Spackling: Smoothing Make-to-Order Production of Mass-Customized Products with Make-to-Stock Production of Standard Items

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Abstract

Consider a mass customizer who produces multiple variants of a product in make-to-order fashion, and who also produces some standard variants as make-to-stock. We evaluate pricing and production issues from a joint marketing and operations perspective. Marketing determines prices while operations chooses between three production strategies as it sets capacity. A key result is that under the logit-based assumption for our marketing model, it is optimal for all mass-customized products to be priced at the same absolute dollar markup, implying all add-on options are priced at cost. Operations considers the following production strategies: 1) a focus strategy where the firm produces custom variants in a flexible plant and standard items in an efficient plant; 2) a pure-spackling strategy where it produces everything in a flexible plant, first manufacturing custom products as demanded each period, and then filling in, or spackling, the production schedule with make-to-stock output of standard products to restock inventory; or 3) a layered-spackling strategy where it uses an efficient plant to make some of its standard items and a flexible plant where it spackles. A second key result is to identify the optimal production strategy as determined by the tradeoff between the cost premium for flexible (as opposed to efficient) production capacity and the opportunity costs of idle capacity. We illustrate our framework with data from a messenger bag manufacturer. Spackling amortizes fixed costs more effectively and thus has the potential to increase profits from mass customization.

Keywords: mass customization, logit, constant markup, spackling, flexible capacity, outsourcing, offshore production
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1. Introduction

Timbuk2, a San Francisco manufacturer of bicycle messenger bags, has an Internet site where customers configure and order mass-customized bags shipped directly from the manufacturer. The Internet channel takes advantage of Timbuk2’s flexible manufacturing capabilities and complements the firm’s traditional retail channel where pre-configured (standard) bags are sold. Timbuk2’s flexible San Francisco factory can produce an individual bag of any configuration and deliver it within days.

At the time of this research, the San Francisco factory was being used to fill demand for both the Internet channel and the traditional retail channel. In order to reduce production costs, Timbuk2 was considering moving production offshore for the standard make-to-stock (MTS) bags sold in its traditional channel. While the off-shore capacity would be less expensive, it would not be able to fill demand for the mass-customized make-to-order (MTO) bags sold in the Internet channel in a timely manner – the San Francisco factory would continue to produce to meet this MTO demand. (We use the term MTS to refer to production in anticipation of demand, with output being added to inventory, and MTO to indicate production that fills specific end-customer orders after they are received.) Timbuk2’s management team wondered what effect a shift to off-shore production of standard bags would have on overall costs, including at the flexible domestic factory where the custom bags would continue to be produced.

This paper developed as we sought to analyze Timbuk2’s situation. Should Timbuk2 utilize efficient (off-shore) capacity? Or, should it continue to produce standard off-the-shelf MTS bags using the flexible San Francisco plant, keeping a mix of MTO and MTS in the same factory?² Under the latter option it would continue to employ a strategy we call spackling, where it first makes custom MTO bags

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² At Timbuk2, standard and custom products are associated with retail stores and internet sales, respectively. In reality, the distribution channel and product type need not be linked – it could also sell standard bags directly to customers and customized bags through retailers. The analysis would be similar.
as demanded each period, and then fills in, or spackles, the production schedule with standard MTS product, to restock inventory. We were also interested in how Timbuk2 should price the mass-customized bags. In particular, how should the multitude of add-on options be priced?

These issues are not unique to Timbuk2 and are strategic in nature. For example, senior managers at automobile manufacturers have been talking for a decade or more about the *five-day car*, a plan for mass-customized cars to be produced only after orders are received, shipping within five days (Robison, 1999). Among many detailed suggestions and analysis, Holweg (2003) cites the need for volume flexibility and a build-to-order system for a five-day (or three-day) strategy to be viable. Volume flexibility is required to meet the day-to-day demand uncertainty that is inherent when implementing mass customization, since customers may prefer not to wait until orders can be batched. Our proposed spackling strategy would assist automobile manufacturers in addressing the higher cost of this volume flexibility.

As another example, consider issues of keen interest to senior management at Dell Inc. Dell focuses on a single direct channel and must determine the optimal production and pricing strategies for its mass-customized desktop personal computers. Should Dell use efficient off-shore production for long lead-time customers along with local flexible capacity for short lead-time customers, or should it use local capacity for both? How should Dell price all the add-on options offered on its mass-customized units?

We model these decisions from a joint marketing and operations perspective. On the demand (i.e., marketing) side, we focus on the issue of pricing the numerous product configurations that arise in a mass-customization setting. We assume that customer choice of product configuration follows the logit representative customer framework as described in Ben-Akiva and Lerman (1985). Similar to Anderson, et al. (1992), we find the optimal pricing policy for the various configurations of a mass-customized product is one of constant markup. That is, the firm adds the same dollar markup to the product, independent of the configuration the customer has chosen (see §3 and Theorem 1), with the markup level

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3 The American Heritage® Dictionary of the English Language: Fourth Edition 2000 defines spackle as “A trademark used for a … paste designed to fill cracks and holes in plaster before painting or papering. This trademark often occurs in lowercase and as a verb…”
effectively set by the product configurations most highly valued by the customer. Assuming the mass-
customized product variants can be thought of as some base configuration plus a set of add-on options
(such as would be identified on the window sticker of a new car), this pricing policy means all add-on
options should be priced at cost (with all margin effectively included in the price of the base
configuration).

On the operations side, we find the optimal levels for efficient and flexible capacities, a decision
which defines the firm’s use of one of three possible strategies: focus, pure spackling, or layered
spackling. A focus strategy (see the top frame in Figure 1) uses both types of capacity in a specialized
(focused) manner: the flexible factory meets daily (or other time period) demand for custom products
using an MTO process, requiring enough capacity to meet a high daily demand realization, while the
efficient factory meets demand for standard products via an MTS process. Demand for standard items is
realized at the end of some planning period $T$, at which time overage and underage costs are assessed.

With the layered-spackling strategy (depicted in the middle frame in Figure 1), the firm again
uses two factories, a flexible one and an efficient (e.g., overseas) factory. With spackling, the firm’s first
priority in the flexible factory is to produce custom products via MTO. However, orders for custom
products are uneven (uncertain), yielding an undesirable production profile compared to smooth
schedules that would allow for higher capacity utilization. The firm spackles (smoothes) the production
schedule by using the same (flexible) production capacity to first produce all custom products and then
using the remaining capacity to produce standard products in MTS fashion. With layered-spackling, in
addition to the flexible factory (that employs spackling) the firm also uses an efficient factory to produce
additional standard products in MTS fashion. The result is that MTO production in the flexible factory
closely tracks demand for customized products, while MTS production of standard products equals the
sum of the daily surplus of flexible capacity plus another layer of output from the efficient factory. MTS
production tracks demand for standard products not over the short-run but rather over the longer-run, with
some inventory temporarily built-up, and meets the level of demand determined by the critical fractile of
a news vendor problem that we develop herein.
Under a pure-spackling strategy, the firm utilizes only one flexible factory (see the bottom frame of Figure 1). Similar to the layered-spackling strategy, each day the flexible factory first produces the custom products demanded that day, and then fills any remaining capacity with production of standard products.
products. The firm sets total capacity of this one flexible factory to meet a critical fractile of a
newsvendor problem (again, as described herein). With both the layered and pure spackling strategies,
the level total production output allows for higher utilization of the flexible capacity and improved
efficiency. A contribution of this paper is to identify the conditions under which the various strategies of
focus, layered spackling, and pure spackling are optimal.

We applied the model to Timbuk2’s situation. We gathered data on customer preferences to use
directly in the marketing model, while on the operations side we used cost and demand information
provided by Timbuk2 to gain insights as to the best use of efficient and flexible capacity. Because it
promised short lead times, Timbuk2 planned to continue to make its mass-customized products in its
domestic factory. Should Timbuk2 continue to use a pure-spackling strategy, producing all custom and
standard products in the flexible domestic factory? Alternatively, should it adopt a layered-spackling
strategy (with some standard products made overseas, in addition to the ones made domestically via the
spackling strategy)? Or, finally, should it go to a focus strategy (with custom products made domestically
and with standard products made strictly overseas)? Timbuk2’s management could calculate readily the
direct unit cost savings the focus strategy proffered from the use of efficient production (reduced unit cost
times units produced). In contrast, the counter-benefits that spackling yielded by making better use of
capacity resulted in lower amortized fixed cost per unit which were less apparent. Surprisingly, we found
that under a reasonable scenario, the current pure-spackling strategy remained optimal—Timbuk2 should
not move any production offshore, in spite of the available lower production costs.

In § 2 we frame our work in the context of other work related to achieving mass customization.
We then build our model—in § 3 we develop the marketing model assuming costs are known, and then
feed the resulting prices into the operations model in § 4. To illustrate some of the insights our model
offers, in § 5 we use the results to analyze Timbuk2’s situation. We conclude with a discussion and
summary in § 6.
2. **Related Literature**

A firm using mass customization strives to produce many variants of the product so that each customer’s needs are individually met while maintaining/retaining the low cost of mass production. Strategies for achieving these (historically) diverging objectives (of customization and low cost) have been a topic of much recent attention. Spackling (in either the pure or layered form) is one tool the firm can employ to supply a wider variety of products at low cost in the face of uncertain demand.

Our notion of spackling contributes to the body of research that demonstrates the benefit of flexible resources. In seeking the optimal mix of less costly, dedicated capacity and more expensive, flexible capacity, our research builds on the model of Van Mieghem (1998). In his model the firm decides between production using only dedicated capacity, only flexible capacity, or a mix of both, as determined by the marginal cost of flexible capacity. All production is MTO, in that capacities are allocated to products after demand is observed. Van Mieghem finds that it may be advantageous to invest in more expensive flexible resources even with perfectly positively correlated product demands. We also find support for expensive flexible resources but in a different framework: we assume that production can be MTO or MTS and that both fixed and variable costs are a function of the type of production. In our setup, flexible resources can improve capacity utilization through production smoothing.

Postponing product differentiation is another form of production flexibility that can reduce the risk of under-producing or over-producing varied product configurations. Graman and Magazine (2002) show that if postponement involves even only a relatively small fraction of production, the benefits are nearly as great as if the firm could delay differentiation for all units. Gupta and Benjaafar (2001) show that postponement helps reconcile the needs of high variety and quick response time. In our setting, risks of under- or over-producing are partly eliminated by the firm’s offering of custom products. However, custom products require more expensive capacity, and there is a risk of acquiring too much or too little. Our proposed spackling strategy mitigates this risk.

Eynan and Rosenblatt (1995) study the tradeoff between lower-cost MTS production and higher-
cost MTO (or assemble-to–order, ATO) production for a single standard product. Rudi (2000) also assumes a single product and considers the tradeoff between low-cost, long-lead time MTS production in the Far East and higher-cost local production with short lead times and pre-positioning of components (ATO). Muckstadt, et al. (2001) investigate the use of MTO production for items (called B/C-type) with highly erratic demand while using MTS production for more predictable A-type items. They develop a computationally efficient approach for setting inventory base-stock levels and for allocating capacity, comparing performance to an alternative where inventory and capacity decisions are made prior to observing demand (MTS). In contrast to the above articles, our model considers a mass-customized product in addition to a standard product, and only the standard products might benefit from lower-cost off-shore production. The customer chooses either a standard product that can be MTS or a custom MTO product, and we determine the appropriate production strategy. By producing both standard and custom products using flexible capacity, the firm more efficiently utilizes the flexible capacity that is needed to produce the custom products.

Arreola-Risa and DeCroix (1998) study the optimality of a strict MTO policy versus a strict MTS policy for a company producing multiple heterogeneous products at a shared manufacturing facility. They model demands as independent Poisson processes with different arrival rates and derive optimality conditions for MTO versus MTS policies considering tradeoffs in inventory holding and backordering costs. They consider whether a specific product should be built MTO or MTS. Rajagopalan (2002) also considers the firm’s portfolio of products using a model to determine which products should be made to order and which should be made to stock. We focus on a different problem: determining whether efficient capacity should be employed to produce MTS items. In our setting, standard products are always produced as MTS and custom products as MTO.

The idea of dedicated capacity and reactive capacity, as put forth by Fisher and Raman (1996), is also related to our approach. Early in the production season, capacity is dedicated to products having low demand variability (essentially, production is MTS). Capacity used late in the season can react to closer-to-actual demand (essentially, production is MTO). In Fisher and Raman, the focus is on the production
sequence given the capacity (i.e., which products should be produced at each stage of the season) while we focus on the optimal structure of the capacity. Additionally, we require MTO production for a subset (i.e., the mass-customized segment) of our demand.

Variety and lead-time are pertinent issues in our setting. Lancaster (1990) offers a broad survey of papers addressing the variety issue. Li (1992) discusses lead-time issues. If customers were always willing to wait, all items would be produced as MTO without any risk. He finds a boundary between MTS and MTO as reflected in newsvendor–type results. Salvador, et al. (2002) discuss how manufacturing characteristics affect the appropriate type of modularity in product family architecture and component sourcing to mitigate the negative impact of product variety. They suggest that when the desired level of product variety is low relative to total production volume, component swapping modularity helps to maximize operational performance. We present an additional strategy to maximize operational performance when there is a mix of high variety custom products along with standard products.

Blackburn et al. (1992) suggest that offering increased variety and shorter response times yields strategic advantages. For a firm that has chosen to offer high variety and quick response at a specific level, our research analyzes alternate strategies by which a firm can fill its orders most efficiently. We address the variety issue by giving the customer a choice between standard and custom products, using the logit framework; the issue of customer willingness-to-wait is folded into a product’s reservation price.

Alptekinoglu and Corbett (2007) study competition between a mass customizer and a mass producer in a game-theoretic setting where customers have heterogeneous preferences on a single taste attribute. The mass producer can choose to invest in more flexible capacity (ultimately being able to match the mass customizer’s flexibility). They find that the mass producer facing competition from a mass customizer chooses to offer lower product variety compared to a monopolist. While our research is not in a competitive setting, we present a strategy to help a mass customizer capitalize on the heterogeneous customer preferences for both standard and custom products.

Our research also contributes to a growing literature that touches on the integration of marketing and operations management decisions; cf., Chase (1996), Eliashberg and Steinberg (1993), Karmarkar
(1996), Lovejoy (1998), van Ryzin and Mahajan (1999), and Verma, et al. (2001). Our focus solution is similar to van Ryzin and Mahajan (1999) who use the multinomial logit model in a newsvendor setting and show that the optimal set of stocking levels has a simple structure.

3. Marketing Model for Pricing Mass-Customized Units

In this section, we consider the pricing decision for a mass-customized product where the customization arises through options added or subtracted from a base product. (We refer to these as add-on options). We use a standard logit formulation to develop a marketing model which yields optimal prices for each product variant. The resulting product margins for the mass-customized products, along with exogenous parameters for the standard products, are fed into the operations model (see § 4) which determines the optimal capacities. We apply the integrated marketing and operations models to the case of Timbuk2 in § 5.

We accept as given the base product and the menu of customizable features (i.e., add-on options) the firm offers on the product. We assume the firm has measured each customer’s willingness-to-pay for each product attribute, scaled into dollars. This might be done, for example, through conjoint analysis and user design, as described by Dahan and Hauser (2002). Thus we can find each customer’s dollar-scaled utility for any of the customized configurations available.

Let product \( j \) denote a specific configuration of a mass-customized product, with \( p_j \) denoting its price. See Table 1 for a summary of notation used in the marketing model. Let \( r^i_j \) denote customer \( i \)’s expected dollar-equivalent utility for product \( j \), excluding price: We refer to \( r^i_j \) as customer \( i \)’s reservation price for product \( j \). If viewed deterministically, then given the choice of buying product \( j \) or buying nothing, customer \( i \) buys at a price below \( r^i_j \). See Schmidt and Porteus (2000) and Smith (1986) for a similar approach.

Each customer’s net utility is scaled by a price-sensitivity parameter \( \beta^i \), such that the net dollar value customer \( i \) attaches to product \( j \) is \( \beta^i (r^i_j - p_j) + \xi^i_j \) where \( \xi^i_j \) is a random error term assumed to be distributed Gumbel, i.e., double exponential, such that \( \xi^i_j - \xi^k_j \) is distributed logistic; hence the
standard logit formulation applies. Let \( q^i_j \) denote customer \( i \)'s probability of purchase for product \( j \); \( q^i_j \) is a function of prices. If the customer has the choice of buying one of \( w \) product configurations, along with the option to buy nothing from the firm (for this “buy nothing” choice we assume \( r^i_j - p_j = 0 \)), then customer \( i \)'s probability of purchase of product \( j \) is given by the logit formulation as:

\[
q^i_j = \left[ \frac{\exp\left( \beta^i \left( r^i_j - p_j \right) \right)}{1 + \sum_{k=1}^{w} \exp\left( \beta^i \left( r^i_k - p_k \right) \right)} \right]
\]

(1)

Table 1. Notation Used in the Marketing Model, § 3 (in the order defined in the text)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_j )</td>
<td>Sales price of customized product ( j ).</td>
</tr>
<tr>
<td>( r^i_j )</td>
<td>The maximum customer ( i ) is expected to pay for product ( j ) (her reservation price).</td>
</tr>
<tr>
<td>( \beta^i )</td>
<td>Price sensitivity parameter for customer ( i ), assumed constant over all ( i ).</td>
</tr>
<tr>
<td>( \xi^i_j )</td>
<td>Random error term.</td>
</tr>
<tr>
<td>( q^i_j )</td>
<td>Customer ( i )'s probability of purchase of product ( j ).</td>
</tr>
<tr>
<td>( \beta, r_j, q_i )</td>
<td>The representative customer’s parameters.</td>
</tr>
<tr>
<td>( c_j )</td>
<td>Product ( j )'s unit cost, assumed constant over volume.</td>
</tr>
<tr>
<td>( m_j )</td>
<td>The markup on product ( j ), ( m_j = p_j - c_j ).</td>
</tr>
<tr>
<td>( W )</td>
<td>The number of customized products offered.</td>
</tr>
<tr>
<td>( d_j )</td>
<td>The maximum markup at which the customer would buy product ( j ), ( d_j = r_j - c_j ).</td>
</tr>
<tr>
<td>( \Pi_M )</td>
<td>Expected profit, a function of prices ( p_1 ) through ( p_w ).</td>
</tr>
<tr>
<td>( \mu )</td>
<td>The number of customers.</td>
</tr>
<tr>
<td>( m^* )</td>
<td>The optimal markup, constant across all products.</td>
</tr>
</tbody>
</table>

We adopt the representative customer approach, whereby the parameters for a single customer are used to approximate the aggregate characteristics of the population. Ben-Akiva and Lerman (1985) describe this approach in their discussion of aggregate forecasting techniques, and Anderson, et al. (1992) further discuss its validity. One interpretation is that \( \xi^i_j \) is not a random error term, but rather accounts for customer preference heterogeneity. That is, the representative customer approach is valid if each customer’s choice is deterministic and the variations in customer evaluations of a given product are distributed Gumbel. From here on, we simply use \( \beta \) and \( r_j \) (without superscripts) to denote the characteristics of the representative customer, and use \( q_j \) to denote the representative customer’s purchase probability for product \( j \). The customer will choose between buying any one of the possible product configurations or nothing at all.
Let $c_j$ denote product $j$’s per-unit cost, assumed to be constant over volume. In reality, costs $c_j$ are a function of the firm’s production strategy (be it one of focus, pure spackling, or layered spackling); for simplicity of exposition we ignore this subtlety. (A possible extension of the analysis would be to iterate between the marketing model described here in § 3 and the operations model in § 4 to find the convergent outcome in terms of marketing’s pricing decision and operations’ capacity decision.)

Let $m_j \equiv p_j - c_j$ denote the firm’s dollar markup on product $j$, and let $w$ denote the number of products marketed such that $j \in \{1, 2, \ldots, w\}$. Define $d_j \equiv r_j - c_j$ as the representative customer’s discriminating markup for product $j$; it is the markup the firm could achieve in selling product $j$ if it could perfectly price discriminate at the individual customer level (to the first degree), under deterministic customer choice. The customer’s net utility from buying product $j$ at price $p_j$ is $(r_j - p_j) = (d_j - m_j)$, since $d_j \equiv r_j - c_j$ and $m_j \equiv p_j - c_j$. The expression $(d_j - m_j)$ replaces the expression $(r_j - p_j)$ in (1), and allows us to highlight the role of markup $m$, leading to a powerful result.

Given costs for each product, marketing’s optimization problem is to set prices (and therefore markups) to maximize expected profit, denoted by $\Pi_M(p_1, p_2, \ldots, p_w)$. The marketing model does not consider demand uncertainty and thus assumes all demand is met from the population of $\mu$ possible customers. (Overage and underage costs could be accounted for by iterating between the marketing and operations models.) Given the logit representative customer approach described earlier, and recalling that $q_j$ is the representative customer’s probability of purchase of product $j$:

$$\Pi_M(p_1, p_2, \ldots, p_w) = \mu \sum_{j=1}^{w} (p_j - c_j) q_j = \mu \sum_{j=1}^{w} m_j q_j$$

(2)

Marketing’s optimal pricing strategy is stated in Theorem 1, representing a version of the constant absolute markup property that follows from this formulation. This property is noted in Anderson, et al. (1992), p. 251, in a different context. We include in Appendix I a proof in the setting of our model.
THEOREM 1. If the representative customer is choosing between buying any one of w products offered by a single firm or buying nothing from the firm, the firm should price all w products at the same absolute dollar markup, denoted by $m^*$. The implicit solution for $m^*$, which exists and is unique, is given by:

$$m^* = \frac{1}{\beta \left[1 - \sum_{j=1}^{w} q_j \right]},$$

where:

$$q_j = \left\{ \frac{\exp\left(\beta(d_j - m^*)\right)}{1 + \sum_{k=1}^{w} \exp\left(\beta(d_k - m^*)\right)} \right\} = \frac{\exp\left(\beta d_j\right)}{\beta m^* \exp\left(\beta m^*\right)}.$$

Theorem 1 demonstrates that when applying the representative customer approach, the firm finds it optimal to price any configuration of its products at a constant absolute dollar markup (but not at the same percent margin). If it deviates from $m^*$ by, say, increasing the markup of product $j$, there are several competing consequences. It gets higher margin on product $j$, but reduces $j$’s expected unit sales (i.e., its probability of purchase). In turn, it increases the expected unit sales (and profit) derived from each of the other products, but diminishes total expected unit sales. It is not clear a priori which effect will dominate. Theorem 1 proves there is always a profit loss by deviating from $m^*$.

Next, consider the firm’s pricing strategy when its product assortment is created by adding or deleting individual features (i.e., add-on options) from a “base unit,” as in a mass-customization setting. For example, automobile manufacturers typically suggest a retail price for a base configuration, along with prices for individual options. (The firm may also quote deductions from the base price if certain features are deleted.) We assume additive costs: the cost of any finished unit is equal to the cost of the base unit plus the cost of the supplemental features.

COROLLARY 1. Applying Theorem 1 to a strategy for pricing add-on options, it is optimal for the firm to price the base unit at cost plus $m^*$, and price each option to be added or deleted at cost.
Corollary 1 provides guidance to firms as they price add-on options in a mass-customized setting. To the extent their demand meets the assumptions of the logit model, Corollary 1 shows that the firm need not optimize the price of each individual option, but only the price of the base unit. The markup \( m^* \) that is included in the base-unit price accounts for the desirability of the assortment of customized products, which depends to some extent on the menu of upgrade options. In other words, this result should not be interpreted to mean that the firm does not make any money on desirable options. The more desirable the add-on options, the higher the optimal markup \( m^* \). Conversely, if the firm eliminated all add-on options, then \( m^* \) would go down; otherwise customers would walk away without buying.

Customers always have a “no-buy” alternative. A further key assumption is that the price sensitivity (\( \phi \)) for each customer is not different across the various product configurations available. This is reasonable only if the resulting product choices are relatively similar.

While the notion of pricing all add-on options at cost seems counterintuitive, some examples of this type of pricing might be an auto dealer that advertises “any car on the lot for $99 over invoice,” or a fast-food restaurant that will super-size your meal for $0.39, where $0.39 is the incremental cost of supersizing. Another example is a warehouse club whose profits are roughly equal to the sum of its annual membership fees: the base product is the membership and the add-on options are the goods purchased by a customer throughout the year, which are effectively sold at cost. Improving the selection of goods increases the optimal membership fee and/or the number of members. The constant-markup pricing solution is an intriguing result, and while outside the scope of this paper, further investigation is merited as to the range of situations in which such pricing is optimal.

The constant-markup result has powerful implications for pricing options in a mass-customization setting. To the extent that customer demand is consistent with a logit framework, and product configurations are additive variations to a base unit, then the constant-markup result provides a simple pricing structure at optimality: from the optimal markup for a base unit, options are added or deleted at cost.
4. **Operations Model**

In this section we develop an operations model to gain insight into the tradeoffs inherent in the decision between focus and spackling. In this section the term “spackling” will apply to either pure- or layered-spackling. Counter to our intuition, we find that with spackling the optimal level of efficient capacity can be zero, even when there is a unit cost advantage to producing standard units in an efficient factory. When the optimal efficient capacity under spackling is zero, then the result is pure spackling; otherwise it is layered spackling.

We assume that demands are realized over a single period, corresponding to the lead time of the firm’s standard retail orders (e.g., a month), and that this period has $T$ subperiods (e.g., days). At the start of the period, prior to the realization of demands in each subperiod, the firm must determine the level of each type of capacity (flexible and efficient) to acquire in order to meet the demands expected over this period. In making its capacity decisions, the firm minimizes expected costs. At the beginning of the period (but after the capacity decision), the firm receives an order for standard products, due at the end of the period. At the beginning of each subperiod (e.g., each day), the firm receives orders for customized products due by the end of that subperiod.

We denote $F_{C}(x)$ and $F_{S}(x)$ as the distribution functions for subperiod demand for custom units and period demand for standard units, respectively, with densities $f_{C}(x_{C})$ and $f_{S}(x_{S})$. Demand for custom units is distributed normally with mean $\mu_{C}$ and standard deviation $\sigma_{C}$. We assume that demand for standard units in a period is independent of demand for custom units and is distributed normally with mean $\mu_{S}$ and standard deviation $\sigma_{S}$. Thus, total demand is also distributed normally with density $f_{T}(x_{T})$, mean $\mu_{T} = T \mu_{C} + \mu_{S}$, and standard deviation $\sigma_{T} = \sqrt{T \sigma_{C}^{2} + \sigma_{S}^{2}}$. See Table 2 for a summary of notation used in the operations model.

The firm’s decisions for production capacities of efficient and flexible resources are denoted $K_{E}$ and $K_{F}$, respectively, where one unit of capacity can make exactly one unit of product per subperiod. Let $\theta_{E}$ denote the fixed cost per subperiod for a unit of efficient capacity (which is used only to make
standard units) and let \( \theta_F \) denote the fixed cost per subperiod for a unit of flexible capacity (which can be used to make a custom unit under the focus strategy, or to make either a standard or custom unit under either spackling strategy), and let \( c_E \) and \( c_F \) denote the variable production costs per unit for efficient and flexible capacities, respectively. Efficient capacity is assumed to be fully utilized (i.e., \( K_E \) units will be built independent of demand), while flexible capacity is assumed to be utilized only to the extent that there are orders. Under a spackling strategy, where flexible capacity is used to produce both custom and standard orders, we assume that in each subperiod custom orders are produced first, with any remaining capacity used to fill standard orders.

### Table 2. Notation Used in the Operations Model, § 4 (in the order defined in the text)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
<td>Number of subperiods in the period.</td>
</tr>
<tr>
<td>( F_C(x_C) )</td>
<td>Distribution function for subperiod demand of custom units (density ( f_C(x_C) )) ~Normal(( \mu_C, \sigma_C )).</td>
</tr>
<tr>
<td>( x_C^\delta )</td>
<td>Realization of demand for custom units in subperiod ( \delta ).</td>
</tr>
<tr>
<td>( F_S(x_S) )</td>
<td>Distribution function for period demand of standard units (density ( f_S(x_S) )) ~Normal(( \mu_S, \sigma_S )).</td>
</tr>
<tr>
<td>( F_T(x_T) )</td>
<td>Distribution function for period demand of both standard and custom units (density ( f_T(x_T) )) ~Normal(( T\mu_C+\mu_S, \sqrt{T\sigma_C^2+\sigma_S^2} )).</td>
</tr>
<tr>
<td>( K_E )</td>
<td>Decision variable: Production capacity (per subperiod) of efficient factory.</td>
</tr>
<tr>
<td>( K_F )</td>
<td>Decision variable: Production capacity (per subperiod) of flexible factory.</td>
</tr>
<tr>
<td>( \theta_E )</td>
<td>Fixed cost per subperiod for a unit of efficient capacity.</td>
</tr>
<tr>
<td>( \theta_F )</td>
<td>Fixed cost per subperiod for a unit of flexible capacity.</td>
</tr>
<tr>
<td>( c_E )</td>
<td>Variable cost per subperiod for a unit of efficient capacity.</td>
</tr>
<tr>
<td>( c_F )</td>
<td>Variable cost per subperiod for a unit of flexible capacity.</td>
</tr>
<tr>
<td>( c_P )</td>
<td>Penalty cost of backlogged standard order (but avoids cost of ( \theta_E + c_E )), ( c_P &gt; \theta_E + c_E ).</td>
</tr>
<tr>
<td>( c_P' )</td>
<td>Penalty cost of backlogged standard order (but avoids cost of ( \theta_F )), ( c_P' &gt; \theta_F ).</td>
</tr>
<tr>
<td>( G(\cdot) )</td>
<td>Expected period costs, including production and penalty costs.</td>
</tr>
<tr>
<td>( n(\cdot) )</td>
<td>Loss function: expected number of backlogged sales for demand with distribution ( F_t(x) ).</td>
</tr>
<tr>
<td>( L(z_i) )</td>
<td>Standard normal loss function with standard variate ( z_i ).</td>
</tr>
</tbody>
</table>

We assume backlogging, with penalty costs as follows. Backlogged orders of standard units are shipped at the higher (penalty) cost \( c_P \) per unit instead of the usual \( \theta_E + c_E \) per unit (and to ignore the trivial scenario, we assume \( c_P > \theta_E + c_E \)). This assumption corresponds to a scenario where backorders of standard units are cleared after the last period, incurring a high penalty cost but avoiding capacity or unit costs. (This might be production from a subcontractor, for example.) Backlogged orders of custom units incur a penalty of \( c_P' \) in addition to the usual variable production costs \( c_F \) while avoiding fixed costs for
capacity of $\theta_F$ per unit (and, we assume $c_p > \theta_F$). The custom units are backordered across subperiods, and so this assumption essentially is that the backorder will consume existing flexible capacity and the usual variable production costs in the next subperiod, but with a backorder penalty. We assume no salvage values (for MTS production) and that all costs are greater than zero. With no salvage values, overage costs arise from fixed production costs.

We first analyze the firm’s optimization problems given that it chooses the focus strategy, and then given that it chooses a spackling strategy.

In the case of focus, production of standard products is $K_E$ each subperiod, while production of custom products in each subperiod is the minimum of demand and capacity. If the firm chooses the focus strategy, expected period costs $G(\cdot)$ are determined as follows:

$$G(K_E) = (\theta_F E K_F + c_p n_S(TK_E))$$

where $n_S(TK_E) = \int_{TK-E}^\infty (x-TK_E) f_S(x)dx = \sigma_S L(z_S)$ and $n_C(K_F) = \int_{K-F}^\infty (x-K_F) f_C(x)dx = \sigma_C L(z_C)$ are the relevant loss functions, with $z_S = \frac{TK_E - \mu_S}{\sigma_S}$ and $z_C = \frac{K_F - \mu_C}{\sigma_C}$. Total cost is the sum $G(K_E) + G(K_F)$.

In the case of spackling—denoted with a superscript “s”—in each of the subperiods exactly $K_E^s$ standard units are produced using efficient capacity while $K_F^s$ units are produced using flexible capacity. The flexible production is used first to meet demand for $\max(Cx_\delta, K_F^s)$ units of customized products, where $x_\delta$ is the realization of demand for custom units in subperiod $\delta$. The remaining capacity $K_F^s - \max(x_\delta, K_F^s)$ is used to produce standard products. Shortfalls of custom units in a subperiod incur a penalty of $c_p$. A shortfall of standard units arises and incurs a penalty cost of $c_p$ per unit if total period demand (for all products) is greater than total capacity $TK_E^s + TK_F^s$. Total expected cost over the period for the spackled-production case is:

$G(K_E^s, K_F^s) = \theta_F E K_F^s + (c_E + \theta_E)TK_E^s + Tc_F \mu_C + c_p n_S^s(TK_E^s) + c_p n_C^s(TK_E^s + TK_F^s) + Tc_p n_C^s(K_F^s)$ where $n_S^s(TK_E^s) = \sigma_S L(z_S^s)$, $n_C^s(TK_E^s + TK_F^s) = \sigma_T L(z_T^s)$, and $n_C^s(K_F^s) = \sigma_C L(z_C^s)$, and $z_S^s = \frac{TK_E^s - \mu_S}{\sigma_S}$, $z_T^s = \frac{TK_E^s + TK_F^s - \mu_T}{\sigma_T}$ and $z_C^s = \frac{K_F^s - \mu_C}{\sigma_C}$. The term $c_p n_S^s(TK_E^s)$ accounts
for the production of standard units built using flexible capacity (since there is insufficient efficient
capacity), while \( c_p n_S^* (TK_E^* + TK_F^*) \) and \( Tc_p n_C^* (K_E^*) \) account for period shortages of standard and custom
products, respectively.

We determine optimal capacities for standard and custom products through first and second order
conditions on the cost functions for the focused- or spackled-production cases. All proofs are in the
Appendix.

**THEOREM 2.**

**Case A (Focus):** The optimal capacities \( K_E^* \) and \( K_F^* \) are:

\[
K_E^* = \frac{1}{T} (\mu_S + z_S^* \sigma_S) \quad \text{and} \quad K_F^* = \mu_C + z_C^* \sigma_C
\]

where the standard variates \( z_S^* = \Phi^{-1} \left( \frac{c_p - \theta_E - c_E}{c_p} \right) \) and \( z_C^* = \Phi^{-1} \left( \frac{c_p - \theta_F}{c_p} \right) \).

**Case B (Spackle):** The optimal capacities \( K_E^{s*} \) and \( K_F^{s*} \) in the spackling case are:

1. layered spackling \( (K_E^{s*} > 0) \)

\[
K_E^{s*} = \mu_C + z_C^{s*} \sigma_C \quad \text{and} \quad K_F^{s*} = \frac{1}{T} \mu_S + \frac{1}{T} z_T^{s*} \sigma_T - z_C^{s*} \sigma_C
\]

with \( z_C^{s*} = \Phi^{-1} \left( \frac{c_p - \theta_F - c_E (1-F_S(TK_E^{s*})) - c_E \theta_E}{c_p} \right) \) and \( z_T^{s*} = \Phi^{-1} \left( \frac{c_p - \theta_E - c_E + c_F (1-F_S(TK_E^{s*}))}{c_p} \right) \).

2. pure spackling \( (K_E^{s*} = 0) \)

\[
K_F^{s*} = \frac{1}{T} \left( \mu_T + z_T^{s*} \sigma_T \right)
\]

with \( z_T^{s*} = \Phi^{-1} \left( \frac{c_p - \theta_T + c_F (1-F_C(K_F^{s*})))}{c_p} \right) \).

We note that the optimal solutions in the focus case are standard newsvendor, and thus closed-
form, explicit functions. In contrast, while the optimal solutions for the spackle case also have a
newsvendor structure, they are closed-form, implicit functions that include the optimal capacity on the right hand side of the equation as well as on the left hand side. Given that the cost function in the spackle case is well-behaved (see the proof of Theorem 2 in the Appendix), the optimal solution can be found easily via a search algorithm.

Case A corresponds to focus (efficient capacity for standard products and flexible for custom) while Case B corresponds to spackling (flexible capacity in a dual role with part of it responding to custom orders while the remainder produces standard items to inventory).

In the case of focus, optimal capacities are determined through traditional newsvendor formulations with net unit underage costs of $c_p - \theta_F$ and $c_p - \theta_E - c_E$ for flexible and efficient capacities, respectively, and overage costs of $\theta_F$ and $c_E + \theta_E$, respectively. In the spackling case, the optimal capacity is determined implicitly through equations (5) and (6) or through equation (7) for the layered- and pure-spackling cases, respectively. Safety capacities for the two cases are $\frac{1}{\sigma} z^*_{s} \sigma_s + z^*_{c} \sigma_c$ for the focus case and $\frac{1}{\sigma} z^*_{T} \sigma_T$ for the spackling cases.

The optimal solutions in Case B1 are implicit with underage and overage costs interpreted as follows. The underage cost for flexible capacity, $c_p - \theta_F - c_F \left(1 - F_S(TK^*_E)\right) + c_E + \theta_E$, includes the penalty cost for a missed sale of a custom order $c_p$ less the premium for flexible versus efficient capacity. The premium is adjusted for the probability that the flexible capacity will be used to fill standard orders, $\left(1 - F_S(TK^*_E)\right)$, since shortages of efficient capacity will be filled using flexible capacity. The average cost is the probability-adjusted premium for flexible versus efficient capacity.

The underage cost for efficient capacity, $c_p - \theta_E - c_E + c_F \left(1 - F_S(TK^*_E)\right)$, includes the penalty cost for a missed sale of a standard unit adjusted by the (sunk) cost of capacity less the expected variable cost of producing the unit using existing flexible capacity, $c_F$. The expectation is determined again from the probability that the flexible capacity will be needed, i.e., $Pr\{x_s > TK^*_E\}$. The interpretation of Case B2 is similar.
Spackling shifts some (or all) of the production of standard units from the efficient facility to the flexible facility. Thus, flexible capacity will be greater (or equal) under spackling vis-à-vis focus, i.e., $K_F^s \geq K_F^s$. Similarly, efficient capacity will be lower (or equal) under spackling, i.e., $0 \leq K_E^s \leq K_E^s$. It is interesting to note, that the optimal level of efficient production $K_E^s$ can be zero (i.e., pure-spackling may be optimal) even when efficient production has a lower unit cost than flexible production. In other words, it is not always the case that at least some units should be produced in an efficient facility.

Having found the optimal capacities and resulting expected costs with the focus and spackling strategies, the optimal strategy (of focus versus spackling) is found by simply choosing the one with the lower expected cost. In the next section we offer some examples to gain further insight into the tradeoffs inherent in this choice.

5. **Empirical Analysis and Examples**

In this section, we describe briefly our analysis of Timbuk2’s situation. In order to address the marketing concerns, we acquired Timbuk2’s customer-related data in a three-stage study. In stage one, 297 MBA students at MIT participated in an adaptive conjoint survey, as described by Toubia, et al. (2003). The conjoint survey involved price plus nine other attributes, including size (medium or large), color (red or black), type of closure of the laptop sleeve (a full flap or a small tab), and six “on-off” attributes—either the bag included the attribute or lacked it—namely, an MIT logo, a holder for a cell phone, a holder for a personal digital assistant, a laptop sleeve, a mesh pocket, a bottom boot, and a carrying handle separate from the shoulder strap. From the conjoint study we were able to determine dollar-scaled part-worths for each attribute. In the second stage, each student was given $100 to spend on a self-configured custom bag. If a student configured a bag priced below $100, she received the difference in cash. The third stage consisted of a follow-up survey in which a student indicated the probability that she would have actually purchased her customized bag, and identified her delay penalty for having to wait for the MTO bag. From this data, we determined each customer’s expected dollar-
equivalent utility for each product and option, excluding price, and estimated the price-sensitivity parameter. Based on this information, we could determine the optimal markup (Theorem 1).

On the operations side, we used Timbuk2’s product cost and demand data to determine the optimal production strategy. At the time of this research, Timbuk2 was producing everything in its flexible San Francisco factory using a pure-spackling strategy, and was considering moving to a focus strategy where standard units would be produced overseas, or to a layered-spackling strategy where some production of standard goods would remain in the flexible domestic factory. We sought to quantify the effects of each strategy.

We used our stylized model to gain insights into the tradeoffs Timbuk2 faced. While Timbuk2’s setting was an ongoing rather than a single-period problem, its situation could be viewed approximately as a sequence of monthly problems with monthly decisions for efficient capacity (production of standard bags in the offshore facility) and flexible capacity. We assumed a subperiod to be one day and a period to be one month ($T = 30$).

Timbuk2 estimated that efficient off-shore production would cost approximately $13 per bag ($1 labor and $12 material) while bags produced in San Francisco cost approximately $18 per bag ($6 labor and $12 material). Leadtime and shipping costs made it impractical to produce mass-customized bags overseas. In particular, shipping bags by sea took four weeks but cost a negligible $0.10 per bag. Air shipments of bags would result in satisfactory leadtimes, but air shipments cost approximately $15 per bag, which was much greater than the cost savings of overseas production.

We assumed that the fixed cost for the flexible capacity was the labor commitment for the upcoming month while the variable cost was the material used in each bag. Production costs were thus $\theta_E + c_E = $13 and $\theta_F + c_F = $18 for efficient and flexible capacities, respectively. We used penalty costs equal to the average prices in each channel. Approximately 20% of demand at Timbuk2 was for custom bags; daily demand for custom bags was approximately 40 bags while daily demand for standard bags was approximately 160. The coefficient of variation in subperiod demand was approximately 33%. 
Recall that management was considering going to a strategy of focus or one of layered spackling. Given a predominance of standard demand, and with efficient overseas production offering nearly 30% in (fixed plus variable) cost reduction, it is not surprising that a layered spackling strategy would be preferred over pure spackling. Figure 2 shows the expected costs for the three strategies over a range of costs for the efficient production. As expected, layered-spackling dominated the strategies with a lower total expected cost across all values. We note that a focus strategy would be preferred to Timbuk2’s pure-spackling strategy if the unit costs of efficient capacity were $16.50 or less—about 8% lower than the unit costs of the $18.00 per unit flexible capacity.

**Figure 2. Given 20% Custom Products, Layered Spackling was the Preferred Strategy**

Note in particular the cost benefit that layered spackling offers over the focus strategy. Even though custom products represent only about 20% of the production volume, layered spackling reduces overall cost by approximately 5% as compared to the focus strategy. Another way of looking at this is that layered spackling reduces the effective cost of the custom units by about 20%. (This is calculated by multiplying the expected daily demand for custom items of 40 by the variable plus fixed cost of $18 yielding a monthly cost of $21,600. Meanwhile, the current unit cost of efficient production of $13 saves
$93,493 – $89,386 = $4,107.) Timbuk2 can significantly reduce the effective cost of mass-customization via the layered spackling strategy.

Interestingly, Timbuk2’s increasing emphasis on mass customization could significantly change the picture. Setting all parameter values at current levels except the fraction of total demand that is custom product, we examined in Figure 3 what happened to expected costs and optimal capacities as the fraction of custom production increased from 20%. The total expected cost gap between layered and pure spackling narrowed rapidly with an increase in the fraction of output that is custom product, and was effectively eliminated at just over 70%. In other words, moving offshore to take advantage of efficient production could be a fleeting savings if mass-customized products were to become a more prominent fraction of the product mix.

Figure 3. Pure Spackling Becomes Favorable as the % of Orders that are Custom Grows

Figure 3 confirmed that at the then current ratio of custom to total demand of around 20%,

Timbuk2 had significant incentive to move to offshore production and use layered spackling. But if the ratio of custom orders were to increase, as had been the trend to date, this incentive would diminish and completely disappear when custom orders comprised 70% or more of total demand. Above 70%, it would be optimal to use a pure-spaking strategy – essentially the then current strategy of utilizing just one flexible domestic factory.

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Summarizing the Timbuk2 results, the layered-spackling strategy offered significant savings for Timbuk2 over the focus strategy that they were considering, reducing overall costs by 4.4% (and reducing effective costs for custom products by about 20%). The layered-spackling strategy was attractive given the high (80%) fraction of output that was currently associated with standard products. However, as mass-customized demand continued to grow, pure-spackling could become optimal. If over 70% of production were to be allocated to custom demand, the firm would no longer find significant advantage for overseas production even though it was nearly 30% less costly per unit.

6. Conclusion

Motivated by the experiences of Timbuk2, we gain insight into a couple of issues arising with mass customization. First, in a marketing model, we find that if customer demand fits into a logit framework, then optimal pricing is achieved through a constant markup. The constant-markup result has powerful implications: when mass customization is achieved by adding options to a base unit, then the base unit should incorporate the full optimal markup, and options should be added or deleted at cost. However, the firm does have an incentive to offer desirable options because they increase the markup commanded by the base unit. For example, at the time of this research, Timbuk2 was charging customers $5 for choosing a different logo color than the standard color. The sewing machine that added the logo had numerous colors of thread mounted to the machine, and there was no difference in cost between adding the standard color or any of the other colors mounted on the machine. A significant fraction of customers paid the $5 for a customized logo color, suggesting this was a desirable option. Assuming the logit model fairly represented their demand, our model suggested that offering the custom logo colors at no extra cost and increasing the price of the base bag (by some amount determined by the marketing model) would increase Timbuk2’s total profit (and possibly their sales) as compared to their current pricing strategy.

On the operations side, we analyze the benefits of spackling, a process involving the use of a flexible production facility to produce both standard and customized products. We compare spackling to
focus, where an efficient resource makes the standard items and a flexible factory focuses on the custom units. With pure spackling the firm has only one (flexible) factory to produce both standard and custom products, while with layered spackling the firm supplements its production of custom and standard items in the flexible factory with additional production of standard units in an efficient factory.

Applying our model to the case of Timbuk2, we found that even though going offshore to produce standard bags could presumably yield a unit cost savings of 30%, it was still optimal for Timbuk2 to keep some production of standard bags in the domestic factory. Specifically, layered spackling reduced overall costs by an additional 4.4% as compared to the focus strategy. Another way of viewing this cost savings is to say that layered spackling reduced the effective cost of custom products by nearly 20%. Additionally, we found it somewhat surprising that the advantage of offshore production completely disappeared when custom demand was over 70% of total demand. Since the demand for mass-customized products had been increasing sharply for Timbuk2, an investment in overseas production (under a layered-spackling strategy) may not have long-term benefits.

In any case, while the benefits of going to efficient production are often visible and easy to calculate, our analysis highlights some of the more subtle benefits arising from better use of capacity given uncertain demands. In other words, if unaware of the benefits of spackling, senior management could be falsely attracted by a phantom savings of lower unit costs under efficient production, and unwisely choose a focus strategy with suboptimal results.

Currently, the basic concept of spackling is employed by some of the firms that successfully implement mass customization, although not always precisely in the fashion presented in our paper. For example, Dell uses flexible manufacturing systems in its U.S. factories to assemble desktop computers according to the customer’s exact configuration and is able to deliver within a few days. Even though Dell assembles to order (and not to stock), not all orders are equally time sensitive. Within the factory, which is local and flexible, orders can be scheduled in such a way that time-sensitive orders get first priority, and then the schedule can be spackled with less time-sensitive orders. For example, the schedule might proceed in the following order: (1) First priority goes to time-sensitive customers. (2) Spackled on
top are normal customers, with about a 5 day lead time. (3) Another spackled layer consists of a class of customers who specify delivery on an exact date – these units could be built after receipt of the order but in advance of the specified delivery date and held as FGI for that customer. (4) The final layer is a class of customers that gets a discount for being flexible on lead time and delivery date. Through such a layeredspackling strategy, Dell can use its capacity most efficiently while meeting the expectations of each class of customers.

There seems to be heightened interest in mass-customized products, possibly due to the successes of companies such as Dell, as well as to the spread of the Internet. The Internet fosters customization because it allows rapid, two-way communication between the firm and its customers, facilitating product configuration and pricing, and media-rich conceptualization of product alternatives by customers (Dahan and Hauser (2002)). This attracts customers who can thus attain a better product fit or who simply prefer to configure their own products, despite having to wait for delivery and/or pay a higher price.

At the same time, some customers still prefer to buy standard products off-the-shelf. Our operations model offers senior management a new way of thinking about how to use this continued demand for standard products to their benefit, in order to reduce the effective cost penalty typically associated with mass customization. Furthermore, the constant markup pricing result stemming from our marketing model offers further insight with regard to the pricing of mass customized products. Jointly, these operations and marketing insights offer the potential to help make mass customization more viable, more profitable, and more ubiquitous in the future.

Appendix: Proofs

Proof of Theorem 1.

We apply the technique of Anderson, et al. (1992) to our context.

Given \( w \) products, the firm maximizes its profit, \( \pi \), given by:
\[ \pi = \mu \sum_{i=1}^{w} m_i q_i \]

Note that \( \frac{\partial q_i}{\partial m_i} = -\beta q_i (1 - q_i) \) and \( \frac{\partial q_j}{\partial m_j} = \beta q_j q_j \) for \( i \neq j \). The first order conditions (FOC), with respect to \( m_i \), result in:

\[
\frac{\partial \pi}{\partial m_i} = \mu q_i \left( 1 + \beta \sum_{j=1}^{w} m_j q_j - \beta m_i \right) = 0 \forall i. \tag{8}
\]

Noting \( \mu q_i \neq 0 \forall i \), manipulation of (8) yields:

\[
m_i^* = \frac{1}{\beta} \left( 1 + \beta \sum_{j=1}^{w} m_j q_j \right) \forall i. \tag{9}
\]

From (9), \( m_i = m_k = m^* \forall i \). Replacing \( m_i \) with \( m^* \forall i \) in (9), \( m^* \) is the solution to:

\[
m^* = \frac{1}{\beta(1 - \sum_{j=1}^{w} q_j)} \quad \forall i. \tag{10}
\]

Note that the left side of (10) is linearly increasing in \( m^* \) while the right side is strictly decreasing, so there exists a unique solution.

To see that the FOC result in a globally optimal solution, note that there exists, for each possible markup \( m_i \), a lower boundary at \( m_i = 0 \) where profit is everywhere strictly increasing in \( m_i \), that is,

\[
\left. \frac{\partial \pi}{\partial m_i} \right|_{m_i=0} = \mu q_i \left( 1 + \beta \sum_{j=1}^{w} m_j q_j \right) > 0 ,
\]

and note that (as we show in the next paragraph) there exists, for each possible markup \( m_i \), an upper boundary at \( m_i = \bar{m} \) where profit is strictly decreasing in \( m_i \) as long as \( m_j \leq \bar{m} \) for \( j \neq i \); that is, there exists an \( \bar{m} \) such that \( \left. \frac{\partial \pi}{\partial m_i} \right|_{m_i=\bar{m}} < 0 \) for all \( m_j \leq \bar{m} \) where \( j \neq i \). Thus the globally maximizing set of markups must lie strictly within the interior defined by these lower and upper boundaries, i.e., all optimal markups must be strictly greater than zero and strictly less than \( \bar{m} \).
Since \( m_i = m_j = m^* \quad \forall \quad i, j \) is a solution, and the only solution, to the FOC, it must lie within this interior and must be globally optimal.

To show that \( \bar{m} \) exists, let \( \hat{m} \geq m_j \; \forall \; j \), and \( \hat{m} = \frac{1}{\beta \left( 1 - \sum_{j=1}^{w} q_j \right)} \) and let \( m_i = \hat{m} \). Then

\[
\frac{\partial \pi}{\partial m_i} \bigg|_{m_i = \hat{m}} = \mu q_i \left( 1 + \beta \sum_{j=1}^{w} m_j q_j - \hat{m} \beta \right) \leq \mu q_i \left( 1 + \beta \hat{m} \sum_{j=1}^{w} q_j - \beta \hat{m} \right) = \mu q_i \left( 1 - \beta \hat{m} \left( 1 - \sum_{j=1}^{w} q_j \right) \right) < 0 .
\]

Note that \( 0 < \beta \left( 1 - \sum_{j=1}^{w} q_j \right) < \beta \). Then \( \bar{m} \) is any number \( > \hat{m} \).

**Proof of Corollary 1.**

Define the base unit to be any product \( j \) with cost \( c_j \). Consider any other product \( k \) with cost \( c_k \). By Lemma 1, \( m^* = p_j^* - c_j = p_k^* - c_k \). Thus \( p_k^* - p_j^* = c_k - c_j \). That is, the optimal incremental price for the feature set that transforms product \( j \) into product \( k \) equals the incremental cost of that feature set.

**Proof of Theorem 2, Case A.**

The focus case is a standard newsvendor problem with total expected costs over the period:

\[
G(K_E) = (\theta_E + c_E)TK_E + c_p n_S(TK_E), \quad \text{and} \quad G(K_F) = T[\theta_F K_F + c_F \mu_c + c_p n_c(K_E)].
\]

First order conditions for \( G(K_E) \) and \( G(K_F) \) readily provide the optimal capacities, and second order conditions ensure optimality. For normal distributions we note that the derivative of the loss function is

\[
n'(Q) = F(Q) - 1.
\]

\[
\frac{dG(K_E)}{dK_E} = (c_E + \theta_E)T + c_p T(F_S(TK_E) - 1)
\]

\[
\frac{dG^2(K_E)}{dK_E^2} = c_p T^2 f_S(TK_E) > 0
\]

\[
K_E^* = \frac{1}{T} F_S^{-1}\left( [c_p - \theta_E - c_E]/(c_p) \right) = \frac{1}{T} (\mu_S + z_S^* \sigma_S) \text{ where } z_S^* = \Phi^{-1}\left( [c_p - \theta_E - c_E]/c_p \right).
\]

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\[
\frac{dG(K_F)}{dK_F} = \theta_F T + c_p T(FCPF(K_F) - 1)
\]
\[
\frac{dG^2(K_F)}{dK_F^2} = c_p T f_c(K_F) > 0
\]
\[
\Rightarrow K_F^* = F_C^{-1}[(c_{p'} - \theta_F)/(c_{p'})] = \mu_C + z_C^* \sigma_C \quad \text{where} \quad z_C^* = \Phi^{-1}[(c_{p'} - \theta_F)/(c_{p'})].
\]

**Proof of Theorem 2, Case B.**

For the spackle case, we minimize total costs
\[
G(K_E, K_F) = \theta_F TK_F + (c_E + \theta_E )TK_E + Tc_F \mu_C + c_F n^S (TK_E^2) + c_p n^S (TK_E + TK_F^2) + Tc_p n^S (K_E^2)
\]
subject to \(K_E \geq 0\) and \(K_F \geq 0\).

We use the Karush-Kuhn-Tucker Method to account for possible cases for the non-negativity constraints. In order to minimize the notation, we do not show the superscript “s” for spackling in this proof.

The Lagrangian function is:
\[
\mathcal{L}(K_E, K_F) = \theta_F TK_F + (c_E + \theta_E )TK_E + Tc_F \mu_C + c_F n^S (TK_E^2) + c_p n^S (TK_E + TK_F^2) + Tc_p n^S (K_E^2) + u_E (-K_E) + u_F (-K_F)
\]

**First order conditions**

\[
\frac{\partial G(K_E, K_F)}{\partial K_F} = 0 \Leftrightarrow T \theta_F + Tc_p \left[F_c(K_F^*) - 1\right] + Tc_p \left[F_T(TK_F^* + TK_E^*) - 1\right] - u_F = 0 \quad \text{(11)}
\]
\[
\frac{\partial G(K_E, K_F)}{\partial K_E} = 0 \Leftrightarrow (c_E + \theta_E ) + Tc_F \left[F_S(TK_E^*) - 1\right] + Tc_p \left[F_T(TK_F^* + TK_E^*) - 1\right] - u_E = 0 \quad \text{(12)}
\]

Equating equations (11) and (12) we obtain
\[
\theta_F + c_p \left[F_c(K_F^*) - 1\right] - u_F = c_E + \theta_F - c_p \left[1 - F_S(TK_E^*)\right] - u_E / T
\]
\[
\Rightarrow \left[F_c(K_F^*) - 1\right] = \frac{\theta_E + c_E - \theta_F - c_p \left[1 - F_S(TK_E^*)\right]}{c_p} - u_E / T + u_F / T
\]

\[ (13) \]
\[ K_F^* = F_C^{-1} \left[ \frac{c_p + \theta_E + c_E - \theta_F - c_F \left[ 1 - F_S \left( TK_F^* \right) \right]}{c_p} \right] - u_E / T + u_F / T \]

\[ K_F^* = \mu_C + z_C^* \sigma_C \text{ where } z_C^* = \Phi^{-1} \left[ \frac{c_p + \theta_E + c_E - \theta_F - c_F \left[ 1 - F_S \left( TK_F^* \right) \right]}{c_p} \right] - u_E / T + u_F / T \]

From equations (11) and (13) we obtain

\[ K_E^* = \frac{1}{T} F_T^{-1} \left[ \frac{c_p - \theta_E - c_E + c_F \left[ 1 - F_S \left( TK_E^* \right) \right] + u_E / T}{c_p} \right] - K_F^* \]

\[ K_E^* = \frac{1}{T} \mu_S + \frac{1}{T} z_T^* \sigma_T - z_C^* \sigma_C \text{ where } z_T^* = \Phi^{-1} \left[ \frac{c_p - \theta_E - c_E + c_F \left[ 1 - F_S \left( TK_E^* \right) \right] + u_E / T}{c_p} \right]. \]

**Case 1, \( u_E = 0 \) and \( u_F = 0 \): Layered spackling**

Orthogonality constraints imply that when \( u_E = 0 \) and \( u_F = 0 \) then \( K_F^* \) and \( K_E^* \) are greater than or equal to zero. In this case,

\[ K_F^* = \mu_C + z_C^* \sigma_C \text{ where } z_C^* = \Phi^{-1} \left[ \frac{c_p + \theta_E + c_E - \theta_F - c_F \left[ 1 - F_S \left( TK_F^* \right) \right]}{c_p} \right] \text{ and} \]

\[ K_E^* = \frac{1}{T} \mu_S + \frac{1}{T} z_T^* \sigma_T - z_C^* \sigma_C \text{ where } z_T^* = \Phi^{-1} \left[ \frac{c_p - \theta_E - c_E + c_F \left[ 1 - F_S \left( TK_E^* \right) \right]}{c_p} \right]. \]

Ignoring the non-interesting case where capacities are zero as well, when both capacities are strictly greater than zero we refer to it as layered spackling.

**Case 2: \( u_E > 0 \) and \( u_F = 0 \): Pure spackling**

In this case, orthogonality constraints force \( K_E^* = 0 \). Thus, equation (11) becomes

\[
\partial G(K_E = 0, K_F) \partial K_F = 0 \Leftrightarrow T \theta_F + T c_p \left[ F_c (K_F^*) - 1 \right] + T c_p \left[ F_T (TK_F^*) - 1 \right] = 0
\]

and thus

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\[ K_F^* = \frac{1}{T} (\mu_T + z_T^* \sigma_T) \text{ where } z_T^* = \Phi^{-1} \left[ \frac{c_p - \theta_F + c_F \left[ 1 - F_C \left( K_F^* \right) \right]}{c_p} \right]. \]

Pure spackling (where no efficient capacity is used) arises when the premium for flexible capacity is sufficiently small that there is no incentive to use efficient capacity.

**Case 3: \( u_E = 0 \) and \( u_F > 0 \): Efficient capacity only**

Orthogonality constraints in this case force \( K_F^* = 0 \). From equation (13) we note that \( u_E = 0 \) and \( u_F > 0 \) leads to a greater value of \( K_F^* \) than case 1, where \( K_F^* > 0 \). This is a contradiction and therefore not feasible.

**Case 4: \( u_E > 0 \) and \( u_F > 0 \): No production**

Orthogonality constraints in this case force \( K_E^* = 0 \) and \( K_F^* = 0 \). Then,

\[
\frac{\partial G(K_E, K_F)}{\partial K_F} = T \theta_F - T c_F - T c_p - u_F < 0.
\]

This is a contradiction with first order conditions, and therefore not feasible.

**Second order conditions**

\[
\frac{\partial^2 G(K_E, K_F)}{\partial K_F^2} = T c_p \left[ f_T \left( K_F^* \right) \right] + 2 T c_p \left[ f_T \left( T K_F^* + T K_E^* \right) \right] > 0,
\]

\[
\frac{\partial^2 G(K_E, K_F)}{\partial K_E^2} = T^2 c_F \left[ f_S \left( K_F^* \right) \right] + 2 T c_p \left[ f_T \left( T K_F^* + T K_E^* \right) \right] > 0, \text{ and}
\]

\[
\frac{\partial^2 G(K_E, K_F)}{\partial K_E \partial K_F} = T^2 c_p \left[ f_T \left( T K_F^* + T K_E^* \right) \right] > 0.
\]

The Hessian is positive definite since

\[
\frac{\partial^2 G(K_E, K_F)}{\partial K_E^2} \frac{\partial^2 G(K_E, K_F)}{\partial K_F^2} - \left( \frac{\partial^2 G(K_E, K_F)}{\partial K_E \partial K_F} \right)^2 > 0.
\]
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