by the model to determine shock-contingent policy paths. We drop the first 2970 periods, leaving us with a panel of 3000 firms with 30 years of data for each firm. The simulated regressions only use 20 years of data, since the construction of some variables requires the use of lagged data. The simulated panel data set is similar in size to those commonly used in empirical testing.

Figure 3.3 plots the equity value function \(v\). Since the \(\theta\) shocks in the simulation are assumed to be i.i.d., there is only one equity value function \((v_H = v_L = v)\). A couple of points are worth noting. First, \(v\) is concave, which implies that there are precautionary motives for savings and capital accumulation. Second, note that the firm continues even when realized net worth is negative. This stems from the fact that there is option value inherent in equity ownership.

Figure 3.4 plots the capital allocations of each type relative to first-best. This figure casts doubt on the conventional wisdom that asymmetric information leads to underinvestment. In the least-cost separating equilibrium, the low type invests roughly 9% more
than the first-best level, regardless of realized net worth. This overinvestment pattern reflects the fact that informational asymmetries create a precautionary motive for capital accumulation. The high type also invests more than first-best in all states, with the extent of the distortion decreasing in net worth. The overinvestment of the high type reflects both signaling and precautionary motives. When net worth is low, the no-mimic constraint binds and the high type overinvests in order to signal private information. When net worth is sufficiently high, the no-mimic constraint becomes slack, but the high type still overinvests a bit due to precautionary motives. The high type has a weaker precautionary motive than the low type, since, ceteris paribus, he will realize higher net worth for any given realized $\varepsilon$.

Figures 3.5 and 3.6 plot the wealth-contingent financing policies for each firm type. Consistent with Proposition 3, the low type uses dividends and equity issuance as the sole means of achieving budget-balance, while retaining a wealth-invariant level of savings. When net worth is low, the dividend is set to zero and the firm issues a large amount of equity. Equity issuance for the low type then declines monotonically in net worth. The debt of the high type declines monotonically in net worth. Effectively, higher net worth allows the high type to reduce its need for costly external funds. By way of contrast, the dollar value of equity issuance is non-monotonic in net worth. The rising part of the equity issuance curve is mechanical. As shown in Figure 3.7, the equity stake ($s_H$) sold by the high type stays roughly constant for low-intermediate net worth. Since the firm’s debt commitment falls with net worth, $\Omega^{HH}$ increases, leading to an increase in $s * \Omega$. 
Eventually, net worth becomes sufficiently high such that the high type begins cutting \( s_H \) which leads to a reduction in the value of equity flotations. Both types of firms only pay dividends if net worth is sufficiently high. This prediction is consistent with the observed positive relation between dividends and firm size.

Figure 3.8 depicts the evolution of the leverage ratio for an arbitrary simulated firm. Note that the leverage ratio always hits the same level, roughly -0.22, when the firm realizes a negative productivity shock \( (\theta_L) \). The leverage ratio rises when the firm experiences a positive shock. The leverage ratio in good states varies, depending upon the firm’s endogenous net worth. For example, the leverage ratio is approximately zero in year 15 despite the fact that the firm has drawn \( \theta_H \). The leverage ratio exceeds 0.20 in year 25, which reflects the fact that the firm has drawn \( \theta_H \) and has low net worth.

Upon seeing a data series like the one depicted in Figure 8, an advocate of trade-off theory might be tempted to view the firm as being governed by trade-off theory cum transactions costs, given that the leverage ratio looks to be mean-reverting. However, this conclusion would clearly be incorrect. In our model, the leverage ratio is mean-reverting. However, this is caused by endogenous fluctuations in net worth.

Table 3.2 presents summary statistics for the simulated firm. The leverage ratio is negative on average, although this property of the model is sensitive to the assumed probability of realizing \( \theta_H \) and the cost of cash retentions \( (\gamma) \). The average leverage ratio increases when we use higher probabilities of \( \theta_H \) and higher values of \( \gamma \). In this set of simulations, the firm issues defaultable debt infrequently, but issues equity frequently.
Consequently, there are frequent violations of the traditional pecking-order. In particular, roughly 26% of the simulated firms issue equity despite having access to default-free debt.

Table 3.3 reports the results of regressions that mimic those commonly found in the literature. In each of the four regressions, the dependent variable is the book leverage ratio. The first row mimics the specification in equation number 2 in Shyam-Sunder and Myers (1999). Essentially, this specification tests the pecking-order prediction that debt is used to fill any financing gaps. In the notation of our model, the financing gap is equal to desired capital plus dividends minus internal resources \((k + d - w)\). According to the pecking-order as traditionally specified, the predicted coefficient on the financing gap is one. Inspecting Table 3.2 we see that the simulated firms fill only 44% of the financing gap with debt. Although this point estimate is sensitive to the assumed probability of \(\theta_H\), the point that we want to stress is that there is no theoretical basis for expecting the coefficient on the financing gap to equal one in an economy with asymmetric information between managers and investors.

The second row of Table 3.3 mimics leverage regressions in Rajan and Zingales (1995). Consistent with empirical observation, and with the prediction of Myers (1984), in our model leverage does indeed decline in lagged liquidity. The causal mechanism in our model is as follows. Leverage is invariant to wealth if the firm draws \(\theta_L\), since the low type is given a second-best cash buffer stock in the least-cost separating equilibrium. However, conditional upon \(\theta_H\) being drawn, the leverage of the firm will decline with net worth.
The third row mimics the specification in equation number 3 in Shyam-Sunder and Myers (1999), regressing leverage on the difference between lagged leverage and the firm’s “target” where the target is computed as the sample average leverage ratio. Consistent with empirical observation, the mean-reversion coefficient is small but positive. In the simulated data, the mean-reversion coefficient is 0.27. By way of contrast, Shyam-Sunder and Myers report a mean-reversion coefficient of 0.33.

The last row mimics the market-timing regression of Baker and Wurgler (2002). In simulated data, the coefficient on their market-timing variable (external finance weighted average q) is actually insignificant. Based on this evidence, we argue that there is no reason to interpret a significant market-timing variable as indicative of managers actually attempting to time the market. In a rational economy with Bayesian updating by investors, market-timing managers do not generate significant market-timing coefficients. Given the behavioral spin Baker and Wurgler place on their results, it is likely that they would take exception with the notion that investors do engage in Bayesian updating. We here note that the well-documented existence of announcement effects is clearly inconsistent with the notion that investors stubbornly cling to their priors. Therefore, their story must be predicated upon some notion of misreaction.
3.5 Conclusions

There is little doubt that the informational asymmetries stressed by Myers and Majluf (1984) play an important role in corporate finance. As evidence, one may point to the large fees paid to underwriters in return for performing due diligence. As additional direct evidence, one may note the existence of announcement effects surrounding changes in corporate financial structure. However, this paper shows that the types of reduced-form regressions commonly found in the literature are uninformative about the validity of corporate financing theories predicated on asymmetric information. In particular, it has been shown that firms operating in an environment with asymmetric information do not necessarily generate the types of regression coefficients that advocates of the pecking-order predict.

In addition to this critique of econometric practice, the model generates theoretical predictions of independent interest. We show that concern over future adverse selection costs effectively converts a risk-neutral manager into a pseudo-risk-averse manager. This risk-aversion weakens the attractiveness of debt relative to what one obtains in a single-period model where (by construction) the firm only faces adverse selection once. This same risk-aversion adds to the signal content of debt. The optimal mix of debt and equity can be viewed as balancing efficient risk-sharing against information revelation. Thus, the results of our dynamic model have clear linkages with static contracting theory. This insight may prove useful in subsequent analysis, as recursive techniques can be used to
convert dynamic contracting problems into simpler static problems.
The equity value function $v$ is plotted as a function of the realized net worth, $w$. The productivity shock $\varepsilon$ is exponentially distributed and both productivity shocks are i.i.d. The parameter choices are reported in Table 3.1.
Optimal capital allocations, $k_i^*$, scaled by the first-best allocations, $k_i^{FB}$, are plotted as a function of the realized net worth, $w$, for both high and low values of $\theta$. The productivity shock $\varepsilon$ is exponentially distributed and both productivity shocks are i.i.d. The parameter choices are reported in Table 3.1.
Optimal financing policies: debt, $\rho_H$, and equity, $s_H^* \Omega^{HH}$, as well as optimal dividend policy, $d_H^*$, are plotted as functions of the realized net worth, $w$, for the case of high value of $\theta$. The productivity shock $\varepsilon$ is exponentially distributed and both productivity shocks are $i.i.d$. The parameter choices are reported in Table 3.1.
Figure 3.6: Optimal financing policies - low value of $\theta$

Optimal financing policies: debt, $\rho_L$, and equity, $s^*_L \Omega^{LL}$, as well as optimal dividend policy, $d^*_L$, are plotted as functions of the realized net worth, $w$, for the case of low value of $\theta$. The productivity shock $\varepsilon$ is exponentially distributed and both productivity shocks are i.i.d. The parameter choices are reported in Table 3.1.
Optimal percentage of equity sold to new shareholders, $s^*_i$, is plotted as a function of the realized net worth, $w$, for cases of both high and low values of $\theta$. The productivity shock $\varepsilon$ is exponentially distributed and both productivity shocks are $i.i.d$. The parameter choices are reported in Table 3.1.
Simulated time series of the firm-level leverage ratio defined as a ratio of debt, $\rho(t)$, to total assets, $k(t)$, is shown. This is a randomly chosen sample from the simulated panel of firms that contains 3,000 firms over 300 time periods, where only the last thirty time periods are kept for each firm. The productivity shock $\varepsilon$ is exponentially distributed and both productivity shocks are $i.i.d.$ The parameter choices are reported in Table 3.1.
Table 3.1: Parameter Choices

This table reports the values of parameters used in simulation. The profit shock $\varepsilon$ is distributed exponentially.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>1/1.065</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\xi$</td>
<td>2.0</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\theta_L$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\pi_H$</td>
<td>0.5</td>
</tr>
</tbody>
</table>
### Table 3.2: Summary Statistics from Simulated Firms

<table>
<thead>
<tr>
<th>Variable</th>
<th>All</th>
<th>High</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Leverage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>-0.0618</td>
<td>0.1032</td>
<td>-0.2236</td>
</tr>
<tr>
<td>median</td>
<td>-0.1126</td>
<td>0.0840</td>
<td>-0.2236</td>
</tr>
<tr>
<td>std</td>
<td>0.1789</td>
<td>0.1042</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Investment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.3258</td>
<td>3.0433</td>
<td>-2.3946</td>
</tr>
<tr>
<td>median</td>
<td>0.3247</td>
<td>3.1386</td>
<td>-2.4682</td>
</tr>
<tr>
<td>std</td>
<td>3.5922</td>
<td>2.3649</td>
<td>2.3617</td>
</tr>
<tr>
<td><strong>Frequency</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Equity Issuance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all debt</td>
<td>0.2633</td>
<td>0.2633</td>
<td>0.0000</td>
</tr>
<tr>
<td>default-free debt</td>
<td>0.2633</td>
<td>0.2633</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Dividend Payment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.7367</td>
<td>0.2368</td>
<td>0.4999</td>
</tr>
<tr>
<td><strong>Repurchasing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Table 3.2 presents summary statistics from simulated panel of firms. The simulated panel of firms is generated from the model and contains 3,000 firms over 300 time periods, where only the last thirty time periods are kept for each firm. The productivity shock $\varepsilon$ is exponentially distributed and both productivity shocks are i.i.d. The parameter choices are reported in Table 3.1. The first column reports statistics for all firms, while the next two columns report the results for firms with “high” and “low” values of $\theta$ separately.

The frequency of the event $A > 0$, $f(A)$, is defined as

$$f(A) = \frac{\sum_{i=1}^{N} \chi(A_i > 0)}{N},$$

where $\chi(A_i > 0)$ is an indicator function.
Table 3.3: Leverage Regressions

This table reports results of several regressions on the simulated data with leverage ratio, $\frac{\rho(t)}{k(t)}$, as the dependent variable. Here $q(t)$ is the Tobin’s $Q$ defined as $q(t) = \frac{v(t) + b(t-1)}{k(t-1)}$. The financing gap is defined as $\frac{d(t) + k(t) - w(t)}{k(t)}$. Operating profits are defined as $\theta(t) \varepsilon(t) k^{\alpha-1}(t)$. EFWAQ($t$) is a weighted average of each of the prior ten year’s $q$ ratios, with the weight on any given lagged $q$ equal to the market value of external finance in that year, $\rho_i(t)(1 - \chi) + s_i(t) \Omega_{ii}(t)$ divided by the total market value of external finance obtained over the entire ten year lag period. The simulated panel of firms contains 3,000 firms over 300 time periods, where only the last thirty time periods are kept for each firm. The productivity shock $\varepsilon$ is exponentially distributed and both productivity shocks are i.i.d. The parameter choices are reported in Table 3.1.

<table>
<thead>
<tr>
<th>Financing Gap</th>
<th>$q(t)$</th>
<th>Lagged Operating Profits</th>
<th>$E[\frac{q}{k}] - \frac{\rho(t-1)}{k(t-1)}$</th>
<th>EFWAQ($t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4416</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( 0.0315 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>0.0299</td>
<td>-0.0071</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>( 0.0131 )</td>
<td>( 0.0028 )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2683</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>( 0.0612 )</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>0.0267</td>
<td>-0.0816</td>
<td>-</td>
<td>-0.0138</td>
</tr>
<tr>
<td>-</td>
<td>( 0.0254 )</td>
<td>( 0.0118 )</td>
<td>-</td>
<td>( 0.0256 )</td>
</tr>
</tbody>
</table>

178
3.6 Appendix A: Proofs

Proof of Lemma 1.

Let \( a^*_L \) denote the solution to Program L. The allocation \( a^*_L \) was in the feasible set when the type-H program was solved. To verify, the \( NM_{LH} \) constraint would be trivial as the allocation would be type-independent. Since \( BC_L \) is satisfied at \( a^*_L \) we know \( BC_H \) would be slack. Optimality then demands that the payoff to the high type must be at least as high as what would he would obtain under \( a^*_L \). Therefore:

\[
d^*_H + (1 - s^*_H)\Omega^{HH} \geq d^*_L + (1 - s^*_L)\Omega^{HL}.
\]

Proof of Lemma 2.

Consider first a firm with lagged type \( \theta_L \). Since \( w > 0 \) at the start of the next period (on the equilibrium path), equity will have positive value even if the next period’s type is also \( \theta_L \). Applying the Envelope Theorem, the value of internal funds if the low type is realized is \( \lambda_L = 1 \). The value of a dollar of internal funds to the high type is given by \( \lambda_H + \mu \). To see this, it must be noted that the low type’s equilibrium payoff (which enters the \( NM_{LH} \) constraint) can be expressed as \( w + \kappa^* \). This explains why the shadow value of internal funds to the high type is not simply \( \lambda_H \). Rather, one must account for the fact that higher wealth adds slack to the \( NM_{LH} \) constraint. Taking a probability weighted average of these values yields (3.20). The derivation of expression (3.21) is identical,
except that one must account for the fact that a dollar of internal funds is worth zero if the low type is drawn following a high type and the equity becomes worthless.

**Proof of Proposition 4.**

We begin by demonstrating

$$\mu(\hat{w}) = 0 \Rightarrow \mu(w) = 0 \ \forall \ w > \hat{w}.$$  

From the Envelope Theorem

$$\frac{\partial}{\partial \hat{w}} \left[ d^*_L + (1 - s^*_L)\beta \int_{-\infty}^{\infty} v_L[(1 - \delta)k^*_L + \theta_L\varepsilon(k^*_L)^{\alpha} - b^*_L]f(\varepsilon)d\varepsilon \right] = 1.$$

Next, consider that \(NM_{LH}\) demands

$$d_L + (1 - s_L)\beta \int_{\hat{w}}^{\infty} v_L[(1 - \delta)k_L + \theta_L\varepsilon(k_L)^{\alpha} - b_L]f(\varepsilon)d\varepsilon \geq d_H + (1 - s_H)\Omega^{LH}.$$  \hspace{1cm} (3.49)

The left-side has a slope of one in wealth. Suppose now that \(\mu(\hat{w}) = 0\) which implies that the high type implements \((b^*_H, k^*_H)\) at \(\hat{w}\). Now consider the slope of the right-side of \(NM_{LH}\) under the conjecture that the constraint remains nonbinding as \(\hat{w}\) increases. Under the hypothesis that \(NM_{LH}\) remains nonbinding, the high type continues to implement \((b^*_H, k^*_H)\) which are invariant to \(\hat{w}\). The slope of the right-side of (3.49) may then be computed as

$$\frac{d}{d\hat{w}}[d_H + (1 - s_H)\Omega^{LH}] = \frac{\partial d_H}{\partial \hat{w}} - \Omega^{LH} \frac{d}{d\hat{w}}s_H.$$  \hspace{1cm} (3.50)

If \(\partial d_H/\partial \hat{w} > 0\), it follows from (3.31) that \(\mu = 0\). Suppose instead that \(\partial d_H/\partial \hat{w} \leq 0\). From \(BC_H\) it follows that

$$\frac{d}{d\hat{w}}s_H = -1 + \frac{\partial d_H}{\partial \hat{w}}.$$  \hspace{1cm} (3.51)
Substituting (3.51) into (3.50) one obtains
\[
\frac{\partial d_H}{\partial \tilde{w}} - \frac{ds_H}{d\tilde{w}} \Omega^{LH} = \frac{\Omega^{LH}}{\Omega^{HH}} + \frac{\partial d_H}{\partial \tilde{w}} \left[ 1 - \frac{\Omega^{LH}}{\Omega^{HH}} \right] < 1.
\]
Thus, the left-side of (3.49) has a steeper slope than the right and the $NM_{LH}$ constraint remains nonbinding as conjectured. The high-type allocation for $\mu = 0$ follows directly from (3.32) and (3.33).

3.7 Appendix B: Details of Algorithm

The computational procedure is based on value function iteration. The individual steps are as follows. The idiosyncratic shock $\varepsilon$ is implemented by discretizing its domain using $N$ possible values. Each maximization is implemented by discretizing the domain of the decision variables.

1. Guess “going-concern” values $w^d_j$ of the firm.

2. Guess value functions $v_j$ of the firm.

3. Solve for the low type allocation $a_L$ that maximizes the value of the low type firm subject to its budget. Pick the policy in the optimal set that minimizes the dividend payout.

4. Solve for the high type allocation $a_H$ that maximizes the value of the high type firm subject to its budget and the incentive constraint that guarantees the low type prefers $a_L$ to $a_H$. 

181
5. Compute new value functions $v'_j$ from

$$
v'_j = \pi_{Hj} \left[ d_H + \beta (1 - s_H) \sum_{n=1}^{N} f(\varepsilon_n) v_H [(1 - \delta) k_H + \theta_H \varepsilon_n k^\alpha_H - b_H] \right]$$

$$+ \pi_{Lj} \left[ d_L + \beta (1 - s_L) \sum_{n=1}^{N} f(\varepsilon_n) v_L [(1 - \delta) k_L + \theta_L \varepsilon_n k^\alpha_L - b_L] \right].$$

6. The functions $v'_j$ are the new guess for $v_j$. The procedure is then restarted from step 2 until convergence.

7. Check the option value inherent in the firm by verifying $v_j(w^d_j) = 0$. If these conditions are not satisfied, update the initial guesses $w^d_j$ and restart the procedure from step 1 until convergence.

### 3.8 References


